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Money in a Theory of Banking

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Abstract

We introduce money in a model of banking in order to examine how open market operations can affect aggregate bank lending and output. We see two important ways changes in money supply affect the banking system. First, by changing prices and interest rates, it affects the wealth of demanders of *real* liquidity, and thus the equilibrium production of liquid assets, aggregate output, and the health of the banking system. Second, it changes the value of money as a means of payment, and thus the attractiveness of moving from deposits to currency (the demand for *financial* liquidity). Again, this impacts the health of the banking system and thus bank lending and aggregate output. We use this understanding to see how changes in money supply can create, alleviate, or be ineffective in banking crises.

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Why do central bank open market operations affect aggregate real economic activity? As puzzles go, this is one of the most important and enduring ones in economics. Not only is the answer important in understanding when output in developed economies can be influenced by manipulating monetary aggregates, it is also important in understanding the value and limits of monetary policy in dealing with financial crises in developing economies. In this paper, we attempt another crack at the answer, based on the microeconomic role of banks.

There are three main ways the literature has sought to explain the monetary transmission mechanism (see Bernanke and Gertler (1995) or Kashyap and Stein (1994, 1997) for excellent surveys). The traditional *money* view focuses on only two financial assets, bonds and money. When the central bank sells bonds for reserves, the banking sector's ability to issue demand deposits is diminished by the traditional multiplier effect. As a result, banks will hold fewer bonds so the public has to hold more bonds and less money (both in nominal units). If prices are sticky, the public will have to hold lower real balances of money. In equilibrium, the cost of holding money will have to be higher – implying higher real interest rates, a higher cost of capital and lower aggregate economic activity.

The shortcomings of the money view are that it is hard to identify a quantitatively significant effect of the cost of capital on aspects of aggregate spending that the theory suggests it should influence. Furthermore, presumably monetary policy should primarily affect short rates, yet it seems to have large effects on long-term investment. These concerns have led some to suggest that additional effects might have to be superimposed on the money view to account for the empirical facts.

The set of theories that have emerged to fill this gap could broadly be termed "credit channel" theories (see Bernanke and Gertler (1995)). One version of this emphasizes the *balance sheet* channel, whereby tight monetary policy weakens the creditworthiness of small firms (for example, because they pay higher rates on their short term borrowing, which reduces their coverage ratios and their ability to service debt) and reduces their ability to raise funds from any provider.

Another version focuses on the *bank lending* channel (see, for example, Bernanke and Blinder (1988, 1992) or Kashyap and Stein (1997)). According to this view, monetary policy specifically affects bank loan supply, which in turn has an independent and significant effect on aggregate economic activity (this does not preclude any effects of movements in the interest rate or changes in the quality of corporate balance sheets). Three assumptions are key to the centrality of banks in the transmission process: (i) binding reserve requirements tie the issuance of bank demand deposits to the availability of reserves (ii) banks cannot substitute between demand deposits and other forms of finance easily so they have to cut down on lending when the central bank curtails reserves (iii) client firms cannot substitute between bank loans and other forms of finance, so they have to cut down on economic activity.

The concern with the bank lending channel is that as reserve requirements have been eliminated for almost all bank liabilities except demand deposits, the argument that banks will find it difficult or expensive to raise alternative forms of financing to demand deposits becomes less persuasive (see, for example, the critique by Romer and Romer (1990)). But there does seem to be strong evidence that monetary policy has effects on bank loan supply (Kashyap, Stein, and Wilcox (1995), Ludvigson (1996)), has greater effect on banks at times when their balance sheets look worse (Gibson (1996) and has the greatest effect on the policies of the smallest and least credit worthy banks (Kashyap and Stein (2000)). Can we reconcile the reduced importance of reserve requirements with the seemingly strong evidence that the monetary propagation mechanism continues to work through banks?

In this paper, we propose a model that, in the spirit of the bank-lending channel view, places banks at the center of the transmission process. Even though the model suggests that under fairly general circumstances demand deposits are the cheapest way banks can finance themselves (so that if reserves had to be maintained against these deposits, the central bank could alter banks' cost of funds -- and thus bank lending -- by changing the quantity of reserves) we aim to see if monetary policy can have effects even if there are no reserve requirements.[1]

We study the interactions between real financial intermediation services and nominal contracting when currency and deposits serve as separate means of payment. Traditional money and banking focused only on banks through their effects on the supply of a single "money." Gurley and Shaw [1960] and Tobin [1963] suggested that changes in the scale of non-bank financial intermediaries (those without monetary liabilities) would also influence real activity. Microeconomic studies on the role of banks and banking crises (Diamond [1984, 1989], Diamond-Dybvig [1983] and Diamond-Rajan [2000, 2001a, 2001b] have analyzed intermediaries in models without payment systems, where there is no role for money.

[1] Stein (1998) has a model where reservable insured demand deposits are cheaper than other bank liabilities because deposit insurance helps mitigate problems of adverse selection that would otherwise be associated with bank liabilities. In our model, bank deposits are cheaper for banks to issue because they help solve problems of commitment – they enable a bank to pledge more of bank value than it could by issuing time deposits, bonds or equity.

We start with a micro-model of a bank that we have developed in previous work (Diamond and Rajan (2001b)). Production is financed through banks, and the way banks fund their loans leads to the effects of monetary policy on real activity. Bank liability holders in our model want to consume in the near term. Aggregate production of near term goods can be increased beyond a certain point only by closing down longer term projects (or equivalently, not initiating them). Therefore the higher the inelastic demand for near term consumption, the higher has to be the short-term real interest rate to call forth the termination of longer term projects.[2]

Now we are ready to describe how open market operations can have real effects. The first route is best described as the *real liquidity* channel – where open market operations alleviate an excess demand for near term consumption. The demand that bank liability holders can express for near term goods depends on the value of their claims on the bank. This in turn depends on the value of bank assets, which includes loans, money, and government bonds. One way the government can alter the value of bank assets is by altering the value of the bank's claim on the government through open market operations.

Even though at the margin, unanticipated open market operations such as buying bonds for money are zero NPV transactions, they affect the value of infra-marginal bank holdings of government claims. In particular, an increase in money accompanied by a decrease in government bonds reduces the aggregate present value of claims on the government. While we will postpone explaining why this happens in our model, the consequence is bank asset values fall. In turn, this reduces the value of bank liabilities

[2] We use the words "longer term" advisedly. All that we require is that these projects mature beyond the horizon at which bank liability holders want to consume. Therefore, the short rate can have significant effects on their viability.

and thus also the demand bank investors can express for near term consumption (*real liquidity*). Real interest rates fall, and more longer-term projects are continued or initiated.

Bank loans and bank liabilities can themselves be sensitive to changes in the composition of government liabilities (money vs bonds). These separate effects can further accentuate or temper the demand for real liquidity that we have described above.[3]

The effect of changes in the demand for real liquidity that we have described above is just one, albeit perhaps the most important, route through which open market operations can affect real activity. There are others. In particular, we examine the *financial liquidity* channel, where open market operations alleviate a shortage of means of payment: We add a payments system to our analysis by adding a "payments in advance constraint" for all goods and assets purchases. This makes banks special in the payments system. Either money or deposits can be used for most purchases (as in Englund-Svensson [1988]). Money is special in our model in that it is the only means of payment for some goods – like the cash goods in the cash in advance literature (see Clower [1967] and Lucas-Stokey [1987]). For concreteness, we call these perishable goods "illegal" (e.g., unreported plumbing services) that cannot be bought with other media, but the results apply to any goods where deposits are a poor substitute for cash.

A shortage of money depresses the price of these goods making it very attractive for bank depositors to withdraw money to buy them. Therefore, because demand deposits are claims on money, a shortage of money will imply a substantial increase in the rate of

[3] For example, the effect can be more pronounced if those who want to consume in the near term primarily hold nominal deposits and bank loans are insensitive to prices. Then an increase in money will also reduce the value of deposits, further reduce the demand for near term consumption, and hence also lower the real rate further.

return banks will have to pay on deposits to keep depositors in the bank. From the bank's perspective, this will make longer-term projects unattractive. At the extreme, such a spike in rates can make the bank insolvent, precipitating runs and significantly curtailing lending. If depositors who run take time (or fear) to redeposit money elsewhere in the banking system, bank runs reduce the money available for transactions, further increasing the implied rate of return on buying illegal goods. This could cause trouble for even otherwise healthy banks. Thus the kind of contagion described in Friedman and Schwartz (1963) can take place in our model if there is a shortage of money.

Open market operations, by injecting money into the system, can boost the prices of illegal goods, reducing the outside option depositors have, and reducing the real rate of return they have to be paid. Banks will then be able to extend more credit, increasing aggregate real output. Open market operations do not work through the bank balance sheet here.

We do not assume a direct relationship between a bank's reserves and its deposits in deriving these results. A reserve requirement would augment the quantitative effects in our model (by the standard multiplier) but is not needed. We also do not need prices to be sticky (they are determined in the model in such a way that they do not automatically offset changes in money). Of course, like the traditional money models, open market operations are ineffective when the demand for real money balances is satiated (bonds and money become perfect substitutes) and the nominal rate is zero.

In sum, this paper aims to make three contributions: First, we introduce money in a fully specified micro-model of a banking system. This is important for it is the structure of the bank, which stems from the functions it performs, that drives our results. Second, we analyze the different ways open market operations can affect real activity. The effects we find are not inconsistent with any of the three traditional channels -- it is because money and bonds are not perfect substitutes that open market operations have effects (money channel), one channel through which they work is by altering the value of bank balance sheets (balance sheet channel), and open market operations have real effects by affecting bank lending (bank lending channel). However, the details of how these channels work is typically different from they way they work in the existing literature. Third, we examine how different ways of infusing (or taking out) money from the system can help resolve financial crises.

The rest of the paper is as follows. We present the framework in section I, solve the model in section II, and conclude with a discussion of how well the model corresponds to the data, especially in understanding financial crises and their resolution.

I. The Framework

1.1. Agents, Preferences, Endowments, Technology.

Consider an economy that lasts from date -1 to date 4.[4] There are four types of agents in the economy: investors, entrepreneurs, bankers, and dealers. Investors get utility only from consumption at date 2 or before – they have a demand for real liquidity (near term consumption) so their utility is the sum of consumption at dates 1 and 2. All other agents get equal utility from consuming at any time, so their utility is the sum of consumptions at all dates.

[4] Odd dates are primarily so that transactions can be initiated or settled. All the action effectively takes place on even dates. Were it not for the cash-in-advance constraint (see later), we would only need two periods.

There are two kinds of goods in the economy: legal goods and illegal goods. Investors are each endowed with a fraction of a unit of legal good at date –1. No other agent is endowed with legal goods. Some dealers are endowed with illegal goods at date 1, others are endowed with illegal goods at date 3. Legal goods can be stored at a gross return of 1 (the net real rate of return is zero). Illegal goods are perishable (such as services) and have to be consumed on the same date as they appear as endowment.[5]

Each entrepreneur has a project idea. An entrepreneur can invest a unit of legal good at date -1. The investment will pay off C legal goods at date 2 if the project produces *early* or C at date 4 if the project is delayed and is *late*. There is a shortage of endowments of legal goods at date –1 relative to projects ideas.

1.2. Financing

Since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur has access to a banker with knowledge about local entrepreneurs or their businesses. The banker's knowledge allows him to collect γC from an entrepreneur whose project just matures.[6] The banker can also *restructure* the project at any time to yield c_1 in date-2 legal goods (for sale at date 1) (and nothing on other dates) – intuitively, restructuring implies stopping half finished projects and salvaging all possible legal goods from them. No other agent has any ability to extract any payment from the entrepreneur or to restructure the project. We have

$$c_1 < 1 < \gamma C < C, \qquad (1.1)$$

[5] We assume illegal goods are perishable both to capture the notion that they offer a fleeting opportunity so that money has a convenience yield, and to avoid unnecessary complications associated with storage.
[6] See Diamond and Rajan (2001a) for an extensive form game with this outcome. The problem with committing human capital that they model appears first in Hart and Moore (1994).

Bankers themselves have no endowment, so they have to persuade investors to trust them with their money. But while the banker can recover payments from the entrepreneur, ordinary investors cannot. The problem then is that having obtained investors' money, the banker can threaten to hold back his collection skills unless investors make concessions. This prospect would seriously impair the amount investors are willing to entrust him with. Therefore, the banker has to find a way to commit to using his skills on behalf of investors, else he will not be able to raise enough to finance the loan he has made.

The way for the banker to finance lending while committing his human capital to the service of investors is to issue uninsured demand deposits. Because of the "first come, first served" aspect of uninsured demand deposits, they cannot be negotiated down because depositors are liable to run if they ever apprehend that they will be paid less than their due (see Diamond-Rajan (2001a) for details). Thus if a bank has promised to pay depositors d_t, they want to consume at date t, and the banker has enough resources at that date, he will make the payment.[7]

There is, however, a dark side to the commitment obtained by issuing demand deposits. If depositors anticipate the bank will not be able to meet its deposit claims at date t, they will run immediately, forcing the bank to restructure loans and sell assets if it does not have cash to pay depositors. Therefore, while the collective action problem inherent in demand deposits enables the banker to commit to repay if he can (that is, avoid strategic defaults), it exposes the bank to destructive runs if he truly cannot pay (it makes non-strategic default more costly).

[7] For other models where runs serve as a source of discipline, see Calomiris and Kahn (1991) and Jeanne (2000). The difference in Diamond-Rajan (2001) is that the run is particularly useful in disciplining an intermediary.

To deal with the latter problem, the bank can also issue some capital (long term bonds or equity) as buffer. The advantage of capital is that payments to it adjust to the residual value of the bank – specifically, Diamond and Rajan (2000) present an extensive form game where if the value of bank assets is v at date t, capital gets $\frac{v-d_t}{2}$, with the rest of the residual value absorbed by the banker as rent. If there is uncertainty about bank asset values, the bank can avoid destructive runs if it raises some money via capital in lieu of deposits. The disadvantage is that the banker, unlike with deposits, will absorb some rents. Trading off the destruction in value from runs against the loss in value because the banker retains rents, when bank asset values are uncertain the banker may be able to pay out the most in expectation at a non-zero level of capital (see Diamond and Rajan [2000]).

We will assume that the un-modeled uncertainty facing any bank on its loans, or an explicit capital requirement, requires it to finance at least fraction *k* of the loans it carries on the books with capital. This will imply, for example, that for every late project the bank finances at date 2, it can promise to pay out only

$$\frac{\gamma C}{(1+k)} \qquad (1.2)$$

of the amount it collects at date 4, with the rest absorbed by the banker as rent.[8]

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[8] From the definition, we have $k=\frac{\frac{1}{2}(\gamma C-d_4)}{\frac{1}{2}(\gamma C+d_4)}$ where the numerator on the right hand side is the date-4 value of capital (per late project), and the denominator is the value of capital plus date-4 maturing deposits. Therefore, the total amount that can be pledged to investors at date 2 out of the amount the bank collects from late entrepreneurs at date 4 is the denominator, which on substituting for d_4, works out to $\frac{\gamma C}{1+k}$.

Since there is a shortage of endowment, at date –1 only a select few entrepreneurs get a loan from their respective banks to buy one unit of the legal good each from investors. Entrepreneurs will have to promise to repay the maximum possible to obtain the loan: they will contract to repay γC on demand.

The local lending markets faced by each bank are identical at the time of lending at date -1, but become heterogeneous at date 0 in that, in each market, a different fraction of the funded projects turn out to be early. Therefore, at date 0 the aggregate state is fully characterized by the realization of the fraction of projects that are early for each bank *i*, α^i. We assume for simplicity that there are two types of banks, G and B, where $\alpha^G > \alpha^B$. The fraction of banks of type G in state s is θ^s. In what follows all quantities will be normalized by the total endowment of legal goods at date -1.[9]

1.3. Government and the Value of Money

The government needs legal goods for un-modeled infrastructure expenditures at date -1.[10] It issues M_0 of money and nominal bonds maturing at date 2 with face value B_2 to the banks, obtains deposits, which it then uses to pay investors for legal goods. When bonds mature, the government extinguishes them by repaying their face value in new money. It can also issue fresh bonds.

The government taxes real production at the rate τ (C is the after-tax quantity produced from a project, so total nominal taxes due on a project that matures at date t are

[9] As we will see, it is without loss of generality to assume that when banks exist, investors will sell all their legal goods and place the proceeds in banks. Therefore banks owe d_t in deposits per unit of legal good.

[10] Equivalently, we could assume that the government just endows investors with bonds and money at date –1. In this case all initial endowment is invested in projects.

$\frac{\tau C}{1-\tau}\ p_t^L$ where p_t^L is the price of legal goods at date t). Taxes are due at the time of production and payable in money.

An explanation is in order. Because we use a finite horizon model, we need to establish a value for money at the last date. Money's only value at this date is to pay taxes. We anchor the price level at that date by equating the nominal value of the government claims outstanding to the value of final tax revenues. Recently, this approach has been termed the fiscal theory of the price level, and in Woodford [1995] and Cochrane [2001] it has been used to value nominal claims even when they provide no payments services (implying that currency and nominal bonds are perfect substitutes). We use the fiscal theory only as an anchor at the terminal date: in most of what we study there is a payments role for currency (see also Schmitt-Grohe and Uribe [2000]). For example, at intermediate dates, the price level is determined both from the requirement that taxes are paid in money and the fact that money also plays a role in payments. It turns out that this will be important in establishing a role for open market operations. Let us turn to therefore to the role of money in payments.

1.4. The Need for Trade, Transactions Technology, and the Value of Money

No one can consume his or her own endowment or production, therefore everyone must trade to consume. This introduces a motive for trade. All trades require payment one period ahead in deposits or cash (payment cannot be made in loans or bonds). Legal and illegal goods are perfect substitutes in consumption; they differ in that only currency is accepted for the illegal good. The names of goods offer one motivation for the currency-only restriction – dealers do not want these transactions recorded by the legal economy. Dealers have a perishable endowment, q_1, of illegal goods at date 1 and q_3 at date 3. So as to maintain consistency with the idea that dealers are outside the formal economy, we assume they cannot deposit in the bank or hold bonds.[11]

The payment-in-advance constraint implies that an agent can use currency or deposits that he holds from the last period as payment instantaneously, but the proceeds from a sale of assets or goods is not cleared and available for transactions by the seller until the next period. Similarly, for depositors to withdraw currency from a bank for use on this date, the bank must have the currency in reserve from the last period. If the bank sells assets for currency, the currency will not be available for transactions until the next period.

Finally, investors can borrow deposits against the value of the bank capital they hold (this could be thought of as a margin loan). They therefore can effectively use their holdings of bank capital also as means of payment.

Let us now summarize the time line and the sequence of events.

1.5. Timing.

Date –1. Investors are endowed with goods. Entrepreneurs need goods to make investments while the government needs goods for public works. Banks intermediate the necessary trades by making loans to entrepreneurs, holding government-issued currency and date-2 bonds, and finance all of this with demand deposits and capital issued to investors. All transactions clear before date 0. Of each unit of legal good, λ is invested in a project, while 1- λ goes to the government. The ex ante identical banks promise to pay depositors d_t, which together with capital is the maximum the banks can pay per unit of

[11] We could do without this assumption without altering the results qualitatively: it only has effect on the date 3 price of illegal goods as nominal rates turn out to be zero from date 1 to 2.

good given up, entrepreneurs offer to pay banks γC (per unit of legal good invested), and the government matches the implied return on bank loans on its bonds and money.

Date 0. Uncertainty is resolved: everyone learns which entrepreneurs' projects are early and which are late, and thus what fraction α^i of a bank *i*'s projects are early. If depositors anticipate that, given the bank's state and aggregate liquidity conditions, the bank will not be able to pay them at date 2, they will run immediately forcing the bank to sell bonds or restructure loans once it runs out of cash. Otherwise, they will wait and use their deposits to pay for legal goods to be delivered at date 2 or withdraw immediately to buy illegal goods from dealers for currency. Dealers arrange to deliver goods at date 1.

Date 1 Investors consume illegal goods. Early entrepreneurs sell legal goods for deposits (and effectively bank capital) or currency to investors.[12]

Date 2. Investors consume legal goods. If their bank has survived, the entrepreneurs with early projects will repay the bank γC (i.e., in units of legal goods) at date 2 (leaving them with (1-γ)C to invest as they will), while entrepreneurs with late projects will default. Depending on the prevailing rate and its need for funds, the bank will then decide how to deal with each late project – whether to restructure it if proceeds are needed before date 4, or perhaps get greater long run value by rescheduling the loan payment till date 4 and keeping the project as a going concern. The bank uses repayments from entrepreneurs whose projects are early, money from selling restructured late projects at date 1, and from deposits reinvested in the banks by early entrepreneurs to fund deposit withdrawals by investors at date 2. Date 2 taxes are paid in currency. Dealers sell illegal

[12] This could occur on date 0 instead, but we need the intermediate date to allow illegal good producers to buy goods for date 2 consumption, similarly date 3 is necessary only to allow illegal goods producers to buy goods for date 4 consumption.

goods for currency. Government repays date-2 maturing bonds in currency and issues fresh bonds maturing at date 4.

Date 3 Late entrepreneurs sell goods for deposits (or bank capital) or currency.

Date 4. Entrepreneurs, dealers, and bankers consume. Late entrepreneurs repay loans and banks repay deposits (to early entrepreneurs). Taxes paid in currency. Money stock retired.

II. Solving the Model.

We are interested characterizing equilibrium prices, interest rates, and real activity. Second, we want to know the consequences of interventions such as open market operations on outcomes. Even though the model is solved by backward induction, it will be a little more illuminating to solve the model forward from date 2 to date 4 and then from date 0 to date 2.

We will examine two sorts of problems. The first is what happens when the "natural" supply of goods produced on or before date 2 (real liquidity) is not enough to meet the demand. Clearly supply has to rise or demand has to fall. We will see how this can happen, and how open market operations alter the equilibrium. The second problem has to do with payments. The value of illegal goods purchased at date 0 or 2 is constrained to be less than the cash held directly in the previous period plus cash withdrawn from cash held from the previous period by one's bank. In this section, we assume that no banks fail, which is sufficient to guarantee that there is no other potentially binding payment in advance constraint. This will allow us to find the conditions where bank failures can be avoided. Because banks can hold bonds and loans

Comment:

and issue demand deposits and capital against them, the value of bonds and loans can effectively be "monetized" and can be used as means of payment for financial transactions and legal good transactions. Therefore, in the normal course, all wealth can be paid out and standard budget constraints apply. However, when banks fail, this correspondence can break down. There may be a "shortage" of means of payment relative to assets for sale – a problem of financial illiquidity. We will see what open market operations and other monetary interventions can do to alleviate this problem. To do all this, we need to first establish how prices and interest rates are determined. We also assume that both bank capital and deposits can be used as a means of payment. This assumption is relaxed below, and it has no qualitative effect (it results in one parameter being reinterpreted).

Comment:

2.1. Money and Prices On and After Date 2

At date 2, money is held to pay the taxes due at date 2 and also to pay in advance for illegal goods to be delivered at date 3. Any money in excess of this is held as a store of value. Thus the money market clears at date 2 when

$$M_1 + B_2 - PV(B_4) = \tau X_2 p_2^L + q_3 p_3^I + m_2 \tag{2.1}$$

The left hand side is the supply of money consisting of the currency available at the end of date 1 (which is the currency available at date 0, M_0, plus any changes made by the government) plus the bonds maturing (and therefore paid off in money) at date 2 less any new date-4 maturing bonds issued for money at date 2. The right hand side consists first of the money paid to the government in taxes, where X_2 is the pre-tax production at date 2. The date-2 price of legal goods, p_2^L, converts the real tax into a cash tax. The second term is the money held to buy illegal goods (p_3^I is the price and q_3 is the endowment of dealers) and the third term is the money held as a store of value.

Let r_{23} be the gross real interest rate (one plus the net real interest rate) paid between date 2 and date 3 and let i_{23} be the gross nominal interest rate (one plus the net nominal rate) paid by bank deposits. We will shortly argue that the gross nominal interest rate in the settlement period between dates 3 and 4 (or dates 1 and 2) is 1. Hence nominal interest rates over the period 2 to 4 are the same as those over the period 2 to 3. A number of arbitrage relationships can be established immediately.

Cash can either be paid at date 2 to buy illegal goods for delivery at date 3, or invested in bank securities earning the nominal interest and used at date 3 to buy legal goods for delivery at date 4. For both transactions to take place in equilibrium, it must be that

$$\frac{1}{p_3^I} = \frac{i_{23}}{p_4^L} \Rightarrow p_3^I = \frac{p_4^L}{i_{23}} \tag{2.2}$$

Thus cash at date 2 provides a "liquidity" return because it permits the purchase of illegal goods, which are discounted in price relative to legal goods. By definition $r_{23} = \frac{i_{23}}{p_4^L / p_2^L}$. Substituting from (2.2), we get

$$p_3^I = \frac{p_2^L}{r_{23}} \tag{2.3}$$

Finally, cash can be held as a store of value to be paid at date 3 instead of being deposited. This will be the case only if the gross nominal interest rate, i_{23}, is 1. Hence if the nominal rate exceeds 1, m_2=0. Substituting from (2.3) in (2.1), when m_2=0 we get

$$p_2^L = \frac{M_1 + B_2 - PV(B_4)}{\tau X_2 + \frac{q_3}{r_{23}}} \tag{2.4}$$

Thus the price at date 2 is determined both by the role of money in tax payments (fiscal theory) and its role in permitting payment for illegal transactions.

At date 3, depositors and holders of cash will pay in advance to entrepreneurs who are yet to produce (entrepreneurs with late projects).[13] Since only legal goods are available for delivery at date 4, date-3 cash and securities maturing at date 4 are equivalent transactions media at date 3. The gross nominal interest rate on deposits is $r_{34}=1$.[14]

The only use of claims on the government at this terminal date is to pay taxes. If X_4 is total pre-tax production at date 4, the cash price, p_4^L, equates the nominal claims on the government to the nominal taxes owed to the government at date 4. Then

$$p_4^L = \frac{q_3 p_3^I + m_2 + B_4}{\tau X_4} \tag{2.5}$$

The first term in the numerator is the cash held by sellers of the illegal good, the second term is the cash held at date 2 as a store of value, and the third term is the cash paid out to maturing bonds. The denominator is the tax denominated in goods.

[13] Capital holders can borrow deposits against the value of their holdings of capital and thus effectively become depositors at date 3. Bankers can also borrow deposits against the rents they expect to get at date 4 so long as they can commit their human capital at date 3 to collecting repayments at date 4. We need such an assumption only because date 4 is the terminal date and all transactions have to be undertaken a period in advance. Else, we could allow the banker to collect his rent at date 4 and spend it on goods for delivery at date 5. Apart from added notational burden, none of the results of the model would change. Thus, for simplicity, we assume that the banker can commit to collect on outstanding loans one period before the end of time.

[14] If money is held at date 3 only by producers of the illegal goods (who cannot deposit), then multiple nominal interest rate equilibria (including 1) are possible. But the interest rate of 1 is the only one that survives a tremble that puts an infinitesimal amount of cash in the hands of anyone who can deposit.

The equilibrium prices and money holdings, given the real interest rate and the quantities of goods produced are now easily obtained:

Lemma 1: Let $i = r_{23} \, {}^{p_4^L}\!\!/_{p_2^L}$ where $p^L{}_2$ is given by (2.4). If $i > 1$, prices of legal goods are given by (2.4) and (2.5), and m_2=0. If $i \le 1$, the price of legal goods for delivery at date 4 is given by (2.5) and $p_2^L = \dfrac{p_4^L}{r_{23}}$. The quantity of money that is held as a store of value is

$$m_2 = M_1 + B_2 - PV(B_4) - \tau X_2 p_2^L - q_3 \frac{p_2^L}{r_{23}} . \ \blacklozenge$$

The intuition is simple. We know that the gross nominal interest rate on deposits cannot be pushed below 1, else money will be held instead of deposits. Furthermore, given the date-4 maturing bonds and money outstanding, the price at date 4 is pegged. This means that the price at date 2 can be given by (2.4) only if the resulting gross nominal rate is greater than 1 (equivalently, the price level deflation between date 2 and date 4 does not exceed the real rate). Otherwise, the price at date 2 will be one that offers a price deflation equal to the real rate of interest, the nominal rate of interest will be 1, and investors will be happy to hold currency as a store of value because it pays a rate equal to that paid by deposits.

2.2. Open Market Operations and Their Effect on the Value of Government Claims

To see what all this implies, consider the government reducing its issue to the public at date 2 of date-4 maturing bonds, thus leaving more cash outstanding. This is a current open market operation (the government issues the bonds to the public and the central bank buys some back with money), which expands the money supply. So long as the nominal interest rate exceeds 1, and assuming for now that the real side of the economy (the fraction of projects restructured) is unaffected, the anticipated increase in date-2 money supply will push up the date-2 price of legal goods. The nominal value of government obligations outstanding at date 4 will fall both because the increased date-2 price level will force more money to be paid in as taxes at date 2, and because the government has swapped a non-interest bearing claim for an interest bearing one. Thus the date 4 price level will fall. Given a constant real rate of interest, the nominal rate will fall. The price of illegal goods at date 3 will rise relative to legal goods at date 4.

Eventually, however, sufficient expansion in money supply will depress the nominal rate to 1. At this point, the price of illegal goods at date 3 will equal the price of legal goods at date 4. Money will no longer provide a "liquidity" return in the sense of enabling its possessors to buy goods cheaply. But none is needed to cause it to be held. Because it promises the same return as bonds, it can simply be used as a store of value. Open market operations will then no longer have any effect on the date-2 price (any additional money that is issued will simply be held, prices do not have to rise to expand money demand). They will also have no further effect on the date-4 price of legal goods (because open market operations will not affect the size of outstanding date-4 claims on the government) or on the price of illegal goods. In short, open market operations eventually cease to have an effect once the nominal rate is driven to one and money and bonds serve the same purpose – this is simply a version of the Keynesian liquidity trap.[15]

Another way to look at this open market operation is to focus on the real quantities supported by these prices. Unanticipated open market operations, even though they involve a zero NPV swap at the margin, and even if they had no effect on real

[15] It is important to distinguish one-time open market operations from "helicopter drops" of money. The former are currently inflationary (up to a point), but risk deflation in the future. The latter are inflationary not just today but also in the future.

activity or real rates, do affect the relative price of legal and illegal goods and the outstanding value of government liabilities at date 2. It is important to understand why because intuitively (and incorrectly) it would seem that in the spirit of the fiscal theory of money without payment frictions (Cochrane [2001]), the real value of government liabilities should equal the real value of government tax receipts, which are unchanged if open market operations do not affect real activity.

What is missing in this intuition is that money also serves as a means of payment for illegal goods in our model. To see this, let us calculate the real value of outstanding government liabilities (the consumption value of legal plus illegal goods that can be purchased). If nominal interest rates are greater than 1 (so that money held as a store of value, m_2, is zero), from (2.4) we get the real value of money balances at date 2 is $tX_2 + \frac{q_3}{r_{23}}$. From (2.5), we get the real value of date-4 maturing bonds, discounted back to date 2 at the real interest rate is $\frac{B_4}{p_4^L r_{23}} = \frac{\tau X_4}{r_{23}} - q_3 \frac{p_3^I}{p_4^L r_{23}}$. So the real value of outstanding government liabilities before date 2 is $tX_2 + \frac{\tau X_4}{r_{23}} + \frac{q_3}{r_{23}}\left[1 - \frac{1}{i_{23}}\right]$. The first two terms are the real present value of tax receipts. The third term is the interesting one. It reflects the additional value of money as a means of payment – the amount that holders of money make by buying goods on the cheap from dealers. Clearly, as the amount of money outstanding increases, and as the nominal rate falls to 1, the real value of government liabilities -- holding real activity and real rates constant -- falls. Value passes from holders of government liabilities to dealers, for the latter no longer have to obtain a deeply discounted price for the goods they sell. Thus open market operations that increase the money stock reduce the real value of claims on the government by increasing the value appropriated by future holders of money. By reducing the artificial shortage of means of payment, the expansion of money reduces the real purchasing power of total government obligations. (Contrast this with the effect on nominal liabilities. Pushing down the nominal interest rate increases the nominal value of bonds and the total nominal value of government claims.) Of course, once the nominal rate falls to one, no further alteration in value is possible, and further open market operations lose all effect.

Note also that the government's real revenues are unchanged if real output does not change (and government expenditure is fixed). So substituting interest-bearing liabilities for non-interest bearing ones does not result in a greater real claim on the government or in lower seigniorage profits. There are no sticky prices in our model. Open market operations simply transfer value from one set of agents to others but do not alter the aggregate real future payments by the government. In summary, *keeping production and real interest rates fixed*,

Theorem 1: Unanticipated open market operations where the central bank issues money and buys back bonds decreases the aggregate real value of government claims and increased their nominal value, so long as the gross nominal interest rate exceeds 1. When the gross nominal interest rate reaches 1, further open market operations have no effect.

The next question, of course, is what effect do open market operations have on real activity. This is what we turn to.

2.2 Bank Credit and Production When Deposits and Loans Are Real

When deposits and loans are real, the potential effects of monetary policy on banks work through the wealth effects of changing the real value of government assets held by the banks, and money's effect on real interest rates. We have established prices of goods and financial assets at dates 2-4 given both the quantity of goods produced and the real interest rate, and assuming that no banks fail. We now determine what happens to these real variables. They will depend on the state of bank balance sheets and banker incentives.

Let us start with the case where banks issue real deposits and capital at date -1. Real deposits are claims on a certain quantity of consumption goods at date 2 (examples of such deposits would be deposits indexed to the price level or deposits denominated in a stable foreign currency). So if d_2 is the face value of real deposits maturing at date 2, the amount of cash that has to be paid out by the bank to depositors at date 2 is $d_2 p_2^L$. This also means that at any other prior date, a depositor can withdraw an amount, which if deposited in the bank at the prevailing nominal interest rate would fetch $d_2 p_2^L$ at date 2.

Assume securities were fairly priced at issue (reflecting all current and future payments in advance constraints) and the bank met capital requirements at date -1. At date 0, the state of nature is revealed. Banks are either type G or B, where a bank's type i reflects the fraction of projects, α^i, that mature early. We now determine what fraction of late projects will be continued by a banker and what the resulting real interest rates would be. We will then examine the effects of open market operations.

Let the banker of type i restructure μ^i of his late projects. Then the real amount he anticipates to have at date 2 to pay off his deposits is

$$v^i(p_2^L, \mu^i, r_{23}) = \frac{M_0 + B_2}{p_2^L} + \lambda\left[\alpha^i \gamma C + \mu^i(1-\alpha^i)c_1 + (1-\mu^i)(1-\alpha^i)\frac{\gamma C}{(1+k)r_{23}}\right] \quad (2.6)$$

The numerator in the first term on the right hand side is the nominal value of financial assets the bank holds, and it has to be divided by the date-2 price of legal goods to get the real value of those assets. Within the square brackets, the first term is the real amount repaid by the α early entrepreneurs whose projects mature at date 2. The second term is the amount obtained by restructuring late projects. The third term is the amount the bank can raise (in new deposits and capital – see (1.2)) against late projects that are allowed to continue without interruption till date 4 (early entrepreneurs finance the bank from the excess they have after repaying loans).

Now consider a banker's choice between continuing and restructuring a project. If a banker chooses to continue financing a late project, he will collect γC at date 4. If he restructures it, he will get $r_{23}\, c_1$ at date 4. Therefore, the banker gets more at date 2 by continuing a project if $r_{23} < \overline{R} = \frac{\gamma C}{c_1}$. However, the banker also needs to raise enough money to pay off his maturing date-2 deposits. Given that he has to finance with a fraction k of capital, he can only raise $\frac{\gamma C}{(1+k)r_{23}}$ in deposits and capital at date 2 against the prospective payment from late entrepreneurs. The bank can obtain c_1 by restructuring late projects. So continuation fetches more date-2 funds than restructuring only if

$r_{23} < R = \dfrac{\gamma C}{(1+k)c_1}$. Note that this is what outsiders such as the owners of capital see, so they get the most value if the bank restructures everything when $r_{23} > R$ and if it restructures nothing when $r_{23} \leq R$.

The bank survives only if it can raise enough money to pay off maturing date-2 deposits – only if $\max\limits_{\mu^i} \; v^i(p_2^L, \mu^i, r_{23}) \geq d_2$. Conditional on surviving, to raise enough to pay all investors, the banker will set μ^i such that

$$v^i(p_2^L, \mu^i, r_{23}) \geq \frac{\max\limits_{\mu^i} \; v^i(p_2^L, \mu^i, r_{23}) + d_2}{2} \tag{2.7}$$

How much the banker restructures then depends on the level of real rates. By inspection, $R < \overline{R}$. Therefore, if r_{23} <R, the banker prefers to continue a late project and he can also raise more from investors against the project than he gets from restructuring. So if r<R, the banker will not restructure any projects (so that μ^i=0) provided he can pay off all date-0 depositors – provided $v^i(p_2^L, 0, r_{23}) \geq d_2$.[16] If $R < r < \overline{R}$, the banker may want to continue because he gets a rent from doing so, but given the interest rate, he will raise more funds per project at date 2 by restructuring. The bank will restructure the minimum fraction μ^i such that $v^i(p_2^L, \mu^i, r_{23}) = \dfrac{v^i(p_2^L, 1, r_{23}) + d_2}{2}$. Finally, if $r \geq \overline{R}$, the banker is best off restructuring all late projects.

[16] This condition will also ensure he can pay off capital, as can be seen by simply substituting $\max\limits_{\mu^i} \; v^i(p_2^L, \mu^i, r_{23}) = v^i(p_2^L, 0, r_{23})$ when r < R into (2.7).

In sum, the fraction of late projects restructured by the type i bank, μ^i, depends on the real interest rate. As we have just seen, the higher the real interest rate the greater the incentive of the banker to restructure projects, and beyond a certain level, the more he needs to restructure to pay depositors. Now let us see how this can help link open market operations and aggregate economic activity.

We focus on a special case which gives us much of the intuition. Let us assume that α^G is high so type G banks always have enough from early projects to repay deposits and capital. Also let θ^G and α^B be high enough that the B type banks survive, but not so high that they can continue all their late projects and not restructure anything. This means that $r_{23} > R$, else the B type banks would not restructure anything. At any such rate, the amount a bank of type i can raise at date 1 is maximized at μ^i=1 and is $v^i(p_2^L, 1, r_{23}) = \frac{M_0 + B_2}{p_2^L} + \lambda\left[\alpha^i \gamma C + (1-\alpha^i)c_1\right]$. The real value of wealth held by depositors will be the sum of deposit claims and capital claims on the bank at date 2 and will then be $\frac{v^i(p_2^L, 1, r_{23}) + d_2}{2}$. In the aggregate, the supply of legal goods is equal to its demand so that

$$\frac{\lambda}{(1-\tau)}\left[\theta^G\left(\alpha^G C + (1-\alpha^G)\mu^G c_1\right) + (1-\theta^G)\left(\alpha^B C + (1-\alpha^B)\mu^B c_1\right)\right] = \theta^G \frac{v^G(p_2^L, 1, r_{23}) + d_2}{2} + (1-\theta^G)\frac{v^B(p_2^L, 1, r_{23}) + d_2}{2}$$

(2.8)- Legal Goods Market Clearing

Also, each bank type has to restructure enough so that it can pay off the deposit and capital claims on it. This means that

$$v^i(p_2^L, \mu^i, r_{23}) \geq \frac{v^i(p_2^L, 1, r_{23}) + d_2}{2} \qquad \text{(2.9) Solvency of type B banks}$$

with equality when $\mu^i > 0$.[17] Since $\mu^G = 0$ and the G type banks can meet their claims without any restructuring, the aggregate demand for legal goods will fully determine the amount the B type banks have to restructure.

In determining general equilibrium, three conditions have to be satisfied. The first is that the money market has to be clear at date 2 so that (2.4) is satisfied. Second, (2.8) should be satisfied so that the market for goods at date 2 clears. Finally, the real interest rate should be such that B type banks cannot afford to pay more without becoming insolvent and (2.9) is satisfied with equality. Call this the credit market (or equivalently, the deposit market) equilibrium.

Our interest is not in the details of the equilibrium (see Diamond and Rajan (2001b) for that) but in how open market operations affect the equilibrium. The channel of transmission is through bank balance sheets. As we have seen, open market operations affect the value of government claims the banks hold. In turn, this affects the purchasing power of investors. One route we will now examine is through the value of bank capital. But there are others, which we will come to shortly.

2.3. Banks and Money with Real Deposits and Loans

Consider an unanticipated contractionary open market operation where, at date 2, the government issues more date-4 maturing bonds thus leaving less money outstanding. If the denominator in (2.4) does not fall significantly (we will return to this shortly), the

[17] Note that even though the amount the bank can raise – its pledgeable value – is maximized when $\mu^i = 1$, it would not want to restructure so much if it can do so without becoming insolvent. So long as $v^i(p_2^L, 1, r_{23}) > d_2$, it will be able to set $\mu^i < 1$. Also, so long as it is doing a positive amount of restructuring to meet all its claims, it would not want to have any excess left over after paying claimants, else it could restructure less. Hence the equality when $\mu^i > 0$.

date-2 price of legal goods will fall so that the demand for money can fall to equal supply.

Moving to the goods market, the real value of a bank's holdings of money and bonds will rise as the price level falls, and so will the value of its capital. The real value of financial assets in the hands of date-2 consumers, which is the sum of bank capital and bank deposits, will rise (the right hand side of (2.8)). For the goods market to clear, banks will have to restructure more loans to provide the necessary date-2 goods. As a result μ^B will go up. Bank credit, which is the fraction of late projects continued, $1-\mu^B$, will fall.

Turning now to the credit market, as B type banks restructure more (at a rate r_{23}>R) and if the price level falls, their asset values increase. This means they can promise more in real terms to new potential depositors, and competition will force them to do so. So the credit market equilibrates at a higher r_{23}. Therefore the natural consequence of an open market operation reducing the supply of money is a fall in prices, a decrease in bank credit, and an increase in real interest rates.

We said natural because we have assumed the denominator in (2.4) does not fall significantly so the price level does fall when the money supply tightens. But in the denominator, $\tau X_2 + \frac{q_3}{r_{23}}$, the first term will rise because more date-2 goods are produced through restructuring (the quantity of goods produced, X_2, is the left hand side of (2.8)) increasing the tax burden, and hence the demand for money to pay those taxes. But the second term falls because the falling money stock reduces the price of illegal goods and hence the real return on holding currency (and thus on bank deposits). The reduced cost of illegal goods, and reduces the money necessary to buy them. If the falling second term outweighs the rising first term, the demand for money can fall when the money supply falls. If demand falls faster than the falling supply, prices can perversely decrease and credit increase with a decline in the supply of money.

Under weak conditions, however, the price level and bank credit do fall when the money supply tightens as a result of Open Market Operations.

Theorem 2: If there is a unique positive solution (p_2^L, μ^B, r_{23}) to the system (2.4), (2.8), and (2.9) and $1-\tau > \frac{PV(B_4)}{M_1+B_2}$ then

$$\frac{dp_2^L}{d\,PV(B_4)} < 0, \quad \frac{d\mu^B}{d\,PV(B_4)} > 0, \quad \frac{dr_{23}}{d\,PV(B_4)} > 0 \tag{2.10}$$

Proof: See Appendix.

We show in the appendix that the unique positive solution obtains under a variety of plausible conditions. So the theorem says that if the issuance of bonds relative to the pre-existing money supply is not too high, so that a sizeable amount of money is left outstanding, the relationship between Open Market Operations and prices, credit, and interest rates are the intuitive ones. But if this condition does not hold, then the relationships can reverse.

Corollary 1:

(i) If there is a unique positive solution (p_2^L, μ^B, r_{23}) to the system (2.4), (2.8), and (2.9) and $1-\tau < \frac{PV(B_4)}{M_1+B_2}$ then

$$\frac{dp_2^L}{d\,PV(B_4)} > 0, \quad \frac{d\mu^B}{d\,PV(B_4)} < 0, \quad \frac{dr_{23}}{d\,PV(B_4)} < 0 \tag{2.11}$$

(ii) There can be, at most, two positive solutions to the system. If so, for one solution, (2.10) holds, while for the other, (2.11) holds.

Proof: See Appendix.

In what follows, we assume that the conditions for Theorem 2 hold, and thus that expansionary open market operations will increase the price level (and hence increase credit and reduce real interest rates).

Discussion

It is useful to reiterate why open market operations have an effect here: Monetary easing reduces the real value of claims on the government, the value of bank assets, the value of bank liabilities, and thus the real demand for legal goods (the demand for real liquidity) at date 2.[18] So long as some restructuring is being undertaken to provide these goods, changes in demand will affect the fraction of late projects continued, the quantity of legal goods produced at date 2, the quantity of legal goods produced at date 4, and the amount of aggregate credit offered. Note, however, that if legal goods exceed the value of deposits plus capital with no restructuring (because, for example, θ^G is high), then open market operations will have no effect. Therefore a necessary condition for open market operations to have effect is that there not be too much money (so that the nominal rate is positive) and there not be too much legal goods produced at date 2 relative to the demand of investors (so that the real interest rate is high and there is some restructuring).

Note also that even though an open market operation can increase total output, this is not a Pareto improvement. Initial investors consume less because their claims are

[18] What is important, therefore, is that holders of bank liabilities disproportionately have a strong preference for near term consumption or real liquidity.

worth less, and early entrepreneurs get less for the amounts they deposit in the bank at date 2 because date-2 real rates are lower. Late entrepreneurs are better off because they are, on average, less likely to have their projects restructured, and bankers are better off because they get more rents. Dealers are better off because the price of illegal goods is higher.

Monetary easing affects real output by changing the aggregate demand for liquidity. The channel we have examined is via its effect on the value of government assets held by the bank-- this effects bank actions and the value of capital claims outsiders hold on the bank. A more direct route is if monetary easing affects the value of bank liabilities or bank loans directly because they are not denominated in real terms, unlike what we have assumed so far. But before we explore that, let us see whether monetary easing is always helpful. When banks are fragile, it may indeed hurt banks.

2.4. Altering Aggregate Demand When Banks are Fragile and Have Real Deposits

We have assumed thus far that both types of banks survive irrespective of the monetary actions. Monetary easing weakens bank balance sheets by reducing the real value of government claims but the general equilibrium effect helps B type banks continue more late projects. The problem, however, is if B type banks get pushed to the brink so that they do not have enough to pay depositors, they will be run.

In other words, there is a limit to the process of monetary easing over and above the one we have discussed of zero net nominal rates. If the B type banks have to restructure some late projects to pay, real rates have to be R or above. But as bank asset values fall due to monetary easing, banks will eventually be able to raise just enough to pay deposits at a rate R. Any further easing will reduce B type bank asset values below the level that is enough to pay deposits, so the banks will be run as soon as such a policy is announced.[19] This will have an immediate adverse effect on production.

The reason for the adverse effect on production is that depositors will seek to withdraw cash immediately. The only way the bank can meet their demand after it pays out the cash reserves it has is by selling its assets to solvent G type banks and passing on the proceeds to depositors. The greatest value the bank can obtain from its projects is by selling them restructured, since it cannot transfer its collection skills. Since the bank will typically not be able to meet depositor claims even after restructuring all projects, all the B type bank's projects, not just late ones, will be restructured. The projects funded by B type banks will produce c_1 at date 1 instead of the much larger $\alpha^B C + (1-\alpha^B)\mu c_1$ at date 2 and $(1-\alpha^B)(1-\mu)C$ at date 4. Bank credit for B type banks, as measured by the fraction of projects funded to maturity, will fall to zero. There are other effects of these bank failures, which occur because failing banks must not only restructure but also sell them. We postpone discussion of these until we introduce nominal deposits.

If there are many weak (type B) banks, this suggests that excessively inflationary monetary policy can cause bank failures and a fall in aggregate output if bank liabilities and bank loans are largely real. So there is no easy prescription when the banking system is undercapitalized after an adverse shock – monetary policy may or may not work in enhancing aggregate output, depending on the fragility of the banking sector. The central

[19] These runs are not "sunspot" runs but runs based on fundamentals. Depositors could always anticipate a very high interest rate that will bring down the banks and therefore run. To rule out such sun spot equilibria, we assume that depositor beliefs at date 0 are the most optimistic ones possible – that the real interest rate and the number of banks that will be insolvent at date 2 will be the lowest consistent number.

authority may decide to accompany a monetary loosening with a recapitalization of the most fragile banks. However, this may weaken the healthier banks as they are forced to match the rates bid by recapitalized banks, perhaps necessitating further recapitalization or causing the strong to fail (see Diamond and Rajan (2002)).

Having examined the effects of monetary loosening at date 2 on aggregate activity when deposits are real, let us collapse back to date 0. While this is for completeness at this point, we will find the analysis very useful shortly when we move to examining nominal deposits.

2.5. Date-0 Prices and Rates

Deposits can be withdrawn at date 0 to buy illegal goods for delivery at date 1. Since the total amount of cash available is M_0, the price of illegal goods, p_1^I, cannot be more than $\frac{M_0}{q_1}$. The nominal interest rate the bank has to offer on deposits so as to keep depositors from withdrawing at date 0 to buy illegal goods is at least $\frac{p_2^L}{p_1^I}$ -- the ratio of the price of legal goods for delivery at date 2 relative to the price of illegal goods for delivery at date 1. This implies that

Lemma 2: If $\frac{p_2^L}{\frac{M_0}{q_1}} \geq 1$, the nominal interest rate offered on deposits, i_{01}, equals $\frac{p_2^L}{\frac{M_0}{q_1}}$ and the price of illegal goods, p_1^I, is $\frac{M_0}{q_1}$. If $\frac{p_2^L}{\frac{M_0}{q_1}} < 1$, then i_{01} is 1, and $p_1^I = p_2^L$. Some money is then held as a store of value at date 0.

III. Nominal Deposits and Loans

Suppose now that deposits (or bank loans) are denominated in nominal terms – they specify a pre-determined monetary payment on withdrawal. Consider an expansionary open market operation at date 2 that increases the price level and reduces the real value of claims on the government. This will reduce the real value of nominal deposits. Even though the value of capital increases as a result, capital captures only half the decrease in the value of deposits (the remainder is a rent to the bankers). Therefore, the real value of claims held by investors, the sum of capital and deposits totaling $\frac{v+\frac{d_2}{p_2^L}}{2}$, falls, and even more so if bank assets also fall in value (because either nominal claims on the government or nominal bank loans fall in value). Thus there is an additional channel through which aggregate demand for liquidity can be reduced through monetary easing- directly reducing the real value of bank liabilities, reducing the excess demand for liquidity. Obviously, the reverse (monetary tightening) can increase the excess demand for liquidity.

It turns out that monetary tightening is particularly dangerous for banks: our model illustrates and provides new light on the danger of the famous debt-deflation problem of Irving Fisher. This occurs when both assets and liabilities are nominal, and there is unanticipated deflation. The reason is that in a period of deflation, the real value of the promised payment on both bank loans and deposits can be high. However, the maximum the bank can recover is γC in real terms from a maturing project. So, bank recovery on loans is capped in real terms. By contrast, the depositors' claims are not limited by an orderly process of negotiation. Instead, the depositor relies on the threat of the collective action problem to collect. Therefore, deposit claims mount until the bank cannot pay at which point depositors are forced to run. Since a sizeable portion of the bank's assets (loans) can be capped in value in a period of unanticipated deflation, while the bank's primary liabilities (deposits) need not be, such periods can be extremely dangerous for the health of the banking system. In sum, unanticipated monetary policies can affect aggregate output in several ways that work through the banking system and its special role.[20]

We now consider another potential path through which the quantity of money can matter, through its role in payments: money affects the real return that banks must offer to deter currency withdrawals and the failure of banks and the associated fire sales when they cannot meet that return can increase the rate that is required. As we will see, this effect is not present when bank deposits are real.

Let banks issue demand deposits to investors at date -1, promising them cash at date 0 equal to δ_0. As always, depositors have the right to withdraw at any date, but they can also roll over their deposit at the prevailing nominal rate (hence these deposits are nominal).

At date 0, depositors can either withdraw to pay for illegal goods for delivery at date 1, or they can hold claims on the bank to buy legal goods for delivery at date 2. Let us remind the reader on how exactly the cash-in-advance constraint works. Cash or deposits have to be pledged at the beginning of a date to buy goods for the next date. If a bank has cash reserves going into a date or obtains it from loan repayments, depositors can withdraw it at the beginning of the date and use that cash in transactions. But if a

[20] Another, more traditional way aggregate demand for liquidity could vary is if some of the bank's depositors had utility for consumption at date 4. They could be bribed through higher interest rates to postpone consumption. Therefore, higher real interest rates would lower the aggregate demand for consumption of date-2 goods.

bank has to sell assets for cash in a fire sale and pays out that cash to depositors, that cash is available to depositors only for transactions at the next date. Intuitively, we want to maintain a strict version of the cash in advance constraint where cash can be used for purchases only once at any date.[21]

As lemma 2 indicates, if the amount of money outstanding is not too high, the price of the illegal good is $p_1^I = \frac{M_0}{q_1}$. Since depositors can withdraw immediately to buy the illegal good, they have to be promised as much to stay in. So the real value of what they have to be promised at date 2 is $\frac{\delta_0}{p_1^I}$. The condition for the B type bank to be solvent at date 2 is then

$$v^B(p_2^L, \mu, r_{23}) \geq \frac{\delta_0}{p_1^I} \tag{3.1}$$

where $\mu=0$ if $r_{23}<R$ and 1 otherwise. This is similar to the condition we encountered with real deposits except that the threshold value the bank has to meet to avoid being run depends directly on the quantity of money available to buy illegal goods at date 0. The reason it differs is that depositors are promised a fixed amount of currency rather than a fixed future real value. If either the quantity of money is low or the quantity of illegal goods available is high, the threshold increases because the opportunity to withdraw cash to buy the highly discounted illegal goods becomes more attractive.

The point is that when the bank offers nominal deposit contracts, there is an intertemporal arbitrage condition it has to meet. Not only does it make the bank highly

[21] An alternative version is that when a bank sells assets on a date, the cash that is paid for the asset is effectively in escrow till the next date. While claims on the cash can be allocated to depositors immediately, the depositors actually get the cash only on the next date.

susceptible to fleeting opportunities available in the cash market even if that market is quite small, it also forces a future real solvency constraint on the bank, even if its deposits are nominal. Because the supply of cash is inelastic, a low level of money in the system can make cash transactions extremely lucrative. Even if these transactions are a minuscule portion of the economy, rates on deposits have to rise to match them because depositors have the right to withdraw cash.

But matters can be worse. Suppose now that (3.1) does not hold so that the type B banks will fail. Then they will be run immediately at date 0. They will pay out their cash reserves to depositors, but once the B type banks run out, they will have to sell assets. If the only asset that their depositors will accept is cash, the bank's assets must be sold for cash. But the sale of these assets will lock up more cash in transactions, leaving less cash to buy illegal goods. This will further depress the price of illegal goods to

$$p_1^{I}{}' = \frac{M_0}{q_1 + \theta^B \left[\lambda c_1 + \frac{B_2}{p_2^L} \right]} \tag{3.2}$$

where the denominator in (3.2) now also includes the real value of assets the B type banks sell. This imposes a cash in advance constraint on these financial asset sales as well, increasing the demand for cash.[22] This will make the purchase of illegal goods even more lucrative, and force the G type banks to pay a yet higher rate to keep their depositors from moving to cash. Given the higher effective payout to deposits at date 2,

[22] Note that the problem is not just the price drop of the assets sold in the fire sale (also present in Diamond [1997] and Allen-Gale [1998], but also the required interest rate effects of diverting money from its use in (illegal) transactions.

these type G banks could also fail if $v^G < \frac{\delta_0}{p_1^I}$. The mechanism here resembles the contagious bank failures described by Friedman and Schwarz (1963).

Key to contagion is that the depositors of failed banks want cash rather than deposit claims on healthy banks (for example because having experienced a run they fear depositing elsewhere or because they have little knowledge of distant banks). If they were willing to take deposit claims on healthy banks, then there would be no additional shortage of cash caused by bank failures. Thus an increase in the currency deposit ratio held by the public, as noted by Friedman and Schwarz, is a key feature of such crises. But the effect in our model is not because of sticky prices combined with a multiplicative contraction of money creation as high-powered money is withdrawn from the banking system. Instead it is because strong banks cannot afford the real interest rate their deposits would need to offer to match the opportunities in the cash market resulting from the diversion of some of the existing money stock to financial transactions.

Finally, note that this kind of problem would not arise if deposits were denominated in real terms. As prices of illegal goods fall, the amount depositors would be able to withdraw would also fall, thus reducing the value of the outside opportunity to depositors.

3.1. Intervention

Clearly, one form of intervention is for the government to buy date-2 maturing bonds for money from the market. At best, however, this can reduce the magnitude of the contagion if the government ends up purchasing the bulk of the bonds from B type banks. The B type banks will still have to sell restructured loans in the market thus “trapping” cash.

An alternative is for discount window lending by the government to the B type banks against their assets (or equivalently, for the government to buy these assets and hold them off the market as exemplified by the working of the Resolution Trust Corporation). While this will stop monetary contagion by preventing these assets from being unloaded on the market, it will not raise the price of the illegal goods above $p_1^I = \frac{M_0}{q_1}$. So discount window lending or Lender of Last Resort operations will not restore the B type banks to health since they would still have to pay a high interest rate to their depositors to deter withdrawal of cash even if a fire sale is avoided.

There are three ways the B type banks can be saved from the financial liquidity crisis. The first is for the government to provide added currency directly to investors (a helicopter drop) to increase the price of illegal goods. This will take the pressure off the B type banks and restore their financial health. In an open economy with floating rates and domestic currency deposits, this is similar to devaluation. The second is for banks to suspend convertibility. While such an action will depress the prices of illegal goods even more (and put immense pressure on banks that do not suspend convertibility, perhaps explaining in part why suspension has to be collective), it will preserve the B type banks till such time as more usable money can be injected into the system alleviating the financial illiquidity. The third is for the type B banks to receive a significant capital infusion so that they can meet the high required rates of interest without defaulting. This will prevent the fire sale, but need not solve the problem of the system unless the real value of claims held by the public is reduced to an amount consistent with the aggregate supply of goods (see Diamond-Rajan [2002]).

3.2. Discussion

In our earlier analysis with real deposits, open market operations worked by altering wealth. Here, open market operations also work by freeing up cash to be used for transactions, increasing the price of illegal goods, and thus reducing the opportunity cost for depositors of staying in the bank. In both cases, however, the value of money in permitting exchanges that cannot be done with deposits is critical to why open market operations have effect.

There are many ways this analysis can be extended. For instance, it would be useful to explore how the central government/bank's willingness to intervene in the future affects banks' decisions to offer credit today. Clearly, the central authority's willingness at date 0 to infuse cash to keep the price of illegal goods from falling too much will give banks less of a need to keep a precautionary hoard of reserve currency, and allow them to make more loans. (Therefore, signals about the central authority's stance on accommodation can influence bank lending.) On the other hand, too great a willingness on the part of the central authority to intervene in the future can also have adverse consequences. For instance, if the central authority indicated a willingness to increase its financing with money at date 2, that would immediately push down anticipated bank asset values. If that was not accompanied by a commensurate willingness to expand money supply going into date 0, banks could face the unwelcome challenge of paying out high rates on deposits even while future assets values were likely to be depressed – a sure recipe for disaster.

IV. Broader Interpretations and Potential Applications

Broadly speaking, our basic point is that the monetary transmission mechanism relies, not on a "cost of capital effect", but on a "cost of liquidity" effect operating through banks. This is why a change in what is essentially a short-term rate can have effects on medium and long-term projects. Key to this mechanism is the commercial bank. There are a number of factors that place the bank at the center of the transmission process.

First, the nature of the bank's lending activities forces it to issue demandable claims. Given that these claims are liquid, we would expect those with a preference for liquidity hold these claims. It is essential for our results that these preferences be relatively inelastic. If so, small changes in the price of liquidity will not clear the market for liquidity. Instead, there will have to be some quantity adjustments if there is an initial mismatch between demand and supply.

The mismatch between supply and demand for liquidity can come from either side. On the supply side, there could be a reduction in available liquidity because projects are delayed (or because there has been overly optimistically investment in projects that turn out to be much longer duration ex post). On the demand side, there could be an increased demand for liquidity because outside opportunities are lucrative. In response to this mismatch, adjustment can take place on the supply side as more projects are restructured or stopped to focus on the near term production of liquidity. It can also take place on the demand side, as monetary policy reduces the claims of those who need liquidity. Finally, the adjustment can also take place in a lumpy way if banks fail, with grave consequences for the economy.

We have overlaid a monetary system over a real economy. It should be clear from our analysis that fundamental problems are invariably real ones – real illiquidity or insolvency -- and changes in money "work" by altering real conditions at the margin. Financial illiquidity may seem like a purely monetary phenomenon, but even it has serious consequences (such as contagious failure) only when banks become insolvent. The point is that in resolving financial crises, it makes sense to first generate policy options by focusing on the underlying real economy, and only then to use tools like (but not limited to) monetary policy to achieve the necessary shifts in wealth and rates (see Diamond and Rajan (2002 b) for examples of what tools might seek to achieve).

While the model we have built is for a closed, bank-dominated economy, it should be relatively straightforward to extend it to study an open economy. Let us offer some conjectures here on how the model could be applied.

Some developing countries have attempted to tie their currencies to international currencies. Perhaps to strengthen the commitment or perhaps confident of it, domestic banks have also issued deposits in foreign currency – resembling the real deposits in our model. This has created a potential problem. If there is a shortage of supply of foreign exchange (likely if the country has limited foreign collateral (see, for example, Caballero and Krishnamurthy (2001)), banks will fail. There is a fundamental mismatch between the supply and the demand for real liquidity here, which the central bank can do little about.[23] It has two options, neither pleasant. The first is to let banks go under, the second is to arrange a collective default on the foreign denominated debt, similar to a suspension of convertibility (we assume the third option, that of seeking loans from international

[23] In addition, there is a basis risk when there are foreign currency deposits because the amount that domestic borrowers can pay is likely in domestic units, even if the contracts are written in foreign currency.

financial institutions has been exhausted). Monetary easing through open market operations can do little here (unless holders of bank capital also seek foreign exchange) since it cannot reduce the demand of deposit holders.

Some have argued that this kind of mismatch can be resolved by limiting the amount banks can borrow in foreign currencies. This may not be true. In fact, it can create the potential for a financial liquidity crisis. A bank run is triggered when the bank cannot offer the real return a depositor can obtain by withdrawing (the depositor's outside option). The units in which future withdrawal rights are specified determine which the rate of return condition must be met, while the units of the current withdrawal determine the value that must be paid today on withdrawal. While domestic currency deposit holders can only withdraw domestic currency, the high rate of return they require when the domestic currency is overvalued (i.e., in all but a fully floating rate regime) can be enough to precipitate a banking crisis

The reason is that depositors in an economy with a convertible currency have the option today of withdrawing cash in domestic currency and converting to foreign exchange. If there is a significantly high probability of devaluation, the rate of return they will demand to stay in the bank at the overvalued exchange rate will be very high, perhaps greater than the bank can pay. Depositors will then run and convert immediately, making the devaluation all the more likely. Thus the point at which banks no longer can credibly pay the high interest rates needed to keep domestic currency deposits serves as the trigger point for the speculative attack.

Note that while the central bank has the option of inflating away bank deposits in the future when they are in nominal terms, our analysis suggests this option may be quite illusory if we introduce sticky prices or managed exchange rates implying that inflation takes time to kick in. In fact, anticipating high inflation, depositors would want to withdraw and convert today. Thus the simple act of changing real deposits to nominal deposits does not prevent the bank from facing a real solvency constraint. Worse still, the severity of that constraint can be worsened by any temporary overvaluation of the domestic currency.

In sum, twin crises (foreign exchange and banking) are likely in a bank-dominated economy with convertibility and a fixed exchange rate, and they may occur even if bank debt is largely denominated in domestic currencies. Any act of defending the overvalued exchange rate may simply accentuate the bank runs. Instead, a better policy for the health of the banks is to let the exchange rate settle quickly at the new lower equilibrium (so that bank depositors do not have the outside option of converting at the overvalued rate), or of suspending convertibility to the foreign currency. Both will reduce the attractiveness of the outside option depositors have, and thus reduce the rate banks have to pay to get them to stay. Key in creating the problem is the fixed exchange rate. If the country has freely floating exchange rates, then the currency's price will adjust until it will be held at an interest rate that someone can commit to. So a short- and long-term solution is to allow the exchange rate to float.

While we have not incorporated a foreign sector in our model, these speculations suggest that there might be some value in doing so. More research is clearly called for.

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Proof of Theorem 2.

Let us write down the three constraints, expand them and collect terms.

For aggregate liquidity demand to be equal to aggregate liquidity supply, we must have

$$\frac{\lambda}{(1-\tau)}\left[\theta^G\left(\alpha^G C+(1-\alpha^G)\mu^G c_1\right)+(1-\theta^G)\left(\alpha^B C+(1-\alpha^B)\mu^B c_1\right)\right]=\theta^G\,\frac{v^G(p_2^L,1,r_{23})+d_2}{2}+(1-\theta^G)\frac{v^B(p_2^L,1,r_{23})+d_2}{2}$$

(3.3)

where $v^i(p_2^L,\mu^i,r_{23})=\dfrac{M_0+B_2}{p_2^L}+\lambda\left[\alpha^i\gamma C+\mu^i(1-\alpha^i)c_1+(1-\mu^i)(1-\alpha^i)\dfrac{\gamma C}{(1+k)r_{23}}\right]$.

Setting $\mu^G=0$ by assumption, and substituting the value of v in (3.3) and expanding, we get

$$\underbrace{\frac{\lambda}{(1-\tau)}\left[\theta^G\alpha^G C+(1-\theta^G)\alpha^B C\right]-\lambda\left[\theta^G\left[\alpha^G\gamma C+(1-\alpha^G)c_1\right]+(1-\theta^G)\left[\alpha^B\gamma C+(1-\alpha^B)c_1\right]\right]-\frac{d_2}{2}}_{k_1}+\frac{\lambda}{(1-\tau)}(1-\theta^G)(1-\alpha^B)c_1\mu^B=\frac{M_0+B_2}{2p_2^L}$$

$$\Rightarrow \qquad k_1+k_2\mu^B=k_3\left(\frac{1}{p_2^L}\right) \qquad (3.4)$$

Next, we have the solvency condition for the B type bank, which requires that

$$\frac{M_0+B_2}{p_2^L}+\lambda\left[\alpha^B\gamma C+\mu^B(1-\alpha^B)c_1+(1-\mu^B)(1-\alpha^B)\frac{\gamma C}{(1+k)r_{23}}\right]=\frac{\frac{M_0+B_2}{p_2^L}+\lambda\left[\alpha^B\gamma C+(1-\alpha^B)c_1\right]+d_2}{2}$$

Rearranging, we have

$$\frac{M_0+B_2}{2p_2^L}+\underbrace{\lambda\left[\alpha^B\gamma C+(1-\alpha^B)\frac{\gamma C}{(1+k)r_{23}}\right]-\frac{\lambda\left[\alpha^B\gamma C+(1-\alpha^B)c_1\right]+d_2}{2}}_{k_4}+\lambda(1-\alpha^B)c_1\mu^B-\lambda(1-\alpha^B)\frac{\gamma C}{(1+k)}\left(\frac{1}{r_{23}}\right)\mu^B=0$$

$$\Rightarrow \quad k_3\left(\frac{1}{p_2^L}\right)+k_4+k_5\mu^B-k_6\left[\frac{1}{r_{23}}\right]\mu^B=0 \qquad (3.5)$$

Finally, for money market equilibrium, we have $\tau X_2+\dfrac{q_3}{r_{23}}=\left(M_1+B_2-PV(B_4)\right)\left(\dfrac{1}{p_2^L}\right)$.

$X_2=\lambda\left[\theta^G\alpha^G\gamma C+\left(1-\theta^G\right)\left(\alpha^B\gamma C+\mu^B(1-\alpha^B)c_1\right)\right]$. Substituting and regrouping, we have

$$k_7 + \tau k_2 \mu^B + q_3 \left(\frac{1}{r_{23}} \right) = \left(k_3 - k_8 \right) \left(\frac{1}{p_2^L} \right) \tag{3.6}$$

Equations (3.4), (3.5), and (3.6) are in 3 unknowns, $\mu^B, \frac{1}{r_{23}}, \frac{1}{p_2^L}$. Solving for $\frac{1}{r_{23}}$ and $\frac{1}{p_2^L}$ from (3.4) and (3.5) and substituting in (3.6), we get

$$(k_1 + k_4) + \left(k_2 + k_5 - k_6 a_1 \right) \mu^B - k_6 a_2 (\mu^B)^2 = 0 \tag{3.7}$$

where $a_1 = \frac{1}{q_3} \left[\left(\frac{k_3 - k_8}{k_3} \right) k_1 - k_7 \right]$ and $a_2 = \frac{1}{q_3} \left[\left(\frac{k_3 - k_8}{k_3} \right) k_2 - \tau k_2 \right]$. Substituting and implicitly differentiating (3.7) w.r.t. k_8 (the present value of bonds issued), we get

$$\frac{d\mu^B}{dk_8} = \frac{-\frac{\mu^B k_6}{q_3 k_3} \left(k_1 + \mu^B k_2 \right)}{(k_2 + k_5 - k_6 a_1) - 2 k_6 a_2 \mu^B} \tag{3.8}$$

The numerator is easily shown to be negative. To sign the denominator, note that it is in the form $a_4 - 2a_5\mu^B$ where the quadratic in (3.7) is $a_5 \mu^{B^2} - a_4 \mu^B - a_3 = 0$. Now $a_5 > 0$ if $1 - \tau > \frac{PV(B_4)}{M_1 + B_2}$. This then implies that one positive solution for μ^B is $\frac{a_4 + \sqrt{a_4^{\ 2} + 4a_3 a_5}}{2a_5}$.
If this is unique, then this implies $a_4 - 2a_5 \mu^B = -\sqrt{a_4^{\ 2} + 4a_3 a_5}$ which is negative.

Therefore, we have $\frac{d\mu^B}{d\,k_8} = \frac{d\mu^B}{d\,PV(B_4)} > 0$.

For a unique positive solution when $a_5 > 0$, a necessary and sufficient condition is that $a_3 a_5 > 0$, which means that $a_3 = k_1 + k_4 > 0$. k_1 is positive if τ high enough and k_4 is negative but does not include τ. Thus $PV(B_4)$ low and τ high will ensure right sign.

The other parts of the theorem and the corollary follow easily.