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Financial Frictions, Foreign Currency Borrowing, and Systemic Risk

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Abstract

Fixed exchange rate regimes that are not fully credible are typically fraught with a peso problem (the low probability expectation of a large currency devaluation). Under these conditions, foreign currency borrowing can ameliorate financial frictions in the economy, while at the same time increasing systemic risk. The lower interest rate on foreign-currency-denominated liabilities improves borrowers' incentives and reduces the agency problem associated with limited liability and the unobservability of borrowers' action. In doing so, it reduces borrower idiosyncratic risk, but exposes the system to the risk of correlated defaults through exchange rate devaluation. The resulting trade-off between average performance and systemic stability provides a rationale for measures aimed at limiting currency mismatches. This becomes stronger when the risk of contagion from widespread bankruptcies is added to the model.

Keywords: liability dollarization, systemic risk, banking crises
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1 Introduction

In this paper we explore how foreign currency borrowing can ameliorate financial frictions in emerging economies while at the same time increasing systemic risk. Under certain conditions (that we interpret as typical of fixed exchange rate regimes), foreign-currency-denominated liabilities improve firms' incentives and reduce the agency problem associated with limited liability and the unobservability of a firm's actions. In doing so, it reduces idiosyncratic risk for firms. However, it exposes the system to the risk of correlated defaults through exchange rate devaluation.

Foreign currency borrowing (or liability dollarization) has been a common feature in several emerging market economies. Typically, this liability dollarization reduces the interest borrowers pay on their loans (these countries generally pay a currency premium) and has been associated with faster credit and economic growth. For instance, in the run-up to the recent global financial crises, among a sample of Eastern European countries, credit growth was the fastest in countries that had a larger share of credit denominated in foreign currency.

Liability dollarization, however, also increases systemic risk. Should the country experience a sharp currency depreciation, firms with unhedged foreign-currency denominated debt would find it difficult to honor their liabilities, resulting in widespread bankruptcies.[1] Indeed, there is a clear link between liability dollarization and the frequency of banking crises. Liability dollarization also appears to be associated with more rigid exchange rate regimes. For example, again in Eastern Europe, countries with currency boards or rigid pegs (such as Bulgaria, Estonia, or Latvia) had a much larger share of credit to the private sector denominated in foreign currency than exchange rate floaters (such as the Czech Republic, Poland, and Slovakia).[2] There is also some evidence that the share of foreign currency lending in domestic credit gradually declined in countries that abandoned a fixed exchange rate regime.[3]

In our model, entrepreneurs borrow in order to invest in productive projects. A project's probability of success depends on the entrepreneur's costly effort. We introduce two basic financial frictions. First, entrepreneurs/firms are protected by limited liability. Second, an entrepreneur's effort is unobservable to lenders and cannot be contracted upon. These two frictions generate an

[1] See Schneider and Tornell (2004).
[2] See Rosenberg and Tirpak (2008).
[3] See Martinez and Werner (2002) for a study on Mexico in the aftermath of the Tequila crisis.

inefficiency in the economy as they entail a backward bending credit supply curve (à la Stiglitz and Weiss, 1981). Higher interest rates reduce the entrepreneur's payoff in case of success and thus also reduce her effort. Then, when the cost of effort is sufficiently high, there does not exist an interest rate at which the lender can break even given the expected probability of repayment. Put differently, projects that could be funded under perfect information are rationed out of credit markets when the entrepreneur cannot commit to a particular level of effort.

We assume that the domestic currency is expected to depreciate, so that the risk free domestic interest rate is higher than the foreign rate. Further, we assume that this spread is due to the expectation of a large devaluation to which markets attach a relatively low probability. We interpret these "peso problem" conditions as typical of exchange rate pegs and currency boards. Under these conditions, we show that foreign currency borrowing reduces the moral hazard associated with limited liability. The reason is that borrowing in foreign currency acts as a bonding mechanism for the firm: since the risk free rate abroad is lower, the firm is able to obtain a lower interest rate loan as foreign lenders have a lower threshold for lending. This creates a higher return for the firm whenever its project is successful, and provides the firm with a greater incentive to ensure that its return actually materializes, i.e., to put in more effort. The trade-off, however, is that borrowing in the foreign currency exposes the firm to more risk since it exposes it to devaluation risk. This depends critically, however, on the distribution of possible exchange rate movements: when the probability of an exchange rate movement is instead high, foreign currency lending worsens, rather than ameliorates, the agency problem. Note as well that in our framework the firm does not borrow in order to hedge foreign currency risk and does not have income in foreign currency, unlike part of the literature on dollarization focusing on large firms, and that borrowing in foreign currency could be from abroad or directly from local banks.

From a policy perspective, the paper supports the view that government intervention, in the form of (macro) prudential regulation and/or capital controls, to curb foreign currency borrowing and the systemic risks associated with it may be socially optimal. The paper points to a trade-off between superior productivity and greater systemic risk (defined as the risk of widespread failures, i.e., a crisis). In the model, we assume risk neutrality. Hence, average performance is all that matters. However, the model easily lends itself to the analysis of the risk/performance trade-off once one introduces a risk-averse regulator/social planner. Alternatively, the analysis can be readily augmented to include a non-linear social cost of failure: if a few borrowers fail the costs are relatively low, but become very large if there are widespread failures and defaults. Under either one of these minor extensions, situations where a large mass of firms default, which constitute systemic crises, have a significant negative effect on social welfare. This means that the social planner may be willing to trade average performance for a reduced probability of a systemic crisis.

We extend the model to consider the possibility that a sufficiently large number of failures and defaults negatively affects firms with successful projects and causes them to fail as well. In other words, we study the possibility of contagion and show that this adds an additional important wrinkle to the problem. The risk of devaluation acts as an externality if widespread bankruptcies have detrimental effects on the ability to repay of firms that would be otherwise sound. If a sufficiently large fraction of firms borrows in foreign currency, others (who would have otherwise borrowed in local currency) may find it optimal to do the same as they are exposed to the risk of devaluation through its effects via the real economy. The possibility of contagion thus affects firms' choice of whether to borrow in the domestic or the foreign currency, and may further exacerbate the likelihood and the severity of a systemic crisis. Under these conditions, regulation aimed at limiting or eliminating foreign currency borrowing may be welfare improving.

The paper relates to a broad literature on how financial imperfections contribute to shaping international capital flows. Our analysis is closest to Ranciere, Tornell, and Westermann (2008) and Schneider and Tornell (2004). As in those papers, foreign currency borrowing can help address an agency problem and increases output in tranquil times at the cost of greater risk of systemic crises. In those papers, however, credit rationing helps to resolve the asymmetric information problem between borrowers and lenders so that, in the absence of bailout guarantees, risk is correctly priced at the margin. Here, while risk is correctly priced in equilibrium, lenders cannot condition their pricing on an entrepreneur's effort. As a result, systemic risk associated with foreign currency borrowing can emerge even in the absence of bailout guarantees. From this point of view, our paper identifies an additional mechanism linking systemic risk and economic performance.

Several papers focus on the interaction between liability dollarization and government behavior. In Jeanne (2009), a sovereign's inability to protect foreign creditors' rights results in a system dominated by short-term loans. This short maturity structure provides governments with incentives to enforce foreign contracts. However, it comes at the cost of risking liquidation (i.e., a "run") triggered by negative productivity shocks (a similar theme is in Tirole, 2003). In Velasco and Chang (2004), foreign currency borrowing emerges as a reaction to the expectation that the central bank will choose a fixed exchange-rate regime. Then, the financial instability that a devaluation would cause through balance-sheet effects induces the central bank to fight exchange rate flexibility, validating expectations. Under these conditions, pre-commiting to exchange-rate flexibility, if feasible, is welfare improving. A similar analysis is in Chamon and Hausmann (2005). In Jeanne (2005), foreign currency borrowing is an outcome of domestic monetary policy. If monetary policy mitigates default risk in the private sector, firms will tend to borrow in domestic currency. If, on the other hand, the monetary environment does not protect firms against low realizations of their domestic currency income, firms will borrow in foreign currency because borrowing in domestic currency can result in unbearably high real debt burdens if the expected domestic monetary policy does not materialize ex post. In Korinek (2011), foreign currency debt emerges from an optimal portfolio choice problem with a risk premium on local currency debt. The advantage of local currency debt is that it mitigates economic volatility. Local currency debt emerges at low levels of volatility of consumption and the exchange rate, as well as when risk premia on local currency debt are low.

The paper proceeds as follows: Section 2 examines a series of stylized facts and the empirical literature on foreign currency borrowing and financial crises. Section 3 presents and analyzes the main model. Section 4 examines the effects of changes in interest rates. Section 5 extends the model to the case of contagious defaults. Section 6 discusses the trade-off between performance and stability. Section 7 concludes and briefly discusses the policy implications of the model.

2 Stylized facts and empirical evidence

In this section, we review the empirical literature on foreign currency borrowing and financial crises, and present some stylized facts that are consistent with the predictions of our model.

Much of the micro-level empirical literature on the determinants of foreign currency borrowing and the balance sheets effects that arise as a result of currency depreciations when firms borrow in foreign currency has focused, due to data limitations, on large and publicly traded firms. Allayanis, Brown, and Klapper (2003) investigate the capital structure of 327 large, publicly traded firms in East Asia around the time of the East Asian financial crisis and collect data on their local, unhedged foreign, and hedged foreign debt. They find that interest rate differentials are a key determinant of the use of foreign currency debt, and that the market value of firms that used financial hedges to synthetically convert foreign debt into local debt were hit particularly hard during the 1997-98 East Asian financial crisis. Bleakley and Cowan (2008) study the currency composition of the debt of 500 publicly traded firms in Latin America during the period 1990 to 1999, a period of substantial exchange rate volatility in this region of the world, and find that the sensitivity of firms' investments does not depend on the currency composition of their debt because firms tend to match the currency composition of their debt with the elasticity of their income to the exchange rate. For a sample of large US firms, Kedia and Mozumdar (2003) find that firms issue foreign currency debt mainly to hedge their exposure to foreign currencies. Similarly, Keloharju and Niskanen (2001) find for a sample of large Finnish firms that hedging features prominently in the decision to borrow in foreign currency, with firms for which exports represent a larger fraction of sales more likely to raise foreign currency debt. At the same time, they also find that firms tend to borrow in foreign currency when the foreign interest rate is relatively low, consistent with carry trade explanations.

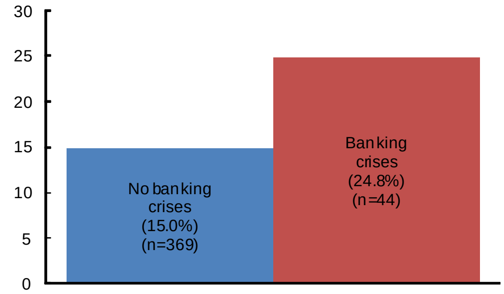
Many of these results may be skewed by the focus on large and stock-exchange listed firms that are often in a better position to use financial hedges compared to small firms (either because of know-how or economies of scale). In fact, Gelos (2003), using data on 500 Mexican firms, shows that firm size is a key determinant of foreign currency borrowing in addition to imports and exports. Moreover, a large fraction of these firms has natural hedges against exchange rate risks because they operate in the tradable sector and have significant foreign currency revenues. Not surprisingly, much of this literature finds relatively small balance sheet effects associated with foreign currency borrowing during currency crises (for reviews, see Galindo, Panizza, and Schiantarelli, 2003, and Kamil, 2008).

Brown, Ongena and Yesin (2009) are the first to study the determinants of foreign currency borrowing for a representative sample of firms that includes small firms using survey data on firms in Eastern Europe. They find that firms that naturally generate a larger fraction of income in foreign currency, such as exporting firms, are more likely to borrow in foreign currency, while interest rate differentials and exchange rate volatility do not explain the use of foreign currency borrowing. Brown, Kirschenmann, and Ongena (2009) study a representative sample of Bulgarian firms and find that foreign currency borrowing is not only driven by demand factors but is partly supply-driven by banks that prefer to lend in foreign currency to minimize currency mismatches in their balance sheets, even when borrowers request loans in domestic currency. Obviously, this still exposes the banks to credit risks arising from balance sheet effects of their borrowers in case of currency depreciation.

Ranciere, Tornell, and Vamvakadis (2010) also study a representative sample of firms in Eastern Europe and focus on foreign currency borrowing by firms with no foreign currency income. They find that currency mismatches reduce interest rates and enhance growth of small firms in non-tradable sectors, thereby contributing to growth in tranquil times, while at the same time increasing the probability of crises. They argue that the expectation of government bailouts in the event of a currency crisis is one of the mechanisms that fosters the use of foreign currency borrowing by firms that face borrowing constraints.

The empirical link between foreign currency borrowing and boom-bust cycles has led countries to implement regulatory policies to slow foreign currency borrowing during credit booms, although these policies have typically met with only limited success, mainly because these policies are generally easy to circumvent, for instance, through direct borrowing from abroad (Rosenberg and Tirpak, 2008). At the macro-level, a large empirical literature links banking and currency crises to credit booms accompanied by an overvalued currency, although most of this literature does not distinguish between local and foreign currency borrowing (see, for example, Kaminsky and Reinhart, 1999). In an exception, Ize and Levy-Yeyati (2003) show that the use of foreign-currency debt can be linked to macroeconomic uncertainty, including the relative volatility of domestic inflation and the real exchange rate.

Using data on foreign currency borrowing from the IMF's Vulnerability Exercise Database (not publicly available), Figure 1 shows a clear link between the degree of foreign currency borrowing in the country and the occurrence of banking crises, as defined in Laeven and Valencia (2008), in a sample of 114 countries. Foreign currency borrowing from banks in countries that experienced banking crises over the period 1970 to 2010 stood at 24.8 percent on average compared to only



Notes: The red (blue) bar denotes the average percentage of foreign currency lending to nominal GDP across country-year observations for those years over the period 1970-2010 during which the country did (not) experience a systemic banking crisis, as defined in Laeven and Valencia (2010). Data on banking crises from Laeven and Valencia (2010), "Resolution of Banking Crises: The Good, the Bad, and the Ugly," IMF working paper 10/146, and data on percentage of foreign currency lending to nominal GDP from the IMF's Vulnerability Exercise Database. Number of country-year observations (n) between brackets. Sample of 114 countries.

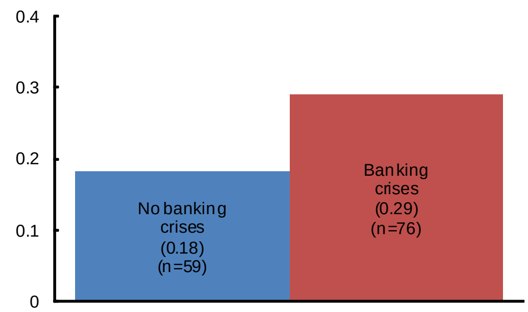
Figure 1: Foreign currency lending to GDP and occurrence of banking crises, 1970-2010

15.0 percent in countries that did not experience a banking crisis over this period. These empirical findings are consistent with the prediction from our model that foreign currency borrowing exposes borrowers to exchange rate risk and exposes lenders to default risk from devaluation-driven balance sheet effects.

On the deposit-taking side of banks, De Nicolo, Honohan, and Ize (2003) show that dollarization is associated with deeper financial development, especially in high inflation environments. This is consistent with the model in Caballero and Krishnamurthy (2003) who argue that limited financial development reduces the incentives for foreign lenders to enter emerging markets.

Using data from Levy-Yeyati (2006) on the degree of dollarization of deposits in the country, Figure 2 shows a clear link between dollarization and the occurrence of banking crises, in line with the findings on the link between foreign currency lending and banking crises shown in Figure 1. The ratio of foreign currency deposits in total deposits is about 24 percent on average for countries that experienced a banking crisis over the period 1970 to 2004 compared to only 18 percent for countries that did not experience a banking crisis over this period.

A related literature studies the link between exchange rate regimes and banking crisis. Burnside, Eichenbaum, and Rebelo (2001) argue that banks in countries with a fixed exchange rate regime do not completely hedge the exchange risk that arises from the currency mismatch between their assets and liabilities in anticipation of government bailouts, and that such open foreign exchange positions makes banks prone to banking crises associated with currency crises. Empirical studies



Notes: Bars denote the average degree of dollarization over the period 1970-2004 across countries depending on whether or not they experienced a systemic banking crisis during the period 1970-2004, as defined in Laeven and Valencia (2008). Degree of dollarization is the ratio of foreign currency deposits in total deposits. Data on the occurrence of banking crises are from Laeven and Valencia (2008), "Systemic Banking Crises: A New Database", IMF Working Paper 08/224, and data on the degree of dollarization are from Levy-Yeyati (2006), "Financial Dollarization: Evaluating the Consequences," Economic Policy, January, pp. 61-118. Number of country observations (n) between brackets.

Figure 2: Degree of dollarization and occurrence of banking crises, 1970-2004

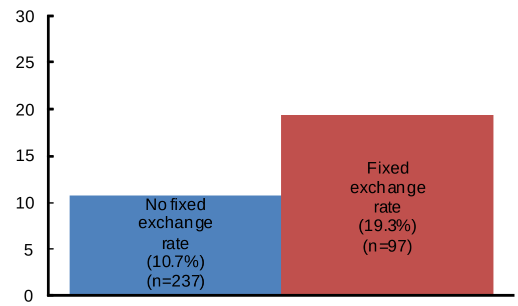
generally find that fixed exchange rate regimes, and especially those with hard pegs, are more prone to banking crises than flexible exchange rate regimes or those with adjustable pegs, and that banking crises in fixed exchange rate regimes are more costly in terms of severity of crisis and output losses (Eichengreen, 2002, Demac and Martinez Peria, 2003, and Husain et al., 2005).

Figure 3 shows that foreign currency borrowing is more pervasive in countries with fixed exchange rate regimes, which together with Figure 1 suggests that the currency composition of borrowing may be a key driver linking fixed exchange rate regimes to crises. In the context of our model this association can be interpreted as fixed exchange rates being associated with higher expected devaluations despite higher probabilities (α) of the exchange rate remaining constant.

3 Model

Consider an economy populated by entrepreneurs/firms that invest \$1 in risky assets that return y when successful and 0 otherwise. A firm's effort determines the probability of success, q, at a cost $\frac{c}{2}q^2$. Firms are heterogenous in their effort cost c. Firms have no initial funds and need to borrow in order to invest. The loan contract specifies the *gross* interest rate (i.e., one plus the net interest rate) r_L to be repaid by the borrower.

This is an open economy and firms can borrow in the domestic or a foreign currency. The two currencies are linked by a standard (simplified) interest parity condition: $r^* = r^{*f} + \widehat{e}$, where r^* is the gross (credit) risk-free interest rate in domestic currency, r^{*f} its equivalent in foreign currency,



Notes: The red (blue) bar denotes the average percentage of foreign currency lending to nominal GDP across country-year observations over the period 1970-2010 during which the country did (not) have a fixed exchange rate regime. Data on classification of exchange rate regimes from Reinhart, Carmen and Kenneth Rogoff, 2004, "The Modern History of Exchange Rate Arrangements: A Reinterpretation," Quarterly Journal of Economics 119(1): 1-48, and data on percentage of foreign currency lending to nominal GDP from the IMF's Vulnerability Exercise Database. We define fixed exchange rate regimes as exchange rate regimes with preannounced or de facto pegs (classification codes 2, 3 or 4 in Reinhart and Rogoff). Data on exchange rate regimes are averaged over the period 1970-2007. Number of country-year observations (n) between brackets.

Figure 3: Exchange Rate Regimes and Foreign Currency Lending, 1970-2010

and $\widehat{e}$ the expected exchange rate change. For simplicity, we assume that exchange rate movements are governed by a binomial distribution: the exchange rate stays constant with probability α, and depreciates (or appreciates) by Δe with probability $1-\alpha$. Thus, we can rewrite the interest parity condition as

$$r^* = r^{*f} + \widehat{e} = r^{*f} + (1-\alpha)\,\frac{\Delta e}{e}. \tag{1}$$

3.1 Domestic currency borrowing

When a firm borrows in the local currency, it's expected profits can be written as

$$\Pi = q(y - r_L) - \frac{c}{2}q^2,$$

which reflects the fact that the firm's project will only pay off with a probability that is increasing in its level of effort q. When the project does pay off, the cash flow from the project is y, and the firm repays the lender the promised amount r_L. It's profit is then $q(y-r_L)$ minus the cost of its effort, $\frac{c}{2}q^2$. Maximizing this with respect to the level of effort gives

$$\widehat{q} = \frac{y - r_L}{c}.$$

The interest rate offered on the loan has to reflect the level of risk associated with the project. Suppose that investors or lenders conjecture a level of effort q^C. This then means that

$$q^C \widehat{r}_L = r^* \Rightarrow \widehat{r}_L = \frac{r^*}{q^C}.$$

In equilibrium, investors' or lenders' beliefs about the amount of effort that will be supplied must be correct, which means that $q^C = \widehat{q}$. We can substitute this into the expression for optimal effort $\widehat{q}$ to obtain $\widehat{q} = \frac{y - \frac{r^*}{\widehat{q}}}{c}$, and then solve for $\widehat{q}$ as

$$\widehat{q} = \min\left\{\frac{y + \sqrt{y^2 - 4cr^*}}{2c}, 1\right\}, \tag{2}$$

where (2) reflects the fact that the positive root that solves for the equilibrium value of effort is Pareto optimal (this can be easily shown). While in principle the negative root may also be part of a Nash equilibrium, we assume going forward that the Pareto dominant solution will be chosen. The constraint that $\widehat{q} \leq 1$ reflects the fact that $\widehat{q}$ is the probability of project success and hence cannot exceed 1. Throughout, we focus on the case where there is an interior solution for the firm's effort, so that $\widehat{q} < 1$. It is straightforward to see that parameter values exist that guarantee $\widehat{q} < 1$ in equilibrium. We also assume that financing is viable, which amounts to assuming that $\widehat{q}$ is a real variable. A sufficient condition to guarantee this is that $y^2 - 4cr^* > 0$. We come back to this issue later when we explore the conditions under which investment, which entails financing, is feasible.

We can now invert the expression for optimal effort to obtain $\widehat{r}_L = y - \widehat{q}c$, which, after substituting for $\widehat{q}$ yields

$$\widehat{r}_L = y - c\frac{y + \sqrt{y^2 - 4cr^*}}{2c} = \frac{y}{2} - \frac{\sqrt{y^2 - 4cr^*}}{2}.$$

Using the optimal value $\widehat{q}$, we can write the equilibrium expected profits as

$$\begin{aligned} \Pi &= \widehat{q}(y - r_L) - \frac{1}{2c}(y - r_L)^2 = \frac{1}{c}(y - r_L)^2 - \frac{1}{2c}(y - r_L)^2 \\ &= \frac{1}{2c}(y - r_L)^2. \end{aligned}$$

Substitute now for the optimal $\widehat{r}_L$ to obtain

$$\widehat{\Pi} = \frac{1}{2c}\left(\frac{y + \sqrt{y^2 - 4cr^*}}{2}\right)^2.$$

Finally, we can use the interest rate parity condition, (1), to write this as a function of the foreign risk free rate, r^{*f}, and the expected expected exchange rate movement, $\widehat{e}$, and the probability of devaluation, $1 - \alpha$:

$$\widehat{\Pi} = \frac{1}{2c}\left(\frac{y + \sqrt{y^2 - 4c\left(r^{*f} + (1 - \alpha)\frac{\Delta e}{e}\right)}}{2}\right)^2. \tag{3}$$

3.2 Foreign currency borrowing

When firms borrow in the foreign currency, they become exposed to exchange rate risk. In particular, in case of a sharp depreciation of the domestic currency, they may be unable to repay their loans even though if their projects are successful (this is because the project is still run domestically, so the return y is denominated in the domestic currency, but the repayment r_L is in the foreign currency). Assume for simplicity (we may generalize this later) that the possible depreciation Δe is large enough that, in the even of a depreciation, the firm would go bust and default on its loan. A condition sufficient to guarantee this is that

$$y - r_L^f \frac{\Delta e}{e} < 0.$$

Similar to above, we can write a firm's expected profit when it borrows in the foreign currency as

$$\Pi^f = q(y - r_L^f)\alpha - \frac{c}{2}q^2,$$

which reflects the fact that the project only pays off with probability q, but also the firm's return net of loan repayment is only positive if the domestic currency remains stable and does not depreciate. We can maximize these profits Π^f with respect to effort to obtain

$$\widehat{q}_f = \min\left\{\left(\frac{y - r_L^f}{c}\right)\alpha, 1\right\}. \tag{4}$$

As above, we will focus on the case where an interior solution exists, so that $\widehat{q}_f < 1$.

Now, since firms only repay when the currency remains stable, for banks/investors to be willing to lend in the foreign currency the interest rate needs to compensate them for both the borrower idiosyncratic risk, $1 - q$, and the devaluation risk, $1 - \alpha$. Assume for now the extreme (and unrealistic) case that under devaluation, the lender receives nothing back from the borrower. This would be consistent, for instance, with a very large devaluation that leaves little on the firm's balance sheet, and which subsequently gets lost as part of bankruptcy proceedings. As we show later, this extreme assumption in fact biases against borrowing in the foreign currency. Under this assumption, we have, given a conjectured level of effort q_f^C, that the promised repayment on the foreign loan, r_L^f, must satisfy

$$q_f^C \alpha r_L^f = r^{*f}.$$

From this we can solve for the equilibrium foreign denominated loan rate, $\widehat{r}_L^f$, as

$$\widehat{r}_L^f = \frac{r^{*f}}{q_f^C \alpha}.$$

As above, we can now substitute $\widehat{r}_L^f$ into the expression for the optimal effort $\widehat{q}_f$ given in (4) and solve for $\widehat{q}_f$ to obtain

$$\widehat{q}_f = \frac{1}{2c}\left(y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}}\right).$$

Noting that $\widehat{r}_L^f = y - \frac{c\widehat{q}_f}{\alpha}$, we can substitute for $\widehat{q}_f$ and obtain

$$\widehat{r}_L^f = y - \frac{c\frac{1}{2c}\left(y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}}\right)}{\alpha} = \frac{y}{2} - \frac{\sqrt{y^2\alpha^2 - 4cr^{*f}}}{2\alpha},$$

which gives us the equilibrium loan rate when the firm borrows in the foreign currency.

Given the optimal loan rate $\widehat{r}_L^f$ and effort level $\widehat{q}_f$, we can replace these in the expression for the firm's expected equilibrium profit as

$$\widehat{\Pi}^f = \widehat{q}_f(y - \widehat{r}_L^f)\alpha - \frac{c}{2}\left(\widehat{q}_f\right)^2 = \left(\frac{y - \widehat{r}_L^f}{c}\right)\alpha(y - \widehat{r}_L^f)\alpha - \frac{1}{2c}\alpha^2\left(y - \widehat{r}_L^f\right)^2.$$

Simplifying, this becomes

$$\widehat{\Pi}^f = \frac{1}{2c}\left(\frac{y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}}}{2}\right)^2, \tag{5}$$

which again expresses the firm's equilibrium profit as a function of the foreign risk free rate.

3.3 Optimal borrowing

One important question of interest now is under what conditions, if any, firms prefer to borrow in foreign rather than domestic currency. We can now state the following result.

Proposition 1 *Keeping the size of the expected devaluation, $(1-\alpha)\frac{\Delta e}{e}$, constant, there exists a value $\underline{\alpha} < 1$ such that if the probability of no devaluation, α, is greater than $\underline{\alpha}$, firms prefer to borrow in the foreign currency rather than the domestic currency.*

Proof: A firm will prefer to borrow in the foreign currency if $\widehat{\Pi}^f > \widehat{\Pi}$. Using (3) and (5), we can write this inequality as

$$\frac{1}{2c}\left(\frac{y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}}}{2}\right)^2 > \frac{1}{2c}\left(\frac{y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}}{2}\right)^2.$$

Rewriting yields

$$y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}} > y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}. \tag{6}$$

If (6) is satisfied, then borrowing in the foreign currency will be optimal for the firm. From here, it is immediate that as α and $\frac{\Delta e}{e}$ increase so as to keep $(1-\alpha)\frac{\Delta e}{e}$ constant, hence keeping the domestic risk-free rate constant, $\widehat{\Pi}^f$ increases while $\widehat{\Pi}$ remains constant. As $\alpha \to 1$, the left hand side converges to $y + \sqrt{y^2 - 4cr^{*f}}$, which is strictly greater than $y + \sqrt{y^2 - 4cr^{*}}$ since $r^{*f} > r^*$ whenever there is a positive risk of a devaluation. □

Proposition 1 establishes that an increase in the size of a (very large) devaluation that occurs with only a small probability - a "peso-problem" - favors foreign currency lending. One simple interpretation of this is as a fixed exchange rate regime where a change in the foreign risk free rate, r^{*f}, is immediately reflected onto an equal change in the domestic rate r^* because of the free flow of capital and the fact that with a (credible) fixed exchange rate regime expectations of a devaluation will not be affected by the change in the foreign interest.

By contrast, if α is low but the expected depreciation is also small (to maintain consistency with the interest rate parity condition), then profits are higher when borrowing in domestic currency. In other words, if there is a relatively high probability of a small devaluation, borrowing in the foreign currency is suboptimal because it exposes the firm to an additional large risk - the probability $1-\alpha$ of a devaluation - but little benefit since they only enjoy the reduced cost of borrowing $\widehat{r}_L^f$ when the project both succeeds and there is no devaluation. As above, this can be interpreted as a flexible exchange rate regime, where adjustments in the exchange rate are more frequent (i.e., lower α) but also typically smaller. We note, however, that (6) is written entirely in terms of the foreign rate r^{*f}, meaning that throughout we are assuming that parity is maintained by adjustments in the domestic rate r^*. In other words, the exercise conducted here cannot readily be interpreted as representing a flexible (i.e., fully floating) exchange rate regime. We discuss this case in more detail later.

As a final point, we show that the assumption that under devaluation the lender receives no repayment actually biases the firm against foreign borrowing. Suppose that instead under devaluation there is some residual amount less than what is promised to the lender, r_L^f, and which the lender can recover in case of default. The expression for the firm's profit will remain unchanged since under devaluation, which occurs with probability $1-\alpha$, there will still be nothing left for the firm. However, since the lender recovers something, the equilibrium loan rate r_L^f should be lower, ceteris paribus. Given that the optimal effort decision for the firm is given by $\widehat{q}_f = \left(\frac{y-r_L^f}{c}\right)\alpha$, this implies that $\widehat{q}_f$ will be higher, so that the firm's project is more likely to payoff. This has an additional feedback effect onto r_L^f, since the loan rate will also be lower when the probability of full repayment, $\widehat{q}_f$, increases. Both of these effects together imply that the firm's equilibrium expected profit $\widehat{\Pi}^f$ will be higher when the lender obtains some recovery in case of devaluation. Therefore, the assumption we used above, that no such recovery exists, in fact biases our results against the optimality of foreign denominated borrowing.

4 Firm risk and capital flows

When capital can move across international boundaries with little frictions, the results above show that firms may sometimes find it optimal to obtain foreign currency denominated loans. The reason is that borrowing in the foreign currency acts as a bonding mechanism for the firm: since the risk free rate abroad is lower, the firm is able to obtain a lower interest rate loan as foreign lenders have a lower threshold for lending. This creates a higher return for the firm whenever its project is successful, and provides the firm with a greater incentive to ensure that its return actually materializes, i.e., to put in more effort. The trade-off, however, is that borrowing in the foreign currency exposes the firm to more risk since it introduces devaluation risk into the firm's expected profit function. Whether or not firms control risk to a greater extent is thus an open question. In this section we study how firm's choice of currency in which to borrow affects their optimal effort decisions and hence the risk of bankruptcy.

Proposition 2 *Whenever it is optimal to borrow in the foreign currency, so that $\widehat{\Pi}^f > \widehat{\Pi}$, then the firm also exerts more effort and reduces risk more when borrowing in the foreign currency than when borrowing in the domestic currency: $\widehat{q}_f > \widehat{q}$.*

Proof: When the firm borrows in the domestic currency, the optimal effort is given by

$$\widehat{q} = \frac{y + \sqrt{y^2 - 4cr^*}}{2c} = \frac{y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}}{2c}.$$

By contrast, when it borrows in the foreign currency, optimal effort is

$$\widehat{q}_f = \frac{y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}}}{2c}.$$

From this, $\widehat{q}_f > \widehat{q} \Leftrightarrow$

$$y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}} > y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)},$$

which is the exact same condition that guarantees $\widehat{\Pi}^f > \widehat{\Pi}$. $\square$

The proposition establishes an equivalence between a firm's effort or, equivalently, risk taking, and its optimal choice of financing arrangement. Specifically, it establishes that the preferred denomination of debt, in the sense of maximizing the firm's profit, is also the one that minimizes risk, implying that firms' decisions to maximize profits go hand in hand with risk reduction.

However, it may still be that firm risk may correlate to a greater extent with the choice of one currency versus the other. As we show in the previous section, changes in devaluation risk will shift a firm's choice between one type of debt and the other, e.g., switching from domestic to foreign borrowing or vice versa. To analyze this issue formally, it is useful to consider the marginal firm that is essentially indifferent between borrowing from a foreign lender versus obtaining a loan from a domestic lender. We first establish the following preliminary result.

Lemma 1 *For large α, there is always a value of c low enough such that $\widehat{\Pi}^f < \widehat{\Pi}$.*

Proof: Recall that $\widehat{\Pi}^f > \widehat{\Pi}$ if and only if

$$y\alpha + \sqrt{y^2\alpha^2 - 4cr^{*f}} > y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}.$$

Assuming an interior solution continues to hold in both cases, it is straightforward to see that a marginal reduction in c increases the right hand side more than the left hand side. Note, however, that as c decreases toward zero, $q, \widehat{q}_f \to 1$. Consider the firm's profit at the limit in both cases:

$$\begin{aligned}
\widehat{\Pi} &= (y - \widehat{r}_L) - \frac{c}{2} \\
\widehat{\Pi}^f &= (y - \widehat{r}_L^f)\alpha - \frac{c}{2}.
\end{aligned}$$

Since $\widehat{r}_L = \frac{r^*}{\widehat{q}} = r^*$ and $\widehat{r}_L^f = \frac{r^{*f}}{\widehat{q}_f \alpha} = \frac{r^{*f}}{\alpha}$, we have that $\widehat{\Pi}^f = (y - \widehat{r}_L^f)\alpha - \frac{c}{2} = (y - \frac{r^{*f}}{\alpha})\alpha - \frac{c}{2} = \alpha y - r^{*f} - \frac{c}{2}$, while $\widehat{\Pi} = (y - \widehat{r}_L) - \frac{c}{2} = y - r^* - \frac{c}{2}$, so that $\widehat{\Pi} > \widehat{\Pi}^f \Leftrightarrow y - r^* - \frac{c}{2} > \alpha y - r^{*f} - \frac{c}{2} \Leftrightarrow y\left(1-\alpha\right) > r^* - r^{*f}$. This will be satisfied for large α, i.e., for α sufficiently close to 1. $\square$

The opposite result may also be true as long as increases in c do not make lending in one market non viable. It is easy to see that parameter values exist so that for high effort costs c, foreign denominated debt is always preferred. The intuition for this stems from noting that the cost parameter c is a measure of the agency cost for the firm. High agency cost firms, i.e., firms with high c, have a greater need for bonding themselves and thus committing to exert more effort. This is achieved by borrowing in the foreign "hard" currency, which is not subject to devaluation.

With this, we can now see that for a wide variety of instances, a value of c exists for which $\widehat{\Pi}^f = \widehat{\Pi}$, so that the firm is indifferent between borrowing in the foreign or in the domestic currency. Call such a value $\widehat{c}$. We can now establish the following.

Proposition 3 *Under fixed exchange rates, so that $\Delta r^* = \Delta r^{*f}$, the cutoff value $\widehat{c}$ is decreasing in the foreign risk free rate r^{*f}: $\frac{\partial \widehat{c}}{\partial r^{*f}} < 0$.*

Proof: Start from the equality between the profits. After defining $F = \frac{\Delta e}{e}$, we can write:

$$Z = -y\left(1-\alpha\right) + \sqrt{y^2\alpha^2 - 4cr^{*f}} - \sqrt{y^2 - 4c\left(r^{*f} + \left(1-\alpha\right)F\right)} = 0$$

Taking the derivative with respect to c, we get:

$$\frac{\partial Z}{\partial c} = \frac{2\left(r^{*f} + F\left(1-\alpha\right)\right)}{\sqrt{y^2 - 4c\left(r^{*f} + \left(1-\alpha\right)F\right)}} - \frac{2r^{*f}}{\sqrt{y^2\alpha^2 - 4cr^{*f}}},$$

and substituting from Z, we get

$$\frac{\partial Z}{\partial c} = \frac{2\left(r^{*f} + F\left(1-\alpha\right)\right)}{-y\left(1-\alpha\right) + \sqrt{y^2\alpha^2 - 4cr^{*f}}} - \frac{2r^{*f}}{\sqrt{y^2\alpha^2 - 4cr^{*f}}} > 0,$$

which is positive since $F > 0$ and $y\left(1-\alpha\right) > 0$.

Taking the derivative with respect to r^{*f} we get:

$$\frac{\partial Z}{\partial r^{*f}} = \frac{2c}{\sqrt{y^2 - 4c\left(r^{*f} + \left(1-\alpha\right)F\right)}} - \frac{2c}{\sqrt{y^2\alpha^2 - 4cr^{*f}}},$$

which after substituting from Z is

$$\frac{\partial Z}{\partial r^{*f}} = \frac{2c}{-y\left(1-\alpha\right) + \sqrt{y^2\alpha^2 - 4cr^{*f}}} - \frac{2c}{\sqrt{y^2\alpha^2 - 4cr^{*f}}} > 0.$$

Using the implicit function theorem, we can then write

$$\frac{d\widehat{c}}{dr^{*f}} = -\frac{\frac{\partial Z}{\partial r^{*f}}}{\frac{\partial Z}{\partial c}} < 0,$$

as desired. □

The proposition establishes that increases in the foreign risk free rate r^{*f} make foreign denominated borrowing relatively more attractive, which translates formally here into the fact that the marginal foreign-currency borrower will have a lower cost of effort c. Put differently, if for each borrower c is assumed to be drawn from some fixed distribution, this would imply that the number of firms that borrow in the foreign currency increases as r^{*f} increases.

At first sight, this result seems counterintuitive since it suggests that firms prefer to borrow in the currency whose loans are becoming relatively more expensive. Note, however, that here we are assuming that changes in r^{*f} are fully and instantly reflected in domestic interest rates, r^*. Therefore, increases in r^{*f} reflect overall increases in the cost of borrowing no matter in which currency the firm borrows. However, as the risk free rate rises, the agency problem associated with limited liability increases since the loan rate r_L will also rise to reflect the increase in the risk free rate. Ceteris paribus, this reduces effort q, and leads to a yet higher loan rate since lenders need to be compensated for this increased risk.

The firm can, however, partially bond itself by borrowing from abroad. This bonding has the most value precisely when effort q is likely to be lowest, since the loan rate, for domestic debt, is $\widehat{r}_L = \frac{r^*}{\widehat{q}}$ and for foreign debt is $\widehat{r}_L^f = \frac{r^{*f}}{\widehat{q}_f \alpha}$. Therefore, a firm is more likely to find it optimal to borrow in the foreign currency when the risk free rates go up everywhere.

As a final point, it is useful to consider the opposite case where there is a floating exchange rate regime, which we define as a situation where the adjustment to a foreign interest rate shock occurs entirely through an adjustment in the expected exchange rate depreciation. We can rewrite Z as

$$Z = -y\left(1 - \alpha(r^{*f})\right) + \sqrt{y^2\left(\alpha(r^{*f})\right)^2 - 4cr^{*f}} - \sqrt{y^2 - 4cr^*} = 0$$

where the domestic interest rate r^* is assumed to remain constant under the assumption of fully flexible exchange rates. The derivative of Z with respect to c is the same as in the case with fixed rates.

$$\frac{\partial Z}{\partial c} = \frac{2\left(r + F\left(1-\alpha\right)\right)}{-y\left(1-\alpha\right) + \sqrt{y^2\alpha^2 - 4cr^{*f}}} - \frac{2r^{*f}}{\sqrt{y^2\alpha^2 - 4cr^{*f}}} > 0$$

The derivative with respect to the foreign interest rate r becomes:

$$\frac{\partial Z}{\partial c} = y\frac{\partial \alpha\left(r^{*f}\right)}{\partial r^{*f}} + \frac{y^2\alpha\left(r^{*f}\right)\frac{\partial \alpha\left(r^{*f}\right)}{\partial r^{*f}} - 2c}{\sqrt{y^2\alpha^2\left(r^{*f}\right) - 4cr^{*f}}}$$

where is α is function of r^{*f}. Now if $\frac{\partial \alpha\left(r^{*f}\right)}{\partial r^{*f}} \leq 0$, then $\frac{\partial Z}{\partial c} < 0$ and $\frac{d\widehat{c}}{dr^{*f}} > 0$. However, if $\frac{\partial \alpha\left(r^{*f}\right)}{\partial r^{*f}}$ is sufficiently positive, the opposite would happen. The second is economically irrelevant given the problem we want to study, allowing us to conclude that in this case $\frac{d\widehat{c}}{dr^{*f}} > 0$.

4.1 Quantity effects

Consider the marginal borrower that just breaks even at the prevailing interest rate conditions (the current level of the risk-free rate). In this economy there will be credit rationing (a la Stiglitz and Weiss, 1981) as borrowers cannot commit to a certain level of effort. Essentially, for borrowers with a high effort cost there is no interest rate at which the lender can break even once they take into account the effect that the interest rate has on effort. Formally, these are the borrowers for which (2) and/or (4) does not admit a real solution. If a solution exists, however, then financing is feasible and the firm is able to obtain credit. An interesting question is whether there credit is more likely to be available, or credit rationing is likely to be less severe, when borrowing in one type of currency versus the other.

To study this issue, we define $\overline{c}$ as the effort cost of the marginal borrower that is able to obtain credit domestically.[4] Likewise, we use $\overline{c}^f$ to denote the effort cost of the marginal borrower that can obtain foreign currency denominated debt.

Proposition 4 *Keeping the expected devaluation, $(1-\alpha)\frac{\Delta e}{e}$, constant, when the risk of devaluation is sufficiently low (i.e., α is large) but the size of the possible devaluation is large ($\frac{\Delta e}{e}$ is large), we have $\overline{c}^f > \overline{c}$.*

Proof: The marginal borrower in domestic currency is one for whom $y^2 - 4cr^* = 0$, which after some rearranging delivers the following threshold value of c:

$$\overline{c} = \frac{y^2}{4\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}.$$

[4]More precisely, given that the cost of effort q is $\frac{c}{2}q^2$, $\overline{c}$ is the threshold value of the parameter for the cost function above which (i.e., for $c > \overline{c}$) firms are unable to obtain credit.

The equivalent threshold value for foreign currency borrowing is:

$$\overline{c}^f = \frac{\alpha^2 y^2}{4r^{*f}}.$$

Comparing the two cutoffs, it is immediate that

$$\overline{c}^f > \overline{c} \Leftrightarrow \frac{\alpha^2}{r^{*f}} > \frac{1}{r^{*f} + (1-\alpha)\frac{\Delta e}{e}},$$

or, rearranging,

$$\overline{c}^f > \overline{c} \Leftrightarrow (1-\alpha)\frac{\Delta e}{e} > \frac{\left(1-\alpha^2\right) r^{*f}}{\alpha^2}. \tag{7}$$

As for the difference in profits, (7) can be always satisfied by increasing α and $\frac{\Delta e}{e}$ so that $(1-\alpha)\frac{\Delta e}{e}$ remains constant. □

Proposition 4 highlights again the effect of "peso-problem" conditions, this time on firms' access to credit. The proposition establishes that, under these conditions where severe devaluations are possible but rare, firms with more severe agency problems will have access to foreign currency credit but not to domestic denominated credit. The reason is that, for firms with severe agency problems for whom the cost of effort c is relatively high, the only way to get financing is to use foreign denominated debt as a bonding mechanism, and take advantage of the lower cost of borrowing, r^{*f}, in the event of no devaluation. However, such a mechanism is only possible when the risk of devaluation is not too large, even if the trade-off is a larger devaluation when and if it occurs.

Consider the related question of who borrows in what currency. From (6) it is evident that, ceteris paribus, relatively high c firms will favor foreign currency borrowing more than relatively low c firms. As an extreme example, consider a firm with $c = 0$ and, hence, $\widehat{q} = 1$ irrespective of the currency denomination of debt, and for whom (6) is never satisfied, so that he always finds it preferable to borrow in the domestic currency. This means that if (6) does not hold for the firms with the highest cost c in our distribution, then there will be no foreign currency lending. If it does, there may still be some domestic currency lending if the range of c gets close enough to zero (i.e., if there are firms that can borrow close to the risk-free rate).

5 Contagion

So far we have examined each borrower's risk of failure in isolation. Entrepreneurs borrowing in domestic currency are exposed exclusively to their own idiosyncratic risk. Those that borrow in foreign currency are also exposed to devaluation risk, but their failure and, hence, the fact that they borrow in foreign currency, does not have any implication for the creditworthiness of other firms. This is obviously a simplification as we can envisage several circumstances under which widespread bankruptcies would have a negative impact on other firms' creditworthiness. Consider, for instance, about real sector (macro) linkages: it may be difficult for a firm to repay its loans when many of its customers have gone bust, even if its own products are technically a success. Or bankruptcies upstream in the supply chain may cause production delays and financial difficulties downstream.

In this section, we modify our simple model to examine these issues. We assume that in the case of widespread bankruptcies, all entrepreneurs are at risk of failure independently of the realization of their idiosyncratic projects. More formally, we assume that if a fraction of borrowers greater than $1-\mu<1$ defaults, firms whose projects were successful may nevertheless default with probability θ. The variable θ represents a form of "contagion risk" which may change the incentive structure in the model. In particular, contagion risk introduces an element of interaction among firms through the externality associated with a sufficiently large number of failures. A firm's effort level now affects another firm's expected payoff through its impact on the probability that a sufficiently large portion of borrowers fails. This externality reduces the individual payoff from succeeding and, thus, lowers the equilibrium monitoring effort. For a broad range of parameters this may also lead to multiple equilibria since an individual's effort depends on her beliefs about the effort of others. A full analysis of this modified model is in the appendix. Here we focus on the most interesting case in which contagion risk is associated with foreign currency borrowing.

Start from the case without foreign currency borrowing. First, note that with a continuum of firms, in a symmetric equilibrium where each firm chooses the same effort q, exactly a portion $1-q$ of firms will fail. Then, given $1-\mu$ is the threshold for contagion risk, if $q<\mu$ each firm faces a contagion risk of θ, i.e., with probability θ the firm fails irrespective of her realization of her project. Then, the expected profits for firm i become:

$$\Pi=\begin{cases} q_i(y-r_L)-\frac{c}{2}q_i^2, \text{ for } q_{-i}>\mu \\ q_i\left(1-\theta\right)\left(y-r_L\right)-\frac{c}{2}q_i^2, \text{for } q_{-i}<\mu \end{cases}$$

where q_{-i} are the entrepreneur's beliefs about the level of effort to be exerted by other firms.

Depending on its beliefs, firm i will choose

$$\widehat{q}_i = \begin{cases} \frac{y-r_L}{c}, \text{ for } q_{-i} > \mu \\ \frac{(y-r_L)(1-\theta)}{c}, \text{ for } q_{-i} < \mu \end{cases}$$

Now consider the case where $\mu < \frac{(y-r_L)(1-\theta)}{c}$. In the absence of foreign currency lending, there is never a symmetric equilibrium that involves contagion because the belief that $q_{-i} < \mu$ cannot be correct (things are different when $\mu > \frac{(y-r_L)(1-\theta)}{c}$, see the appendix for details).

Now let us introduce foreign currency lending. Suppose that the firms are heterogenous in their effort cost c. In particular, assume that a mass φ of firms has low effort costs c_1, and a mass $1-\varphi$ high effort costs c_2, with $c_2 > \widehat{c} > c_1$, where $\widehat{c}$ is the threshold value of the cost below which firms borrow in the domestic currency and above which they borrow in the foreign currency in the absence of contagion risk (see Section 4). This means that, absent the possibility of contagion, a mass φ entrepreneurs borrow in domestic currency and the rest in foreign currency (fx). As above, assume that $\mu < \frac{(y-r_L)(1-\theta)}{c_2}$, which implies that in the absence of devaluation there cannot be contagion. Under these assumptions, consider again the expected profits expressions for borrowing in foreign and domestic currency. The former remains identical to the case without contagion. This is obvious, since contagion occurs only conditional on devaluation and conditional on devaluation firms that borrowed in fx fail anyway. As we show below, however, the firm's expected profit when borrowing in the domestic clear is not identical to that in case without contagion since domestic firms may now be at risk of failure as a result of a devaluation that causes all the firms that borrowed in the foreign currency to fail. The question is, then, whether there are entrepreneurs that are induced to switch from domestic-currency to fx borrowing by the presence of contagion risk associated with a devaluation.

By construction, if firms ignore the risk of contagion and behave as if contagion were not possible, then indeed there would be no equilibrium involving contagion risk in this economy if φ, the fraction of firms with relatively lost effort cost, is very high. However, when φ is relatively low, domestic currency borrowers become exposed to contagion risk through the correlated default of fx borrowers. In this case, the expected profit for borrowing in domestic currency becomes

$$\Pi_C = q\left(\theta\alpha + 1 - \theta\right)\left(y - r_L\right) - \frac{c_1}{2}q^2,$$

where the subscript C refers to the profits under the risk of contagion. We can write the first order condition for effort q when borrowing in domestic currency as

$$(\theta\alpha + 1 - \theta)(y - r_L) - c_1 q = 0,$$

which yields

$$\widehat{q} = \min\{\frac{(\theta\alpha + 1 - \theta)(y - r_L)}{c_1}, 1\}.$$

Lenders will price these loans according to their probability of repayment, so that, for a conjectured effort level q^C, the loan rate must satisfy

$$\widehat{r}_L = \frac{r^*}{q^C(\theta\alpha + 1 - \theta)}. \tag{8}$$

We can immediately see that contagion affects the fx/local currency choice through three channels: 1) it directly affects the expected profits from borrowing in domestic currency; 2) it reduces the optimal level of effort when borrowing in domestic currency; and 3) it increases the interest rate for loans in domestic currency beyond the amount caused by the reduction in effort q (the probability of repayment drops from q to $\alpha q + (1 - \alpha)(1 - \theta) q$).

For ease of exposition define $\xi = \theta\alpha + 1 - \theta$. Then, by substituting 8 into the expression for $\widehat{q}$ and solving we obtain

$$\widehat{q} = \min\{\frac{1}{2c_1}\left(y\xi + \sqrt{y^2\xi^2 - 4c_1 r^*}\right), 1\}.$$

Noting that, from this, $\widehat{r}_L = y - \frac{c_1\widehat{q}}{\xi}$, we can substitute for $\widehat{q}$ and obtain

$$\widehat{r}_L = \frac{y}{2} - \frac{\sqrt{y^2\xi^2 - 4c_1 r^*}}{2\xi},$$

which gives us the equilibrium loan rate when the firm borrows in the domestic currency under the risk of contagion.

As in the previous section, we can replace $\widehat{r}_L$ and $\widehat{q}$ in the expression for the firm's expected equilibrium profit and obtain

$$\widehat{\Pi}_C = \frac{1}{2c_1}\left(\frac{y\xi + \sqrt{y^2\xi^2 - 4c_1 r^*}}{2}\right)^2, \tag{9}$$

from which it is immediate that, since $\theta > 0$ implies $\xi < 1$, we must have $\widehat{\Pi}_C < \widehat{\Pi}$. That is, the risk of contagion reduces the expected profits from borrowing in domestic currency. We can now derive a simply implication related to firms' choice of borrowing in the domestic versus the foreign currency.

Lemma 2 *The threshold value of effort cost c below which borrowing in the domestic currency is optimal, denoted by $\widehat{c}$, is lower when $\theta > 0$, so that contagion is possible, than when $\theta = 0$ and no possibility of contagion exists.*

Proof: The threshold value $\widehat{c}$ comes from comparing a firm's profit when it borrows domestically, $\widehat{\Pi}$, to what it obtains when borrowing in fx, $\widehat{\Pi}^f$: $\widehat{c}$ is the value of c that satisfies $\widehat{\Pi} = \widehat{\Pi}^f$. As established in (9), however, $\widehat{\Pi}_C < \widehat{\Pi}$, while profits when borrowing in fx are invariant to the possibility of contagion. Therefore, if at $c = \widehat{c}$ we have $\widehat{\Pi} = \widehat{\Pi}^f$, we must also then have $\widehat{\Pi}_C < \widehat{\Pi}^f$, so that fx borrowing is strictly preferred. This implies that the threshold value must be lower in order for firms to be indifferent. □

The lemma describes a relatively straightforward implication of the analysis above, which is that the threshold value of the effort cost below which borrowing in the domestic currency is optimal decreases once contagion risk is present. One stark example of the possible effect associated with this shift in firms' preferences for domestic versus foreign currency borrowing can be obtained by considering the case when $c_1 = \widehat{c}$, where $\widehat{c}$ is as described above and represents the threshold cost when contagion is not possible. For this case, low cost firms in the absence of contagion are exactly indifferent between foreign currency and domestic currency borrowing. However, for any contagion risk $\theta > 0$, these firms will strictly prefer fx borrowing. By continuity, there will exist a θ such that for firms with $c \in [\underline{c}, \widehat{c})$ we will have $\widehat{\Pi} > \widehat{\Pi}_f > \widehat{\Pi}_C$: once contagion is possible, firms that would otherwise have borrowed in the domestic currency will instead choose to borrow in the foreign currency. In the context of our example here where there are only two types of firms, we get that the risk of contagion causes all firms to borrow in fx and as a result increases the degree of systemic risk to which all firms are exposed, increasing the likelihood that all firms fail at once in the event of a devaluation.

The implication of this result is that there are conditions under which measures aimed at preventing or limiting foreign currency borrowing can be welfare enhancing. Note, however, that these measures may not lead to Pareto improvements. Restrictions on foreign currency borrowing can prevent the risk of contagion and, thus, improve the incentives and profits of entrepreneurs that would otherwise switch to fx borrowing. But they do so at the cost of worse incentives and lower profits for those that would prefer to borrow in foreign currency irrespective of contagion risk.

6 Average Performance and Systemic Risk

The results in our model have a natural interpretation from the point of view of a trade-off between average performance and systemic risk. We assume risk neutrality throughout the model. Hence, average performance is all that matters. However, the model easily lends itself to the analysis of the risk/performance trade-off once one introduces a "risk-averse" regulator/social planner. Alternatively, the model could be augmented with a non-linear social cost of failure: if a few borrowers fail the costs are relatively low, but become very large if there are widespread failures and defaults.

With either one of these modifications, realizations involving a large mass of borrowers in default (systemic crises) weight negatively on the social welfare functions. This means that the social planner will be willing to trade average performance for a reduced probability of systemic crisis.

For simplicity, consider an economy where borrowers have a uniform effort cost coefficient c. Also, assume that condition (6) holds, so that if allowed, all entrepreneurs will borrow in foreign currency. It is easy to see that, under these conditions, a trade-off emerges. In the absence of foreign currency borrowing, the model delivers a predictable proportion of borrowers that default, $1-\widehat{q}$ (indeed, with a continuum of borrowers and no aggregate risk, the realized number of failures will be identical to the expected one). By contrast, when all entrepreneurs borrow in foreign currency, there will be a mass $1-\widehat{q}_f < 1-\widehat{q}$ of failures when the currency does not depreciate. But everybody will default (call this a systemic crisis) when it does. It follows that a social planner allowing foreign currency borrowing can obtain a reduction in "tranquil-times" failures of $\widehat{q}_f - \widehat{q}$ at the cost of a probability $1-\alpha$ of systemic crisis.

The rationale for government intervention and limits on foreign currency mismatches has an additional dimension when widespread defaults entail the possibility of contagion. As shown in the previous section, in cases where the threat of contagion induces a large fraction of borrowers to switch to foreign currency borrowing, limiting the use of fx can increase average performance (reduce defaults) while at the same time reducing systemic risk taking. The reason is that, through limiting currency mismatches, regulation can eliminate inefficiencies stemming from the externality associated with the risk of contagion. Doing so may be particularly important when contagion risk is high, since then the devaluation risk can trigger a larger systemic problem either directly through contagion, or through firm's increased desire to borrow in fx rather than in domestic currency.

7 Discussion and Conclusions

This paper presents a model where foreign currency borrowing, through a lower interest rate, may ameliorate agency problems between firms and lenders relative to borrowing in domestic currency. The reduction in idiosyncratic risk, however, comes at the cost of exposure to the risk of default should the currency devalue sharply. A trade-off emerges between average performance in tranquil times and systemic risk: foreign currency borrowing reduces the average number of failures in the economy, but will lead to widespread bankruptcies when the currency devalues. In addition, if widespread defaults can lead otherwise successful entrepreneurs to default (by triggering a deep recession, for instance), then a sufficiently large fraction of firms borrowing in foreign currency may induce others to switch to foreign currency borrowing as well. In this case, foreign currency borrowing may actually be welfare reducing beyond its effect through systemic risk.

Our results support the view that government intervention to curb foreign currency borrowing and the contagion risks associated with it may be socially optimal in certain circumstances. Such government intervention could come in the form of capital controls or prudential regulation, or some combination of the two.[5]

The analysis of specific measures for intervention is beyond the reach of our stylized model. In practice, however, the optimal response depends on the type of risk and firms that are being targeted. When the problem is primarily with banks and other intermediaries funding themselves in hard currency on international markets and lending domestically in local currency, bank regulatory measures aimed at limiting foreign currency mismatches - such as tightening open position limits, in relation to bank capital, and stepping up of foreign currency-related liquidity requirements - may be effective.

When the ultimate borrower (a firm or household) takes on foreign currency debt from a bank but its income is in local currency, so that the borrower is therefore unhedged, banks are exposed to devaluations through credit risk. In this case, however, especially in small open economies,

[5] See Ostry et al. (2011) for an overview of the considerations and tradeoffs involved in determining the optimal mix of macroeconomic policies, capital controls, and prudential regulation to manage foreign currency lending and capital inflows more generally.

regulatory measures such as higher capital requirements on foreign currency loans or limits on loans to borrowers who cannot demonstrate a natural hedge may be only partially effective. They can reduce the direct exposure of the local banking system to currency risk, but to the extent that real sector borrowers switch to foreign lenders, which may be possible through cross-border flows, they will not protect the economy as a whole. Then, the rationale for broader-reaching capital controls would have to be evaluated.

While we cast the analysis in the context of domestic- versus foreign-currency borrowing, several insights from our framework apply more broadly. In particular, the central finding that a reduction in idiosyncratic risk, and the related efficiency gains, may come at the cost of greater systemic risk applies to other contexts. For instance, consider the trade-off between fixed- and variable-rate debt contracts. Under normal conditions, short-term rates will be lower than long-term ones, allowing for better borrower incentives, much the way that foreign currency borrowing does in our model. However, such short term contracts will leave firms exposed to potentially sharp increases in their debt burden, in a similar fashion to how devaluation affects firms in our model. While interest rate changes will typically be small and gradual, unlike devaluation in our model, there are cases in which even marginal changes will imply payment difficulties for certain borrowers. For example, this kind of effect was observed for a large fraction of subprime borrowers when their contracts reset, suggesting that the basic ideas here may be applied to a broader context.

8 Appendix

Here, we describe more fully the case where contagion may arise, as presented in Section 5. Suppose that a fraction ϕ of firms borrow domestically, and the remaining fraction $1-\phi$ borrow in the foreign currency. Each firm borrowing domestically will fail with probability $1-\widehat{q}$, which implies that of the fraction ϕ firms that borrow domestically, a fraction $(1-\widehat{q})$ of them will fail. Of the foreign firms, each firm's individual probability of failure is $1-\alpha\widehat{q}^f$, where α is the probability of no devaluation. This also translates, in expecation, into the population at large: with probability α there is no devaluation, in which case a fraction $\widehat{q}^f$ of firms survive. With probability $1-\alpha$ there is a devaluation, in which case none of these firms survive. Therefore, the expected fraction of firms that survive is $\alpha\widehat{q}^f$.

Putting this together, it means that, with no devaluation, exactly a portion $1-Z=\phi(1-\widehat{q})+(1-\phi)\left(1-\widehat{q}^f\right)$ will fail. With devaluation, the portion becomes $1-z=\phi(1-\widehat{q})+(1-\phi)$. Suppose that if a fraction $1-\mu$ of firms fail, the rest may also suffer due to contagion, which arises with probability θ. Note that, if $Z<\mu$, a large fraction of firms fail even without a devaluation and the remainder will fail with probability θ as a result of contagion. If $z>\mu$, then even with a devaluation only a few firms fail, so there is no possibility of contagion. The more interesting case arises if $Z>\mu>z$, which means that the possibility of contagion only arises when there is a devaluation.

Suppose that all firms first simultaneously choose whether to borrow in either the domestic or the foreign currency. Then, after the distribution of borrowing denominations is observed by everyone, firms decide how much effort to put in.

We solve by backward induction and consider firm i's choice of effort q_i given the distribution of borrowing ϕ. Suppose first that firm i borrowed in the domestic currency. The profit function for firm i is

$$\Pi=\begin{cases} q_i(y-r_L)-\frac{c}{2}q_i^2 & \text{for } z>\mu \\ q_i(y-r_L)\left(\theta\alpha+1-\theta\right)-\frac{c}{2}q_i^2 & \text{for } Z>\mu>z \\ q_i\left(1-\theta\right)\left(y-r_L\right)-\frac{c}{2}q_i^2 & \text{for } Z<\mu \end{cases}$$

So depending on its beliefs on other firms' effort choices, firm i will choose

$$\widehat{q}_i=\begin{cases} \frac{y-r_L}{c} & \text{for } z>\mu \\ \frac{y-r_L}{c}\left(\theta\alpha+1-\theta\right) & \text{for } Z>\mu>z \\ \frac{(y-r_L)(1-\theta)}{c} & \text{for } Z<\mu \end{cases}$$

We can now impose symmetry and assume that all borrowers in the domestic currency behave symmetrically (those in the foreign currency may do something diferent).

To find the loan rate r_L, we analyze each case in turn. Suppose that $z > \mu$. Then we need $q_i r_L = r^*$. Substituting for $q_i = \frac{y-r_L}{c}$ gives us $\frac{y-r_L}{c} r_L = r^*$, or equivalently

$$-r_L^2 + y r_L - c r^* = 0,$$

which can be solved for r_L:

$$r_L = \frac{-y \pm \sqrt{y^2 - 4(-1)(-cr^*)}}{-2} = \frac{y \pm \sqrt{y^2 - 4cr^*}}{2}.$$

We can also plug into the equation for q_i to obtain

$$q_i = \frac{y - r_L}{c} = \frac{y - \frac{r^*}{q_i}}{c}.$$

This has the usual solution

$$q_i = \frac{y + \sqrt{y^2 - 4cr^*}}{2c}.$$

We can plug this back into the profit expression to get

$$\widehat{\Pi} = q_i (y - r_L) - \frac{c}{2} q_i^2 = \frac{c}{2} q_i^2 = \frac{c}{2} \left(\frac{y + \sqrt{y^2 - 4cr^*}}{2c} \right)^2.$$

Now consider the case where $Z < \mu$. Now we need $q_i (1-\theta) r_L = r^*$, or $r_L = \frac{r^*}{q_i(1-\theta)}$. Substitute into

$$\begin{aligned} q_i &= \frac{(y - r_L)(1-\theta)}{c} = \frac{\left(y - \frac{r^*}{q_i(1-\theta)}\right)(1-\theta)}{c} \\ &= \frac{(1-\theta) y - \frac{r^*}{q_i}}{c}. \end{aligned}$$

Can solve for q_i as

$$q_i = \frac{(1-\theta) y + \sqrt{(1-\theta)^2 y^2 - 4cr^*}}{2c}.$$

Now plug into the profit expression,

$$\widehat{\Pi} = q_i (1-\theta)(y - r_L) - \frac{c}{2} q_i^2 = \frac{c}{2} q_i^2 = \frac{c}{2} \left(\frac{(1-\theta) y + \sqrt{(1-\theta)^2 y^2 - 4cr^*}}{2c} \right)^2.$$

Finally, take the case where $Z > \mu > z$. We know that loan interest rates must reflect the probability of default, or in other words $E[\widehat{q}]\,(\theta\alpha + 1 - \theta)\,r_L = r^*$or

$$r_L = \frac{r^*}{E[\widehat{q}]\,(\theta\alpha + 1 - \theta)}$$

Here it is convenient to define $\xi = \theta\alpha + 1 - \theta$, so that we can write $r_L = \frac{r^*}{E[\widehat{q}]\xi}$. We can then substitute into the expression for $\widehat{q}$ and solve to get

$$\widehat{q} = \frac{1}{2c}\left(y\xi + \sqrt{y^2\xi^2 - 4cr^*}\right)$$

with expected profits of

$$\widehat{\Pi} = \frac{1}{2c}\left(\frac{y\xi + \sqrt{y^2\xi^2 - 4cr^*}}{2}\right)^2 = \frac{c}{2}\left(\frac{y\,(\theta\alpha + 1 - \theta) + \sqrt{y^2\,(\theta\alpha + 1 - \theta)^2 - 4cr^*}}{2c}\right)^2$$

If borrowing in the foreign currency, the profit function for firm i is

$$\Pi^f = \begin{cases} q_i(y - r_L^f)\alpha - \frac{c}{2}q_i^2 & \text{for } z > \mu \\ q_i(y - r_L^f)\alpha - \frac{c}{2}q_i^2 & \text{for } Z > \mu > z \\ q_i\,(1-\theta)\,(y - r_L^f)\alpha - \frac{c}{2}q_i^2 & \text{for } Z < \mu \end{cases}$$

Note that the first and the second cases are the same since in the event of a devaluation, which occurs with probability $1 - \alpha$, the firm fails independently of whether contagion spreads to other firms or not. From this, we can solve for the equilibrium level of effort:

$$\widehat{q_i}^f = \begin{cases} \frac{(y - r_L^f)\alpha}{c} & \text{for } z > \mu \\ \frac{(y - r_L^f)\alpha}{c} & \text{for } Z > \mu > z \\ \frac{(1-\theta)(y - r_L^f)\alpha}{c} & \text{for } Z < \mu \end{cases}$$

Now again solve for the loan rate, r_L^f. In case $z > \mu$ or $Z > \mu > z$, we have that $q_i\alpha r_L^f = r^{*f}$, or $r_L^f = \frac{r^{*f}}{q_i\alpha}$. Substitute into q_i to find

$$\widehat{q}_i^f = \frac{(y - r_L^f)\alpha}{c} = \frac{(y - \frac{r^{*f}}{\widehat{q}_i^f\alpha})\alpha}{c} = \frac{\alpha y - \frac{r^{*f}}{\widehat{q}_i^f}}{c},$$

which can be solved for $\widehat{q}_i^f$ as

$$\widehat{q}_i^f = \frac{\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}.$$

Profits are

$$\widehat{\Pi}^f = \frac{c}{2}q_i^2 = \frac{c}{2}\left(\frac{\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}\right)^2.$$

Finally, the case where $Z < \mu$ can be solved analogously. The loan rate is determined by $q_i (1-\theta) \alpha r_L^f = r^{*f}$, or $r_L^f = \frac{r^{*f}}{q_i(1-\theta)\alpha}$. Substitute into the equation for effort to obtain

$$\widehat{q}_i^f = \frac{(1-\theta)\,\alpha(y - r_L^f)}{c} = \frac{(1-\theta)\,\alpha(y - \frac{r^{*f}}{\widehat{q}_i^f (1-\theta)\alpha})}{c} = \frac{(1-\theta)\,\alpha y - \frac{r^{*f}}{\widehat{q}_i^f}}{c},$$

which solving yields

$$\widehat{q}_i^f = \frac{\alpha\,(1-\theta)\,y + \sqrt{\alpha^2\,(1-\theta)^2\,y^2 - 4cr^{*f}}}{2c}.$$

Finally, plug into the profits to get

$$\widehat{\Pi}^f = \frac{c}{2} q_i^2 = \frac{c}{2}\left(\frac{\alpha\,(1-\theta)\,y + \sqrt{\alpha^2\,(1-\theta)^2\,y^2 - 4cr^{*f}}}{2c}\right)^2.$$

Summarizing, we have, for domestic borrowing:

$$\widehat{\Pi} = \begin{cases} \frac{c}{2}\left(\frac{y+\sqrt{y^2-4cr^*}}{2c}\right)^2 & \text{for } z > \mu \\ \frac{c}{2}\left(\frac{y(\theta\alpha+1-\theta)+\sqrt{y^2(\theta\alpha+1-\theta)^2-4cr^*}}{2c}\right)^2 & \text{for } Z > \mu > z \\ \frac{c}{2}\left(\frac{(1-\theta)y+\sqrt{(1-\theta)^2y^2-4cr^*}}{2c}\right)^2 & \text{for } Z < \mu \end{cases}$$

For foreign currency borrowing:

$$\widehat{\Pi}^f = \begin{cases} \frac{c}{2}\left(\frac{\alpha y+\sqrt{\alpha^2y^2-4cr^{*f}}}{2c}\right)^2 & \text{for } z > \mu \\ \frac{c}{2}\left(\frac{\alpha y+\sqrt{\alpha^2y^2-4cr^{*f}}}{2c}\right)^2 & \text{for } Z > \mu > z \\ \frac{c}{2}\left(\frac{\alpha(1-\theta)y+\sqrt{\alpha^2(1-\theta)^2y^2-4cr^{*f}}}{2c}\right)^2 & \text{for } Z < \mu \end{cases}$$

We can now use these optimal effort and equilibrium profit functions to characterize the equilibrium as a function of the fraction of firms borrowing in the domestic currency versus in fx. Given beliefs that $z > \mu$, a firm borrowing in domestic currency does not believe that enough firms will fail and will choose $\widehat{q} = \frac{y - r_L}{c}$, believing that foreign firms will choose $\widehat{q}^f = \frac{(y - r_L^f)\alpha}{c}$. Using these equilibrium effort levels and the equilibrium interest rates r_L and r_L^f, we can substitute into the expression above to obtain z, and check that $z > \mu$. If so, then this is an equilibrium, and under this equilibrium there is no contagion.

If not, and $z < \mu$, then this is not an equilibrium, and we consider the other extreme beliefs, that $Z < \mu$, in which case enough failure for contagion to occur is expected regardless of whether or not there is a devaluation. In that case, domestic borrowers choose $\widehat{q} = \frac{(y-r_L)(1-\theta)}{c}$, believing that foreign borrowers choose $\widehat{q}^f = \frac{(1-\theta)(y-r_L^f)\alpha}{c}$. Again using the equilibrium loan rates r_L and r_L^f, we can obtain the value for Z and check to see whether $Z < \mu$. If so, then this is an equilibrium, and contagion is possible whether or not there is a devaluation.

If not, and $Z > \mu$, then this is not an equilibrium, and we must have $Z > \mu > z$. The solutions for $\widehat{q}$ and $\widehat{q}^f$ are also given above. Note that, since the optimal domestic effort in this case is $\frac{y-r_L}{c}(\theta\alpha + 1 - \theta) > \frac{(y-r_L)(1-\theta)}{c}$ for $\theta > 0$, and the same is true for the firms borrowing the foreign currency, this immediately implies that $Z > \mu$ at the optimal level of effort. Likewise, $\frac{y-r_L}{c}(\theta\alpha + 1 - \theta) < \frac{y-r_L}{c}$, implying that $z < \mu$. Therefore, this confirms that $Z > \mu > z$, so that contagion only occurs in the event of a devaluation.

The next step is to then consider a firm's choice of currency in which to borrow. Recall that $1 - Z = \phi(1-\widehat{q}) + (1-\phi)\left(1-\widehat{q}^f\right)$, or in other words that $Z = 1 - \phi(1-\widehat{q}) - (1-\phi)\left(1-\widehat{q}^f\right)$. Likewise, for the devaluation case we have $1 - z = \phi(1-\widehat{q}) + (1-\phi)$, or in other words $z = 1 - \phi(1-\widehat{q}) - (1-\phi)$. There are now a number of possibilities given firms' conjectures about the borrowing strategy of other firms. To simplify the characterization, assume that $\mu < \widehat{q}$. This is a joint restriction on θ and y in addition to μ which simply implies that contagion risk is not so high that if everyone ($\phi = 1$) borrows domestically, there will be no contagion. Under this assumption, as $\phi \to 1$ we will satisfy $z > \mu$ and there will be no contagion. We also assume that $\mu < \widehat{q}^f$, so that absent a devaluation, if all firms borrow in the foreign currency there will also be no contagion.

Suppose therefore that $z > \mu$. Then, $\Pi^f > \Pi^d \Leftrightarrow$

$$\frac{c}{2}\left(\frac{\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}\right)^2 > \frac{c}{2}\left(\frac{y + \sqrt{y^2 - 4cr^*}}{2c}\right)^2,$$

which is equivalent to

$$\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}} > y + \sqrt{y^2 - 4cr^*}$$

Using the parity condition, $r^* = r^{*f} + (1-\alpha)\frac{\Delta e}{e}$, we can write this as

$$\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}} > y + \sqrt{y^2 - 4c\left(r^{*f} + (1-\alpha)\frac{\Delta e}{e}\right)}.$$

Note that this is the same condition as for the base case where contagion was not possible.

Claim: Combining the two assumptions that $\widehat{q}, \widehat{q}_i^f > \mu$ imply that $Z > \mu$, so that we can ignore the third case above, $Z < \mu$, which is the case where contagion arises in the event of no devaluation.

With this we can conclude that for ϕ large enough, $z > \mu$ will be the relevant regime, so that no contagion occurs even if there is a devaluation. By contrast, for ϕ low enough, $Z > \mu > z$ will be the relevant regime, so that contagion occurs if and only if a devaluation occurs.

Since each firm's decision of currency in which to borrow depends on its beliefs of other firms' borrowing decisions, it is straightforward to see that there are multiple equilibria that are possible. In particular, believing that all other firms borrow domestically, so that $z > \mu$, if it is optimal for the firm to also borrow domestically, there will be no contagion risk. The more interesting cases arise when a particular firm i believes that a significant portion of other firms will borrow in the foreign currency, and as a result may give rise to contagion if a devaluation takes place, irrespective of firm i's borrowing decision. For this case, the presence of contagion risk may lead firm i to borrow in fx even if in the absence of contagion risk it would have borrowed in the domestic currency. Rather than characterizing all the possible equilibria, we focus just on this latter case.

To see how this case arises, suppose that the distribution of effort costs c is such that a sufficiently large number of firms prefer to borrow in the foreign currency (i.e., $\phi >> 0$) in the absence of contagion risk. For the foreign firms, their behavior under $z > \mu$ or $Z > \mu > z$ is the same, and is given by effort $\widehat{q}_i^f = \frac{\alpha y\sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}$ and profits $\widehat{\Pi}^f = \frac{c}{2}\left(\frac{\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}\right)^2$. Worried about contagion (i.e., $Z > \mu > z$), firms that might borrow in the domestic currency with no contagion risk ($z > \mu$) now compare their profits when contagion risk is possible and will prefer to borrow in the foreign currency if $\Pi^f > \Pi^d \Leftrightarrow$

$$\frac{c}{2}\left(\frac{\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}}}{2c}\right)^2 > \frac{c}{2}\left(\frac{y\left(\theta\alpha + 1 - \theta\right) + \sqrt{y^2\left(\theta\alpha + 1 - \theta\right)^2 - 4cr^*}}{2c}\right)^2$$

$\Leftrightarrow$

$$\alpha y + \sqrt{\alpha^2 y^2 - 4cr^{*f}} > y\left(\theta\alpha + 1 - \theta\right) + \sqrt{y^2\left(\theta\alpha + 1 - \theta\right)^2 - 4cr^*}.$$

Note, however, that the left hand side is the exact same as for the case where there is no contagion risk. By contrast, for the right hand side, we have

$$\begin{aligned} & y\left(\theta\alpha + 1 - \theta\right) + \sqrt{y^2\left(\theta\alpha + 1 - \theta\right)^2 - 4cr^*} - \left(y + \sqrt{y^2 - 4cr^*}\right) \\ = \; & y\theta\left(\alpha - 1\right) + \sqrt{y^2\left(\theta\alpha + 1 - \theta\right)^2 - 4cr^*} - \sqrt{y^2 - 4cr^*} < 0 \end{aligned}$$

for $\left(\theta\alpha + 1 - \theta\right)^2 < 1$. Therefore, the profit to a firm borrowing in the domestic currency is lower under contagion risk than when there is no risk of contagion. This immediately implies that the conditions for borrowing in the foreign currency are strictly less stringent when there is contagion risk than when there is not.

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