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Self-Fulfilling Credit Market Freezes✧

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This paper develops a model of a self-fulfilling credit market freeze and uses it to study alternative governmental responses to such a crisis. We study an economy in which operating (nonfinancial) firms are interdependent, with their success depending on the ability of other operating firms to obtain financing. In such an economy, we show, inefficient credit market freeze may arise in which banks abstain from lending to operating firms with good projects because (and only because) of their (self-fulfilling) expectations that other banks will not be lending. We show how inefficient credit freeze equilibria may result from the arrival of information about fundamentals or a negative shock to the banking system's capitalization. While such equilibria result from the arrival of information about fundamentals, they do represent a "coordination failure:" banks' separate and fully rational decisions produce an outcome that would have been avoided had they been able to choose a coordinated action.

Our model enables us to study the effectiveness of alternative measures for getting an economy out of an inefficient credit market freeze. In particular, we study the effectiveness of (1) interest rate cuts, (2) infusion of capital into financial firms, (3) direct lending to operating firms by the government, (4) lending to operating firms by funds owned by the government and managed by private agents compensated with a share of the profits generated by the fund, and (5) provision of guarantees by the government against losses incurred by banks on loans to operating firms. Throughout, we discuss the implications of our analysis for understanding and responding to the credit crisis of 2008.

Key words: Credit freeze, credit crunch, credit thaw, self-fulfilling crisis, run on the economy, global game, coordination failure, bank capital, lending, strategic complementarities.

JEL Classification: D21, E44, E50, E58, E62, G18, G21, G28, H50, P11.

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1. Introduction

An important aspect of the economic crisis of 2008-2009 has been the “freezing” of credit to nonfinancial firms.[1] During the fall of 2008, despite government efforts to provide substantial liquidity and additional capital to the financial sector, financial firms have displayed considerable reluctance to extend loans to nonfinancial firms (as well as households). Because governments have traditionally left to the financial sector the role of lending to nonfinancial firms, financial firms’ reluctance to lend to nonfinancial firms, and their election to hoard their capital for the time being, can have severe consequences for the economy. Some observers have attributed the reluctance of financial firms to lend to irrational fear, while others have attributed it to a rational assessment of the fundamentals of the economy which can be expected to make it difficult for operating firms to repay extended loans.

This paper develops a model of how coordination failure among financial institutions, and self-fulfilling rational expectations, can lead to inefficient “credit markets freeze” equilibria. In such equilibria, financial institutions rationally avoid lending to nonfinancial firms (operating firms) that have projects that would be worthy if banks did not withdraw from the lending market en masse, doing so out of self-fulfilling fear, validated in equilibrium, that other financial institutions would withhold loans and that operating companies would not be able to succeed in an environment in which other operating firms fail to obtain financing.

The paper then analyzes which government policies can best get the economy out of an inefficient credit market freeze equilibrium. This analysis identifies the role and potential limitations of interest rate cuts and infusion of capital into the financial sector. It analyzes less traditional forms of intervention – involving government direct intervention in lending to nonfinancial companies or provision of incentives to financial firms to lend to such companies – and discusses when they may be preferable.

Our analysis is based on the premise that operating firms, or at least a significant fraction of such firms, are interdependent – that is, that the returns they will make on capital they borrow depend on whether other operating firms are able to obtain financing.

[1] For a description of the crisis and the events leading to it, see Brunnermeier (2008) and International Monetary Fund (2008).

The success of a given operating firm might depend on other firms' operations to the extent that the other firms provide necessary inputs or that the other firms, or those generating income from them, provide demand for the firm's output. This interdependence makes the decision of any given financial institution whether to lend to a given operating firm depend not only on the financial institution's assessment of the firm's project but also on its expectations as to whether other financial institutions will lend money to other operating firms. (Below we refer to financial institutions as banks for simplicity.)

This interdependence can give rise to multiple equilibria when each of many operating firms has a project that would be worth financing if other operating firms obtained financing for their project but not otherwise. In an efficient "lending" equilibrium, banks expect other banks to lend to operating firms with worthy projects, and these expectations are self-fulfilling. In an inefficient "credit freeze" equilibrium, banks have self-fulfilling expectations that other banks will withdraw from the lending market, and they rationally avoid lending to operating firms.

We use a global game methodology to identify which equilibrium will arise. The unique equilibrium is determined by the information that banks get about macroeconomic fundamentals (with each bank getting a noisy signal) and about the capital available to the financial sector. We identify a certain threshold of unfavorable information about fundamentals (or about depletion of banks' capital) below which a credit freeze equilibrium results. If the information about fundamentals is bleak enough, the withholding of credit would be efficient, reflecting the inability of firms to produce sufficient returns even if no banks were to withhold from lending. However, there is a range within which the credit freeze equilibrium would be inefficient, with banks not lending to operating firms whose projects would be worth financing if banks were all to lend to such firms.

During the economic crisis of 2008, the Fed and other central banks around the world slashed interest rates. Although interest rate cuts by the central bank make a credit market freeze less likely, by reducing the payoff to banks that avoid lending and invest in government bonds, we show that such cuts, however large, cannot be relied on to get the economy out of a credit market freeze equilibrium. To the extent that lending to a given operating company would produce a negative return if other operating firms fail to obtain financing, a bank would prefer to avoid lending to an operating company when other banks do not lend even if the economy's interest rate is barely positive, and an interest rate cut might consequently fail to produce a credit thaw.

Our analysis indicates that a shock to the banking system that depletes the amount of capital banks have makes an inefficient credit market freeze equilibrium more likely. This is because banks are more concerned that operating firms will not have enough capital to succeed and thus they are reluctant to lend them even the capital that they have. As a result, intervention through the infusion of capital into financial firms, which governments in the US, UK, and other countries did during 2008, can be beneficial. However, we show that, like interest rate cuts, the effectiveness of capital infusion in producing a credit market thaw is limited. The reason is that, as long as other banks are expected to avoid lending to operating firms, banks that have ample capital will still choose to park it in government bonds rather than lend it to operating firms that are expected to fail to return it in the economic conditions that result from a credit freeze equilibrium.

We then turn to examine the possibility of the government's providing loans directly to operating firms. Should the government serve as "lender of last resort" to operating firms? The problem with such direct lending by the government is that it is reasonable to assume that the government does not have the same ability as private financial firms to distinguish between operating firms with good and bad projects. Thus, while direct government lending can provide financing to some firms with good projects that could otherwise fail to get financing due to an inefficient credit freeze equilibrium, it might also provide financing to some operating firms with bad projects that should not be financed and would not get funding even in an efficient lending equilibrium. The superior ability that financial firms have in screening operating firms seeking credit is presumably the reason why governments have focused on shoring up the financial sector in the hope that it will in turn provide efficient financing to operating firms.

The problem that our analysis highlights, however, is the existence of circumstances in which the financial firms, acting on their own in ways that are individually rational, and armed with sufficient information about operating firms and with sufficient capital, will produce an outcome that is collectively suboptimal even though it is individually rational due to coordination failure. This leads us to examine yet another approach in which the government assumes the risk of lending to operating firms but uses the expertise of private financial firms to screen which operating firms will get credit and which will not.

Under one version of this approach, the government places capital in funds managed by private agents that will use them to extend loans to operating firms and will receive a reimbursement of their expenses and a cut of the profits (i.e., the returns above the riskless rate) generated by the funds. Hence, the government shares the profits generated by a portfolio of loans to operating firms with private players, and bears the risk in the event that the portfolio ends up in the red. This guarantees that the government's money will be lent to good firms. We show that the use of this approach can improve the government's ability to produce a credit thaw. But this approach is costly when the economy ends up in a credit freeze, as it involves wasting the government's resources on unsuccessful projects (even projects of good firms fail in a credit freeze).

Under an alternative version of this approach, the government sells guarantees to financial institutions that extend new loans to operating firms, such that the government covers part of their losses to unsuccessful real projects. This approach can avoid the waste of resources when a credit freeze happens as the banks do not lend and thus the guarantees never get materialized. However, it is not as efficient in reducing the probability of a freeze.

Our paper is related to the large literature on bank runs, where depositors rush to demand early withdrawal from the bank because they believe that other depositors are going to do the same. The seminal paper on bank runs is by Diamond and Dybvig (1983), and it was followed by much subsequent work on the subject (see, e.g., Allen and Gale (1998), Peck and Shell (2003), and Goldstein and Pauzner (2005)). The ideas in the bank-run literature have subsequently been applied to describe also runs by investors on currencies (Morris and Shin (1998)), financial markets (Bernardo and Welch (2004) and Morris and Shin (2004a)), and other contexts. Our paper, which builds on the analytical insights of this literature, focuses on a different context. We do not consider a run by depositors or investors on financial institutions, financial markets, or governments, but rather a run by financial institutions on the nonfinancial firms of the real economy.

Furthermore, our focus is on analyzing alternative government responses that can be used in this context.

Several papers analyze policies of deposit insurance or 'lender of last resort' to prevent runs on financial institutions and markets. These include the papers by Rochet and Vives (2004), Corsetti, Guimaraes, and Roubini (2006), and Morris and Shin (2006). The policy problem we consider here is fundamentally different. Because, in our model, coordination failures arise among banks in their decision to lend to operating firms, banks end up not using capital that they have for lending purposes. Hence, capital infusion to banks might not be sufficient to eliminate an inefficient credit market freeze. This leads to our discussion on the role of direct government intervention in lending to operating firms, and the various ways of implementing it without losing the informational advantage that banks have in lending to such firms.

The source of coordination failures among banks in our model is the interdependence among firms in the real economy that makes the investment in a firm profitable only if other firms are able to invest and produce. Such strategic complementarities in the macro economy were motivated in an influential paper by Cooper and John (1988), and have been used in other papers (e.g., Goldstein and Pauzner (2004)). Our paper complements this literature by showing how such complementarities can cause a credit freeze and analyze government policy in such context.

Models of strategic complementarities usually yield multiple equilibria and thus do not lend themselves naturally to policy analysis. To overcome this problem, we follow recent work on self-fulfilling crises and rely on global-games techniques. The global-games literature has been pioneered by Carlsson and van Damme (1993) and Morris and Shin (1998) and is reviewed in Morris and Shin (2003)). In particular, we build here on the model in Morris and Shin (2004b).

The remainder of this paper is organized as follows. Section 2 describes our framework of analysis. Section 3 provides an equilibrium analysis, identifying the conditions under which inefficient credit freeze equilibria will arise. Section 4 analyzes several governmental policies that may be used to produce a credit thaw, identifying their potential benefits and limitations. Section 5 concludes.

2. The Model

There is a continuum *[0,K]* of identical financial firms, which we call banks for simplicity. Each bank has 1 dollar of capital. Banks can choose whether to invest their capital in a risk-free asset, such as a deposit with the central bank, generating $1+r$ (>1) dollars next period, or lend it to operating (nonfinancial) firms. Banks are risk neutral and hence make their choices so as to maximize expected payoffs.

Operating firms have access to investment projects that require investment of 1 dollar, but do not have any capital to finance them. They rely on bank lending to invest in their projects. There are two types of operating firms. Some operating firms have bad projects that always generate a gross return of 0. Others have good projects, generating a gross return of $1+R$ ($>1+r$) when the macroeconomic fundamentals are strong and a sufficient number of operating firms get the required financing to invest. Specifically, the return on a good project is assumed to take the following form:

$$\begin{cases} 1+R & if \quad aL+\theta \geq b \\ 0 & if \quad aL+\theta < b \end{cases}. \tag{1}$$

Here, θ is a macroeconomic fundamental that can represent various factors, such as firms' productivity, consumers' demand, the cost of imported oil, etc. The variable L represents the mass of firms that received loans from banks to invest in their projects. In the basic model, $L = nK$, where $n \in [0,1]$, whose value is determined endogenously in the model, is the proportion of banks that decide to lend to firms. Hence, the macroeconomic fundamentals and the proportion of firms investing in their projects are together responsible for the profitability of good projects. a is a parameter capturing the importance of complementarities vs. fundamentals in making projects profitable, and b is a parameter capturing the threshold needed to become profitable.

The effect of L reflects the interdependence among operating firms in the economy. This interdependence can be due to several reasons. For example, many firms can prosper only when there are other firms in the economy that can provide them with adequate inputs. In addition, many firms sell some or all of their output to other firms, and thus depend on the operation of other firms. Even firms that sell their output solely to individuals might suffer from declining sales if other firms do not operate and thus are not able to employ these individuals. In sum, the success of the economy in our model requires the coordination among various operating firms and the banks that finance them. Such coordination issues in the macro economy were proposed before by other authors, e.g., by Cooper and John (1988).

We assume that banks can tell the difference between firms with bad projects ("bad firms") and firms with good projects ("good firms"), and thus can choose to lend only to firms with good projects.[2] Moreover, we assume for simplicity that the mass of firms with good projects is greater than the mass of banks *K*, and thus banks are able to extract the full return *R* from lending to good firms, whose projects were successful.[3]

We assume that the fundamental θ is not publicly known. It is normally distributed around a mean of y. We consider y to be the commonly-known strength of the economy. The standard deviation of θ around y is σ_θ, and we use $\tau_\theta = \frac{1}{(\sigma_\theta)^2}$ to denote the precision of the distribution of θ. Each bank i receives a private signal regarding the value of θ, given by $x_i = \theta + \varepsilon_i$. Here, the individual specific noise terms ε_i are independently normally distributed with mean 0 and standard deviation σ_p. We use $\tau_p = \frac{1}{(\sigma_p)^2}$ to denote the precision of banks' signals. Banks make their decisions whether to invest in the riskless asset or to lend to operating firms after observing these signals.

Because the profitability of operating firms depends on macroeconomic conditions and the availability of financing to other firms, a bank's incentive to lend to a given operating firm with a good project is higher when the economy's fundamentals are favorable and when the number of banks who are going to lend is high. While the optimal behavior of a bank usually depends on its belief regarding the behavior of other banks, there are ranges of macroeconomic fundamentals in which banks have a dominant strategy. More specifically, when the fundamental θ is above b, a bank will prefer to lend to an operating firm no matter what it believes other banks will do. This is because in this range the return on lending is guaranteed to be $1+R$. Similarly, when the funda-

[2] The firms with bad projects will have an explicit role in the model later when we consider the possibility of the government extending direct loans to operating firms.

[3] We are thus able to show that a credit freeze equilibrium may arise even when the competitive conditions enable banks to extract the full surplus from lending and are thus as favorable to lending activity as possible.

mental is below $b - aK$, the bank will invest in a government bond even if it believes that all the other banks will lend to operating firms.

Since θ is drawn from an unbounded distribution, there are signals at which banks choose to lend to operating firms independently of their beliefs regarding other banks' behavior, as well as signals at which they choose not to lend independently of their beliefs. As to banks that receive a signal in the intermediate range, however, their optimal decision depends on their expectations about whether other banks will lend to operating firms. This calls for an equilibrium analysis to which we turn next.

3. Equilibrium Analysis

3.1. Credit Freeze

We solve the model using global-games techniques. In particular, we follow here Morris and Shin (2004b). Proposition 1 states the basic equilibrium result.

Proposition 1: *Suppose that the information in banks' signals is precise relative to prior information, so that $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK}$. Then, there is a unique Bayesian Nash Equilibrium in which all banks lend to operating firms if they observe a signal above x^* and withdraw from lending if they observe a signal below x^*. Investment projects then succeed if and only if the fundamentals are above the threshold θ^*, between $b - aK$ and b, which is characterized by the following equation:*

$$\theta^* = b - aK + aK\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right), \qquad (2)$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal.

Proof: The proof follows Morris and Shin (2004b). The arguments in their proof (which we don't repeat here, for brevity) establish that there can only be a threshold equilibrium, where banks lend if and only if their signal is above some common x^*. Given this result, we now characterize the threshold equilibrium and show that it is unique.

Given x^*, there is a unique threshold fundamental θ^*, at which investment projects are on the margin between failure and success. This is given by:

$$\theta^* = b - aK\left(1 - \Phi\left(\sqrt{\tau_p}(x^* - \theta^*)\right)\right)$$

Here, $\Phi\left(\sqrt{\tau_p}(x^* - \theta^*)\right)$ is the proportion of banks receiving a signal below x^* and withdrawing from lending when the fundamental is exactly θ^*.

This gives us the first equation for the two unknowns x^* and θ^*. The second equation comes from the fact that at the threshold signal x^* a bank has to be indifferent between lending to firms and investing in the risk-free asset. When bank i observes signal x_i, his posterior distribution of θ is normal with mean $\frac{\tau_\theta y + \tau_p x_i}{\tau_\theta + \tau_p}$ and precision $\tau_\theta + \tau_p$. He knows that lending to firms yields $(1 + R)$ if and only if the fundamental is above θ^*, while not lending yields $(1 + r)$ with certainty. The indifference condition is then given by:

$$\left(1 - \Phi\left(\sqrt{\tau_\theta + \tau_p}\left(\theta^* - \frac{\tau_\theta y + \tau_p x^*}{\tau_\theta + \tau_p}\right)\right)\right)(1 + R) = 1 + \mathrm{r}$$

Which can be developed as follows:

$$\theta^* - \frac{\tau_\theta y + \tau_p x^*}{\tau_\theta + \tau_p} = \frac{\Phi^{-1}\left(1 - \frac{1 + \mathrm{r}}{1 + R}\right)}{\sqrt{\tau_\theta + \tau_p}}$$

Leading to:

$$\theta^* - x^* = \frac{-\tau_\theta(\theta^* - y)}{\tau_p} + \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1 + \mathrm{r}}{1 + R}\right)}{\tau_p}$$

Plugging this in the first equation, we get:

$$\theta^* = b - aK\left(1 - \Phi\left(\sqrt{\tau_p}\left(\frac{\tau_\theta(\theta^* - y)}{\tau_p} - \frac{\sqrt{\tau_\theta + \tau_p}\Phi^{-1}\left(1 - \frac{1 + \mathrm{r}}{1 + R}\right)}{\tau_p}\right)\right)\right)$$

Which yields the equation in the proposition statement:

$$\theta^* = b - aK + aK\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right)$$

The left-hand side is the 45-degree line with respect to θ^*, and the right-hand side is increasing in θ^*, and is bounded between $b - aK$ and b. A unique solution for θ^* is guaranteed when the right-hand side has a slope of less than 1 everywhere. The slope of the right-hand side is given by $aK\phi(\cdot)\frac{\tau_\theta}{\sqrt{\tau_p}}$, where $\phi(\cdot)$ is the density of the standard normal evaluated at the appropriate point. Since $\phi(\cdot) \leq \frac{1}{\sqrt{2\pi}}$, a sufficient condition for a unique solution is $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK}$. **QED**. ■

Remarks: (i) *Intuition:* The intuition behind the result of Proposition 1 can be explained as follows. Due to strategic complementarities, when banks do not know that the fundamentals are below $b - aK$ or above b, they do not have a dominant action to choose. In this case, they simply want to do what other banks do. In a model with common knowledge about the fundamental θ, this would result in multiple equilibria, as both the case where all banks lend to operating firms and the case where none of them does so can be supported by equilibrium beliefs. The assumption that banks observe slightly noisy information about θ combined with the presence of extreme regions where they have dominant actions pins down the threshold equilibrium characterized by equation (2) as the unique equilibrium here.

Intuitively, with noisy information, banks that observe a signal slightly below the upper dominance region know that the fundamental may well be higher than their signal and thus choose to lend. Knowing this, banks with even lower signals will also choose to lend. This rationale can be repeated again and again, guaranteeing a range of signals below the upper dominance region, where banks choose to lend. Similarly, due to the noisy information, there will be a range of signals above the lower dominance region, where banks will choose to invest in government bonds. The proof of equilibrium with global-game techniques demonstrates that this procedure exactly separates the real line, so that banks lend above x^* and do not lend below it, leading to success of real projects above θ^* and failure below it.

(ii) *The Credit Freeze Threshold:* Equation (2) characterizes the threshold fundamental θ^* below which investment projects fail. To gain some intuition for what determines this threshold, it is useful to consider the limit, as banks' private signals become infinitely precise, i.e., as τ_p approaches infinity. In this case, x^* and θ^* converge to the same value, which is given by:

$$\theta^* = b - aK + aK\frac{1+r}{1+R} \tag{3}$$

This expression clearly reveals that θ^* is between $b - aK$ and b. Intuitively, it is determined by an indifference condition: A bank observing the signal θ^* is indifferent between lending to operating firms and investing in the risk-free asset under the belief that the proportion of other banks lending to operating firms is uniformly distributed between 0 and 1. This implies that lending to operating firms will be profitable with probability $\left(1 - \frac{b-\theta^*}{aK}\right)$, which yields the following indifference equation:

$$1 + r = \left(1 - \frac{b-\theta^*}{aK}\right)(1 + R).$$

Rearranging this equation, we get (3). The rationale behind the uniform-distribution belief is that each bank perceives a uniform distribution on the proportion of banks getting lower signals than its own. Given that the bank observed θ^* and that other banks lend if and only if they obtained a signal above θ^*, the bank perceives a uniform distribution on *n*. Because banks' signals have infinitesimally small noise, the equilibrium result is that all banks lend when the fundamental is above θ^* and do not lend when the fundamental is below θ^*. Hence, below θ^*, the economy ends up in a *credit freeze*.

(iii) *Efficient and Inefficient Credit Freezes:* When macroeconomic fundamentals are so bleak that we are below $b - aK$, the refusal of banks to lend is efficient because firms' projects will not produce payoffs exceeding the economy's riskless rate even if no banks withdraw from the lending market. In this case, funding the operating firms' project would be inefficient and reduce social wealth. Outside this set of circumstances, however, there exists a range of circumstances, when fundamentals lie between $b - aK$ and θ^*, that the economy will be in an inefficient credit freeze equilibrium. In this interval, banks will withdraw from lending even though, were banks all willing to lend, firms' projects will produce returns exceeding the riskless rate and the banks will be all better off relative to the credit freeze equilibrium.

(iii) *Inefficient Credit Freezes as a Coordination Failure*: When fundamentals lie between $b - aK$ and θ^*, the inefficient credit freeze can be viewed as due to coordination failure. Here, banks do not lend to operating firms just because they fear that other banks will not lend to operating firms. The fundamentals uniquely determine banks' expectations regarding what other banks are going to do and thus (indirectly) uniquely determine whether a credit freeze will arise, but the credit freeze is still inefficient. If the banks could have concluded among themselves an enforceable agreement on how they will act (or otherwise acted in a concerted fashion), they would have agreed on a coordinated strategy of lending to firms. However, as long as the banks make their decisions separately, based on their expectations as to how other banks will act, an inefficient credit freeze equilibrium may ensue. This raises the question, which section 4 will study in detail, what policy measures by the government could get the economy out of a credit freeze equilibrium.

(iv) *The 2008 Credit Crunch:* The credit crunch of 2008 was preceded by the arrival of bad economic news about macroeconomic fundamentals. For one thing, the substantial decline in housing prices considerably reduced the wealth of households, and such a reduction could have been expected to produce a subsequent decrease in consumer spending and thus the demand for firms' output. Our model indicates that the arrival of bad macroeconomic news might trigger a credit freeze that will lead to the refusal of banks to lend to firms even though the firms would still be worth financing notwithstanding the deterioration in macroeconomic fundamentals absent a self-fulfilling withdrawal of banks from the lending market. Such triggering of a credit freeze will of course further reinforce and exacerbate the effects of the deterioration in fundamentals that triggered it in the first place.

3.2. Can Reduction in Banks' Capital Trigger A Credit Freeze?

The credit crunch of 2008 was preceded by a perceived deterioration in the capital positions of financial institutions as a result of losses from real estate mortgage assets. This subsection examines whether a reduction in the banks' capital can trigger a credit freeze. To be sure, the decline in housing prices, which caused the losses from real estate mortgage assets, can directly produce a deterioration in the macroeconomic fundamentals represented by θ in our model, and such deterioration may by itself trigger a credit freeze. Our interest in this section, however, is whether, holding θ constant, a reduction in banks' capital can lead to, or make more likely, a credit freeze equilibrium. Our model indicates that a negative shock to the capital of the banking sector can indeed shift the economy to an inefficient credit freeze.

To study this issue, let us introduce the parameter *l* (between 0 and 1), which denotes the proportion of capital lost by banks in the economy due to bad past investments (say, because capital has been invested in "toxic" real estate paper). For simplicity of exposition, we assume that capital has been lost uniformly across banks, that is, each bank in the economy lost a fraction *l* of its capital. With this parameter introduced into the model, the capital of a single bank (*1-l*) does no longer suffice to finance a firm's project. Hence, each firm will have to pool resources from more than one bank. Eventually, if a fraction *n* of banks decide to lend the capital they have to operating firms, the total capital that will be provided as loans to such firms will be only a fraction *n(1-l)* of *K*, and hence $L = n(1-l)K$.

Proposition 2 characterizes the new equilibrium results and the effect that the parameter *l* may have on the realization of a credit freeze.

Proposition 2: *(a) In the unique Bayesian Nash Equilibrium, investment projects succeed if and only if the fundamentals are above the threshold $\theta^*(l)$. The threshold $\theta^*(l)$ is characterized by the following equation:*

$$\theta^* = b - aK(1-l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right), \quad (4)$$

(b)The threshold $\theta^*(l)$ is an increasing function of the parameter l; hence, an increase in the fraction of bank capital that was lost, l, with no change in the fundamental θ, can shift the economy from an efficient lending equilibrium to an inefficient credit freeze.

Proof: Proving the first part of the proposition is straightforward given the proof of Proposition 1. The proof just replaces K with $K(1-l)$ to reflect the fact that when a proportion *n* of the banks lend, only $nK(1-l)$ capital makes its way to operating firms.

Note that the condition for uniqueness is now $\frac{\tau_\theta}{\sqrt{\tau_p}} \leq \frac{\sqrt{2\pi}}{aK(1-l)}$, which always holds when the condition in Proposition 1 holds.

The second part is proved with the implicit function theorem. Denote

$$F(\theta^*, l) = \theta^* - b + aK(1-l) -$$
$$aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right) = 0$$

Then,

$$\frac{d\theta^*}{dl} = -\frac{dF(\theta^*, l)\big/_{dl}}{dF(\theta^*, l)\big/_{d\theta^*}}$$

We know that

$$\frac{dF(\theta^*, l)}{dl} = -aK\left(1 - \Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right)\right) \leq 0$$

$$\frac{dF(\theta^*, l)}{d\theta^*} = 1 - aK(1-l)\frac{\tau_\theta}{\sqrt{\tau_p}}\phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta + \tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right)$$
$$\geq 1 - aK(1-l)\frac{\tau_\theta}{\sqrt{\tau_p}}\frac{1}{\sqrt{2\pi}} \geq 0$$

It follows that $\frac{d\theta^*}{dl} \geq 0$. **QED**. ■

Remark: The intuition behind the result of Proposition 2, which indicates that a reduction in the banking sector's capital raises the threshold, below which banks elect to withdraw from lending, is as follows. A reduction in the banking sector's capital makes each bank "less sure" that other banks will provide enough capital to operating firms to guarantee adequate return from extending loans to operating companies. Hence, such a reduction makes each bank more concerned that, in the event it provides a loan to a given operating company, the firm will nonetheless suffer from the inability of many other operating companies to obtain financing. Technically, in equilibrium, a higher fundamental θ is required to make banks indifferent between providing credit to operating companies and investing in the riskless asset, which leads to an increase in the threshold θ^* and thus in turn to a larger range of fundamentals at which an inefficient credit freeze ensues.

Thus, our results indicate that banking losses can drive the economy into a credit freeze even without any accompanying change in other macroeconomic fundamentals. What is important to stress is that such reduction in capital will make operating firms less likely to receive financing not only because of the direct effect that some capital that could have been available for loans is no longer in place but also because of the indirect effect, which our result identifies, that it might deprive operating firms even of the capital that remains in place. By influencing banks' expectations as to how many operating firms will be able to obtain financing, the disappearance of some capital can make banks more reluctant to lend the capital that still remains.

4. Government Policy

4.1. Interest Rate Reduction

One governmental measure that is natural to examine as an instrument for addressing a credit freeze is a cut in interest rates. During the credit crisis of 2008, governments around the world have made substantial use of interest rate cuts. During 2008, in a series of moves, the Federal Reserve Board cut the federal rate considerably, bringing the Federal funds rate down from 4.25% in January to 1% in October. Similar steps have been taken by other central banks around the world. In October 2008, facing a worldwide contraction in lending, twenty one countries around the world, including the US and the UK, simultaneously cut interest rates.

Under normal market conditions, a cut in a country's interest rate can be expected to spur lending. To what extent can a cut in interest rate, however, be relied on to eliminate a coordination failure that results in an inefficient credit freeze equilibrium? Can any self-fulfilling credit freeze crisis be prevented by a sufficiently large interest rate cut? This section uses our model to consider these questions. As we explain below, a cut in interest rate may – but does not have to – produce a credit thaw. Such a cut may move the economy from a credit freeze equilibrium to a lending equilibrium in some circumstances, but there are circumstances in which a credit freeze will persist despite an interest rate cut however large.

In the language of our model, a reduction in interest rate amounts to reduction in the parameter *r*. Inspection of Equation (4) reveals the potential of this policy measure to help banks coordinate on a desirable credit thaw. The result is summarized in the following proposition.

Proposition 3: *(a) For every level of bank losses l, a decrease in the interest rate r on government bonds reduces the threshold θ^*, below which a credit freeze occurs, and hence reduces the likelihood of a credit freeze.*
(b) Yet, for every r ≥ 0 and l (between 0 and 1), there are realizations of the fundamental θ, at which an inefficient credit freeze occurs.

Proof: Proving the first part of the proposition is again done using the implicit function theorem. Denote:

$$F(\theta^*,r) = \theta^* - b + aK(1-l) -$$

$$aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right) = 0$$

Then,

$$\frac{d\theta^*}{dr} = -\frac{dF(\theta^*,r)\big/_{dr}}{dF(\theta^*,r)\big/_{d\theta^*}}$$

Given that $\frac{dF(\theta^*,r)}{dr} \leq 0$ and $\frac{dF(\theta^*,r)}{d\theta^*} \geq 0$, it follows that $\frac{d\theta^*}{dr} \geq 0$.

To see why the second part holds, note that, given the capital available to banks $K(1-l)$, not lending to operating firms is efficient only when the fundamental θ is below $b - aK(1-l)$. Since $\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1+\mathrm{r}}{1+R}\right)\right)\right) > 0$ (unless y approaches infinity), $\theta^* > b - aK(1-l)$. Hence, there is a range of fundamentals for which banks do not lend and projects fail, even though this is inefficient. **QED**. ■

Remarks: (i) *The Reduction in the Likelihood of Credit Freeze*: The intuition behind the first part of Proposition 3 is as follows. A reduction in *r* makes investment in the riskless asset less attractive and thus lowers the expected return that will be necessary to induce banks to lend to operating firms, which in turn lowers the threshold θ^* above which banks will lend to such firms rather than withdraw from the lending market. It is interesting to note that the effect of the reduction in *r* on the decision of an individual bank is more than just the direct effect on this bank's payoff. Because the reduction in interest rate can be expected to affect other banks' decisions, it also affects the individual bank's decision through its effect on the bank's expectation concerning how other banks will act. Thus, the coordination aspect remains important when thinking about the effect of interest-rate policy in this model.

(ii) *The Limits of Interest Rate Cuts*: The second part of the proposition says that interest rate reductions cannot eliminate all inefficient credit freezes. Even if the government reduces *r* all the way to 0 (or to a very low level just above zero), θ^* will remain above $b - aK(1-l)$, which implies that inefficient credit freezes may occur in the interval between $b - aK(1-l)$ and θ^*. The intuition goes back to the coordination-failure aspect of credit freezes in our model. Even if the net return on the riskless asset is close to zero, banks will prefer to invest in it rather than lending to operating firms when they expect that other banks will all do so. When all other banks are expected to withhold funds from operating firms, a bank may conclude that lending to a given operating firm will produce a loss and thus will be dominated by a safe investment yielding no positive return.

Thus, while governmental reduction in interest rates can shift the threshold that triggers coordination failure and credit freezes, it cannot completely eliminate such coordination failures. This result might be thought of as similar in spirit to the well-known liquidity trap in monetary economics. When the economy finds itself in a liquidity trap, even if the government reduces the interest rate all the way to 0, such rate cuts will fail to provide sufficient stimulus to the economy to bring it back to the desired level of activity. Similarly, the result indicates that there exists a range of circumstances in which interest rate cuts, however large, will fail to produce a credit thaw.

4.2. Infusion of Capital to the Banking System

During the financial crisis of 2008, governments around the world infused a large amount of capital into banks to shore up banks' capital positions, which have eroded due to losses from real estate mortgage assets and other investments. In the US, following the adoption of legislation in October 2008, the US Treasury infused into financial firms about $250 billion in additional equity capital. During the same period, the UK invested about $90 billion in several major banks. In addition to providing additional equity capital to financial firms, the Federal Reserve Board also provided additional capital to financial intermediaries by purchasing large amounts of their commercial paper.[4]

Infusion of capital into banks is a policy measure that is natural to consider in financial crises. Infusion of capital, e.g., in the form of a lender of last resort, has been used to prevent or stop bank runs in which depositors seek to withdraw their deposits en masse from a bank. When a solvent bank faces a problem of a bank run, providing the bank with capital may ensure depositors that their money is safe and prevent a run on the bank. Infusion of capital has also been used in the case of insolvent banks when governments felt that making sure such banks can meet their obligations to depositors is necessary to prevent a contagion effect that would lead to runs by depositors on other banks.

The subject we examine using our model is different because it does not involve potential runs by depositors on banks (or financial institutions more generally). Rather, it is the banks that may "run on the economy" by not extending loans to operating firms. In our context, therefore, capital infusion will not be designed to enable banks to meet their obligations toward their creditors. Rather, in our context, capital infusion may be used to facilitate lending by banks to operating firms in two ways: first, the direct and straightforward way of providing banks with additional capital that they may use for the purposes of extending loans; and, second, the indirect effect, which our model highlights,

[4] The Fed established the Commercial Paper Funding Facility in October 2008, and it purchased during the subsequent several weeks hundreds of billions of dollars worth of commercial paper from financial intermediaries such as Morgan Stanley, GMAC, and American Express.

of encouraging banks to lend to operating firms capital that they already have but that they might elect not to lend in the absence of the capital infusion to the banking sector and the shift in expectations produced by it.

To analyze governmental infusion of capital into the banking sector, let us assume that the government has or can obtain capital that would be sufficient to cover part of banks' losses. In particular, let us assume that the government has an amount $Z = \alpha l K$, enabling it to inject a proportion α of the lost capital *l* to all banks in the economy. If the government injects the capital, each bank will have a total capital of $1-(1-\alpha)l$. In return, the government will get a share of $\alpha l/(1-(1-\alpha)l)$ in the banks. It can be shown that banks will not object to receive such capital infusion from the government. With the allocation of shares mentioned here, banks maintain the claim on the proceeds of their original capital. They now have to make an investment decision on the sum of their original capital and the government's injected capital. This does not change the return on their capital, aside from the (positive) indirect effect that investing more capital has on the returns in the economy.

Banks will again make a decision whether to lend to operating firms or invest in the riskless asset. The first option yields a gross return of $1+R$ if firms' investment projects succeed, which happens as long as the proportion of banks lending to firms is above $\frac{b-\theta}{a(1-(1-\alpha)l)K}$, while the second one yields a certain gross return of *1+r*. To focus on capital infusion, we will assume from now that *r*=0, so that the government has already reduced the interest rate as much as possible. The following proposition analyzes the effect of injecting capital to the banking system.

Proposition 4: *(a) The threshold θ^*, below which a credit freeze occurs when the government covers proportion α of bank losses is implicitly determined by:*

$$\theta^* = b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1}{1+R}\right)\right)\right), \qquad (5)$$

(b) This likelihood decreases in α .Yet, for every $\alpha \leq l$, there are realizations of the fundamental θ at which an inefficient credit freeze will occur.

Proof: The proof is analogous to the proof of Proposition 3, and thus omitted. **QED** ■

Remarks: (i) *The Reduction in the Likelihood of Credit Freeze*: By providing capital to the banking system, the government creates externalities that make the projects of operating firms more profitable. This is because banks have more capital to lend to operating firms, and so when they decide to lend, operating firms will produce greater returns. This encourages banks to lend to operating firms, making a credit thaw more likely to occur. Technically, at the threshold, below which a credit freeze occurs, banks will require a lower fundamental θ to be indifferent between lending and not lending to operating firms when the government injects more capital to the banking system (α is higher). This is because a higher α implies that under a uniform distribution of banks that decide to lend, the returns from lending increase. This pushes the threshold θ^* lower and increases the likelihood of a credit thaw.

(ii) *The Limits of Capital Infusion:* Even when the government covers all the losses that banks accumulated, banks will be reluctant to lend if they believe other banks are not going to lend. Hence, this policy of the government cannot fully eliminate coordination-based credit freezes. This sharpens the difference between infusion of capital to banks in our model, where crises reflect a run of banks on operating firms, and infusion of capital in a model of a run on the bank. Because, in our model, coordination failures arise among banks in their decision to lend to operating firms, banks end up not using capital that they have for lending purposes. Hence, capital infusion might not be sufficient to eliminate an inefficient credit market freeze. It should be noted, however, that, while capital infusion might not eliminate all inefficient credit freeze equilibrium, it will leave intact efficient credit freeze equilibria, as it will never lead banks to lend when the economy is below $b - aK(1-(1-\alpha)l)$.

4.3. Does Direct Lending to Operating Firms Provide A Better Solution?

As explained above, the difficulty that the government faces in breaking the credit freeze by providing capital to banks is that banks might take the capital and not lend it to operating firms due to the fear that other banks will not lend. An alternative to providing capital to banks is for the government to lend directly to operating firms. This would be the truly equivalent policy to a traditional lender of last resort, as it would have the government directly providing capital to those that need it, who in our model are the operating firms.

The problem with such policy is that the government does not have the ability that banks have to identify good firms from bad firms. Thus, providing capital to firms without using the intermediation services of banks would lead to lending to some firms that have bad projects and should not get financing.

To examine the efficiency of direct lending formally, we have to explicitly describe the bad operating firms in our model. So far, there was no need to consider them and how many of them exist, as the assumption was that banks can tell good firms from bad firms, and thus bad firms would always be avoided. If the government attempts to lend to operating firms directly, however, it will have to consider the consequences of not being able to tell good firms from bad firms.

For the formal analysis, let us denote the mass of bad (good) operating firms in the economy as B (G). Recall that G is greater than K (the mass of banks). Suppose that the government has capital at the amount of $Z = \alpha l K$ (as in Section 4.2) and it has to decide whether to inject it directly to operating firms or to the banks. When the government lends capital to operating firms, the capital is randomly allocated between good or bad firms. We denote the proportion of the capital that finds its way to bad firms as $\beta \equiv B/(B+G)$. For simplicity, we assume that the government does not know the realization of the fundamental θ (and does not get any signal about it). Initially, we will assume that the operation of firms with bad projects, while producing no returns for the lending bank, still provides a positive externality for other operating firms (as firms with bad projects do purchase inputs from other firms etc.); below we will discuss how our conclusions will change if we were to assume that such externalities flow only from the operation of firms with good projects.

Below we show that using the government's capital for direct lending to operating firms is more effective, compared with infusing this capital into the banks, in lowering the threshold θ^* above which banks will lend to operating firms. Thus, there are circumstances in which direct lending would produce a credit thaw where a capital infusion into the banks would not. Even in these circumstances, however, direct lending might be overall less efficient due to the waste associated with lending to bad firms. Furthermore, outside these circumstances, direct lending would produce worse results.

Proposition 5: *If the government lends αlK directly to operating firms, there is a credit freeze equilibrium if and only if the fundamental θ is below the threshold θ^*, which is implicitly defined by:*

$$\theta^* = b - aK(1-(1-\alpha)l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1}{1+R}\right)\right)\right), \quad (6)$$

Denoting the threshold under capital injection to banks (defined in equation (5)) as θ^*_{Bank} and the one under direct lending to firms (defined in equation(6)) as θ^*_{Direct}, we get that for every α and l, $\theta^*_{Bank} > \theta^*_{Direct}$, implying that the probability of a credit freeze is higher under capital injection to banks than under direct lending to operating firms.

Proof: Equation (6) is based on the same principles behind the construction of equilibrium in Propositions 1 and 2. The only thing to note in equation (6) is that all the government's capital is lent and generates the positive externality. Hence, investment projects fail when the proportion *n* of banks that decide to lend is below $\frac{b-\theta-a\alpha lK}{a(1-l)K}$. Having established equation (6) and comparing it with (5) (using the implicit function theorem, as in Propositions 2 and 3) reveals that $\theta^*_{Bank} > \theta^*_{Direct}$. **QED** ■

Remark: The intuition for why directly lending the government's capital will reduce the lending threshold more than infusing the capital into banks is simple. When the government injects capital to banks, some of this capital might remain "stuck" in the banking system as banks fail to coordinate on lending it to operating firms. When the government lends the capital directly to operating firms, banks know that it will generate the desired externalities. As a result, lending directly to operating firms more effectively increases the returns to banks from lending and encourages banks to lend, and thus is more likely to bring the economy to a credit thaw. Focusing attention on the limit case where banks' private signals become infinitely precise, i.e., as τ_p approaches infinity, the comparison between the two cases becomes very transparent. Following (3), we can express the thresholds under the two regimes in the limit case as:

$$\theta^*_{Bank} = b - aK(1-(1-\alpha)l) + aK(1-(1-\alpha)l)\frac{1}{1+R} \tag{7}$$

$$\theta^*_{Direct} = b - aK(1-(1-\alpha)l) + aK(1-l)\frac{1}{1+R} \tag{8}$$

Equations (7) and (8) clearly reveal that $\theta^*_{Bank} > \theta^*_{Direct}$.

But, as noted above, the fact that direct lending is more likely to generate a credit thaw is not enough to make this policy measure more efficient. We now carry out a full comparison between the two measures. For a sharp comparison, we focus attention on the limit case considered above. This is easier to work with because at the limit either all banks lend or none of them does, and then we do not have to consider cases where some banks lend but projects fail and vice versa. The following proposition characterizes which policy ends up producing better results for different levels of the fundamentals.

Proposition 6: *(a) When the fundamental θ is below θ^*_{Direct} or above θ^*_{Bank}, the overall wealth in the economy is higher under injection of capital to the banking system than under direct lending to operating firms.*

(b) When the fundamental θ is between θ^*_{Direct} and θ^*_{Bank}, the comparison between the two regimes yields ambiguous results. For a sufficiently large β and/or small R the wealth is higher under injection of capital to the banking system.

(c) Ex-ante, when choosing the policy, the government should choose to inject capital to the banking system when β is sufficiently high, R is sufficiently low, and y is either sufficiently high or sufficiently low (i.e., outside an intermediate range).

Proof: The overall wealth in the economy under injection of capital to the banking system is given by $(1-(1-\alpha)l)K$ when the economy is in a credit freeze, and by $(1-(1-\alpha)l)K(1+R)$ when the economy is in a credit thaw.

The overall wealth in the economy under direct lending to operating firms is given by $(1-l)K$ when the economy is in a credit freeze, and by $\big(1-l+\alpha l(1-\beta)\big)K(1+R)$ when the economy is in a credit thaw. Note that in a credit thaw, only $(1-\beta)$ of the projects financed by the government succeed, as the government cannot tell the difference between good firms and bad firms. In a credit freeze, all the projects financed by the government fail, as even the good firms cannot succeed given that too many of them do not receive financing (if this was not the case, then banks would lend, and there would not be a credit freeze).

Based on these results, we now prove the different parts of the proposition.

(a) Now, when the fundamental θ is below θ^*_{Direct}, we know that there is a credit freeze under both regimes. Then, since $(1-(1-\alpha)l)K > (1-l)K$, the wealth in the economy is higher under infusion of capital to the banking system than under direct lending to firms.

When the fundamental θ is above θ^*_{Bank}, there is a credit thaw under both regimes. Then, since $(1-(1-\alpha)l)K(1+R) > \big(1-l+\alpha l(1-\beta)\big)K(1+R)$, the wealth in the economy is again higher under capital injection to banks than under direct lending to operating firms.

(b) When the fundamental θ is between θ^*_{Direct} and θ^*_{Bank}, the economy is in a credit thaw under the regime of direct lending and in a credit freeze under the regime of injection of capital to banks. Then, there is no obvious ranking between the levels of wealth in the two regimes: Capital injection to banks yields $(1-(1-\alpha)l)K$ and direct lending yields $\big(1-l+\alpha l(1-\beta)\big)K(1+R)$. Overall, a high enough β and/or a small enough R makes capital injection better than direct lending.

(c) For the choice of regime, the government should consider all possible realizations of θ, weighted by their prior probabilities, and the difference in wealth they generate between the two policy measures. Based on the results above, the expected difference between wealth under capital injection and wealth under direct lending can then be expressed as:

$$
\begin{aligned}
&\alpha l K \Phi\left(\frac{\theta^*_{Direct} - y}{\sigma_\theta}\right) \\
&\qquad + \big(\alpha\beta l - \big(1 - l + \alpha l(1-\beta)\big)R\big)K\left[\Phi\left(\frac{\theta^*_{Bank} - y}{\sigma_\theta}\right) - \Phi\left(\frac{\theta^*_{Direct} - y}{\sigma_\theta}\right)\right] \\
&\qquad + \alpha\beta l K(1+R)\left[1 - \Phi\left(\frac{\theta^*_{Bank} - y}{\sigma_\theta}\right)\right]
\end{aligned}
$$

The statement in (c) then follow directly based on (a) and (b). **QED**. ■

Remarks: (i) *When* θ *is below* θ^*_{Direct} *or above* θ^*_{Bank}: The first part of the Proposition indicates that in these circumstances, direct lending is clearly undesirable. In these circumstances, direct lending does not have an advantage over capital infusion into banks in terms of inducing banks to lend to operating firms, but the costs of direct lending are still borne. When θ is above θ^*_{Bank}, capital infusion into banks will be sufficient to produce a credit thaw, as would direct lending program, and thus there is no reason to bear the costs of lending to firms with bad projects the latter involves. When θ is below θ^*_{Direct}, direct lending by the government will not induce banks to do their own lending to operating firms, and, given this, the government will produce low returns not only on loans to operating firms with bad projects but also on operating firms with good projects, as not enough of them are getting loans to generate success.

(ii) *When* θ *is between* θ^*_{Direct} *and* θ^*_{Bank}: In these circumstances, which are the focus of the second part of the proposition, infusion of capital into the banks will fail to induce them to lend even though lending to operating firms with good projects is efficient. Direct lending by the government will accordingly have two benefits in these circumstances: first, it will provide financing to some operating firms with good projects that should receive financing; second, direct lending will induce banks to lend to operating firms. On the other hand, direct lending by the government will involve the wasteful provision of financing to firms with bad projects. If β is sufficiently large – that is, when the government's screening ability is sufficiently poor that it will not be able to provide financing to operating firms with good projects without providing financing also to a sufficiently large number of firms with bad projects – this cost of a direct lending program may make it overall undesirable. The same is true when the return on successful good projects R is sufficiently low.

(iii) *Ex-ante choice between the two policy measures*: As noted above, the government does not know the realization of θ. Hence, it should make its decision between the two policy measures based on the characterization provided above of what will happen for different realizations of θ and on the prior distribution of θ. Clearly, based on the above, we can see that for sufficiently high β and/or low R, the government should not go with direct lending. In addition, y – the prior mean of the fundamentals – matters for the decision. Given that direct lending may only be desirable at an intermediate range of the fundamentals, the government should not choose it when y is either too high or too low, only when it is in an intermediate range.

(iv) *The Case in which only Operating Firms with Good Projects have Beneficial Spill-over Effects*: Finally, it should be noted that our analysis in this section was conducted under the assumption that capital that is lent to bad firms still creates positive externalities to other firms even though it generates no direct return. It might be argued, however, that some bad projects create no or lower spill-over benefits for other firms. To examine the consequences of this factor, let us assume that the payoffs of operating firms do not depend on the number of other firms in operation but on the number of other firms in operation with good projects. Making this assumption weakens the attractiveness of direct lending to operating firms by the government.

To see this, note that if only good firms getting capital from the government created synergies to other firms, than the equation that determined the threshold θ^*_{Direct}, below which a credit freeze occurs in a regime of direct lending, would change from equation (8) to the following (we consider the limit again, for simplicity):

$$\theta^*_{Direct} = b - aK\big(1 - \big(1 - \alpha(1-\beta)\big)l\big) + aK(1-l)\frac{1}{1+R} \qquad (9)$$

Clearly, this would increase the likelihood of a credit freeze under direct lending, making this regime overall less desirable.

4.4. Government Funds Managed by Private Firms

While the direct lending program analyzed in the preceding section could ensure that the government's capital will flow to operating firms, it is disadvantaged by the government's inability to distinguish between operating firms with good and bad projects. Accordingly, a direct lending program could benefit if it were designed to utilize the expertise of private parties in screening operating firms with good projects from operating firms with bad projects.

Consider the following mechanism. The government places the capital $Z = \alpha l K$ in a number of funds dedicated to lending to operating firms, with any amounts not so invested placed in the economy's riskless asset. The funds will be managed by banks or by other private agents that have the same expertise. The managers of the funds will be paid, in addition to reimbursement of the expenses of processing loans which for simplicity are assumed to be zero in our model, a proportion γ on any profit that they generate on the capital invested by the fund they manage – that is, the excess of the return they generate over the riskless return. Note that, whereas the managers will capture a share of the profits if any that the fund will generate, they (like hedge fund managers) will not bear any share of the losses generated, if any, and such losses will be borne by the government.[5] The following proposition characterizes the consequences of this mechanism.

Proposition 7: *(a) If the government invests $Z = \alpha l K$ in funds dedicated to lending to operating firms, and managed by private agents promised a proportion γ on any return they generate above 1, then (i) the funds' capital will be fully lent to operating firms with good projects, and (ii) the threshold defining whether banks will lend to operating firms will be θ^*_{Direct}, as characterized in equation (6).*

(b) Consider the case where τ_p approaches infinity: Compared with infusing the capital Z into banks, the setting of government funds proposed here will (i) produce the same total wealth if θ exceeds θ^*_{Bank}, (ii) produce a higher total wealth if θ is between θ^*_{Direct} and θ^*_{Bank}, and (iii) produce a lower total wealth if θ is lower than θ^*_{Direct}.

[5] For a fuller discussion of the institutional details involved in implementing this mechanism, see Bebchuk (2008b).The mechanism is similar to the one proposed by Bebchuk (2008a) for the government's purchase of troubled assets through funds using governments fund and run by private agents compensated with a cut of the profits generated by the funds.

(c) Ex-ante, when choosing the policy, the government should choose to inject capital to the banking system, rather than to place it with private funds, when R is sufficiently low, and y is sufficiently low.

Proof: (a) When choosing whether to lend the government's capital to operating firms or not, banks managing the government's funds always prefer to lend. This is because their only chance to get a return above 1, on which they are compensated, is when they lend. Moreover, given that the noise, with which the banks observe the fundamentals is unbounded (even though it can be very small), they always perceive some probability that lending will generate a return above 1, which will provide compensation for them, while they know that there is no cost in generating a return below 1. Hence, the government's capital always flows to operating firms, generating the threshold θ^*_{Direct} characterized in equation (6). Finally, it is straightforward that banks lend to good firms and not to bad firms, as a positive return only comes from the former and the banks can distinguish between the two types.

(b) The second part of the proposition follows similar lines to those used in the proof of Proposition 6. The difference is that when the government sets private investment funds, as opposed to when it lends directly to operating firms, the government's capital does not go to bad firms. Hence, the overall wealth in the economy when the government lends the money via private funds is given by $(1-l)K$ when the economy is in a credit freeze (here the government's capital still gets wasted because even good firms fail to produce returns), and by $(1-(1-\alpha)l)K(1+R)$ when the economy is in a credit thaw. Then, comparing this with the overall wealth levels under infusion of capital to the banks, we get the result stated in part (b) of the proposition.

(c) Based on the results above, the expected difference between wealth under capital injection to banks and wealth under lending via private funds can be expressed as:

$$\alpha l K \Phi\left(\frac{\theta^*_{Direct}-y}{\sigma_\theta}\right)-(1-(1-\alpha)l)RK\left[\Phi\left(\frac{\theta^*_{Bank}-y}{\sigma_\theta}\right)-\Phi\left(\frac{\theta^*_{Direct}-y}{\sigma_\theta}\right)\right]$$

The statement in (c) then follow directly. **QED**. ■

Remarks: (i) *The Decisions of the Government Funds' Managers*: The design of the mechanism ensures that the government's capital invested in the funds will be fully provided to operating firms with good projects. Because the government will fully bear the losses, the managers will have no reason to avoid lending the funds given to them. Furthermore, because the managers will be promised a cut of the profits, they will have an incentive to screen operating firms with good projects from operating firms with bad projects, and their dominant strategy will be to lend funds only to firms with good projects.

(ii) *The Effect on Banks' Lending Threshold*: Because the government funds program, like the direct lending program, will ensure that an amount of $Z = \alpha lK$ will be lent to operating firms, the threshold for banks' lending to operating firms will be the same as the threshold, defined in equation (6), that would result from the direct lending program under the assumption that all operating firms have positive externalities for other operating firms.

(iii) *Comparison with the Direct-Lending Program*: Relative to the direct-lending program, the government funds mechanism has the advantage of not involving loans to operating firms with bad project; as a result, the government funds program performs better in a comparison with infusion of capital into banks. While the government funds program does not have this cost of the direct-lending program, it does, like the direct-lending program, provide capital to firms in circumstances in which θ *is below* θ^*_{Direct}, which are circumstances in which even funding good operating firms is inefficient (because not enough of them are being funded).

(iv) *Comparison with Infusion of Capital into Banks:* In circumstances in which θ *exceeds* θ^*_{Bank} , where infusion of capital will be sufficient to produce a credit thaw, the government funds mechanism will perform neither better nor worse than infusion of capital. In this case, both mechanisms will lead to providing *K* to operating firms with good projects. (In these circumstances, the direct-lending program performs worse than capital infusion into banks, because it involves lending to operating firms with bad projects.)

In circumstances in which θ is between θ^*_{Direct} and θ^*_{Bank}, the government funds mechanism will be superior to infusion of capital to banks. In these circumstances, the infusion of capital will not eliminate an inefficient capital freeze, and no funding will be provided to operating firms. In contrast, in these circumstances, under the government funds mechanism, both the capital in the government funds and the capital in the hands of the banks will be provided to operating firms.

Finally, when θ is lower than θ^*_{Direct}, the government funds mechanism will produce inferior results. In this case, the operation of government funds will not lead banks to lend to operating firms, and the lending by the government funds will produce losses.

(v) *Ex-ante choice between the two policy measures*: Given that the government funds mechanism is worse than infusion of capital to banks only when the fundamental is relatively low, the government would prefer injecting capital to banks only when y – its prior expectation about the fundamental – is sufficiently low.

4.5. Government Guarantees

A related mechanism the government may use to get the banking sector out of a credit freeze is to provide banks with guarantees. In this case, the government does not provide any capital upfront, but rather just commits to cover banks' losses in case the return on their loans falls below 1.

Suppose that the government guarantees a proportion γ of a bank's losses. In this case, a lending bank will receive the return $1+R$ when projects succeed and $\gamma < 1$ when projects fail. The following proposition characterizes the consequences of using this mechanism.

Proposition 8: *(a) Suppose that the government provides a guarantee covering a proportion γ (between 0 and 1) of banks' losses, that is, the government pays γ when a bank lends and real projects fail. Then, the threshold θ^*, below which a credit freeze occurs is given by:*

$$\theta^* = b - aK(1-l) + aK(1-l)\Phi\left(\frac{\tau_\theta}{\sqrt{\tau_p}}\left(\theta^* - y + \frac{\sqrt{\tau_\theta+\tau_p}}{\tau_\theta}\Phi^{-1}\left(\frac{1-\gamma}{1+R-\gamma}\right)\right)\right), \quad (10)$$

which is decreasing in γ.

(b) At the limit, as τ_p approaches infinity, the threshold (denoted as $\theta^*_{Guarantees}$) is given by:

$$\theta^*_{Guarantees} = b - aK(1-l) + aK(1-l)\frac{1-\gamma}{1+R-\gamma} \tag{11}$$

Then, when the government provides full guarantees ($\gamma = 1$), all banks lend and projects fail only when they are inefficient ($\theta < b - aK(1-l)$). Otherwise, banks do not always lend, and projects sometimes fail even though they are efficient ($\theta > b - aK(1-l)$).

Proof: The proof follows similar steps to those in Propositions 1, 2, and 3. **QED**. ■

Remarks: (i) *The Nature of the Mechanism:* Government guarantees reduce the threshold below which crises occur because they make it more attractive for banks to lend. Considering the case where banks' signals are very precise (consider the limit case of part (b)), the attraction in this mechanism is that the government essentially does not need to provide any capital. Above $\theta^*_{Guarantees}$, where banks lend, the government's guarantee of providing capital if loans fail is sufficient to get the economy out of a credit freeze. Hence, banks lend and loans do not fail, so the government does not need to provide the capital. Below $\theta^*_{Guarantees}$, where banks do not lend, there are no loans made, and hence no loans that fail. This implies that the government's guarantees again do not lead to any capital being spent. In sum, this mechanism leads to an improvement in the threshold below which a credit freeze occurs without any actual cost.

(ii) *Comparing this Mechanism with Previous Ones*: It is hard to provide a sharp comparison. Such comparison depends on the extent to which the guarantees can reduce the threshold θ^*. This, in turn, depends on the level of the guarantees γ. In principle it is tempting to conclude that the government should increase γ very close to 1,[6] but this is not so easy. Essentially, while the mechanism does not lead to actual costs, its validity depends on the credibility of the government in providing the guarantees. That is, banks have to believe that the government will indeed be able to pay back a proportion γ of the losses. Hence, there is a budget constraint in the background that has to be considered.

[6] There is a problem in setting $\gamma = 1$, because at that level of guarantees, banks always lend and the government will have to bail them out sometime. Setting γ very close but still below 1 ensures that banks (who have infinitesimally precise signals) never lend when projects fail.

The solution is for the government to increase γ (still below 1) until this budget constraint becomes binding. A reasonable case to consider is where the maximum guarantee provided by the government is equal to its available capital $Z = \alpha l K$. The maximum that the government will have to pay is when all banks lend and fail. This will cause a liability of $\gamma(1-l)K$, implying that γ cannot exceed $\frac{\alpha l}{(1-l)}$. The following proposition compares the government guarantees mechanism with the government funds mechanism (considered in the previous section) for this level of guarantees (assuming that $\frac{\alpha l}{(1-l)} < 1$, i.e., that the government's available capital is smaller than that in the banking sector).

Proposition 9: *(a) Suppose that the government provides a guarantee $\gamma = \frac{\alpha l}{(1-l)} < 1$, and that τ_p approaches infinity. Then the threshold under the guarantee regime ($\theta^*_{Guarantees}$) is higher than under the funds regime (θ^*_{Direct}), implying that the latter is more effective in preventing a credit freeze.*

(b) Compared with the government funds regime, providing guarantees as proposed here will (i) produce a lower total wealth if θ exceeds $\theta^*_{Guarantees}$, (ii) produce a lower total wealth if θ is between θ^*_{Direct} and $\theta^*_{Guarantees}$, and (iii) produce a higher total wealth if θ is lower than θ^*_{Direct}.

(c) Ex-ante, when choosing the policy, the government should choose to provide guarantees, rather than to place capital with private funds, when y is sufficiently low.

Proof: (a) We need to show that:

$$b - aK(1-(1-\alpha)l) + aK(1-l)\frac{1}{1+R} < b - aK(1-l) + aK(1-l)\frac{1-\gamma}{1+R-\gamma}$$

This can be developed as follows:

$$-(1-(1-\alpha)l) + (1-l)\frac{1}{1+R} < -(1-l) + (1-l)\frac{1-\gamma}{1+R-\gamma}$$

$$-\alpha l + (1-l)\frac{1}{1+R} < (1-l)\frac{1-\gamma}{1+R-\gamma}$$

$$(1-l)\left(\frac{1+R-\gamma-(1+R)(1-\gamma)}{(1+R)(1+R-\gamma)}\right) < \alpha l$$

$$\frac{\gamma R}{(1+R)(1+R-\gamma)} < \frac{\alpha l}{(1-l)}$$

Plugging in $\gamma = \frac{\alpha l}{(1-l)}$, we get:

$$R < (1+R)(1+R-\gamma)$$

$$0 < (1+R)(1-\gamma) + R^2$$

which is always true.

(b) The government guarantees regime generates an overall wealth of $(1-(1-\alpha)l)K$ when the economy is in a credit freeze (here the government's capital doesn't gets wasted), and of $\left((1-l)(1+R)+\alpha l\right)K$ when the economy is in a credit thaw (here the government's money doesn't get invested in the real projects). Then, comparing this with the overall wealth levels under the government funds program (in Proposition 7), we get the result stated in part (b) of the proposition.

(c) Based on the results above, the expected difference between wealth under the funds regime and wealth under the guarantees regime can be expressed as:

$$-\alpha l K \Phi\left(\frac{\theta^*_{Direct} - y}{\sigma_\theta}\right) + (1-(1-\alpha)l)RK\left[\Phi\left(\frac{\theta^*_{Guarantees} - y}{\sigma_\theta}\right) - \Phi\left(\frac{\theta^*_{Direct} - y}{\sigma_\theta}\right)\right]$$
$$+ \alpha l R K\left[1 - \Phi\left(\frac{\theta^*_{Guarantees} - y}{\sigma_\theta}\right)\right]$$

The statement in (c) then follows directly. **QED**. ■

Remark: The advantage of the guarantees regime over the funds regime is that it avoids the waste of government's capital when there is a credit freeze. This is because in the guarantees regime, the government's capital does not get invested, while in the funds regime it gets invested in projects that end up failing because of the coordination failure. The disadvantage, however, is that under the guarantees regime, there are overall more realizations of the fundamental with a credit freeze – the government's investment turns out to be more efficient in breaking the coordination failure – and also under a credit thaw the government's capital does not get invested. Overall, government guarantees offer a better solution when y is relatively low. In that, the guarantees regime is closer to the injection of capital to banks considered in Section 4.2. Direct comparison between the two (not reported here, for brevity) reveals that capital injection to banks dominates the guarantees regime when $R > \gamma$, while the guarantees regime may be preferred when $R < \gamma$.

5. Concluding Remarks

This paper has developed a model of credit freezes that are inefficient but arise from the rational and self-fulfilling expectations of financial institutions. In this equilibrium, banks would be collectively better off if they were all willing to extend loans to a set of operating firms, but each of them avoids doing so out of self-fulfilling expectations that others will do as well. In such circumstances, efficiency will be served by getting the economy out of the inefficient credit freeze equilibrium, and the developed model has been seen to be useful for studying and assessing government policies that can be considered for this purpose.

Our analysis has shown that interest rate cuts, and infusion of capital into the financial sector might but also might not produce a credit thaw. Even with very low interest rates, and with ample capital, banks will not extend loans to operating firms when they believe that their projects, even though worthy in an environment in which other such firms obtain financing, will fail in an environment in which credit to other firms is frozen. If such circumstances arise, the government will have to look beyond interest rate cuts and capital enhancement to get the economy out of the credit freeze.

An alternative or supplemental approach that may be considered would involve the government's getting more directly involved in assuming risks generated by portfolios of loans to operating companies. We have shown how a credit thaw could be facilitated by the government's investing in government-owned funds that will extend loans to operating firms and be run by private managers. Provision of guarantees by the government to lending banks may also be preferred in some circumstances.

Our work has implications for the current economic crisis. With government intervention producing a substantial increase in the financial system's capital, some observers suggest that lack of expansion in the credit extended to operating firms will imply that the current economic environment has made such expansion no longer efficient. Our analysis indicates that this inference cannot be made. While banks' failure to extend additional credit may be efficient, it may also be an inefficient outcome due to coordination failure. Our paper provides a framework for examining this possibility and potential government responses to it.

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