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Financial Frictions and Unconventional Monetary Policy in Emerging Economies

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Abstract

This paper discusses conventional and unconventional monetary policies in a dynamic small open economy model with financial frictions. In the model, financial intermediaries or banks borrow from the world market and lend to domestic households. The external debt of banks is limited by a multiple of their equity; in turn, households hold equity in banks subject to a limit, reflecting domestic frictions. As a result, there is an economy wide credit constraint determined by a combination of external and domestic frictions, and an endogenous interest rate spread arises. Financial frictions are shown to add amplification and persistence to exogenous shocks. Fixed exchange rates are contractionary and flexible exchange rates are expansionary (although less so in the presence of currency mismatches). Unconventional policies, including central bank direct credit, discount lending, and equity injections to banks, have real effects only if financial constraints bind. Because of bank leverage, central bank discount lending and equity injections are more effective than direct credit. Sterilized foreign exchange intervention is equivalent to one of the preceding operations. Unconventional policies are feasible only to the extent that the central bank holds a sufficient amount of international reserves.

Keywords: Monetary Policy, Exchange Rates, Financial Frictions

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1. Introduction

Weakening commodity prices and the prospect of increasing interest rates in the U.S. have been followed by capital outflows from emerging economies, raising concerns about macro and financial vulnerabilities in these economies. This evolving scenario is also fueling debate on how policy should respond to these and similar adverse shocks when they materialize.

The central issue, in our view, is whether and how ill-functioning financial markets, often loaded with sizeable stocks of dollar-denominated debt, will act as amplifiers of external shocks. And whether, in the extreme, the financial sector could itself become a source of shocks. That is what occurred in advanced economies during the world financial crisis of 2008-09. Could it happen this time around in emerging markets? And, if so, what are the implications for monetary policies of both the "conventional" and "unconventional" kinds? [1]

This paper attempts to answer these questions by building a simple macro model of an emerging economy in which financial imperfections take center stage, and using it to analyze the implications of various real and financial shocks under alternative monetary polices. While the model is mostly standard, dynamic, and built from first principles, we derive results analytically. To do so, we impose some special assumptions, so that our model may not be as general as others in the literature. But we hope it offers a compensating payoff in terms of insight and understanding, especially on the mechanics of conventional and unconventional policy.

We show how domestic financial frictions, combined with external financial frictions, can translate into an economy-wide foreign debt limit, with significant implications for aggregate demand and monetary policy. More precisely, in our model domestic residents cannot borrow abroad directly. Instead, they borrow from domestic financial intermediaries or banks which, in turn, borrow abroad. Foreign credit to banks, however, faces a limit that depends on the size of their equity capital. [2]

[1]For a discussion of unconventional policies in advanced economies, see Gertler and Kiyotaki (2010). For emerging economies, see Céspedes, Chang, and Velasco (2014).

[2]Notice that we refer casually to the model's financial intermediaries as "banks", but they not need to be banks in the standard sense. In fact, recent work by economists at the Bank of International Settlements (Avdjief, Chui, and Shin 2014) stresses the degree to which, in some emerging markets, large corporates have become de facto financial intermediaries, borrowing in international markets, often from their offshore subsidiaries, to fund carry trades. In the context of Latin America, however, IADB (2014) finds that such phenomenon is mostly restricted to

A crucial question, therefore, is the determination of the financial sector equity base. We assume that intermediaries obtain equity capital from households, but the typical household faces an exogenous upper limit to the amount of equity it can hold. This *equity constraint* is quite consequential; presumably it can be derived from more fundamental domestic frictions, such as informational or enforcement imperfections that result in an incomplete transfer of equity from households to financial intermediaries.[3] For our purposes, the exact source of the equity constraint is not as critical as its implications.

The most interesting implication is that a binding domestic equity constraint becomes a binding international borrowing constraint, so that an endogenous spread between foreign and domestic interest rates emerges. This is a key effect of financial frictions, as Woodford's (2010) canonical model emphasizes. Woodford's model is one of a closed economy in which shocks affect aggregate demand not only through the standard channels but also through changes in the spread between lending and borrowing rates. In our open economy context, the relevant spread is that between the borrowing rate abroad and the lending rate at home. Movements in the spread reflect changes in the demand and supply for funds in the domestic loan market. The spread increases when the external debt constraint becomes tighter.

The model sheds light on several policy-relevant issues. One is amplification. We show how the effects of standard shocks –for instance, a temporary drop in foreign export demand– are magnified by the financial frictions. When the borrowing constraint binds, domestic agents cannot smooth out the effects of the temporary shock by running up debt. This means that, on impact, domestic consumption has to fall by more than it would without frictions, and domestic interest rates shoot up to induce this fall in consumption. Relative prices (in this case the real exchange rate) have to adjust further than they would in a world with perfectly functioning capital markets.

Shocks can also arise in the financial sector itself. This is the case, for instance, if the equity constraint tightens, so that financial intermediaries suddenly have less capital. Or, alternatively, if foreign lenders are now willing to lend less to domestic financial intermediaries, for a given amount of equity in the financial sector. These two alternatives are conceptually different (one can be regarded as a

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Brazil and Mexico.

[3]Here our model is reminiscent of the work by Caballero and Krishnamurthy (2003, for example). But their model is static and their analysis is concerned with a very different set of questions.

domestic shock, the other one as external), but they are both equivalent to forced deleveraging: financial intermediaries, and by implication domestic households, have to reduce their debt abruptly. This captures what Dornbusch and Werner (1994) and Calvo (1998) termed *sudden stops*: overnight capital inflows become capital outflows, and the economy requires a quick adjustment.

In our model, under the assumption of nominal price flexibility, the shock causes consumption and exports to drop, and the real exchange rate depreciates sharply. Adding the assumption of nominal price stickiness, we derive implications for monetary and exchange rate policies, of both the conventional and the unconventional kind. The implications depend naturally on some specific features of the model –particularly on whether foreign loans and/or domestic capital are denominated in domestic or foreign currency, possibly giving rise to so-called currency mismatches.

We take up the traditional question of fixed versus flexible exchange rates. Under a policy of fixing the nominal exchange rate (and, by implication, the real exchange rate), we show that the burden of adjusting to exogenous shocks falls squarely on aggregate demand and production. To fix ideas, consider what happens if the exogenous component of exports drop temporarily or there is a sudden deleveraging shock of the kind described above. Since borrowing to ride through the storm is not possible, the external sector must adjust either by increasing exports, cutting imports or both. But given that the exchange rate is fixed by policy, external adjustment can only occur via a fall in imports and therefore in domestic consumption demand, caused by a spike in home interest rates. In turn, since output is determined by demand, output drops. So in the face of adverse shocks, fixed exchange rates are contractionary. And there is amplification: the resulting movements in consumption, interest and output are larger than they would have been without financial frictions.

Things are rather different if the exchange rate floats and, instead, there is a policy of fixing the nominal (and, under sticky prices, real) interest rate. Adjusting to the same shocks requires a real depreciation which, in turn, and provided that export demand is not too price inelastic, raises the dollar value of exports. Since the interest rate is constant so is consumption, but the dollar value of consumption and of imports both fall. This ensures external adjustment to the shock, even though the economy cannot borrow more in order to smooth out the consequences of the shock. Output expands, since consumption is constant and exports rise. Under flexible exchange rates, therefore, these adverse shocks are expansionary.

All of this analysis holds with equity claims and debts, both foreign and domestic, denominated in foreign currency, so it does not depend on dollarization per se. But currency mismatches are indeed consequential: matters are quite different if the equity of banks is denominated in domestic currency while foreign loans remain denominated in dollars. In that case, an adverse shock that results in a real depreciation reduces the value of banks' equity, also cutting the capacity of the financial intermediary to borrow abroad. This causes further deleveraging which, in turn, requires an even larger real depreciation. In that sense, a currency mismatch is responsible for added magnification of the effects of adverse shocks. These valuation effects would be eliminated if the exchange rate is fixed, which may account for central bankers' alleged *fear of floating.*

Finally, we turn to more recently fashionable policies of the unconventional kind. Following Gertler and Kiyotaki (2010), we evaluate recent central bank facilities that provide lending to firms and households (*direct lending*, in the Gertler-Kiyotaki terminology) or to financial intermediaries (*liquidity facilities*). To organize the analysis, ee focus on the following question: Suppose that an emerging economy is hit by an adverse shock, but its central bank holds a stock of international reserves (or, equivalently, has access to a credit line abroad in international currency, say dollars). What should the central bank do with those dollars? What kind of unconventional policy, if any, should it undertake?

Three main conclusions emerge. First, direct lending and liquidity facilities make a difference if and only if private sector borrowing constraints bind. This is intuitive, since otherwise the central bank would be offering credit that is no superior to that which domestic agents can already get from private sources abroad.

Second, when borrowing constraints bind, liquidity facilities have a general advantage over direct lending. The intuition has to do with one simple factor: leverage. If loans from the central bank improve the capacity of domestic financial intermediaries to borrow abroad, then a favorable multiplier effect kicks in: for every dollar the central bank lends, the intermediaries can lend more than one dollar to domestic households. Hence, and in contrast with direct lending, financial intermediaries leverage the resources advanced to them by the monetary authority. In a situation of constrained borrowing this is highly beneficial.

Third, the feasibility of direct lending and liquidity facilities is limited by the amount of foreign exchange reserves at the central bank. This is because, ultimately, such policies work by alleviating the external debt constraint. The question of optimal accumulation and utilization of reserves in a dynamic context emerges as a central issue (although we do not tackle it here).

Several other unconventional policies turn out to be similar or even completely isomorphic to direct lending or liquidity facilities. This is the case, specifically, of central bank purchases of banks' equity: we show that the impact of such policies depends crucially on how equity held by the central bank equity affects the borrowing constraint of the banking sector, which in turn reflects how foreign lenders evaluate central bank equity vis a vis privately-held equity. Indeed, if the two kinds of equity are treated in the same way by foreign creditors, equity injections are equivalent to liquidity facilities.

Finally, we study sterilized foreign exchange intervention. In our analysis, sterilized intervention can be understood as an unconventional attempt at alleviating the effects of financial constraints. Intervention is effective if and only if financial constraints bind. And, that case, sterilized foreign exchange operations are equivalent to increases in central bank credit, either to households or banks. A corollary is that sterilized intervention can matter only because of the central bank credit required to sterilize, through which the central bank makes its foreign liquidity available to private agents.

This explanation for the real effects of sterilized intervention falls directly out from our analysis, and deserves special mention for at least two reasons: from the point of view of theory, it is new and different from others in the literature, such as portfolio balance or signaling effects; and from the point of view of policy, it may help explain why central banks are prone to exchange market intervention at times of financial stress.

The paper is organized as follows. Section 2 describes the model. We characterize steady states in section 3. Section 4 discusses dynamic adjustment to exogenous shocks under flexible prices. Adding nominal price rigidity, section 5 focuses on conventional monetary policy. Unconventional policies are the subject of section 6. Section 7 expands on sterilized foreign exchange intervention. Section 8 concludes. Some peripheral technical derivations are delayed to an Appendix.

2. The Model

Consider a small open economy inhabited by households, firms, and domestic financial intermediaries or "banks" for short. The model is standard, except for the details of financial intermediation. To smooth consumption, households borrow from banks, which in turn borrow from the rest of the world. Because of financial frictions, banks' external debt will be limited by their equity capital. Households acquire equity in the banks, subject to an exogenous limit that captures domestic financial frictions: this limit, or *equity constraint* for short, plays a crucial role by matching scarce loans with the credit demands of households. The combination of these assumptions results in an economy-wide endogenous collateral constraint, with interesting implications for dynamics and policy.

2.1. Commodities and Production

Time is discrete and indexed by $t = 0, 1, 2, \ldots$. There are two traded goods, home and foreign. The foreign good has an exogenous price of one in terms of a world currency, or *"dollar"*. We can therefore talk about foreign goods or dollars interchangeably.

In order to allow for nominal rigidities and a role for monetary policy, we assume that the home good is the usual Dixit-Stiglitz aggregate of varieties, with elasticity of substitution ϵ. Each variety is produced by one of a large number of monopolistically competitive firms, via a linear technology in which a unit of domestic labor yields a unit of output. Each variety producer takes wages as given and sets prices, in terms of domestic currency (*"peso"*), one period in advance. Standard markup pricing then yields $P_{h,t} = W_t/(1-\epsilon^{-1})$, where W_t is the wage and $P_{h,t}$ the price of the domestic aggregate, both in pesos.

We assume the Law of One Price. Then, letting E_t denote the nominal exchange rate (number of pesos per dollar), the world relative price of the domestic aggregate, or *real exchange rate,* is

$$e_t \equiv \frac{E_t}{P_{h,t}}.$$

Note that with this definition the optimal markup condition becomes

$$w_t = (1-\epsilon^{-1})e_t^{-(1-\alpha)},$$

where $w_t = W_t/P_t$ is the real wage.

We assume foreign demand for the domestic good is simply a function xe_t^{χ} of its relative price, with x a shift coefficient and χ a positive elasticity parameter. Domestic demand, on the other hand, we derive from the demand for consumption goods. Consumption is a Cobb Douglas aggregate of the home aggregate and foreign goods. Under the usual assumptions, the price of consumption (the CPI) is

$$P_t = P_{h,t}^{\alpha} E_t^{1-\alpha},$$

and the demand for the home aggregate is

$$c_{h,t} = \frac{\alpha P_t}{P_{h,t}} c_t = \alpha e_t^{1-\alpha} c_t,$$

where c_t is total consumption demand.

The market-clearing condition for home output is

$$y_t = \alpha e_t^{(1-\alpha)} c_t + x e_t^{\chi}, \tag{1}$$

so that total output demand is split between domestic consumption demand and exports.

2.2. Banks

We assume that domestic households cannot borrow nor lend directly in the world market. But they can borrow or lend from a large number of domestic financial intermediaries or banks. Banks, in turn, can obtain funding from foreigners, possibly subject to financial frictions.

The representative bank lives for only one period. A typical bank in period t has some capital or net worth of k_t dollars which, as we will see shortly, is transferred from the households to the banks at the beginning of the period. It is probably best to think of k_t as equity sold to households in exchange for a share of the bank's profits. Alternatively, one can think of k_t as deposits in the domestic banking system. The bank can also borrow d_t dollars from foreigners, at a fixed interest rate of $\rho \geq 0$. Then, the bank can issue domestic loans worth l_t dollars subject to

$$l_t = k_t + d_t. \tag{2}$$

The bank's mandate is to maximize profits, given by

$$\pi_t = (1 + \varrho_t) l_t - (1 + \rho) k_t, \tag{3}$$

where ϱ_t is the rate of interest on domestic bank loans between periods t and $t+1$. Banks are competitive and take interest rates as given.[4]

The representative bank is subject to a collateral assumption of the form

$$d_t \leq \theta k_t$$

[4]Note that we are allowing for $k_t > l_t$, i.e. for d_t to be negative. If so, the interpretation is that the bank can invest excess funds abroad at rate ρ.

where θ is a constant between zero and one. One can rationalize this constraint in various ways. For example, it may reflect the temptation that after borrowing d_t the banker can "run away", and take with him an amount equal to θ times equity. So naturally the banker's debt cannot exceed θk_t.

Note that the constraint can be rewritten as

$$l_t \leq (1+\theta)k_t. \tag{4}$$

This says that the bank's loans are limited by a multiple of its equity. Note also that a bank's profit π_t can be written as the sum of a "normal" return on its equity plus an excess return on loans:

$$\pi_t = (1+\rho)k_t + (\varrho_t - \rho)l_t\,. \tag{5}$$

Excess returns are non-zero only if if $\varrho_t > \rho$ –that is, if the rate of return on loans exceeds the world interest rate, which is the bank's cost of foreign finance.

Hence the bank's problem has an easy solution. If $\varrho_t = \rho$, there are no supra-normal returns, so l_t and d_t are indeterminate as long as

$$l_t = k_t + d_t \leq (1+\theta)k_t$$

and $\pi_t = (1+\rho)k_t$.

In contrast, if $\varrho_t > \rho$, the bank will lend as much as it can. The collateral constraint must then bind. Loan volume l_t is then given by $(1+\theta)k_t$ and the bank's foreign debt is $d_t = \theta k_t$.

Finally, note that the return to equity will be given by π_t/k_t, which can be rewritten as

$$(1+\rho) + (\varrho_t - \rho)(1+\theta) \equiv (1+\omega_t)(1+\rho)$$

2.3. Households

There is a continuum of households that choose how much to consume and work. Recall they can also borrow from banks at rate ϱ_t and choose how much equity k_t to send to banks, collecting the return on that investment next period. Last but not least, households can invest in a government bond that pays interest r_t in terms of the final consumption good. The government bond is in zero net supply, but introducing it will allow us to define an interest rate that will be the main lever of monetary policy.

The representative household maximizes

$$\sum_{t=0}^{\infty}\beta^t U(c_t,n_t)=\sum_{t=0}^{\infty}\beta^t[\log(c_t)-\frac{\eta}{2}n^2]$$

subject to the sequence of budget constraints (expressed in dollars)

$$e_t^{-\alpha}b_t+k_t-l_t=(1+r_{t-1})e_t^{-\alpha}b_{t-1}+(1+\omega_{t-1})\,(1+\rho)k_{t-1}-(1+\varrho_{t-1})l_{t-1}\;+e_t^{-\alpha}(w_t n_t+v_t)+z-e_t^{-\alpha}c_t,$$

where v_t denotes profits from domestic firms and z is an exogenous endowment of foreign goods (dollars), which we can interpret as oil or another commodity. This bit will be useful later when we examine the dynamics of adjustment.

The household's utility function is admittedly restrictive, but this is for simplicity of exposition. Much of what follows can be generalized if the period utility is of the form $u(c)-v(n)$, with $u(.)$ and $v(.)$ satisfying usual properties. A more crucial assumption is that there is an exogenous limit to the amount of bank equity that the typical household can hold, so

$$k_t\leq\widetilde{k} \tag{6}$$

where $\widetilde{k}>0$ is some constant. This domestic *equity constraint* is the result of unmodeled domestic distortions. It could, for example, capture agency problems between households and firms, or imperfections in domestic equity markets.

The appendix presents the solution to the households' dynamic problem. Here we simply note that solution can be summarized by an optimal labor supply condition

$$(1-\epsilon^{-1})e_t^{-(1-\alpha)}c_t^{-1}=\eta y_t, \tag{7}$$

the consumption Euler equation

$$c_{t+1}=c_t\beta(1+r_t), \tag{8}$$

and the arbitrage equation

$$1+r_t=(1+\varrho_t)\left(\frac{e_{t+1}}{e_t}\right)^{\alpha}, \tag{9}$$

all of which are standard and have intuitive interpretations.

Finally, the appendix shows that the equity constraint must be binding if $\varrho_t>\rho$. If the equity constraint is binding, the bank's external debt constraint must also bind; correspondingly, the latter constraint is slack if the former one is. Without loss of generality, then, we impose below that $k_t=\tilde{k}$ always, while the constraint $d_t\leq\theta k_t=\theta\tilde{k}$ will be binding if $\varrho_t>\rho$ and slack if $\varrho_t=\rho$

3. Equilibrium and steady states

In this section we first lay out the equilibrium conditions of this model and then analyze different types of steady states.

3.1. Equilibrium conditions

In equilibrium, the household budget constraint reduces to

$$e_t^{-\alpha} c_t - d_t = -(1+\rho) d_{t-1} + e_t^{-1} y_t + z. \tag{10}$$

And, as discussed, the equilibrium amount of external debt is limited by the equity constraint, with complementary slackness:

$$0 \leq d_t \leq \theta \widetilde{k} \quad \text{if} \quad \varrho_t = \rho \tag{11}$$

$$d_t = \theta \widetilde{k} \quad \text{if} \quad \varrho_t > \rho \tag{12}$$

A perfect foresight equilibrium is given by sequences $c_t, y_t, e_t \quad r_t, \varrho_t, d_t$ that satisfy (1), (7)-(11) for all $t = 0, 1, 2, \ldots$ (We take d_{-1} as given).[5] This definition assumes that shocks are absent. Below, we will examine the consequences of unexpected shocks at some period t.

Before leaving this subsection, it is useful to note that inserting (1) into (10) and simplifying, the latter equation can be rewritten as

$$(1-\alpha) e_t^{-\alpha} c_t = d_t - (1+\rho) d_{t-1} + z + x e_t^{\chi - 1} \tag{13}$$

This equation shows the economy's *external balance* in dollars and has an intuitive interpretation. The dollar value of total imports (on the LHS) can be financed by further borrowing abroad (net of interest payments), or exporting either foreign or domestic goods. This external balance condition will play a key role in what follows.

3.2. Steady States

It is useful to adopt the following convention: we say that the economy is *constrained in period* t if $\varrho_t > \rho$, and *unconstrained* if $\varrho_t = \rho$. As discussed, the household's equity constraint is slack when the economy is unconstrained. Likewise, the bank is unconstrained in the sense that it is (in equilibrium) indifferent

[5] And, to be rigorous, the complementarity condition (33) in the appendix.

to lend any amount as long as its leverage ratio does not exceed θ^{-1}. If the economy is constrained in period t, the opposite must hold: in particular, the bank lends as much as it can, with maximal leverage.

In what follows we characterize steady states in the usual way. Steady state variables are identified with an overbar.

3.2.1. Unconstrained Steady State

If the SS is unconstrained, all rates of return are equalized:

$$1 + \overline{r} = 1 + \overline{\varrho} = 1 + \rho$$

At the same time, the Euler condition requires that $1+\overline{r} = \beta^{-1}$. Hence a necessary condition for an unconstrained steady state is $(1 + \rho)\,\beta = 1$. Variables $\overline{l}$ and $\overline{d}$ are indeterminate if $\overline{d} \leq \theta\widetilde{k}$. This is just the usual indeterminacy result for a small open economy with the rate of time preference equal to the world rate of interest. What is "given" at the beginning of time is the debt of banks, d. In an unconstrained steady state, the households send enough equity to the banks so that the latter can lend freely.

Given debt levels satisfying the above restriction, the other steady state variables are pinned down by the economy's budget constraint

$$\overline{c} = -\overline{e}^{\alpha}\rho\overline{d} + \overline{e}^{-(1-\alpha)}\overline{y} + \overline{e}^{\alpha} z, \tag{14}$$

the output supply function

$$(1 - \epsilon^{-1})\overline{e}^{-(1-\alpha)}\overline{c}^{-1} = \eta\overline{y}, \tag{15}$$

and the market clearing condition

$$\overline{y} = \alpha\overline{e}^{(1-\alpha)}\overline{c} + x\overline{e}^{\chi}. \tag{16}$$

These three equations determine $\overline{c}, \overline{e}$, and $\overline{y}$, given d.

3.2.2. Constrained Steady State

Let us now examine a *constrained* steady state. As we have seen, this means that $\overline{\varrho} > \rho$, which in turn implies $\overline{k} = \widetilde{k}$. In such a steady state, it is still the case that

$$1 + \overline{r} = 1 + \overline{\varrho} = \beta^{-1}.$$

Therefore, a necessary condition for a constrained steady state is that $\beta(1+\rho) < 1$.

As discussed, in a constrained steady state the household's equity constraint binds. The external debt constraint correspondingly binds, reflecting the economy's inability of the economy to transfer enough "international collateral" to the banks, which are the only agents that can borrow abroad. In a constrained steady state, the bank's debt is $\overline{d} = \theta\widetilde{k}$ –not indeterminate, but a multiple of the equity bound $\widetilde{k}$. The steady state stock of debt *only* depends on θ and $\widetilde{k}$.

The steady state values of y, e, and c are determined by (14)-(16), just as in an unconstrained steady state. The economic implications, however, are different. If the equity bound $\widetilde{k}$ is larger, in a constrained steady state, debt will be larger. Hence, if $\rho > 0$, the economy needs to generate a larger trade surplus every period to service the debt. This requires consumption to be smaller or the real exchange rate to be more depreciated. The interpretation is that the economy is more impatient than the rest of the world, so a permanent increase in $\widetilde{k}$ allows for the banks to borrow more. In equilibrium, this means consumption increases in the short run but falls in the long run.

A special case to which we will pay special attention is $\rho = 0$. Then equations (14)-(16) reduce to

$$\overline{e}^{(1-\alpha)}\overline{c} = \overline{y} + \overline{e}z,$$

$$(1-\epsilon^{-1})\overline{e}^{-(1-\alpha)}\overline{c}^{-1} = \eta\overline{y},$$

$$\overline{y} = \alpha\overline{e}^{1-\alpha}\overline{c} + x\overline{e}^{\chi}.$$

This means that SS real allocations do not depend on debt nor the equity bound $\widetilde{k}$. On the other hand, the external debt $\overline{d}$ and the quantity of bank loans do depend on that bound. How is equilibrium attained, then? Intuitively, the loan interest rate $\overline{\varrho}$ increases above the world interest rate (of zero) to bring demand for bank credit down.

4. Short Term Adjustment with Flexible Prices

In this section we study short term adjustment under flexible prices, which highlights the basic workings of this model. We focus on the case $\beta(1+\rho) < 1$, so that the steady state is constrained and financial imperfections play a crucial role. In later sections we turn to the case of sticky prices, in which monetary policy also comes into play.

4.1. Adverse Shocks: Real and Financial

Consider three kinds of shocks. The first two are financial in nature: a fall in the commercial bank's debt constraint parameter θ to $\theta' < \theta$, or a drop the equity bound, from $\widetilde{k}$ to $\widetilde{k}' < \widetilde{k}$. We assume that the shocks are unanticipated and, for concreteness and simplicity, permanent. Clearly, the two shocks will have a similar impact and call for deleveraging on the part of the representative bank and the economy as a whole. But they are different in nature. The fall in θ can be regarded as an external event, equivalent to the "sudden stop" and "reversal of capital flows" discussed by Dornbusch and Werner (1994) and Calvo (1998), and to the "deleveraging shock" discussed by Krugman and Eggertson (2012). The fall in $\tilde{k}$, on the other hand, has been mostly ignored in the literature, but realistically captures domestic distortions that impede the capitalization of the banking system.

The third shock is an unexpected, temporary fall in z. In particular, assume that z is constant, except that in some period it falls unexpectedly to $z' < z$ for that period only. An unconstrained economy would normally borrow abroad to smooth the effects of this shock. But with a binding borrowing constraint, the pattern of adjustment will be very different.

These three shocks are different but, under the simplifying assumption that $\rho = 0$, they have the same effects: they all imply a drop in available dollar resources in the period of the shock, calling, as we shall see, for a cut in domestic consumption and a real devaluation. And, as long as the real interest rate is zero, these shocks do not change the long run resting position of the economy. To see this result formally, let the new steady state be denoted by overbars. The new steady state must have $\bar{d} = \theta' \widetilde{k}'$, but other steady state variables are unchanged. Transition to the new steady state must take only one period. We use c, e, y, etc. to denote values during that period, which is the period of the shock.

The pattern of adjustment is driven by the external balance condition, which is (13) extended to include the three shocks. It can be rewritten as:

$$(1 - \alpha)\, e^{-\alpha} c - (x e^{\chi - 1} + z) = s, \tag{17}$$

where $s \equiv \tilde{k}'(\theta' - \theta) + \theta(\widetilde{k}' - \widetilde{k}) + (z' - z) < 0$ is a composite of the three shocks under study. The expression is intuitive: the LHS is the trade deficit, the difference between the value of imports $((1 - \alpha)\, e^{-\alpha} c\ )$ and exports $(x e^{\chi - 1} + z\ )$, both measured in dollars. The RHS gathers together the shocks that necessitate a reduction of the trade deficit. This is clearly the case if exports fall. But in a financially constrained economy, the trade deficit must also fall in response to financial shocks.

Financial constraints can, in fact, amplify the size of the needed adjustment on impact. This is clearly the case of the fall in exports, from z to z'. In a financially unconstrained economy, the trade deficit would fall in the period of the shock, but the economy would also spread the cost of adjustment over time by borrowing in the world market, increasing foreign debt. Here, financial constraints prevent further borrowing, and hence the trade deficit must shrink immediately fully to compensate for the fall in exports. In the case of financial shocks, external balance directly implies that the foreign debt must fall and the trade deficit must shrink on impact. This is necessary because financial constraints bind and the debt is at its upper bound, so it must fall from $\theta\widetilde{k}$ to $\theta'\widetilde{k}'$.

The external balance condition (17) can be seen as a locus of combinations (c, e), each of which are consistent with external adjustment to the shock s. In other words, the condition implies that the shock must be met with a reduction in the trade deficit, which requires some combination of a fall in consumption and a real depreciation (an increase in e). The exact combination is pinned down by the other equilibrium conditions. With flexible prices, the relevant conditions are the optimal labor supply condition (7) and the market clearing condition (1). If there are nominal rigidities, as in subsequent sections, the optimal labor supply condition does not hold *ex post*, and it is replaced by a condition determined by monetary policy.

Once the short term values of c and e are determined in the manner just described, output and labor supply are given by demand, that is (1). The response of output is ambiguous in principle, since consumption falls but real depreciation switches demand towards domestic produce. In our case, however, the latter effect dominates and output must increase under flexible prices, as long as $s < z$ (which must be the case unless the financial shock is too large relative to z). This is shown in the Appendix, with also discusses more formally some of the assertions of this subsection.

Finally, the loan rate adjusts to clear the domestic credit market in the short run, according to:

$$\overline{c} = c\beta(1+\varrho)(\frac{\overline{e}}{e})^{a} \tag{18}$$

This says that the loan rate (and the spread between it and the international rate ρ) increases when consumption falls or the real exchange rate depreciates. Recalling that an adverse shock s must result in a combination of falling c and higher e, it follows that the shock must increase the loan rate. This is natural: in the face of the shock, households would like to smooth out the adjustment by borrowing; but the economy cannot come up with the necessary funds (on the contrary, deleveraging is necessary). The domestic loan rate must then increase to choke off this increased demand for loans.

4.2. Favorable Shocks

The nonlinearities in the model raise the possibility that financial constraints may be slack in the adjustment to a favorable shock which calls for a *reduction* in external debt. To see this, consider a temporary *increase* in exports from z to $z' > z$. Intuitively, the economy would like to increase savings to propagate the beneficial impact of the shock to future periods. On the other hand, we know that the steady state does not change.

The adjustment must be as follows: suppose that the economy reaches the steady state one period after the shock (this will be the case if the shock is small enough, as we will see). Then, the three last equations of the last subsection, with $s = z' - z > 0$, determine c, y, and e. Therefore consumption must increase, the exchange rate must appreciate, and output must fall on impact. The loan rate ϱ must fall *below* its steady state value. This is just the reverse of our argument at the end of the previous subsection.

But ϱ cannot drop too much –that is, it cannot fall below ρ. This means that if the increase in z' is large enough, the economy *cannot* remain financially constrained in the period of the shock, and therefore $\varrho = \rho$. By the same reasoning, debt must fall below $\theta\widetilde{k}$ in the short run.

To be more precise: there must be a value of z', call it z^1, such the economy ceases to be financially constrained. For such a value, $\varrho = \rho$, and the Euler equation becomes

$$\bar{c} = c(1+\rho)\beta(\frac{\bar{e}}{e})^{\alpha}.$$

This and

$$(1-\epsilon^{-1})e^{-(1-\alpha)}c^{-1} = \eta y,$$

$$y = \alpha e^{(1-\alpha)}c + xe^{\chi},$$

determine c, y, and e. Given these values, z^1 must then be pinned down by the external constraint:

$$e^{1-\alpha}c = y + ez^1.$$

So if the shock is small enough, in the sense that $z' \leq z^1$, the economy remains financially constrained and converges to the new steady state in one period.

What happens if the shock is larger, so that $z' > z^1$? Clearly, the external debt carried to the period after the shock must be less than its steady state level, so the economy must take at least two periods to converge to the steady state. The same reasoning as above suggests that there must be a $z^2 > z^1$ such that, if $z' \in (z^1, z^2)$, the economy goes back to the steady state after two periods. In this case, the economy is unconstrained in the period of the shock but constrained thereafter; the loan rate rises in the period after the shock to some value higher than ρ but lower than its steady state level. In that period, consumption and the real exchange rate start converging towards their steady state values. Two periods after the shock, the initial debt reduction is completely reversed, and the economy settles back in the steady state.

For even greater values of z', adjustment to the steady state can take successively three periods, four periods, etc. Note the contrast with negative shocks, which led to an abrupt adjustment, involving a move to the steady state after only one period, regardless of the size of the shock.

5. Conventional Monetary Policy

Next we study the role of monetary policy, assuming that prices are fixed one period in advance. With nominal rigidities, optimal labor supply condition (7) does not hold ex post. What replaces it? We take the view that the monetary authority can control one of the short term real variables in the model by an appropriate setting of available instruments, although we do not model the specific link between instruments and real variables.[6] To make the analysis as simple as possible, focus on the case of a constrained steady state in which $\rho = 0$, so that consumption, output and the real exchange rate are independent of debt levels. Of course the monetary and exchange rate policy regime makes a crucial difference, so we analyze the effects of shocks under two alternative polar regimes: an exchange rate peg in which the central bank fixes the real exchange rate at its steady state level $\bar{e}$, and a floating exchange rate regime in which the interest rate is held at its steady state level by central bank policy.

[6] In this we follow e.g. Romer (2013).

5.1. External shocks under an exchange rate peg.

Suppose first that the central bank pegs the real exchange rate at its steady state level $\bar{e}$. Then, suppose there is an unanticipated, temporary fall in z and/or a permanent drop in the equity ceiling $\widetilde{k}$ or the bank's leverage coefficient θ. As in the earlier case of flexible prices, these three shocks have identical effects, and we denote the overall shock by s. Keeping the same notation as before, variables in the period of the shock have no subscript or overbar, while steady-state variables carry an overbar.

If policy keeps e at $\bar{e}$, consumption and output must fall. This is because the external balance constraint (17) becomes

$$(1-\alpha)\bar{e}^{-\alpha}c - (z + x\bar{e}^{\chi-1}) = s.$$

As stressed in the previous section, the shock requires a reduction in the trade deficit. Because the exchange rate is fixed by policy, the trade deficit can only fall if consumption falls. Consumption must, in fact, contract more than under flexible prices, since exchange rates cannot aid in the adjustment.

In turn, since output is determined by demand, we have

$$y = \alpha\bar{e}^{(1-\alpha)}c + x\bar{e}^{\chi},$$

so that output falls unambiguously along with the fall in consumption.

It is easy to show that the loan interest rate rises, and by more than it would under flexible prices. This is intuitive, because the exchange rate peg requires a sharper consumption fall, and therefore the demand for loans increases more than under flexible prices.

In summary, the combination of price stickiness and a binding borrowing constraint produce an abrupt adjustment, in which consumption and output fall sharply, and the domestic interest rate spikes up.

5.2. External shocks under an interest rate peg

Alternatively, suppose that monetary policy keeps the real interest rate at its steady state value $1+\bar{r} = \beta^{-1}$. The shocks are the same.The Euler condition implies that in the period of the shock, the interest rate peg implies that consumption is constant at its steady state value: $c = \bar{c}$. The external balance equation therefore becomes

$$(1-\alpha)e^{-\alpha}\bar{c} - (z + xe^{\chi-1}) = s$$

The LHS is a decreasing function of e as long as χ is larger than one. Therefore (recalling s is negative) the shock causes a real depreciation (e goes up).

The intuition, clearly, is that the trade adjustment required by an adverse shock cannot be met by a fall in consumption, which is fixed by the interest rate policy. Hence the dollar value of imports must fall or exports must increase, both of which are accomplished by a real depreciation. In fact, it is easy to see that the real depreciation must be steeper than under flexible prices, since consumption does not help with external adjustment.

In this case the market-clearing condition is:

$$y = \alpha e^{(1-\alpha)}\overline{c} + xe^{\chi}.$$

So output increases in response to the shock, since consumption does not move but e is higher than in steady state.

Finally, arbitrage condition (9) in this case is

$$1 + \overline{r} = (1 + \varrho)(\frac{\overline{e}}{e})^{a}.$$

Since e rises above $\bar{e}$, the loan rate ϱ increases above its steady state value. This is necessary to keep the real interest rate $\overline{r}$ (which is defined in terms of the consumption aggregate) from falling due to the temporary depreciation.

So under an interest rate peg and a floating exchange rate we have a very different pattern of adjustment than under a fixed exchange rate. As long as export demand is not too price inelastic, a real depreciation raises both the dollar value of exports and the level of output. Consumption is constant, but the dollar value of consumption falls, and so does the dollar value of imports. Both of these factors ensure external adjustment to the shock, even though the economy cannot borrow more in order to smooth out the consequences of the shock.

5.3. Currency Mismatches

So far we have assumed that the equity capital made available to banks by households is denominated in foreign currency. But this does not have to be so, nor is it necessarily so in the real world. Alternatively, let us assume that the equity constraint is not $k_t \leq \widetilde{k}$, but instead

$$e_t^{\alpha} k_t \leq \widetilde{k},$$

so that implicitly we now assume that the equity capital is denominated in domestic currency. Since foreign and domestic loans are denominated in foreign goods (or dollars), the new assumption captures the possibility of a currency mismatch. This means that as relative prices change, in particular as the real exchange rate depreciates (an increase in e_t), the equity constraint tightens.

The necessary amendments to the model are straightforward. The bank's problem in subsection (2.2) is untouched, while the household's problem and its solution remains the same except for the obvious correction to the complementarity condition. As a consequence, the definition of equilibrium is the same except that (11)-(12) is replaced by

$$e_t^{\alpha} d_t \leq \theta \widetilde{k}, \tag{19}$$

$$e_t^{\alpha} d_t = \theta \widetilde{k} \quad \text{if} \quad \varrho_t > \rho \tag{20}$$

The analysis of steady states also remains essentially untouched. Focusing in the financially constrained case, (14)-(16) must still hold. These equations depend on d, which in this case requires that

$$\overline{e}^{\alpha} \overline{d} = \theta \widetilde{k}.$$

Hence, if $\rho > 0$, the preceding equation plus (14)-(16) simultaneously determine $\overline{y}, \overline{c}, \overline{e}$, and $\overline{d}$. If $\rho = 0$, on the other hand, (14)-(16) remain independent of d, and hence suffice to pin down $\overline{y}$, $\overline{c}$, and $\overline{e}$. In this case, $\overline{e}^{\alpha} \overline{d} = \theta \widetilde{k}$ determines $\overline{\text{d}}$.

Now consider the implications of currency mismatches for shocks and alternative monetary policies under prices that are sticky (for one period only). Continue the analysis of a shock s as before, which we now interpret exclusively as a temporary adverse shock to z, the endowment of the foreign good, and ask how the analysis of the preceding two subsections must change. Assume $\rho = 0$ for simplicity. Then the old and new steady state values of output, consumption, the real exchange rate, and debt are the same; let us denote them by $\bar{y}, \bar{c}, \bar{e}, \bar{d}$.

Under a *fixed exchange rate* policy, the analysis is just the same as without currency mismatches. This should be evident because a fixed exchange rate eliminates any additional tightening of the equity constraint that would result from a real depreciation. The shock has to be acommodated with a contraction in aggregate demand and output, as before.

With a *fixed real interest rate* policy, the analysis here is considerably more involved because, given that the shock results in an exchange rate depreciation, the external debt ceiling tightens so that on impact the debt must fall below its steady state level. As a consequence, it is no longer the case that the economy goes back to steady state after just one period. Instead, it turns out that the return to the steady state is only asymptotic even if the economy is constrained in every period after the shock.

We can say more, however, assuming that the shock is small enough, so that the economy remains financially constrained in every period. Because of the perfect foresight dynamics starting the period after the shock, consumption after the period of the shock must be a decreasing function of the debt level d determined the period of the shock, with the intuition being that the lower is accumulated debt, the higher consumption can afford to be thereafter. Note also that the fixed interest rate policy implies that consumption in the period of the shock must be the same as consumption one period after (recall the Euler equation). Both must be the same decreasing function of d.

What does this mean for the behavior of the real exchange rate? Note that now the external constraint can be written as

$$(1-\alpha)c - e^{\alpha}(d-\overline{d}) = e^{\alpha}z' + xe^{\chi-(1-\alpha)} \tag{21}$$

This is the same equation as in the case without currency mismatches, except that in that earlier case $c = \overline{c}$ (because of the fixed interest rate policy) and $d = \overline{d} = \theta\widetilde{k}$ (because of the debt constraint), so that the LHS was simply equal to $(1-\alpha)\overline{c}$. In this case, by contrast, the exchange rate depreciation implies that $d < \overline{d}$, which together with the interest rate policy implies that $c > \overline{c}$. Hence the LHS must be greater than $(1-\alpha)\overline{c}$ regardless of the value of e, and in fact it must be an increasing function of e (since $e^{\alpha}d = \theta\widetilde{k}$). It is then apparent that currency mismatches imply that the shock must result in a *steeper* depreciation of the currency (see Figure 1). The intuition is simple: the shock tightens the equity constraint, which together with interest rate policy implies that the required external adjustment is larger than without currency mismatches. Hence the exchange rate has to depreciate by more to generate the additional expansion of exports.

The lesson is that currency mismatches add amplification and persistence to shocks, because of the effects of exchange rate movements on financial constraints. Such effects are eliminated under fixed exchange rates, which therefore gain some appeal relative to flexible rates. But, as stressed, fixed exchange rates remain ineffective to prevent an aggregate demand contraction in response to the adverse shocks.

6. Unconventional Policies

Since the great financial crisis of 2008-09, advanced country central banks have engaged in all sorts of unconventional monetary policies, which have been studied at length in the academic and the policy literature. Much less studied is the fact that emerging market central bank have also engaged in unconventional, though not always novel, monetary and financial policies.

Before the recent crisis, many emerging market central banks had been targetting the inflation rate. At this simplest, this approach implied, in the open economy, using the short domestic interest rate to target some forward-looking measure of inflation, while letting the exchange rate float. Yet to a limited extent before the crisis, and with abandon after the big crash, most EM central banks deviated from this simple orthodoxy. As a first step they often engaged in foreign exchange intervention, whether sterilized or not.[7] They also fiddled with reserve requirements (sometimes different ones for domestic and foreign currency assets and liabilities) in order to control the growth of domestic credit or domestic monetary aggregates.[8] And as the credit crunch caused by the crisis made itself felt, they did as their developed country counterparts, lending to banks and households, buying bonds and other kinds of paper of different maturities, and sometimes going as far as to take equity positions in domestic financial intermediaries.[9]

Several questions arise. Should we understand these unconventional policies as an attempt to get around the borrowing constraints that play a central role here? If in fact policies do affect financial constraints, what are the effects on key macro-economic variables of interest? What if conventional and unconventional policies are applied simultaneously? These are some of the questions we study in what follows.

6.1. Direct Lending

Start the discussion by assuming that the central bank has $f > 0$ dollars as foreign exchange reserves. A passive policy might then be to invest them in the foreign exchange market, earning the world rate ρ, and to transfer ρf to households every period.

[7] Chang (2007).

[8] Montoro and Moreno (2011).

[9] Céspedes, Chang, and Velasco (2014).

In the spirit of Gertler and Kiyotaki (2010), we consider an alternative *direct lending* policy: in period t the central bank lends l_t^g to households at the market rate ϱ_t. The amount l_t^g is a policy decision. We assume that $0 \leq l_t^g \leq f$, and that the central bank makes a transfer τ_t to the household at the beginning of each period, so as to keep the amount of foreign reserves constant at f.

The commercial bank's decision problem is the same as before. The household's problem remains almost unchanged, too, except that the budget constraint is now

$$b_t+e_t^{\alpha}(k_t-l_t^h) = (1+r_{t-1})b_{t-1}+e_t^{\alpha}(1+\omega_{t-1})(1+\rho)k_{t-1}-e_t^{\alpha}(1+\varrho_{t-1})l_{t-1}^h+w_t n_t+v_t+e_t^{\alpha}(z+\tau_t)-c_t,$$

where $l_t^h = l_t+l_t^g$ is the sum of bank loans plus central bank loans to the household. Note that the RHS includes the central bank transfer τ_t.

The household's first order conditions are exactly as before. But to derive the associated equilibrium, we need to be more explicit about the central bank transfer τ_t. As mentioned, at the end of each period t, the central bank lends l_t^g to the household. We assume that the remainder, $f - l_t^g$, is invested in the world market. In period $t + 1$, therefore, the central bank's transfer must be

$$\tau_{t+1} = (1 + \varrho_t)l_l^g + (1 + \rho)(f - l_t^g) - f = \rho f + (\varrho_t - \rho)l_t^g \qquad (22)$$

This amount is the world return on foreign reserves plus the supranormal profit on central bank lending. One can check that, with this assumption, the central bank starts every period with the same amount of reserves. It should also be noted that this is not an innocuous assumption, in the sense that alternative uses of these "quasi-fiscal" profits may change equilibria.

With this and our previous assumptions, it follows that, in equilibrium, the external constraint reduces to

$$(1 \ -\alpha)\, e_t^{1-\alpha} c_t - e_t(d_t + l_t^g) = -e_t(1 + \rho)(d_{t-1} + l_{t-1}^g) + x e_t^{\chi} \ + e_t(z + \rho f). \qquad (23)$$

Compare this expression with the original (13). The new form of the external constraint is intuitive: direct lending by the central bank allows the economy to circumvent the external credit limit, at a cost ρ, which is the opportunity cost of reserves to the economy.

The other equilibrium conditions remain exactly the same, so that the path of l_t^g affects equilibrium *only* through the immediately preceding external constraint (23). This reveals a crucial aspect of credit policies. Take any equilibrium in which the financial constraint is slack in a period t, that is, $0 < d_t < \theta\widetilde{k}$. Then it is easy to see that, for alternative values $\hat{d}_t, \hat{l}_t^g$ such that $0 < \hat{d}_t < \theta\widetilde{k}$ and $\mathrm{d}_t + l_t^g = \hat{d}_t + \hat{l}_t^g$, the *same* equilibrium obtains. (To see this, one only needs to check that the equilibrium conditions depend only on the sum $d_t + l_t^g$ and not on d_t and l_t^g separately, as long as $0 < d_t < \theta\widetilde{k}$.) The intuition is simple: suppose that the financial constraint does not bind in period t. Then, if the central bank extends additional credit to the commercial bank, the latter simply reduces its external debt by an offsetting amount, leaving the total supply of loans in the economy unchanged. Since the financial constraint does not bind, the same equilibrium obtains with the loan interest rate equal to the world interest rate.

In other words, the amount of *central bank credit in period t is irrelevant if financial constraints do not bind in that period.* This is an instance of a more general result, discussed by Céspedes, Chang, and Velasco (2015) and others, which applies to a large set of unconventional monetary policies, including many that have been tried in practice.

To move forward, suppose l_t^g is a constant, and consider steady states. The external constraint (23) becomes

$$(1-\alpha)\,\overline{e}^{1-\alpha}\overline{c} = -\overline{e}\rho(\theta\widetilde{k} + l^g) + x\overline{e}^{\chi} + \overline{e}(z + \rho f),$$

the case in which the steady state is financially constrained, so that $\overline{d} = \theta\widetilde{k}$. In steady state, central bank lending allows the economy to effectively borrow more than that limit.

The case $\rho = 0$ is instructive. Then, as before, the steady state is independent of d and of l^g. But direct lending matters in the short run: the external constraint can be rearranged to read

$$(1-\alpha)\,e_t^{1-\alpha}c_t - e_t d_t = x e_t^{\chi} - e_t d_{t-1} + e_t \hat{z}_t, \tag{24}$$

where

$$\hat{z}_t = z + l_t^g - l_{t-1}^g.$$

In this case, central bank credit is isomorphic to control of z. Notice it is the change in l_t^g, not the level, that matters. If l_t^g is constant, the policy is irrelevant. This is intuitive, since z is a flow variable, while l_t^g is a stock variable. Notice also that if f is large enough relative to the size of the shock, this policy can be used to offset completely a temporary fall in z or other equivalent shocks.

To sharpen intuition consider, in particular, a temporary fall in z. Normally (without borrowing constraints), the household would like to borrow abroad in order to smooth out the consumption effects of the temporary shock. If the real international interest rate is positive, so that borrowing is costly, the new feasible level of consumption has to adjust downward to reflect the carrying cost of the additional debt. But if, as we have assumed here, $\rho = 0$, so the additional debt has no carrying cost, the level of consumption can afford to remain the same it would have been without the shock.

Note, however, two related aspects of the policy: we have assumed that the increase in credit, $l_t^g - l_{t-1}^g$, is permanent; and we have assumed that $l_t^g \leq f$ or, equivalently, that the credit increase is no more than the available amount of reserves, given by $f - l_{t-1}^g$. This implies that a credit increase to offset a temporary fall in z will not be feasible if $f - l_{t-1}^g$ is small. This would be the case if f is small, but also if l_{t-1}^g is large, due to analogous credit operations in the past.

Here, then, our restriction to one time shocks oversimplifies the analysis. In a situation in which, realistically, z is continuously buffeted by shocks, the question of appropriate reserves accumulation and how it limits credit policies emerges as an important one.

6.2. Liquidity Facilities

Suppose that, instead of lending to households, the central bank lends part or all of its f resources to banks. This is akin to what Gertler and Kiyotaki (2010) term liquidity facilities, or discount-window lending.

Such lending alters the bank's problem. The amount that the bank can lend is now

$$l_t = k_t + d_t + d_t^g,$$

where d_t^g is the loan from the central bank. We assume that central bank loans carry the world rate of interest, and that the size of d_t^g is determined by policy, subject to $0 \leq d_t^g \leq f$.

The impact on the bank's incentive constraint turns out to be key. We assume that discount lending affects the commercial bank's foreign debt limit in the following way:

$$d_t \leq \theta k_t + \phi d_t^g \tag{25}$$

with $0 \leq \phi \leq 1$. The idea is that a banker can "abscond" after obtaining external credit d_t, as before, but now the cost is not only a fraction θ of equity but also a fraction ϕ of its debt to the central bank. If $\phi = 1$, in particular, the assumption is that the banker cannot cheat on the central bank.

Combining the two expressions, the associated limit on bank loans is now

$$l_t \leq (1 + \theta)k_t + (1 + \phi)d_t^g.$$

This emphasizes that, if $\phi > 0$, using central bank liquidity facilities banks can leverage up and multiply d_t^g in the world market. In this sense, lending to banks delivers more bang for the buck than lending to households, just like an increase in private equity. The crucial assumption, of course, is that the existence of a central bank loan increases the cost to the bank of reneging on its foreign debt.

The commercial bank now chooses l_t and d_t to maximize profits, given by

$$\pi_t = (1 + \varrho_t)l_t - (1 + \rho)(d_t + d_t^g) = (\varrho_t - \rho)l_t + (1 + \rho)k_t, \tag{26}$$

as before. Hence the bank's problem has a similar solution as before: if $\varrho_t = \rho$, l_t and d_t are indeterminate as long as (25) is satisfied; if $\varrho_t > \rho$, (25) must hold with equality, so $d_t = \theta k_t + \phi d_t^g$.

The household's problem is as before, except that we assume that the central bank transfers the difference between receipts from its past loans plus any interest earned on the resources not lent, minus the amount needed for new investments. But because the central bank changes ρ for its credit, the transfer is equal to the world return on foreign reserves:

$$\tau_t = (1 + \rho)d_{t-1}^g + (1 + \rho)(f - d_{t-1}^g) - [d_t^g + (f - d_t^g)] = \rho f.$$

With this assumption, the external constraint reduces to

$$(1 - \alpha)\, e_t^{1-\alpha} c_t - e_t(d_t + d_t^g) = -e_t(1 + \rho)(d_{t-1} + d_{t-1}^g) + x e_t^\chi + e_t(z + \rho f)$$

which is the same as in the case of the previous subsection, except that d_t^g replaces l_t^g. In contrast with the direct lending policy, however, the liquidity facility affects the external credit limit, and the bank can now borrow more abroad and lend more to the household.

One implication is that, as with direct lending, the amount of liquidity provided by the central bank d_t^g does not matter if and only if financial constraints are slack (meaning $0 < d_t < \theta \tilde{k} + \phi d_t^g$ in this case). Another implication is that financially constrained steady states are given by

$$(1 - \alpha)\, \overline{e}^{1-\alpha} \overline{c} = -\rho[\theta \widetilde{k} + (1 + \phi)d^g] + x\overline{e}^\chi + \overline{e}(z + \rho f).$$

This emphasizes that the liquidity facility is more effective than direct lending in increasing the economy's capacity to borrow abroad. The intuition, again, is leverage.

In the case of $\rho = 0$, the external constraint is as in (24), with $\hat{z}_t = z + (d_t^g - d_{t-1}^g)$. This is just as with direct lending. However, the additional consideration is that, if constraints bind so that $\varrho_t > \rho$, $d_t = \theta\widetilde{k} + \phi d_t^g$. To illustrate the differences further, suppose that $\rho = 0$ and the economy is in a financially constrained steady state with zero central bank loans (in either direct credit or discount credit). Consider then an unexpected, temporary fall in z to $z' < z$. The analysis of the previous subsection implies that the central bank would be able to prevent the shock from affecting real allocations by extending direct credit to households in the amount $l^g = z - z'$ (this would be possible provided $z - z' \leq f$). Alternatively, with a liquidity facility, the amount needed for the same purpose would be $d^g = (z - z')/(1 + \phi)$. This is because the liquidity facility would allow commercial banks to borrow an amount ϕd^g over and above the original credit limit of θk. If $\phi > 0$, then, the liquidity facility requires fewer resources than direct credit. In fact, if $f < z - z' \leq (1 + \phi) f$, the former is feasible but the latter is not.[10]

6.3. Equity Injections

Lending is not the only operation that the central bank can engage in vis à vis the bank. Instead of lending at rate ρ, the central bank can choose to take an equity position in the commercial bank. In so doing, the central receives an equiproportional share of the commercial bank's profits. Under some conditions, the analysis is the same as with liquidity facilities. But it is interesting to see the details, if only to identify the necessary conditions for this operation to play a useful role in offsetting the effects of shocks under financial distortions.

Let k_t^g denote the central bank equity position in the commercial bank. Then the total equity of the commercial bank is $\kappa_t = k_t + k_t^g$. The commercial bank's problem is then exactly as described in subsection (2.2), except that κ_t replaces k_t in all the obvious expressions.

The household's problem is as before, and its solution is the same, except that the budget constraint reduces to

$$e_t(k_t - l_t) = -e_t(1 + \varrho_{t-1})l_{t-1} + e_t\left(\frac{k_{t-1}}{\kappa_{t-1}}\right)\pi_t + xe_t^{\chi} + e_t z - (1-\alpha)\, e_t^{(1-\alpha)} c_t + e_t \tau_t$$

where the RHS emphasizes that, in equilibrium, the household receives a fraction k_{t-1}/κ_{t-1} of the commercial bank's profits, and also a transfer τ_t from the government.

[10]Note, in the previous example, that we have assumed that the amount of central bank credit (either to households or banks) increases permanently. This is for simplicity of exposition: the assumption ensures that the economy returns to the original steady state in the period of the shock.

The central bank finances the equity injection using reserves (or, equivalently, a dollar credit line). In each period t, the central bank takes an equity position $0 \leq k_t^g \leq f$ in the commercial bank, and invests $f - k_t^g$ in the world market. As before, we assume that the central bank transfers its profits to the household, so that

$$\tau_t = \left(\frac{k_{t-1}}{\kappa_{t-1}}\right)\pi_t + (1+\rho)(f - k_{t-1}^g) - f = \pi_t - (1+\rho)k_{t-1}^g + \rho f.$$

Combining the last two expressions with the definition of profits π_t, and recalling that $l_{t-1} = d_{t-1} + \kappa_{t-1} = d_{t-1} + k_{t-1} + k_{t-1}^g$, we have the equilibrium version of the crucial external constraint:

$$-e_t(d_t + k_t^g) = -e_t(1+\rho)(d_{t-1} + k_{t-1}^g) + xe_t^{\chi} + e_t z - (1-\alpha)\, e_t^{(1-\alpha)} c_t - e_t \rho f.$$

In turn, the credit limit is

$$d_t \leq \theta\kappa_t = \theta(\widetilde{k} + k_t^g).$$

These expressions are the same as in the previous subsection, except that k_t^g has replaced d_t^g and that $\phi = \theta$. The latter fact reflects, of course, our assumption that central bank equity in the commercial bank is treated the same as the household's equity.

The conclusions of the previous subsection then apply, in particular that operations involving banks –whether liquidity lending or equity injections– are more effective than direct credit in relaxing financial constraints because of the bank's leverage. But our discussion also suggests how equity injections might differ from liquidity facilities. Gertler and Kiyotaki (2010), for example, suggested that the central bank might pay more than the market price for its equity position. They stressed that this should be understood as a transfer from the central bank from the commercial bank. This is clearly also the case in our framework.

6.4. Combining Credit Policy and Monetary Policy

For monetary policy to have an effect one must assume sticky prices. In that case, as we have seen above, the optimal labor supply condition (7) does not hold ex post in the event of shocks. Instead, monetary policy provides an additional short run condition for equilibrium. In the cases we have examined, this means that either the real exchange rate or the real interest rate are fixed at their steady state levels.

Given these observations, it is not hard to characterize the implications of combining credit policy and monetary policy. Consider, for example, a temporary fall in z to $z' < z$ met with an increase of central bank credit to the commercial bank, and assuming either fixed exchange rates or a fixed interest rate policy. As we have seen, the economy converges after one period to the steady state, which under the simplifying assumption $\rho = 0$ is independent of the debt and credit policy. Hence, in the period of the shock, the external constraint is

$$(1-\alpha)\, e^{1-\alpha} c = e(\Delta d + \Delta d^g) + \, x e_t^{\chi} + e^{\alpha} z' = x e_t^{\chi} + e[(1+\theta^g)\Delta d^g - \Delta z] \quad (27)$$

where $\Delta z = z - z'$ is the fall in exports and Δd and Δd^g denote the increase in external debt and central bank liquidity, respectively, in the period of the shock. The last equality emphasizes that the credit response Δd^g essentially offsets the shock Δz by the amount $(1+\theta^g)\Delta d^g$.

The short run equilibrium is determined by the preceding equation together with the domestic market-clearing condition

$$y = \alpha e^{1-\alpha} c + x e^{\chi},$$

and the assumption that $e = \bar{e}$ (under a fixed exchange rate) or $c = \bar{c}$ (under a fixed interest rate).

This all means that the analysis of monetary alternatives complemented by credit policies is as usual, once one takes into account how credit policy effectively reduces the size of the exogenous shock from Δz to $(1+\theta^g)\Delta d^g - \Delta z$. Again, a crucial issue is the availability of international reserves, which places an upper bound on Δd^g.

To summarize, unconventional policies can offset exogenous shocks fully but are limited by international reserves. In contrast, monetary policy faces no such limits, but involve tradeoffs involving output, consumption and the real exchange rate, all of them operating through the external balance condition. Further research is warranted in this regard, especially on the optimal accumulation and utilization of foreign reserves in (potentially) financially-constrained open economies.

7. Sterilized intervention

Many emerging markets claim to target inflation. A standard version of that policy requires that monetary and interest rate policy be targeted at the rate of inflation, while letting the exchange rate float freely. In that framework there is no direct feedback from the exchange rate back to (say) the policy interest rate. The level of the nominal (and real) exchange rate only matters for policy to the extent that it affects the expected inflation rate and/or the output gap. Moreover, the policy response to movements in the expected inflation rate and the output gap is supposed to involve the interest rate only, excluding by design active intervention in the foreign exchange market.

This theory stands in sharp contrast to what many economies (emerging and also advanced) have actually done during and since the 2008-09 crisis. Many have pursued standard foreign exchange intervention, both sterilized and unsterilized. Attempts to affect the exchange rate via the derivatives market have also been common. In Céspedes, Chang and Velasco (2014) we provide an account of such policy responses in Latin America. Similar policies have been put into place in other emerging market economies, particularly in Asia.

In what follows we ask what effects, if any, sterilized foreign exchange intervention has in our model. We deliver one conclusion up front: such intervention has effects if and only borrowing constraints bind. In this sense, foreign exchange intervention can be understood as an "unconventional" atttempt to lessen the effects of such borowing constraints.

When discussing sterilized foreign exchange intervention, it is necessary to add to the model some view on the supply and demand of domestic money. The specific details are not important: one could append a quantity equation or one could assume that domestic money is in the household's utility or production function; there are other, well-known alternatives. For concreteness and simplicity also, we suppose for the rest of this subsection that money exogenous, so that equilibrium in the money market only determines the price level (the argument is modified in an straightforward way if the price level is determined in alternative ways, such as interest rate rules, and money market equilibrium determines the equilibrium money stock endogenously).

Consider the impact of a *sterilized* central bank purchase of one dollar with domestic currency. Without sterilization, nominal money supply M_t would increase by E_t (the nominal exchange rate); central bank foreign reserves would correspondingly increase by one dollar. However, sterilization means that the central bank must adjust its asset position so as to keep M_t the same. To do this, in particular, the central bank can increase its credit to either households or commercial banks by the equivalent of one dollar (it does not matter here if such an increase is given in domestic currency or foreign currency, as households or banks can rearrange their currency holdings accordingly). The net result, in this case, is that at the end of the process M_t will not have changed, central bank foreign reserves will have fallen by one dollar, and either l_t^g or d_t^g will have increased by one dollar.

Hence foreign exchange intervention affects the equilibrium conditions only through their effect on l_t^g or d_t^g. By design, it does *not* affect equilibrium in the money market, which is why the exact specification of the demand for and supply of domestic money is not important. While this argument is simple, it has significant implications for our views of sterilized foreign exchange intervention. We have discussed the issue at length in Céspedes, Chang and Velasco (2013), in the context of another model that also involves financial constraints. Using the model here, one can easily show that:

- Sterilized foreign exchange operations are equivalent to increases in central bank credit, either to households or banks.
- Such operations matter only because of the central bank credit required to sterilize, through which the central bank makes its foreign liquidity available to private agents.
- Sterilized foreign exchange intervention matters only when financial constraints are binding. Under other circumstances, as we have seen in earlier sections, central bank credit, either to households or banks, has no real effects.

From the point of view of theory, these arguments for the real effects of sterilized intervention are new and independent of others in the literature, such as portfolio balance effects or signaling effects. From the point of view of policy, they help explain why central banks are prone to intervention at times of financial stress, precisely when borrowing constraints bind. More broadly, if financial crisis involve a scarcity of liquidity, and in our perspective exchange market intervention is precisely a means for providing liquidity, it is only natural that monetary authorities will display what Calvo and Reinhart (1999) termed "fear of floating", in the sense of keep in intervention among the policy tools to be used in times of stress. One can even go further, and argue that a well-designed inflation-targeting-cum-floating regime should have a mechanism akin to an escape clause that, when triggered, allows the central bank to intervene in the foreign exchange market to provide dollar liquidity and lessen the pressure of borrowing constraints.

8. Final Remarks

Our analysis suggests several directions for research. Some of them, such as exploring the consequences of alternative parameterizations and checking empirical adequacy, are straightforward. Others are less obvious but, to us, equally urgent and promising.

One is to explore alternative assumptions about finance constraints. For instance, we assumed but did not provide microfoundations for the equity constraint. While we do not believe that to be a serious shortcoming for the analysis in this paper, it may turn out to be important for studying some other questions, such as financial regulation. Perhaps more significantly, we assumed finance constraints of a very simple form. Examining the robustness of our results to other forms, such as dynamic ones, is warranted.

A second direction is the study of optimal accumulation and utilization of foreign exchange reserves. From the perspective of this paper, foreign exchange reserves represent a key restriction on the availability of unconventional policies in emerging markets. Since we have shown that the latter have real effects when financial frictions bind, an appropriate policy of accumulation of reserves emerges as a priority in the debate.

9. Appendix

We derive here the first order conditions of the household's problem in the main text. Let $\beta^t\lambda_t$ and $\beta^t\gamma_t$ denote Lagrange multipliers associated with the budget constraint and the equity constraint. The first order conditions for maximization are

$$l_t : \lambda_t e_t^\alpha = \beta\lambda_{t+1}(1+\varrho_t)e_{t+1}^\alpha \tag{28}$$

$$b_t : \lambda_t = \beta\lambda_{t+1}(1+r_t) \tag{29}$$

$$k_t : \lambda_t e_t^\alpha + \gamma_t = \beta\lambda_{t+1}(1+\omega_t)(1+\rho)e_{t+1}^a \tag{30}$$

$$c_t : c_t^{-\sigma} = \lambda_t \tag{31}$$

$$n_t : \eta n_t^\phi = \lambda_t w_t \tag{32}$$

The complementarity condition

$$\gamma_t \geq 0, = 0 \quad \text{if} \quad k_t < \bar{k} \tag{33}$$

must also hold. Note that these expressions require that, in any equilibrium,

$$1 + r_t = (1+\varrho_t)\frac{e_{t+1}^\alpha}{e_t^\alpha} \tag{34}$$

Note also that if $\varrho_t = \rho$ these first order conditions require $\gamma_t = 0$ –that is, that the household's equity constraint not bind.

Next, we justify the assertions at the end of the subsection on adjustment to adverse shocks with flexible prices. Rewrite the external condition (17) in terms of the domestic good as

$$es + xe^\chi + ez = m \tag{35}$$

where $m \equiv (1-\alpha)e^{1-\alpha}c$ is the value of imports in units of the domestic good. Assuming $z > s$, this gives m as an increasing function of e. Given e, a larger (more negative) shock drives imports m down. At the same time, combining the labor supply condition and market clearing gives

$$\mu m^{-1} - \alpha m = (1-\alpha)xe^\chi \tag{36}$$

where $\mu \equiv (1-\alpha)^2(1-\epsilon^{-1})\eta^{-1}$ The LHS is a decreasing function of m and the RHS is an increasing function of e, so this whole expression represents a negatively-sloped relation between m and e.

Short-run equilibrium is given by the intersection of these two schedules: this is depicted in Figure 2, where XX is the external constraint (35) and MM is the graph of (36). An adverse shock means that s becomes negative, displacing the XX downwards and resulting in a fall in m and an increase in e –that is, a drop in imports and a real depreciation. Then, by the definition of m, c must also fall: consumption drops along with the drop in imports and in the real value of the currency. This is all necessary to restore external equilibrium, given the two adverse external shocks.

In this case, the output supply condition can be written as

$$(1-\alpha)\, y = \mu m^{-1}.$$

So as m falls, y must increase. The shock is expansionary, for supply (not demand) reasons: as consumption (and the real exchange rate) drop, the marginal utility of consumption rises. Labor supply and output must then rise to keep utilities equated at the margin.

We know that c falls and e increases, while $\bar{c}$ and $\bar{e}$ are the same as in the original steady state. So, from Euler equation (18) we can see that ϱ increases above its steady state value.

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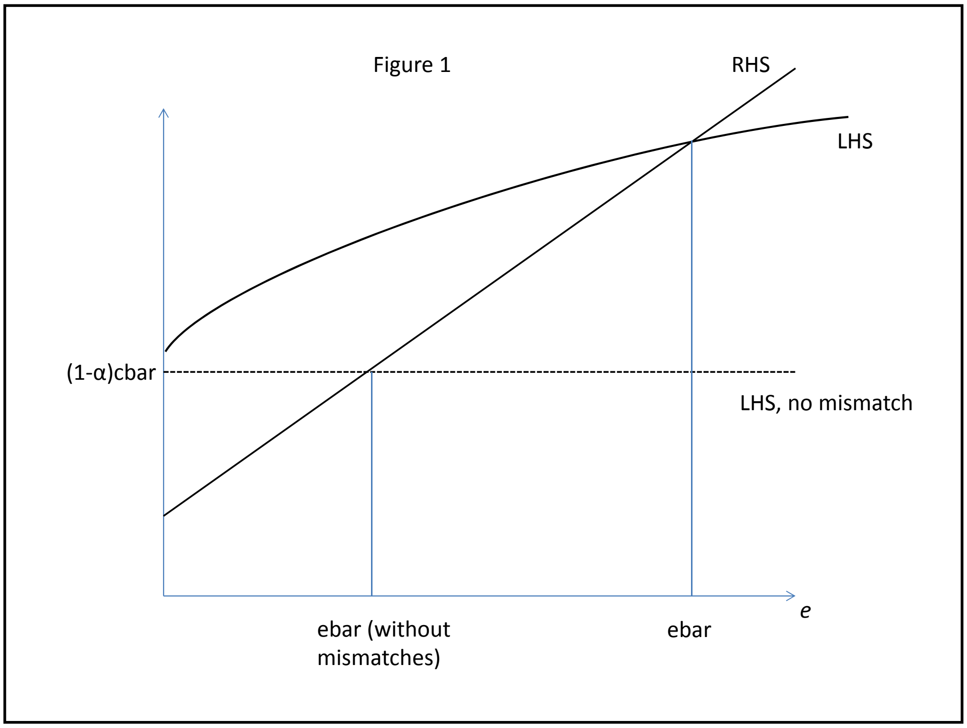
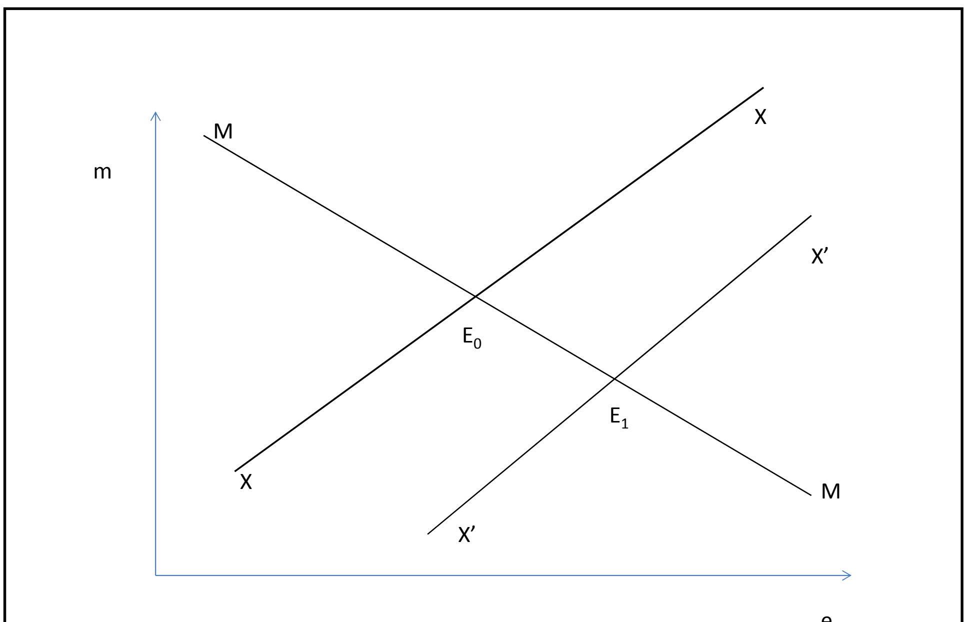


Figure 1
RHS
LHS
(1-α)cbar
LHS, no mismatch
e
ebar (without mismatches)
ebar


m
M
X
X'
E0
E1
X
M
X'
e
Figure 2