The Shifting and Steepening of Phillips Curves During the Pandemic Recovery: International Evidence and Some Theory

Tryggvi Gudmundsson, Chris Jackson and Rafael Portillo

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Prepared by Tryggvi Gudmundsson, Chris Jackson and Rafael Portillo

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ABSTRACT: We study the global inflation surge during the pandemic recovery and the implications for aggregate and sectoral Phillips curves. We provide evidence that Phillips curves shifted up and steepened across advanced economies, and that differences in the inflation response across sectors made the price of goods relative to services behave procyclically rather than a-cyclically as during previous cycles. We present a two-sector new-Keynesian model where all three features (shifting, steepening, and relative price procyclicality) emerge endogenously once we introduce unbalanced recoveries that run against a supply constraint in the goods sector. A calibrated exercise shows that the resulting changes to the output-inflation relation are quantitatively important and greatly improve the model's ability to replicate the inflation surge, both in magnitude and composition. These changes to the Phillips curve are also temporary: they disappear once supply constraints are no longer increasingly binding, leading to a sharp fall in goods inflation but a more gradual disinflation in services.¹

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WORKING PAPERS

The Shifting and Steepening of Phillips Curves During the Pandemic Recovery: International Evidence and Some Theory

Prepared by Tryggvi Gudmundsson, Chris Jackson and Rafael Portillo

		Contents	Page
I.		uction	
II.	A. B. T. 1 2 2 3 4	Limited labor market slack and increased frictions	. 10 . 14 . 14 . 14 . 14
III.	Phillip A. I B. F	The inflation response before and after Covid	. 15 . 16
IV.	model	Households Distribution Sector Market clearing Aggregation and exogenous processes Phillips curves under different business cycles Phillips curves under balanced and unconstrained business cycles Phillips curves with unbalanced and unconstrained business cycles Phillips curves with unbalanced and unconstrained business cycles and initially high/low relative goods prices Phillips curves under unbalanced and constrained business cycles and initially high/low relative goods prices Alternative Results Alternative model-based Phillips curves	. 21 . 21 . 22 . 22 . 23 . 24 . 24 . 24 . 25 . 25 . 27 . 29 . 31 . 31
V. VI.	Appen A. M B. A	Model estimations and robustness checks A two sector new-Keynesian model with supply constraints B Households Distribution Services Wholesale Goods and Services Sectors	. 41 . 45 . 45 . 45 . 45

5.	Steady state														4	47
6.	Log-linearization.														2	48

I. Introduction

The world economy experienced an unprecedented surge in inflation during the pandemic recovery that surprised most economists and policy-making institutions.² The magnitude of the surprise raises the question of whether standard analytical frameworks used for forecasting and policy analysis—variations of the new-Keynesian model used in policy making institutions—lack crucial features for thinking about the Covid period. Specifically, the Phillips curve—the relationship between inflation and activity which lies at the heart of New Keynesian models—was widely thought to be flat before the pandemic. These models have, however, struggled to explain the recent surge in inflation (Gopinath 2022). While models can always match data through shocks, an important question is whether they have the right structure so that the increase in inflation can arise endogenously. This requires a clear understanding of what has been different this time around.

In this paper we make several contributions to the ongoing debate on these questions and to an already burgeoning literature. First, we document the nature the post-Covid recovery and inflation surge, emphasizing features that were common to many economies but which differed from past economic cycles. Second, we compare the cross-country relationship between inflation and activity before Covid and during the pandemic recovery, using both local projection methods and estimates of various specifications of the New Keynesian Phillips curve. We investigate how Phillips curves have changed for core goods and services as well as for (aggregate) core inflation. Distinguishing core goods and services allows us to zoom in on the cyclical behavior of the relative price of goods and how it has differed from the pre-Covid period. Third, we show what additional features can help new-Keynesian models replicate these patterns, building on recent insights from the literature.

We find that the post-pandemic period differs from pre-Covid recessions in four key respects, several of which are well understood by now, but which help set the stage for the modeling exercise.³ First, the recovery was much more rapid. Second, there was a global rotation in demand away from services and toward goods. Third, the pandemic period featured unprecedented supply disruptions. Fourth, there were limited signs of labor market slack even though activity was below its pre-Covid trend. These features contributed to inflation dynamics that were markedly different to previous cycles. Inflation fell initially at the onset of the pandemic but also accelerated much more rapidly, rising well above pre-Covid rates. The inflation surge was also unusually skewed towards goods rather than services.

The differences with the pre-pandemic period are most telling when looking at the Phillips curve. In line with previous studies, we find that in past cycles Phillips curves looked relatively flat. The relationship was strongest between activity and services inflation. During the pandemic recovery instead, the Phillips curve has looked very different. First, it has become much steeper, so that movements along the recovery path led to a greater increase in inflation. Across specifications the slope of the curve has increased by 170-270 percent relative to pre-Covid. Second, the Phillips curve has also shifted outwards so that similar levels of activity (relative to pre-shock trends) have been associated with higher inflation rates. The estimated shift is around 130 basis points for core infla-

²See Koch and Noureldin 2023 who look at recent inflation forecast errors at the IMF and the factors that may have contributed to their rise in the current cycle.

³For an overview of the recent inflation surge in the US, see Blanchard and Bernanke 2023 and Ball et al. 2022.

7

tion, after controlling for other drivers of inflation including expectations and different estimates of slack. Third, there are notable differences between Phillips curves at the sector level. The shift up and steepening was most prominent in the goods sector, consistent with the price of goods relative to services becoming much more sensitive to the recovery.

We show that these empirical patterns (steepening, shifting, and relative goods price pro-cyclicality) emerge in a two-sector new-Keynesian model when we introduce unbalanced recoveries that run against a supply constraint in the goods sectors. The model builds on Guerrieri et al. 2021. It features two sectors, goods and services, roundabout production, and distribution costs that create a wedge between wholesale and retail goods prices. Firms in the two sectors face the same degree of nominal rigidities, while nominal wages are flexible. The structure is standard except for three features. First, there are shocks to sectoral preferences that leave aggregate spending unchanged. Second, the goods sector is subject to a production constraint, which binds if demand exceeds a given threshold. The constraint captures in a simple way the effects of disruptions to global supply chains which, by disrupting manufacturing capacity upstream and lengthening delivery of intermediate inputs and final goods, limited the ability of firms in the goods sector to respond to increased demand during the pandemic. Third, as the model is non-linear, we also make the simplifying assumption that activity, i.e., the output gap, follows an exogenous process. This allows us to solve the model analytically.

We use the model to derive the relationship between inflation and activity under two types of business cycles. When activity is balanced and unconstrained, the model generates a "standard" Phillips curve relationship in which the inflation response is the same across sectors, as firms face the same marginal costs and have the same nominal rigidities. The relative goods price does not vary. We can think of this specification of the model as a simplified version of the pre-Covid period. One implication is that the sectoral structure and the relative price of goods are not important for aggregate inflation dynamics and the shape of the aggregate Phillips curve.

When demand is unbalanced toward goods, however, there is the possibility that production in the goods sector runs against the supply constraint. If this is the case, then the constraint replaces the sectoral Phillips curve as the supply curve for the sector. Instead of an equation that links goods inflation to the level of aggregate activity, the model now features a fixed supply of traded goods. As a result, the relative price of goods must now increase as much as is needed to equilibrate strong demand and limited supply. We assume the threshold depends on the level of potential output. The greater the recovery—relative to potential—the greater the increase in the relative goods price as the constraint becomes increasingly binding. Relative goods prices thus become markedly pro-cyclical. This mechanism was identified by Guerrieri et al. 2021; we show it has important implications for the slope of sectoral and aggregate Phillips curves as well.

The increase in the relative price of goods changes inflation dynamics in the rest of the economy (the services sector). The intuition is that constrained recoveries increase nominal marginal costs through their effect on the relative price of goods. There are two channels at play. First, higher goods prices directly raise firms' marginal costs because goods are used as intermediate inputs in production. Second, all else equal, higher goods prices raise the CPI and therefore nominal wages. Both channels put pressure on pro-

⁴See Alessandria et al. 2023 for an explicit treatment of increases in delivery times and stock-outs in an international model with input-output linkages.

ducers in the services sector to also raise their nominal prices to preserve their markups. As constrained recoveries raise the pro-cyclicality of marginal costs, they steepen the Phillips curve in the services sector.

In addition, while the original Phillips curve in the goods sector is no longer relevant, a new Phillips curve emerges in equilibrium. This is because goods inflation must move one-to-one with services inflation in order to implement the required change in relative goods price, as shown below:

$$\hat{\pi}_t = \frac{1}{(1-\bar{\alpha})_{\text{changes in relative goods prices}}} + \hat{\pi}_{St},$$
 goods inflation

where $\bar{\alpha}$ is the share of goods in core inflation. As relative goods prices are now procyclical, a rapid recovery implies a positive and equally rapid rate of change of these prices $(\Delta \hat{p}_{G,t} >> 0)$, which shifts goods inflation up relative to services. In addition, the steepening of the services Phillips curve carries over to goods inflation. A similar change (steepening and shifting) affects the Phillips curve for aggregate (core) inflation, in line with the empirical evidence:

$$\hat{\pi}_t = \frac{\bar{\alpha}}{(1-\bar{\alpha})} \Delta \hat{p}_{G,t} + \hat{\pi}_{St}.$$
 core inflation =
$$\frac{1}{2} \Delta \hat{p}_{G,t} + \hat{\pi}_{St}.$$
 services inflation.

While the model is stylized, it has several stark quantitative implications when calibrated to match the structure of an average advanced economy. The elasticity of substitution between goods and services consumption is a key parameter. In the baseline calibration it is set so that the model can replicate the behavior of relative goods prices observed during this period. When recoveries are unbalanced and constrained, the model generates an increase in the slope of the Phillips curve of between 50 and 60 percent, depending on the representation of the Phillips curve (dynamic versus static). The model also generates a shift in the Phillips curve for core inflation of about 80 basis points. We find that the roundabout production structure and distribution costs are crucial for the magnitude of the steepening, as they amplify the impact on "upstream" marginal costs for a given degree of relative (retail) price procyclicality.

While both steepening and shifting are sizable in the model, the steepening is smaller than the empirical estimates in the first part of the paper. One possible reason is that the observed increase in relative prices may not fully reflect the scarcity of goods during this period—the true (shadow) cost of goods may have been higher and more pro-cyclical as there were other non-price mechanisms, for example stock-outs. An alternative calibration of the model where the "true" relative price of goods is twice as pro-cyclical as observed in the data generates a steepening of the Phillips curve of between 90 and 110 percent, closer to the empirical estimates. The alternative calibration is also in line with standard values of the elasticity of substitution between goods and services in the literature (one), unlike the baseline calibration which requires an implausible high elasticity (four).

We also assess the model's ability to replicate the inflation surge through mid-2022, when core inflation for a group of advanced economies reached a peak of about six percentage points higher than the pre-Covid average (qoq sa). This is done by feeding a path of output into the model that broadly matches the recovery seen in our sample. The exercise also requires having an explicit view on the level of potential output during the

Covid recovery. We assume that potential output is four percentage points lower than the pre-Covid trend during this period, consistent with the lack of labor market slack observed during the Covid period.⁵ We show binding supply constraints significantly improve the model's quantitative performance. The version of the model where the recovery is unconstrained predicts an increase in core inflation of less than two percent (from steady state), well below the inflation surge in the data. The constrained version of the model that replicates the observed pro-cyclicality of relative goods prices generates a much larger increase in core inflation, of around four percent above steady state, while the constrained version of the model that assumes a larger increase in the "true" relative price of goods generates an increase in core inflation of close to six percent, broadly matching the data. Our simulations also show that it is the steepening of the Phillips curve that contributes the most to the amplification of the inflation surge.

Finally, the model is used to study the dynamics of inflation once supply constraints are no longer increasingly binding. This is the case if the output gap is gradually reduced, for example if potential output increases, or if demand is re-balanced toward services. In this case relative goods prices stay flat or start to decline $(\Delta \hat{p}_{G,t} \leq 0)$, and so the model predicts a sharp fall in goods inflation. Disinflation in the services sector is instead more gradual, with differences in inflation across sectors leading to a gradual normalization of relative goods prices. These predictions are in line with the inflation dynamics seen in the more recent period, providing further support to the channels presented in our model.

A. Brief review of the literature

This paper adds to the growing literature on the drivers of the post-Covid surge in inflation and the interpretation of the Phillips curve. Our empirical findings on the steepening of Phillips curves are consistent with the results of Hobijn et al. 2023. The findings are also consistent with two recent assessments of the US (Blanchard and Bernanke 2023 and Ball et al. 2022). The mechanism in our model helps formalize the "shocks to prices given wages" story put forward in the first paper and the "pass-through of headline into core" story put forward in the second paper.

Some papers have emphasized the impact of sectoral reallocation but via alternative mechanisms. For example, Ferrante et al. 2023 estimate a rich multi-sector model and argue that that the shift in demand from goods to services raised inflation because of reallocation costs and the greater flexibility of goods prices (see also Kindberg-Hanlon and Portillo forthcoming for a related argument). Gagliardone et al. 2023 emphasize instead the interaction of oil price shocks and real wage rigidities as a key driver of the inflation surge. Other papers have argued that the inflation surge was driven in part by the de-anchoring of inflation expectations (see Reis 2022 for suggestive evidence based on measures of distribution of household expectations and the model of adaptive expectations in Alvarez and Dizioli 2023).

The discussion of the non-linearity of the aggregate Phillips curves pre-dates the pandemic (Forbes et al. 2021; Hooper et al. 2019, Debelle and Laxton 1996) but has also been offered as an explanation for the post-Covid inflation surge. Benigno and Eggertsson 2023 embed non-linearities in the labor supply curve in a model of search and matching, while Harding et al. 2023 emphasize instead changes in the extent of strategic complementarities between firms, which reduce the degree of real rigidities in the model as

⁵See Blanchard, Domash, et al. 2022, among others.

the output gap becomes increasingly positive. These models imply that the non-linearity of the Phillips curve is a deeper feature that became apparent in the Covid period. While we find some empirical support for a non-linear Phillips curve before Covid, we find a subsequent steepening and shift up in the Phillips curve over and above this non-linearity. In our paper it is the combination of overheating and the unbalanced recovery that helps produce this change in the Phillips curve.

Supply constraints were identified early on as an important feature of the Covid period, for example in Gourinchas et al. 2021. The impact on inflation was also stressed by Giovanni et al. 2022 for the euro area, building on the multi-sector framework in Baqaee and Farhi 2022. Our model comes closest to two papers. First, it builds on Guerrieri et al. 2021. These authors present a two sector model with a similar mechanism: in the presence of a permanent shift in sectoral preferences, a more expansionary monetary policy pushes activity in the benefited sector up against its full-employment-consistent level. Once this happens the relative price of this sector goes up, which shifts the Phillips outward. There are important differences, however: these authors are interested in the optimal monetary policy response and their model features a stylized production structure, wage rigidities, and costly factor reallocation. Our model features flexible wages and a richer treatment of production and firms' marginal costs, which allows us to emphasize the steepening of sectoral and aggregate Phillips curve and assess the model's quantitative performance. Our paper is also related to Comin et al. 2023. These authors introduce potentially binding constraints in an estimated multi-sector model and show that these can shift Phillips curve up. They also argue that these shifts can account for half of the increase observed in inflation during the Covid period. Their paper does not assess the steepening of the Philips curve.

The rest of the paper is organized as follows. Section 2 provides a summary of the global surge in inflation. Section 3 compares the evolution of the economy to Covid shock with pre-Covid recessions. Section 4 evaluates the change in the Phillips curve before and after Covid. Section 5 presents the two-sector model and shows how it can replicate the patterns in the data. Section 6 provides areas for future work and concludes.

II. THE COVID SHOCK VERSUS PREVIOUS RECESSIONS

A. A local projections framework

To illustrate the unique nature of the Covid recovery, we compare the behavior of key variables following the onset of the pandemic with their average behavior during past recessions. We employ local projection methods (Jordà 2005) using the following specification:

$$y_{i,t+h} - y_{i,t} = \beta^h \operatorname{recession}_t + \theta^h \operatorname{Covid}_t + \gamma^h X_{i,t} + \mu_i^h + \epsilon_{i,t}^h$$

The term $y_{i,t+h} - y_{i,t-1}$ represents the change in the variable of interest h quarters from the start of a recession ($h = 0 \dots 12$). β^h indicates the response during the average pre-Covid recession and θ^h the response during Covid, both at horizon h. $X_{i,t}$ is a set of controls that includes the lagged dependent variable and the lagged quarterly change in the variable of interest; for equations estimating the impact on inflation we also include lagged oil price growth and lagged changes in the exchange rate. μ^h_i are country fixed

effects to control for time-invariant country characteristics. Implicitly, these capture the average change at horizon h in the variable of interest for each country.

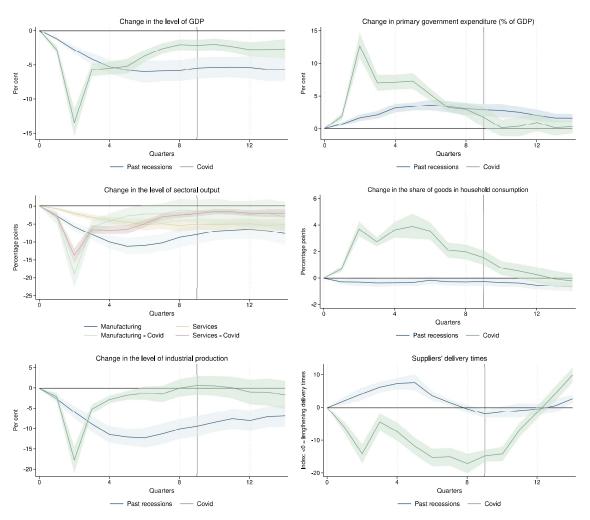
The impulse responses are estimated on an unbalanced panel of 25 OECD Advanced Economies from 1990Q1 to 2023Q1.⁶ We use robust standard errors clustered by country. The Covid dummy is set to one in 2020 Q1 for all countries. Pre-covid recessions are identified using the Harding and Pagan algorithm (2002) with a minimum cycle length of 3 years (ie the minimum period between two peaks or two troughs). The algorithm identified 68 recessions during the pre-Covid period, with 35 recessions occurring before 2007, 23 during the GFC and 10 during the European debt crisis.⁷ 40 percent of recessions in our sample occurred during the global financial crisis and therefore the latter weighs heavily on the pre-Covid results, with many variables displaying permanent declines following a recession.

The results are shown in Figures 1 and 2 and are discussed briefly in the following section.

⁶AEs in the sample are the following: Australia, Austria, Belgium, Canada, Czechia, Germany, Greece, Denmark, Spain, Estonia, Finland, France, UK, Italy, Japan, Korea, Latvia, Lithuania, Netherlands, Norway, Portugal, Slovakia, Slovenia, Sweden, USA.

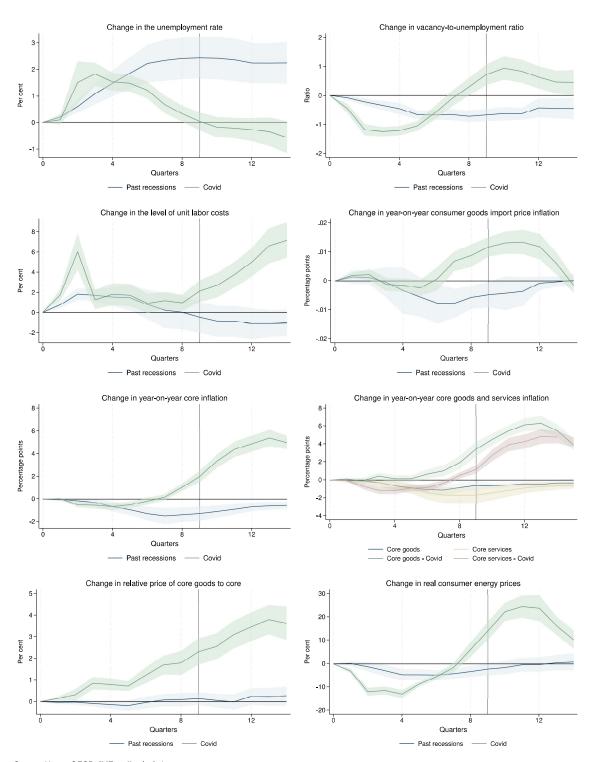
⁷The algorithm identifies some recessions as beginning in 2019 due to quarterly falls in GDP in that year. These are not considered so as not to overlap with the Covid period.

Figure 1. Impulse responses to past recessions and Covid: activity



Source: Haver, OECD, IMF staff calculations Note: Sample includes AEs within the OECD from 1990-2023Q1. Shaded areas represent 90% confidence interval. Vertical line denotes the start of Russia's war in Ukraine

Figure 2. Impulse responses to past recessions and Covid: labor market and prices



Source: Haver, OECD, IMF staff calculations Note: Sample includes AEs within the OECD from 1990-2023Q1. Shaded areas represent 90% confidence interval. Vertical line denotes the start of Russia's war in Ukraine

B. The Post-Covid Recovery: Four key facts

1. A rapid recovery in activity

The post-lockdown recovery in GDP was sharp (see Figure 1). Widespread concerns at the onset of the pandemic that global output would remain significantly depressed for a lengthy period—consistent with the pattern observed in previous recessions—did not materialize. Unlike past recessions, which were typically caused by a sustained contraction in aggregate demand, the Covid recession was the result of a temporary shutdown of large parts of the economy and efforts to reduce exposure to the virus. The fall in GDP was thus larger and more sudden, but also temporary. As the shutdowns were phased out, the subsequent recovery was swift, albeit still incomplete at the time of writing. The recovery was also boosted by decisive fiscal and monetary support. For instance, the increase in primary nominal government expenditures in response to Covid was materially larger than in other past recessions, in addition to support from lower interest rates and the expansion of central bank balance sheets.

2. A global rotation in demand towards goods (and then back toward services)

While the recovery in demand was swift, it was also uneven. Consumer spending initially rotated away from services and towards goods, prompted by behavioral responses to the pandemic and the continuation of some restrictions after the initial lockdowns. The consumption of goods recovered to or above pre-pandemic levels in many economies during 2021, but services consumption remained well below pre-Covid levels. The average composition of demand gradually rotated back towards services by the end of 2022. In contrast, the composition of spending during pre-Covid recessions/recoveries was much more stable.

3. The emergence (and fading) of supply chain disruptions

The recovery in industrial production and trade was more pronounced than the recovery in overall activity, in line with the surge in demand for goods. The recovery in industry and trade was associated with unprecedented bottlenecks in global supply chains, in stark contrast with the average pre-Covid recessions. Indicators of supply constraints, such as the S&P PMI index of suppliers' delivery times, usually improve during a downturn as demand for goods falls, freeing up spare capacity. Delivery times then return to normal as demand picks up. In contrast, supply delivery times worsened significantly during the Covid period, both during the lockdowns and in the reopening phase. The latter reflects a combination of factors: (i) recurrent Covid outbreaks and associated factory shutdowns, most notably in Asia in the summer of 2021, (ii) shortages of key components such as semiconductors, and (iii) shipping delays and a large increase in the cost of shipping.

4. Limited labor market slack and increased frictions

Labor markets recovered quickly in comparison to past recessions (see Figure 2).⁸ While the policy response varied widely across countries, unemployment on average rose sharply

⁸For a more detailed analysis of labor market dynamics in advanced economies during the pandemic, see Duval et al. 2022.

15

at the onset of the pandemic but recovered to near pre-pandemic levels after a few quarters. Alternative indicators of labor market slack, such as the vacancy-to-unemployment ratio (v/u), suggest that the labor market became even tighter than the pre-Covid period, reflecting changes in working patterns, labor participation, and sectoral demand. This is in clear contrast to the protracted slump in both unemployment and v/u in past recessions.

Lack of labor market slack in advanced economies can also be seen in the behavior of nominal unit labor costs (ULCs). Previous recessions feature a protracted decline in labor costs. Instead, ULCs rose sharply at the start of the pandemic reflecting labor hoarding and short-time work or job retention schemes, which kept workers employed even as their output dropped sharply. Changes in the sectoral allocation of work are likely also to have affected the data. As these effects faded, ULCs dropped back sharply but picked up again in the second half of the post-pandemic recovery as tight labor markets and high headline inflation increased labor costs and put upwards pressure on core inflation.

C. The inflation response before and after Covid

Differences between Covid and previous shocks are evident in the response of inflation (see Figure 2). The decrease in inflation is gradual in previous downturns, and it takes around two years for inflation to reach its trough. During the pandemic, core inflation fell more quickly than usual given the more sudden reduction in activity but it also reached a trough after only two quarters. The size of the initial fall in inflation was modest relative to past recessions, particularly given the size of the fall in GDP. Inflation started to accelerate a year into the shock and rose well above pre-Covid rates soon thereafter. Finally, the pickup in inflation was initially skewed towards goods inflation whereas the composition of inflation was usually much more balanced in the average pre-Covid recession. ⁹ The goods-heavy nature of inflation dynamics in turn highlights the global nature of the surge. This is reflected in the surge in imported inflation seen on average across advanced economies. This is again in stark contrast with the pre-Covid period.

III. PHILLIPS CURVES BEFORE AND AFTER COVID

The previous section suggests that the relationship between activity and inflation was markedly different during the post-Covid recovery, particularly at the sectoral level. We investigate this using two methodologies. First, we trace out the relationship between activity and inflation captured in the impulse responses in the previous section. This approach has the advantage of flexibility as we do not need to impose parametric restrictions on the relationship between activity and inflation. It does not, however, control for the movements in other possible drivers such as inflation expectations or commodity prices. We therefore complement this by estimating a standard New Keynesian Phillips curve on our panel of economies and investigate how the Phillips curve has changed after Covid, at an aggregate level and by sector.

⁹Core goods inflation is defined as goods excluding energy and food, also commonly referred to as non-energy industrial goods. Korea and Japan do not publish series of non-energy industrial goods and therefore durable goods prices are used instead.

A. Local Projections

This section uses the impulse response functions estimated from the local projection methods in the previous section to trace out Phillips curves before and after Covid. These are shown in Figure 3, which consists of four charts. The first chart plots the response of GDP at different quarters against the response of core inflation at the same horizon. The second chart plots this for unemployment against core inflation. The bottom two charts plot the impulse responses of GDP against services and core goods inflation, respectively. Evidence from the average pre-Covid recession in shown in blue; data from the Covid period is shown in red.

Deviation in core inflation (qoqar, %)

Change in unemployment (pp)

Deviation in core goods inflation (qoqar, %)

Deviation in core goods inflation (qoqar, %)

Deviation in services inflation (qoqar, %)

Past recessions

Covid

Figure 3. Deviation in activity versus inflation

Hollow dots indicate data points subsequent to the start of Russia's war in Ukraine: 2022Q2-2023Q1

Our exercise assumes that recessions/recoveries can be interpreted as movements along a Phillips curve, for a given level of potential output, which could nonetheless vary across recessions. We make two adjustments to ensure this is a reasonable interpretation of the data. For pre-Covid recessions, we trace out the segment of the impulse response for the first eight quarters of the recovery. The reason is that in pre-Covid recessions inflation starts to recover eight quarters after the onset of the recession while GDP stays

permanently lower, likely because the inflationary effect of lower potential output becomes increasingly evident. For the Covid period, we do not include the first two quarters of 2020, as the impact on activity in both quarters is dominated by the lockdowns. Therefore, these two quarters are not very informative about the slope of the Phillips curve. We also show the dots from the period following the Russia war with Ukraine as hollow to differentiate them from the pre-war period.

The difference in the shape of the Phillips curves before and after Covid is stark. The pre-Covid recessions are consistent with a clearly downward sloping but relatively flat aggregate Phillips curve in which inflation falls when activity falls below trend during the recession, or equivalently when unemployment increases above its trend. This is consistent with the pre-pandemic view that the slope of the Phillips curve was fairly flat (Simon et al. 2013, among many others).

During the pandemic recovery, the relationship has shifted up and become steeper. Core inflation was at or above trend throughout the pandemic period, even though average activity was below trend for most countries. This suggests that supply has fallen after the pandemic. In addition, core inflation accelerated as the recovery gathered steam, reaching its highest value in 2023Q1 at the end of our sample. A similar relation emerges when we replace activity with unemployment. Unlike for activity there is clearer evidence of some tightening, with unemployment falling below its trend by the first quarter of 2022 and remaining low after that. Again, the relation between unemployment and inflation is again starkly different from the pre-Covid period.

At the sectoral level there is clear evidence of a shift in the Phillips curves for both goods and services. Pre-Covid, both curves appear fairly flat. Since the pandemic, however, core goods inflation increased steeply as activity recovered. Services inflation, while relatively subdued early in the recovery, increased to rates higher than what would be implied by the pre-Covid Phillips curves. It also accelerated as the recovery gathered steam.

These changes at the sectoral level also imply a stark change in the behavior of the relative price of goods. Figure 4 shows that in past recessions, the relative price of goods was relatively a-cyclical. But after Covid, the relative price of goods shifted up and increased with the level of activity. As we show in Section 4, this increased cyclicity of the relative price of goods has important implications for inflation dynamics across the economy, especially in the services sector.

Deviation in the relative price of goods to core

Deviation in the relative price of goods to core

1
2
1
5
0
5
0
5
0
5
0
Change in unemployment (pp)

Past recessions

Covid

Figure 4. Deviation in activity versus relative price of goods

Hollow dots indicate data points subsequent to the start of Russia's war in Ukraine: 2022Q2-2023Q1

B. Phillips curves estimates

We next test formally whether the Phillips curve relationship changed after Covid by estimating a standard reduced-form hybrid New Keynesian Phillips curve (IMF 2021); (Gali and Gertler 1999) on a panel of advanced economies. The specification is the following:

$$\pi_{i,t} = \alpha_i + \beta gap_{i,t} + \gamma Covid_t * gap_{i,t} + \delta Covid_t + \gamma_1 \pi_{i,t-1} + \gamma_2 \pi_{i,t-2} + \gamma_3 \pi_{i,t}^e + \sum_{j=0}^{q} \zeta_{t-j} X_{i,t-j} + \epsilon_{i,t}$$

 $\pi_{i,t}$ is the annualized quarterly core inflation rate in country i at time t. We estimate separate models for core, core goods and services inflation. $gap_{i,t}$ is slack in the economy. In the baseline specification, we use the unemployment gap defined as the unemployment rate minus the estimate of the natural rate of unemployment in the April 2023 World Economic Outlook¹⁰. We consider alternative measures of slack in following section. The model includes country fixed effects, α_i , two lags of core inflation, 1-year inflation expectations from Consensus and a vector of other controls, $X_{i,t}^{11}$. The model is estimated on a panel of 25 advanced economies over the period from 2000Q1 to 2023Q1, using robust standard errors clustered at the country level. We test for whether Covid affected the relationship between slack and inflation by including a dummy which takes a value of one over 2020Q1-2023Q1 and zero otherwise and interact that Covid dummy with the measure of slack. We also include a separate dummy for 2020Q2-Q3 as those quarters were

¹⁰The estimates of the natural rate of unemployment are annual but slow moving. We use a cubic spline to create a quarterly series. We use OECD estimates for three countries where IMF estimates are unavailable. We use the June 2020 forecast, as after this date the OECD switched to publishing trend rates of total employment.

¹¹The bias associated with using fixed effects in a dynamic panel model is likely to be small given *T* is reasonably large. The controls include the detrended level of real consumer energy prices and real total import prices and lagged quarterly changes in real energy prices.

dominated by the initial lockdowns. Detailed results are shown in Table 3 in the Appendix.

Figure 5 shows that the Phillips curve both steepened and shifted up since the start of the pandemic, using our preferred specification (with the unemployment gap). The slope of the Phillips curve has become around two and a half times steeper than before Covid, while the curve has shifted up by over a percentage point, after controlling for other covariates. In other words, the fall in unemployment during the post-Covid recovery has been associated with a larger increase in inflation than before the pandemic, but the level of unemployment relative to trend needed to keep inflation at target has also increased.

A similar specification is used to test whether relative core goods prices have became more pro-cyclical after the pandemic. Relative goods prices de-trended using a Hodrick-Prescott filter are regressed on different measures of slack interacted with a dummy for the Covid, controlling for real consumer energy prices and real import prices. Consistent with the observation from the location projections results, the results in Table 4 indicate that relative goods prices became more pro-cyclical after Covid, although the statistical significance of the increase varies by specification and measure of slack. Slack as measured by the output gap and vacancy-to-unemployment ratio indicate a statistically significant increase, but somewhat less so when using the unemployment gap.

Core inflation (qoqar)

4

3

2

4

Unemployment gap

Core goods inflation (qoqar)

4

4

Unemployment gap

Pre-Covid

Post-Covid

Figure 5. Phillips curve slopes: linear model

Note: Phillips curves evaluated at series average values of inflation expectations and lagged inflation

C. Robustness checks and non-linearities

We next check the robustness of our baseline results to alternative specifications. First, we check whether the results are robust to alternative measures of slack. Estimating slack has become particularly challenging after Covid due to the many shocks to supply. We consider three alternative measures of slack: the GDP gap estimated with a Hodrick-

20

Prescott filter, estimates of the output gap produced by the IMF and the vacancy-to-unemployment ratio ¹². The latter is chosen as alternative measures of labor market tightness may better reflect slack in the economy given persistent changes in labor force participation, sectoral labor demand and worker preferences (Duval et al. 2022). In addition, reduced form estimates of the Phillips curve are likely to be biased downwards if monetary policy offsets demand shocks and the economy is also subject to trade-off inducing supply shocks. Following McLeay and Tenreyro 2020, we therefore estimate the baseline model with only euro area economies or those with a fixed exchange rate with the euro and time fixed effects, which controls for the endogenous response of monetary policy to aggregate supply shocks. As shown in Table 5 in Appendix A, these alternative specifications also point to a statistically significant steepening and shift up in the aggregate Phillips curve, albeit with more mixed support for a steepening in the slope of the curve in the services sector.

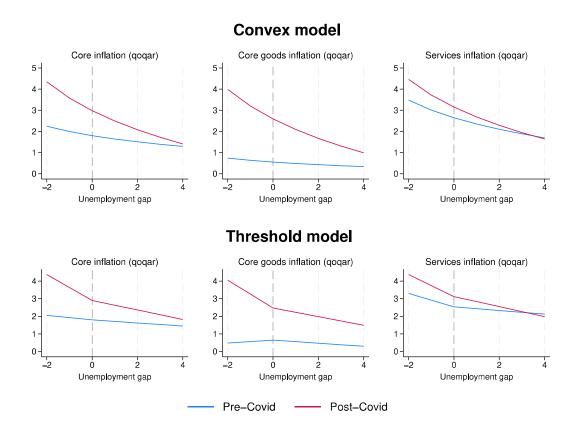
Next, we test for alternative specifications of inflation expectations. The baseline specification uses 1-year ahead inflation expectations, but these may be endogenously determined with current inflation or longer-term expectations may be a stronger determinant of inflationary pressures. Table 6 in Appendix A indicates that the results remain robust to using lagged values of slack and expectations as instruments and to using longer-term (5-year) measures of inflation expectations.

Finally, as discussed in Section I, the argument that the Phillips curve is non-linear has been made in several papers (Forbes et al. 2021; Gagnon and Sarsenbayev 2022; Hooper et al. 2019; Debelle and Laxton 1996). The apparent steepening of the Phillips curve after Covid may therefore reflect the fact that prior to the pandemic unemployment was generally at or above its natural rate and so on the flat portion of the Phillips curve. But the recovery from Covid may have pushed many economies onto the steep portion of the Phillips curve. What seems like a post-Covid steepening in the curve may reflect overheating economies moving along a non-linear Phillips curve, as opposed to a change in the curve itself. We therefore consider two forms of non-linear Phillips curve discussed in Hooper et al. 2019. First, we consider a convex Phillips curve which uses the log of the ratio of unemployment to the natural rate. Second, we consider a threshold model in which the slope of the Phillips curve is allowed to vary depending on whether unemployment is above or below its natural rate. In both cases, we interact these terms with the Covid dummy to investigate whether this non-linearity has changed. Detailed tables are show in Table 7 in Appendix A.

Even allowing for a non-linear specification, we find evidence that the Phillips curve has shifted up and steepened. Figure 6 shows that in the convex model, the slope of the Phillips curve was flat before Covid but became significantly steeper after the pandemic. This was most pronounced in the goods sector and less so for services. In the threshold model, we recover a kinked Phillips curve in the pre-Covid period in the services sector, consistent with other research on pre-Covid Phillips curves. But we also find that the Phillips curve has shifted up and become steeper at all levels of activity. The observed shift and steepening in the Phillips curve therefore does not appear simply to reflect the economy overheating, but a change in the relationship between activity and inflation relative to the pre-Covid period.

 $^{^{12}}$ To mitigate end-point problems associated with the filter, we project forward GDP using the forecasts in the July WEO

Figure 6. Phillips curve slopes: non-linear models



IV. INFLATION DYNAMICS DURING CONSTRAINED RE-COVERIES IN A TWO-SECTOR NEW-KEYNESIAN MODEL

We now present a two-sector new-Keynesian model with roundabout production, distribution services, sectoral preference shocks, and potentially binding supply constraints. The equations of the model are shown below in log-linearized form (denoted with a hat); a full derivation is provided in Appendix B. To relate the model to the stylized facts in earlier sections, variables should be interpreted as deviations from pre-Covid trends.

1. Households

A representative household consumes goods ($\hat{c}_{G,t}$) and services ($\hat{c}_{S,t}$):

$$\hat{c}_{G,t} = \hat{c}_t - \hat{\eta} \hat{p}_{G,t} + \hat{\alpha}_t,$$

$$\hat{c}_{S,t} = \hat{c}_t - \hat{\eta}\hat{p}_{S,t} - \frac{\bar{\alpha}}{1 - \bar{\alpha}}\hat{\alpha}_t = \hat{c}_t - \hat{\eta}\hat{p}_{S,t} - \vartheta\hat{\alpha}_t.$$

Household demand depends on overall consumption (\hat{c}_t) and on prices relative to the CPI, $\hat{p}_{G,t}$ and $\hat{p}_{S,t}$, with the elasticity of substitution given by η . Relative prices move in

opposite direction, with the mapping pinned down by their shares in the CPI basket ($\bar{\alpha}$ being the steady state share of goods consumption):

$$\hat{p}_{S,t} = -\frac{\bar{\alpha}}{1 - \bar{\alpha}} \hat{p}_{G,t} = -\vartheta \hat{p}_{G,t}.$$

Sectoral demand is also subject to preference shock $\hat{\alpha}_t$, which leaves overall demand unchanged. Households also supply labor:

$$\hat{w}_t = \hat{c}_t + \hat{\xi}_t,$$

where \hat{w}_t denotes real wages and $\hat{\xi}_t$ is a shock to labor supply.

2. Distribution Sector

Goods consumption requires a mix of wholesale goods $(\hat{c}_{GW,t})$ and distribution services $(\hat{c}_{GS,t})$ in fixed proportions. The retail goods price faced by consumer $(\hat{p}_{G,t})$ is therefore a weighted sum of the wholesale goods price $(\hat{p}_{W,t})$ and the price of services:

$$\hat{p}_{G,t} = (1 - \theta)\hat{p}_{W,t} + \theta\hat{p}_{S,t},$$

where θ is the share of distribution costs in final (retail) goods consumption. Consumer demand for wholesale goods and distribution services move one-to-one with retail goods consumption: $\hat{c}_{GW,t} = \hat{c}_{GS,t} = \hat{c}_{G,t}$.

3. Services and wholesale goods sectors

Monopolistic firms in the services (S) and wholesale goods (W) sectors set prices, hire workers, and demand intermediate inputs. Firms in both sectors rely on a Cobb-Douglas production function with the same factor shares and subject to economy-wide variations in labor productivity (\hat{a}_t) . Firms compete for the same pool of labor and face the same nominal wage. As a result of these assumptions real marginal costs are the same for firms in both sectors:

$$\hat{mc}_t = \gamma \hat{p}_{W_t} + \omega \hat{p}_{S_t} + (1 - \gamma - \omega)(\hat{w}_t - \hat{a}_t).$$

Parameters γ and ω denote the share of intermediate wholesale goods and services in gross production, respectively. Profit maximization leads to the following demand for intermediate consumption of good i in sector j, denoted $\hat{x}_{ii.i}$:

$$\hat{x}_{ji,t} = -(\hat{p}_{i,t} - \hat{mc}_t) + \hat{y}_{j,t},$$

for i, j = W, S, and where $\hat{y}_{j,t}$ denotes gross output in sector j.¹³ Firms set prices subject to price adjustment costs a la Rotemberg (1982). For the services sector, these assumptions result in a standard (sectoral) new-Keynesian Phillips curve:

$$\hat{\pi}_{S,t} = \kappa(\hat{mc}_t - \hat{p}_{S,t}) + \beta E_t[\hat{\pi}_{S,t+1}],$$

 $^{^{13}}$ The presentation of the model assumes for simplicity all individual firms in sector j produce the same level of output—which is the case in equilibrium due to the Rotemberg price setting specification.

where $\hat{\pi}_{S,t}$ is the inflation rate in the services sector. The key difference relative to the Phillips curve derived from a single-sector model is that movements in relative prices $(\hat{p}_{S,t})$ also influence inflation dynamics. What matters for sectoral inflation dynamics is whether marginal costs are rising or falling relative to prices in the sector, i.e., whether sectoral markups are increasing or decreasing relative to their desired value.

For the wholesale goods sectors, we assume that firms' production cannot exceed a time-varying-threshold: ¹⁴

$$\hat{y}_{W_t} \leq \hat{q}_{W_t}$$

To specify $\hat{q}_{W,t}$, it is useful to define unconstrained potential aggregate output \hat{y}_t^p . Roundabout production creates a distinction between aggregate value added, \hat{c}_t , and gross output. As will be shown below when production is not constrained, there is a level of value added \hat{c}_t^p that stabilizes marginal costs (and inflation) across the economy:

$$\hat{c}_t^p = \hat{a}_t - \hat{\xi}_t$$

 \hat{c}_t^p is therefore potential output in value added space. When demand is balanced, \hat{c}_t^p also corresponds to (log-linearized) potential output in gross output space, in the aggregate and for each sector. We therefore define $\hat{q}_{W,t}$ as \hat{c}_t^p plus a threshold v:

$$\hat{q}_{W,t} = \hat{a}_t - \hat{\xi}_t + v$$

The calibration of v is informed by the sectoral dynamics seen during the pandemic recovery. With the introduction of this constraint the supply curve in the goods sector becomes non-linear. If the constraint does not bind $(\hat{y}_{W,t} < \hat{q}_{W,t})$ we obtain the standard sectoral Phillips curve:

$$\hat{\pi}_{W,t} = \kappa(\hat{mc}_t - \hat{p}_{W,t}) + \beta E_t[\hat{\pi}_{W,t+1}],$$

where we have assumed for simplicity the degree of nominal rigidity is the same in both sectors (as implied by having the same k). ¹⁵ If the constraint does bind sectoral supply is instead:

$$\hat{y}_{W,t} = \hat{a}_t - \hat{\xi}_t + v.$$

4. Market clearing

Market clearing in both sectors requires that overall production equals the sum of final and intermediate use:

$$\bar{y}_W \hat{y}_{W,t} = \bar{x}_{WW} \hat{x}_{WW,t} + \bar{x}_{SW} \hat{x}_{SW,t} + \bar{c}_{GW} \hat{c}_{GW,t},$$

$$\bar{y}_S \hat{y}_{S,t} = \bar{x}_{WS} \hat{x}_{WS,t} + \bar{x}_{SS} \hat{x}_{SS,t} + \bar{c}_S \hat{c}_{S,t} + \bar{c}_{GS} \hat{c}_{GS,t}$$

where a bar () over a variable denotes its steady state value.

¹⁴The specification used here can be thought of as a simplification of the putty-clay model used in Boehm and Pandalai-Nayar 2020.

¹⁵For a related analysis in which the degree of price rigidity differs between the two sectors, see Kindberg-Hanlon and Portillo forthcoming.

5. Aggregation and exogenous processes

Aggregate inflation is the weighted sum of inflation in the two sectors at the consumer level:

$$\hat{\pi}_t = \bar{\alpha}\hat{\pi}_{G,t} + (1 - \bar{\alpha})\hat{\pi}_{St},$$

and relative prices relate to aggregate and sectoral inflation rates through the definition below:

$$\hat{\pi}_t = \hat{\pi}_{S,t} + \vartheta(\hat{p}_{G,t} - \hat{p}_{G,t-1}) = \hat{\pi}_{S,t} + \vartheta \Delta \hat{p}_{G,t}$$

The model is complete with the description of the shocks to this economy:

$$\hat{a}_t = \rho \hat{a}_{t-1} + e_{A,t},$$

$$\hat{\xi}_t = \rho \hat{\xi}_{t-1} + e_{\xi,t},$$

$$\hat{\alpha}_t = \rho_{\alpha} \hat{\alpha}_{t-1} + e_{\alpha,t}.$$

A key simplification is to assume value added also follows an exogenous process:

$$\hat{c}_t = \rho \hat{c}_{t-1} + e_{v,t}.$$

 $(\hat{\alpha}_t, \hat{\xi}_t, \hat{c}_t)$ have the same persistence ρ , as these variables combine linearly to create the aggregate output gap:

$$ygap_{t} = \hat{c}_{t} - \hat{c}_{t}^{P} = c - (\hat{a}_{t} - \hat{\xi}_{t}).$$

 ρ is therefore the persistence of the output gap. The persistence of preference shocks ρ_{α} affects inflation dynamics through its potential impact on the supply constraint, as will be shown below.

A. Phillips curves under different business cycles

We now solve the model to illustrate how the relation between the output gap $ygap_t$ and inflation $\hat{\pi}_t$ varies depending on whether movements in aggregate activity are unbalanced and constrained.

1. Phillips curves under balanced and unconstrained business cycles

When there are no shocks to sectoral preferences ($\hat{a}_t = 0$) and provided the aggregate output gap is not too large, the supply constraint does not bind ($\hat{y}_{W,t} < \hat{q}_{W,t}$). Inflation in both sectors moves in tandem as firms in each sector face the same marginal costs. This implies:

$$\hat{p}_{S,t} = \hat{p}_{G,t} = \hat{p}_{W,t} = 0,$$

$$\hat{\pi}_{W,t} = \hat{\pi}_{G,t} = \hat{\pi}_{S,t} = \hat{\pi}_{t}.$$

In this case, since wages net of productivity are equivalent to the output gap $(\hat{w} - \hat{a}_t = ygap_t)$, the Phillips curves in each sector can be recast as the standard forward-looking relation between inflation and the output gap alone:

$$\hat{\pi}_t = \hat{\pi}_{W,t} = \hat{\pi}_{S,t} = \hat{\pi}_{G,t} = \kappa_U^D ygap_t + \beta E_t \left[\hat{\pi}_{t+1} \right],$$

where

$$\kappa_{II}^{D} = \kappa (1 - \gamma - \omega).$$

 κ_U^D measures the sensitivity of inflation to variations in the output gap, taking expectations of inflation as given. The Phillips curve can also be solved forward to be restated as a static relation between the current aggregate output gap and inflation:

$$\hat{\pi}_t = \hat{\pi}_{W,t} = \hat{\pi}_{S,t} = \hat{\pi}_{G,t} = \kappa_U^S ygap_t$$

where k_U^S is given by:

$$\kappa_U^S = \frac{\kappa (1 - \gamma - \omega)}{1 - \beta \rho}$$

Since relative prices do not change and there are no sectoral shocks, activity in each sector comoves perfectly with aggregate output and so the relation between sectoral output and sectoral inflation is the same as in the aggregate. This version of the model can be thought of as broadly capturing the pre-Covid inflation dynamics presented in previous sections.

2. Phillips curves under unbalanced and unconstrained business cycles

If there are sectoral shocks ($\hat{\alpha}_t > 0$) but activity remains unconstrained ($\hat{y}_{W,t} > \hat{q}_{W,t}$), then it is still the case that relative prices do not change ($\hat{p}_{s,t} = \hat{p}_{G,t} = \hat{p}_{W,t} = 0$). This is because, in this stylized model, marginal costs in each sector are independent of the amount of sectoral output produced. Costs depend instead on the economy-wide level of activity, through the aggregate labor supply curve. They also depend on real costs of intermediate inputs, which do not play a role if relative prices do not vary. In addition, the constraint is not expected to bind in the future if it does not bind today. As a result, sectoral inflation dynamics continue to be determined by equation (1). However, the relation between sectoral inflation and sectoral output shifts sideways with shocks to $\hat{\alpha}_t$ (not shown).

3. Phillips curves with unbalanced and unconstrained business cycles and initially high/low relative goods prices

The previous sections have shown that the relative price of goods does not adjust in response to output when supply constraints do not bind. If supply constraint do not bind but relative prices are initially either above or below zero ($\hat{p}_{G,t-1} \neq 0$), for some unspecified reason, then sectoral and aggregate inflation dynamics will be affected, as lagged relative goods prices are one of the state variables in the model. We present this intermediate case below as it will prove useful for solving the model when constraints are initially binding.

If the relative price is initially either greater or lower than zero, its dynamics are pinned down by subtracting the two "unconstrained" Phillips curves, solving forward, and choosing the non-explosive solution. This yields a solution for $\hat{p}_{G,t}$ as an AR(1) process:

$$\hat{p}_{G,t} = \chi \hat{p}_{G,t-1}, \qquad \chi = \frac{1 + \kappa + \beta - \sqrt{((1 + \kappa + \beta)^2 - 4\beta)}}{2\beta} < 1$$

This equilibrium condition implies that movements in relative prices will take time to unwind, even if the original shock has faded, as firms find it optimal to smooth the rate at which relative prices become realigned with real marginal costs. This "unconstrained" rate of price decay will be used in the next section.

Relative price movements have implications for the aggregate inflation rate through the effect on marginal costs. Restating marginal costs as a function of $ygap_t$ and $\hat{p}_{G,t}$ we obtain:

$$\begin{split} \hat{mc}_t &= (1 - \gamma - \omega) ygap_t + \varsigma \hat{p}_{G,t}, \\ \varsigma &= [\frac{\gamma(1 + \theta\vartheta)}{1 - \theta} - \omega\vartheta] \gtrless 0. \end{split}$$

 ζ can be positive or negative. Positive relative goods prices $(\hat{p}_{G,t} > 0)$ raise marginal costs in the unconstrained case if ζ is positive. This is the case when (wholesale) goods are used more intensively in production than in consumption, either because γ is sufficiently high $(\frac{\gamma}{\gamma + \omega} > \alpha)$ or if the final consumption of goods has a large services component $(\theta >> 0)$. In the calibration we use below, this is always the case $(\zeta > 0)$.

Replacing the revised equation for marginal costs into the sectoral Phillips curves, aggregating them, and solving forward, we obtain a static relation between aggregate inflation, the output gap and relative goods prices:

$$\begin{split} \hat{\pi}_t &= \kappa_U^S y g a p_t + \tau_U^S \hat{p}_{G,t}, \\ \tau_U^S &= \frac{\kappa \zeta}{1 - \beta \, \chi}. \end{split}$$

Relative price movements also have obvious implication for sectoral inflation rates, which can also be solved as a function of $ygap_t$ and $\hat{p}_{G,t}$:

$$\begin{split} \hat{\pi}_{W,t} &= \kappa_U^S y g a p_t + \tau_{UW}^S \hat{p}_{G,t}, \\ \hat{\pi}_{G,t} &= \kappa_U^S y g a p_t + \tau_{UG}^S \hat{p}_{G,t}, \\ \hat{\pi}_{S,t} &= \kappa_U^S y g a p_t + \tau_{US}^S \hat{p}_{G,t}, \\ \tau_{UW}^S &= \frac{\kappa \varsigma_W}{1 - \beta \chi}, \ \varsigma_W &= \varsigma - \frac{1 + \theta \vartheta}{1 - \theta} < 0, \\ \tau_{US}^S &= \frac{\kappa \varsigma_S}{1 - \beta \chi}, \ \varsigma_S &= \varsigma + \vartheta > 0, \\ \tau_{UG}^S &= \frac{\kappa \varsigma_G}{1 - \beta \chi}, \ \varsigma_G &= \varsigma - 1 < 0, \end{split}$$

Differences in sectoral inflation rates implement the gradual convergence of relative prices discussed above: goods inflation will be below (above) services and aggregate inflation if relative goods prices are initially above (below) zero. Unlike for ζ , the term ζ_S , which captures the impact of relative goods prices on pricing decisions in the services sector, is unambiguously positive, while the terms ζ_W and ζ_G , which capture the impact of relative goods prices on pricing decisions in for goods, are unambiguously negative. The term ζ_S will also play an important role in the analysis below.

4. Phillips curves under unbalanced and constrained business cycles

Inflation dynamics change considerably if sectoral demand is large enough to push the goods sector against its supply constraint. In this case the Phillips curve in that sector no longer holds and is replaced by:

$$\hat{y}_{W,t} = \hat{a}_t - \hat{\xi}_t + v.$$

It is useful to combine the above relation with the demand for wholesale goods, for both intermediate and final use, using the market clearing condition. After some algebra and rearranging terms, the relative price of goods at the retail level $\hat{p}_{G,t}$ is now a function of the output gap, the sectoral demand shock, and the threshold v:

$$\hat{p}_{G,t} = \Gamma_{vgap} ygap_t + \Gamma_{\alpha} \hat{\alpha}_t - \Gamma_{v} v > 0,$$

$$\Gamma_{ygap} = \Gamma^{-1} \left[\alpha (1 - \theta) (1 - \gamma - \omega) + 2\gamma \right], \qquad \Gamma_{\alpha} = \Gamma^{-1} \left[\alpha (1 - \theta) (1 - \gamma - \omega) \right],$$

$$\Gamma_v = \Gamma^{-1} \left[\alpha (1 - \theta) (1 - \gamma - \omega) + \gamma \right], \ and \qquad \Gamma = \alpha (1 - \theta) (1 - \gamma - \omega) \eta + \frac{\gamma (1 + \theta \vartheta)}{1 - \theta}.$$

In the first period, a binding supply constraint is equivalent to having relative goods prices higher than steady state, $\hat{p}_{G,t} > 0$, to balance goods demand and supply. The constraint is activated initially if the combination of sufficiently positive output gaps $(ygap_t \gg 0)$ and sufficiently large shocks to sectoral demand $(\hat{\alpha}_t \gg 0)$ becomes larger than the threshold coefficient v.

The above relation between relative goods prices and the output gap is the first modeling result of the paper, though as already mentioned it was first emphasized by Guerrieri et al. 2021. It is also a central channel of inflation dynamics during the Covid recovery. As the output gap increases, it pushes the economy further up against the supply constraint, thus requiring an increase in the relative price of goods. It is also worth stressing that negative shocks to potential output $((\hat{a}_t - \hat{\xi}_t) < 0)$ can be an important driver of the increase in goods prices, through its impact on the output gap.

Implications for the Phillips curve in the services sector

We return to the Phillips curve in the services sector. After replacing the wholesale price of goods $\hat{p}_{W,t}$ with $\frac{1}{1-\theta}\hat{p}_{G,t} - \frac{\theta}{1-\theta}\hat{p}_{St}$ and \hat{p}_{St} with $-\theta\hat{p}_{G,t}$, and rearranging terms we obtain:

$$\hat{\pi}_{St} = \kappa \left((1 - \gamma - \omega) ygap_t + \zeta_S \hat{p}_{G,t} \right) + \beta E_t [\hat{\pi}_{St+1}],$$

where the definition of ζ_S is shown again below:

$$\varsigma_S = \frac{\gamma(1+\theta\vartheta)}{1-\theta} + \vartheta(1-\omega)$$

Unbalanced and constrained recoveries now generate higher inflation in the non-constrained sector (services) through the effect on $\hat{p}_{G,t}$. The intuition is that constrained

recoveries result in higher nominal marginal costs. There are two channels at play. First, higher goods prices directly raise firms' marginal costs because goods are used as intermediate inputs in production. Second, all else equal, higher goods prices raise the CPI and nominal wages equally (real wages are determined by the level of activity through the labor supply curve). Both channels put pressure on producers in the services sector to raise their nominal prices to preserve their markups. Moreover, some of these channels are amplified by the presence of distribution costs, as these imply that the upstream pressure on marginal costs is higher than what is implied by retail goods prices.

Replacing $\hat{p}_{G,t}$ with equation (2) and rearranging terms we can restate the Phillips curve in the services sector as:

$$\hat{\pi}_{St} = \kappa_C^D ygap_t + \beta E_t[\hat{\pi}_{St+1}] + \kappa \varsigma_S(\Gamma_\alpha \hat{\alpha}_t - \Gamma_v v)$$

$$\kappa_C^D = \kappa_U^D + \kappa \varsigma_S \Gamma_{ygap} > \kappa_U^D$$

Services inflation is now more sensitive to the aggregate output gap through the procyclicality of the relative price of goods. This is the second and main modeling result of the paper.

To derive a static Phillips curve relation as in the previous subsection, it must be acknowledged that the supply constraint eventually stops binding. This is the case at some time T when the relative price implied by the constraint is lower than the "unconstrained" rate of decay presented in the previous subsection:

$$\Gamma_{ygap}ygap_T + \Gamma_{\alpha}\hat{\alpha}_T - \Gamma_v v < \chi \hat{p}_{G,T-1}$$

Once the constraint stops binding, relative prices decrease at the rate implied by χ . Period T also varies depending on how high the current price of goods is, or how high the output gap is. The static version of the Phillips curve is now given by:

$$\hat{\pi}_{St} = \kappa_{Ct}^S ygap_t + \omega_t$$

The term $\kappa_{C,t}^{\mathcal{S}}$ is the time-varying slope of the static Phillips curve during constrained recoveries:

$$\kappa_{C,t}^{S} = \kappa_{U}^{S} + \kappa \varsigma_{S} \left[\frac{1 - (\beta \rho)^{T-t}}{1 - \beta \rho} + \frac{\beta^{T-t} \rho^{T-t-1} \chi}{1 - \beta \chi} \right] \Gamma_{ygap} > \kappa_{U}^{S}$$

while ω_t is given by:

$$\omega_t = \kappa \varsigma_S \left[\frac{1 - (\beta \rho_\alpha)^{T-t}}{1 - \beta \rho_\alpha} + \frac{\beta^{T-t} \rho_\alpha^{T-t-1} \chi}{1 - \beta \chi} \right] \Gamma_\alpha \hat{\alpha}_t - \kappa \varsigma_S \left[\frac{1 - \beta^{T-t}}{1 - \beta} + \frac{\beta^{T-t} \chi}{1 - \beta \chi} \right] \Gamma_v \upsilon.$$

The term ω_t should be thought of as shifter of the static Phillips curve relation.

Implications for the Phillips curve in the goods sector

During unconstrained business cycles, goods and services inflation are determined by Phillips curves in each sector, and the relative price of goods follows from sectoral inflation dynamics. Instead, as just shown, during unbalanced and constrained business cycles the relative price of goods adjusts to balance surging goods demand and constrained supply. While services inflation continues to be determined by the Phillips curve in the

sector (albeit impacted by pro-cyclical relative goods prices), goods inflation now follows from the combination of services inflation and changes in relative goods prices:

$$\hat{\pi}_{G,t} = \hat{\pi}_{S,t} + \frac{\vartheta}{\bar{\alpha}} \Delta \hat{p}_{G,t} = \kappa_{C,t}^S ygap_t + \frac{\vartheta}{\bar{\alpha}} \Delta \hat{p}_{G,t} + \omega_t.$$

To understand how the supply constraint changes inflation dynamics in the goods sector, it is instructive to compare the above Phillips curve with the Phillips curve from the previous subsection:

$$\kappa_U^S ygap_t + \tau_{UG}^S \hat{p}_{G,t} \\ \text{Unconstrained Goods Phillips Curve} << \kappa_{C,t}^S ygap_t + \frac{\vartheta}{\bar{\alpha}} \Delta \hat{p}_{G,t} + \omega_t \,.$$
 Constrained Goods Phillips Curve

When the supply constraint is not binding firms in the goods sector would like to lower goods inflation when the relative price of goods is high (τ_{UG}^S << 0), while the sensitivity of inflation to the output gap depends solely on the impact on real wages. When the supply constraint is instead binding, goods inflation increases with the change in relative goods prices (the term $\frac{\theta}{\bar{a}}\Delta\hat{p}_{G,t}$). Goods inflation now also moves one-to-one with services inflation, and so it inherits the greater sensitivity of services inflation to the output gap ($\kappa_{C,t}^S > \kappa_U^S$). As a result the new Phillips curve is both steeper and has a upward shift, at least while relative goods prices are increasing. This is the third key result of the paper.

Finally, a similar relation can be derived for aggregate inflation. The three Phillips curves are shown below for convenience:

$$\hat{\pi}_t = \kappa_{C,t}^S ygap_t + \vartheta \Delta \hat{p}_{G,t} + \omega_t,$$

$$\hat{\pi}_{G,t} = \kappa_{C,t}^S ygap_t + \frac{\vartheta}{\bar{\alpha}} \Delta \hat{p}_{G,t} + \omega_t,$$

$$\hat{\pi}_{S,t} = \kappa_{C,t}^S ygap_t + \omega_t.$$

All Phillips curves experience the same steepening but the extent of the shift varies depending on the sector. The term ω_t shifts all Phillips curve, while $\Delta \hat{p}_{G,t}$ shifts the goods and aggregate Phillips curves but not the services curve.

B. Calibration

The baseline calibration is shown in Table 1. Share of goods in core CPI (α) is based on the average value for the countries in our sample. The share of distribution costs (θ) is based on Burstein et al. 2003. Shares of intermediate goods and services in gross output come from the World Input Output Database (Timmer et al. 2015): they imply a value-added share in gross output of 60 percent. The slope of the Phillips curve κ is set sufficiently low so that the unconstrained Phillips curve is broadly similar to the pre-Covid empirical estimate, while ρ is set to 0.9 in line with standard business cycle persistence.

The remaining parameters and the size of the shock are calibrated to match the dynamics during the Covid recovery. The choice of $\hat{\alpha}_t$ implies that consumption of goods

Table 1. Baseline Model Calibration

Parameter	Value	Parameter	Value
α	0.333	ρ	0.9
θ	0.4	$ ho_{lpha}$	0.95
γ	0.15	K	0.03
ω	0.25	\hat{lpha}_t	10 %
v	3 %	$\Delta \hat{p}_{G,t}$	0.3 %
η	4		

increases by 1.4 percent of steady-state value-added, broadly in line with what was observed during Covid. Threshold v implies that the goods sector becomes constrained once output is 3 percent above potential. It is chosen so that the choice of $\hat{\alpha}_t$ is enough to generate a binding constraint. For the baseline calibration, the elasticity of substitution η has been calibrated so that Γ_{vgap} matches the sensitivity of relative goods prices to activity seen during the Covid recovery. This can be seen by plotting the impact on relative goods prices during the Covid period against the impact on activity (both obtained from the local projections exercise) as shown in Figure 4. The elasticity ranges from 0.4 to 0.65 depending on whether the 2020Q4-2022Q1 period is used (0.4) or the entire post Covid period (0.65); we, therefore, set $\Gamma_{ygap} \approx 0.5$. Calibrating η this way leads to a high value ($\eta = 4$), higher than standard values in the literature, typically one or less than one, see Mendoza 1995 among others. We revisit the choice of η below. Persistence ρ_{α} implies that supply constraints are expected to bind for 9 quarters if the initial output gap is 2 percent, which seems reasonable given measures of supply disruptions during this period. Finally, $\Delta \hat{p}_{G,t}$ is chosen to match the average quarterly growth rate in relative prices over this period (non-annualized).

C. Quantitative Results

We run three simulation exercises. First, we calculate the quantitative changes to the Phillips curve relation when the recovery is unbalanced and constrained, in either its dynamic form (holding inflation expectations constant) or in static form (solving the model forward). Second, we assess the model's ability to replicate the surge in inflation observed during the pandemic recovery. Third, we simulate the model forward starting from the third quarter of 2022, under the assumption that the supply constraint is no longer increasingly binding.

1. Alternative model-based Phillips curves

The panels in Figure 7 plot headline, goods, and services inflation, and the relative price of goods against the output gap. This is done for two versions of the model—when recoveries are unconstrained and balanced and when they are constrained and unbalanced. The output gap ranges from 0 to 5 percent. As is evident from the chart, unbalanced and constrained recoveries both shift and steepen the Phillips curve. The shift is visible in aggregate inflation and specially in goods inflation, while the steepening is the same for all measures of inflation.

Table 2 compares model-based changes to the slope and shift in the Phillips curve with the empirical estimates for the output gap in Table 5.¹⁷ Model-based measures are annualized for comparison. The model generates quantitatively meaningful changes to the Phillips curve relation. The magnitudes fall short of the empirical results however. Whereas the dynamic slope of the Phillips curve increases by 170-270 percent in the post-Covid data, the model can generate a steepening of 61 percent, or about 23-35 percent of the steepening seen in the data.

Several factors likely account for the difference between the data and the model. A possible explanation relevant for our model is that the observed increase in goods price may not reflect the true scarcity of goods during this period, which may have been reflected along, other, non-price dimensions, such as waiting times and stockouts (see Alessandria et al. 2023 and Cavallo and Kryvtsov 2021). An implication of such rationing schemes is that the increase in marginal costs may have been higher than what was implied by the increase in wholesale goods prices.

 $^{^{16}}$ The calibration of v implies that sufficiently large output gaps generate a binding supply constraint, even in the absence of sectoral shocks. We do not take this into account when comparing the two versions of the Phillips curve for ease of comparison.

¹⁷We refer to the GDP-based measures of slack as these are most comparable to the set-up of the model

Figure 7. Model Based static Phillips curves during unconstrained/constrained business cycles

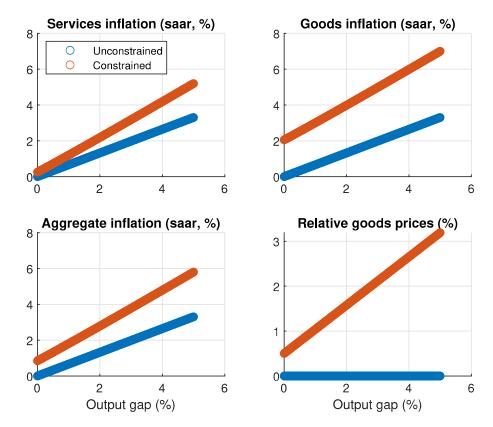


Table 2. Comparison between models and empirical estimates

		Dynamic Slope	Static Slope	Shift (ppt qoqar)
	Pre-covid	0.07		
Empirical	Covid	0.19-0.26		1.3-1.4
	Difference in Percent	171-271%		
	Unconstrained	0.07	0.66	
Model (baseline)	Unbalanced / Constrained	0.12	0.99	0.79
	Difference in Percent	61%	49%	
	Unconstrained	0.07	0.66	
Model (alternative)	Unbalanced / Constrained	0.15	1.26	0.96
	Difference in Percent	112%	91%	

To assess the implication of a higher shadow price of goods that is more sensitive to the recovery, we run an alternative specification of the model where the elasticity of substitution between goods and services consumption is reduced from 4 to 1. This calibration is more in line with the literature. We use the shadow price when solving for the slope of the Phillips curve; however, we continue to use the observed change in relative goods prices when calculating the shift, in order to be consistent with the empirical evidence. In this case the elasticity of \hat{p}_G to the output gap increases doubles ($\Gamma_{ygap} \approx 1$). As a result, the steepening of the Phillips curve is much greater, between 90 to 112 percent depending on the specification. The shift of the curve is also slightly larger: while $\Delta \hat{p}_{G,t}$ is the same, ω_t is now higher.

Other factors may have also played a role in addition to the supply disruptions in goods. Greater sensitivity of commodity prices to the post-Covid recovery may account for some of the steepening; greater pro-cyclicality of commodity prices would operate along similar channels to those presented here. Gagliardone et al. 2023 also emphasize oil prices but through a different mechanism (interaction with real wage rigidities). Changes in the pricing behavior of firms, due to changes in the extent of strategic complementarities brought about by the pandemic may have also played a role, as argued in Harding et al. 2023. Finally, non-linearities in the dynamics of wages due to increased frictions in labor markets may have also contributed as in Benigno and Eggertsson 2023. The recovery may have also coincided with the de-anchoring of expectations, as argued by Reis 2022. A quantitative assessment of the relative importance of all possible drives is a promising area for future research. While these factors may have played an important role during the recovery, it is striking how much of the observed changes to the Phillips curve can be captured in a relatively simple framework such as ours, not just qualitatively but also quantitatively.

2. Replicating the inflation surge during the pandemic recovery

We assess the path of inflation generated by the model following a stylized recovery that is broadly similar to what is observed for the average economy in our sample, starting in 2020Q3 and ending in 2022Q3. The average recovery is based on the local projection results in Section 2. We do so for four versions of the model:

- **A**: A version where the recovery is balanced and there is no hit to potential output $(\hat{c}^p = 0)$.
- **B**: A version where the recovery is balanced but potential output is 4 percent lower than the pre-Covid trend through the entire simulation ($\hat{c}^p = -4\%$),
- C: A version where the recovery is unbalanced and constrained, with $\hat{c}^p = -4\%$, and where the increase in relative prices is consistent with what was observed during the pandemic recovery,
- **D**: version C but where the true increase in relative prices is twice what was observed during the pandemic recovery.

The inflation paths generated in the four versions of the model are then compared with the average inflation observed during this period, also taken from the local projection results in section II and shown in deviation from pre-Covid levels.

The calibration of potential output reflects the assessment that potential must be lower for the output gap to be positive, i.e., for there to be overheating in the first place. It is also consistent with the analysis in Blanchard, Domash, et al. 2022 for the US, among others, that estimates of the NAIRU have been higher during the pandemic recovery.

Other than the differences in model versions listed above, the calibration is the same as in the previous exercise, except the initial shock to sectoral preferences, which is set higher ($\hat{\alpha}_t = 13\%$). This is so that activity is constrained even when the output gap is somewhat negative, as will be the case early on in the recovery in cases **C** and **D**. The calibration of η also varies from version **C** to version **D**: as in the previous section it is lowered from 4 to 1, which implies a doubling in the sensitivity of relative goods prices to the recovery.

The simulations are shown in Figure 8. The economy starts from a level of activity (value added) that is about -5.5 percent below pre-Covid trend, corresponding to the average level of activity in our sample of countries in 2020Q3. The recovery is swift: after 7 quarters (2022Q1) the level of activity is only -2.5 percent lower than the pre-Covid trend. In the simulation, activity continues to grow in the last two quarters, whereas in the data it stabilizes and declines somewhat, again relative to pre-Covid trend.

Whether the recovery is balanced or not has implications for the relative price of goods. When the recovery is balanced, relative goods prices do not respond at all (versions **A** and **B**). In version **C** of the model, where the recovery is unbalanced, relative prices behave similarly to what is observed in the data; in version **D**, the true increase in relative goods prices is twice as large as what is observed in the data.

Regarding the inflation performance, by 2022Q3, core inflation in the average country was 5.2 percent higher than the pre-Covid period version. A cannot replicate the inflation rates seen in the data, as it generates inflation that is lower than average throughout the simulation. Version **B** improves the inflation performance and is a necessary step toward replicating the data, but still falls quite short: inflation peaks at only 1.7 percent by the end of the period.

Version C—when the recovery is unbalanced and constrained—does a much better job in replicating the inflation performance observed during this period. Aggregate core inflation increases by as much as 3.7 percent by the end of the sample. This version of the model also comes closer to matching inflation dynamics for goods and services. Finally, the model replicates the inflation surge when the true cost of goods is higher than what is observed in the inflation data (version **D**). In this case, core inflation increases by 4.9 percent by the end of the sample.

Our assessment from this exercise that the model does a good job of capturing inflation dynamics during the Covid period. In order to do so however, potential output must be significantly lower than pre-Covid.

35

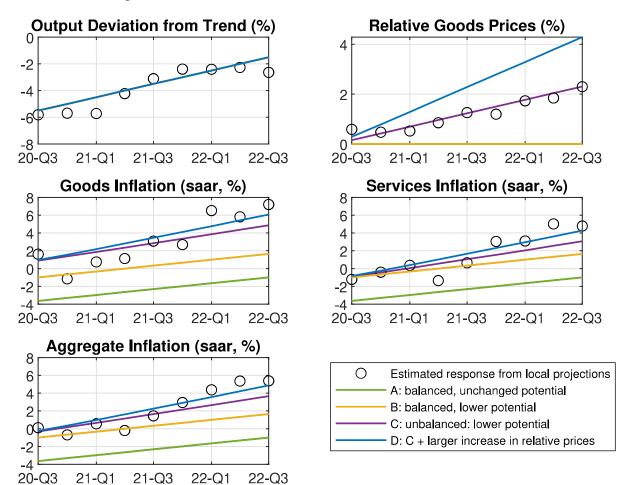
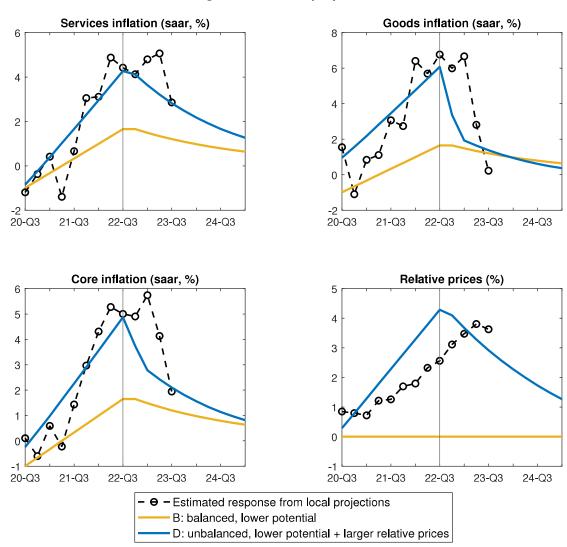


Figure 8. Inflation simulations under alternative model versions

3. Model-based projections with decreasingly binding supply constraints

We conclude the quantitative evaluation with a projection in which supply constraints are decreasingly binding. The projection is done with version **D** of the model; it starts right after the last period of the simulation in the previous section, which corresponds to 2022Q3, and ends 9 quarters later (2024Q4). Whereas inflation in the previous section was generated under the assumption that the output gap was growing over time, here the output gap stays flat for one quarter and then starts decaying at the rate implied by ρ . The shock to preferences $\hat{\alpha}_t$ declines at rate ρ_{α} .

Figure 9. Inflation projections



Inflation projections are shown in Figure 9, which also includes data for the average advanced economy in the sample, and for version **B**. In version **D**, relative goods prices stop increasing and actually decline, mildly in the first quarter and more noticeably after that, as gradual decreases in the output gap and in $\hat{\alpha}_t$ decrease the extent to which the supply constraint is binding. The shift from increasing to decreasing relative goods prices is a source of immediate disinflation. Goods inflation decreases rapidly, from a peak of 6 percent above pre-Covid levels by 2022Q3 to 2 percent two quarters after that. Similarly, aggregate core inflation also decreases rapidly, from close to 5 percent above pre-Covid to about 2.5 percent in two quarters.

While goods inflation immediately declines, services inflation remains elevated: starting from about 4 percent in 2022Q3 it remains above 3.5 percent by 2023Q1 and gradually decreases over the entire period. The reason for the gradual disinflation in services is because the relative price of goods is too high, which implies the relative price of services is too low. A period of higher services inflation than goods inflation is necessary for

firms in the services sector to raise their markups back to their desired level. The convergence of $\hat{p}_{G,t}$ back to steady state is gradual.

The comparison of version **D** with version **B** helps illustrate the role of the supply constraint. As discussed in the previous subsection, the supply constraint was a key source of amplification of inflationary pressures in the run-up period, when a growing output gap made the constraint increasingly binding. This amplification allowed version **D** to match the inflation, surge, whereas a version of the model with a balanced and unconstrained recovery (**A**) could not. Similarly, when the output gap starts to decay the constraint becomes decreasingly binding, resulting in a rapid disinflation. This is the case even thought the output gap remains elevated, i.e., the economy remains overheated. Such rapid disinflation is not possible if supply constraints are not taken into account. The disinflation in version **A** is very gradual and consistent with the gradual closing of the output gap. Relatedly, the absence of supply constraints generates the same pace of disinflation in both sectors.

It is also worth stressing that the projected disinflation is broadly consistent with what has been observed in the data since 2022Q3. An important difference is that the disinflation happens immediately in the model projections whereas it happens two to three quarters later in the data. This delay is seen most clearly in the behavior of goods inflation. As a result, relative goods prices keep increasing in the data for several quarters and only start declining in 2023Q2, the last quarter for which data is available. A plausible reason for the difference between model and data is that advanced economies were hit with an additional shock, this time coming from energy prices. The latter shock hit European countries, which are heavily represented in our sample of countries, most severely. This shock delayed the disinflation from the fading of supply constraints. The latter become more visible once the effects of the energy shock faded.

V. CONCLUSION

The post-Covid recovery has been unusual in several respects. It was rapid, skewed towards goods consumption and ran up against significant supply constraints. This combination of factors led to a shift up and steepening in the Phillips curve. Relative goods prices became markedly procyclical whereas before they had varied little with the economic cycle. Introducing sectoral supply constraints demonstrates how an unbalanced and constrained recoveries can produce both the shift up and the steepening in the Phillips curve, as well the observed increased pro-cyclicity of relative goods prices. While steepening is common to both sectors, the shift is largest in the goods sector. The richness of this stylized model is its ability to match several of the unusual features of the post-Covid recovery, both qualitatively and quantitatively.

By emphasizing the role of sectoral factors, we also add to the explanation of why policy institutions failed to forecast the post-Covid inflation surge. Most frameworks for thinking about inflation are based on aggregate output focused on cyclical fluctuations in demand – understandable as the composition of most past businesses cycles was balanced and inflation thought mostly a demand-side issue. ¹⁸ Therefore, even when supply constraints were thought to be increasing inflation in one sector, these models allowed little role for non-linearities or shifts in the Phillips curve, or for dynamics in one sector to affect inflation elsewhere in the economy. Building on insights from the recent literature, our model shows how these features are crucial for understanding the unusual nature of the post-Covid recovery and inflation surge.

The assumption of an exogenous process for the output gap allowed us to derive analytical results. In future research we plan to endogenize output gap dynamics, which will help assess quantitatively the role of various factors in driving the Covid recovery and hence inflation in the presence of supply constraints, most notably fiscal and monetary policy.

¹⁸Koch and Noureldin 2023 find that inflation was stronger than expected even accounting for the fact that forecasts were generally too pessimistic on activity when using a standard slope of the Phillips curve.

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VI. APPENDIX

A. Model estimations and robustness checks

Table 3. Baseline Phillips curve estimate

	(1)	(2)	(3)
	Core	Core goods	Services
Unemployment gap	-0.10***	-0.06	-0.16***
	(-5.61)	(-1.60)	(-4.34)
Covid x unemployment gap	-0 .2 9***	-0.32**	-0.23***
	(-4.15)	(-2.71)	(-3.29)
Covid dummy	1.28***	2.13***	0.59***
	(8.09)	(9.72)	(3.82)
Lagged inflation (-1)	0.26***	0.20***	0.28***
	(5.34)	(2.85)	(5.56)
Lagged inflation (-2)	0.18***	0.29***	0.07**
	(6.44)	(5.20)	(2.08)
Inflation expectations	0.55***	0.37***	0.82***
	(9.32)	(4.83)	(7.97)
Constant	-0.17*	-0.60***	0.03
	(-1.99)	(-3.67)	(0.17)
N	2234	2169	2234
Countries	25	25	25
Adjusted R-sq	0.61	0.51	0.51
rmse	1.25	1.88	1.64

t statistics in parentheses

^{*} p < 0.10, ** p < 0.05, *** p < 0.010

Table 4. Impact of slack on the relative goods price gap

	Pre-Covid sample		Post-Covid sample			Covid interaction			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Output gap	0.02			0.11**			0.03		
	(0.99)			(2.78)			(1.12)		
Covid x Output gap							0.08**		
							(2.53)		
Unemployment gap		-0.00			-0.20			-0.02	
		(-0.03)			(-1.58)			(-0.54)	
Covid dummy x unemployment gap								-0.12*	
								(-1.81)	
Vacancy-to-unemployment gap			-0.19			0.46**			-0.23
			(-1.29)			(2.72)			(-1.58)
Covid dummy x V-U gap									0.78***
									(3.37)
Covid dummy							0.52***	0.47***	0.53***
							(5.35)	(4.48)	(4.79)
Constant	-0.02	-0.04*	-0.09***	0.49***	0.44***	0.45***	-0.04*	-0.05	-0.10***
	(-1.55)	(-1.72)	(-21.01)	(11.69)	(14.43)	(13.35)	(-1.77)	(-1.59)	(-4.21)
N	2400	2212	1211	325	325	325	2225	2176	1476
Countries	25	25	25	25	25	25	25	25	25
Adjusted R-sq	0.015	0.010	0.021	0.42	0.42	0.46	0.061	0.057	0.11
RMSE	0.97	0.99	1.03	0.78	0.78	0.75	1.00	1.01	1.01

t statistics in parentheses

^{*} p < 0.10, ** p < 0.05, *** p < 0.010

Table 5. Phillips curve estimates: alternative estimates of slack

		GDP gap			Output gap				
	(1)	(2)	(3)	(4)	(5)	(6)			
	Core	Core goods	Services	Core	Core goods	Services			
Gap	0.07***	0.01	0.13***	0.07***	0.03	0.10***			
	(5.13)	(0.48)	(5.39)	(6.18)	(1.22)	(6.18)			
Covid x gap	0.12**	0.34***	-0.02	0.19***	0.31***	0.08			
	(2.39)	(5.14)	(-0.41)	(4.92)	(4.85)	(1.68)			
Covid dummy	1.30***	2.13***	0.70***	1.42***	2.34***	0.74***			
	(10.15)	(13.09)	(4.57)	(9.53)	(11.46)	(4.47)			
Constant	-0.28***	-0.68***	-0.17	-0.15	-0.59***	0.00			
	(-3.90)	(-5.25)	(-1.15)	(-1.58)	(-4.01)	(0.02)			
N	2283	2218	2283	2283	2218	2283			
Countries	25	25	25	25	25	25			
Adjusted R-sq	0.61	0.52	0.50	0.61	0.52	0.50			
RMSE	1.25	1.86	1.65	1.24	1.86	1.64			

t statistics in parentheses

	Vacancy-to-unemployment ratio			Euro economies only			
	(1)	(2)	(3)	(4)	(5)	(6)	
	Core	Core goods	Services	Core	Core goods	Services	
Unemployment gap				-0.07***	-0.09*	-0.08**	
				(-4.56)	(-1.89)	(-2.60)	
Covid dummy x unemployment gap				-0.21***	-0.19*	-0.16**	
				(-3.41)	(-2.12)	(-2.57)	
Vacancy-to-unemployment gap	0.16***	0.03	0.37**				
	(2.85)	(0.30)	(2.77)				
Covid dummy x V-U gap	0.62***	1.05***	0.15				
	(5.04)	(4.24)	(0.81)				
Covid dummy	1.41***	2.29***	0.92***	2.20***	4.18***	1.74**	
	(11.40)	(14.76)	(5.18)	(3.56)	(4.15)	(2.52)	
Constant	-0.20*	-0.74***	-0.06	-0.14	-0.31	-0.43	
	(-1.86)	(-5.36)	(-0.26)	(-0.40)	(-0.48)	(-1.56)	
Time fixed effects	No	No	No	Yes	Yes	Yes	
N	1496	1473	1496	1330	1330	1330	
Countries	25	25	25	16	16	16	
Adjusted R-sq	0.63	0.57	0.46	0.72	0.63	0.65	
RMSE	1.29	1.88	1.73	1.12	1.51	1.46	

t statistics in parentheses

^{*} p < 0.10, ** p < 0.05, *** p < 0.010

^{*} p < 0.10, ** p < 0.05, *** p < 0.010

Table 6. Phillips curve estimates: alternative inflation expectations specifications

	IV inflation expectations			Long-term inflation expectations			
	(1) Core	(2) Core goods	(3) Services	(4) Core	(5) Core goods	(6) Services	
Unemployment gap	-0.06***	-0.02	-0.12***	-0.06**	-0.06	-0.14***	
	(-3.86)	(-0.45)	(-3.00)	(-2.21)	(-1.13)	(-5.17)	
Covid dummy x unemployment gap	-0.31***	-0.32***	-0.21***	-0.32***	-0.35***	-0.37***	
	(-3.91)	(-2.65)	(-3.22)	(-4.58)	(-3.06)	(-4.16)	
Covid dummy	1.25***	2.13***	0.34**	1.42***	2.18***	1.37***	
	(7.60)	(10.17)	(2.35)	(9.29)	(14.92)	(7.87)	
Lagged inflation (-1)	0.17***	0.18***	0.19***	0.35***	0.24***	0.31***	
	(3.34)	(2.62)	(3.41)	(5.47)	(4.78)	(4.58)	
Lagged inflation (-2)	0.12***	0.28***	0.01	0.27***	0.31***	0.11*	
	(3.97)	(5.14)	(0.27)	(8.38)	(6.15)	(1.90)	
1-year ahead inflation expectations	0.98***	0.53***	1.36***				
	(8.49)	(5.20)	(5.80)				
Long-term inflation expectations				0.17	0.34	0.29	
				(0.83)	(1.31)	(1.45)	
Constant				0.25	-0.48	0.64	
				(0.56)	(-0.92)	(1.70)	
N	2212	2148	2212	1243	1210	1243	
Countries	25	25	25	25	25	25	
Adjusted R-sq	0.59	0.51	0.48	0.64	0.58	0.43	
RMSE	1.29	1.88	1.69	1.25	1.87	1.63	

Table 7. Phillips curve estimates: non-linear models using unemployment gap

		Convex			Threshold			
	(1)	(2)	(3)	(4)	(5)	(6)		
	Core	Core goods	Services	Core	Core goods	Services		
Unemployment gap				-0.09***	-0.09**	-0.10***		
				(-5.33)	(-2.40)	(-5.60)		
Covid x unemployment gap				-0.18***	-0.16*	-0.18**		
				(-2.99)	(-1.82)	(-2.11)		
log(u/u*)	-0.87***	-0.36	-1.62***					
	(-5.35)	(-1.04)	(-7.90)					
Covid x log(u/u*)	-1.81***	-2.37**	-0.95*					
	(-3.88)	(-2.62)	(-1.94)					
Unemployment gap <0				-0.03	0.17**	-0.28***		
				(-0.66)	(2.66)	(-3.07)		
Covid x unemployment gap <0				-0.43	-0.71**	-0.06		
				(-1.44)	(-2.25)	(-0.18)		
Covid dummy	1.18***	2.04***	0.50***	1.10***	1.83***	0.59**		
	(8.32)	(10.20)	(3.40)	(5.26)	(6.81)	(2.39)		
N	2234	2169	2234	2234	2169	2234		
Countries	25	25	25	25	25	25		
Adjusted R-sq	0.61	0.51	0.51	0.61	0.51	0.51		
RMSE	1.25	1.88	1.64	1.25	1.87	1.63		

t statistics in parentheses p < 0.10, *** p < 0.05, **** p < 0.010

t statistics in parentheses p < 0.10, *** p < 0.05, **** p < 0.010

B. A two sector new-Keynesian model with supply constraints

1. Households

The representative agent consumes goods and services, provides labor to firms, and holds a nominal asset. The agent maximizes lifetime utility:

$$\max \sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t[\ln(c_{\tau}) - \xi_{\tau} l_{\tau}]$$

where c_{τ} denotes aggregate consumption, l_{τ} denotes labor supply, β is the intertemporal discount factor, and ξ_{τ} is a shock to labor supply. Aggregate consumption is a CES basket of goods $c_{G,\tau}$ and services $c_{S,\tau}$:

$$c_{\tau} = \left[\alpha_{\tau}^{1/\eta} c_{G,\tau}^{(\eta-1)/\eta} + (1-\alpha_{\tau})^{1/\eta} c_{S,\tau}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between goods and services, and α_{τ} is the time-varying share of goods in total consumption. Utility maximization is subject to the agent's budget constraint:

$$P_{S,t}c_{S,t} + P_{G,t}c_{G,t} + B_{t+1} = W_tl_t + \Omega_{S,t} + \Omega_{G,t} + B_tR_{t-1} - T_t$$

 $P_{S,t}$ and $P_{G,t}$ denote prices of services and goods at the consumer level, respectively, W_t is the nominal wage and $\Omega_{S,t}$ and $\Omega_{G,t}$ are the profits in the goods and services sectors. B_{t+1} is a nominal bond that pays rate R_t and T_t denotes lump-sum taxes. Utility maximization leads to the following first order conditions:

$$\begin{split} c_{\mathrm{G,t}} &= \alpha_t \left(\frac{P_{\mathrm{G,t}}}{P_t}\right)^{-\eta} c_t = \alpha_t p_{G,t}^{-\eta} c_t, \\ c_{\mathrm{S,t}} &= (1 - \alpha_t) \left(\frac{P_{\mathrm{S,t}}}{P_t}\right)^{-\eta} c_t = (1 - \alpha_t) p_{\mathrm{S,t}}^{-\eta} c_t, \\ P_t &= \left[\alpha_\tau P_{\mathrm{G,t}}\right]^{1-\eta} + (1 - \alpha_\tau) P_{\mathrm{S,t}}\right]^{1-\eta} \\ \xi_t &= \frac{W_t}{P_t} c_t^{-1} = w_t c_t^{-1}, \\ c_t^{-1} &= \beta R_t E_t \left[c_{t+1}^{-1} \pi_{t+1}^{-1}\right]. \end{split}$$

 π_t is the gross inflation rate $(\pi_t = \frac{P_t}{P_{t-1}})$, w_t is the real wage, and $(p_{G,t}, p_{S,t})$ are the real prices of goods and services at the consumer level.

2. Distribution Services

The consumption good $c_{G,t}$ is a Leontieff bundle of wholesale goods $c_{GW,t}$ and distribution services $c_{GS,t}$:

$$c_{G,t} = min(\frac{c_{GW,t}}{1-\theta}, \frac{c_{GS,t}}{\theta}).$$

 θ is the share of distribution services. The sector is populated by perfectly competitive firms, leading to the following first order conditions from profit maximization:

$$\begin{split} c_{GW,t} &= (1-\theta)c_{G,t}, \quad c_{GS,t} = \theta c_{G,t}, \\ P_{G,t} &= \theta P_{GW,t} + (1-\theta)P_{GS,t}. \end{split}$$

3. Wholesale Goods and Services Sectors

The wholesale goods and services sectors consist of a continuum of monopolistic producers. Producer i in sector j produces variety $y_{ij,t}$ and faces demand $y_{ij,t} = \left(\frac{P_{ij,t}}{P_{j,t}}\right)^{-\epsilon} y_{j,t}$, where $P_{ij,t}$ is the price set by that producer, and $P_{j,t}$ and $y_{j,t}$ are the price index and total demand in sector j, respectively.

Variety i in sector j are produced using a mix of labor $(l_{ij,t})$ and intermediate wholesale goods $(x_{ijW,t})$ and services $(x_{ijS,t})$:

$$y_{ij,t} = z(a_t l_{ij,t})^{(1-\omega-\gamma)} x_{ijW,t}^{\gamma} x_{ijS,t}^{\omega}$$

 γ and ω denote the share of goods and services in gross production, a_t is time-varying labor productivity which is common to all firms in the economy, and z is a scaling parameter. All producers face the same wage rate and intermediate input costs, with cost minimization leading to the following demand for factors of production:

$$\begin{split} l_{ij,t} &= (1 - \omega - \gamma) \left(\frac{W_t/a_t}{MC_t}\right)^{-1} y_{ij,t}, \\ x_{ijW,t} &= \gamma \left(\frac{P_{W,t}}{MC_t}\right)^{-1} y_{ij,t}, \\ x_{ijS,t} &= \omega \left(\frac{P_{S,t}}{MC_t}\right)^{-1} y_{ij,t}, \end{split}$$

and where MC_t denotes nominal marginal costs:

$$MC_t = \left(\frac{W_t}{a_t}\right)^{(1-\gamma-\omega)} P_{W,t}^{\gamma} P_{S,t}^{\omega}.$$

Producers also face Rotemberg-type quadratic costs of adjusting their own prices $\frac{\phi \bar{y}_j}{2} \left(\frac{P_{ijt}}{P_{ijt-1}} - 1 \right)^2$, where ϕ is common to both sectors, and \bar{y}_j is the steady-state value of sectoral production. Each producer sets a price that maximizes its discounted sum of nominal profits.

$$\max_{P_{ij,t}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} E_t \left[\frac{u'(c_{\tau})}{u'(c_t)} \frac{P_t}{P_{\tau}} \left(P_{ij,t} - (1-\varpi) M C_t \left(\frac{P_{ij,t}}{P_{j,t}} \right)^{-\epsilon} y_{j,t} - P_t \frac{\phi \bar{y}_J}{2} \left(\frac{P_{ij,t}}{P_{ij,t-1}} - 1 \right)^2 \right) \right],$$

where ϖ is a production subsidy financed with lump-sum taxes. Firms are also subject to a potentially binding production constraint, which holds separately for each sector:

$$y_{i,i,t} \leq q_{i,t}$$

Profit maximization leads to the following Phillips curve, removing sub-index i as all firms in sector j set the same price:

$$(\pi_{j,t}-1)\pi_{j,t} = \frac{(\epsilon-1)y_{jt}}{\phi \bar{y}_j} \left[\frac{\epsilon}{\epsilon-1} (1-\varpi)mc_t + \mu_{j,t} - p_{j,t} \right] + \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (\pi_{j,t+1}-1)\pi_{j,t+1} \right]$$

with either $\mu_{j,t} = 0$ or $y_{j,t} = q_{j,t}$. Here $\pi_{j,t}$ is the rate of gross inflation in sector j in period t ($\pi_{j,t} = \frac{P_{ij,t}}{P_{ij,t-1}}$), $p_{j,t}$ denotes the price in sector j deflated by the CPI, mc_t are nominal marginal costs deflated by the CPI, and $\mu_{j,t}$ is related to the Lagrange multiplier from the production constraint (the shadow value), also deflated by the CPI.

We assume the production constraint in each sector depends on the economy's productive capacity:

$$q_{jt} = \bar{y}_j \left(a_t \xi_t^{-1} + v \right),\,$$

where \bar{y}_j is the level of each sector's gross output at steady state, and v denotes the threshold beyond which production becomes constrained. We limit our analysis to cases where at most one sector is constrained.

4. Market clearing, exogenous processes, and model closure

Markets for goods, services, and labor must clear, which implies:

$$l_{t} = l_{W,t} + l_{S,t},$$

$$y_{W,t} = c_{GW,t} + x_{WW,t} + x_{SW,t},$$

and

$$y_{S,t} = c_{S,t} + c_{GS,t} + x_{WS,t} + x_{SS,t},$$

We assume that labor productivity, labor supply shocks, and the goods share in consumption, follow AR(1) processes in logs:

$$\begin{split} \ln(a_t) &= \rho \ln \left(a_{t-1}\right) + e_{a,t} \\ \ln(\xi_t) &= \rho \ln \left(\xi_{t-1}\right) + e_{\xi,t} \\ \ln(\alpha_t) &= (1 - \rho_\alpha) \ln(\bar{\alpha}) + \rho_\alpha \ln \left(\alpha_{t-1}\right) + e_{\alpha,t} \end{split}$$

Finally, instead of directly specifying a monetary policy reaction function we assume that consumption (value added) also follows an AR(1) in logs around its steady state value:

$$\ln(c_t) = (1 - \rho)\ln(\bar{c}) + \rho\ln(c_{t-1}) + e_{c,t}$$

Although monetary policy is not specified directly, it is assumed that it keeps long-term inflation expectations anchored at the inflation target $(\bar{\pi})$. We assume the same persistence in the AR process for productivity, labor supply and consumption, which simplifies the derivation of analytical results.

5. Steady state

Steady-state variables are denoted with a bar (\bar{a}). From the above processes, it follows that $\bar{a} = \bar{\xi} = 1$. Steady-state inflation equals its target, which is set to 1 ($\bar{x} = 1$). These

conditions imply:

$$\begin{split} & \bar{m}c = \bar{w} = \bar{p}_G = \bar{p}_S = \bar{p}_W = 1, \\ & \bar{c} = 1, \quad \bar{c}_G = \bar{\alpha}, \quad \bar{c}_S = 1 - \bar{\alpha}, \quad \bar{c}_{GW} = (1 - \theta)\bar{\alpha}, \quad \bar{c}_{GS} = \theta\bar{\alpha}, \\ & \bar{y}_W = (1 - \theta)\bar{\alpha} + \frac{\gamma}{1 - \gamma - \omega}, \quad \bar{y}_S = \theta\bar{\alpha} + (1 - \bar{\alpha})\frac{\omega}{1 - \gamma - \omega}, \end{split}$$

and

$$\bar{x}_{WW} = \gamma \bar{y}_W, \quad \bar{x}_{WS} = \gamma \bar{y}_S, \quad \bar{x}_{WS} = \omega \bar{y}_W, \quad \bar{x}_{SS} = \omega \bar{y}_S.$$

Production constraints at steady state are given by $\bar{q}_W = \bar{y}_W(1+v)$ and $\bar{q}_S = \bar{y}_S(1+v)$, and hence neither sector is constrained at steady state.

6. Log-linearization

The model is log-linearized around the steady state, with log-linearized variables denoted with a hat (^):

$$\hat{c}_{G,t} = \hat{c}_t - \eta \hat{p}_{G,t} + \hat{\alpha}_t$$

$$\hat{c}_{S,t} = \hat{c}_t - \eta \hat{p}_{S,t} - \frac{\bar{\alpha}}{1 - \bar{\alpha}} \hat{\alpha}_t = \hat{c}_t - \eta \hat{p}_{S,t} - \vartheta \hat{\alpha}_t$$

$$\hat{p}_{S,t} = -\vartheta \hat{p}_{G,t}$$

$$\hat{w}_t = \hat{c}_t + \hat{\xi}_t$$

$$\hat{\pi}_t = \bar{\alpha} \hat{\pi}_{G,t} + (1 - \bar{\alpha}) \hat{\pi}_{S,t}$$

$$\hat{p}_{G,t} = \hat{p}_{G,t-1} + \hat{\pi}_{G,t} - \hat{\pi}_t$$

$$\hat{p}_{G,t} = \theta \hat{p}_{S,t} + (1 - \theta) \hat{p}_{W,t}$$

$$\hat{m}\hat{c}_t = (1 - \gamma - \omega)(\hat{w}_t - \hat{a}_t) + \gamma \hat{p}_{W,t} + \omega \hat{p}_{S,t}$$

$$\hat{\pi}_{W,t} = \kappa(\hat{m}\hat{c}_t - \hat{p}_{W,t}) + \beta E_t[\hat{\pi}_{W,t+1}] \quad or \quad \hat{y}_{W,t} = \hat{a}_t - \hat{\xi}_t + v$$

$$\hat{\pi}_{S,t} = \kappa(\hat{m}\hat{c}_t - \hat{p}_{S,t}) + \beta E_t[\hat{\pi}_{S,t+1}] \quad or \quad \hat{y}_{S,t} = \hat{a}_t - \hat{\xi}_t + v$$

$$\hat{l}_{W,t} = \hat{y}_{W,t} - (\hat{w}_t - \hat{a}_t - \hat{m}\hat{c}_t), \quad \hat{l}_{S,t} = \hat{y}_{S,t} - (\hat{w}_t - \hat{a}_t - \hat{m}\hat{c}_t)$$

$$\begin{split} \hat{x}_{WW,t} &= \hat{y}_{W,t} - (\hat{p}_{W,t} - \hat{m}c_t), & \hat{x}_{SW,t} &= \hat{y}_{S,t} - (\hat{p}_{W,t} - \hat{m}c_t) \\ \hat{x}_{WS,t} &= \hat{y}_{W,t} - (\hat{p}_{S,t} - \hat{m}c_t), & \hat{x}_{SS,t} &= \hat{y}_{S,t} - (\hat{p}_{S,t} - \hat{m}c_t) \\ \bar{y}_{W} \hat{y}_{W,t} &= \bar{x}_{WW} \hat{x}_{WW,t} + \bar{x}_{SW} \hat{x}_{SW,t} + \bar{c}_{GW} \hat{c}_{GW,t} \\ \bar{y}_{S} \hat{y}_{S,t} &= \bar{x}_{WS} \hat{x}_{WS,t} + \bar{x}_{SS} \hat{x}_{SS,t} + \bar{c}_{S} \hat{c}_{S,t} + \bar{c}_{GS} \hat{c}_{GS,t} \\ & (\bar{y}_{S} + \bar{y}_{W}) \hat{l}_{t} = \bar{y}_{S} \hat{l}_{S,t} + \bar{y}_{W} \hat{l}_{W,t} \\ & \hat{p}_{W,t} &= \hat{p}_{W,t-1} + \hat{\pi}_{W,t} - \hat{\pi}_{t} \\ & \hat{a}_{t} &= \hat{p} \hat{a}_{t-1} + e_{a,t} \\ & \hat{c}_{t} &= \hat{p} \hat{c}_{t-1} + e_{y,t} \\ & \hat{\alpha}_{t} &= \hat{p}_{a} \hat{\alpha}_{t-1} + e_{a,t} \end{split}$$

with $\kappa = \frac{\epsilon - 1}{\phi}$. Finally, it is useful to introduce aggregate potential output \hat{c}_t^P :

$$\hat{c}_t^P = \hat{a}_t - \hat{\xi}_t,$$

and the output gap:

$$ygap_t = \hat{c}_t - \hat{c}_t^p.$$

