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Managerial Incentives and Financial Contagion

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Managerial Incentives and Financial Contagion

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Abstract

This paper proposes a framework for comovements of asset prices with seemingly unrelated fundamentals, as an outcome of optimal portfolio strategies by fund managers. In emerging markets, dedicated managers outperforming a benchmark index and global managers maximizing absolute returns lead to systematic interactions between asset prices, without asymmetric information. The model determines optimal portfolio weights, the incidence of relative value strategies, and the systematic deviation of prices from fundamentals with limits to arbitraging this differential. Managerial compensation contracts, optimal at the firm level, may lead to inefficiencies at the macroeconomic level. Conditions are identified when shocks in one emerging market affect others.

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I. Introduction

The phenomenon of financial contagion has achieved considerable attention in both academic and policy circles in recent years. The *tequila* crisis of 1994-95, the Asian crisis of 1997, the Russian default and the collapse of Long Term Capital Management in 1998, the boom and bust related to the Internet bubble in the late 1990s, the response of international markets in the immediate aftermath of September 11, and the run-up to the Argentine debt default in late 2001, all were accompanied by the transmission of financial market volatility across borders. In the case of emerging markets, the prices of assets of countries which were not related through direct macroeconomic links (e.g. trade channels, linked exchange rates, or vulnerability to similar commodity prices) showed comovements in excess of what could be explained through traditional macroeconomic linkages.

The literature on contagion can broadly be classified into its empirical and theoretical strands. The empirical strand has focused on definitions of contagion to account, for instance, for simultaneous increases in volatility which show up as increased correlations, or the impact of common external factors. The theoretical strand has tried to identify the possible channels of contagion, including the herding behavior of investors, the transmission of panic, and automated risk management procedures. This paper will focus on contagion as a transmission of a negative or positive shock to another country where financial markets are not linked by economic fundamentals but affected by the behavior of different types of fund managers to parameters such as index weights, volatility of the returns of the assets, the level of risk aversion, and trading strategies.

Policy approaches to contagion have relied mainly on the argument that informational asymmetry drive excess comovements of prices as investors watch each others' actions and often tend to reinforce each others actions. This has prompted calls for greater disclosure both of market positioning and key financial and economic data from countries vulnerable to contagion. In a related vein, policymakers and researchers have also focused on the role of particular investor groups in driving market prices. Particularly after the Asian crisis, hedge funds and other highly leveraged institutions were the subject of much debate on the causes of contagion and policies were targeted at reducing the sources of contagion from such sources.

A key assumption in much of the literature is that the main financial markets are efficient and that contagion is a deviation from the norm. This could be driven by information asymmetry, market manipulation through size, or the destabilizing effects of leverage. This paper aims to analyze the phenomenon of contagion by showing that the institutional structure of markets can play a significant role in creating market architectures that may lead to contagion.

In particular, the incentives fund managers face can lead to contagion even in a market with no asymmetric information dominated by certain classes of institutional investors—a key feature both of emerging debt markets as well as major equity markets. The different compensation mechanisms of different classes of fund managers, themselves an outcome of optimal principal-agent relationships between fund managers and their clients, are a root cause of deviations of asset prices from what may be the efficient market outcome. This also suggests that asset prices may continue to significantly deviate from underlying "fundamentals" and the behavior of fund managers is optimally guided not just by the fundamentals, but by their expected compensations for taking on risky positions. The paper finds that given the domination of markets by distinct types of portfolio managers, who are distinguished by their mandates and compensation mechanisms, the optimal responses of these investor classes to the same information set and market conditions vary considerably. While groups of investors behave in well-defined ways in response to shocks, the paper finds that the impact on equilibrium market prices and fund managers' rebalancing of their portfolio weights is based on the type of shock and the relative sizes of the two fund manager classes, and the initial conditions in the market.

While this model was motivated by emerging markets, this framework can also be used to analyze comovements in prices in different assets within the borders of the same country, for example between stocks and the bond market. A key conclusion that emerges from this paper is that managerial compensation systems are a key source of distortions in financial markets, and may be the source for long-term deviations of prices from the so-called fundamentals. This also leads to the conclusion that the opportunity to arbitrage away such deviations may be limited for long periods of time, and markets may be over- or undervalued and be perceived as such for extended periods.

The paper considers two types of fund managers—dedicated and opportunistic—in the model. Dedicated managers are compensated based on deviations from an emerging market index and are not allowed to borrow cash or short any asset. Opportunistic managers are compensated based on the absolute return on their portfolio and are allowed to short any asset and borrow cash. First, the optimal weights for each asset for each type of investor are derived. The paper finds that dedicated investors tend to rebalance their portfolios towards the index when asset volatility or their risk aversion increases. It also finds that opportunistic managers decrease the amount of leverage in response to increased asset volatility or increase in risk aversion. Second, the paper derives equilibrium expected asset returns and prices. The paper finds that a demand shock in one asset affects the expected price of the other asset. Specifically, the relative contribution of one type of trader to contagion depends on underlying market conditions.

The paper is structured as follows. The next section provides a brief overview of the literature. Section III presents the basic framework of the paper and discuss features of the demand functions of three types of fund managers. In Section IV, equilibrium prices are calculated and the impacts of changes in parameter values are investigated. Section V offers some concluding thoughts.

II. A Review of the Literature

This paper best fits in the theoretical literature about contagion where the reallocation of assets by investors is not necessarily based on market fundamentals.[2] Calvo and Reinhart (1996) distinguish between fundamentals-based contagion and "true" contagion where channels of potential interconnection are not present (also see Kaminsky and Reinhart, 2000). Contagion is defined as the propagation of a shock to another country's asset when there are no fundamental linkages between the country hit by the shock and the other countries, and the comovement of asset prices across borders is based on the behavior of global investors.

Calvo and Mendoza (2000) suggest that information regarding investments in a portfolio may be expensive and investors may choose to "optimally" mimic market portfolios. They find that financial globalization in an environment of imperfect information may increase contagion where investors face high costs to gather information on market fundamentals and rely on the actions of other investors. Kyle and Xiong (2001) construct a continuous-time model with two risky assets and three types of traders—noise traders, long-term investors, and convergence traders. When convergence traders suffer large capital losses in one market, they liquidate positions in both markets. The liquidation of the portfolio amplifies and transmits the shock from one asset to another. Contagion in their model is generated through the wealth-effect of convergence traders. Kodres and Pritsker (2002) construct a multiple asset model to study contagion through cross-market rebalancing when one country faces an idiosyncratic shock. Countries may be weakly linked in terms of macroeconomic risks. They also find that asymmetric information increases a country's vulnerability to contagion. Schinasi and Smith (1999) suggest an alternative view to contagion from those based on market imperfections such as asymmetry of information. They construct a partial equilibrium framework to study different portfolio management rules and rebalancing events and their effects on contagion. They find that a shock to an asset in one country may have effects on risky assets in other countries because of the underlying portfolio management rules and the parameterization of the joint distribution of asset returns. Furthermore, they find that rebalancing may be affected by whether or not the investor is leveraged. Leveraged investors will reduce their exposure to risky assets if the return on the leveraged portfolio is less than the cost of funding.

This paper extends the literature by considering the case where investors optimally rebalance their portfolios based on an idiosyncratic shock to one market in terms of increased volatility and a demand shock to an emerging market asset potentially resulting in contagion. Unlike the previous literature, the focus is on the managerial incentives of fund managers and their role in dampening or exacerbating contagion. Fund managers are often restricted in the amount that they can invest in emerging markets. In addition, they may also be compensated on the relative return on the portfolio to the emerging market index. The paper considers two

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[2] Some of these models discuss herd behavior as a possible explanation. For a general discussion about herd behavior, see Banerjee (1992) and Scharfstein and Stein (1990).

types of international fund managers, dedicated and opportunistic fund managers, which are discussed in detail below.

III. The Model

This paper considers a simple discrete time model with two risky emerging market assets (*A* and *B*) a mature market asset (*Z*), and cash (*M*). The emerging market and mature market assets can be viewed as long-term bonds. There are three types of traders: dedicated emerging market fund managers (investing in only emerging market assets and cash), global opportunistic fund managers (investing in emerging markets and mature markets), and noise traders (local investors). Risk averse managers will attempt to maximize their risk-adjusted compensation.

Local Investors trade in asset *A* or asset *B*, and do so based on conditions in other asset markets in that country. They do not invest outside of their respective country, and hence only choose between asset *A* (or *B*). For purposes of this model, noise traders add a random element to the demand of assets *A* and *B*.

Dedicated fund managers allocate their capital between two risky assets *A* and *B* and a risk-free asset (cash), and can only invest in these assets (their mandate does not allow investing in the mature market asset *Z*). The compensation of dedicated fund managers is tied to the performance of the funds under their management relative to the benchmark index for emerging market assets.[3]

Opportunistic fund managers are allowed to invest in all three assets *A*, *B* and *Z*. While their main investment universe is defined as mature market assets, they have the opportunity to invest in the emerging market asset class to enhance their overall returns. Thus, their decision is whether to invest a small amount of their portfolio in emerging market assets or mature market assets. Opportunistic mangers may either increase or reduce their exposure to assets *A*, *B* and *Z* depending on the relative returns/volatilities of mature and emerging market assets.[4] Asset Z can be interpreted as a risk-free asset such as U.S. Treasuries with fluctuations in secondary market prices. Unlike dedicated managers, opportunistic managers may sell assets short to finance long positions in other assets.[5]

Since comprehensive data on the composition of the investor base is difficult to compile, one has to rely on the evidence presented by international banks who are the main market makers

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[3] Typical benchmarks are the JP Morgan's Emerging Market Bond Index Plus (EMBI+) and EMBI Global indices.

[4] Such investors are often linked to broader indices such as the Lehman Universal or Lehman Aggregate or Salomon's Broad Investment Grade (BIG) index.

[5] The model allows for short selling to examine the behavior of hedge funds as one type of global investor.

for emerging market debt, in gauging the relative size of investor classes. The total sovereign emerging bond market universe investible by international investors is estimated at some $225 billion. While the size of outstanding bond market capitalization is somewhat larger, the above estimates exclude smaller and illiquid sovereign bond issuances, and emerging market corporate issuances of about $100 billion, and others not meeting the criteria for inclusion in the major market indices. Of this pool of available assets, between 40-50 percent is thought to be held by dedicated investors including both emerging market mutual funds as well as emerging market funds managed independently but belonging to a larger family of funds. Hedge funds typically comprise between 10 and 20 percent of the investor base. The remainder is dominated by global investors who either invest in the whole emerging markets index or who selectively and opportunistically "cross over" into emerging markets. Direct retail investors do not form a significant proportion of overall emerging market investor bases.

A. The Investment Horizon

For the purposes of modeling portfolio managers' behavior, this paper considers a time horizon consisting of three periods as below (see figure 1):

$$t = \begin{cases} 0 \\ T \\ T+1. \end{cases}$$

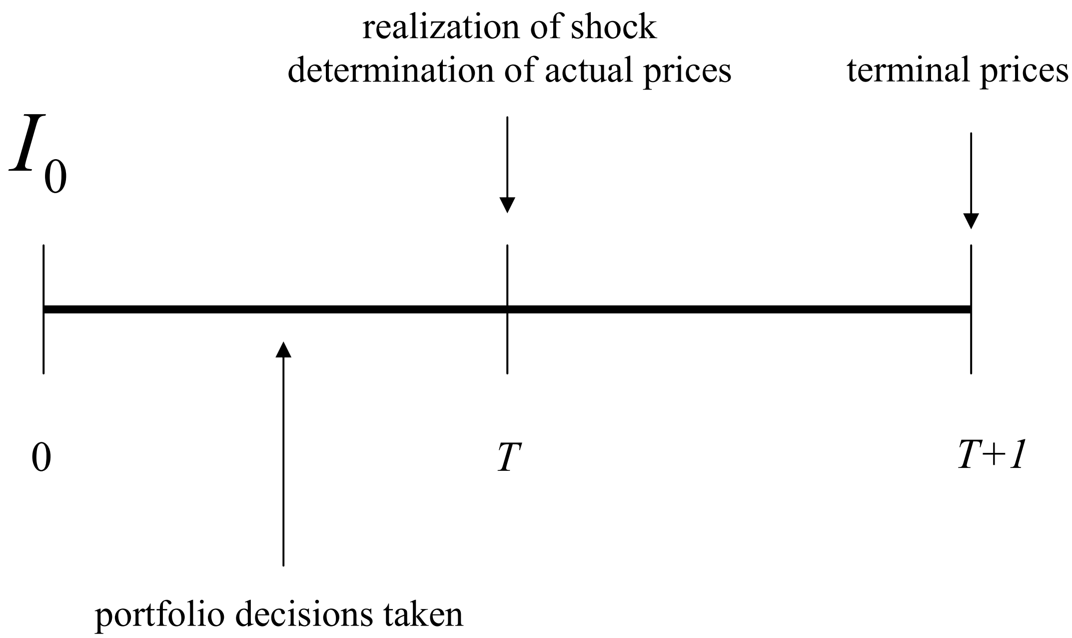
Period 0 is the initial period, where fund managers begin with a certain portfolio allocation, and a certain knowledge of prices and returns, which is an outcome of the previous period's portfolio decisions and shocks. They then update their information set I_0 in this period based on which they form their expectations of the future demand of local investors for each asset, and the variance (distribution) of all assets. Based on that, in a rational expectations framework, they make a decision on their new optimal portfolio, based on expectations of the variables.

Period T is when portfolio managers, based on I_0 and their initial conditions, put in place their new portfolios, and when the realization of the random variable takes place. The actual outcome of equilibrium prices and returns in period T will be the result of the realization of the random variable on the new portfolio positions. These equilibrium prices have to be compared against the prices under alternative scenarios to analyze the dynamics of contagion. Since expected returns are the inverse of prices, as will be shown, the allocation of a proportion of a portfolio to an asset will help determine its price, and hence expected return.[6]

[6] The derived demand curves can be seen as analogous to an auction mechanism wherein investors put in their bids for assets along a price schedule, and depending on the equilibrium price will be allocated a particular amount of the asset.

Period *T+1* is a terminal condition on prices. The terminal condition is significantly beyond the time period focused on in the model. The terminal price is based on asset and economy-wide fundamentals is fixed and known. These assets may be viewed as long-term bonds where the terminal payout is known but the price in secondary markets fluctuates.

Figure 1: The Timeline



The model will be based on the rates of return of various assets, which is the inverse of their prices. The model will determine the total demand for each asset, and set that against a fixed supply of each asset to determine equilibrium prices. Note that the rates of return will be computed as the difference between the equilibrium prices determined in the model and the terminal prices.

B. The Benchmark Index

Let r^I denote the return on the benchmark portfolio in period ($T+1$) as:

$$r^I \equiv \alpha\left(\frac{P_{T+1}^A}{P_T^A}-1\right)+(1-\alpha)\left(\frac{P_{T+1}^B}{P_T^B}-1\right),$$

where:

$$\alpha \in (0,1).$$

As is usually the case, fund managers take α as given exogenously, as the weights of the components of the index are determined by the proprietor of the benchmark index, and are only modified periodically.[7]

C. Local Investors as the Source of Uncertainty

Local investors add uncertainty to the demand (and hence equilibrium prices) of assets A and B. Their demands are given by: [8]

$$\begin{aligned} D^{L,A} &\sim N(0,\sigma_A^2) \\ D^{L,B} &\sim N(0,\sigma_B^2) \end{aligned} \tag{5.1}$$

The market clearing conditions then are as follows:

$$\begin{aligned} D^{D,A} + D^{O,A} + D^{L,A} &= S^A \\ D^{D,B} + D^{O,B} + D^{L,B} &= S^B \end{aligned}$$

Note that the only source of uncertainty is the demand for assets by the local investor, with a fixed supply of an asset, the uncertainty on the equilibrium price will be equal to the uncertainty associated with the demand by the local investor. This will be true for any shape of the aggregate demand curve.[9]

D. Dedicated Fund Manager's Compensation Structure

Let r^D denote the net return on the portfolio held by the index investors from period *T* to period (*T + 1*), where λ is the proportion of their wealth invested in asset *A* and τ is the proportion of their wealth invested in asset *B*, with $(1-\lambda-\tau)$ being the proportion invested in cash. Then, the net return on the dedicated manager's portfolio is:

$$r^D = \lambda\left(r^A\right) + \tau\left(r^B\right) + \left(1-\lambda-\tau\right)(r^M),$$

where:

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[7] An extension of the model could study the effects of changes in benchmark weights in a longer-time horizon model.

[8] For simplicity it is assumed that the both assets share the same distribution properties though not necessarily the same parameters.

[9] This is easy to see diagrammatically.

$$\frac{P_{T+1}^{A}+P_{T}^{A}}{P_{T}^{A}} \equiv r^{A},$$
$$\frac{P_{T+1}^{B}+P_{T}^{B}}{P_{T}^{A}} \equiv r^{B},$$
$$\frac{P_{T+1}^{M}+P_{T}^{M}}{P_{T}^{A}} \equiv r^{M}.$$

Let $r^D - r^I$ denote the total *excess* return of the dedicated fund manager's portfolio at time (t $+1$). The excess return is defined as the return of the managed portfolio over a portfolio which simply tracks the market index. The fund manager's compensation is a fixed proportion k of the excess return she earns for the portfolio, and her utility is increasing in his expected income and decreasing in the variability of his income (with (a) denoting the coefficient of constant absolute risk aversion). Assuming that each fund manager's initial portfolio value is normalized to one, the dedicated fund manager's optimization problem is as follows:

$$\max_{\lambda,\tau} \left\{ kE[U(r^D - r^I)] \right\},$$

where:

$$U(r^D - r^I) = -e^{(-a(r^D - r^I))}$$

and

$$E[U(r^D - r^I)] = -e^{(-a[E[r^D - r^I] - \frac{1}{2} a Var[r^D - r^I])}.$$

The excess return of the portfolio is given by:

$$r^D - r^I = (\lambda - \alpha) r^A + (\tau - 1 + \alpha) r^B + (1 - \lambda - \tau) r^M.$$

Then,

$$E(r^D - r^I) = (\lambda - \alpha) E(r^A) + (\tau - 1 + \alpha) E(r^B) + (1 - \lambda - \tau) r^M$$

and

$$Var(r^D - r^I) = \left((\lambda - \alpha)^2 \sigma_A^2 + (\tau - 1 + \alpha)^2 \sigma_B^2 \right).$$

The return on cash is a known constant r^M. To isolate the effects of index-linked investing on comovement of asset prices, it is assumed that $Cov(r^A, r^B) = 0$, i.e. it is assumed there is nothing inherent in asset prices of *A* and *B* that already has contagion incorporated in it. Maximizing the expected utility of wealth (since the fund manager gets a fixed percentage *k* of the excess returns on the portfolio, he will maximize his utility by maximizing the excess returns on the portfolio) is equivalent to maximizing:

$$E\left(r^D - r^I\right) - \frac{1}{2} a Var\left(r^D - r^I\right).$$

The following function is maximized with respect to λ and τ:

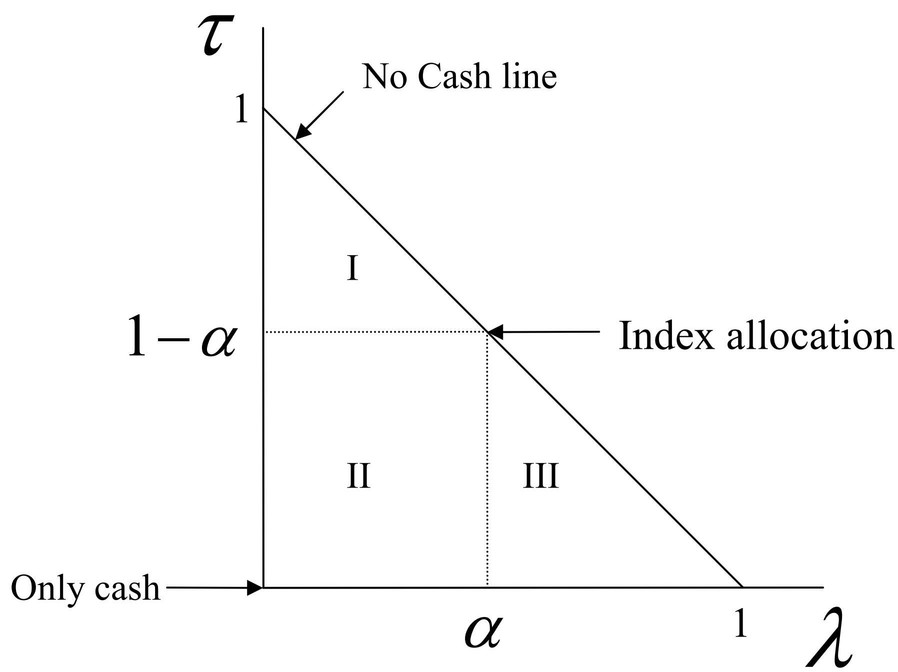
$$\max_{\lambda,\tau} \begin{Bmatrix} (\lambda-\alpha)\left(E(r^A)\right) + (\tau-1+\alpha)\left(E(r^B)\right) + (1-\lambda-\tau)r^M \\ -\frac{a}{2}[(\lambda-\alpha)^2\sigma_A^2 + (\tau-1+\alpha)^2\sigma_B^2] \end{Bmatrix} \qquad (5.2)$$

$$\text{subject to: } \begin{cases} \lambda \geq 0, \\ \tau \geq 0, \\ \lambda + \tau \leq 1. \end{cases}$$

Note that dedicated managers are not allowed to short either asset *A* or *B*, or borrow cash.

The dedicated fund managers' demand space for assets *A* and *B* is diagrammed in Figure 2. When cash holdings are zero, the manager is on the diagonal line. When cash holdings are positive, the manager is below the diagonal line. Because dedicated managers are not allowed to short either asset or borrow cash, their allocations are bounded from below by the λ and τ axes. If the manager is underweight asset *A* but overweight asset *B*, then she will be in the triangle labeled I. If the manager is overweight asset *A* and underweight asset *B*, she will be in the triangle labeled III. If she is underweight both assets she will be in rectangle II. Even if the manager knows an asset is likely to bring negative returns, the compensation and indexation structure results in her holding some amount of the asset under certain conditions as elaborated below.

Figure 2: Dedicated Investor Demand Space



Proposition 1: *The solution of the dedicated fund manager's optimization problem (5.2) is as follows:*[10]

For the region of parameter values where $\dfrac{E(r^A)-r^M}{a\sigma_A^2} \geq 0$ and $\dfrac{E(r^B)-r^M}{a\sigma_B^2} \geq 0$ *, the optimal portfolio weights* (λ^*, τ^*) *are:*

$$\lambda^* = \frac{E(r^A)-E(r^B)}{a\left(\sigma_A^2+\sigma_B^2\right)} + \alpha \quad \textit{and} \quad \tau^* = \frac{E(r^B)-E(r^A)}{a(\sigma_A^2+\sigma_B^2)} + (1-\alpha)$$

For these parameter values, cash holdings are zero. The investor will be along the "no-cash" line in Figure 2 above.

For the region of parameter values where $\dfrac{E(r^A)-r^M}{a\sigma_A^2} < 0$ and/or $\dfrac{E(r^B)-r^M}{a\sigma_B^2} < 0$*, the optimal portfolio weights* $(\lambda^{**}, \tau^{**})$ *are:*

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[10] All derivations and proofs of propositions appear in the appendix.

$$\lambda^{**} = \begin{cases} \dfrac{E(r^A) - r^M}{a\sigma_A^2} + \alpha, \text{ whenever } \dfrac{E(r^A) - r^M}{a\sigma_A^2} + \alpha > 0 \\ \\ 0, \text{ whenever } \dfrac{E(r^A) - r^M}{a\sigma_A^2} + \alpha \leq 0 \end{cases}$$

and

$$\tau^{**} = \begin{cases} \dfrac{E(r^B) - r^M}{a\sigma_B^2} + (1-\alpha), \text{ whenever } \dfrac{E(r^B) - r^M}{a\sigma_B^2} + (1-\alpha) \geq 0 \\ \\ 0, \text{ whenever } \dfrac{E(r^B) - r^M}{a\sigma_B^2} + (1-\alpha) < 0 \end{cases}$$

For these parameter values, cash holdings are:

$$(1-\lambda-\delta)^{**} = \begin{cases} \dfrac{r^M - E(r^A)}{a\sigma_A^2} + \dfrac{r^M - E(r^B)}{a\sigma_B^2}, \text{ whenever } 0 < \lambda^{**} < 1 \text{ and } 0 < \tau^{**} < 1, \\ 1 - \dfrac{E(r^A) - r^M}{a\sigma_A^2} \text{-} \alpha, \text{ whenever } 0 < \lambda^{**} < 1 \text{ and } \tau^{**} = 0, \\ 1 - \dfrac{E(r^B) - r^M}{a\sigma_B^2} \text{-(1-}\alpha\text{)}, \text{ whenever } \lambda^{**} = 0 \text{ and } 0 < \tau^{**} < 1, \\ 1, \text{ whenever } \lambda^{**} = 0 \text{ and } \tau^{**} = 0. \end{cases}$$

Proposition 1 demonstrates that the index weights α and $1-\alpha$ are key determinants of a dedicated managers portfolio allocation towards an asset. Other things equal, a country with a greater weight in the index will automatically get a greater allocation of funds in an optimal behavioral framework. Note also that the deviation of the allocation from the index weight is independent of that weight.

Proposition 2 describes the behavior of dedicated managers when one or both emerging market assets underperform cash.

Proposition 2:

Suppose that the risk-adjusted excess return of an emerging market asset underperforms cash:

a) If $E(r^A) > r^M$ *and* $E(r^B) < r^M$ *or* $E(r^B) > r^M$ *and* $E(r^A) < r^M$*, the manager will go overweight asset that outperforms cash. Conversely, if* $E(r^A) < r^M$ *or* $E(r^B) < r^M$ *or both, the manager will be underweight asset A (* $\lambda < \alpha$ *) and/or asset B (* $(1-\lambda) < (1-\alpha)$ *), but will not necessarily hold zero of either asset.*

b) As the weight of asset A in the benchmark index α *rises, a manager who is overweight the asset will increase her exposure further by maintaining the overweight. A manager who is underweight asset A will also increase her exposure, but maintain the underweight.*

c) As the risk aversion coefficient (a) rises, the demand for asset A or B falls, if the manager is overweight the asset. If the manager is underweight the asset, an increase in (a) results in her reducing her underweight position. In other words, a higher degree of risk aversion causes "hugging of the index."

d) As σ_A^2 *or* σ_B^2 *rises, the demand for asset A or B falls if the manager is overweight the asset. If the manager is underweight the asset, as* σ_A^2 *or* σ_B^2 *increases, the manager reduces her underweight. In other words, an increase in* σ_A^2 *or* σ_B^2 *results in greater "hugging of the index."*

e) An increase in (a), σ_A^2 *or* σ_B^2*, may reduce the demand for cash and greater hugging of the index.*

Proposition 2 states that dedicated managers may hold positive values of an emerging market asset even when it underperforms cash. Intuitively, it is easy to see that while lower weights to an asset with lower returns than cash would increase utility, the low weight relative to the benchmark increases the risk of underperforming the index and hence lowering utility. For some ranges, the return element dominates and hence a zero allocation may be optimal, but in other ranges, the risk element dominates leading to a positive allocation.

This result can be easily generalized to more than two emerging market assets. When the dedicated manager rebalances her portfolio weights closer to the index, the demand for all assets where she was underweight will increase and the demand for all the assets where she was overweight will decrease. Thus, the behavioral characteristics of the dedicated investor results in linkages between otherwise unrelated markets based on whether the portfolio weight is greater or less than the market index.

Proposition 2 also states that dedicated managers tend to hug the index more closely when volatility of returns on emerging market assets and risk aversion increase. If the manager is underweight an asset and the volatility of that asset increases, she will increase her holdings of that asset. Interestingly, dedicated managers reduce their cash holdings when volatility and risk aversion increase.

The model next considers the case when both emerging market assets outperform cash.

Proposition 3:

Considering the case when $\lambda + \tau = 1$.

a) The dedicated manager is overweight the asset with the higher expected return and is underweight the asset with the lower expected return.

b) An increase in risk aversion coefficient (a) would result in "hugging of the index" or allocations closer to the index. If the manager is underweight an asset, an increase in (a) would result in the dedicated manager increasing her exposure of that asset and decreasing her exposure of the other asset. Similarly, if the dedicated manager is overweight an asset an increase in (a) would result in a decrease in exposure of that asset and an increase in exposure of the other asset.

c) An increase in σ_A^2 *or* σ_B^2 *reduces the size of the overweight/underweight positions as well, forcing the dedicated manager to move closer to the benchmark index.*

When dedicated managers do not hold cash, they increase their holdings of an underweight asset when its volatility increases and decrease their holdings of the other emerging market asset. In other words, an increase in the volatility of an underweight asset results in a decrease in the demand for the other emerging market asset when there are only two assets. If there are more than two assets, the demands for all the underweight assets vis-à-vis the index increase while the demands for all the overweight assets decrease. In this sense, an increase in the volatility of one asset spills over into the demand for the other asset.

Propositions 2 and 3 state that changes in the expected returns, level of risk aversion, and variance of the emerging market assets may lead to changes in the demand for the underlying assets. It is also found that increases in σ_A^2, σ_B^2 or a would result in managers choosing allocations closer to the index.

E. Global Opportunistic Managers

This subsection considers opportunistic fund managers that maximize their expected portfolio value from holding assets A, B, and Z and do not follow any index or benchmark. The global opportunistic fund manager's optimization problem is:

$$\max_{\phi,\delta} jr^O W^O,$$

where r^O is the return on the opportunistic fund manager's portfolio, W^O is the initial wealth of the opportunistic manager, j is the percentage of compensation for the opportunistic manger, ϕ is the proportion allocated to asset A, δ is the proportion allocated to asset B, and $(1-\phi-\delta)$ is the proportion allocated to asset Z. The return on the opportunistic manager's portfolio is:

$$r^O = \phi r^A + \delta r^B + (1-\phi-\delta) r^Z .$$

The return on the mature market index, r^Z, is stochastic and exogenous for the opportunistic manager.[11]

$$E(r^O) = \phi E(r^A) + \delta E(r^B) + (1-\phi-\delta) E(r^Z)$$

and

$$Var(r^O) = \phi^2 \sigma_A^2 + \delta^2 \sigma_B^2 + (1-\phi-\delta)^2 \sigma_Z^2 .$$

As before, it is assumed that all covariance terms are zero. The opportunistic fund manager maximizes the following problem with respect to ϕ and δ:

$$\max_{\phi,\delta} \phi E(r^A) + \delta E(r^B) + (1-\phi-\delta) E(r^Z) - \frac{a}{2}\left[\phi^2 \sigma_A^2 + \delta^2 \sigma_B^2 + (1-\phi-\delta)^2 \sigma_Z^2\right]. \qquad (5.3)$$

Unlike the dedicated manager, the opportunistic manager is allowed to short any asset to finance positions in other assets.

Proposition 4:

The solution of the opportunistic fund manager's optimization problem (5.3) is as follows.

The optimal portfolio weights $(\phi^*, \delta^*, (1-\phi-\delta)^*)$ *are:*

$$\phi^* = \frac{\sigma_Z^2}{U}\left[\frac{E(r^A) - E(r^B)}{a}\right] + \frac{\sigma_B^2}{U}\left[\sigma_Z^2 + \frac{E(r^A) - E(r^Z)}{a}\right],$$

[11] The mature market asset can be interpreted as a return on mature market bonds where the opportunistic investor is a price taker.

$$\delta^* = \frac{\sigma_Z^2}{U}\left[\frac{E(r^B)-E(r^A)}{a}\right] + \frac{\sigma_A^2}{U}\left[\sigma_Z^2 + \frac{E(r^B)-E(r^Z)}{a}\right],$$

and

$$(1-\phi-\delta)^* = 1 - \frac{\sigma_B^2}{U}\left[\sigma_Z^2 + \frac{E(r^A)-E(r^Z)}{a}\right] + \frac{\sigma_A^2}{U}\left[\sigma_Z^2 + \frac{E(r^B)-E(r^Z)}{a}\right],$$

where:

$$U = \sigma_A^2\sigma_B^2 + \sigma_A^2\sigma_Z^2 + \sigma_B^2\sigma_Z^2 .$$

Some behavioral characteristics of opportunistic managers to changes in parameter values are considered next.

Proposition 5:

The opportunistic manager reacts to changes in the underlying parameters in the following ways:

a) The opportunistic manager will hold increasing amounts of an emerging market asset if the expected return on that asset increases. This increase in exposure will come at the expense of her exposure to both the other emerging market asset and the mature market asset.

b) The proportions of reallocation away from the other emerging market asset and from the mature market asset will depend on the relative volatilities of the two assets. If the emerging market asset is more volatile than the mature market asset, then the reduction will be greater for the mature market asset, and vice versa.

c) If $E(r^B) > E(r^A)$ *and* $E(r^M) > E(r^A)$*, the opportunistic manager would short asset A and go long at least one other asset that has higher positive expected returns if:*

$$\sigma_Z^2\left[\frac{E(r^B)-E(r^A)}{a}\right] + \sigma_B^2\left[\frac{E(r^Z)-E(r^A)}{a}\right] > \sigma_B^2\sigma_Z^2 .$$

This is the relative value strategy (also known as the long-short strategy) of hedge funds.

Note that returns do not have to be negative to short the asset, just less than that of the other two.

d) If $E(r^A) > E(r^B)$ *and* $E(r^A) > E(r^M)$*, the opportunistic manager would go long asset A.*

e) If $E(r^A) > E(r^B)$ and $E(r^M) > E(r^A)$, then the manager will short asset A if:

$$\sigma_Z^2\left[E(r^A) - E(r^B)\right] < \sigma_B^2\left[E(r^Z) - E(r^A) - a\sigma_Z^2\right].$$

f) As (a) increases, the opportunistic manager would reduce her exposure to the highest yielding asset, and increase her exposure to the lowest yielding asset.

As can be seen, the opportunistic investor may hold negative quantities (i.e. go short) of both emerging market assets if the expected return on mature market asset is sufficiently high relative to emerging market assets and the product of the volatilities of the other emerging market asset and the mature market asset are sufficiently low. Conversely, the investor may short the mature market asset if emerging market assets offer sufficiently high expected returns. The opportunistic manager may also go long one emerging market asset and go short the other, a strategy commonly employed by relative value hedge funds. Similarly, it is observed that shorting the mature market asset implies taking a leveraged position in emerging markets, with the optimal amount of such leverage given above. In real life, the mature market asset return in such a case would be the cost of borrowing for the hedge fund. Again, the amount of leverage would be endogenous and a function of the cost of leverage. As the cost of leverage rises, overweight positions in emerging markets assets are reduced ceteris paribus, which is consistent with the evidence that a rise in global interest rates induces a selloff in emerging markets often based purely on technical considerations of reduction of leverage in the market.

IV. The Equilibrium

The previous sections derived the optimal behavior of two main classes of fund managers in emerging market bond markets, namely dedicated emerging market managers and global opportunistic managers. Now the equilibrium returns (and implicitly prices) that are derived from the interaction of these two classes of managers are computed.

The supply of asset *A*, (S^A), and asset *B*, (S^B), are known and fixed. $D^{D,A}$ and $D^{D,B}$ denote the dedicated mangers' demand for assets *A* and *B*, respectively. Similarly, $D^{O,A}$ and $D^{O,B}$, denote the opportunistic managers' demand for assets *A* and *B*, respectively, and $D^{L,A}$ and $D^{L,B}$ denote the local investors' demand for assets *A* and *B*, respectively.

For dedicated managers, their compensation mechanism is linked to the performance of their portfolio relative to a benchmark portfolio. Most dedicated investors are benchmarked to either the EMBI+ or the EMBI Global index. In equity markets, they are typically benchmarked to Morgan Stanley Capital International Emerging Markets Free index.

Hedge funds and the proprietary desks of commercial and investment banks act like the global opportunistic managers described above. They essentially are focused on the absolute risk-adjusted returns of their portfolios, and have access to both emerging and mature market assets, and can go long or short assets, thereby allowing significant expansions of their balance sheets. What the model shows is that such managers look at the relative risk-adjusted returns for all assets. The main determining factor for their positioning, including whether to go long or short any asset, is their expected excess return over other assets they can invest in, for given levels of volatilities. Therefore, whether they will treat two emerging market assets similarly or differently will depend on how the returns compare with that of the mature market asset in a three-asset case.

Defining contagion as a comovement of asset prices (and hence returns) in the same direction, and reverse contagion as the offsetting movements (in the opposite direction) of two asset prices, contagion can be analyzed by comparing the returns on the two assets when subject to a shock. The shocks of particular interest are when investor expectations of local traders in a particular country changes and its effect on the expected return on the other emerging market's asset via the trading strategies of cross-border managers.

The impact on emerging market bond prices from the interaction of dedicated and opportunistic managers can be seen from the computation of equilibrium prices. For this, the total demand of assets *A* and *B* from two types of managers is set equal to their respective supplies and compute equilibrium prices. Suppose that there are *n* number of dedicated investors and *q* number of global investors. When dedicated and opportunistic managers along with local investors are present, the market clearing conditions are:

$$S_A = nD^{D,A} + qD^{O,A} + D^{L,A} \tag{6.1}$$

$$S_B = nD^{D,B} + qD^{O,B} + D^{L,B} \tag{6.2}$$

A. Dedicated (Positive Cash Holdings) and Opportunistic Managers

This subsection considers the equilibrium expected returns for assets *A* and *B* when there are dedicated managers that hold cash in their portfolios and opportunistic managers.

Substituting the optimal portfolio allocations to each asset for each type of investor and plugging into (6.1) and (6.2) yields:

$$S_A = n\left[\frac{E(r^A)}{a\sigma_A^2} + \alpha\right] + \frac{q}{aU}\left[\sigma_Z^2\left[E(r^A) - E(r^B)\right] + \sigma_B^2\left[a\sigma_Z^2 + E(r^A) - E(r^Z)\right]\right] + D^{L,A} \tag{6.3}$$

and

$$S_B = n\left[\frac{E(r^B)}{a\sigma_B^2} + (1-\alpha)\right] + \frac{q}{aU}\left[\sigma_Z^2\left[E(r^B) - E(r^A)\right] + \sigma_A^2\left[a\sigma_Z^2 + E(r^B) - E(r^Z)\right]\right] + D^{L,B} \tag{6.4}$$

where:

$$U = \sigma_A^2\sigma_B^2 + \sigma_A^2\sigma_Z^2 + \sigma_B^2\sigma_Z^2 .$$

Rearranging equations (6.3) and (6.4) and solving for $E(r^A)$ and $E(r^B)$, yields :

$$E(r^A) = \frac{\left[\frac{n}{a\sigma_B^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_A^2\right]\right]\left[S_A - D^{L,A} - n\alpha + \frac{q\sigma_B^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a\sigma_A^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_A^2\right]\right] - \left[\frac{q\sigma_Z^2}{aU}\right]^2} + \frac{\left[\frac{q\sigma_Z^2}{aU}\right]\left[S_B - D^{L,B} - n(1-\alpha) + \frac{q\sigma_A^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a\sigma_A^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_A^2\right]\right] - \left[\frac{q\sigma_Z^2}{aU}\right]^2} \tag{6.5}$$

$$E(r^B) = \frac{\left[\frac{n}{a\sigma_A^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_B^2\right]\right]\left[S_B - D^{L,B} - n(1-\alpha) + \frac{q\sigma_A^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a\sigma_A^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_A^2\right]\right] - \left[\frac{q\sigma_Z^2}{aU}\right]^2} + \frac{\left[\frac{q\sigma_Z^2}{aU}\right]\left[S_A - D^{L,A} - n\alpha + \frac{q\sigma_B^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a\sigma_A^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2} + \frac{q}{aU}\left[\sigma_Z^2 + \sigma_A^2\right]\right] - \left[\frac{q\sigma_Z^2}{aU}\right]^2} \tag{6.6}$$

Proposition 6: *If dedicated and opportunistic managers along with local investors comprise the types of investors demanding assets A and B, the effects of changes in the expectations of local investor demand will affect the returns (and prices) of both assets, leading to contagion from one country to another.*

In other words, if local investors are expected to buy assets in country A (or B), portfolio rebalancing will force equilibrium prices of both assets A and B to rise and their expected returns to fall. Conversely, if local investors are expected to sell assets in country A (or B), equilibrium prices of both A and B will fall. This is a simple yet powerful result that shows that local investors in one market can impact prices in assets in countries unrelated through fundamentals, with the propagation of contagion arising purely from the investors in the market.

The model is also able to study the magnitude of each type of manager's contribution to expected prices in the market with the shock and the market without the shock. While the total effect of a reduction in demand of either asset results in a decrease in the price of both assets, the magnitude of the fall in price depends on the type of investor. If q (no opportunistic managers) is equal to zero, equations (6.5) and (6.6) show that neither asset is affected by a change in expected demand of local investors of the other asset. In other words, when at least one emerging market asset underperforms cash, portfolio rebalancing by dedicated managers does not lead to contagion or reverse contagion. However, from equations (6.5) and (6.6), it is observed that the rebalancing of dedicated managers rebalancing from an expected change in the local investors' demand affects the price of that asset more than the opportunistic managers.

The model also predicts that the equilibrium expected price for both assets falls when there is an increase in the expected return of the mature market asset. Intuitively, all else equal an increase in the return of the mature market asset would result in an outflow of emerging market assets. It is observed in equations (6.5) and (6.6) that if q = 0, then a change in the expected return of the mature market asset does not affect the expected price of either asset. While this result is not surprising given that dedicated managers are not allowed to invest in mature market assets, it illustrates that restricting fund managers' set of investments can also have affects in markets that would otherwise be unrelated.

B. Dedicated Manager (Zero Cash Holdings) and Opportunistic Manager

This section examines the equilibrium expected prices when dedicated managers do not hold cash. Substituting the optimal portfolio allocations to each asset for each type of investor and plugging into (6.1) and (6.2) yields:

$$S_A = n\left[\frac{E(r^A)-E(r^B)}{a(\sigma_A^2+\sigma_B^2)}+\alpha\right]+\frac{q}{aU}\left[\sigma_Z^2\left[E(r^A)-E(r^B)\right]+\sigma_B^2\left[a\sigma_Z^2+E(r^A)-E(r^Z)\right]\right]+D^{L,A}$$

and

$$S_B = n\left[\frac{E(r^B)-E(r^A)}{a(\sigma_A^2+\sigma_B^2)}+(1-\alpha)\right]+\frac{q}{aU}\left[\sigma_Z^2\left[E(r^B)-E(r^A)\right]+\sigma_A^2\left[a\sigma_Z^2+E(r^B)-E(r^Z)\right]\right]+D^{L,B}$$

Solving for the expected returns for assets A and B yields:

$$E(r^A) = \frac{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[S_A - D^{L,A} - n(\alpha) + \frac{q\sigma_B^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right] - \left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]^2}$$
$$+ \frac{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]\left[S_B - D^{L,B} - n(1-\alpha) + \frac{q\sigma_A^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right] - \left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]^2} \tag{6.7}$$

$$E(r^B) = \frac{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right]\left[S_B - D^{L,B} - n(1-\alpha) + \frac{q\sigma_A^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right] - \left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]^2}$$
$$+ \frac{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]\left[S_A - D^{L,A} - n(\alpha) + \frac{q\sigma_B^2}{aU}\left[E(r^Z) - a\sigma_Z^2\right]\right]}{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right] - \left[\frac{n}{a(\sigma_A^2+\sigma_B^2)} + \frac{q\sigma_Z^2}{aU}\right]^2} \tag{6.8}$$

Proposition 7: *If dedicated and opportunistic managers along with local investors comprise the types of investors demanding assets A and B, changes in the expectations of local investors demand for an emerging market asset will affect the returns (and prices) of both assets, leading to contagion from one country to another.*

While this result is similar to the previous result, both dedicated managers and opportunistic managers contribute to contagion. The coefficients of the local investor demand of the other asset has *n* and *q* in equations (6.7) and (6.8), implying that both managers portfolio rebalancing results in contagion. Unlike the previous case, the contribution to contagion by the dedicated manager is greater than the opportunistic manager. Furthermore, the impact of changes in the local investor demand of an asset on its own price is affected more by the opportunistic investor.

The equilibrium analysis has shed light on the macroeconomic effects of trading strategies of fund managers. It is seen that underlying relationships between the risk-adjusted expected returns of a set of assets affects the contribution of each type of manager to contagion. The model suggests that it is difficult to isolate a particular type of player that would increase contagion.

V. Conclusion

This paper develops a model for modeling the investment strategies of two main classes of investment managers—dedicated and opportunistic—in emerging markets and their interaction in determining the equilibrium prices of financial assets. It demonstrates that the aggregation of optimal micro-level behavior of fund managers leads to market equilibria that may deviate from what efficient markets may suggest, even in the absence of asymmetric information or regulatory distortions. In particular, assets of countries unrelated by fundamental economic links or even by common external shocks may become related through the channel of managers' optimizing behavior and the trade-offs they face. This suggests that contagion is often linked to the institutional structure of markets.

This paper makes a few key points which are consistent with market practioners' experience in the comovement of asset prices and its link with the investor base. First, different types of investment managers with different investment objectives have differential effects on price dynamics in asset markets even in the absence of informational asymmetries or transactions costs. Second, the presence of incentives for fund managers can lead to the systematic deviation of prices from their long-term fundamentals with no room for arbitraging away the difference. Third, the presence of leveraged investors who can both go long and short has a significant impact on market valuations, as well as on price dynamics as the cost of that leverage increases. Fourth, while common external factors are also shown to have an impact on two emerging market assets, pure contagion arising from noise trading in one country spilling over to another country not linked through macroeconomic fundamentals is an outcome of the optimal behavior of international investors. Fifth, one type of fund manager does not always create more cross-border contagion than another type. The model predicts that both types of managers may contribute to contagion. In sum, this paper concludes that fund managers' compensation and investment systems bear in them the seeds of contagion arising from "technical" factors, and do not eliminate all sources of contagion even in the presence of full information.

The framework of this paper could be applied to other markets dominated by institutional investors, such as markets within one country. For example, the interaction between high-yield fund managers and broader fixed income managers, and between equity managers and comingled stock and bond fund managers, could shed further light on the comovement of seemingly unrelated equity prices or high yield bonds, and their interaction with broader bond market prices.

Policy responses that improve the efficiency and transparency of markets, as well as those that help cope with volatility, will alleviate but may not eliminate the phenomenon of contagion. Areas of future research could focus on the optimal incentive contracts for different classes of fund managers, as well as the optimal construction of market indices as benchmarks for managerial compensation.

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APPENDIX: PROOFS OF PROPOSITIONS

Proof of Proposition 1:

The Lagrangian for the optimization problem for the dedicated investor can be written as follows:

$$L = (\lambda - \alpha)\left(E(r^A)\right) + (\tau - 1 + \alpha)\left(E(r^B)\right) + (1 - \lambda - \tau) r^M$$
$$- \frac{a}{2}[(\lambda - \alpha)^2 \sigma_A^2 + (\tau - 1 + \alpha)^2 \sigma_B^2] + \varphi(1 - \lambda - \tau).$$

Assuming $\lambda > 0$ and differentiating L with respect to λ, yields:

$$\lambda = \frac{E(r^A) - r^M - \varphi}{a\sigma_A^2} + \alpha \ .$$

Assuming $\tau > 0$ and differentiating L with respect to τ, yields:

$$\tau = \frac{E(r^B) - r^M - \varphi}{a\sigma_B^2} + (1 - \alpha)\ .$$

Cash holdings will be:

$$1 - \lambda - \tau = \frac{r^M - E(r^A) + \varphi}{a\sigma_A^2} + \frac{r^M - E(r^B) + \varphi}{a\sigma_B^2}\ .$$

The complementary slackness condition and the non-negativity constraint for the Lagrange multiplier associated with the "no borrowing constraint" are:

$$\varphi(1 - \lambda - \tau) = 0 \text{ and } \varphi \geq 0.$$

Thus, if the constraint does not bind, i.e. $\lambda + \tau < 1$, then the multiplier must be $\varphi = 0$. Alternatively, if the multiplier is positive $\varphi > 0$, the constraint must be binding, i.e. $\lambda + \tau = 1$.

Suppose that $\varphi > 0$ and $\lambda + \tau = 1$. The optimal value of φ can be derived as:

$$\varphi = \frac{(E(r^A) - r^M)\sigma_B^2 + (E(r^B) - r^M)\sigma_A^2}{\sigma_A^2 + \sigma_B^2},$$

which is positive whenever:

$$\frac{E(r^A)-r^M}{a\sigma_A^2}+\frac{E(r^A)-r^M}{a\sigma_B^2}>0\,.$$

Then, solving for the optimal portfolio weights, yields:

$$\lambda^* = \frac{E(r^A)-E(r^B)}{a\left(\sigma_A^2+\sigma_B^2\right)}+\alpha\,,$$

$$\tau^* = \frac{E(r^B)-E(r^A)}{a(\sigma_A^2+\sigma_B^2)}+(1-\alpha)\,.$$

Cash holdings will be zero because $\lambda+\tau=1$.

Now, suppose that $\lambda+\tau<1$ and $\varphi=0$, which is equivalent to:

$$\frac{E(r^A)-r^M}{a\sigma_A^2}+\frac{E(r^B)-r^M}{a\sigma_B^2}<0\,. \tag{6.9}$$

This condition holds only if the expected return on at least one of the emerging market assets is lower than the return on cash. On the other hand, $\lambda>0$ and $\tau>0$ imply that:

$$\frac{E(r^A)-r^M}{a\sigma_A^2}+\alpha>0\,, \tag{6.10}$$

$$\frac{E(r^B)-r^M}{a\sigma_B^2}+(1-\alpha)>0\,. \tag{6.11}$$

When condition (6.9) is satisfied along with conditions (6.10) and (6.11), the optimal portfolio weights are:

$$\lambda^{**} = \frac{E(r^A)-r^M}{a\sigma_A^2}+\alpha\,,$$

$$\tau^{**} = \frac{E(r^B)-r^M}{a\sigma_B^2}+(1-\alpha)\,,$$

$$(1-\lambda-\delta)^{**}=\begin{cases}\dfrac{r^M-E(r^A)}{a\sigma_A^2}+\dfrac{r^M-E(r^B)}{a\sigma_B^2}, \text{ whenever } 0<\lambda^{**}<1 \text{ and } 0<\tau^{**}<1,\\ 1-\dfrac{E(r^A)-r^M}{a\sigma_A^2}\text{-}\alpha, \text{ whenever } 0<\lambda^{**}<1 \text{ and } \tau^{**}=0,\\ 1-\dfrac{E(r^B)-r^M}{a\sigma_B^2}\text{-(1-}\alpha\text{)}, \text{ whenever } \lambda^{**}=0 \text{ and } 0<\tau^{**}<1,\\ 1, \text{ whenever } \lambda^{**}=0 \text{ and } \tau^{**}=0.\end{cases}$$

Finally, one needs to verify that the value of the objective function $V\left(\lambda^{**},\tau^{**}\right)$ is indeed greater than $V(0,0)$ when $\dfrac{E(r^A)-r^M}{a\sigma_A^2}+\alpha>0$ and $\dfrac{E(r^B)-r^M}{a\sigma_B^2}+\left(1-\alpha\right)>0$.

The value of the objective function when $\lambda=0$, $\tau=0$ is:

$$V\left(0,0\right)=r^M-\left(\alpha E(r^A)+\left(1-\alpha\right)E(r^B)\right)-\frac{1}{2}a\left(\left(\alpha\right)^2\sigma_A^2+\left(1-\alpha\right)^2\sigma_B^2\right),$$

and the value of the objective function when $\lambda>0$ and $\beta>0$:

$$V\left(\lambda,\tau\right)=\lambda\left(E(r^A)-r^M\right)+\tau\left(E(r^B)-r^M\right)+r^M-\left(\alpha E(r^A)+\left(1-\alpha\right)E(r^B)\right)$$
$$-\frac{1}{2}a\left(\left(\lambda-\alpha\right)^2\sigma_A^2+\left(\tau-1+\alpha\right)^2\sigma_B^2\right).$$

Note that $V\left(\lambda,\tau\right)>V\left(0,0\right)$ whenever:

$$\lambda\left[\left(E(r^A)-r^M\right)-\frac{1}{2}a\left(\lambda-2\alpha\right)\sigma_A^2\right]+\tau\left[\left(E(r^B)-r^M\right)-\frac{1}{2}a\left(\tau-2(1-\alpha)\right)\sigma_B^2\right]>0.$$

Knowing that $\lambda>0$ and $\tau>0$, then:

$$\left(E(r^A)-r^M\right)-\frac{1}{2}a\left(\lambda-2\alpha\right)\sigma_A^2>0 \text{ whenever } \frac{\left(E(r^A)-r^M\right)}{a\sigma_A^2}+\alpha>\frac{1}{2}\lambda.$$

Plugging in $\lambda^{**}=\dfrac{E(r^A)-r^M}{a\sigma_A^2}+\alpha$ results in $\dfrac{E(r^A)-r^M}{a\sigma_A^2}+\alpha>0$

which holds by assumption.

$\left(E(r_B)-r_M\right)-\frac{1}{2}a\left(\tau-2(1-\alpha)\right)\sigma_B^2>0$, whenever $\frac{E(r^B)-r^M}{a\sigma_B^2}+(1-\alpha)>\frac{1}{2}\tau$

Plugging in $\tau^{**}=\frac{E(r^B)-r^M}{a\sigma_B^2}+\left(1-\alpha\right)$ results in $\frac{E(r^B)-r^M}{a\sigma_B^2}+(1-\alpha)>0$
which holds by assumption.

Proof of Proposition 2:

When at least the return on one emerging market asset is negative, the optimal portfolio weights are:

$$\lambda^*=\frac{E(r^A)-r^M}{a\sigma_A^2}+\alpha\,, \tag{6.12}$$

$$\tau^*=\frac{E(r^B)-r^M}{a\sigma_B^2}+(1-\alpha)\,, \tag{6.13}$$

$$(1-\lambda-\tau)^*=\frac{r^M-E(r^A)}{a\sigma_A^2}+\frac{r^M-E(r^B)}{a\sigma_B^2}\,. \tag{6.14}$$

The behavioral characteristics of dedicated managers to changes in parameter values are summarized as the following:

a. *If $E(r^A)>r^M$ or $E(r^B)>r^M$, the manager will go overweight asset A ($\lambda>\alpha$) or asset B ($(1-\lambda)>(1-\alpha)$), respectively. Conversely, if $E(r^B)<r^M$ or $E(r^B)<r^M$, or both, the manager will be underweight asset A ($\lambda<\alpha$) and/or asset B ($(1-\lambda)<(1-\alpha)$), but will not necessarily hold zero of either asset.*

From equation (6.12), observe that if the $E(r^A)>r^M$ and $E(r^B)<r^M$, $\lambda>\alpha$. If $E(r^A)<r^M$, the dedicated manager holds positive quantities of asset *A* if:

$$-\frac{E(r^A)-r^M}{a\sigma_A^2}<\alpha\,.$$

Similarly, if $E(r^B) > r^M$ and $E(r^A) < r^M$, the dedicated manager is overweight asset *B* ($\tau > (1-\alpha)$), as seen in equation (6.13). If $E(r^B) < r^M$, the dedicated manager holds positive quantities of asset *B* if:

$$-\frac{E(r^B) - r^M}{a\sigma_B^2} < (1-\alpha).$$

b. *As the weight of asset A in the benchmark index α rises, a manager who is overweight the asset will increase her exposure further by maintaining the overweight. A manager who is underweight the asset will also increase her exposure, but maintain the underweight.*

From equation (6.12), if α increases so does λ. If $\lambda > \alpha$, the first term in equation (6.12) is positive. If α increases, the manger increases her holdings of asset *A*. If $\lambda < \alpha$, the first term in equation (6.12) is negative, the manager increases her exposure to asset *A* but $\lambda < \alpha$ still holds. Similarly, an increase in α would lead the manager to decrease her holdings of asset *B* as seen from equation (6.13). If the manager is underweight or overweight asset *B*, the manager maintains the underweight or overweight.

c. *As the risk aversion coefficient (a) rises, the demand for asset A or B falls, if the manager is overweight the asset. If the manager is underweight the asset, an increase in (a) reduces the underweight.*

As (*a*) increases the magnitude of the first term in equations (6.12) and (6.13) decreases confirming that as (*a*) increases, the manager will rebalance her portfolio towards the index.

d. *As σ_A^2 or σ_B^2 rises, the demand for asset A or B falls if the manager is overweight the asset. If the manager is underweight the asset as σ_A^2 or σ_B^2 increases, the manager reduces her underweight.*
From equations (6.12) and (6.13), as σ_A^2 or σ_B^2 increases, the magnitude of the first term decreases confirming that a manager will rebalance her portfolio towards the index.

e. *An increase in (a), σ_A^2 or σ_B^2 ,may reduce the demand for cash resulting in greater hugging of the index.*

From equation (6.14), observe that $(1-\lambda-\tau)$ is only positive when $E(r^A) < r^M$ or $E(r^B) < r^M$. The partial derivative of $(1-\lambda-\tau)$ with respect to (*a*) is:

$$\frac{\partial(1-\lambda-\tau)}{\partial a} = \frac{-(r^M - E(r^A))}{(a\sigma_A^2)^2} + \frac{-(r^M - E(r^B))}{(a\sigma_B^2)^2} < 0, \qquad (6.15)$$

when $E(r^A) < r^M$ and $E(r^B) < r^M$.

Equation (6.15) is also negative when $E(r^A) < r^M$ and:

$$\frac{-(r^M - E(r^A))}{(a\sigma_A^2)^2} > \frac{(r^M - E(r^B))}{(a\sigma_B^2)^2} .$$

Finally, equation (6.15) is negative when $E(r^B) < r^M$ and:

$$\frac{-(r^M - E(r^B))}{(a\sigma_B^2)^2} > \frac{(r^M - E(r^A))}{(a\sigma_A^2)^2} .$$

The partial derivative of $(1-\lambda-\tau)$ with respect to σ_A^2 is:

$$\frac{\partial(1-\lambda-\tau)}{\partial\sigma_A^2} = \frac{-(r^M - E(r^A))}{(a\sigma_A^2)^2} < 0 ,$$

if $E(r^A) < r^M$.

The partial derivative of $(1-\lambda-\tau)$ with respect to σ_B^2 is:

$$\frac{\partial(1-\lambda-\tau)}{\partial\sigma_B^2} = \frac{-(r^M - E(r^B))}{(a\sigma_B^2)^2} < 0 ,$$

if $E(r^B) < r^M$.

Thus, increases in (*a*), σ_A^2 or σ_B^2 may result in a reduction of cash holdings by managers under certain conditions.

Proof of Proposition 3:

When the sum of the risk-adjusted excess returns on emerging markets is positive, the optimal portfolio weights are:

$$\lambda^* = \frac{E(r^A) - E(r^B)}{a\left(\sigma_A^2 + \sigma_B^2\right)} + \alpha , \qquad (6.16)$$

$$\tau^* = \frac{E(r^B) - E(r^A)}{a(\sigma_A^2 + \sigma_B^2)} + (1-\alpha). \qquad (6.17)$$

a. *The dedicated manager overweights the asset with the higher expected return and underweights the asset with the lower expected return.*

If $E(r^A) > E(r^B)$, the first term on the right hand side of equation (6.16) is positive and similarly the first term on the right hand side of equation (6.17) is negative.

b. *An increase in (a) would result in allocations closer to the index. If the manager is underweight an asset, an increase in (a) would result in the manager increasing her exposure of that asset and decreasing her exposure of the other asset. Similarly, if the fund manager is overweight an asset an increase in (a) would result in a decrease in exposure of that asset and an increase in exposure of the other asset.*

In equations (6.16) and (6.17), the first term on the right hand side (the magnitude away from the index) decreases in magnitude implying that the manager would rebalance towards the index allocations.

c. *An increase in σ_A^2 or σ_B^2 reduces the size of the overweight/underweight positions, forcing the manager to move closer to the benchmark index.*

In equations (6.16) and (6.17), the first term on the right hand side decreases in magnitude as σ_A^2 or σ_B^2 increases confirming that managers would rebalance towards the index allocations.

Proof of Proposition 4:

The opportunistic manger solves the following optimization problem:

$$\max_{\phi,\delta} \phi E(r^A) + \delta E(r^B) + (1-\phi-\delta)E(r^Z) - \frac{a}{2}\left[\phi^2\sigma_A^2 + \delta^2\sigma_B^2 + (1-\delta-\phi)^2\sigma_Z^2\right]. \qquad (6.18)$$

The first order conditions for the optimization problem (6.18) with respect to ϕ and δ are:

$$\frac{E(r^A) - E(r^Z)}{a} = \phi\sigma_A^2 + (\phi+\delta-1)\sigma_Z^2$$

and

$$\frac{E(r^B) - E(r^Z)}{a} = \delta\sigma_B^2 + (\phi+\delta-1)\sigma_Z^2.$$

Solving for the optimal portfolio allocations, ϕ^*, δ^*, and $(1-\phi-\delta)^*$ yields:

$$\phi^* = \frac{\sigma_Z^2}{U}\left[\frac{E(r^A)-E(r^B)}{a}\right] + \frac{\sigma_B^2}{U}\left[\sigma_Z^2 + \frac{E(r^A)-E(r^Z)}{a}\right], \tag{6.19}$$

$$\delta^* = \frac{\sigma_Z^2}{U}\left[\frac{E(r^B)-E(r^A)}{a}\right] + \frac{\sigma_A^2}{U}\left[\sigma_Z^2 + \frac{E(r^B)-E(r^Z)}{a}\right], \tag{6.20}$$

and

$$(1-\phi-\delta)^* = 1 - \frac{\sigma_B^2}{U}\left[\sigma_Z^2 + \frac{E(r^A)-E(r^Z)}{a}\right] - \frac{\sigma_A^2}{U}\left[\sigma_Z^2 + \frac{E(r^B)-E(r^Z)}{a}\right], \tag{6.21}$$

where:

$$U = \sigma_A^2\sigma_B^2 + \sigma_A^2\sigma_Z^2 + \sigma_B^2\sigma_Z^2 .$$

Proof of Proposition 5:

a. *The opportunistic manager will hold increasing amounts of an emerging market asset if the expected return on that asset increases.*

The partial derivatives of ϕ and δ with respect to $E(r^A)$ and $E(r^B)$, respectively, are:

$$\frac{\partial\phi}{\partial E(r^A)} = \frac{\sigma_Z^2 + \sigma_B^2}{aU} > 0$$

and

$$\frac{\partial\delta}{\partial E(r^B)} = \frac{\sigma_Z^2 + \sigma_B^2}{aU} > 0,$$

Confirming that as $E(r^A)$ and $E(r^B)$, the manager increases her allocation of that asset in her portfolio.

This increase in exposure will come at the expense of her exposure to the other emerging market asset and the mature market asset.

Let's consider an increase in $E(r^A)$. The partial derivatives of δ and $(1-\phi-\delta)$ with respect to $E(r^A)$, respectively, are:

$$\frac{\partial\delta}{\partial E(r^A)} = \frac{-\sigma_Z^2}{aU} < 0$$

and

$$\frac{\partial(1-\phi-\delta)}{\partial E(r^A)} = \frac{-\sigma_B^2}{aU} < 0,$$

confirming that an increase in $E(r^A)$ will result in the opportunistic manager reducing her allocation to the other two assets.

b. *The proportions of reallocation away from the other emerging market asset and from the mature markets will depend on the relative volatilities of the two assets. If the emerging market asset is more volatile than the mature market asset, then the reduction will be greater for the mature market asset, and vice versa.*

As can be seen from equations (6.19)-(6.21), the coefficient of the terms in the demand function relating to the returns of the assets are σ_A^2, σ_B^2 and σ_Z^2. Suppose for example in equation (6.19) that $E(r^A)$ rises for given returns of other assets. Of the total increase in allocations to *A*, $\frac{\sigma_Z^2}{aU}$ times the change in $E(r^A)$ will come at the expense of asset *B* (as can be seen from equation (6.19)), while $\frac{\sigma_B^2}{aU}$ times the change in $E(r^A)$ will come from asset *Z*. If $\sigma_B^2 > \sigma_Z^2$, then it can be seen that more of the reallocation will be from *Z* and less from *B*. If $\sigma_B^2 = \sigma_Z^2$, the reduction in demand for assets *B* and *Z* will be identical.

c. *If* $E(r^B) > E(r^A)$ and $E(r^M) > E(r^A)$, *the opportunistic manager would short that asset A and go long at least one of the assets with higher expected returns if:*

$$-\sigma_Z^2\left[\frac{E(r^A)-E(r^B)}{a}\right]-\sigma_B^2\left[\frac{E(r^A)-E(r^Z)}{a}\right]>\sigma_B^2\sigma_Z^2 \tag{6.22}$$

Plugging in condition (6.22) into equation (6.19), yields:

$$\phi=\frac{\sigma_Z^2}{U}\left[\frac{E(r^A)-E(r^B)}{a}\right]+\frac{\sigma_B^2}{U}\left[\sigma_Z^2+\frac{E(r^A)-E(r^Z)}{a}\right]<0,$$

confirming that the manager will short the asset when condition (6.22) is satisfied. Part d demonstrates that the opportunistic manager will take a long position.

d. *If* $E(r^A) > E(r^B)$ *and* $E(r^A) > E(r^M)$, *the investor would go long asset A.* Plugging these conditions into equation (6.19), yields

$$\phi=\frac{\sigma_Z^2}{U}\left[\frac{E(r^A)-E(r^B)}{a}\right]+\frac{\sigma_B^2}{U}\left[\sigma_Z^2+\frac{E(r^A)-E(r^Z)}{a}\right]>0,$$

confirming that the manager will be long asset *A*.

If $E(r^A) > E(r^B)$ and $E(r^M) > E(r^A)$, and lower than the mature market asset, then the manager will short asset A if:

$$\sigma_Z^2\left[E(r^A) - E(r^B)\right] < \sigma_B^2\left[E(r^Z) - E(r^A) - a\sigma_Z^2\right].$$

Plugging this condition into equation (6.19), yields:

$$\phi = \frac{\sigma_Z^2}{U}\left[\frac{E(r^A) - E(r^B)}{a}\right] + \frac{\sigma_B^2}{U}\left[\sigma_Z^2 + \frac{E(r^A) - E(r^Z)}{a}\right] < 0,$$

confirming that the manager will be short asset *A*.

Conversely, the manager would be long asset *A* only if:

$$\sigma_Z^2\left[E(r^A) - E(r^B)\right] > \sigma_B^2\left[E(r^Z) - E(r^A) - a\sigma_Z^2\right].$$

e. *As (a) increases, the opportunistic manager would reduce her exposure to the highest yielding asset and increase her exposure to the lowest yielding asset.*

The partial derivative of ϕ with respect to (a) is:

$$\frac{\partial \phi}{\partial a} = -\left[\frac{\sigma_Z^2(E(r^A) - E(r^B))}{(aU)^2}\right] - \left[\frac{\sigma_B^2(E(r^A) - E(r^Z))}{(aU)^2}\right]. \qquad (6.23)$$

Suppose $E(r^A) > E(r^B)$ and $E(r^A) > E(r^Z)$. Now, equation (6.23) will be negative confirming that increases in (a) would result in the manger reducing her holdings of asset *A*. Alternatively, suppose $E(r^B) > E(r^A)$ and $E(r^M) > E(r^A)$. Now, equation (6.23) will be positive confirming that increases in (a) would result in the manger increasing her holdings of asset *A*.

Proof of Proposition 6:

If dedicated and opportunistic managers along with local investors comprise the types of investors demanding assets A and B, the effects of changes in the expectations of local investor demand will affect the returns (and prices) of both assets, leading to contagion from one country to another.

If $\frac{\partial E(r^A)}{\partial(D^{L,A})} < 0$ and $\frac{\partial E(r^B)}{\partial(D^{L,B})} < 0$, a decrease in the expected demand of local investors of a given asset would result in a lower expected price of that asset. This is confirmed from:

$$\frac{\partial E(r^A)}{\partial(D^{L,A})}=\frac{-\left[\frac{n}{a\sigma_B^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_A^2\right]\right]}{\left[\frac{n}{a\sigma_A^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_A^2\right]\right]-\left[\frac{q\sigma_Z^2}{aU}\right]^2}<0$$

$$\frac{\partial E(r^B)}{\partial(D^{L,B})}=\frac{-\left[\frac{n}{a\sigma_A^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_B^2\right]\right]}{\left[\frac{n}{a\sigma_A^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_A^2\right]\right]-\left[\frac{q\sigma_Z^2}{aU}\right]^2}<0$$

Contagion would occur from one market to the other if $\frac{\partial E(r^A)}{\partial(D^{L,B})}<0$ and $\frac{\partial E(r^B)}{\partial(D^{L,A})}<0$. This is confirmed by:

$$\frac{\partial E(r^A)}{\partial(D^{L,B})}=\frac{\partial E(r^B)}{\partial(D^{L,A})}=\frac{-\left[\frac{q\sigma_Z^2}{aU}\right]}{\left[\frac{n}{a\sigma_A^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_B^2\right]\right]\left[\frac{n}{a\sigma_B^2}+\frac{q}{aU}\left[\sigma_Z^2+\sigma_A^2\right]\right]-\left[\frac{q\sigma_Z^2}{aU}\right]^2}<0$$

Proof of Proposition 7:

If dedicated and opportunistic managers along with local investors comprise the types of investors demanding assets A and B, changes in the expectations of local investors demand for an emerging market asset will affect the returns (and prices) of both assets, leading to contagion from one country to another.

Differentiating with respect to the expected return of an asset with respect to the change in local investor demand of the other asset yields:

$$\frac{\partial E(r^A)}{\partial(D^{L,B})}=\frac{\partial E(r^B)}{\partial(D^{L,A})}$$

$$=\frac{-\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)}+\frac{q\sigma_Z^2}{aU}\right]}{\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)}+\frac{q}{aU}(\sigma_Z^2+\sigma_A^2)\right]\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)}+\frac{q}{aU}(\sigma_Z^2+\sigma_B^2)\right]-\left[\frac{n}{a(\sigma_A^2+\sigma_B^2)}+\frac{q\sigma_Z^2}{aU}\right]^2}<0$$