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The Inverted Fisher Hypothesis: Inflation Forecastability and Asset Substitution

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The Inverted Fisher Hypothesis: Inflation Forecastability and Asset Substitution

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Abstract

This paper examines the implications of inflation persistence for the *inverted* Fisher hypothesis that nominal interest rates do not adjust to inflation because of a high degree of substitutability between money and bonds. It is emphasized that the substitutability between nominal assets and capital renders the hypothesis inconsistent with the data when inflation persistence is high. Using a *switching regression* model, the analysis allows the reflection of inflation in interest rates to vary according to the degree of inflation persistence or forecastability. The hypothesis is supported by U.S. data only when inflation forecastability is below a certain threshold.

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I. Introduction

This paper examines the implications of inflation persistence for the *inverted* Fisher hypothesis (IFH) proposed by Carmichael and Stebbing (1983, henceforth CS). The standard Fisher hypothesis implies that the *real* interest rate is not affected by inflation because of the substitutability between bonds and capital. In contrast, CS argue that this hypothesis becomes inverted if there is a high degree of substitutability between money and bonds and if government regulation precludes the payment of interest on money.[2] They propose that the *nominal* interest rate does not adjust to inflation, with the real interest rate moving inversely one-for-one with inflation. However, they presuppose that the inflation process is stationary (that is, nonpersistent).

CS deemphasize the potential substitutability between nominal assets (money and bonds) and capital, but this substitutability matters when the inflation process is highly persistent (that is, realized inflation is informative in predicting future inflation movements). As a result, the IFH is likely to be supported only when the inflation process is stationary. CS themselves conjecture that the hypothesis is expected to hold only in countries with moderate or low inflation rates. Also, Barth and Bradley (1988) attribute the failure of the hypothesis in samples extending beyond 1978 to the crucial dependence of the hypothesis on the assumption of moderate inflation.

A plausible interpretation of CS's conjecture is that economic agents are more concerned about asset substitutions between money and capital when inflation is high than otherwise, since changes in inflation are usually sizable and thus have important consequences for the rate of return on money. In this case the degree of willingness to adjust the asset portfolio in response to changes in inflation could be positively correlated with the level of inflation. Higher inflation should then be associated with less reflection of inflation in real interest rates. This interpretation of CS's conjecture relies on asset substitutability varying across inflation levels. In contrast, if substitutability among assets is constant across inflation levels, the failure of the IFH can be attributed to the effect of inflation persistence on asset substitutions. This paper suggests that, if inflation is persistent, and thus a direction or trend in future inflation is largely anticipated, changes in inflation lead to asset substitutions between money and capital, which in turn affect the rate of return on money. Thus, this paper proposes that higher inflation persistence should be associated with less reflection of inflation in real interest rates, despite constant asset substitutability.

Despite evidence on the persistence of inflation in the post–World War II period (for example, Klein, 1975; Evans and Wachtel, 1993), no previous studies have explicitly taken into account the influence of inflation persistence in testing the IFH. This paper sheds light

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[2] CS argue that one should take into account the substitutability between *money* and bonds in determining the effect of inflation on interest rates, especially because the data mainly pertain to the return on financial assets rather than to the return on capital.

on the link between the persistence of inflation and the effect of asset substitutions on interest rates. Numerous studies of hyperinflations and other rapid inflations (for example, Laidler, 1993) suggest that expected inflation affects money holdings. Although most economies do not have such high and rapid inflation, many do experience highly persistent inflation over a substantial length of time. If inflation is highly persistent, economic agents can largely forecast inflation using their own most recent experience. When higher inflation is expected, agents will substitute capital for nominal assets. The resulting change in real balances alters the implicit marginal return on money, which CS, however, assumed to be approximately constant.

The more persistent the inflation rate, the higher its short-term forecastability becomes. Changes in the monetary regime affect the persistence of inflation and thus its forecastability. Friedman (1977) views regime changes as an important source of inflation uncertainty. Klein (1975) provides an appealing explanation, subsequently emphasized by Friedman and Schwartz (1982, chapter 10), of how inflation forecastability is influenced by fundamental changes in the character of the monetary system. Klein argues that, with the collapse of the Bretton Woods system and the explicit step toward a fiat money system in the 1970s, changes in institutional arrangements made inflation more predictable in the short term than it had been earlier.[3] Also, a shift in emphasis on whether monetary policy targets money growth, interest rates, or inflation results in changes in the inflation process (see, for example, Rudebusch and Svensson, 1998). The anti-inflationary policy pursued by Federal Reserve Chairman Paul Volcker kept U.S. inflation in check beginning around 1982, and thereafter steady policy aimed at keeping inflation low. After the mid-1980s inflation became lower and less persistent than before.

In order to make a judgment about the inflation path that affects their asset portfolio decisions, agents look for past evidence that they regard as most relevant to the current situation.[4] Rather than restrict the IFH by the use of a chronological time scale, this paper attempts to identify a regime that fits the situation on which the hypothesis is based. It is proposed that the IFH holds only when inflation is nonpersistent.

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[3] Klein argues that, under specie standards that provided an anchor for the price level, there was considerable short-term unpredictability but much less long-term unpredictability of inflation. Under the fiduciary monetary system of the post–World War II period, there is no anchor for the price level. Agents have come to regard prices as largely affected by policy, and short-term unpredictability is less than before. For example, Hutchison and Keeley (1989) show that inflation evolved from a white noise process in the pre–World War I period to a highly persistent, nonstationary process in the post-1960 period, which strengthened the Fisher effect.

[4] As Friedman and Schwartz (1982) note, improvements in short-term forecasting methods and in the sophistication and breadth of financial and capital markets shortened the forecast horizon of agents and increased their concern about whether or not high rates of inflation persist.

CS tested the IFH by regressing the actual after-tax real interest rate (defined as the after-tax nominal rate of return on short-term bonds minus the actual inflation) on the actual inflation. This testing procedure does not involve, under the IFH, an errors-in-variables problem that can make parameter estimates inconsistent due to measurement error on the regressor. The merit of this procedure is that it does not require the use of arbitrary data about inflation expectation. To test whether the validity of the IFH depends on regimes, we estimate *switching regressions* that allow the reflection of inflation in interest rates to vary over regimes. Two regimes are assumed to be distinguishable according to whether the degree to which inflation can be forecast is above or below a threshold value. Two schemes are considered for switching between regimes. First, a switching regression with a smooth transition is used to reflect the notion that agents assign different weights to possible regimes according to their judgment, with time-varying confidence, about the inflation process. This scheme controls to some extent the errors-in-variables problem in testing the IFH under alternative regimes. Second, a switching regression with perfect discrimination is used to treat two regimes separately. This scheme is immune to the errors-in-variables problem in testing the IFH. On the other hand, for comparison purposes, this paper also examines the alternative proposition that the IFH holds only when the inflation level rather than its forecastability is below a certain threshold.

Linearity testing supports the evidence for the threshold effect in the CS equation. This paper finds new evidence that the IFH is supported by the U.S. data only when inflation forecastability is below a certain (estimated) threshold. This finding reflects the notion that the implicit marginal return on money and the nominal interest rate adjust to inflation through substitution between nominal assets and capital in periods of persistent inflation, but do not adjust to inflation when inflation is nonpersistent and thus largely unpredictable.

The paper proceeds as follows. Section II presents a brief review of the IFH and sets out a model for the augmented version of the IFH. Section III reports the estimation and testing procedures for the model. Section IV reports the empirical results, and robustness checks on the results are offered in Section V. The model and alternative specifications are discussed in Section VI. The final section provides the concluding remarks.

II. Conceptual Background and the Model

The IFH is based on the premise that optimizing agents hold money and financial assets up to the point at which their after-tax real yields are equal. The after-tax real rate of return on a bill is given by

$$r_{Nt} = i_{Nt} - \pi_{t+1}, \tag{1}$$

where i_{Nt}, the after-tax nominal rate of return on the bill, is defined as $i_{Nt} = (1-\theta_t)i_t$ with θ_t being the marginal tax rate, and π_{t+1} is the inflation rate at the maturity of the bill. The after-tax real return on money is given by $r_{mt} = z_t - \pi_{t+1}$, where z_t is the *implicit* marginal return on money. In equilibrium, $r_{Nt} = r_{mt}$, therefore, $r_{Nt} = z_t - \pi_{t+1}$, or, equivalently, $z_t = i_{Nt}$.

Assuming a close substitutability between money and financial assets, CS argue that z_t is approximately constant.

Taking (conditional) expectations of equation (1) on both sides,

$$E(r_{Nt}) = E(i_{Nt}) - E(\pi_{t+1}), \tag{1'}$$

where E is the expectations operator conditional on the information set available in period t. CS assume that expectations are unbiased, that is,

$$\pi_{t+1} = E(\pi_{t+1}) + \varepsilon_{t+1}. \tag{2}$$

where ε_{t+1} is a mean-zero homoscedastic error and independent of ξ_t. Also, the after-tax nominal interest rate is assumed to be known at all points in time, that is, $E(i_{Nt}) = i_{Nt}$.

To test the IFH, CS present the following equation:

$$E(r_{Nt}) = \alpha_0 + \alpha_2' E(\pi_{t+1}) + \xi_t, \tag{3}$$

where α_0 is a constant, and ξ_t is homoscedastic error with a mean of zero. Under the IFH, $E(r_{Nt})$ moves inversely one for one with $E(\pi_{t+1})$, that is, $\alpha_2' = -1$. Substituting equations (1) and (1′) into equation (3) and eliminating the unobservable $E(\pi_{t+1})$ using equation (2), CS obtain in the testable form

$$r_{Nt} = \alpha_0 + \alpha_2' \pi_{t+1} + \xi_t - (1 + \alpha_2')\varepsilon_{t+1}. \tag{3'}$$

Under the null hypothesis $\alpha_2' = -1$,[5] an errors-in-variables problem in equation (3′) vanishes. The CS estimation avoids the use of arbitrary data on inflation expectations in determining the effect of inflation on interest rates.[6] (This procedure cannot be used to test the standard Fisher hypothesis because of an errors-in-variables problem (see Graham, 1988).)

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[5] Gallagher (1986) points out that this testing pertains to the maintained hypothesis that π_{t+1} and ξ_t are only contemporaneously uncorrelated. Following earlier studies including CS, this paper takes the IFH as claiming that inflation and nominal interest rates are *contemporaneously* uncorrelated in the sense of Gallagher.

[6] Under the standard Fisher hypothesis ($\alpha_2' = 0$), the bias in the ordinary least-squares (OLS) estimator of α_2' equals $-\Sigma(\pi - \bar{\pi})\varepsilon/\Sigma(\pi - \bar{\pi})^2$, where the bars denote sample means. The bias becomes -1 if π is perfectly correlated with ε (for example, if π is a martingale difference sequence). Then a problematic case arises, since testing $\alpha_2' = -1$ cannot discriminate between the inverted and standard Fisher hypotheses. A similar situation is illustrated in Thies and Crawford (1997).

If inflation is close to a martingale difference sequence with respect to the agents' information set (that is, the conditional expectation of inflation based on the information set available in the period is almost the same as the unconditional expectation of inflation), there is little persistence in inflation. As such, current changes in inflation will have little effect on agents' expectations of future inflation and thus on the substitution between nominal assets and capital. This would fit CS's presumed environment. However, the CS proposition is consistent with the principles of rational agents only when the persistence of inflation is low enough. If inflation is highly persistent and if agents utilize recent experience in updating their expectations of inflation, substitution between nominal assets and capital occurs upon changes in inflation.

To see specifically how inflation persistence affects the link between interest rates and inflation, two equations are set up. The demand for real balances, M_t^D / P_t, is given by

$$\frac{M_t^D}{P_t} = \Phi(\overset{(+)}{Y_t},\ \overset{(-)}{i_t - z_t},\ \overset{(-)}{\pi_{t+1}^e - z_t}) = \hat{\Phi}(\overset{(+)}{Y_t},\ \overset{(-)}{i_t},\ \overset{(+)}{z_t},\ \overset{(-)}{\pi_{t+1}^e}), \tag{4}$$

where Y_t is real output and π_{t+1}^e represents the expected nominal rate of return on real assets. That is, the demand for real balances is an increasing function of real output and a decreasing function of relative returns on bonds and real assets, similar to Friedman and Schwartz's (1982, pp. 37–40) money demand except that the role of wealth and equity is abstracted. Another equation states that the implicit marginal return on money is negatively related to real balances

$$z_t = \Lambda\left(\frac{M_t}{P_t}\right),\ \Lambda_{\frac{M}{P}} < 0. \tag{5}$$

In equilibrium, $M_t^D = M_t$ and $z_t = \Lambda(M_t / P_t)$. For any π_{t+1}^e, there is an equilibrium value for M_t / P_t and, hence, for z_t (from equation (5)). In deciding whether to shift from one type of assets to other types, it is important to know whether inflation is expected to rise or to fall. Thus, the forecastability of inflation matters. When π is highly persistent, if agents expect future π to rise, they will hold less money and more capital. Due to this substitution between money and capital, z_t rises and, hence, $r_{N\,t}$ should rise, too. In contrast, when the persistence of π is low, agents will not expect future π to change in a certain direction, and so they will not alter their money holdings. As a result, both z_t and $r_{N\,t}$ will be unaffected when changes in inflation occur.

The next step is to take explicitly into account the idea that the response of real interest rates to inflation can vary with the persistence of inflation. We assume two distinct regimes, classified according to whether or not inflation persistence, ρ_t^{π}, is above a certain threshold. We set out an augmented version of equation (3′) as

$$r_{Nt} = \alpha_0 + \alpha_2' \pi_{t+1} + \xi_{1t} - (1+\alpha_2')\varepsilon_{t+1} \qquad \text{if } \rho_t^\pi \le \tau, \tag{6a}$$

$$r_{Nt} = \alpha_0 + \gamma_2' \pi_{t+1} + \xi_{2t} - (1+\gamma_2')\varepsilon_{t+1} \qquad \text{if } \rho_t^\pi > \tau, \tag{6b}$$

where model errors are assumed to be heteroscedastic to reflect the evidence from Hamilton (1988) and Choi (1999a) that the interest rate process involves heteroscedastic error. Let D_t = 0 if $\rho_t^\pi \le \tau$ and D_t = 1 otherwise, where the threshold value, τ, is taken to be a fixed parameter.

Combining the two regimes by premultiplying equation (6a) by $(1-D_t)$, equation (6b) by D_t, and adding yields

$$r_{Nt} = \alpha_0 + (1-D_t)\alpha_2' \pi_{t+1} + D_t \gamma_2' \pi_{t+1} + (1-D_t)\eta_{1t} + D_t \eta_{2t}, \tag{7}$$

where $\eta_{1t} \equiv \xi_{1t} - (1+\alpha_2')\varepsilon_{t+1}$ and $\eta_{2t} \equiv \xi_{2t} - (1+\gamma_2')\varepsilon_{t+1}$, which are distributed as $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, respectively. The model error is regime-wide heteroscedastic. In contrast with CS's IFH ($\alpha_2' = \gamma_2' = -1$), it is argued that $\alpha_2' = -1$ holds but $\gamma_2' = -1$ does not. We expect $\gamma_2' > -1$ because substitutability between nominal assets and capital raises the nominal interest rate with higher inflation under the high-persistence regime.

So far we have considered how inflation forecastability affects the link between interest rates and inflation, on the presumption that the degree of substitutability among assets is fixed. From a different viewpoint, in line with CS's conjecture that the IFH is expected to hold only when inflation is low, we suppose that agents are more concerned about asset substitution between money and capital if inflation is high than otherwise. Then the degree of substitutability is positively correlated with inflation. Shifts in the link between interest rates and inflation will occur across low- and high-inflation regimes. To test this proposition, we replace the forecastability index with the level of inflation.

III. Empirical Methodology

To make estimating equation (7) tractable, we incorporate "deterministic switching based on other variables" as proposed by Goldfeld and Quandt (1972, pp. 258–77, 1973). The estimation procedure is as follows. First, we use an inflation forecastability index as a proxy for the switching variable ρ_t^π.[7] The forecastability index F_t is defined as the rolling weighted R^2 of an inflation forecast model

[7] As inflation moves more persistently, it can be better forecast using the past history of inflation. Autocorrelations up to several orders of the residual, which contain information about persistence, will be reflected in this index. In general, agents would utilize the information set including indicators for inflation as well as inflation persistence.

$$F_t = 1 - \sum_{i=0}^{w-1} h^i \, \hat{\varepsilon}_{t+1-i}^2 / \sum_{i=0}^{w-1} h^i (\pi_{t+1-i} - \bar{\pi})^2, \; 0 < h \leq 1, \tag{8}$$

where $\hat{\varepsilon}_{t-i}$(for $i = 0,1,...,w-1$) is the inflation forecast error of the rolling regression with window size w, $\bar{\pi}_t = \frac{1}{w}\sum_{i=0}^{w-1}\pi_{t+1-i}$, and h is a weight parameter. The rolling regression is employed to reflect the idea that agents learn how the inflation process changes by looking back over their recent experience. Also, the rolling regression is preferred to an entire-period regression, because the latter would provide forecasts based on information that agents would not have had unless the model parameters were constant over time. Notice that F_t is negatively related to the ratio of the variance of unanticipated inflation (that is, the conditional variance of inflation) to the variance of inflation. For instance, if inflation is highly persistent, the variance of inflation far outweighs the variance of unanticipated inflation; that is, the value of F_t becomes high.

Second, we define the standardized *unobservable* switching index as $F_t^* = \widetilde{F}_t + v_t$, where $\widetilde{F}_t$ is the standardized F_t and v_t is an identically and independently distributed (i.i.d.) drawing from $N(0, 1)$ and is assumed to be independent of $\eta_{1\,t}$ and $\eta_{2\,t}$. Also, using the switching index, the discrete indicator function can be approximated by a continuous function

$$D_t = \int_{-\infty}^{\widetilde{F}_t - \tau} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}v_t^2) dv_t. \tag{9}$$

This smoothing function allocates weights to observations according to the degree of forecastability. The log-likelihood function for equation (7) up to a constant term is given by

$$\ln L = \tfrac{1}{2}\sum_{t=1}^{T} \ln[\sigma_1^2(1-D_t)^2 + \sigma_2^2 D_t^2] - \tfrac{1}{2}\sum_{t=1}^{T} \frac{(r_{N\,t} - \alpha_0 - [\alpha_2'(1-D_t) + \gamma_2' D_t]\pi_{t+1})^2}{\sigma_1^2(1-D_t)^2 + \sigma_2^2 D_t^2}. \tag{10}$$

Substituting equation (9) into equation (10), a maximum likelihood estimation (MLE) can be performed by maximizing the log-likelihood function with respect to $\alpha_0, \alpha_2', \gamma_2'$, σ_1, σ_2, and τ.

To determine whether the threshold effect is statistically significant, we conduct the Lagrange multiplier (LM) test for linearity, following a procedure suggested by Granger and Teräsvirta (1993). The procedure is described in Appendix I. Since the threshold parameter is unknown a priori and not identified under the null hypothesis of linearity ($\alpha_2' - \gamma_2' = 0$), classical tests have nonstandard distributions. This is the so-called Davies problem (Davies, 1987). Following Hansen (1996), we compute three types of final statistics that are functionals of the collection of LM test statistics over the grid set: the supremum (SupLM), the average (AveLM), and the exponential average (ExpLM) of all LM statistics.[8] The grid set is comprised of 101 grids that evenly divide the range from the 10th to the 90th percentile of the empirical distribution of the (standardized) switching index. Their significance levels are calculated using a simulated empirical distribution of these statistics.

Testing either $\alpha_2' = -1$ or $\gamma_2' = -1$ separately in equation (7) may involve a measurement error problem. That is, a correlation remains between the regressor and the error term under the null hypothesis of $\alpha_2' = -1$ or of $\gamma_2' = -1$, if the two regimes are imperfectly discriminated ($0 < D_t < 1$). To deal with the errors-in-variables problem in the testing, we first test $H_0(\alpha_2' = \gamma_2' = -1)$ and then $H_0(\gamma_2' = -1 \mid \alpha_2' = -1)$, assuming that the IFH holds under the low-forecastability regime. Both hypotheses should not be rejected if CS's IFH holds.

In addition, we test either $\alpha_2' = -1$ or $\gamma_2' = -1$ separately by estimating the switching regression under the assumption of perfect discrimination. To do this, we estimate a piecewise linear regression for equation (7), setting $D_t = 0$ if $\widetilde{F}_t \leq \tau$, and $D_t = 1$ otherwise. This testing procedure is robust to the errors-in-variables problem because the estimator under different regimes uses mutually exclusive observations. Estimating this threshold model should account for the Davies problem, as τ is not identified under the null hypothesis of no threshold effect.

We test the IFH under alternative regimes, controlling the nuisance parameter problem in the context of Hansen (1999, 2000; see Appendix II). First, we estimate the threshold model by weighted least squares (WLS) to control heteroscedastic errors and obtain a consistent estimate for τ by minimizing the sum of squared residuals over a grid set. Second, we perform the likelihood ratio test for $\alpha_2' = \gamma_2'$ using p-values constructed from a bootstrap procedure. Third, we form the confidence interval for τ by forming the no-rejection region using the likelihood ratio statistic for tests on τ. Finally, as in Hansen (1999), we make an inference about the slope estimate as if the threshold estimate were the true value, since Chan (1993) and Hansen (2000) show that the dependence on the threshold estimate is not of first-order asymptotic importance.

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[8] Davies (1987) and Granger and Teräsvirta (1993) suggest using the supremum of statistics over a grid set, whereas Andrews and Ploberger (1994) suggest using the average and the exponential average of statistics.

IV. Empirical Results

A. Measuring Variables

We use the U.S. time series data taken from Federal Reserve Economic Data (FRED) over the period from January 1947 (1947:01) to 1997:12. The three-month Treasury bill rate is used as the nominal interest rate. To measure inflation we use the consumer price index (CPI) with no adjustment for housing costs for the whole period or, alternatively, the implicit deflator for personal consumption expenditure (PPC) from 1959:01 to 1997:12.[9] In keeping with earlier studies on the IFH, we estimate a quarterly model to match the maturity of Treasury bills, although both interest and inflation rates are available monthly.[10] The inflation rate in period t is defined as $\pi_t \equiv 400(P_t / P_{t-1} - 1)$, where P_t is the price level in period t. To account for the tax effect on interest rates (Darby, 1975; Feldstein, 1976), we assume that the marginal tax rate on interest income is fixed at 0.3, following Evans and Lewis (1995).[11] For comparison purposes, we also use Sahasakul's (1986) marginal tax rate data.[12]

Figure 1 depicts the tax-adjusted (ex-post) real rate of return on three-month Treasury bills and the CPI inflation rate at the maturity of the bill. It is quite discernible that inflation was rather persistent in the late 1960s, and high and highly persistent from the 1973 oil shock until 1981. Volcker's anti-inflation policy kept inflation in check in 1982 and, thereafter, policy has consistently aimed at keeping inflation low. The real rate on Treasury bills dipped in the 1970s but changed in the opposite direction by less than one for one with changes in

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[9] Using the mnemonics on the FRED of the Federal Reserve Bank of St. Louis, the variable definitions are TB3MS (the three-month Treasury bill rate), CPIAUSL (the CPI-U: whole items), and PCE/PCEC92 (the personal consumption expenditure deflator). The average for each quarter is used to measure the quarterly series. All except interest rates are seasonally adjusted data.

[10] The use of a monthly model may cause serial correlation in errors because the maturity of yield on a bill is longer than the data frequency. Suppose that the error term of equation (3′) (in a difference form) for the one-month rate is serially uncorrelated. Based on this equation, the monthly model of the three-month rate can be expressed, for example via the expectations hypothesis. Time aggregation results in serial correlation in errors.

[11] Evans and Lewis argue that it is extremely difficult to know the appropriate tax rate on interest income for the overall economy, since the effective tax rate on interest varies tremendously across individuals, firms, and institutions. Friedman and Schwartz (1982, p. 572) suggest that, to judge from the differential return on taxable and tax-exempt securities, the relevant tax rate exceeds one-third and is quite stable over time.

[12] The marginal tax rate was calculated as the weighted average of marginal tax rates from the social security tax, the individual income tax, and other federal taxes.

inflation, implying a drift in the implicit marginal return on money. With the downward movement in inflation and with financial deregulation in the early 1980s, agents increased their holdings of cash and interest-earning deposits. This may have led to a sharp rise in the real rate on the one hand and a decrease in the implicit marginal return on money on the other. After the mid-1980s the movement in the real rate became quite steady, apparently reverting to the pattern of periods other than those of episodic inflation.

The switching variable is defined as the forecastability index that is measured by the quarterly average of the monthly index, to capture the variability of inflation within a quarter. For comparison purposes, the switching variable is alternatively defined as the lagged quarterly inflation rate, which performs better than inflation with a different timing. To construct the forecastability index, this paper uses a monthly inflation forecast model, which includes nine lags of the dependent variable and the nominal interest rate as a predictor of future inflation (see, for example, Nelson and Schwert, 1977). The nominal interest rate is significant at the 5 percent level for CPI inflation but largely insignificant for PPC inflation. Inflation forecast errors are computed from the rolling estimation of the model ($w = 60$) to account for possible structural changes in the inflation process and for the role of learning.[13] The monthly index is then measured by equation (8) by setting $h = 0.9$ for the CPI ($h = 0.98$ for the PPC); when the CPI is used, the most recent year accounts for the 72 percent of the weight given to all observations within each window (the corresponding figure is 31 percent for the PPC).

Figure 2 displays the standardized inflation forecastability index ($\widetilde{F}_t$) along with the smoothing function (D_t) given the estimate of τ (reported in Table 3 below). The indices for both the CPI and the PPC show relatively high values particularly in the late 1960s, for a few years after the first oil shock, and in the 1979–86 period. These were mostly periods of relatively high and persistent inflation; the indices show quite low values in most periods after 1986. Movements in the real interest rate, inflation, and the forecastability index suggest that the IFH can hardly be reconciled with the data during persistent inflationary eras.

This paper uses first differences of the variables to test the IFH, following CS, Barth and Bradley (1988), and Gupta (1991), since estimating the level-form equation involves the use of nonstationary time series.[14] The starting date of regressions with the CPI is 1953:Q2,

[13] Different lag lengths are suggested by different information criteria (over rolling samples). However, choosing an alternative lag length (13 lags, for example) does not affect the estimation results of this paper qualitatively.

[14] Both CS and Gupta obtained virtually identical results regardless of whether the levels or the first differences of the variables were chosen, with correction for first-order serial correlation for the former specification. The present investigation also found that the estimation of equation (2) using levels with correction for first-order serial correlation using the Beach-McKinnon or the Cochrane-Orcutt method provided almost the same results.

whereas that of regressions with the PPC is 1959:Q3 or 1965:Q2, dictated by the starting date of the switching index. The ending date of the regressions is 1997:Q3 or 1982:Q4, the latter being dictated by the availability of Sahasakul's tax rate data. Throughout the paper the results for the first-difference-form regressions without an intercept are reported; regressions including the intercept provide almost identical results.

B. CS Regressions and Linearity Testing

To begin with, the conventional linear regression for equation (3′) is estimated assuming a single regime for the entire sample. Table 1 summarizes the estimation results by the ordinary least squares (OLS) method. The sample period is 1953:Q2 to 1997:Q3 when the CPI is used as the price variable (1959:Q3 to 1997:Q3 when the PPC is used). Despite the difference in the sample periods, the results with a constant marginal tax rate are virtually the same as those with a time-varying tax rate.[15] The IFH ($\alpha_2' = -1$) is rejected at the 5 percent significance level in all cases. For example, with a constant marginal tax rate adjustment, the α_2' estimate is –0.896 with the CPI (–0.931 with the PPC). The null hypothesis of $\alpha_2' = -1$ is rejected by the Wald test at the 5 percent level regardless of the choice of the price variable, although it is less strongly rejected in the PPC case. The last row of the top panel of Table 1 reports the LM test for the null hypothesis of homoscedastic errors in a linear model against an alternative specification in which the variance of residuals depends on the lagged nominal interest rate (as in Marsh and Rosenfeld, 1983, and Hamilton, 1988). The test results provide strong evidence against homoscedasticity. The bottom panel of the table reports the CS equation estimated by WLS to account for heteroscedasticity. The weight applied to each observation is the square root of the lagged nominal interest rate. The results with WLS estimation also provide evidence against the IFH.

Next, linearity testing is performed to determine whether there exists a threshold effect. Table 2 reports the test results assuming a constant marginal tax rate. Two alternative measures are used as the switching variable. The top panel reports results from the use of the forecastability index constructed from the inflation forecast model, and the bottom panel reports results from the use of the lagged inflation level. In each panel, errors are assumed to be either homoscedastic (upper row) or heteroscedastic (lower row). In the case of heteroscedastic errors, it is assumed that the variance of errors depends on the lagged

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[15] Barth and Bradley (1988) and Gupta (1991) find that estimation results for equation (3′) are not sensitive to whether or not the real rate is adjusted for taxes. Indeed, as Barth and Bradley point out, "since taxes are levied on the nominal interest rate, whether the real rate is adjusted for taxes or not, the real interest rate should vary inversely and one-for-one with the inflation rate (under the hypothesis)," unless the tax rate is correlated with inflation.

nominal interest rate as in Marsh and Rosenfeld (1983) and Hamilton (1988).[16] The p-values for the test statistics tend to be higher under the heteroscedasticity assumption. Thus, on the basis of p-values with heteroscedastic errors, one can conservatively decide whether linearity is rejected.

All the test statistics—SupLM, AveLM, and ExpLM (columns 1–3 of Table 2)—indicate that linearity is strongly rejected in most cases (that is, there is a significant threshold effect) except for the case of the PPC with the forecastability index, where the evidence against linearity is rather weak (with the significance level 9–16 percent). The evidence against linearity becomes stronger with the level of inflation as the switching variable: in particular, in the case of the PPC, the null hypothesis of linearity is rejected at the 5 percent level. Also performed is a specification test for parameter constancy across given subsamples as if the threshold estimate (τ reported in Table 3) were the true value as in Durlauf and Johnson (1995) and Choi (1999b). The last column of Table 2 provides evidence against parameter constancy across the subsamples. Measuring the after-tax real interest rate with the time-varying marginal tax rate, or measuring the forecastability index from an autoregressive model, provides qualitatively the same results (not reported). Taken together, these results give credence to nonlinearity in the relationship between interest rates and inflation.

C. Switching Regressions with Smooth Transition

The switching regression with smooth transition defined by equations (7) and (9) was estimated by MLE. Table 3 summarizes the regression results with a forecastability index (upper panel) and, alternatively, with the inflation level (lower panel). Columns 1–2 pertain to the tax rate adjustment with a constant marginal tax rate, and columns 3–4 pertain to that with a time-varying marginal tax rate using Sahasakul's (1986) data. For all cases, the α_2' estimate ranges from -1.12 to -1.01, whereas the γ_2' estimate ranges from -0.84 to -0.76, consistent with the argument that the reflection of inflation in the real interest rate varies across regimes. The estimate of τ, for example, is -0.073 with the CPI (0.392 with the PPC) when the forecastability index and constant marginal tax rate are used: the observations for the high-forecastability regime are 93 out of 178 for the CPI (55 out of 130 for the PPC). The estimates of σ_1 and σ_2 indicate that the conditional variance of the real interest rate under the high-forecastability regime is about three to four times as high as that under the other regime, consistent with the assumption of regime-wide heteroscedasticity.

The Wald test results run counter to the IFH when inflation forecastability is above the estimated threshold value. Specifically, in the upper panel of Table 3 the Wald test strongly rejects the null hypothesis of $\gamma_2' = \alpha_2' = -1$ in most cases, indicating that the IFH

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[16] The time-varying variance fully consistent with the regime-wide heteroscedasticity, $(1 - D_t)^2\sigma_1{}^2 + D_t{}^2\sigma_2{}^2$, is not directly applicable. The identification of the time-varying variance weighted by D_t requires the ML estimation of the model for each value of τ from the grid set.

does not hold for the whole sample. In addition, the Wald test rejects the null hypothesis of $\gamma_2' = -1$ given $\alpha_2' = -1$, at the 1 percent level in the CPI case and at about 5 percent level in the PPC case. Considering how close the α_2' estimate is to -1.0 (although no direct test is available because of an errors-in-variables problem), these results suggest that the data do not support the IFH under the high-forecastability regime. This finding is insensitive to the choice of the price variable and to including the marginal tax rate. The same implications can be drawn from the results with the inflation level as the switching index (lower panel of Table 3), although the p-value of the Wald test for the null hypothesis of $\gamma_2' = -1$ given $\alpha_2' = -1$ is rather high (0.113) in the PPC case. The threshold value for inflation ranges from 4 to 6 percent.

D. Switching Regressions with Perfect Discrimination

Table 4 reports the results of switching regressions with perfect discrimination, which are estimated by WLS to account for heteroscedasticity, using the square root of the nominal interest rate as the weight applied to each observation. The likelihood ratio test statistic for a threshold effect, F_1, is highly significant in all cases as shown by the p-values, strongly indicating a threshold effect. The samples are perfectly discriminated according to the LS estimate of τ. As reported, however, the (90 percent) confidence interval for the τ estimate is not very tight, indicating some degree of uncertainty about the threshold value.[17] When the inflation level is used as the switching index, more observations tend to be assigned to the low-inflation regime.

The point estimate of α_2' is very close to -1, and the t-value for $\alpha_2' = -1$ is too low to reject the IFH under the low-forecastability regime. In contrast, the point estimate of γ_2' is less than 0.90, and the t-value for $\gamma_2' = -1$ is greater than 2.8, indicating that the IFH is rejected only under the high-forecastability regime. In addition, the IFH is rejected under the high-inflation regime but not under the low-inflation regime (lower panel of Table 4).

In sum, tests for linearity provide the evidence of the threshold effect across the different regimes. The results from switching regressions with smooth transition and those with perfect discrimination indicate that the IFH is supported by U.S. data only when inflation forecastability is below a certain estimated threshold. These results are robust to the choice of the price variable, marginal tax rate adjustment, and measurement of the switching variable.

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[17] The plots of the likelihood ratio as a function of τ indicate a unique major dip in the likelihood ratio around the estimate, suggesting that two regimes are sufficient to describe the nature of the threshold effect.

V. Robustness Checks

This section provides further checks for the sensitivity of the results in several dimensions. We examine, first, whether the results are affected when the analysis follows CS's timing convention that r_{Nt} is the after-tax real return on a three-month bill held from the beginning to the end of quarter t. The interest rate should reflect inflation expectations over the life of the bill. Unfortunately, however, data for inflation are available only on a monthly basis and do not pertain to a specific date in the month (say, the first business day). Thus, the underlying maturity cannot be exactly matched with the inflation horizon. Also, the interest rate with this timing is subject to daily seasonality (for example, as shown in Hamilton, 1997) as reserve requirements with the two-week maintenance period affect the daily funds rate and thus other short-term rates.[18] For comparison purposes, switching regressions are nonetheless estimated following the CS timing (with the CPI). The starting date of regressions (1962:Q3) is dictated by the availability of daily data for the three-month Treasury bill rate (DTB3 from FRED). As shown in Table 5, the results with the forecastability index (column 1) and the inflation level (column 2) deliver the same message as those in Tables 3 and 4.

We also examine whether accounting for shifts in institutional factors affects the results. First, since CS argued that the IFH is primarily a regulatory phenomenon, an attempt was made to check whether accounting for financial deregulation in the early 1980s affects the result. This was done by estimating the switching regression model separately for the pre-deregulation (before 1980) and post-deregulation (after 1980) periods. Qualitatively the same result is found as in Tables 3 and 4 from the post-deregulation period regression; similar implications are obtained from the pre-deregulation period regression, in which most observations belong to the low-forecastability regime, so that the result is more favorable to the CS argument. Second, an attempt was made to account for price controls that affect inflation and possibly the interest rate setting. In measuring the forecastability index, a Korean War dummy (set to equal 1 for each of the years 1951–53) was included in the inflation forecast equation. A step function taken from Gordon (1990), designed to capture the effects of imposing and then eliminating price controls in the Nixon era (1971–74), was also used. Using this forecastability measure has little effect on the estimation results, as shown in column 3 of Table 5. Alternatively, the regressions were also estimated leaving out the Korean War period and the era of the Nixon price controls. The results are very similar except that the cutoff value rises.

Several tests were conducted to check whether the results are robust to alternative measures of the forecastability index. First, the forecastability index can be measured with

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[18] This paper deals with these issues by using a monthly (or quarterly) average rate of interest rather than the rate at a specific date for quarter t, taking the rate of inflation as the average figure of the period.

the use of a quarterly model. The forecast model uses the three-month Treasury bill rate and includes four lags of the dependent variable, where the lag length of four for the CPI is selected by the information criteria of Akaike, Schwarz, and Hannan-Quinn (and that for the PPC by Akaike and Hannan-Quinn criteria). With this forecastability index, linearity tests gave similar results. As reported in the last column of Table 5, the use of the quarterly inflation forecast model for the CPI provides qualitatively the same results, although it assigns more samples to the high-forecastability regime. Second, the use of alternative window sizes (four or six years) or alternative values of h provides similar results and supports the main conclusion in most cases. Further, the forecastability measure is largely consistent with Evans and Wachtel's (1993) result from a Markov switching model that allows for two inflation processes, a stationary process and a random walk. Their result suggests that the probability of being in the random walk state hovered near 100 percent in the late 1970s and did not fall appreciably until 1985.

Finally, a more direct measure of inflation persistence was sought. For this purpose, an attempt was made to model a potentially nonstationary process for inflation and estimate the fractional root in the inflation process using a method from Phillips (1998). Figure 3 depicts the long memory parameter (d) with two standard error bands from the rolling estimation of the model for the quarterly inflation rate (window size = 20) with the CPI and the PPC. The rolling estimate of d for the 1967–86 period hovers close to a unit root ($d = 1$) or a nonstationary fractional root ($d > 0.5$). The inflation process is stationary with short memory for the first half of the 1960s and the most recent two years of the period. It is marginally stationary but with long memory (for example, $d > 0.3$) for some fraction of the other years, although confidence intervals include some short-memory alternatives for the second half of the 1980s. This measure, although less volatile, is largely consistent with the inflation forecastability index, with a correlation coefficient of 0.40.[19] Table 6 reports the regression results using the rolling estimate of d in place of F_t. The results are similar to the main results. Specifically, the Wald test results suggest that the IFH under high inflation persistence ($d > 0.40$ with the CPI and $d > 0.54$ with the PPC) is strongly rejected with the CPI but weakly rejected with the PPC. When the perfect discrimination scheme is employed, for both the CPI and PPC cases, the IFH under high inflation persistence ($\gamma_2' = -1$) is strongly rejected, whereas the IFH under low inflation persistence ($\alpha_2' = -1$) is not rejected.

Finally, the sensitivity of the results to serial correlation in the residual was checked. The single-regime regression results are affected little by the use of MLE, allowing for serial correlation in errors or the Newey-West correction for serial correlation and heteroscedasticity. If there is serial correlation in the errors, unfortunately, existing procedures are not readily applicable to linearity testing and statistical inference for a threshold parameter. Nonetheless, equations (7) and (9) were estimated by MLE, allowing

[19] The use of the rolling estimate of d presumes that the current state of inflation persistence is determined by the *average* sample characteristic over the most recent five years.

for the first-order serial correlation in the errors following Goldfeld and Quandt (1972, pp. 258-77). Table 7 shows that this procedure yields strong evidence for this paper's argument regarding the IFH, although the significance level is rather higher. The serial correlation coefficient is around 0.5 under the low-forecastability (inflation) regime and nil under the other regime, perhaps indicating a threshold effect in serial correlation as well. Also, since the WLS point estimates under the perfect discrimination scheme are consistent, the *t*-test with the Newey-West correction for serial correlation and heteroscedasticity was applied for the IFH under alternative regimes. Again, the results are little affected.

VI. Discussion

Comparison of Figures 1–3 suggests that inflation tends to be positively related to short-term inflation forecastability (or inflation persistence): e.g., based on CPI, the correlation coefficient between the forecastability index (rolling estimate of fractional root) and lagged inflation is 0.51 (0.57). Friedman (1977) suggests that high inflation leads to greater inflation uncertainty. As Ball and Cecchetti (1990) point out, however, the inflation-uncertainty link depends largely on the forecast horizon. Indeed, high inflation due to policy under a fiduciary monetary system raises long-term uncertainty but may make it easier to predict short-term inflation, as argued by Klein (1975). High inflation in the U.S. during the 1970s and early 1980s raises the variability of inflation (or long-term inflation uncertainty), on the one hand, and may (but does not necessarily) raise short-term uncertainty on the other.[20] Recall that the forecastability measure used in this paper is inversely related to the ratio of short-term uncertainty to long-term uncertainty (see equation (8)). It is highly plausible that long-term uncertainty outweighs short-term uncertainty during high-inflation periods and, hence, that the inflation level is positively related to inflation forecastability.

If economic agents infer inflation to be nonpersistent, as in a martingale difference sequence, they will use the sample mean of inflation under similar situations they have experienced. Conversely, if agents infer inflation to be highly persistent, they will utilize their most recent experience. In most cases, however, agents are not sure about the characteristics of the monetary system in judging whether inflation is persistent or not. They plausibly assign different weights to a stationary regime than to a persistent-inflation regime, looking back for evidence on the degree to which inflation can be forecast under alternative

[20] Fischer (1981) argues that high inflation raises the variability but not necessarily the uncertainty of inflation. According to Ball and Cecchetti (1990), inflation uncertainty pertains to the variance of unanticipated inflation, whereas inflation variability pertains to the variance of inflation. Since high inflation tends to be largely anticipated, unanticipated inflation will not be large relative to inflation, supporting Fischer's argument. Also, Ball and Cecchetti show that inflation has much larger positive effects on inflation uncertainty at long horizons.

regimes. This idea is captured in the use of the smoothing function, so that there can be a continuum of states between two extreme regimes. Also, the forecastability index, measured from the rolling regression of an inflation equation, accounts for agents' learning about how the inflation process evolves over time. Thus, this paper has attempted to mirror the notion that agents, learning from their recent experience, assign different weights to the two regimes on the basis of their judgment, with time-varying confidence, about the inflation process.[21]

Defining the switching variable in terms of inflation forecastability (or persistence) requires a model estimation to construct the switching variable. Defining the switching variable in terms of the inflation level, in contrast, is motivated by the possibility of varying substitutability over regimes. The estimated results reconcile both arguments and do not distinguish one from the other. In the data, periods of high inflation tend to be associated with high forecastability of inflation. Given a high correlation between inflation persistence and the inflation level, agents may use actual inflation as a proxy for (unobservable) inflation persistence.[22] This notion strengthens this paper's main argument that the validity of the IFH depends on inflation persistence rather than the inflation level.

There is a line of research that suggests a link of the inflation effect on interest rates to the level of inflation from a different viewpoint. Azariadis and Smith (1998) provide a theoretical framework that suggests a threshold effect in the relation between inflation and returns. They assume that real balances and bank deposits (or loans) are substitutes in household portfolio (indirect financing for capital investments). Since both assets should earn the same return in equilibrium, the zero nominal return on real balances anchors the rate of return on bank deposits. In a low inflation economy in which credit is not rationed, an increase in inflation leads to lower real rates (and in turn higher capital-labor ratio).[23] In a

[21] One may alternatively consider the possibility that there are more than two regimes. For instance, observations with moderate forecastability may be grouped between the two extreme regimes. The smooth transition treats these observations as a mix of the two regimes, weighted by the distance from each. Thereby the degree to which inflation is reflected in interest rates is a monotonic function of inflation persistence.

[22] Consider Germany, which had significant regulation on money until very recently. In the 1957–97 period, Germany experienced low inflation (−1.3 to 8.5 percent a year) compared with the United States (−1.5 to 15.8 percent a year), and inflation showed no remarkable persistence in most periods. Again, inflation persistence and inflation are positively correlated. In this case we find that linearity testing provides no evidence for nonlinearity in the CS equation.

[23] This is similar to the Mundell-Tobin effect that represents a portfolio substitution effect of inflation on the steady-state capital-labor ratio. However, the Mundell-Tobin effect assumes that *real balances and capital are substitutes* (direct financing for capital investments) so that an increase in inflation increases the portfolio demand for capital and thus the capital-labor ratio, which in turn lowers the real rate of return on assets.

high inflation economy in which credit must be rationed, in contrast, an increase in inflation leads to lower steady-state capital stock. In this approach, whether or not the economy has credit rationing is crucial for the link of the inflation effect on asset returns to the inflation level. Also, its assumption of a constant implicit real rate of return on real balances anchors the nominal interest rate. Using cross-country data, Barnes, Boyd and Smith (1999) find that nominal interest rates are only weakly positively correlated for low-to-moderate inflation economies whereas inflation has a positive and significant effect on nominal interest rates for high inflation economies. This line of research reconciles our framework in the sense that the substitutability between financial assets and money is emphasized if the implicit real rate of return on real balances is approximately constant. However, our framework emphasizes the stochastic process of inflation and its link to asset substitutions, assuming that the implicit real returns to real balances rises with inflation if inflation is persistent.

Finally, an existing strand of the literature examines the direct effect of inflation uncertainty on interest rates. For example, Lahiri, Teigland, and Zaporowski (1988) regress interest rates on the moments of the probability distribution of forecasts constructed from the American Statistical Association–National Bureau of Economic Research survey of the implicit GNP deflator. Others (for example, Makin, 1983) do so on the semiannual Livingston uncertainty (or disagreement) measure, computed as the standard deviation of the inflation forecasts of respondents to the Livingston survey. In contrast, the present study examines how the link between interest rates and inflation is altered by inflation forecastability (persistence) with a threshold effect. Inflation forecastability, in line with the uncertainty concept based on forecast horizons (Ball and Cecchetti, 1990), reflects short-term variability relative to long-term variability in the spirit of Klein (1975).[24]

VII. Conclusions

The IFH, strongly supported by CS's finding using U.S. data for 1953–1978, was confirmed later by Gupta's (1991) finding for the 1968–1985 period. Barth and Bradley (1988), however, report that the hypothesis no longer holds when samples are extended beyond 1978. They attribute this reversal to the crucial dependence of the hypothesis on relatively stable inflation (and moderate regulatory changes), but they do not test directly the effect of inflation or its persistence. The analysis reported in this paper finds that the estimation of the CS equation using quarterly U.S. data for 1953–1997 lends little support to the IFH.

These results, however, do not imply a complete rejection of the hypothesis, since the support for the hypothesis depends on the persistence of inflation. Specifically, agents can

[24] Inflation persistence is not directly related to survey measures of inflation uncertainty, which depend on the short-term uncertainty of inflation, since a longer-term trend in inflation will largely be anticipated.

largely forecast changes in inflation during persistently inflationary periods, and asset substitutions brought about by such changes alter the implicit marginal return on money and, hence, the nominal interest rate. As a result, the IFH will receive less support from a sample that substantially includes persistently inflationary periods.

To test the proposition that confines the IFH to the regime of low inflation forecastability, this paper explicitly takes into account the idea that the reflection of inflation in interest rates varies with the forecastability or persistence of inflation. We estimated switching regressions with a smooth transition, reflecting the notion that agents assign different weights to the two regimes on the basis of their judgments, with time-varying confidence, about the inflation process. Also estimated were switching regressions with perfect discrimination for the direct test for the IFH under alternative regimes. Analyses using the U.S. data reject linearity in the CS equation and provide new evidence that the IFH is supported by the U.S. data if inflation forecastability is below a certain threshold, but not otherwise. This finding reconciles the argument that the relative importance of the substitutability between money and bonds and that between nominal assets and capital varies with inflation forecastability. It would be interesting to use a demand system analysis (like that of Deaton and Muellbauer, 1980) to test whether the relative importance of the two margins of substitutability varies with inflation forecastability. Such a test is left for future study.

Table 1. Single-Regime Regressions

	Constant Marginal Tax Rate [1]		Time-Varying Marginal Tax Rate [1]	
Parameter	CPI (53:Q2–97:Q3)	PPC (65:Q2–97:Q3)[2]	CPI (53:Q2–82:Q4)	PPC (65:Q2–82:Q4)[2]
OLS				
α_2'	–0.896 (0.037)**	–0.933 (0.045)**	–0.853 (0.044)**	–0.872 (0.070)**
$\bar{R}^2/DW$	0.89 / 1.72	0.79 / 1.67	0.88 / 1.89	0.73 / 1.84
LM(1)[3]	9.37 (0.002)	6.61 (0.010)	8.48 (0.004)	5.40 (0.020)
WLS				
α_2'	–0.928 (0.021)**	–0.957 (0.033)**	–0.905 (0.027)**	–0.904 (0.056)**
$\bar{R}^2/DW$	0.92 / 1.54	0.84 / 1.57	0.91 / 1.65	0.78 / 1.71

Notes: The table presents the results of estimating equation (3) in first-difference form. **, *, and + denote significance at the 0.01, 0.05, and 0.10 level, respectively. Standard errors (in parentheses) for the OLS estimator are based on White's correction for heteroscedasticity.

[1] For the tax-adjusted interest rate, a constant average marginal tax rate ($\theta = 0.3$) is assumed in columns 1 and 2, and the average marginal tax rate (θ_t) from Sahasakul (1986) is used in columns 3 and 4.

[2] The starting date is set the same as those for switching regressions with the forecastability index.

[3] Lagrange multiplier statistic for the null hypothesis that the variance of residuals depends on the lagged nominal interest rate (p-values in parentheses).

Table 2. Tests for Linearity

Switching variable	Test Statistic			
	SupLM	AveLM	ExpLM	$LM(\hat{\tau}_f)$[3]
Forecastability				
index[1]	6.81 (0.012)[2]	6.05 (0.005)	3.08 (0.006)	6.26 (0.012)
CPI	6.26 (0.045)	5.35 (0.024)	2.73 (0.028)	5.31 (0.021)
(53:Q2–97:Q3)	3.06 (0.148)	2.75 (0.086)	1.39 (0.095)	3.00 (0.083)
PPC	2.95 (0.158)	2.66 (0.097)	1.34 (0.106)	2.93 (0.087)
(65:Q2–97:Q3)				
Inflation level				
CPI	14.78 (0.000)	13.75 (0.000)	7.03 (0.000)	15.11 (0.000)
(53:Q2–97:Q3)	14.50 (0.000)	12.46 (0.000)	6.56 (0.000)	13.39 (0.000)
PPC	7.53 (0.011)	5.38 (0.016)	2.91 (0.013)	5.04 (0.019)
(59:Q3–97:Q3)	7.54 (0.015)	5.01 (0.017)	2.79 (0.017)	5.04 (0.025)

Notes: The table reports the results of estimating regressions in first-difference form. A constant average marginal tax rate ($\theta = 0.3$) is assumed for the tax-adjusted interest rate. The switching variable is the forecastability index or lagged inflation.

[1] Measured from the monthly inflation forecast model that includes nine lags of the dependent variable and the three-month Treasury bill rate (two-period-lagged monthly figures).

[2] Figures in the upper row for each statistic are based on the homoscedasticity assumption, and those in the lower row on the heteroscedasticity assumption. The asymptotic p-values (in parentheses) for SupLM, AveLM, and ExpLM are computed from simulations ($J = 1{,}000$) over the grid set ($\#\Gamma = 102$).

[3] This statistic follows a $\chi^2(1)$ distribution under the null hypothesis that the coefficient on inflation is constant across the subsamples grouped by $\hat{\tau}$, which is obtained from the joint estimation of equations (7) and (9) by MLE.

Table 3. Switching Regressions with Smooth Transitions

	Constant Marginal Tax Rate [1]		Time-Varying Marginal Tax Rate [1]	
Parameter	CPI (53:Q2–97:Q3)	PPC (65:Q2–97:Q3)	CPI (53:Q2–82:Q4)	PPC (65:Q2–82:Q4)
Forecastability [2]				
α_2'	−1.021(0.024)**	−1.042 (0.034)**	−1.028 (0.032)**	−1.119 (0.100)**
γ_2'	−0.834 (0.035)**	−0.810 (0.082)**	−0.767 (0.049)**	−0.656 (0.145)**
τ	−0.073 (0.202)	0.392 (0.275)	−0.036 (0.218)	0.167 (0.573)
σ_1	0.255 (0.032)**	0.291 (0.028)**	0.293 (0.037)**	0.433 (0.083)**
σ_2	0.852 (0.088)**	1.165 (0.184)**	0.949 (0.120)**	1.247 (0.340)**
$\bar{R}^2/DW$	0.90 / 1.70	0.79 / 1.68	0.88 / 1.82	0.73 / 1.77
$[\gamma_2' = \alpha_2' = -1]$ [3]	22.7 (0.000)	5.39 (0.067)	22.8 (0.000)	9.56 (0.008)
$[\gamma_2' = -1 \mid \alpha_2' = -1]$	22.3 (0.000)	3.79 (0.051)	22.7 (0.000)	5.13 (0.024)
No. of obs. (low, high)	85, 93	75, 55	60, 59	41, 30
Inflation level		(59:Q3–97:Q3)		(59:Q3–82:Q4)
α_2'	−1.029 (0.021)**	−1.066 (0.037)**	−1.010 (0.032)**	−1.059 (0.055)**
γ_2'	−0.765 (0.051)**	−0.838 (0.070)**	−0.762 (0.050)**	−0.802 (0.079)**
τ	0.493 (0.172)** <5.65%> [4]	0.145 (0.144) <4.60%>	0.157 (0.222) <4.99%>	−0.165 (0.177) <4.20%>
σ_1	0.294 (0.021)**	0.259 (0.028)**	0.300 (0.032)**	0.239 (0.046)**
σ_2	1.078 (0.129)**	1.054 (0.105)**	1.002 (0.128)**	1.063 (0.116)**
$\bar{R}^2/DW$	0.90 / 1.75	0.80 / 1.67	0.88 / 1.83	0.75 / 1.75
$[\gamma_2' = \alpha_2' = -1]$	21.1 (0.000)	5.77 (0.056)	24.2 (0.000)	6.33 (0.042)
$[\gamma_2' = -1 \mid \alpha_2' = -1]$	17.3 (0.000)	2.52 (0.113)	23.6 (0.000)	5.10 (0.024)
No. of obs. (low, high)	135, 43	100,53	77, 42	47, 47

Notes: The table reports results of jointly estimating equations (7) and (9) in first-difference form by MLE. Standard errors are in parentheses. The switching variable is the forecastability index (upper panel) or lagged inflation (lower panel). **, *, and + denote significance at the 0.01, 0.05, and 0.10 level, respectively.

[1] See note 1 to Table 1.

[2] See note 1 to Table 2.

[3] The Wald test for the null hypothesis is indicated in the brackets (p-values are in parentheses).

[4] The threshold value of the (annual) inflation rate (in angled brackets) is converted from the τ estimate.

Table 4. Switching Regressions with Perfect Discriminations

Parameter	Constant Marginal Tax Rate [1] CPI (53:Q2–97:Q3)	Constant Marginal Tax Rate [1] PPC (65:Q2–97:Q3)	Time-Varying Marginal Tax Rate [1] CPI (53:Q2–82:Q4)	Time-Varying Marginal Tax Rate [1] PPC (65:Q2–82:Q4)
Forecastability [2]				
F_1 [3]	9.47 (0.000)	8.15 (0.000)	11.2 (0.000)	6.61 (0.020)
τ	−0.168	0.230	−0.205	−0.386
	[−0.600, 0.092] [4]	[−0.012, 0.689]	[0.532, 0.671]	[n.a.,[5] 0.907]
α_2'	$-1.005(0.032)^{**}$	$-1.036\ (0.049)^{**}$	$-1.006(0.040)^{**}$	$-1.107\ (0.096)^{**}$
γ_2'	$-0.877\ (0.026)^{**}$	$-0.829\ (0.054)^{**}$	$-0.833\ (0.034)^{**}$	$-0.808\ (0.066)^{**}$
$\bar{R}^2/DW$	0.92 / 1.54	0.83 / 1.54	0.91 / 1.64	0.80 / 1.65
t-value for $\alpha_2' = -1$	−0.15 (0.561)	−0.74 (0.770)	−0.15 (0.562)	−1.11 (0.860)
t-value for $\gamma_2' = -1$	4.68 (0.000)	3.16 (0.001)	4.98 (0.000)	2.89 (0.003)
No. of obs. (low, high)	78, 100	69, 61	47, 72	21, 50
Inflation level		(59:Q3–97:Q3)		(59:Q3–82:Q4)
F_1	15.5 (0.000)	15.8 (0.000)	9.70 (0.000)	10.7 (0.000)
τ	0.018 <4.18%> [6]	1.095 <4.32%>	−0.099 <4.58%>	0.759 <4.86%>
	[−0.116, 0.542]	[1.008, 1.257]	[−0.510, 0.955]	[0.690, 0.965]
α_2'	$-0.989\ (0.026)^{**}$	$-1.006\ (0.034)^{**}$	$-0.974\ (0.034)^{**}$	$-1.004\ (0.049)^{**}$
γ_2'	$-0.826\ (0.032)^{**}$	$-0.644\ (0.085)^{**}$	$-0.812\ (0.039)^{**}$	$-0.651\ (0.097)^{**}$
$\bar{R}^2/DW$	0.92 / 1.53	0.85 / 1.45	0.91 / 1.64	0.84 / 1.52
t-value for $\alpha_2' = -1$	0.42 (0.338)	−0.19 (0.575)	0.75 (0.227)	−0.09 (0.535)
t-value for $\gamma_2' = -1$	5.31 (0.000)	4.19 (0.000)	4.75 (0.000)	3.60 (0.001)
No. of obs. (low, high)	114, 64	133, 20	68, 51	74, 20

Notes: The table reports results of estimating equation (7) in first-difference form by WLS. Standard errors are in parentheses. **, *, and $^{+}$denote significance at the 0.01, 0.05, and 0.10 level, respectively.

[1] See note 1 to Table 1.

[2] See note 1 to Table 2.

[3] The statistic is for the likelihood ratio test for the null hypothesis of no threshold effect. P-values in square brackets are obtained from a bootstrap procedure with 1,000 replications of bootstrap samples, following Hansen (1999).

[4] The 90 percent confidence interval for τ is computed using the likelihood ratio statistics for tests on τ (Hansen, 1999).

[5] The lower bound is not available (n.a.) since no statistics on $\tau < -0.386$ are greater than its critical value.

[6] See note 4 to Table 3.

Table 5. Switching Regression Results with Alternative Measures of Timing and Inflation Forecast

Parameter	CS Timing [1] Forecastability index (62:Q3–97:Q3)	CS Timing [1] Inflation level (62:Q3–97:Q3)	Inflation Forecast [2] Monthly forecast with price controls (53:Q2–97:Q3)	Inflation Forecast [2] Quarterly forecast model (53:Q2–97:Q3)
Smooth transition				
α_2'	−0.987 (0.035)**	−1.018 (0.034)**	−1.025 (0.024)**	−1.005 (0.035)**
γ_2'	−0.858 (0.033)**	−0.788 (0.043)**	−0.835 (0.034)**	−0.851(0.032)**
τ	−0.533 (0.199)	0.087 (0.146)	−0.122 (0.181)	−0.112 (0.207)
σ_1	0.216 (0.056)**	0.217 (0.030)**	0.246 (0.030)**	0.461 (0.037)**
σ_2	0.801 (0.070)**	1.023 (0.109)**	0.836 (0.079)**	0.728 (0.067)**
$\bar{R}^2/DW$	0.88 / 1.75	0.89 / 1.71	0.90 / 1.70	0.90 / 1.73
$[\gamma_2' = \alpha_2' = -1]$ [3]	25.3 (0.000)	24.8 (0.000)	24.5 (0.000)	26.5 (0.000)
$[\gamma_2' = -1 \mid \alpha_2' = -1]$	24.1 (0.000)	23.1 (0.000)	23.1 (0.000)	26.5 (0.000)
No. of obs. (low, high)	38, 103	93,48	80,98	83, 95
Perfect discrimination				
F_1 [4]	6.03 (0.010)	8.50 (0.000)	8.52 (0.000)	9.02 (0.000)
τ	−0.363	0.320 <5.79%> [6]	−0.097	0.330
	[−0.918, 0.598] [5]	[−0.703, 0.864]	[−0.885, 0.303]	[−0.242, 0.598]
α_2'	−1.031(0.059)**	−0.969 (0.034)**	−0.997 (0.031)**	−0.983 (0.028)**
γ_2'	−0.873 (0.027)**	−0.826 (0.035)**	−0.877 (0.027)**	−0.860 (0.030)**
$\bar{R}^2/DW$	0.91 / 1.65	0.91 / 1.63	0.92 / 1.53	0.92 / 1.53
t-value for $\alpha_2' = -1$	−0.53	0.91	0.09	0.60
t-value for $\gamma_2' = -1$	4.67	4.90	4.58	4.59
No. of obs. (low, high)	53, 88	100, 41	82,96	96, 82

Notes: Results of estimating regressions in first-difference form with the CPI, assuming a constant marginal tax rate (θ = 0.3). Standard errors are in parentheses. In the upper panel, equations (7) and (9) with smooth transition are jointly estimated by MLE. In the lower panel, equation (7) with perfect discrimination is estimated by WLS. **, *, and $^+$ denote significance at the 0.01, 0.05, and 0.10 level, respectively.

[1] For the CS timing, the interest rate is the yield on the first business day of a quarter (data available from 1962:Q3 on), and inflation (π_{t+1}) is the annualized rate in the last month of quarter t. The switching variable is the last month's value in each quarter from the monthly forecastability index, and the inflation level is the last month's inflation in quarter t−1.

[2] The *monthly* forecast model includes nine lags of the dependent variable, the three-month Treasury bill rate (two-month-lagged figures), a dummy variable for the Korean War period, and a step function for the Nixon-era price controls (Gordon, 1990). The *quarterly* forecast model includes four lags of the dependent variable and the one-quarter-lagged three-month Treasury bill rate (h = 0.72).

[3] See note 3 to Table 3.

[4] See note 3 to Table 4.

[5] See note 4 to Table 4.

[6] See note 4 to Table 3.

Table 6. Estimation Results with the Fractional Root Estimate as the Inflation Persistence Measure

Parameter	CPI (53:Q2–97:Q3)	PPC (64:Q2–97:Q3)
Smooth transition		
α_2'	−1.090 (0.020)**	−1.021 (0.033)**
γ_2'	−0.824 (0.028)**	−0.871 (0.068)**
τ <d> [1]	−0.351 (0.157)* <0.396>	0.051 (0.347) <0.543>
σ_1	0.145 (0.023)**	0.311 (0.034)**
σ_2	0.752 (0.058)**	0.993 (0.150)**
$\bar{R}^2/DW$	0.90 / 1.73	0.79 / 1.68
$[\gamma_2' = \alpha_2' = -1]$ [2]	50.8 (0.000)	3.77 (0.152)
$[\gamma_2' = -1 \mid \alpha_2' = -1]$	21.8 (0.000)	3.38 (0.066)
No. of obs. (low, high)	63, 115	68, 67
Perfect discrimination		
F_1 [3]	19.1 (0.000)	19.8 (0.000)
τ <d>	0.350 <0.600> [0.251, 0.701] [4]	0.633 <0.711> [0.393, 0.941]
α_2'	−1.000 (0.026)**	−1.021 (0.042)**
γ_2'	−0.824 (0.031)**	−0.774 (0.063)**
$\bar{R}^2/DW$	0.93 / 1.52	0.85 / 1.55
t-value for $\alpha_2' = -1$	−0.01	−0.50
t-value for $\gamma_2' = -1$	5.67 **	3.61**
No. of obs. (low, high)	110, 68	88, 47

Notes: Results of estimating equations (7) and (9) in first-difference form by MLE. Standard errors are in parentheses. A constant average marginal tax rate ($\theta = 0.3$) is assumed for the tax-adjusted interest rate. The switching variable is the rolling estimate of the fractional root of an inflation process (Phillips, 1998). **, *, and $^{+}$ denote significance at the 0.01, 0.05, and 0.10 level, respectively.

[1] The threshold value of d (in angled brackets) is converted from the τ estimate.

[2] See note 3 to Table 3.

[3] See note 3 to Table 4.

[4] See note 4 to Table 4.

Table 7. Switching Regression Results with Serial Correlation

Parameter	Switching Index: Forecastability (53:Q2–97:Q3)	Forecastability (53:Q2–82:Q4)	Fractional root (53:Q2–97:Q3)	Inflation level (53:Q2–97:Q3)
α_2'	−1.027 (0.016)**	−1.042 (0.024)**	−1.057 (0.012)**	−1.035 (0.019)**
γ_2'	−0.893 (0.036)**	−0.820 (0.049)**	−0.843 (0.036)**	−0.786 (0.047)**
τ	0.182 (0.145)	0.104 (0.183) < 0.539>[1]	−0.140 (0.143)	0.341 (0.194) <5.16%>[1]
σ_1	0.228 (0.022)**	0.261 (0.033)**	0.143 (0.015)**	0.249 (0.028)**
σ_2	0.951 (0.097)**	0.988 (0.122)**	0.906 (0.088)**	1.008 (0.119)**
φ_1 [2]	0.572 (0.091)**	0.471 (0.125)**	0.715 (0.090)**	0.451 (0.096)**
φ_2 [2]	0.029 (0.118)	−0.037 (0.151)	−0.124 (0.132)	−0.088 (0.141)
$\overline{R}^2$	0.89	0.88	0.90	0.90
$[\gamma_2' = \alpha_2' = -1]$ [3]	8.92 (0.012)	13.5 (0.001)	30.6 (0.000)	20.8 (0.000)
$[\gamma_2' = -1 \mid \alpha_2' = -1]$	5.96 (0.015)	10.4 (0.001)	6.83 (0.009)	15.0 (0.000)
No. of obs. (low, high)	103, 74	62, 56	86, 91	129, 48

Notes: Results of estimating regressions in the first-difference form with the CPI. Switching regressions with smooth transition are estimated by MLE that allows for the first-order serial correlation in errors. Standard errors are in parentheses. **, *, and + denote significance at the 0.01, 0.05, and 0.10 level, respectively. A constant average marginal tax rate (θ = 0.3) is assumed for columns 1, 3, and 4, and a time-varying marginal tax rate is assumed for column 4.

[1] The threshold value of d or inflation (in angled brackets) is converted from the τ estimate.

[2] The serial correlation coefficients are φ_1 and φ_2 under the low forecastability (persistence or inflation) regime and the high forecastability (persistence or inflation) regime, respectively.

[3] See note 3 to Table 3.

Figure 1. Short-Term Real Interest Rates and Inflation

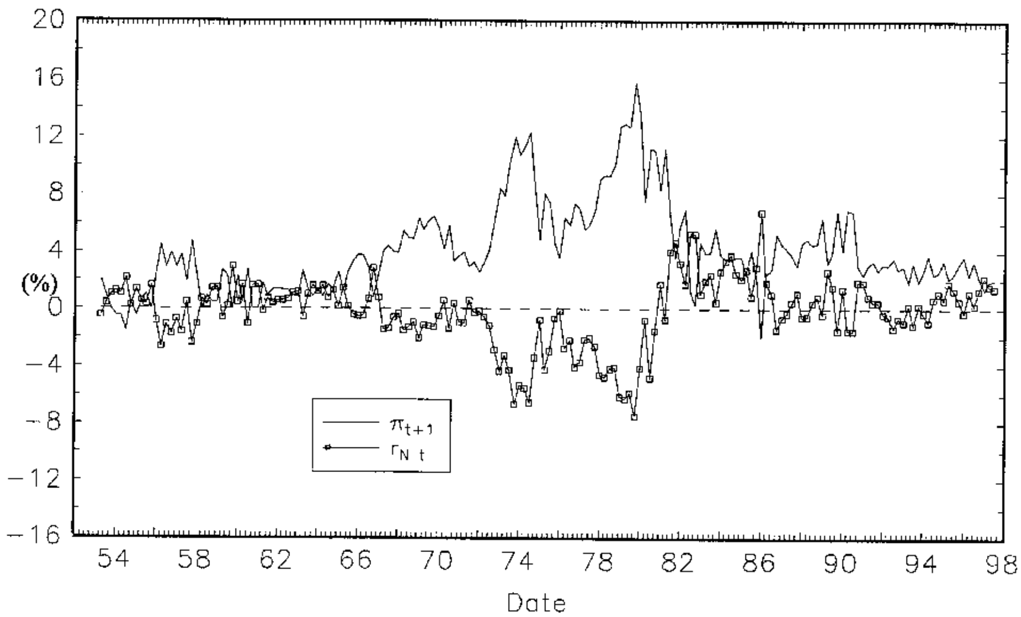


Figure 2. Standardized Forecastability Index and Smoothing Function Values

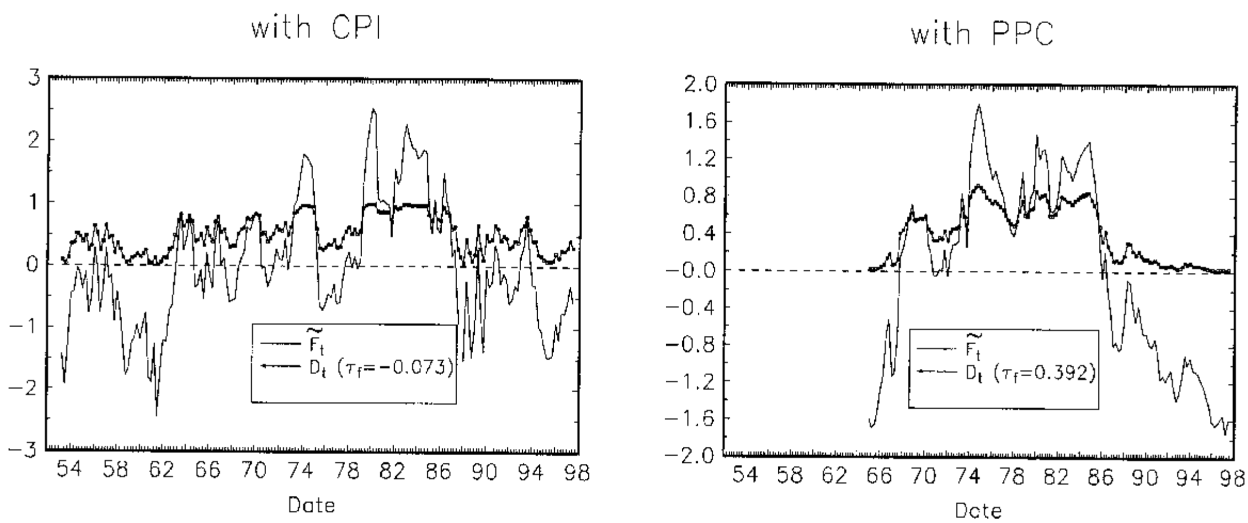
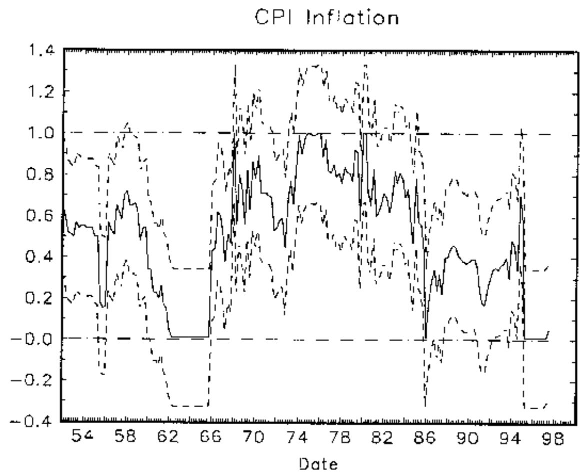
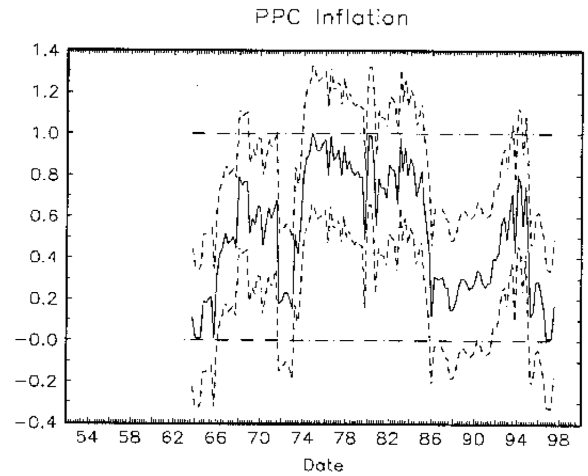


Figure 3. Rolling Estimate of Fractional Root for the Quarterly Inflation

Dashed lines represent two standard error bands





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Linearity Testing

Equation (7) in a different form can be rewritten in a convenient form for testing linearity (Table 2)

$$\Delta r_{Nt} = \alpha_2' \Delta\pi_{t+1} + D_t(\gamma_2' - \alpha_2')\Delta\pi_{t+1} + (1 - D_t)\eta_{1t} + D_t\eta_{2t}\,.$$

A test of linearity tests the null hypothesis of $\alpha_2' = \gamma_2'$. As in Granger and Teräsvirta (1993), the test is carried out in the following steps. First, run the following regression by least squares (LS) $\Delta r_{Nt} = \beta\Delta\pi_{t+1} + e_t$; then compute the residual ($\hat{e}_t = \Delta r_{Nt} - \hat{\beta}\Delta\pi_{t+1}$) and the sum of squared residuals $SSR_0 = \Sigma\hat{e}_t^2$. Next run the following regression by LS: $\hat{e}_t^* = \delta\Delta\pi_{t+1}^* + \lambda D_t \Delta\pi_{t+1}^* + \varsigma_t$, where $\hat{e}_t^* = \hat{e}_t / \sqrt{g_t}$, $\pi_{t+1}^* = \pi_{t+1} / \sqrt{g_t}$ and, given τ, D_t is defined by equation (7). Now define $g_t = 1$ if homoscedastic error is assumed, and $g_t = i_{t-1}$ if heteroscedastic error is assumed. Compute $SSR_1 = \Sigma\hat{\varsigma}_t^2$. Finally, compute the test statistic $LM = T(SSR_0 - SSR_1)/SSR_0$, where T is the number of observations.

Following the idea of Hansen (1996), J realizations of the LM statistics are generated for each grid in the grid set Γ. To do this, we generate (ω_t^j, t=1,…T) i.i.d. $N(0,1)$ random variables; under the assumption of homoscedastic error, we generate the artificial series given by $\Delta\tilde{r}_{Nt}^j = \hat{\beta}\Delta\pi_{t+1} + \omega_t^j\hat{\sigma}_e$, where $\hat{\sigma}_e$ is the standard deviation of the residual $\hat{e}_t$; or, under the assumption of heteroscedastic errors, we generate the artificial series given by $\Delta\tilde{r}_{Nt}^j = \hat{\beta}\Delta\pi_{t+1} + \omega_t^j\{\sqrt{g_t}/\hat{\sigma}_g\}\hat{\sigma}_e$, where $\hat{\sigma}_g$ is the standard deviation of g_t; and the steps in the preceding paragraph are repeated for j = 1, …, J over the grid set. Then we construct empirical distributions for three functionals of the collection of the statistics over the grid set

$$SupLM = \sup_{\tau\in\Gamma} LM(\tau)\,, \qquad AveLM = \tfrac{1}{\#\Gamma}\sum_{\tau\in\Gamma} LM(\tau)\,, \qquad ExpLM = \ln\{\tfrac{1}{\#\Gamma}\sum_{\tau\in\Gamma}\exp(LM(\tau)/2)\}\,,$$

where $\#\Gamma$ is the number of grid points in Γ. The p-value is the percentage of the test statistics derived from the artificial series that exceed the statistics computed from the actual data.

Estimating Switching Regressions with Perfect Discrimination

The switching regression with perfect discrimination in a difference form can be rewritten as

$$\Delta r^{*}_{N\,t} = \alpha_2' \Delta \pi^{*}_{t+1} \cdot I_{(F_t \le \tau)} + \gamma_2' \pi^{*}_{t+1} \cdot I_{(F_t > \tau)} + u^{*}_{t} ,$$

where $\Delta r^{*}_{N\,t} = \Delta r^{*}_{N\,t} / \sqrt{g_t}$, $\pi^{*}_{t+1} = \pi_{t+1} / \sqrt{g_t}$, $I_{(\bullet)}$ is an indicator function, and $g_t = i_{t-1}$. As in Hansen (1999, 2000), testing and estimating of the threshold model proceed as follows. First, the following regression is run by LS: $\Delta r^{*}_{N\,t} = \alpha_2' \Delta \pi^{*}_{t} + u_t$; the residual $\tilde{u}_t = \Delta r^{*}_{N\,t} - \tilde{\alpha}_2' \Delta \pi^{*}_{t+1}$ and the sum of squared residuals $S_0 = \Sigma \tilde{u}_t^2$ are computed. Next, for any given τ, the slope coefficient can be estimated by LS. The residual $\hat{u}^{*}_t(\tau) = \Delta r^{*}_{N\,t} - \hat{\alpha}_2' \Delta \pi^{*}_{t+1} \cdot I_{(F_t \le \tau)} + \hat{\gamma}_2' \pi^{*}_{t+1} \cdot I_{(F_t > \tau)}$ and the sum of squared residuals $S_1(\tau) = \Sigma \hat{u}^{*}_t(\tau)$ are computed. The LS estimate of τ is given by $\hat{\tau} = \arg\min_{\tau \in \Gamma} S_1(\tau)$. Then, the likelihood ratio test for the null hypothesis of no threshold effect is based on $F_1 = (T-1)[S_0 - S_1(\hat{\tau})]/S_1(\hat{\tau})$. Since the asymptotic distribution of F_1 is nonstandard, asymptotically valid *p*-values are constructed by a bootstrap procedure as described in Hansen (1996, 1999). Finally, to test the null hypothesis of $\tau = \tau_0$, construct the likelihood ratio statistic to reject for large values of $LR_1(\tau_0)$, where $LR_1(\tau) = (T-1)[S_1(\tau) - S_1(\hat{\tau})]/S_1(\hat{\tau})$. The no-rejection region of τ at confidence level $1-\alpha$ is the set of values of τ such that $LR_1(\tau) \le c_\alpha$, where $c_\alpha = -2\ln(1-\sqrt{1-\alpha})$ (for example, the 10 percent critical value is 6.53).