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Price of Risk—Recent Evidence from Large Financials

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Strategy, Policy, and Review Department

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Abstract

Probability of default (PD) measures have been widely used in estimating potential losses of, and contagion among, large financial institutions. In a period of financial stress however, the existing methods to compute PDs and generate loss estimates may vary significantly. This paper discusses three issues that should be taken into account in using PD-based methodologies for loss or contagion analyses: (i) the use of “risk-neutral probabilities” vs. “real-world probabilities;” (ii) the divergence between movements in credit and equity markets during periods of financial stress; and (iii) the assumption of stochastic vs. fixed recovery for financial institutions’ assets. All three elements have nontrivial implications for providing an accurate estimate of default probabilities and associated losses as inputs for setting policies related to large banks in distress.

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I. INTRODUCTION

Measures for the probability of default (PD) of financial institutions have been widely used in estimating potential losses of, and contagion among, large financial institutions.² However, different methodologies used to arrive at such estimates have not necessarily produced uniform results. During the recent financial crisis, two types of PDs (based on CDS spreads and Moody's KMV, respectively) have differed markedly for large banks, and the resulting loss estimates have also varied significantly. In order to properly identify policies with respect to large banks in distress, a closer review of the key differences arising from the various methods to extract PDs is necessary. Indeed, the difficulties in harmonizing the results of the methodologies discussed need to be spelled out, as they potentially bear an effect on authorities' reactions and subsequent policy advice.

These differences start with the underlying market signals used to calculate the PDs. Credit default swap (CDS) spreads providing signals from debt and/or credit markets—given an assumed level of recovery—have been used to arrive at a PD measure. By design, it is risk neutral because it does not take into account investors' varying degrees of risk aversion. Risk neutrality allows us to bypass the need to calibrate a real world measure of investors' utility by assuming that all investors are risk neutral. That is to say, risk neutral methods assign greater probabilities to worse outcomes. PDs derived via the risk neutrality assumption are widely accepted when pricing credit instruments, or assessing the impact of default risk on a portfolio of assets with similarly priced components.

The Moody's KMV methodology, which accounts for investors' risk aversion by extracting signals from equity markets to arrive at a "real world" measure of risk have also been used to extract PDs. In contrast to risk neutral PDs, which use only market prices as inputs, risk measures based on the real world approach also use balance sheet inputs. It is generally accepted that real world measures provide for a better approximation of investors' risk aversion and are as such better suited to carrying out scenario analysis to calculate potential future losses caused by defaults (see Hull, 2009.) Nevertheless, the nature of the inputs used for real world measures also provide for the potential of missing important market signals (especially during distress).

The resulting implication is that losses computed from risk neutral PDs may need to be adjusted downward to arrive at the real world probabilities, while during periods of market stress, the assumptions underlying some of the models yielding real world PDs may become tenuous. The difficulties associated with the transformation of risk neutral PDs to real world PDs are discussed below, along with issues that need to be considered and explored further. In

²Probability of default or distress is used here in a broader context, to include conditional probabilities of default, joint probability of default, distance to distress, and joint default dependence (i.e., via the off-diagonal elements of the distress dependence matrix).

particular, in adjusting the risk neutral probabilities with a conversion factor (“the price of risk”), we explore the importance of: (i) deviation between credit and equity prices during periods of financial market stress; and (ii) the role of the assumption of stochastic vs. fixed recovery for financial institution assets.

II. ADJUSTING PROBABILITIES: THE PRICE OF RISK

The price of risk can be defined as the ratio needed to convert risk-neutral probabilities (associated with CDS spreads) to real world probabilities. The recent literature on this topic converges on the methodology of Amato (2005), which proxies the conversion factor as follows:

$$\text{Price of Risk} = \frac{\text{CDS Spread}}{\text{Equity Market Signal}}$$

An example of an Equity market signal would be taking the Moody’s KMV expected default frequency (EDF) as a real world measure. An enhancement to this would be to proxy the conversion factor by also accounting for the recovery (R) expected at default (40 percent is a common assumptions for R), that is to say,

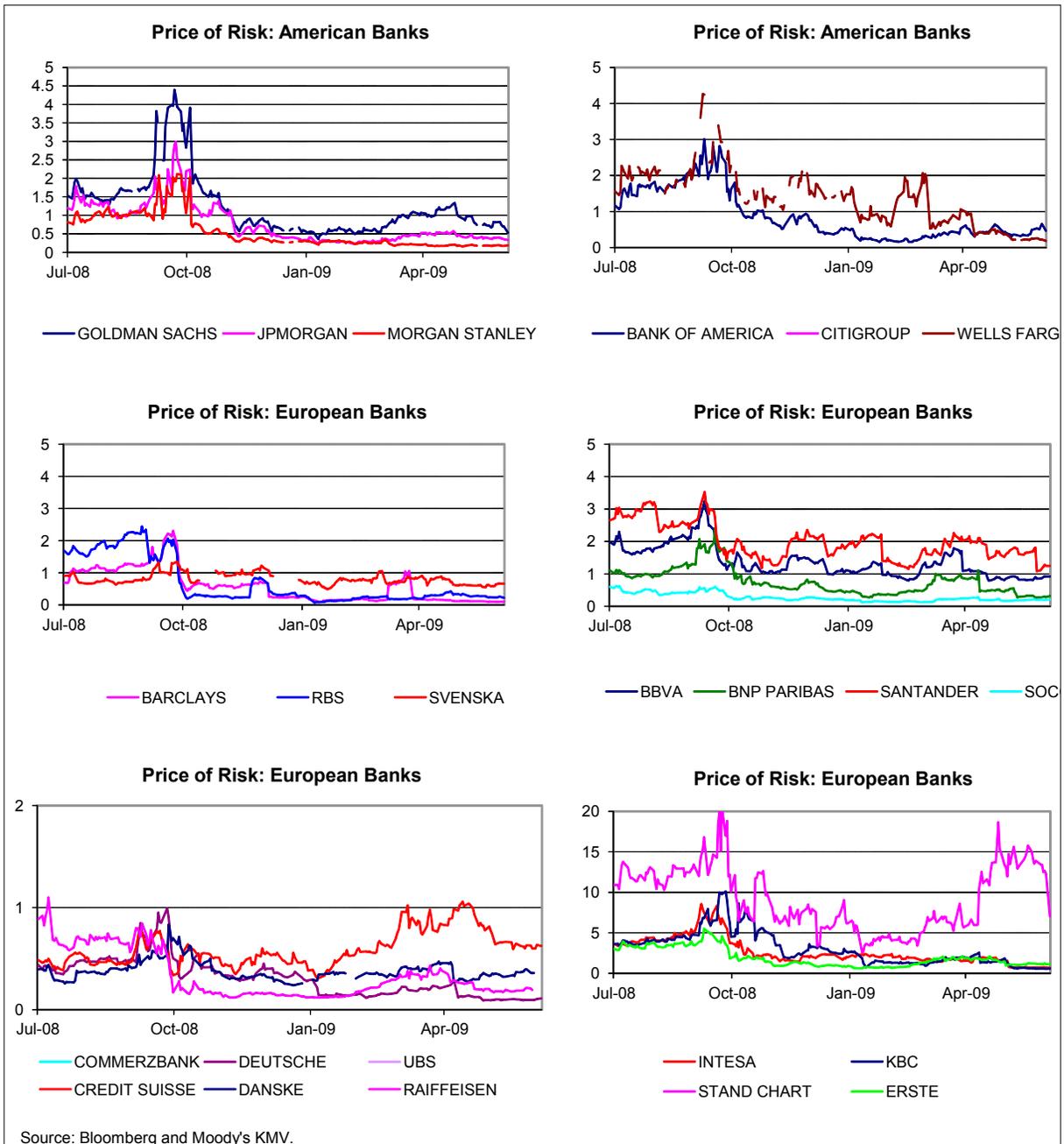
$$\text{Adjusted Price of Risk} = \frac{\text{CDS Spread}}{\text{EDF} (1 - R)}$$

The BIS Quarterly Report (March 2009) uses this approximation to show that the price of risk during the 2007–08 period had fluctuated from an average of about 4 to 12. In other words, risk neutral probabilities derived from CDS spreads would need to be adjusted by a large and significant factor to determine real-world probabilities. For example, if CDS spreads were implying a PD of 0.9 percent, and the associated price of risk conversion factor for a given corporate entity was 10, then the relevant adjusted PD would be 0.09 percent.

The price of risk for large global banks has indeed been sizable and varies across institutions. Our results suggest that at the time of Lehman bankruptcy in September 2008, the price of risk for many large banks was about 5, and for European banks, in particular, higher than 10 in some cases (see Figure 1.)

There have been efforts to use Moody’s model to adjust real world probabilities into risk neutral measures. This is mainly done via the use of the Sharpe Ratio and a correlation coefficient between individual returns and market returns. However, it should be highlighted that this framework assumes that investors treat financial *and* nonfinancial firms in a similar fashion (even during the recent crisis). Additionally, in this framework, the Sharpe Ratio is updated only once a year, which presents an inconsistency with most asset allocation models, especially during the distress periods of 2008. The price of risk approach, which avoids these complications, may better reflect the transformation from risk neutral to the real world probability of default.

Figure 1. Price of Risk

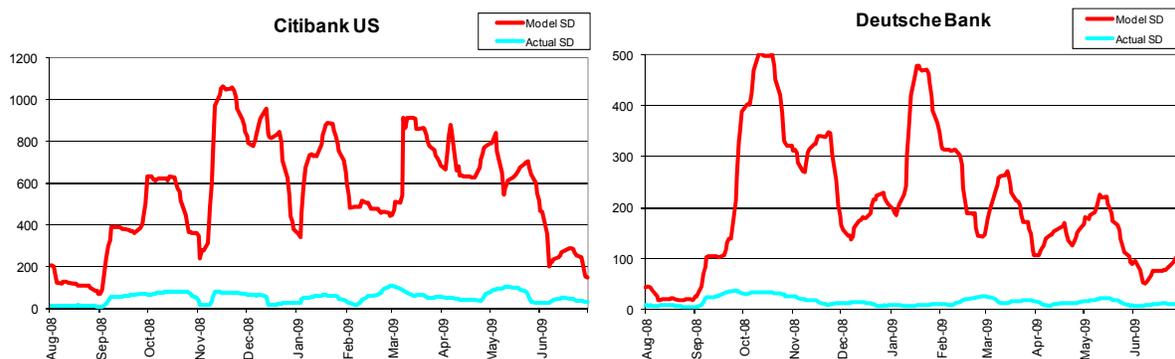


III. DIVERGENCE BETWEEN CREDIT AND EQUITY MARKETS DURING THE RECENT CRISIS

In transforming risk neutral probabilities to real life probabilities, the implicit assumption is the co-movement of equity and bond market signals that drive EDF and CDS spreads, respectively. However, in the case of large banks, the equity market has been far more volatile compared with the bond markets since 2008. In most cases, CDS spreads for the large banks have remained subdued given the perception of “too-large-to-fail.” Compared with nonfinancial firms, such as GM or Chrysler, where bondholders have recently taken a haircut and losses, bondholders of large complex financial institutions have so far been kept whole. As a result, the variations in prospective returns have been reflected more immediately in the equity market, relative to the volatility in their bond prices (see Figure 2 below derived from Bloomberg’s OVCR function.)³ The OVCR function is described in Appendix 1.

The asymmetric signals from debt and equity markets, in turn, have implications for estimating losses or guarantees from the implied balance sheet components. Disentangling the implications of this differential in volatility needs to be considered in probability models. Moreover, the asymmetry in signals from credit and equity markets is important to consider in models using distance-to-distress *where debt and equity market volatilities are important variables in determining the final results* (see Appendix II.) Therefore, from a policy perspective, the estimates of losses and guarantees need to be interpreted with caution when the models do not account for dynamics of these relationships.

Figure 2. Volatility Divergence in Equity and Credit Markets for LCFIs



Sources: Bloomberg and Moody’s KMV.

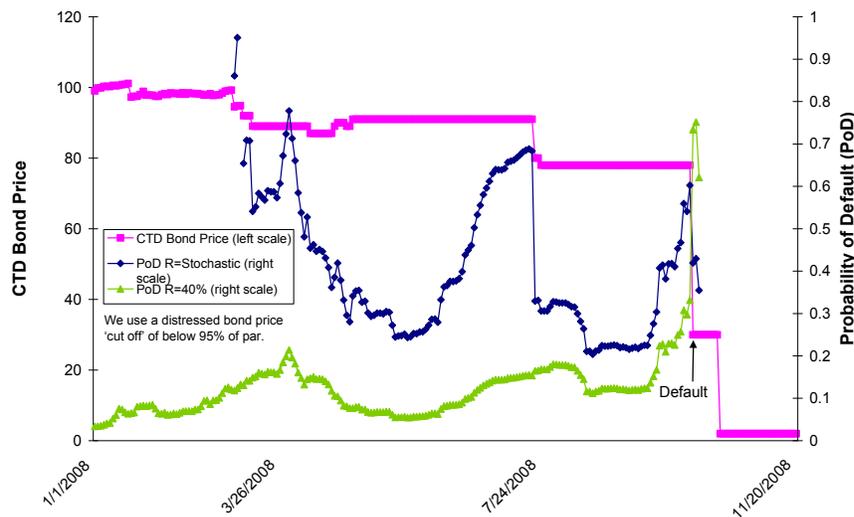
IV. ASSUMPTION BETWEEN FIXED AND VARIABLE RECOVERY

Models estimating PDs have commonly assumed fixed recovery values. However, stochastic recovery value assumptions may be necessary during distress episodes. Unlike sovereigns or

³Bloomberg’s OVCR function (Equity Volatility and Credit Risk) converts equity prices, leverage, and implied volatility to a CDS spread. This ‘theoretical’ equity implied CDS spread can be compared to actual CDS spread.

corporates, financial institutions have few tangible assets, and recovery during the credit crisis was very different from the 40 percent assumption (Lehman and Landsbanki were roughly 8 cents and 1 cent on the dollar respectively, while Fannie Mae and Freddie Mac were both above 90 cents on the dollar). Hence, the use of a time-varying or stochastic recovery rate is all the more important in the case of distressed financial institutions. In cases, where cash bonds trade below par, the cheapest-to-deliver (CTD) bond is a good proxy for stochastic recovery (Duffie, 1999; Singh, 2003, 2004; Singh and Spackman, 2009), as it reflects a more realistic inference of the value of debt obligations than the fixed recovery assumption. Moreover, the use of CTD is also in line with the physical settlement covenants of the ISDA contracts.⁴ In the case of Iceland's Landsbanki Bank, for example, probabilities stemming from using a fixed recovery rate (green line) versus a stochastic recovery (blue line) proxied by cheapest-to-deliver bond are markedly different (Figure 3 below).⁵

Figure 3. Landsbanki, Iceland: PDs from Stochastic and Fixed Recovery Assumptions



Sources: Bloomberg and Fund Staff estimates, and Singh and Spackman (2009).

The use of PDs with a fixed recovery assumption has implications also for assessing institutional interconnectedness through joint probability of distress (JPoD). Inaccurate estimates of conditional probabilities may result if the independent probabilities are biased.

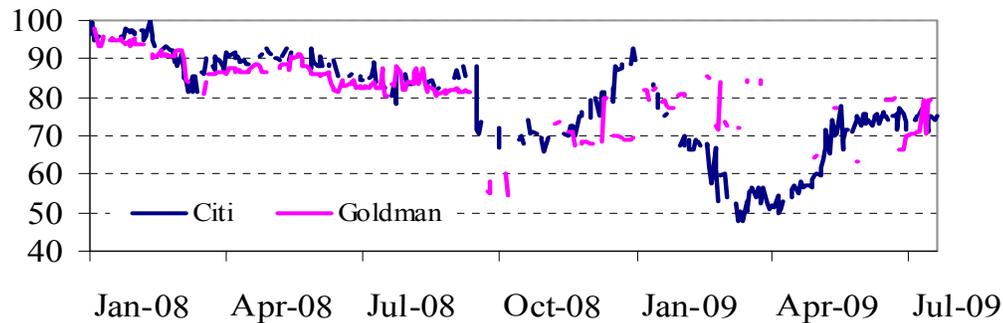
⁴In most models, including those using CDS and Moody's EDF data, the general assumption has been to hold recovery value constant (in the range of 20–40). The probability of default (i.e., the hazard rate) and the recovery value more or less offset each other when bonds trade near par. Such approximation works poorly when bonds trade at high spreads.

⁵To further augment the use of stochastic recovery, the cheapest priced Citi and Goldman bonds illustrate that their bond prices have traded well below par in the recent crisis, despite the implicit forbearance offered to bondholders of large financial institutions, unlike the bondholders of GM, Chrysler or even Fannie Mae and Freddie Mac (Figure 4).

For example, if we are to estimate the probability of default for Goldman conditional on Citi's default, incorrect estimation of the probability for Citi would contaminate the conditional probabilities for Goldman.⁶

Figure 4. U.S. Banks: Citi and Goldman's Bond Prices

Citi and Goldman Cheapest-to-deliver Bonds



Source: Bloomberg.

V. CONCLUSION AND POLICY IMPLICATIONS

This paper has argued that during periods of stress, measures for the probability of default of financial institutions should be adjusted to reflect the price of risk and address potential divergence of credit/equity market relationships and the stochastic nature of asset recovery. Not taking these elements into consideration may result in different, and perhaps misguided, results and policy recommendations.

From a policy angle, until we have a more precise idea of the magnitude of the biases and how best to revise the existing models, loss estimates based on distance-to-distress models should be interpreted with caution for large banks. Also, modeling the degree of interconnectedness of large banks based on joint probabilities of distress should incorporate the low recovery rates observed in the context of the recent credit events that involved large financials, so as to avoid over- or under-estimation of the degree of connectedness.

⁶See IMF Working Paper No. 08/258 (page 14, second paragraph) states: "using CDS data after Lehman's default will require the use of variable recovery value assumption, or in its absence, CTD bonds." There may be other factors such as funding costs during crisis that can contribute towards probability estimates.

Appendix I. Bloomberg's Equity Volatility and Credit Risk (OVCR) Function⁷

Merton (1974) assumes that the value of a firm, A , follows a geometric Brownian motion. Under constant interest rate r assumption, there exists a risk-neutral measure Q under which we can write the dynamics of the firm value as,

$$dA_t / A_t = rdt + \sigma_A dW. \quad (0.1)$$

Merton further assumes that the firm's debt is in the form of a zero-coupon debt with face value, D , and a single fixed expiry date T . At time T , the firm pays off the debt if its firm value is higher than the face value of the debt and claims bankruptcy if its firm value is below the face value of the debt, $A < D$. The equity holder claims the remainder of the firm value.

Under these assumptions, the equity of the firm can be regarded as a call option on the firm value with the strike equal the face value of the debt and the maturity being the maturity of the debt. Since the firm value follows a geometric Brownian motion, we can value the equity of the firm based on the option pricing formula developed by Black and Scholes (1973) and Merton (1973). The time-0 value of the firm's equity, E_0 , can be written as,

$$E_0 = A_0 N(d_1) - De^{-rT} N(d_2), \quad (0.2)$$

with

$$d_1 = \frac{\ln A_0 / D + rT + \frac{1}{2} \sigma_A^2 T}{\sigma_A \sqrt{T}}, \quad d_2 = \frac{\ln A_0 / D + rT - \frac{1}{2} \sigma_A^2 T}{\sigma_A \sqrt{T}}. \quad (0.3)$$

In particular, $N(d_2)$ represents the risk-neutral probability that the call option will finish in the money and hence the firm will not default. Therefore, $1 - N(d_2)$ denotes the risk-neutral probability of default.

In principle, we can use this pricing relation to estimate the default probability, if we are willing to accept the simplifying assumptions of Merton (1974). In reality, there are several more assumptions and choices that we need to make. The equity value can be observed from the stock market if we regard the stock price as the equity value per share and ignore the added value of warrants. The stock return volatility, σ_E , can also be estimated either from the time series data or stock options. Furthermore, the face value of the debt can be obtained from the balance sheet information if we are willing to make further simplifying assumptions regarding the debt structure and their maturities. Nevertheless, the firm value A_0 and the firm volatility σ_A are normally regarded as not observable. These two quantities can be solved from the following two equations,

$$E_0 = A_0 N(d_1) - De^{-rT} N(d_2), \quad \sigma_E = N(d_1) \sigma_A A_0 / E_0, \quad (0.4)$$

where the second equation is a result of Ito's lemma. These two equations contain the two unknowns (A_0, σ_A) if we can obtain estimates on (D, E_0, σ_E, r, T) . Therefore, one can solve for the two unknown quantities using standard numerical nonlinear least square procedures.

⁷See Bloomberg's "Inferring Default Probabilities from Capital Structure Information," version 1.0.

Appendix II. The Moody's KMV Model⁸

The concept of using an option theoretic framework combined with stock prices to estimate default risk was controversial, when first developed. Consider a holding company that owns stock in another company and that the market value of these holdings is V . Further, the company has a debt payment of D due at a fixed point in time, T . Owning the equity of such a holding company is equivalent to owning an option to buy the stock at a price of D with an expiration date of T . Owning the debt is equivalent to owning a risk-free bond that pays D at time T and being short a put option on the stock with an exercise price of D and an expiration date of T . In this example, the firm defaults if the value of assets, V , is below D at the expiration date T . One can use the simple Black-Scholes option formula to determine the value of equity. The four inputs to this equation are the debt payment, D which we refer to as the default point, the market value of the firm's assets, V , the volatility of assets, $A \sigma$, and the risk-free interest rate, r . The probability that the obligations will not be met is a function of the firm's DD, which represents the number of standard deviations that the firm is away from the default point. DD can also be viewed as a volatility-adjusted market-based measure of leverage. As the VK model is a barrier model, the model relates the asset value, the default point and volatility to the default probability via a first passage through time formula. Vasicek has noted that the probability of default for a first passage through time model is approximately equal to:

$$2 \Phi(-DD)$$

where DD is the so-called Distance-to-Default and Φ is the cumulative normal distribution. Distance-to-Default can be defined as:

$$DD(V, X_T, \sigma_A, T, \mu, \delta) = \frac{\log(V/(X_T + aT)) + (\mu - \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}}$$

Where V is the value of a firm's assets, $T X$ is the default point to the horizon, μ is the drift term, $A \sigma$ is the volatility of assets, T is the horizon and a represents cash leakages per unit time due to interest payments, coupons and dividends. The value of the firm's assets and volatility is computed as described above. The default point is computed as current liabilities plus a portion of long-term debt. For longer horizons, a larger portion of long-term debt is included in the default point to reflect that long-term debt becomes more important at longer horizons. Note that the DD varies considerably with the horizon under consideration. At longer horizons, the weight on volatility increases relative to the default point.

⁸See Moody's EDFTM 8.0 Model Enhancements.

Empirically, there is a strong relationship between DD and the observed default rates—firms with a larger DD are less likely to default. Nevertheless, the actual default rate found in the data differs from the literal predictions of the model. Taken literally, the Brownian motion assumption on asset value implies a Gaussian relationship between DD and the EDF credit measure. Specifically, for a DD greater than 4, a Gaussian relationship predicts that defaults will occur 6 out of 100,000 times. This would lead to one half of actual firms being essentially risk-free. This implication is not found in the data. Consequently, when implementing the model, we depart from the Gaussian assumption by implementing an empirical mapping.

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