## Basic Mathematics Aptitude Test

(Full score: 40)

## Please Note:

> You have 60 minutes to complete.
> No calculators are allowed.
> Please show all your work and write your answers in the designated space.

Thank you.

Country:
Reference Number:
Name:
( Please show all your work here and write your answers in the designated space. )
Answer the following questions ( 2 point for each question ):

1. Calculate the following: $2 \frac{5}{14} \div\left\{2 \frac{1}{4} \times\left(2 \frac{3}{7}-1 \frac{2}{9}\right)-1 \frac{27}{35}\right\}$.

## Answer:

2. Calculate the following: $\log _{2} 2 \sqrt{6}-\log _{2} \sqrt{3}$.

Answer:
3. Solve for $x$ in the following equation.

$$
\left(\log _{2} x\right)^{2}-\log _{2} x^{3}-4=0
$$

4. Obtain the approximate value for $x$ in the following equation using $\log _{\mathrm{e}} 55 \cong 4$.

$$
55^{x}=e^{x-1}
$$

Answer:
5. Solve for $x$ in the following equation.

$$
0.2 x^{2}+1.2 x+1.6=0
$$

## Answer:

6. Solve for $(x, y)$ in the following two simultaneous equations.

$$
\left\{\begin{array}{l}
2 x+y=3 \\
3 x-y=7
\end{array}\right.
$$

Answer:
7. Find the total derivative $d z / d y$, given $z=(x+y)(x-2 y)$ where $x=2-7 y$.
8. Find the natural number(s) of $x$ that satisfy the following inequality: $x^{2}-5 x+4<0$.

Answer:
9. Given the three sets $S_{1}=\{2,4,6\}, S_{2}=\{2,6,7\}$, and $S_{3}=\{4,6\}$, find: $\left(S_{1} \cup S_{2}\right) \cap S_{3}$,
where $U$ and $\cap$ denote union and intersection of sets, respectively.

## Answer:

10. Two dice are rolled; find the probability that the sum is equal to 6 .

Answer:
11. A random variable $X$ can take only three values, 1,2 , and 3 . And probabilities are given as $\operatorname{Pr}(X=1)=0.5, \operatorname{Pr}(X=2)=0.3$, and $\operatorname{Pr}(X=3)=0.2$, respectively. Calculate the Expected value of $X$.

Answer:
12. Find the first derivative of the following function.

$$
y=x \log _{\mathrm{e}} x
$$

Answer:
13. Find the quadratic approximation of the following function at the point $x=0$.

$$
f(x)=2 \log _{\mathrm{e}}(1+x)
$$

14. Evaluate the following integral. $\int_{0}^{1}\left(1-x^{2}\right) d x$

## Answer:

15. Evaluate the following sum. $2+4+6+8+\cdots+100$
16. Find the local maximum of the following function (Find both (i) the critical point $(x, y)$ that maximizes the function and (ii) the maximum).

$$
\mathrm{f}(x, y)=1-2 x^{2}-y^{2}
$$

## Answer:

17. Find the solution of the following constrained maximization problem (Find both (i) the critical point ( $x, y$ ) that maximizes the function and (ii) the maximum).

$$
\max _{x, y} f(x, y)=x y^{2} \text { subject to } 2 x+4 y=8 .
$$

## Answer:

18. Given a matrix $X=\left(\begin{array}{cc}1 & -1 \\ 1 & 0 \\ 1 & 2\end{array}\right)$, find the matrix $X^{\prime} X$.
19. Find the solution of the following equation system.

$$
\binom{x}{y}=\left(\begin{array}{cc}
3 & -3 \\
-3 & 5
\end{array}\right)\binom{x}{y}+\binom{1}{-1}
$$

Answer:
20. What is the present value ( PV ) of an investment that will pay $\$ 400$ in one year's time, and $\$ 400$ every year after that, when the annual interest rate is $\mathbf{5 \%}$ ?

Answer:

