The Tradeoffs in Leaning Against the Wind*

– Conference Draft –

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October 28, 2016

Abstract

Credit booms sometimes lead to financial crises which are accompanied with severe and persistent economic slumps. Does this imply that monetary policy should "lean against the wind" and counteract excess credit growth, even at the cost of higher output and inflation volatility? We study this issue quantitatively in a standard small New Keynesian dynamic stochastic general equilibrium model which includes a risk of financial crisis that depends on "excess credit". We compare monetary policy rules that respond to the output gap to rules that respond to excess credit. We find that leaning against the wind may be attractive, depending on several factors, including (1) the severity of financial crises; (2) the sensitivity of crisis probability to excess credit; (3) the volatility of excess credit.

1 Introduction

The question of whether financial stability concerns should play a role in the setting of monetary policy has generated a vigorous debate. In this paper we investigate the wisdom of what has come to be known as "leaning against the wind" (LAW), that is having monetary policy react to perceived financial imbalances such as excess credit growth.

There are broadly two types of arguments that have been put forward to reason against LAW. One critique observes that financial imbalances are difficult to detect and that they are best addressed with other tools such as prudential (or perhaps even time-varying macro-prudential) policy.

*The views expressed in this paper do not necessarily reflect the views of the Federal Reserve System, the Federal Reserve Board, the Federal Reserve Bank of Chicago, or the Bank of England. We are responsible for all errors.

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Monetary policy is presumed to have small and uncertain effects on financial imbalances and when weighed against the usual costs of slowing the economy the trade-off is unfavorable (Bernanke and Gertler (1999)).[1]

Svensson (2016) goes further and argues that it is appropriate to account for the fact that the costs of a crisis can differ depending on the initial conditions that prevail prior to the crisis onset. If the economy is starting from a fragile position, the crisis can be more costly. Because LAW weakens the economy, this is an additional argument against it. He argues that monetary policy should remain focused on inflation and output stability.

There are also at least two alternative rationales that have been offered in favor of LAW.[2] As Reinhart and Rogoff (2009) and Cerra and Saxena (2008) emphasized even before the most recent crisis, recovery from crises are typically slow so that the hangover from a crisis is different than from a regular recession. It is routine to see charts in the popular press showing that there seems to be a large downward shift in the level of GDP. Preventing a crisis may, therefore, bring different benefits than those associated with smoothing out inefficient business cycle fluctuations (See Barro (2009)). As will see this consideration features prominently in our analysis.

The second concern, as recently expressed by Dudley (2015), is that the appeal to use macro-prudential tools as a first line of defense against financial imbalances is much easier said than done. Many countries such as the United States have a limited set of macro-prudential tools, and these tools are difficult to adjust; and monetary policy has broad effects (it "gets in all the cracks" as Stein (2013) famously noted) while macro-prudential tools are perhaps too narrow (e.g. they lead to a migration of activities from the regulated banking system to the unregulated shadow banking system). This consideration motivates our focus of monetary policy.

Our main contribution is to propose a stylized Dynamic Stochastic General Equilibrium (DSGE) to assess the efficacy of LAW. We depart from the usual model in two ways. First, we follow Gourio (2012) and introduce a standard capital structure choice based on the trade-off theory. Capital is accumulated by firms that face costs of issuing equity, while debt is subsidized by the tax code

[1] A distinct argument states that it is preferable to "mop up after the crash", but this argument seems less compelling now in light of the difficulties in stabilizing the economy in the aftermath of the most recent financial crises - for instance, the zero lower bound and reduced potency of monetary policy when agents want to deleverage.

[2] Peek, Rosengren, and Tootell (2015) also add a political economy consideration that is outside the scope of our analysis. As they note, crises often involve bank rescues that generate political backlash which can make it difficult for central banks to conduct monetary policy going forward.

and in the event of default there are deadweight costs to bankruptcy. The combination of the tax subsidy and the equity issuance costs leads the firms to over-rely on debt financing. In what follows we refer to the over-reliance as "excess credit" or "inefficient credit".

We introduce a "financial shock" in this economy by assuming that the tax benefit varies over time. As we explain below, this is a shorthand for various forms of inefficient credit use. We view the tradeoff theory as providing a compact way to introduce variation in the use of debt financing (that could in fact arise for many reasons). What is more important is the potential problems that can arise from the excess credit due to our second modification.

The second modification is to introduce the possibility of a large financial crisis that can hit the economy. This is similar to the rare disasters that have received much attention recently in the asset pricing literature.[3] We assume that the financial crisis leads to a significant, permanent reduction in total factor productivity and a one-off shock to the capital stock. We view this modeling as a convenient device to capture that financial crises lead to large and highly persistent declines in output and consumption. Given that there is not much of a consensus as why crises are so costly, this simplification seems like a reasonable way to model then without taking a stand on a particular reason why losses seem to be so persistent.

We study two types of collapses. The first kind of financial crisis, as in Gourio (2012) and Gourio (2013), occurs exogenously. The second supposes that the probability of the crisis depends on the amount of inefficient credit. By comparing the two alternatives we can isolate the policy consequences of living with possibility of large crises from the possibility that leaning against the wind might be able to change the likelihood of a crisis.

The model allows for the usual productivity and demand shocks in addition to the financial shocks. The centerpiece of the analysis is a comparison of different monetary policy rules that vary with respect to the signals on which the central bank's policy rate is set. In our baseline, we compare policies that rely on perfectly measuring variables and then in some extensions analyze what happens when the central bank must rely on imperfectly measured proxies. In this respect we follow in the long line of papers starting with Bernanke and Gertler (1999) and Gilchrist and Leahy (2002) that ask whether monetary policy should take account of asset price movements. A common conclusion in that literature is that after accounting for movements in inflation, and possibly output,

[3] See Barro (2006), Gabaix (2012), Tsai and Wachter (2015) and Gourio (2012).

there is no need to respond to asset prices. We explore whether the same conclusion holds in our environment.

Our main finding is that gains from responding to credit movements depend importantly on the relative importance of the shocks hitting the economy and the nature of the financial crisis risk. In some versions of the model, for instance when only productivity and demand shocks are present, the possibility of a crisis (endogenous or not) makes little difference for policy. In this environment, stabilizing inflation is optimal. Loosely speaking, once the central bank eliminates demand shocks and accommodates productivity shocks, it can stabilize inflation and simultaneously control crisis risk to the extent possible. In this setup, even if financial crises are endogenous it will make little difference in the policy choices because when the central bank controls demand it will also control credit and limits crisis risk. This result is consistent with the previous literature, in particular Bernanke and Gertler (1999).

On the other hand, when there are also shocks to credit, then failure to respond to credit build ups leads to larger crisis risk than when policy responds to credit developments. Because crises are very costly, the optimal policy tradeoffs leaning against the wind to reduce crisis risk against the costs of enduring larger fluctuations in output and inflation. We emphasize that this result does not require assuming that monetary policy has a very strong effect on the risk of financial crisis. Moreover this result is symmetric: the central bank responds to excessive credit tightening by loosening as well as to excessively loose credit by tightening policy. By smoothing the probability of financial crisis, the central bank generates welfare gains. In part these welfare gains come about because it reduces the average probability of crisis. This reduction is small but the welfare benefits are significant given the large cost of financial crises. We detail these mechanisms in more detail below and provide a preliminary quantitative assessment.

The remainder of the paper proceeds as follows. In the next section, we provide a very brief literature review. In section 3, we introduce the model. Most elements are very standard and are common to many New Keynesian models. In presenting the model, therefore, we concentrate on the two novel aspects mentioned above. Section 4 discusses the parameters used and examines basic properties of the model economy. Finally, in section 5, we compare the performance of a number of policy rules for different versions of the model, and illustrate how several key parameters affect our results. Section 6 concludes.

2 Literature Review

Smets (2014) provides an excellent survey of most of the research on leaning against the wind through 2014, so we summarize his main conclusions and then focus on the most notable papers since then in our review. We refer interested readers to his paper and here just briefly summarize the main points that he emphasizes. He starts with a common point that is agreed on most participants in this debate (including us), that the case for using monetary policy to promote financial stability depends in part on the availability and effectiveness of other tools. He reviews a number of analyses, most notably Lim, Costa, Columba, Kongsamut, Otani, Saiyid, Wezel, and Wu (2011), that study the experience using macroprudential tools and reaches two important conclusions: that "the empirical literature tentatively supports the effectiveness of macroprudential tools in dampening procyclicality" and "to what extent such measures are effective enough to significantly reduce systemic risk is, however, as yet unclear."

Given the ambiguity over whether financial stability can be delivered without appealing to monetary policy, he then turns to the question of what the evidence says regarding the effectiveness of monetary policy in limiting the build up of financial vulnerabilities. Here again he finds mixed evidence. On the one hand, there are a variety of studies that link higher risk-taking by banks with looser monetary policy. He stresses that the risk-taking can occur on both the asset-side of the banks' balance sheet as they reach for yield and through funding choices that entail extra reliance on short-term financing. He argues that although there is ample evidence of risk-taking, the question of whether actively using monetary policy to head it off creates too much collateral damage remains open. He cites several articles that suggest, for instance, that using monetary policy to forestall property price booms would have created a recession. Overall we read his paper as suggesting that there may be scope for leaning against the wind, but doing so would entail non-trivial risks.[4]

Perhaps the most prominent paper written after the Smets survey is Svensson (2016). He provides a simple and transparent framework for evaluating LAW policies. He starts with empirical estimates of the effects of higher interest rates on the likelihood of a crisis (obtained by combining estimates of the effect of interest rates on credit, and of credit growth on the likelihood of crisis

[4]Smets also stresses that if the central bank is given responsibility for financial stability and fails to achieve it, that the bank's monetary independence could be compromised. Though as Peek, Rosengren, and Tootell (2015) mention, central banks that are simply acting as a lender of last resort can also fact this kind of pressure.

(Schularick and Taylor (2012))) and on inflation and output in the short run as well as the cost of a financial crisis (a temporary though long-lasting recession). Svennson emphasizes that on the one hand, tighter policy reduces the risk of financial crisis in the short-run but increases it later on since the effect of tighter policy works through the growth rate of credit (and the long-term level of real credit is assumed to be independent of monetary policy because of long-run neutrality). On the other hand, tighter policy reliably reduces growth and inflation in the short-run. Overall, the costs of slowing down the economy are much higher than the gains from only marginally reducing the risk of a crisis. Indeed, if one accounts for the fact that crisis are to a certain extent inevitable and unavoidable, then a policy that steers the economy to be above potential during non-crisis periods is optimal. Hence, Svensson argues that a careful treatment of this problem calls for leaning with the wind.

The IMF 2015 staff study (IMF (2015)) reaches similar conclusions to Svensson. On their reading of the empirical literature, a 100 basis point increase in the central bank policy rate for one year is needed to reduce the probability of a crisis by only 0.02 percent per quarter. There is obviously much uncertainty around this estimate, but they argue that even using the largest reported estimates of a 0.3 percent per quarter reduction in crisis risk the costs of a slowdown are likely to exceed the gains from preventing a crisis. Ajello, Laubach, Lopez-Salido, and Nakata (2016) similarly argue that the optimal response is small for the median estimate of the effect of monetary policy on risk of crisis, but may be significant if the policymaker takes into account the uncertainty surrounding the estimate, and focuses on the worst-case scenario

Our approach cannot be easily mapped into the Svensson style calculation. There are several differences and we are not certain yet which ones matter most for our different conclusion. First, in terms of methodology we estimate a policy rule in a DSGE model while Svennson conducts a one-time cost/benefit analysis.[5] Second, our objective function is utility while he bases his analysis on a quadratic loss function. Third, we model crises as permanent effects on output while he considers them a temporary "gap" in unemployment or output. Finally, there appears to be a difference between the way the models approach long-run monetary neutrality. In our model, monetary policy shocks have only transient effects on credit and other variables, similar to Svennson. Even so, some

[5] See also Filardo(2016) who also examines policy rules in models with different types of crises. He also concludes that leaning against the wind can be desirable.

monetary policy rules manage to reduce slightly the average probability of crisis by offsetting credit shocks.

IMF (2015), like Smets, questions whether monetary policy is the right tool to address these problems and proposes a three part test that should be considered before monetary policy should be used to lean against the wind. First, are financial risks in the economy excessive? If they are not, then adjusting monetary policy is unnecessary. Second, can other tools be used, particularly macroprudential ones, be used instead of monetary policy? Finally, will monetary policy if set in a conventional fashion based on inflation and output developments take care of financial stability concerns?

Our model allows us to partially address two of the three considerations. We suppose that monitoring financial risks is challenging. Inefficient credit movements may not be observable, so we can study policies that can only be based on noisy indicators of financial risk. Our model has multiple shocks, so we can also study which ones give rise to scenarios where there is a genuine tradeoff between managing the near term inflation and output fluctuations and preventing crises; as will be clear, there are some shocks where a standard inflation targeting central bank will contain financial risks just as a by-product of following its mandate.

We do not discuss macroprudential tools. Partly, this is a tractability issue. There is no consensus model that integrates macroprudential policy levers in a standard monetary model. As Smets (2014) emphasizes even the empirical evidence how this might work is mixed. Developing that kind of framework is beyond the scope of our paper.

More importantly, in many countries the scope for deploying macroprudential tools is limited. The case study developed by Adrian, de Fontnouvelle, Yang, and Zlate (2015) highlights some of the challenges in the U.S. context. In their hypothetical scenario, that they dub a "tabletop exercise", the Federal Reserve is facing a situation where commercial real estate prices are rising sharply, while its inflation and employment objectives are close to being met. Most of the funding fueling the boom are coming from small banks and through capital markets (via securitization).[6] When confronted with this scenario, the the four Federal Reserve Bank Presidents who were attempting to implement policies to manage the situation concluded that "from among the various tools considered, tabletop

[6] This funding constellation matters because in the U.S. the central bank can use some tools, such as stress tests, to steer decisions for very large banks. Restricting the behavior of small banks and stopping securitization is more difficult.

participants found many of the prudential tools less attractive due to implementation lags and limited scope of application. Among the prudential tools, participants favored those deemed to pose fewer implementation challenges, in particular stress testing, margins on repo funding, and supervisory guidance. Nonetheless, monetary policy came more quickly to the fore as a financial stability tool than might have been thought before the exercise."

3 Model

The model economy consists of a representative household, a continuum of monopolistic competitors, a representative investment good producer, and a continuum of financial intermediaries. All firms, including the intermediaries are owned by the household and therefore discount future cash flow using the stochastic discount factor of the representative household.

3.1 Households

The representative household has preferences

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, N_s)$$

where

$$U(C_t, N_t) = \frac{C_t^{1-\tau}}{1-\tau} - \frac{N_t^{1+\upsilon}}{1+\upsilon}. \tag{1}$$

The household consumption bundle is made up of differentiated products,

$$C_t = \left(\int_0^1 C_t(i)^{\frac{1}{1-\eta}} di \right)^{1-\eta}.$$

The dual problem of cost minimization gives rise to a good-specific demand,

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\eta} C_t$$

where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\eta} di \right]^{1/(1-\eta)}$

The representative household earns wage income ($w_t N_t$), the profits of intermediate-goods firms

(Π_t^F) and the profits of financial intermediaries (Π_t^I). The household saves by holding securities issued by financial intermediaries and government bonds (B_t^G), which are zero in net-supply. The bonds issued by the intermediaries are unsecured risky bonds. We denote the price of a bond by q_t. If the bond issuer avoids default, the bond returns one unit of consumption tomorrow. In default, the household earns a partial recovery. Since there is a continuum of issuers, the law of large number applies and the household can form rational expectations about how many bonds fail and how many deliver the promised payment. We denote the probability of default by H_t and the average recovery rate conditional upon default by R_t^D. We can then express the budget constraint of the household as

$$C_t = w_t N_t + \Pi_t^F + \Pi_t^I - q_t B_t + [(1 - H_t) + H_t R_t^D] B_{t-1} + R_{t-1} B_{t-1}^G - B_t^G \tag{2}$$

We denote the Lagrangian multiplier associated with the budget constraint by Λ_t. The household's efficiency conditions are summarized as

$$C_t : \Lambda_t = U_C(C_t, N_t) \tag{3}$$

$$N_t : \Lambda_t w_t = -U_N(C_t, N_t) \tag{4}$$

$$B_t^G : 1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1} \right] \tag{5}$$

and

$$B_t : q_t = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 - H_{t+1}) + H_{t+1} R_t^D \right] \tag{6}$$

A few remarks are in order. First, the two static FOCs together also imply the following efficiency condition.

$$w_t = -\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)}. \tag{7}$$

Second, we assume that the economy is subject to Smets and Wouters (2007)' risk premium shock. Following Fisher (2015), we interpret this as the shock to the demand for safe asset. We denote the shock by Ξ_t, assume that Ξ_t follows an AR(1) process and modify the FOC as

$$1 = \beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \Xi_t R_{t+1} \right]. \tag{8}$$

These shocks do not affect the flexible economy and hence are considered be an inefficient source of business cycle fluctuations (in contrast to technology shock which gives rise to efficient fluctuations).

Third, the FOC for intermediary bond holding plays the role of the pricing equation for intermediary problem. We will provide more details on this, including the determinants of the recovery rate R_t^D, when we discuss the intermediary problem. For later purposes, we define the stochastic discount factor of the household as

$$M_{t,t+1} \equiv \beta \frac{\Lambda_{t+1}}{\Lambda_t}. \tag{9}$$

3.2 Investment Goods Producers

We assume that there exists a continuum of competitive firms indexed by $k \in [0, 1]$. These firms produce an identical composite good I_t using a linear technology subject to an adjustment cost related to the level of investment. We parameterize the costs to be $\kappa/2\,(I_t/I_{t-1} - 1)^2\, I_{t-1}$. The composite good I_t is sold at a price Q_t to be used in the production of capital. Production of the composite good requires the use of all varieties of intermediate goods. Since the industry is competitive, the size of an individual firm is indeterminate. Hence we assume a representative firm that is a price taker. The profit maximization problem of the investment goods producers can be cast as choosing the input level given the cost of adjusting investment level, i.e.,

$$\max_{I_s} \mathbb{E}_t \sum_{s=t}^{\infty} M_{t,s} \left\{ Q_s I_s - \left[I_s + \frac{\kappa}{2} \left(\frac{I_s}{I_{s-1}} - 1 \right)^2 I_{s-1} \right] \right\}.$$

The FOC of the problem is given by

$$Q_t = 1 + \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) - \mathbb{E}_t \left\{ M_{t,t+1} \frac{\kappa}{2} \left[\left(\frac{I_{t+1}}{I_t} \right)^2 - 1 \right] \right\}. \tag{10}$$

3.3 Retailers

There exists a continuum of monopolistic competitors indexed by $i \in [0,1]$. These retailing firms combine labor and capital using a Cobb-Douglas production technology

$$Y_t(i) = Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}$$

where Z_t is the aggregate technology. The retailers are subject to quadratic costs of adjusting prices

$$\frac{\varphi}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t = \frac{\varphi}{2}\left(\Pi_t \frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 Y_t,$$

where $p_t(i) \equiv P_t(i)/P_t$ and $\Pi_t \equiv P_t/P_{t-1}$ is gross inflation. Hence, the firm's static profit is given by

$$\Pi_t(i) = p_t(i)Y_t(i) - w_t N_t(i) - r_t^K K_t(i) - \frac{\varphi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}\Pi_t - 1\right)^2 Y_t.$$

where $w_t \equiv W_t/P_t$ is the real wage. The retailers are owned by the representative household, and hence discount future cash flow using the stochastic discount factor of the household. Pricing maximizes the present value of expected profits

$$\mathcal{L} = E_t \sum_{s=t}^{\infty} M_{t,s}\{\Pi_s(i) + \mu_s(i)[Z_s K_s(i)^{\alpha} N_s(i)^{1-\alpha} - Y_s(i)] + \nu_s(i)[p_s(i)^{-\eta} Y_s - Y_s(i)]\}$$

where $\nu_s(i)$ and $\mu_s(i)$ are the shadow values of the demand constraint and technological constraints. The efficiency conditions in a symmetric equilibrium are:

$$w_t = (1-\alpha)\mu_t(i)\frac{Y_t(i)}{N_t(i)} \tag{11}$$

$$r_t^K = \alpha \mu_t(i) \frac{Y_t(i)}{K_t(i)} \tag{12}$$

$$\nu_t = 1 - \mu_t \tag{13}$$

$$0 = 1 - \varphi\Pi_t\left(\Pi_t - 1\right) - \eta\nu_t + \varphi\mathbb{E}_t\left[M_{t,t+1}\Pi_{t+1}\left(\Pi_{t+1} - 1\right)\frac{Y_{t+1}}{Y_t}\right] \tag{14}$$

3.4 Financial Intermediaries

This part of the model follows the setup in Gourio (2012). We assume that there exists a continuum of financial intermediaries indexed by $s \in [0,1]$. The financial intermediaries combine debt and equity capital to invest in the financial claims on non-financial firms, denoted by $S_t^K(s)$. In our symmetric equilibrium, all intermediaries make identical decisions and hence in equilibrium $S_t^K(s) = S_t = K_{t+1}$. From now on we omit the intermediary index.

If intermediary invests Q_tK_{t+1} at time t, then at time $t+1$ its return on the asset will be

$$\begin{aligned} R_{t+1}^K &= \varepsilon_{t+1}R_{t+1}^K \\ &= \varepsilon_{t+1}\frac{r_{t+1}^K + (1-\delta)Q_{t+1}}{Q_t}, \end{aligned}$$

where ε_{t+1} is an idiosyncratic risk associated with the intermediary. The shocks are iid across time and producers, have a cdf $H(\cdot)$, and a pdf $h(\cdot)$. (In practice we assume that ε_{t+1} follows a lognormal destribution, $\log\varepsilon_{t+1} \sim N(-0.5\sigma^2, \sigma^2)$).

The intermediary here can be thought of integrating a set of financially unconstrained borrowers with a banking system. In a more complete set up where even borrowers are subject to financial constraints, we could have richer financial accelerator mechanism that comes both from the borrowers and the lenders. Here we collapse the actors together so that when the banks expand, they directly create more physical capital (as in Gertler and Karadi (2011)).

The choice of debt vs equity is driven by the standard trade-off model from corporate finance. For now, we assume that debt is set in real terms[7] and has a tax advantage $\chi_t > 1$. We first start with the case without equity issuance cost, then add it later for clarity. This means that for each unit of debt issued at time t, the corporation receives a subsidy equal to $\chi_t - 1 > 0$. This subsidy may be thought as a stand-in for many rationales that make debt issuance attractive. For instance, it is commonly argued that the presence of debt is beneficial as it gives stronger incentives on

[7] This is rather innocuous since our financial crises will not have deflation, so changing this assumption would not materially affect the results.

managers to maximize profits, and to avoid engaging in empire building. One can view χ_t as a shortcut for such an "agency benefit" to debt. The capital producer's problem is to choose capital and debt (and hence equity) to maximize its expected present discounted value.[8] The maximization problem of the entrepreneur can then be expressed as

$$\max_{B_{t+1}, S_t, Q_t K_{t+1}} \mathbb{E}_t[M_{t,t+1} \max\left(V_{t+1} - B_{t+1}, 0\right)] - S_t.$$

where $V_{t+1} = \varepsilon_{t+1} R^K_{t+1} Q_t K_{t+1}$ is the value at time $t+1$, where S_t is equity issuance today. The maximization is subject to the funding constraint:

$$Q_t K_{t+1} = \chi_t q_t B_{t+1} + S_t,$$

where q_t is the price of the bonds and the debt pricing equation is

$$q_t = \mathbb{E}_t\left[M_{t,t+1}\left(1_{V_{t+1}<B_{t+1}} \zeta_t \frac{V_{t+1}}{B_{t+1}} + 1_{V_{t+1} \geq B_{t+1}}\right)\right]$$

where $1_{V_{t+1}<B_{t+1}}$ is a dummy indicating default, and θ is the recovery rate. The intermediary decides on debt and capital, taking into account that higher leverage will lead to higher spreads. This framework directly implies an "externality" since higher leverage increases default risk which leads to larger losses.[9] We can obtain the first-order conditions for capital and debt holdings. To that end, we rewrite the bond pricing function as

$$q_t B_{t+1} = \mathbb{E}_t M_{t+1}[\Omega(\varepsilon^*_{t+1}) \zeta_t R^K_{t+1} Q_t K_{it+1} + (1 - H(\varepsilon^*_{t+1})) B_{t+1}].$$

where $\Omega(x) \equiv \int_0^x \varepsilon dH(\varepsilon) = xh\left(x\right)$, and $\varepsilon^*_{t+1} \equiv \frac{B_{t+1}}{R^K_{t+1} Q_t K_{it+1}}$, i.e., the default threshold.

For some purposes it is helpful to think about what would happen if there were no equity

[8] This equation assumes that the producer only maximizes its one-period ahead value. It is easy to see that this corresponds to maximizing its long-term value because the present value of rents is zero due to free entry.

[9] This assumes that χ is indeed a tax subsidy, which is inefficient. The first best would be simply to set $\chi = 1$, in which case capital structure would be determined by balancing the bankruptcy costs against the equity issuance costs.

issuance cost, in this case the objective function is equivalent to

$$\max_{B_{t+1},K_{t+1}} \mathbb{E}_t M_{t+1}[(1-(1-\chi_t\theta_t)\Omega(\varepsilon^*_{t+1}))R^K_{t+1}Q_tK_{t+1}$$
$$-(1-\chi_t)(1-H(\varepsilon^*_{t+1}))\varepsilon^*_{t+1}R^K_{t+1}Q_tK_{t+1}]-Q_tK_{t+1}.$$

The first-order condition for capital is given by

$$\mathbb{E}_t\left(M_{t+1}R^K_{t+1}\lambda_{t+1}\right)=1, \tag{15}$$

where

$$\lambda_{t+1}=1+(\chi_t-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)-(1-\zeta_t\chi_t)\,\Omega\left(\varepsilon^*_{t+1}\right). \tag{16}$$

This equation simply states that the user cost of capital is adjusted for risk and for the cost of debt finance (which includes the subsidy χ_t and the default costs ζ_t). The second equation equates the marginal benefit of debt with the marginal cost, the former having to do with marginal subsidies and the latter with marginal distress costs:

$$\frac{\chi_t-1}{\chi_t}\mathbb{E}_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)=(1-\zeta_t)\,\mathbb{E}_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right). \tag{17}$$

In our benchmark specification we assume that the issuance of equity is costly and that the cost per unit of equity issuance is an increasing function of the equity share relative to the size of the project:

$$\gamma_t=\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right),\ \ \gamma(0)=0,\ \gamma'(\cdot)>0 \text{ and } \gamma''(\cdot)\geq 0$$

With the presence of equity issuance cost, the objective function becomes

$$\max_{B_{t+1},S_t,K_{t+1}} \underbrace{\mathbb{E}_t[M_{t+1}\max\left(V_{t+1}-B_{t+1},0\right)]-S_t}_{\text{identical to the problem absent equity issuance costs}} -\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)S_t.$$

Hence the objective function can be written as

$$\begin{aligned}\max_{B_{t+1},K_{t+1}} &\mathbb{E}_t M_{t+1}[(1-(1-\chi_t\zeta_t)\Omega(\varepsilon^*_{t+1}))R^K_{t+1}Q_tK_{t+1} \\ &-(1-\chi_t)(1-H(\varepsilon^*_{t+1}))\varepsilon^*_{t+1}R^K_{t+1}Q_tK_{t+1}]-Q_tK_{t+1}\left[1+\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)\frac{S_t}{Q_tK_{t+1}}\right]\end{aligned}$$

where we continue to omit the intermediary index since in equilibrium, all intermediaries will make identical choices. The last term can be rewritten as

$$1+\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)\frac{S_t}{Q_tK_{t+1}}=1+\gamma\left(1-\chi_t\frac{q_tB_{t+1}}{Q_tK_{t+1}}\right)\left(1-\chi_t\frac{q_tB_{t+1}}{Q_tK_{t+1}}\right)$$

where

$$\frac{q_tB_{t+1}}{Q_tK_{t+1}}=\mathbb{E}_tM_{t+1}[\Omega(\varepsilon^*_{t+1})\zeta_tR^K_{t+1}+(1-H(\varepsilon^*_{t+1}))\varepsilon^*_{t+1}R^K_{t+1}]\equiv L\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right).$$

Substituting $L(\varepsilon^*_{t+1})$, the last term of the objective function can be re-expressed as

$$\begin{aligned}&Q_tK_{t+1}\left[1+\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)\frac{S_t}{Q_tK_{t+1}}\right]\\ &=Q_tK_{t+1}\left\{1+\gamma\left[1-\chi_tL\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right)\right]\left[1-\chi_tL\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right)\right]\right\}\equiv Q_tK_{t+1}\Gamma\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right).\end{aligned}$$

Importantly, $\Gamma(B_{t+1}/Q_tK_{t+1})$ depends only on leverage and not separately on Q_tK_{t+1}. Hence, the FOC for capital is now given by

$$1=\Gamma\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right)^{-1}\mathbb{E}_t\left(M_{t+1}R^K_{t+1}\lambda_{t+1}\right) \tag{18}$$

where λ_{t+1} is the same as (16). The efficient level of leverage will be determined by

$$0=\mathbb{E}_tM_{t+1}\left[(\chi_t-1)(1-H(\varepsilon^*_{t+1}))-(1-\chi_t\zeta_t)\varepsilon^*_{t+1}h(\varepsilon^*_{t+1})-(\chi-1)\varepsilon^*_{t+1}h(\varepsilon^*_{t+1})\right]-\Gamma'\left(\frac{B_{t+1}}{Q_tK_{t+1}}\right).$$

This expression can be shown equivalent to

$$\begin{aligned}&\mathbb{E}_t\left\{M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\left[\frac{\chi_t-1}{\chi_t}+\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)+\gamma'\left(\frac{S_t}{Q_tK_{t+1}}\right)\right]\right\}\\ &=(1-\zeta_t)\,\mathbb{E}_t\left\{M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\left[1+\gamma\left(\frac{S_t}{Q_tK_{t+1}}\right)+\gamma'\left(\frac{S_t}{Q_tK_{t+1}}\right)\right]\right\}.\end{aligned} \tag{19}$$

Note that when equity issuance is not costly, $\gamma(\cdot) = \gamma'(\cdot) = 0$, $\Gamma(\cdot) = 1$ and (18) and (19) collapse into (15) and (17).

3.5 Financial Crises

We now describe how the aggregate technology Z_t evolves over time. We assume that the technology a random walk process subject to two kinds of shocks:

$$\frac{Z_{t+1}}{Z_t} = e^{X_{t+1}b_c} e^{\xi_{t+1}}, \ \ b < 0, \tag{20}$$

where ξ_{t+1} are the "usual" shocks and X_{t+1} is the "financial crisis" shock; specifically $X_{t+1} = 0$ with probability $1 - p_t$ an $X_{t+1} = 1$ with probability p_t. When a crisis occurs, the level of technology discretely jumps to a b_c percent lower level. We assume a following reduced-form law of motion for the probability of disaster:

$$\log p_t = b_0 + b_1 \log(B_t / B_t^f) \tag{21}$$

where B_t^f is the efficient level of credit that prevails in an economy without price distortion. The reduced-form assumes that the probability of disaster is an increasing function of the level of inefficient credit.[10] We refer to this as "inefficient credit" and hence implicitly assume that the steady-state distortion that favors debt (that, is the steady-state tax subsidy $\chi > 1$) does not create a risk of financial crisis.

We also assume that the capital accumulation process is also affected by the financial crisis in the same way: financial intermediaries invest I_t and "expect" to obtain

$$K_t^w = (1 - \delta) K_t + I_t,$$

but their capital stock that is realized at beginning of time $t + 1$ is actually

$$K_{t+1} = K_t^w e^{X_{t+1} b_c}.$$

[10] While this is a convenient short-cut, Cairo and Sim (2016) provides a structural model that delivers the same prediction. In order to study the relationship between price stability and financial stability, Cairo and Sim (2016) endogenizes the production and income distribution in the financial crisis model of Kumhof, Ranciere, and Winant (2015). Cairo and Sim (2016) also allows for nominal rigidities and labor market frictions. Cairo and Sim (2016) shows that in this structural model of financial crisis, the correlation between debt and the probability of financial crisis is as high as 0.92. This is one way to justify our reduced form specification for the crisis risk.

That is, in the (unlikely) event of a financial crisis, the capital stock is not what the intermediaries expected it to be.

Finally we further assume that the utility function is affected by a disaster realization. We do so because the preferences we use are not compatible with balanced growth, so that a one-time decline in productivity may lead to a change in hours. For tractability, we assume that

$$U(C_t, N_t) = \frac{C_t^{1-\tau}}{1-\tau} - J_t^{1-\tau}\frac{N_t^{1+\upsilon}}{1+\upsilon}$$

where J_t is the cumulative disaster effect,

$$J_t = e^{X_t b_c} J_{t-1}.$$

We then redefine variables by detrending by Z_t, e.g. $\widetilde{Y}_t = Y_t/Z_t$, etc. Under the assumptions above, the system of equations of detrended variables does not depend on X_t. That is, the detrended system has no jumps. This implies it can be solved using standard perturbation techniques. For the details of transforming the original system of equations into the detrended system, see the appendix. Also see Gourio (2012), Isor and Szczerbowicz (2015) and Gabaix (2011) for detailed detrending methodology for disaster models.

4 Basic model properties

We first discuss the parameters used for our model, then illustrate the model dynamics using impulse response analysis.

4.1 Calibration

Table 1 summarizes the calibration of the model parameters. We set the time discount factor $\beta = 0.99$ such that the implied annual real rate is equal to 4 percent. Capital share of production α is set equal to 36 as is standard in the literature. The depreciation rate δ is calibrated equal to 0.025. We set investment adjustment cost κ equal to 5 to produce roughly three time more volatile investment than output.

We assume that the constant relative risk aversion is equal to 2 while assuming 1/3 of inverse

Table 1: Structural Parameters

Parameters	Interpretation	Value
β	Discount factor	0.99
γ_1	Linear cost of equity issuance	0.03
γ_2	Quadratic cost of equity issuance	0.15
α	Capital share	0.36
τ	Constant relative risk aversion	2
κ	Investment adjustment cost	5
δ	Depreciation rate	0.025
η	Elasticity of substitution between goods	2.0
$\bar{\chi}$	Steady state tax benefit	1.03
ζ	Steady state bankruptcy cost	0.50
υ	Inverse of Frisch elasticity of labor supply	1/3
$\bar{\Pi}$	Gross inflation rate target	1
φ	Price adjustment cost	130
σ	Idiosyncratic volatility	0.25
p_{ss}	Average probability of financial crisis	0.005
b_1	Sensitivity of log prob to credit deviation	5
b_d	Size of output drop if financial crisis	0.10
ρ_Z	Persistence of the technology shock	0.90
ρ_χ	Persistence of the tax benefit shock	0.90
ρ_Ξ	Persistence of the risk premium shock	0.90

Frisch elasticity of labor supply. Regarding the elasticity of substitution between goods, we choose $\eta = 2$ to be consistent with the finding on differentiated goods in Broda and Weinstein (2006). With this choice, we set the price adjustment cost $\varphi = 130$ to match the fact that micro studies suggest that prices adjust about once a year.

We set the linear (γ_1) and quadratic (γ_2) coefficients of the equity issuance cost equal to 0.03 and 0.15, respectively. With these settings, the model generates a 1.3 cent dilution effect per each dollar of issuance. We view this as being a conservative estimate of issuance costs. We presume that there is also a modest tax advantage for debt, we set the steady state tax benefits parameter $\bar{\chi} = 1.03$. $\bar{\chi} - 1$ matches the difference between the corporate income tax rate and the interest income tax rate in the U.S., which governs the size of the tax shield. We choose the bankruptcy cost of default to be $\zeta = 0.5$. This value allows us to match the recovery rate on U.S. corporate bonds. We set the idiosyncratic volatility $\sigma = 0.25$ in line with the estimation results of Christiano, Ilut, Motto, and Rostagno (2010).

The average probability of a financial crisis is set to 2% per year or 0.5% per quarter, corresponding to two crises per century. The size of the output drop is set to 10%. This number is significantly smaller than that of the asset pricing literature on disasters. This number is consistent, for instance, with the recent US experience (and conservative relative to other countries' experience). The sensitivity of the financial crisis probability to excess credit is 5, so that a 20% increase in inefficient credit doubles the probability of financial crisis. We study extensively the sensitivity of our results to these parameters below.

Regarding the aggregate shock processes, we take an agnostic approach and set all the persistence parameters equal to 0.9. We then calibrate the standard deviation of technology shock equal to 0.01. We then choose the other two shock volatilities such that the variance decomposition share of output can be allocated to technology shock, demand shock and financial shock with 40-40-20 shares, respectively.

4.2 Model Properties With a Standard Policy Rule

As a first step, we illustrate how our model economy behaves in response to the three fundamental impulses that we consider - a productivity shock, an aggregate demand shock, and the financial shock. To solve the model, we assume an inertial Taylor (1999) rule:

$$R_t = 0.85 \times R_{t-1} + 0.15 \times (R^* + 1.5 \times (\pi_t - \pi^*) + \widetilde{y}_t),$$

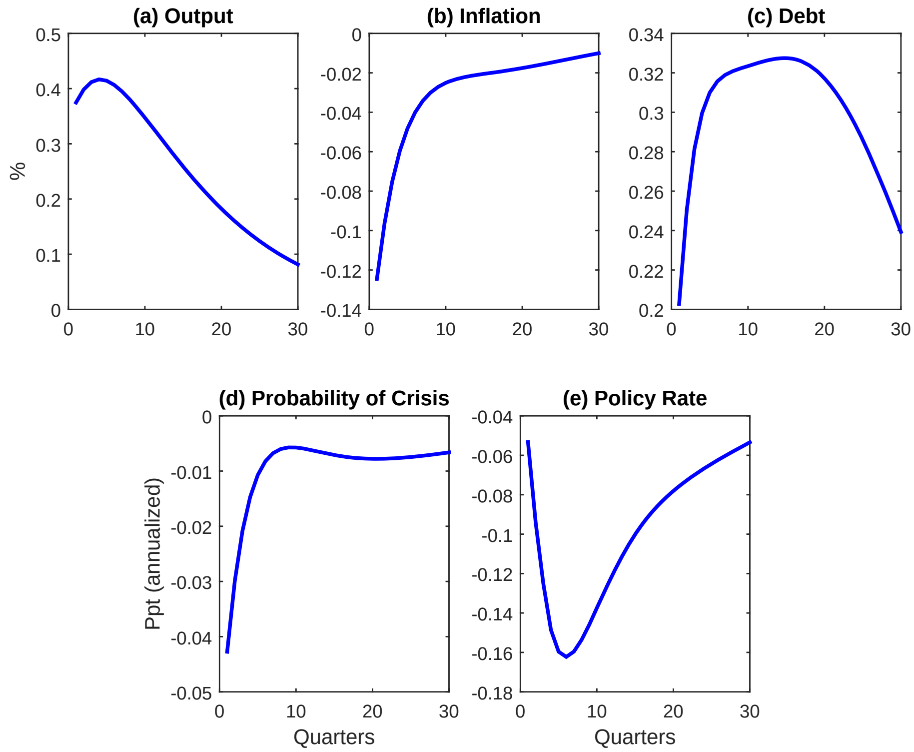
where $\widetilde{y}_t$ is the output gap[11], and π_t is the year-over-year inflation rate. We summarize the main mechanisms in the model by explaining what happens to output, inflation, debt, the policy rate, and the probability of a crisis. As a further diagnostic we also report the effect that a shock to the policy rule has on the same variables.

A productivity shock, shown in Figure 1, leads to higher output and lower inflation as is common in New Keynesian models. The policy rule leads the central bank to cut the policy rate but not sufficiently to stabilize inflation or to allow output to rise in line with potential. Put differently, lower inflation reflects the decline in current and future marginal costs that arise from higher productivity and the fact that monetary policy does not bring demand in line with this higher supply.

The output surge leads to higher borrowing to finance investment, but because output does not keep up with growth in potential, credit actually rises less than in the frictionless benchmark. As a

[11]We define this gap to be the difference between the level of output and the one that would prevail in an economy without nominal rigidities and without financial shocks.

Figure 1: Impulse Response to Productivity Shock: Baseline



result, the probability of crisis falls modestly (by 4 basis points, so that the probability drops from 2% per year to 1.96%).

The response to a demand shock (modeled following Smets and Wouters (2007) as a change in agents' discount rate for nominal bond), shown in Figure 2, leads to lower output and inflation (given the assumed interest rate rule). The lower output in turn leads to lower debt and lower risk of financial crisis.

Next, in Figure 3, we show the effect of a financial shock, which reflects an inefficient shock to credit supply. This type of shock leads to a large expansion of credit which reduces the user cost of capital and leads to a boom in investment and, to a lesser extent, also in output. The lower user cost feeds through to lower inflation. The spike in debt (that is permitted with this policy rule) significantly increases the risk of financial crisis, from 2% per year to 2.28% per year.

Finally we illustrate how a "monetary shock" affects this model economy. Although we most interested in optimal monetary policy rules, showing the impact of deviation from the rule is informative about two aspects of the model. Figure 4 displays the responses of our main variables to a 100 basis point (1%) increase in the (annualized) policy rate. One important takeway from

Figure 2: Impulse Response to Demand Shock: Baseline

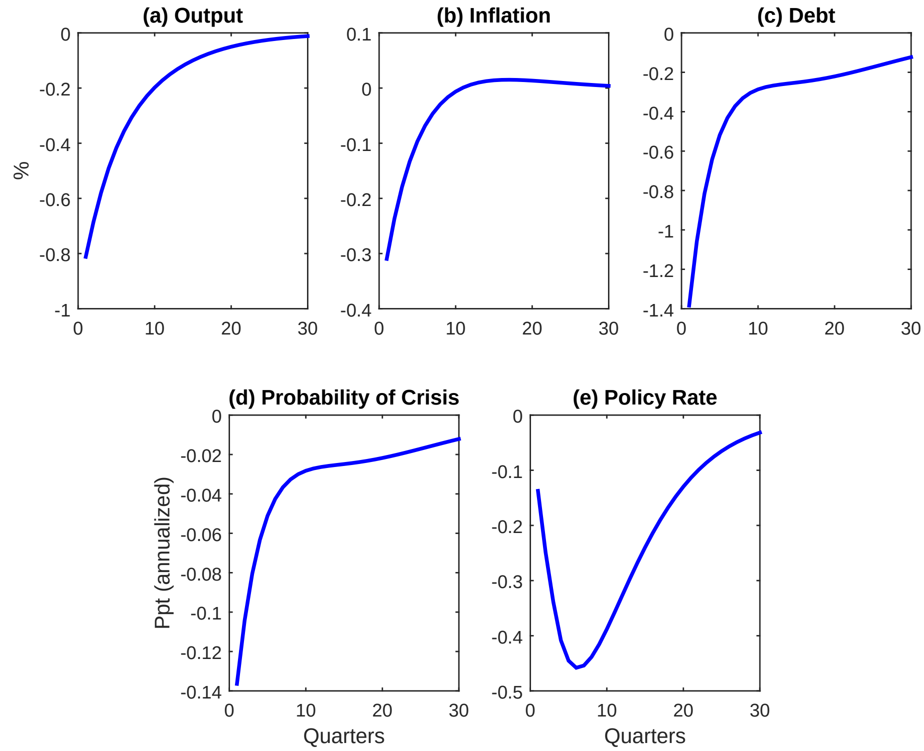


Figure 3: Impulse Response to Financial Shock: Baseline

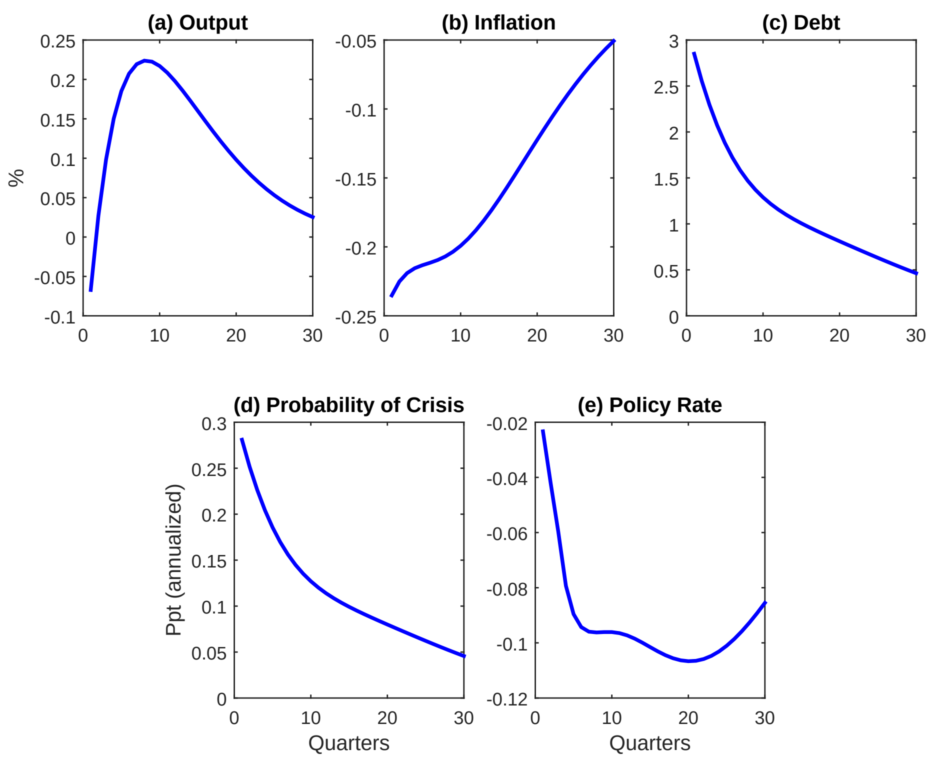
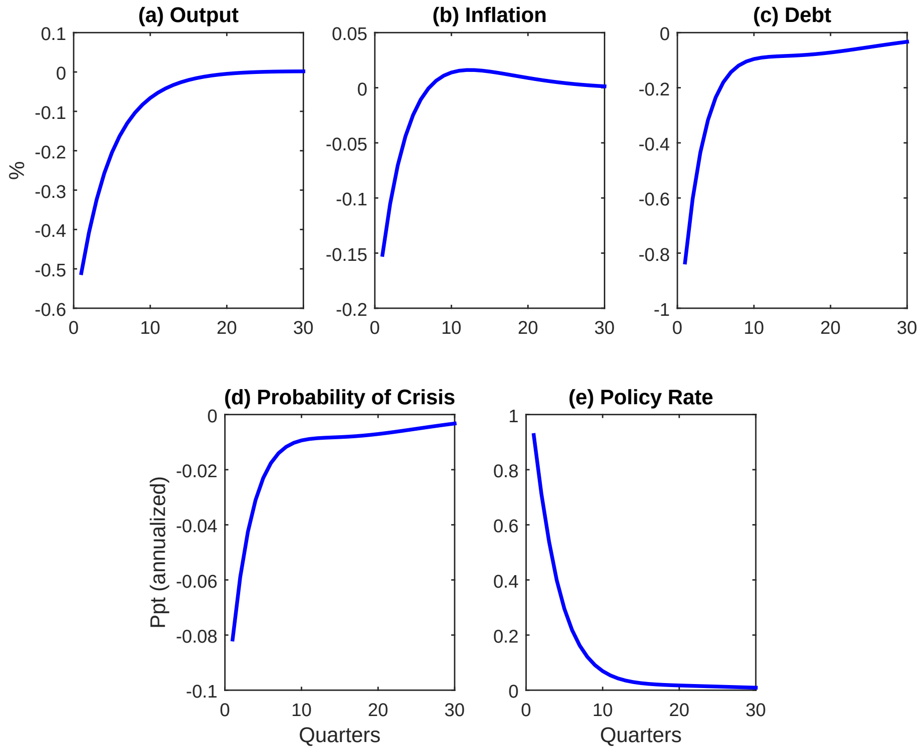


Figure 4: Impulse Response to Monetary Policy Shock: Baseline



the figure is that the shock leads to a decline in output and inflation.[12] The output drop leads to a decline of credit and hence the probability of crisis. The second important conclusion from this exercise is that the sensitivity of the risk of crisis to an increase in the policy rate is by no means extreme - this fairly large monetary shock only generates on impact a reduction of 8 basis points in the annual probability of crisis, i.e. moving it from 2% to 1.92%. This is magnitude of the change is consistent with the empirical estimates reviewed by IMF (2015). We share the view of IMF (2015) that these estimates are somewhat uncertain, but it is important to note that our subsequent conclusions about the desirability of leaning against the wind are not driven by a presumption that monetary policy has powerful effects on the risk of a crisis.

[12]Our model does not generate hump-shapes in response to this shock because it lacks some of the propagation mechanisms introduced by Christiano Eichenbaum and Evans (2005) or Smets and Wouters (2007) such as inflation indexation or consumer habits. We believe this is not critical for our results.

5 Optimal simple rules

Having established the basic model properties, we consider policy rules that specify the interest rate as a function of last period's interest rate, inflation, the output gap and/or the "credit gap", i.e. B_t/B_t^f, the deviation of credit from the level that would prevail with only productivity shocks and flexible prices. Past research have shown such rules typically perform well in models like ours. Because real time measurement of the output and credit gaps is difficult,[13] we also study rules that rely on imperfectly measured version of these variables, namely deviations of output and credit from their steady state values. Our goal is to establish the conditions when responding to credit may be beneficial. The benchmark for comparisons is the welfare of a representative consumer who cares not only about the usual fluctuations in output and inflation, but also about risks that bring large persistent drops in output and consumption. As we will see, in some configurations of the model the central bank finds it optimal to respond to credit so as to smooth the risk of financial crisis, even though this leads to higher output *and* inflation volatility.

Our main result is that leaning against the wind can be beneficial provided that three conditions are met: (1) financial crises have important output effects; (2) financial shocks are important, i.e. the variance of the financial shocks and the associated swing in inefficient credit are large enough, and (3) financial crises are endogenous, i.e. they are caused in part by inefficient credit. In contrast, if there are no financial shocks, even with other financial imperfections present, we obtain the standard result that stabilizing inflation is a sufficient condition for maximizing welfare. In this latter case, a simple Taylor rule that puts enough weight on the output gap can maximize welfare. In the absence of price markup shock, so called divine coincidence holds, and the same outcome can be achieved by maximizing the inflation coefficient in this case. If there are financial shocks, but financial crises are exogenous, a simple rule that puts weight on the output gap still outperforms credit-based rules, because targeting the output gap is a more direct way to eliminate undesirable fluctuations in output and inflation.

Obviously, these results depend on parameter choices. For instance, it is clear that if financial crises have small effects, or the variance of financial shocks is small, responding to output may still be preferable to responding to credit. In the results that follow we have calibrated the financial shocks

[13] See Orphanides and Williams (2002) and Edge and Meisenzahl (2011).

so that they account for a bit less than 20% of the variance of output, and demand and productivity shocks equally account for the remainder (i.e., 40% each). We discuss some robustness exercises after we introduce our main findings. However, because we have not estimated the model and we view these results as being indicative rather than dispositive. Put differently, rather than giving a definitive answer to the question of whether leaning against the wind is desirable, we think our framework is useful precisely because it permits us to understand, within a fairly standard DSGE model, which parameters and model features govern whether responding to credit conditions is beneficial.

5.1 Methodology

We consider policy rules of the following form:

$$R_t = \rho R_{t-1} + (1-\rho)(R^* + \phi_\pi(\pi_t - \pi^*) + \phi_y \tilde{y}_t + \phi_b b_t)$$

where π_t is again the year-over-year inflation rate, $\widetilde{y_t}$ is the output gap and b_t is the credit gap, i.e. $\log\left(B_t/B_t^f\right)$. Note that b_t is the variable which determines the probability of a financial crisis, according to (21). Throughout this exercise we set $\rho = 0.85$ and $\phi_\pi = 1.5$. Our motivation for imposing these restrictions is to make analysis transparent, and to require that the policy rule to resemble the kind that broadly describes actual central bank decisions. We then consider the welfare consequences of policy rules with different coefficients for ϕ_y or ϕ_b. Specifically, we rank rules according to the expected utility they provide to the representative consumer and find the value of ϕ_y and/or ϕ_b that maximizes this expected utility.[14,15] We first consider the simple case where only one gap matters so that $\phi_b = 0$ or $\phi_y = 0$. We then discuss results when we optimize over ϕ_b and ϕ_y jointly.

[14]In contrast, many papers maximize a quadratic loss function of inflation and unemployment. In our case this approach would not capture the cost of financial crises, which permanently lower productivity. It is also a priori attractive to use a micro-founded welfare criterion.

[15]In practice, we first rewrite the system of equations that determines the equilibrium around the stochastic trend induced by disaster. This system can then be solved using standard perturbation methods since it has no jumps. We then use a second-order approximation of the utility. See appendix for details, and https://sites.google.com/site/fgourio/ for the code used to solve the paper (to be posted).

5.2 Main Result

Table 2 summarizes the outcomes that lead to our main conclusion. When we select the best rule that depends solely on a correctly measured output gap, the optimal sensitivity is very high[16], around 100, so that monetary policy eliminates all inefficient fluctuations of output. As can be seen, this monetary policy rule generates also a relatively small volatility of inflation. When this rule is followed, crises occur about 2.08 percent of the time, though the standard deviation of the probability of crises is 0.61 percent, so households face the risk that crises can be more frequent than that.

When we select the best rule that depends solely on the correctly measured credit gap, we obtain a coefficient of 1.88 on the credit gap. This rule generates significantly greater volatility of output and inflation than the one based on the output gap.[17] Yet, the credit-gap based rule outperforms the output-gap based rule in terms of welfare. The difference in utility is equivalent to a permanent increase of consumption of 0.06%, a small but significant number. In all of the comparisons that follow, we report the consumption equivalent change between a rule based only on the output gap and those that depend on the credit gap or both gaps; by this convention, the consumption equivalent for the rule that focusing on output gap only is always zero.

The gain in welfare occurs because the LAW policy is sacrificing volatility in order to limit the financial crisis risk: the probability of a financial crisis is now both smaller and substantially less volatile. The reduction in the mean probability of crisis is driven, in part, by the functional form we use to insure that the crisis probability lies between zero and one.[18] While this effect may seem at first mechanical, it reflects the real constraint that financial crisis probability is bounded below (by zero). As such, decreasing the volatility of financial crisis leads to lower mean because the mean is driven by the occasional upswings.

Figure 5 depicts the response of macroeconomic aggregates to the three fundamental shocks under the standard Taylor (the solid blue line), the rule that responds using only the output gap (the dashed red line), and the rule that responds using only the credit gap (the dotted green line). Our main conclusion is best understood by comparing what the different rules imply about policy

[16]We set an upper bound of 100, and a lower bound of 0, to ensure that the optimization problem well-posed. Allowing for values higher than 100 does not materially alter the results.

[17]The output volatility measure does not take into account financial crises.

[18]We specify a process for the log of the probability and that implies that lower volatility also brings a lower mean.

Table 2: Benchmark Model

	Output gap only	Credit gap only	Both gaps
Welfare	-161.439	-161.334	-161.308
Consumption equivalent (%)	0	0.065	0.081
Coefficient ϕ_y	100	–	100
Coefficient ϕ_b	–	1.88	83.37
100×SD(Π)	0.30	0.51	0.48
100×SD(Y)	1.71	2.84	2.52
100×SD(Y^f)	1.72	1.72	1.72
400×E(P)	2.08	2.02	2.02
400×SD(P)	0.61	0.22	0.26

in the aftermath of a financial shock. The credit-gap rule tightens policy which leads to a much lower debt expansion - and consequently a lower risk of a crisis. The cost of this policy is large in terms of the deviation of inflation and output from target. This policy, nevertheless, delivers higher welfare because it smooths substantially the probability of a financial crisis. In contrast, the output gap based policy cuts interest rates because inflation is low and lower rates help keep output close to its target. The cost of this choice is a rise in the financial crisis risk.

A standard Taylor rule would lead the central bank to gradually cut rates and tradeoff inflation rate undershooting against a modest output boom. In this case, debt also accumulates so that the crisis probability rises even more substantially.

Figures 6 and 7 show the performance of the different rules in face of demand and productivity shocks. Another cost of the credit-gap policy rule is that it does less well than the output-gap based rule in response to standard demand and productivity shocks. While the output-gap based policy offsets completely the demand shock and accommodates almost perfectly the productivity shock, the credit-gap based policy responds less aggressively to both of these shocks. The reason why the optimized credit rule leads to smaller interest rate cuts for these shocks is because if it were more aggressive in these cases, it would be even more responsive to financial shocks too; a higher coefficient on the credit gap would be a positive in responding to these shocks, but would exaggerate even more the output and inflation deviations in response to the financial shock.

Figure 5: Impulse Response to Financial Shock: Optimal Simple Rule

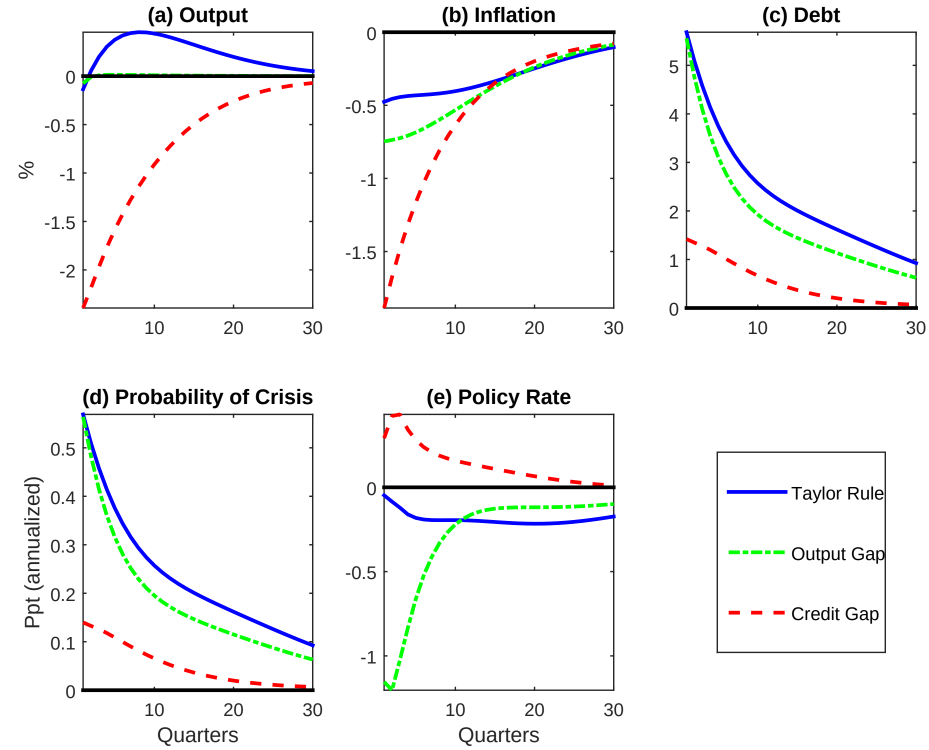


Figure 6: Impulse Response to Demand Shock: Optimal Simple Rule

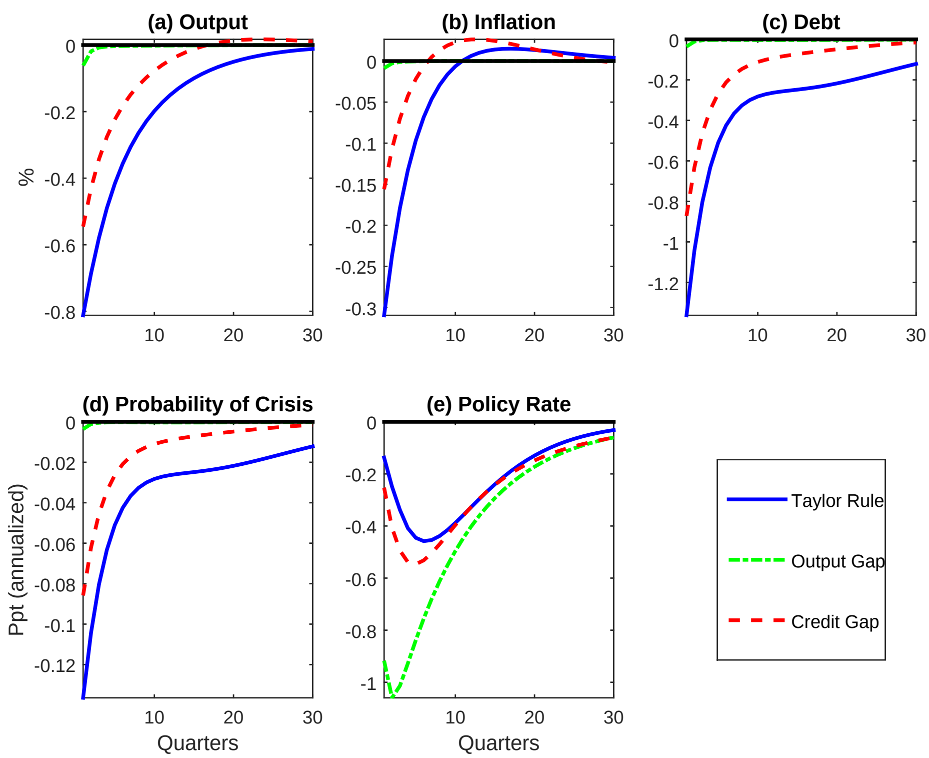
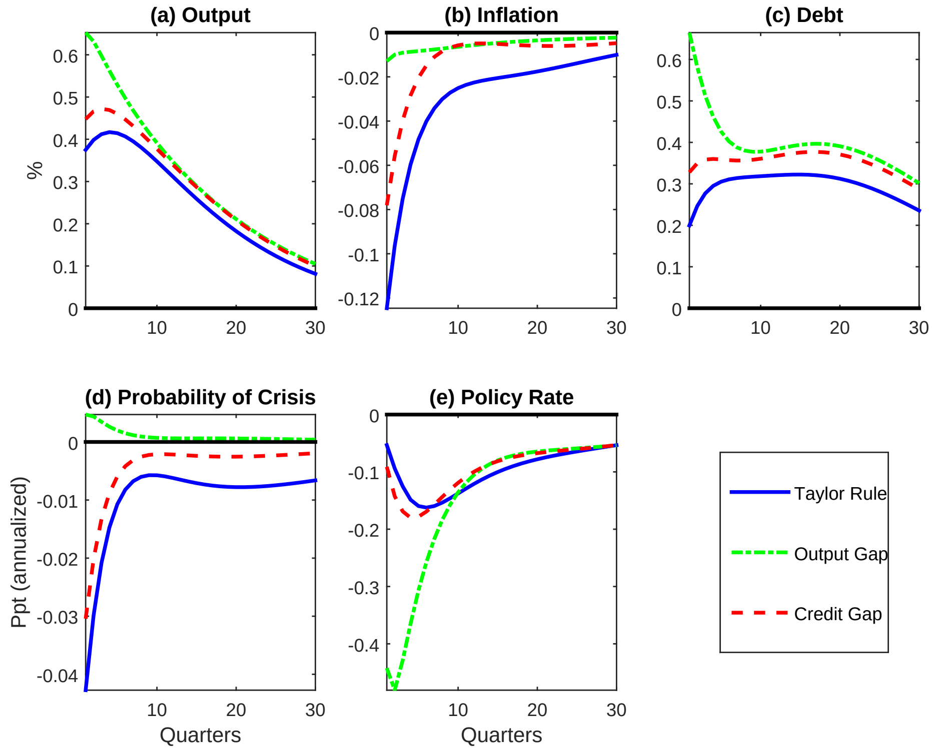


Figure 7: Impulse Response to Technology Shock: Optimal Simple Rule



5.3 Understanding the result

To confirm the interpretation that we have offered for the main findings, it is instructive to shutdown various features of the model to see how they change the results. A particularly helpful experiment is to turn off the financial shocks (i.e. set $\sigma_\chi = 0$) and make the financial crises exogenous events (e.g. $b_1 = 0$). The environment then amounts to a standard New Keynesian model that includes a debt-equity tradeoff in capital structure and exogenous crises. The main findings are summarized in Table 3. In this environment, a policy that responds enough to either the output or credit gap can essentially perfectly stabilize inflation. After a demand shock, monetary policy offsets the shock to fully stabilize output and inflation. On the other hand, when a productivity shock occurs, the policy keeps inflation on target and lets output respond fully to the shock. This result is standard in New Keynesian models - the divine coincidence property (Blanchard and Gali (2007)) applies and so there is no trade-off between output and inflation volatility, and this optimal policy can be (approximately) implemented by either simple rule provided they are sufficiently aggressive.[19]

[19] Note that there is no intrinsic reason as to why one simple rule should perform better than the other in terms of welfare in this case. Also note that output and inflation volatility as well as mean and standard deviation of financial crisis probability are quite close.

Table 3: No Financial Shocks, Exogenous Financial Crises

	Output gap only	Credit gap only	Both gaps
Welfare	-161.023	-160.989	-160.989
Consumption equivalent (%)	0	0.021	0.021
Coefficient ϕ_y	100	–	0
Coefficient ϕ_b	–	95.78	95.78
100×SD(Π)	0.01	0.01	0.01
100×SD(Y)	1.71	1.67	1.66
100×SD(Y^f)	1.72	1.72	1.72
400×E(P)	2	2	2
400×SD(P)	0	0	0

Table 4: No Financial Shocks, Endogenous Financial Crises

	Output gap only	Credit gap only	Both gaps
Welfare	-160.985	-160.988	-160.985
Consumption equivalent (%)	0	-0.002	0
Coefficient ϕ_y	100	–	100
Coefficient ϕ_b	–	96.1	0
100×SD(Π)	0.01	0.01	0.01
100×SD(Y)	1.71	1.66	1.71
100×SD(Y^f)	1.72	1.72	1.72
400×E(P)	1.99	2.00	1.99
400×SD(P)	0.01	0.01	0.01

To further build intuition, we now relax the assumption of exogenous financial crises. The main results are reported in Table 4. The findings are nearly identical to the prior case. This is because there is no reason to offset the credit fluctuations driven by the productivity shock, which are efficient and do not contribute to financial risk. As for demand shocks, the credit fluctuations they create are actually eliminated once output volatility is eliminated. Hence, there is no trade-off between credit stabilization and output/inflation stabilization, and the same policies as in the previous case can implement an efficient allocation without creating any inefficient credit movements.

As a third point of comparison, we now reintroduce financial shocks, though we now suppose that crises are exogenous. In this version of the model both rules continue to perform about as well. The main findings are summarized in Table 5. The novelty compared to the previous cases is that the response to the output gap is diminished. This occurs because the response that would be required to offset demand and productivity shocks is not consistent with the response needed to respond to the financial shock. But the credit gap rule suffers from the same issue and has to

Table 5: Financial shocks, Exogenous Financial Crises

	Output gap only	Credit gap only	Both gaps
Welfare	-160.965	-161.082	-160.965
Consumption equivalent (%)	0	-0.073	0
Coefficient ϕ_y	1.82	–	1.82
Coefficient ϕ_b	–	0.27	0
100×SD(Π)	0.25	0.36	0.25
100×SD(Y)	1.81	2.21	1.81
100×SD(Y^f)	1.72	1.72	1.72
400×E(P)	2	2	2
400×SD(P)	0	0	0

trade off the response against the different shocks. Overall, there is little difference between these two rules. Perhaps more surprisingly, combining the credit and output gap does not allow a better outcome either as the optimization program chooses the boundary value of zero for the credit gap.

5.4 When is leaning against the wind optimal?

We next ask how certain parameters affect the desirability of leaning against the wind. For simplicity, in these comparisons we focus here on rules that depend either only on the (correctly measured) output gap or credit gap.

5.4.1 The cost of financial crises

Our benchmark model assumes that a financial crisis leads to a permanent decline in the level of GDP of 10%. Table 6 illustrates how our results change as we vary this cost from 6% to 14% with all other parameters kept constant. Several points emerge. First, the welfare benefit to targeting credit gap rather than output gap increases monotonically with the size of the financial crisis. Indeed, for a crisis that triggers a 6% output loss, the best output gap rule outperforms the best credit gap rule. Second, the bigger is the crisis, the stronger is the response to credit, with the coefficient rising from 0.75 to 4.06. Third, this higher responsiveness to credit implies noticeably higher volatility of output and, slightly higher inflation volatility, as the central bank puts more weight on financial stability and drives down the volatility of probability of financial crisis (as well as marginally reduces the mean crisis risk).[20]

[20] The best output gap rule is nearly constant as we change the disaster size and leads to 0.3% volatility of inflation, 1.7% volatility of output, and a probability of financial crisis of 2.1% with volatility 0.63%.

Table 6: Effect of Financial Crisis Size on Optimal Credit Policy

Financial crisis size (b_c)	6%	8%	10% (benchmark)	12%	14%
Optimal coeff. on credit ϕ_b	0.75	1.17	1.88	2.88	4.06
Welfare difference	-0.010	0.038	0.105	0.191	0.295
Consumption equivalent (%)	-0.006	0.023	0.065	0.118	0.182
SD(Y) under LAW	2.36	2.59	2.85	3.05	3.20
SD(π) under LAW	0.45	0.48	0.51	0.53	0.54
Mean(P) under LAW	2.05	2.03	2.02	2.01	2.00
SD(P) under LAW	0.40	0.31	0.22	0.16	0.13

Table 7: Effect of Sensitivity of Crisis to Excess Credit on Optimal Policies

Sensitivity of crisis to excess credit (b_1)	2	3	5 (benchmark)	7	8
Optimal coeff. on credit ϕ_b	0.30	0.46	1.88	5.41	7.70
Welfare difference	-0.089	-0.059	0.105	0.491	0.751
Consumption equivalent (%)	-0.055	-0.036	0.065	0.303	0.463
SD(Y) under LAW	2.20	2.21	2.85	3.28	3.38
SD(π) under LAW	0.37	0.41	0.51	0.54	0.55
Mean(P) under LAW	2.01	2.02	2.02	2.0	2.00
SD(P) under LAW	0.25	0.31	0.22	0.14	0.12

5.4.2 The sensitivity of crises to excess credit

A key parameter for our results is how much excess credit matters for financial crises, represented by the coefficient b_1. A large value of b_1 means that excess credit has a strong effect on the risk of crisis and hence on welfare. This naturally gives rise to a stronger motive to lean against excess credit. Table 7 confirms this intuition. First, for low values of b_1, LAW is outperformed by the output gap rule. Second, the coefficient on credit optimally rises with b_1. This allows to offset to some extent the increase in the volatility of financial crisis probability that would otherwise occur mechanically. Third, this policy is chosen despite a clear cost in terms of higher output and inflation volatility.

5.4.3 The importance of financial shocks

Perhaps most basically, the magnitude of the (inefficient) financial shocks is critical for our results. We already illustrated that if there are no financial shocks, leaning against the wind brings essentially no benefits relative to standard policies. Table 8 provides more details on the importance of this consideration. Here too, we see that the welfare difference between the best credit gap policy

Table 8: Effect of Standard Deviation of Financial Shocks on Optimal Policy

Standard dev. of financial shocks (relative to benchmark)	33%	66%	100% (benchmark)	133%	166%
Optimal coeff. on credit ϕ_b	96.1	2.96	1.88	1.55	1.41
Welfare difference LAW-OG	0.004	0.037	0.105	0.205	0.336
Consumption equivalent (%)	0.002	0.022	0.065	0.126	0.207
SD(Y) under LAW	1.97	2.38	2.85	3.34	3.87
SD(π) under LAW	0.19	0.36	0.51	0.65	0.81
Mean(P) under LAW	2.00	2.00	2.02	2.04	2.08
SD(P) under LAW	0.7	0.13	0.22	0.31	0.40

and the best output gap policy is increasing in the variance of financial shocks. The effects on output and inflation volatility as well as the financial crisis probability are more subtle because they result both from (i) the higher variance of financial shocks and (ii) the change in policy rule in response to this higher variance.

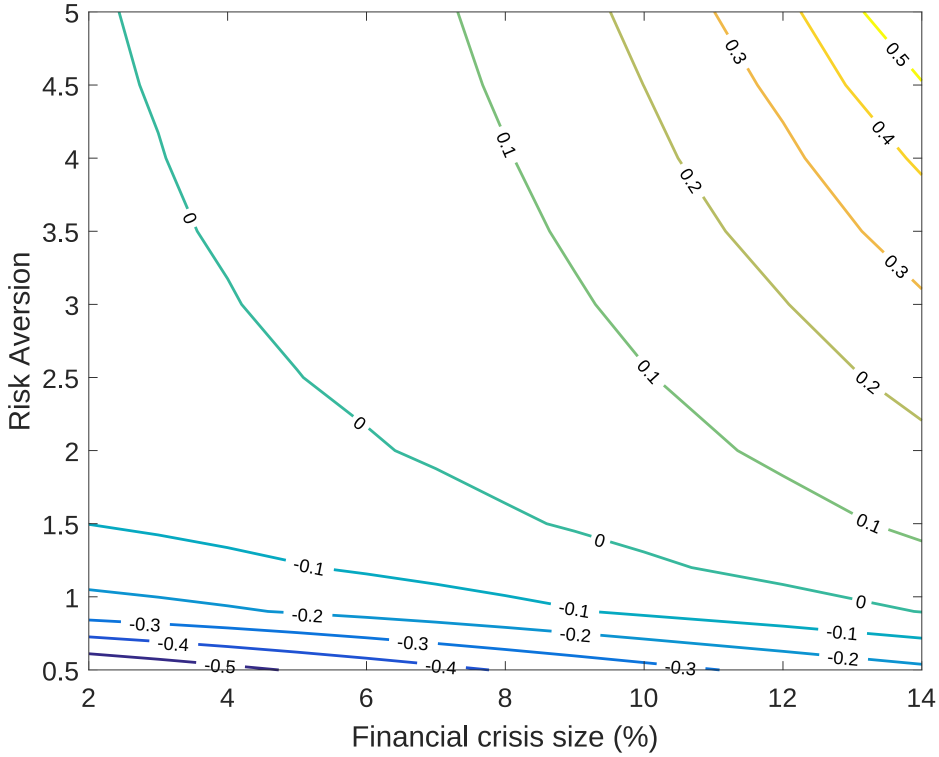
5.4.4 The role of risk aversion

We next explore how the willingness of households to bear macroeconomic risk affects our results. On one side, higher risk aversion makes agents more fearful of financial crises. On the other hand, higher risk aversion also makes agents less willing to tolerate the higher business cycle volatility implied by LAW. Moreover, with expected utility preferences, a higher risk aversion implies a lower elasticity of substitution, which affects the response of the economy to monetary policy (as well as the dynamics of the model more generally). Table 9 reveals that the first effect seems to dominate - the higher the risk aversion, the larger the benefits from leaning against the wind. With a risk aversion of 0.5, an output-gap rule outperforms a credit-gap rule, but the benefits of using the credit gap rule rise with risk aversion. The optimal policy largely stabilizes fluctuations in financial crisis risk.

Figure 8 summarizes many of the central findings of the paper. On the horizontal axis, we vary the size of the financial crisis. On the vertical axis we vary risk aversion. The lines that are drawn trace out isoquants in units of equivalent consumption between the best LAW policy (that responds only to the credit gap on top of inflation) and the best monetary policy rule that responds only to the output gap (on top of inflation).[21] The zero consumption equivalence curve traces out all

[21]Each policy is optimized with respect to the coefficient on the credit gap or output gap, as in the exercises above.

Figure 8: Welfare Gain from LAW: the Effects of Risk Aversion and the Size of Financial Crisis



the combination of the size of the crisis and the representative household's level of risk aversion where the two policies deliver equivalent welfare. Points to the right and above the zero curve show the regions where LAW delivers higher welfare and below and to the left show combinations where the output gap rule performs better. In the benchmark model, described in Table 2 with risk aversion of two and a crisis that brings a permanent ten percent output loss, LAW is better. The results from Table 6 described how welfare varied when we fixed risk aversion at two and varied the size of the crisis. This figure fills in the rest of the parameter space. Not surprisingly, as risk aversion rises, LAW's relative performance improves. For instance, if risk aversion is three, then LAW is the preferred policy even for a crisis that involves a one-time output loss of as little as five percent. Conversely, if risk aversion is much lower, say one, then even a crisis that drops output by 14 percent is not enough to justify a LAW policy.

5.5 Trading off financial stability vs. macroeconomic stability

Our model results demonstrate a significant trade-off between the traditional mandates of monetary policy - output and inflation stability - and financial stability - stabilizing, and if possible reducing

Table 9: Risk Aversion and Leaning Against the Wind

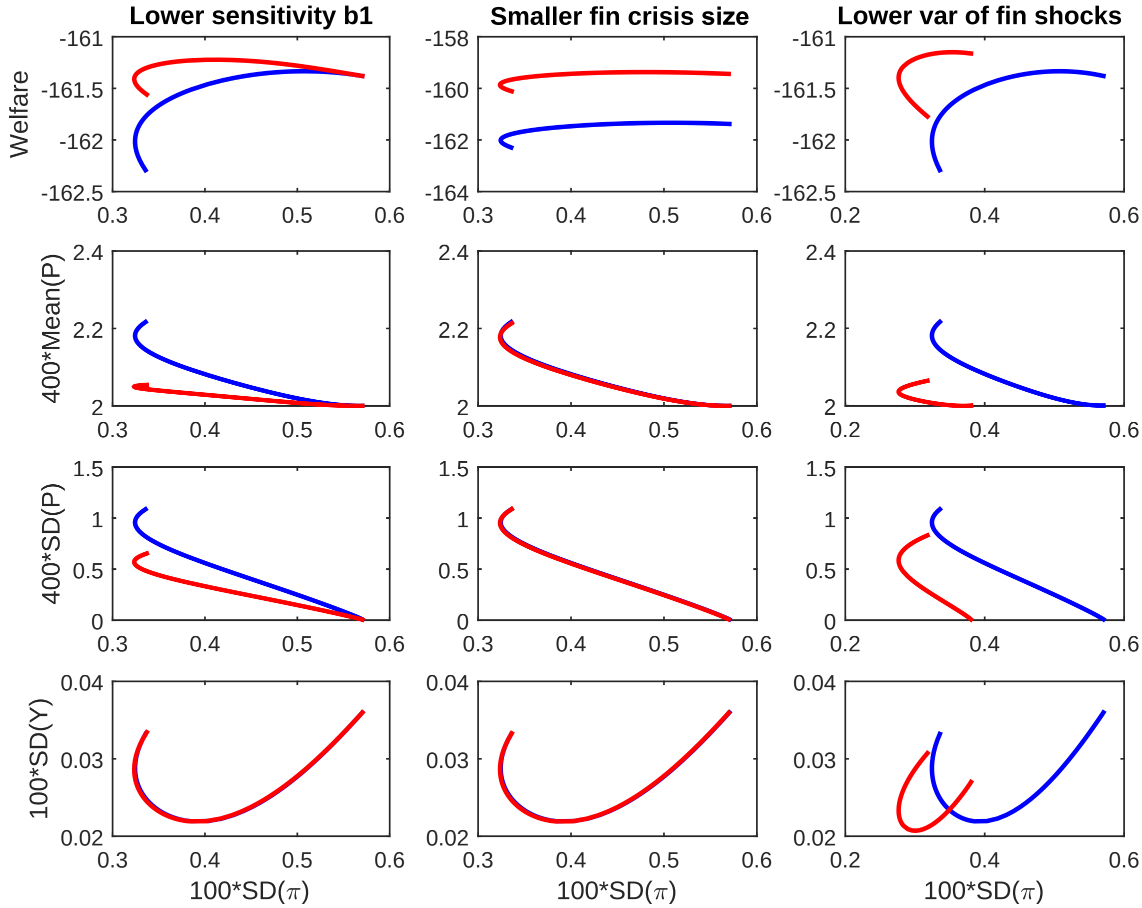
CRRA	0.5	1.5	2 (benchmark)	3	4
Optimal coeff. on credit ϕ_b	87.67	1.15	1.88	6.61	97.72
Welfare difference LAW-OG	-0.214	0.032	0.105	0.273	0.514
Consumption equivalent (%)	-0.33	0.02	0.06	0.12	0.17
SD(Y) under LAW	5.00	2.99	2.85	2.70	2.53
SD(π) under LAW	0.64	0.48	0.51	0.54	0.55
Mean(P) under LAW	2.00	2.02	2.02	2.00	2.00
SD(P) under LAW	0.01	0.31	0.22	0.09	0.01

the probability of financial crisis. To illustrate this trade-off, we consider a range of values for ϕ_b and trace out the frontiers that exist among welfare, volatility of output, volatility of inflation rate, mean and volatility of probability of financial crisis. The policy maker can choose a point on these frontiers - a particular rule - depending on the weights she puts on the relative volatilities.

In our case, the frontiers are somewhat more complex, because there are more than two objectives. Figure 9 is an attempt at illustrating these trade-offs. The blue line depicts the outcomes in our benchmark model as we vary the coefficient on the credit gap. (The zero weight on credit is the leftmost end point of each line.) The red line depicts the same frontier as we change a parameter: the left column depicts the case for a lower sensitivity of financial crisis to credit (b_1); the middle column shows the frontier in the case of a smaller output decline should a financial crisis occur; and the right column shows the effect of changing the variance of financial shocks.

The figure shows that policies that respond weakly to credit (left side of plots) generate lower inflation volatility but much higher volatility of the probability of financial crisis, and slightly higher mean crisis risk, generating a clear trade-off. The effect on output volatility is U-shaped as a small response on credit can eliminate some undesirable output fluctuations due to demand shocks, but much stronger ones generate excessive output fluctuations (as we saw earlier). When the sensitivity b_1 is lowered, the frontier flattens as the trade-off between financial stability and inflation volatility becomes much less acute. Similarly, when the losses from a financial crisis are smaller the trade-off becomes flatter. A lower variance of financial shocks similarly shifts the frontier to higher welfare, and a lower volatility of the financial crisis probability.

Figure 9: Tradingoff of Leaning Against the Wind



5.6 Mismeasurement

Finally, an important practical consideration is that neither the output gap nor the credit gap is actually observable. The importance of inefficient credit fluctuations depends on several factors. One is the size of the financial shocks, but the observed level of borrowing depends also on changes in fundamental financing needs that come from technology shocks. In our baseline, there are no other fundamental reasons why credit varies. In reality, deregulation, changes in property rights and many other factors could lead to a benign surge in credit and a central bank would need to be able to separate those swings from the inefficient ones.

To quantify this, we search again for the best policy rules in our baseline specification where the central bank is restricted to just observing actual output and credit - that is, it uses the deviation from the steady-state rather than the deviation from the efficient benchmark. The results are shown in Table 9 (and these should be compared to the findings in Table 2). The output gap rule

Table 10: Optimal Policy Rules with Mismeasured Gaps

	Output gap only	Credit gap only	Both gaps
Welfare	-161.646	-161.426	-161.426
Consumption equivalent (%)	0	0.124	0.124
Coefficient ϕ_y	100	0	0.02
Coefficient ϕ_b	0	1.59	1.61
100×SD(Π)	0.39	0.53	0.53
100×SD(Y)	0.00	2.41	2.40
100×SD(Y^f)	1.72	1.72	
400×E(P)	2.09	2.01	2.01
400×SD(P)	0.70	0.34	0.34

now eliminates all fluctuations in output, including the efficient fluctuations. This leads to higher inflation volatility and noticeably lower welfare. Relying on the mismeasured credit gap still leads to a similar tradeoff as in the baseline model. The central bank delivers less frequent and less volatile crises, in exchange for higher inflation and output volatility. The relative performance of the rule based on mismeasured credit is bigger in this scenario than in the one with both gaps are perfectly measured. In fact, the best rule when both gaps are considered puts almost no weight on the output gap (and the welfare is about the same as when only the credit gap is used).

The welfare level is actually slightly higher when the mismeasured credit gap is used instead of the perfectly measured output gap; this conclusion depends on all the foregoing factors that have been shown to determine the relative attractiveness of leaning against the wind. For instance, in parameter configurations where the gains from leaning against the wind are low to begin with, then tying the policy rate to mismeasured credit gap would not necessarily lead to higher welfare than a rule that can be set based on a perfectly measured output gap. In these cases, however, the mismeasured credit gap rule would still outperform the mismeasured output gap rule.

6 Conclusion

Conventional discussions about the links between monetary policy and financial stability typically start by saying that one can appeal to different tools for different jobs. Macro-prudential regulation can address stability concerns, while monetary policy can attend to managing inflation. We agree that this would be the ideal arrangement, however, in practice in many countries this is easier said than done. Macro-prudential policymaking is in its infancy and for some countries the tools barely exist. These practical concerns motivate our analysis.

On the question of whether central banks should alter monetary policy to contain financial stability risks IMF staff study (IMF (2015)) says "Based on our current knowledge, and in present circumstances, the answer is generally no." We believe this conclusion is too strong.

The model we have presented is highly stylized and the parameters are not estimated. Nonetheless, we believe it does capture the ingredients that many of the advocates of leaning against the wind believe support the case for doing so. In particular, the model presumes that financial crises are very costly, and are partly driven by credit conditions which monetary policy can affect. The fact that the model can easily uncover circumstances where leaning against the wind is welfare improving is, therefore, not surprising.

The model points to a number of factors that will determine the efficacy of leaning against the wind. Our main hesitation in endorsing the IMF staff study (IMF (2015)) conclusion is that many of these factors are difficult to measure and our reading of existing empirical work is that we still do not have much guidance about how to calibrate certain of these key elasticities. Perhaps subsequent work will confirm the IMF conclusion but for now we believe it is too early to say that the question is settled.

One powerful conclusion from the model is that the case for leaning against the wind likely rests on accepting somewhat higher volatility of inflation and output, in exchange for smoothing out the risk of crises. If central banks are going to embrace this policy, they will need to invest substantially in explaining this tradeoff to the public and to legislatures.

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Appendices

A System of Equations

The system has 31 variables:

$$\mathbf{Y}_t = \begin{bmatrix} \nu_t & \lambda_t^K & \mu_t & C_t & L_t & \Pi_t & l_t & \Omega_t & M_{t-1,t} & \Lambda_t \\ N_t & w_t & \Xi_t & R_t & Q_t & K_t & I_t & Y_t & R_t^K & S_t^K \\ q_t & B_t & \varepsilon_t^* & \chi_t & K_t^w & r_t^K & Z_t & h_t & z_t^* & H_t \\ \Gamma_t & & & & & & & & & \end{bmatrix}$$

The corresponding system of equations are:

$$\nu_t = 1 - \mu_t \tag{A.1}$$

$$0 = 1 - \varphi \Pi_t \left(\Pi_t - 1\right) - \eta \nu_t + \varphi \mathbb{E}_t \left[M_{t,t+1} \Pi_{t+1} \left(\Pi_{t+1} - 1\right) \frac{Y_{t+1}}{Y_t} \right] \tag{A.2}$$

$$M_{t-1,t} = \beta \frac{\Lambda_t}{\Lambda_{t-1}} \tag{A.3}$$

$$\Lambda_t = C_t^{-\tau} \tag{A.4}$$

$$\Lambda_t w_t = N_t^{\upsilon} \tag{A.5}$$

$$1 = E_t \left(M_{t,t+1} \Xi_t \frac{R_t}{\Pi_{t+1}} \right) \tag{A.6}$$

$$K_t^w = (1-\delta) K_t + I_t \tag{A.7}$$

$$K_{t+1} = e^{X_{t+1} b} K_t^w \tag{A.8}$$

$$C_t + I_t = Y_t - \frac{\psi}{2} \left(\Pi_t - \overline{\Pi}\right)^2 Y_t \tag{A.9}$$

$$w_t = (1-\alpha) \mu_t \frac{Y_t}{N_t} \tag{A.10}$$

$$r_t^K = \alpha \mu_t \frac{Y_t}{K_t} \tag{A.11}$$

$$Y_t = Z_t K_t^{\alpha} N_t^{1-\alpha} \tag{A.12}$$

$$R_t^K = \frac{(1-\delta) Q_t + r_t^K}{Q_{t-1}} \tag{A.13}$$

$$S_t^K = Q_t K_t^w - \chi_t q_t B_{t+1} \tag{A.14}$$

$$\varepsilon_t^* = \frac{B_t}{R_t^K Q_{t-1} K_t} \tag{A.15}$$

$$z_t^* = \sigma^{-1}(\log \varepsilon_t^* + 0.5\sigma^2) \tag{A.16}$$

$$H_t = \Phi(z_t^*) \tag{A.17}$$

$$h_t = \phi(z_t^*) \tag{A.18}$$

$$\Omega_t = \Phi(z_t^* - \sigma_t) \tag{A.19}$$

$$q_t = E_t\left(M_{t+1}\left(1 - H(\varepsilon_{t+1}^*) + \frac{\zeta}{B_t} R_{t+1}^K Q_t K_{t+1} \Omega(\varepsilon_{t+1}^*)\right)\right) \tag{A.20}$$

$$\Gamma_t = E_t\left(M_{t+1} R_{t+1}^K \lambda_{t+1}^K\right) \tag{A.21}$$

$$\lambda_t^K = 1 + (\chi_t - 1)\,\varepsilon_t^*\,(1 - H(\varepsilon_t^*)) - (1 - \zeta\chi_t)\,\Omega(\varepsilon_t^*) \tag{A.22}$$

$$\Gamma_t = 1 + \gamma\,(1 - \chi_t L(l_t))\,(1 - \chi_t L(l_t)) \tag{A.23}$$

$$L(l_t) = E_t M_{t+1}[\Omega(\varepsilon_{t+1}^*)\zeta R_{t+1}^K + (1 - H(\varepsilon_{t+1}^*))\varepsilon_{t+1}^* R_{t+1}^K] \tag{A.24}$$

$$\begin{aligned} &E_t\left\{M_{t+1}\left(1 - H\left(\varepsilon_{t+1}^*\right)\right)\left[\frac{\chi_t - 1}{\chi_t} + \gamma\left(\frac{S_t^K}{Q_t K_{t+1}}\right) + \gamma'\left(\frac{S_t^K}{Q_t K_{t+1}}\right)\right]\right\} \\ &= (1-\zeta)\,E_t\left\{M_{t+1}\varepsilon_{t+1}^* h\left(\varepsilon_{t+1}^*\right)\left[1 + \gamma\left(\frac{S_t^K}{Q_t K_{t+1}}\right) + \gamma'\left(\frac{S_t^K}{Q_t K_{t+1}}\right)\right]\right\} \end{aligned} \tag{A.25}$$

$$Q_t = 1 + \kappa\left(\frac{I_t}{I_{t-1}} - 1\right) - \mathbb{E}_t\left\{M_{t,t+1}\frac{\kappa}{2}\left[\left(\frac{I_{t+1}}{I_t}\right)^2 - 1\right]\right\} \tag{A.26}$$

$$l_t = \frac{B_t}{Q_t K_t^w} \tag{A.27}$$

$$\log R_t = (1 - \rho_R)\log R_{t-1} + \rho_R[\log R + \rho_\Pi \log(\Pi_t/\bar{\Pi}) + \rho_Y \log(Y_t/Y_t^F)] \tag{A.28}$$

$$\log \chi_t = (1 - \rho_\chi)\log \chi + \rho_\chi \log \chi_{t-1} + \sigma_\chi \epsilon_{\chi,t} \tag{A.29}$$

$$Z_{t+1} = e^{X_{t+1}b} e^{\xi_{t+1}} Z_t \tag{A.30}$$

$$\log \Xi_t = (1 - \rho_\Xi)\log \Xi + \rho_\Xi \log \Xi_{t-1} + \sigma_\Xi \epsilon_{\Xi,t} \tag{A.31}$$

B Detrending

Define $\tilde{Y}_t = Y_t/Z_t$, $\tilde{K}_t = K_t/Z_t$,etc. - we can then rewrite the system of equations:

$$\begin{aligned} M_{t,t+1} &= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{\upsilon(1-\sigma)} \\ &= \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t}\right)^{-\sigma} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{\upsilon(1-\sigma)} \end{aligned}$$

and we define

$$\tilde{M}_{t,t+1} = \beta \left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)^{-\sigma} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{\upsilon(1-\sigma)}$$

$$M^{\$}_{t,t+1} = \frac{M_{t,t+1}}{\Pi_{t+1}}$$

$$R_t = R^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}^*_t}\right)^{\phi_y},$$

$$\tilde{w}_t = (1-\alpha) MC_t \frac{\tilde{Y}_t}{L_t}$$

$$r^K_t = \alpha MC_t \frac{\tilde{Y}_t}{\tilde{K}_t}$$

$$\tilde{w}_t = \frac{1}{Z_t}\frac{U_2(C_t, N_t)}{U_1(C_t, N_t)} = \frac{\upsilon \tilde{C}_t}{1-N_t}$$

$$\tilde{C}_t + \tilde{I}_t = \tilde{Y}_t - \frac{\psi}{2}\left(\Pi_t - \overline{\Pi}\right)^2 \tilde{Y}_t$$

$$\tilde{Y}_t = \tilde{K}^\alpha_t N^{1-\alpha}_t$$

$$\tilde{R}^K_t = 1 - \delta + r^K_t$$

$$\tilde{K}^w_t = (1-\delta)\tilde{K}_t + \tilde{I}_t$$

$$\tilde{K}^w_t = \chi q_t \tilde{B}_t + \tilde{S}_t,$$

The law of motion for the crisis probability is given by

$$\log p_{t+1} = \rho_p \log p_t + (1-\rho_p) \log \overline{p} + u_{p,t+1}$$

There are five intertemporal equations:

$$\frac{\chi - 1}{\chi} E_t \left(M_{t+1}\left(1 - H\left(\varepsilon^*_{t+1}\right)\right)\right) = (1-\theta) E_t \left(M_{t+1}\varepsilon^*_{t+1} h\left(\varepsilon^*_{t+1}\right)\right).$$

$$E_t \left(M_{t+1} R^K_{t+1} \lambda_{t+1}\right) = 1$$

where

$$\lambda_{t+1} = 1 + (\chi - 1)\,\varepsilon^*_{t+1}\left(1 - H\left(\varepsilon^*_{t+1}\right)\right) - (1 - \theta\chi)\,\Omega\left(\varepsilon^*_{t+1}\right)$$

$$E_t\left(M^{\$}_{t,t+1} R_t\right) = 1.$$

$$0 = \left(1 - \varepsilon + \varepsilon MC_t - \psi(\Pi_t - \overline{\Pi})\Pi_t\right) Y_t + \psi E_t\left(M_{t+1}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1} Y_{t+1}\right)$$

$$q_t = E_t\left(M_{t+1}\left(1 - H(\varepsilon^*_{t+1}) + \frac{\theta}{B_t} R^K_{t+1} K^w_t \Omega(\varepsilon^*_{t+1})\right)\right)$$

We can now rewrite them in detrended terms and eliminate the crisis probability from the equations. Importantly, there are effectively two future possible ε^*_{t+1}, one that is relevant if there is no crisis, and one if there is a crisis:

$$\varepsilon^{*nd}_{t+1} = \frac{\tilde{B}_t}{R^K_{t+1}\tilde{K}^w_{t+1}}$$

$$\varepsilon^{*d}_{t+1} = \varepsilon^{*nd}_{t+1} \times Z_d$$

with $Z_d = e^{-b_c}$. (higher ε so higher defaults)

as a result, we obtain:

(i) $E_t\left(M^{\$}_{t,t+1} R_t\right) = 1$ - just as in the NK model without financial frictions above,

$$\begin{aligned}
1 &= E_t\left(M_{t,t+1}\frac{R_t}{\Pi_{t+1}}\right) \\
&= (1 - p_t) E_t M^{nd}_{t+1}\frac{R_t}{\Pi^{nd}_{t+1}} + p_t E_t M^d_{t+1}\frac{R_t}{\Pi^d_{t+1}} \\
&= \left((1 - p_t) + p_t e^{-\sigma b_c}\right) E_t \frac{R_t}{\Pi_{t+1}}\tilde{M}_{t+1}
\end{aligned}$$

(ii) Phillips curve - also unchanged

$$0 = \left(1 - \varepsilon + \varepsilon MC_t - \psi(\Pi_t - \overline{\Pi})\Pi_t\right) Y_t + \psi E_t\left(M_{t+1}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1} Y_{t+1}\right)$$

leading to:

$$0 = \left(1 - \varepsilon + \varepsilon MC_t - \psi(\Pi_t - \overline{\Pi})\Pi_t\right)\tilde{Y}_t + \psi\left((1 - p_t) + p_t e^{(1-\sigma)b}\right) E_t\left(\tilde{M}_{t+1}(\Pi_{t+1} - \overline{\Pi})\Pi_{t+1}\tilde{Y}_{t+1} e^{u_{z\,t+1}}\right)$$

(iii) debt pricing

$$\begin{aligned}
q_t &= E_t\left(M_{t+1}\left(1 - H(\varepsilon^*_{t+1}) + \frac{\theta}{B_t} R^K_{t+1} K^w_t \Omega(\varepsilon^*_{t+1})\right)\right) \\
&= (1 - p_t) E_t\left(\tilde{M}_{t+1}\left(1 - H(\varepsilon^*_{t+1}) + \theta R^K_{t+1}\frac{\tilde{K}^w_t}{\tilde{B}_t}\Omega(\varepsilon^*_{t+1})\right)\right) \\
&\quad + p_t e^{-\sigma b} E_t\left(\tilde{M}_{t+1}\left(1 - H(\varepsilon^*_{t+1} Z_d) + \theta e^b R^K_{t+1}\frac{\tilde{K}^w_t}{\tilde{B}_t}\Omega(\varepsilon^*_{t+1} Z_d)\right)\right)
\end{aligned}$$

[note that in this case we can't simply factor everything out - hence this shock is not equivalent to a shock to β]

(iv) Euler capital

(iv)-1 Without equity issuance cost

$$E_t\left(M_{t+1}R^K_{t+1}\lambda_{t+1}\right)=1$$

where $\lambda_{t+1}=1+(\chi-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)-(1-\theta\chi)\,\Omega\left(\varepsilon^*_{t+1}\right)$

leads to

$$\begin{aligned}1=&(1-p_t)E_t\left(\tilde{M}_{t+1}\tilde{R}^K_{t+1}\left(1+(\chi-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)-(1-\theta\chi)\,\Omega\left(\varepsilon^*_{t+1}\right)\right)\right)+\\&p_te^{(1-\sigma)b_c}E_t\left(\tilde{M}_{t+1}\tilde{R}^K_{t+1}\left(1+(\chi-1)\,\varepsilon^*_{t+1}Z_d\left(1-H\left(\varepsilon^*_{t+1}Z_d\right)\right)-(1-\theta\chi)\,\Omega\left(\varepsilon^*_{t+1}Z_d\right)\right)\right)\end{aligned}$$

(iv)-2 With equity issuance cost

$$1=\Gamma\left(\frac{B_t}{K^w_t}\right)^{-1}E_t\left(M_{t+1}R^K_{t+1}\lambda_{t+1}\right)$$

Since the equity issuance cost already scales, the detrended Euler equation is simply

$$\begin{aligned}\Gamma\left(\frac{\tilde{B}_t}{\tilde{K}^w_t}\right)=&(1-p_t)E_t\left(\tilde{M}_{t+1}\tilde{R}^K_{t+1}\left(1+(\chi-1)\,\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)-(1-\theta\chi)\,\Omega\left(\varepsilon^*_{t+1}\right)\right)\right)+\\&p_te^{(1-\sigma)b}E_t\left(\tilde{M}_{t+1}\tilde{R}^K_{t+1}\left(1+(\chi-1)\,\varepsilon^*_{t+1}Z_d\left(1-H\left(\varepsilon^*_{t+1}Z_d\right)\right)-(1-\theta\chi)\,\Omega\left(\varepsilon^*_{t+1}Z_d\right)\right)\right)\end{aligned}$$

where

$$\Gamma\left(\frac{\tilde{B}_t}{\tilde{K}^w_t}\right)=1+\gamma\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}^w_t}\right)\right)\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}^w_t}\right)\right)$$

and

$$\begin{aligned}L\left(\frac{\tilde{B}_t}{\tilde{K}^w_t}\right)=&(1-p_t)E_t\left(\tilde{M}_{t+1}\left((1-H(\varepsilon^*_{t+1}))\frac{\tilde{B}_t}{\tilde{K}^w_t}+\theta R^K_{t+1}\Omega(\varepsilon^*_{t+1})\right)\right)\\&+p_te^{-\sigma b}E_t\left(\tilde{M}_{t+1}\left((1-H(\varepsilon^*_{t+1}Z_d))\frac{\tilde{B}_t}{\tilde{K}^w_t}+\theta e^bR^K_{t+1}\Omega(\varepsilon^*_{t+1}Z_d)\right)\right).\end{aligned}$$

Regarding the functional form of $\gamma(\cdot)$, we assume a quadratic form:

$$\gamma(x)=\gamma_1x+\frac{\gamma_2}{2}x^2.$$

(v) Optimal leverage:

(v)-1 Without equity issuance cost

The optimal levelage satisfies

$$\frac{\chi-1}{\chi}E_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)=(1-\theta)\,E_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right),$$

which leads to

$$\begin{aligned}&\frac{\chi-1}{\chi}E_t\left(\tilde{M}_{t+1}\left((1-p_t)\left(1-H\left(\varepsilon^*_{t+1}\right)\right)+p_te^{-\sigma b}\left(1-H\left(Z_d\varepsilon^*_{t+1}\right)\right)\right)\right)\\&=(1-\theta)\,E_t\left(\tilde{M}_{t+1}\left((1-p_{dt})\,\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)+p_te^{-\sigma b}\varepsilon^*_{t+1}Z_dh\left(\varepsilon^*_{t+1}Z_d\right)\right)\right).\end{aligned}$$

(v)-1 With equity issuance cost

The optimal leverage satisfies

$$
\begin{aligned}
&E_t\Bigg\{\tilde{M}_{t+1}\left((1-p_t)\left(1-H\left(\varepsilon_{t+1}^*\right)\right)+p_t e^{-\sigma b}\left(1-H\left(Z_d\varepsilon_{t+1}^*\right)\right)\right)\\
&\times\left[\frac{\chi-1}{\chi}+\gamma\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right)\right)+\gamma'\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right)\right)\right]\Bigg\}\\
=&\,(1-\theta)\,E_t\Bigg\{\tilde{M}_{t+1}\left((1-p_t)\,\varepsilon_{t+1}^* h\left(\varepsilon_{t+1}^*\right)+p_t e^{-\sigma b}\varepsilon_{t+1}^* Z_d h\left(\varepsilon_{t+1}^* Z_d\right)\right)\\
&\times\left[1+\gamma\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right)\right)+\gamma'\left(1-\chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right)\right)\right]\Bigg\},
\end{aligned}
$$

which leads to

C Non-Stochastic Steady State

We guess p and r^K. This guess implies

$$R^K = (1-p+p\exp(b))(r^K+1-\delta)$$

and

$$\tilde{R}^K = r^K + 1 - \delta$$

Also the guess of p implies

$$\frac{\tilde{B}}{\tilde{K}^w} = p^{1/\zeta}.$$

We then construct the default threshold as

$$\varepsilon^* = \frac{p^{1/\zeta}}{\tilde{R}^K}.$$

We define the standardized default threshold as

$$z^* = \sigma_\varepsilon^{-1}(\log\varepsilon^* + 0.5\sigma_\varepsilon^2).$$

We can then write

$$H\left(\varepsilon^*\right) = \Phi(z^*),$$

$$h\left(\varepsilon^*\right) = \phi(z^*)$$

and

$$\Omega(\varepsilon^*) = \Phi(z^* - \sigma_\varepsilon).$$

Similarly we define

$$\varepsilon Z_d^* = \varepsilon^* \exp(-b)$$

and

$$zZ_d^* = \sigma_\varepsilon^{-1}(\log\varepsilon Z_d^* + 0.5\sigma_\varepsilon^2).$$

We then have

$$H\left(\varepsilon Z_d^*\right) = \Phi(zZ_d^*),$$

$$h\left(\varepsilon Z_d^*\right) = \phi(z Z_d^*)$$

and

$$\Omega(\varepsilon Z_d^*) = \Phi(z Z_d^* - \sigma_\varepsilon).$$

We then use the bond pricing equation to determine the price of bond:

$$q = (1-p)\beta\left(1 - H(\varepsilon^*) + \theta\frac{\Omega(\varepsilon^*)}{\varepsilon^*}\right) + pe^{-\sigma b}\beta\left(1 - H(\varepsilon Z_d^*) + \theta\frac{\Omega(\varepsilon Z_d^*)}{\varepsilon Z_d^*}\right).$$

Using the FOC of the retailers' cost minimization problem with respect capital, we derive the output capital ratio as

$$\frac{\tilde{Y}}{\tilde{K}} = \frac{r^K}{\alpha MC}$$

where the steady state marginal cost is pinned down by the Phillips curve as

$$MC = \frac{\varepsilon - 1}{\varepsilon}.$$

The two FOCs of the retailers' cost minimization problem implies

$$\frac{\tilde{w}}{r^K} = \frac{1-\alpha}{\alpha}\frac{\tilde{K}}{N}$$

Using the production function, we have

$$\begin{aligned}\frac{\tilde{Y}}{N} &= \left(\frac{\tilde{K}}{N}\right)^\alpha \\ &= \left(\frac{\tilde{w}}{r^K}\frac{\alpha}{1-\alpha}\right)^\alpha.\end{aligned}$$

Substituting this in the FOC for retailers with respect to labor yields

$$\tilde{w} = (1-\alpha)MC\left(\frac{\tilde{w}}{r^K}\frac{\alpha}{1-\alpha}\right)^\alpha.$$

Solving this equation for the real wage leads to

$$\tilde{w} = [(\alpha/r^K)^\alpha(1-\alpha)^{1-\alpha}MC]^{\frac{1}{1-\alpha}}.$$

Using the FOC of the household with respect to labor, we derive

$$\tilde{C} = \frac{\tilde{w}}{\upsilon}(1-N).$$

The resource constraint implies

$$\frac{\tilde{Y}}{N} = \frac{\tilde{C}}{N} + \delta\frac{\tilde{K}}{N}.$$

This implies

$$\frac{\tilde{C}}{N} = \left(\frac{\tilde{w}}{r^K}\frac{\alpha}{1-\alpha}\right)^\alpha - \delta\tilde{w}\frac{\alpha}{1-\alpha} = \frac{\tilde{w}}{\upsilon}\left(\frac{1}{N} - 1\right).$$

where the last equality is due to the FOC of the representativ household with respect to labor. Solving this condition for N yields

$$N = \left\{ 1 + \frac{\upsilon}{\tilde{w}} \left[\left(\frac{\tilde{w}}{r^K} \frac{\alpha}{1-\alpha} \right)^{\alpha} - \delta \tilde{w} \frac{\alpha}{1-\alpha} \right] \right\}^{-1}.$$

We define two zero functions as

$$\begin{aligned} h^p(p, r^K) \equiv & \frac{\chi - 1}{\chi} \left\{ \beta \left[(1-p)\left(1 - H\left(\varepsilon^*\right)\right) + pe^{-\sigma b}\left(1 - H\left(\varepsilon Z_d^*\right)\right) \right] \right\} \\ & - (1-\theta) \left\{ \beta \left[(1-p)\,\varepsilon^* h\left(\varepsilon^*\right) + pe^{-\sigma b} \varepsilon Z_d^* h\left(\varepsilon Z_d^*\right) \right] \right\} \end{aligned}$$

and

$$h^r(p, r^K) \equiv 1 - \beta \lambda \tilde{R}^K$$

where

$$\begin{aligned} \lambda = & (1-p)\left[1 + (\chi - 1)\,\varepsilon^*\left(1 - H\left(\varepsilon^*\right)\right) - (1-\theta\chi)\,\Omega\left(\varepsilon^*\right)\right] \\ & + pe^{(1-\sigma)b}\left[1 + (\chi-1)\,\varepsilon Z_d^*\left(1 - H\left(\varepsilon Z_d^*\right)\right) - (1-\theta\chi)\,\Omega\left(\varepsilon Z_d^*\right)\right]. \end{aligned}$$

We find a pair (p, r^K) that satisfies $0 = h^p(p, r^K) = h^r(p, r^K)$.

With the presence of equity issuance cost, these functions must be modified into

$$h^r(p, r^K) \equiv \Gamma\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right) - \beta \lambda \tilde{R}^K$$

and

$$\begin{aligned} h^p(p, r^K) = & \beta \left((1-p)\left(1 - H\left(\varepsilon^*\right)\right) + pe^{-\sigma b}\left(1 - H\left(Z_d \varepsilon^*\right)\right) \right) \\ & \times \left[\frac{\chi - 1}{\chi} + \gamma \left(1 - \chi L\left(\frac{\tilde{B}}{\tilde{K}^w}\right) \right) + \gamma' \left(1 - \chi L\left(\frac{\tilde{B}}{\tilde{K}^w}\right) \right) \right] \\ & - (1-\theta)\,\beta \left((1-p)\,\varepsilon^* h\left(\varepsilon^*\right) + pe^{-\sigma b} \varepsilon^* Z_d h\left(\varepsilon^* Z_d\right) \right) \\ & \times \left[1 + \gamma \left(1 - \chi L\left(\frac{\tilde{B}}{\tilde{K}^w}\right) \right) + \gamma' \left(1 - \chi L\left(\frac{\tilde{B}}{\tilde{K}^w}\right) \right) \right], \end{aligned}$$

where

$$\Gamma\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right) = 1 + \gamma \left(1 - \chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right) \right) \left(1 - \chi L\left(\frac{\tilde{B}_t}{\tilde{K}_t^w}\right) \right)$$

and

$$\begin{aligned} L\left(\frac{\tilde{B}}{\tilde{K}^w}\right) = & (1-p)\beta \left((1 - H(\varepsilon^*))\frac{\tilde{B}}{\tilde{K}^w} + \theta R^K \Omega(\varepsilon^*) \right) \\ & + pe^{-\sigma b}\beta \left((1 - H(\varepsilon^* Z_d))\frac{\tilde{B}}{\tilde{K}^w} + \theta e^b R^K \Omega(\varepsilon^* Z_d) \right). \end{aligned}$$