



IMF Working Paper

Systemic Risk and Asymmetric Responses in the Financial Industry

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Monetary and Capital Markets Department

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Abstract

To date, an operational measure of systemic risk capturing non-linear tail comovement between system-wide and individual bank returns has not yet been developed. This paper proposes an extension of the so-called CoVaR measure that captures the asymmetric response of the banking system to positive and negative shocks to the market-valued balance sheets of individual banks. For the median of our sample of U.S. banks, the relative impact on the system of a fall in individual market value is sevenfold that of an increase. Moreover, the downward bias in systemic risk from ignoring this asymmetric pattern increases with bank size. The conditional tail comovement between the banking system and a top decile bank which is losing market value is 5.4 larger than the unconditional tail comovement versus only 2.2 for banks in the bottom decile. The asymmetric model also produces much better estimates and fitting, and thus improves the capacity to monitor systemic risk. Our results suggest that ignoring asymmetries in tail interdependence may lead to a severe underestimation of systemic risk in a downward market.

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I. Introduction

After the collapse of Lehman Brothers in 2008, many of the largest financial institutions in the U.S. were massively bailed out by the Federal government. The unusual market circumstances led to an unprecedented public intervention fearing that shocks in the subprime mortgage market could spread and take down the whole banking industry. Similar initiatives were later taken by local authorities in many countries around the world, aiming to preserve the integrity of the financial system even at a formidable cost to the public sector and the associated increase in moral hazard. These events highlighted the sheer vulnerability of the global financial system to shocks in large-scale banks and the magnitude of the shockwaves propagated through the financial system into the real economy from the failure or impairment of large financial institutions. As a direct consequence, a revamped international regulatory framework aimed at strengthening global capital and liquidity rules has recently emerged to prevent future episodes of systemic contagion and mitigate moral hazard concerns.

The need to quantify systemic risk and identify the economic drivers of systemic spillovers in the financial industry has motivated a fast growing academic literature in this field; see, among others, Goodhart and Segoviano (2009), Huang *et al.* (2009), Acharya *et al.* (2010), Zhou (2010), Brownlees and Engle (2011), and Diebold and Yilmaz (2011) for recent studies. In this context, Adrian and Brunnermeier (2011) have recently proposed a statistical approach to estimate systemic risk contributions. The so-called CoVaR is a bilateral measure of downside risk that, in the spirit of the well-known Value at Risk (VaR), determines the expected loss of the whole financial sector conditional on the VaR of an individual financial institution. The marginal contribution of a firm to the system, termed ΔCoVaR , is the incremental value of CoVaR conditional on the firm being in distress with respect to the CoVaR conditional on the average or normal state of the firm. The ‘stressed’ and ‘average’ states of nature are quantitatively represented by the $\tau \times 100$ percent and 50 percent VaR levels of the firm, respectively, with the probability τ taking usual values in downside risk analysis.

In this paper, we discuss the suitability of the general modeling strategy implemented in Adrian and Brunnermeier (2011) and propose a direct extension which accounts for nonlinear tail comovements between individual bank returns and financial system returns. Like most VaR models, the CoVaR approach builds on semi-parametric assumptions that characterize the dynamics of the time series of returns. Among others, the procedure requires the specification of the functional form that relates the conditional quantile of the whole financial system to the returns of the individual firm. The model proposed by Adrian and Brunnermeier (2011) assumes that system returns depend linearly on individual returns, so changes in the latter would feed proportionally into the former. This assumption is simple, convenient, and to a large extent facilitates the estimation of the parameters involved and the generation of downside-risk comovement estimates. On the other hand, this structure imposes certain limitations, as it

neglects nonlinear patterns in the propagation of volatility shocks and of perturbations to the risk factors affecting banks' exposures. Both patterns feature distinctively in downside-risk dynamics.

There are strong economic arguments that suggest that the financial system may respond nonlinearly to shocks initiated in a single institution. A sizeable, positive shock in an individual bank is unlikely to generate the same characteristic response (*i.e.*, comovement with the system) in absolute terms than a massive negative shock of the same magnitude, particularly if dealing with large-scale financial institutions.² The disruption to the banking system caused by the failure of a financial institution may occur through direct exposures to the failing institution, through the contraction of financial services provided by the weakening institution (clearing, settlement, custodial or collateral management services), or from a shock to investor confidence that spreads out to sound institutions under adverse selection imperfections (Nier, 2011). Indeed, an extreme idiosyncratic shock in the banking industry, will not only reduce the market value of the stocks affected, but may also spread uncertainty in the system rushing depositors and lending counterparties to withdraw their holdings from performing institutions and across unrelated asset classes, precipitating widespread insolvency. Historical experience suggests that a confidence loss in the soundness of the banking sector takes time to dissipate and may generate devastating effects on the real economy. Bernanke (1983) comes to the conclusion that bank runs were largely responsible of the systemic collapse of the financial industry and the subsequent contagion to the real sectors during the Great Depression. Another channel of contagion in a downward market is through the fire-sales of assets initiated by the stricken institution to restore its capital adequacy, causing valuation losses in firms holding equivalent securities. This mechanism, induced by the generalized collateral lending practices that are prevalent in the wholesale interbank market, can exacerbate price volatility in a crisis situation, as discussed by Brunnermeier and Pedersen (2009). The increased complexity and connectedness of financial institutions can generate 'Black Swan' effects, morphing small perturbations in one part of the financial system into large negative shocks on seemingly unrelated parts of the system. These arguments suggest that the financial system is more sensitive to downside losses than upside gains. In such a case, the linear assumption involved in Adrian and Brunnermeier (2011) would neglect a key aspect of downside risk modeling and lead to underestimate the extent of systemic risk contribution of an individual bank.

We propose a simple extension of this procedure that encompasses the linear functional form as a special case and which, more generally, allows us to capture asymmetric patterns in systemic spillovers. We shall refer to this specification as asymmetric CoVaR in the sequel. This approach retains the tractability of the linear model, which ensures that parameters can readily be

²In microprudential risk management, the reasons explaining this asymmetric pattern are mainly related to how investors who own bank stocks and/or deposits perceive risk. Broadly speaking, stockholders care differently about large downside losses than they do about upside gains, showing greater sensitiveness to reductions in their level of financial wealth. This behavior is consistent with loss aversion or decreasing absolute risk aversion preferences; see, among others, the theoretical models in Gul (1991), Barberis et al. (2001), Berkelaar and Kouwenberg (2009), and the empirical evidence in McQueen and Vorkink (2004), Ang et al. (2005) and Bali et al. (2009).

identified by appropriate techniques, and produces ΔCoVaR estimates which are expected to be more accurate. Furthermore, given the resultant estimates, the existence of nonlinear patterns that motivate the asymmetric model can be addressed formally through a standard Wald test statistic. In this paper, we analyze the suitability of the asymmetric CoVaR in a comprehensive sample of U.S. banks over the period 1990-2010. We find strong statistical evidence suggesting the existence of asymmetric patterns in the marginal contribution of these banks to the systemic risk. Neglecting these nonlinearities gives rise to estimates that systematically underestimate the marginal contribution to systemic risk. Remarkably, the magnitude of the bias is tied to the size of the firm, so that the bigger the company, the greater the underestimation bias. This result is consistent with the too-big-to-fail hypothesis which stresses the need to maintain continuity of the vital economic functions of a large financial institution whose disorderly failure would cause significant disruption to the wider financial system.³ Ignoring the existence of asymmetries would thus lead to conservative estimates of risk contributions, more so in large firms which are more likely to be systemic. Accounting for asymmetries in a simple extension of the model would remove that bias.

The remainder of the paper is organized as follows. Section 2 introduces the main features of the CoVaR framework and discusses the symmetric model suggested by Adrian and Brunnermeier (2011). Section 3 proposes a direct extension of this setting in which the functional form that characterizes the conditional quantile of the banking system is allowed to depend nonlinearly on positive and negative individual returns. Section 4 discusses the main features of the sample data analyzed in this paper. Section 5 presents the main evidence related to the sample fitting of the models and discusses the main implications, demonstrating the outperformance of the asymmetric CoVaR model. Finally, Section 6 summarizes and concludes.

II. Modeling Systemic Risk: CoVaR

In this section, we briefly describe the CoVaR methodology in Adrian and Brunnermeier (2011) (AB henceforth). We start our discussion by firstly introducing notation and relevant variables. Let $X_{t,S}$ and $X_{t,i}$, $t = 1, \dots, T$, be the simple returns of the whole financial system and of the individual institution, respectively. Since our focus is on the impact of the deleveraging process in the financial system, it will be convenient to define these returns as the growth rate of market-valued total assets held by either the whole financial system or each individual institution. Our main aim in Section 5 is to capture tail comovement originated by the propagation of distress associated with a decline in the market value of the assets held by an individual institution, a shock that we dub balance sheet contraction. However, the specific definition of return is not essential for the CoVaR methodology, and the procedure described below applies naturally on the

³As of September 30, 2011, 37 bank holding companies with assets over \$50 billion, held \$4.14 trillion insured deposits accounting for 61 percent of all insured deposits in the United States.

returns of other representative portfolios.

For a certain probability level $\tau \in (0, 1)$, the $\tau \times 100$ percent VaR of a portfolio, denoted $VaR_{\tau,t}^P$, can be defined as the implicit value z that solves $\{\min_{z \in \mathbb{R}} : \Pr(X_{t,P} \leq z) \geq \tau\}$, where $X_{t,P}$ denotes the returns of such portfolio. Typically, τ is a small probability and z is a (negative) value in the left tail of the distribution, so VaR can be seen as the τ -quantile of the loss probability distribution or, equivalently, the $(1 - \tau)$ percent confidence interval of the maximum loss. Paralleling this definition, the CoVaR between the system and an individual institution, here denoted $CoVaR_t^{S|i}(\tau, \tau^*)$, can generally be defined as the τ -quantile of the conditional distribution of $X_{t,S}$ given the event $X_{t,i} = VaR_{\tau^*,t}^i$, *i.e.*, the real value which implicitly solves

$$\min_{z \in \mathbb{R}} \Pr(X_{t,S} \leq z | X_{t,i} = VaR_{\tau^*,t}^i) \geq \tau \quad (1)$$

with $\tau^* \in (0, 1)$ denoting an arbitrary probability associated to the individual VaR level. For a certain small target probability τ related to downside risk events (*e.g.*, $\tau = 0.01$), and setting $\tau^* = \tau$, the marginal contribution of the individual institution to the overall systemic risk, here denoted $\Delta CoVaR_t^{S|i}(\tau)$, is defined in AB as

$$\Delta CoVaR_t^{S|i}(\tau) = CoVaR_t^{S|i}(\tau, \tau) - CoVaR_t^{S|i}(\tau, 0.5) \quad (2)$$

i.e., the vertical distance between the τ -th conditional quantile function of $X_{t,S}$ evaluated at $X_{t,i} = VaR_{\tau,t}^i$ and that evaluated at $X_{t,i} = VaR_{0.5,t}^i$. Under the restriction $\tau^* = \tau$, the CoVaR function is completely characterized by the shortfall probability τ . Hence, we shall simply denote $CoVaR_t^{S|i}(\tau) = CoVaR_t^{S|i}(\tau, \tau)$ for notational convenience.

Since the individual $VaR_{\tau,t}^i$ dynamics are unobservable, in practice the conditioning is made on the sample estimates of this process. This stage is exogenous and any VaR methodology in the vast literature devoted to this field would produce such estimates; see, for instance, McNeil *et al.* (2005) for a review. The crucial point in this procedure is to specify the functional form of $CoVaR_t^{S|i}(\tau)$ as a function of the individual bank' returns, *i.e.*, to characterize the conditional quantile of $X_{t,S}$ as a function of $X_{t,i}$. Let $\mathbf{\Omega}_t = (\mathbf{Z}'_t, X_{t,i})'$ be a set of relevant observable information, with \mathbf{Z}_t denoting a vector of suitable variables. The conditional quantile function of $X_{t,S}$ given $\mathbf{\Omega}_t$, denoted $Q_{X_{t,S}}(\tau | \mathbf{\Omega}_t)$, can generally be represented as $Q_{X_{t,S}}(\tau | \mathbf{\Omega}_t) = f(X_{t,i}, \mathbf{Z}_t; \boldsymbol{\theta}(\tau))$, where $f(\cdot)$ is a measurable function and $\boldsymbol{\theta}(\tau)$ is a vector of unknown parameters possibly depending on τ . Like other features of the conditional distribution of returns, conditional quantiles are not observable directly, so the functional form $f(\cdot)$ that characterizes this process is formally unknown. Different econometric specifications, building on different assumptions, may give rise to fairly different estimates, so it is important to assume a sensible representation. AB assume a linear model characterized by

$$Q_{X_{t,S}}(\tau | \mathbf{\Omega}_t) = \mathbf{Z}'_{t-1} \boldsymbol{\theta}_M(\tau) + \theta_i(\tau) X_{t,i} \quad (3)$$

where \mathbf{Z}_{t-1} is a vector of variables containing predictors of the conditional mean and variance of returns. In this model, the coefficient $\theta_i(\tau) \neq 0$ captures systemic spillovers and measures the

average response of the τ -th quantile given the unconditional distribution of $X_{t,i}$ after controlling for other effects.

Since (3) is linear, the unknown parameters $\boldsymbol{\theta}(\tau) = (\boldsymbol{\theta}'_M(\tau), \theta_i(\tau))'$ can be estimated consistently in a linear quantile regression model under standard assumptions; see Koenker and Bassett (1978) and Koenker (2005). Given the resultant estimates, the $CoVaR_t^{S|i}(\tau)$ function is then predicted as

$$\widehat{CoVaR}_t^{S|i}(\tau) = \mathbf{Z}'_{t-1} \widehat{\boldsymbol{\theta}}_M(\tau) + \widehat{\theta}_i(\tau) \widehat{VaR}_{\tau,t}^i \quad (4)$$

noting that the unknown parameters have been replaced with their consistent estimates and $X_{t,S}$ is evaluated at the (estimated) value at risk level, $\widehat{VaR}_{\tau,t}^i$. The different notation given to the estimates involved emphasizes that CoVaR predictions build on previous forecasts of the individual VaR process.⁴ Then, according to (2), the predicted value of $\Delta CoVaR_t^{S|i}(\tau)$ is given by

$$\begin{aligned} \Delta \widehat{CoVaR}_t^{S|i}(\tau) &= \widehat{CoVaR}_t^{S|i}(\tau) - \widehat{CoVaR}_t^{S|i}(\tau, 0.5) \\ &= \widehat{\theta}_i(\tau) \left[\widehat{VaR}_{\tau,t}^i - \widehat{VaR}_{0.5,t}^i \right] \end{aligned} \quad (5)$$

Some comments are in order. The representation that links $X_{t,S}$ to $X_{t,i}$ is not necessarily causal and may be driven by common latent factors; see AB for a discussion. The lagged state variables control for effects that capture variations in tail-risk not directly related to the financial system risk exposure, while contemporaneous comovements are captured through the $\delta_i(\tau)$ parameter. Hence, this parameter critically determines the contribution of the individual bank to the systemic risk, with $\Delta \widehat{CoVaR}_t^{S|i}(\tau)$ being proportional to this term. Note that two banks may have the same individual downside risk, as measured by $VaR_{\tau,t}$, and nevertheless cause a different impact on the system. For the main aim of this paper, the most striking feature is that the conditional quantile of the system depends linearly on the idiosyncratic returns $X_{t,i}$. This seems particularly restrictive because the magnitude of comovements is most likely to be larger (in absolute terms) in downside periods than in upside times, as discussed previously. In the econometric specification, $\widehat{\theta}_i(\tau)$ captures an average response given the unconditional distribution of $X_{t,i}$. Hence, if comovements are larger when $X_{t,i}$ is negative, then (3) would lead to conservative estimates of the true systemic link, causing downward biased estimates of $\Delta CoVaR_t^{S|i}(\tau)$.

III. Asymmetric CoVaR

We now propose a straightforward generalization of the previous model to accommodate asymmetries that most likely characterize systemic interrelations. Our approach largely builds on

⁴ Given $\widehat{VaR}_{\tau,t}^i$, $\widehat{CoVaR}_t^{S|i}(\tau)$ could be obtained from the quantile regression $Q_{X_{t,S}}(\tau|\boldsymbol{\Omega}_t) = \mathbf{Z}'_{t-1} \boldsymbol{\theta}_M(\tau) + \theta_i(\tau) \widehat{VaR}_{\tau,t}^i$. However, this model introduces measurement errors which may have a nontrivial impact and should be taken care of properly in the estimation. The procedure suggested in AB circumvents this problem.

the AB setting and is a direct extension of this methodology. In the absence of systemic interdependences, the system's return $\{X_{t,S}\}$ could generally be generated according to the following linear factor structure,

$$X_{t,S} = \alpha + \mathbf{M}'_{t-1}\boldsymbol{\gamma} + \sigma_t\eta_t, \quad \eta_t \sim iid(0, 1) \quad (6)$$

where \mathbf{M}_t is a p -vector of conditioning variables that characterize a time-varying expected return, η_t is an independent term with innovations, and σ_t is the volatility process. In practice, \mathbf{M}_t may contain valuation and corporate ratios, bond yields, default premiums, and any other variable that may capture the dynamics of the expected return. If $\boldsymbol{\gamma} = 0$, returns will behave as a martingale difference.

Define the signed processes $X_{t,i}^- = X_{t,i}\mathbb{I}_{(X_{t,i} < 0)}$ and $X_{t,i}^+ = X_{t,i}\mathbb{I}_{(X_{t,i} \geq 0)}$, where $\mathbb{I}_{(\cdot)}$ is the indicator function. A simple and convenient variation of (6) in which the distribution of $X_{t,S}$ may be affected by systemic shocks in the individual asset is then given by

$$X_{t,S} = \alpha + \mathbf{M}'_{t-1}\boldsymbol{\gamma} + \delta_{1,i}X_{t,i}^- + \delta_{2,i}X_{t,i}^+ + \sigma_t\eta_t \quad (7)$$

with $\sigma_t = \sigma_t(X_{t,i}) + \sigma_{t,S}$ such that

$$\sigma_t(X_{t,i}) = \sigma_{0,i} + \sigma_{1,i}X_{t,i}^- + \sigma_{2,i}X_{t,i}^+ \quad (8)$$

and

$$\sigma_{t,S} = \sigma_S + \mathbf{N}'_{t-1}\boldsymbol{\xi} \quad (9)$$

where \mathbf{N}_t denotes a vector of suitable predictors of the conditional volatility, and $(\alpha, \boldsymbol{\gamma}', \delta_{1,i}, \delta_{2,i})'$, $(\sigma_{0,i}, \sigma_{1,i}, \sigma_{2,i})'$, and $(\sigma_S, \boldsymbol{\xi}')$ are unknown parameter vectors characterizing the conditional mean and variance of $X_{t,S}$. Some features of this model are worth discussing in detail.

First, $\delta_{1,i}$ can be seen as a downside beta-type coefficient that captures comovements of the system portfolio with the individual portfolio when this is falling. Reciprocally, $\delta_{2,i}$ captures upside comovements. Consequently, if $(\delta_{1,i}, \delta_{2,i})' > 0$, a sudden change in the individual asset will translate nonlinearly into the system shifting the conditional mean of $X_{t,S}$ in the same direction as $X_{t,i}$, with $\delta_{j,i}$ capturing the average comovement with negative ($j = 1$) and positive ($j = 2$) shocks, respectively. This functional form is reminiscent of the the semi-variance models used, among others, by Hogan and Warren (1974), Bawa and Lindenberg (1977) and Harlow and Rao (1989).⁵ These asset pricing models are theoretically founded in loss-aversion behaviors for which utility increases from gains are not necessarily the same as the utility decreases from losses, as discussed previously; see, for instance, Ang *et al.* (2005) and references therein for a recent survey of this literature. From a more intuitive perspective, it perhaps suffices to note that $X_{t,i}$ is a natural proxy of the information flow on the individual bank, with $X_{t,i}^+$ and $X_{t,i}^-$ being obviously

⁵Note that we can reverse the model and focus on the returns of the individual bank as a function of the sytem in a more familiar representation closer to the CAPM and, particularly, the semi-variance models. Under this representation, the CoVaR would measure the exposure of the individual bank to the system.

related to ‘good’ and ‘bad’ news. As shown in Andersen *et al.* (2007) and Beber and Brandt (2010), the returns of financial assets exhibit asymmetric patterns when investors are confronted with different types of news, particularly, at different phases of the business cycle. More importantly, Longin and Solnik (2001) and Andersen *et al.* (2012) show that cross-asset dependencies of returns show sizable asymmetries with correlations between negative returns significantly larger than those between positive returns. Also this asymmetry seems to increase for extreme events. This empirical observation is particularly relevant given our focus on quantile conditional distributions. Model (7) allows us to capture asymmetric patterns and predict tail comovements.

Second, the asymmetric model also captures systemic comovements via volatility. The total volatility of the system is decomposed into $\sigma_t(X_{t,i})$, a term which captures volatility spillovers with the individual bank, and $\sigma_{t,S}$, an autonomous component driven by the market. The volatility term $\sigma_t(X_{t,i})$ could be seen as an affine function of the volatility of $X_{t,i}$ and, therefore, can exhibit asymmetric patterns as a function of the sign of $X_{t,i}$, one of the most well-known and documented features of financial volatility; see, for instance, Nelson (1991).⁶ The economic reasons for asymmetries in volatility are not entirely clear. They were early related to leverage effects (Black 1976), but the measured effect of stock price changes on volatility is too large to be explained solely by financial leverage changes (Christie 1982). Asymmetries were also justified in terms of time-varying risk premiums (see, for instance, French *et al.* 1987), but this hypothesis only enjoys partial success because it fails to explain why stock volatility typically increases after the arrival of good news; see Bekaert and Wu (2000). Trade-induced effects and loss-aversion preferences may generally provide a more consistent explanation; see McQueen and Vorkik (2004) and Berkelaar and Kouwenberg (2009) for a discussion.

The market-related component $\sigma_{t,S}$ can be predicted by a suitable set of state variables. If \mathbf{N}_t contains at least one persistent predictor, $\sigma_{t,S}$ could be seen as a long-term or permanent component, with $\sigma_t(X_{t,i})$ capturing short-term deviations arising from systemic spillovers. Given an observable proxy of market volatility, such as the implied volatility index, VIX, a simple predictive model would state the affine function $\sigma_{t,S} = \sigma_S + \xi \text{VIX}_{t-1}$, with $\sigma_S > 0$, $\xi \geq 0$. Note that the behavior of implied volatility has been documented to exhibit the stylized features of volatility, namely, persistence, clustering, and asymmetric patterns such that volatility is much higher following negative market return shocks. This motivates this simple model, noting that further parametric alternatives are plausible, yet at a cost of higher complexity.

Finally, it is interesting to note that, with certain independence of the economic arguments that generally back up the existence of asymmetries, nonlinear patterns in the returns of the banking industry may arise as consequence of regulatory capital requirements and market practices.

⁶The volatility equation can be reparameterized trivially as $\sigma_t(X_{t,i}) = \sigma_{0,i} - \sigma_{1,i}|X_{t,i}| \times \mathbb{I}_{(X_{t,i} < 0)} + \sigma_{2,i}|X_{t,i}| \times \mathbb{I}_{(X_{t,i} \geq 0)}$. Absolute-valued returns sample at a high-frequency are an unbiased (but particularly noisy) proxy of the latent volatility process, so $\sigma_t(X_{t,i})$ can be seen as a function of the volatility of the individual returns.

Under the standardized approach proposed by international standards setters on financial regulation, banks must use external assessments from credit rating institutions on their exposures to individual banks to determine the proper risk weights. These weights are determined according to a nonlinear criterion. For instance, the risk weight on a claim held on a certain bank is 50 percent if its credit rating is BBB. One notch rating upgrade to A leaves the risk weight unchanged at 50 percent, yet one notch rating downgrade to BB raises the risk weight of the claim to 100 percent. Hence, negative individual shocks can trigger negative upgrades, forcing the remaining banks to hold a larger capital buffer against claims on this individual bank, thereby depressing the return of the system as a whole. This effect is compounded by the asymmetric behavior of provisioning that kicks in when repayment of a claim on a specific bank remains past due for a specific period of time, amplifying the comovement in asset returns in bad times.

A. Estimation and Inference

In model (7)-(9), changes in $X_{t,i}$ can lead to tail comovements with the system because they feed into the conditional mean and/or the variance of $X_{t,S}$. To see this more explicitly, note that the quantile function of $X_{t,S}$ conditional on the set of observable information

$\mathbf{\Omega}_t = \left(1, \mathbf{M}'_{t-1}, \mathbf{N}'_{t-1}, X_{t,i}^-, X_{t,i}^+\right)'$ can be characterized as

$$\begin{aligned} Q_{X_{t,S}}(\tau|\mathbf{\Omega}_t) &= \alpha(\tau) + \mathbf{M}'_{t-1}\boldsymbol{\gamma} + \mathbf{N}'_{t-1}\boldsymbol{\xi}(\tau) + \delta^-(\tau) X_{t,i}^- + \delta^+(\tau) X_{t,i}^+ \\ &= \mathbf{\Omega}'_t\boldsymbol{\theta}(\tau) \end{aligned} \tag{10}$$

where the vector of unknown parameters $\boldsymbol{\theta}(\tau) = (\alpha(\tau), \boldsymbol{\gamma}', \boldsymbol{\xi}'(\tau), \delta^-(\tau), \delta^+(\tau))'$ is defined implicitly, noting that $\alpha(\tau) \equiv \alpha + [\sigma_S + \sigma_{0,i}] F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$, $\xi^-(\tau) \equiv \delta_{1,i} + \sigma_{1,i} F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$, $\xi^+(\tau) \equiv \delta_{2,i} + \sigma_{2,i} F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$, and $\boldsymbol{\xi}(\tau) \equiv \boldsymbol{\xi} F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$, with $F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$ denoting the conditional τ -quantile of η_t .

If $\delta_{1,i} > 0$ and $\sigma_{1,i} < 0$, a negative shock in $X_{t,i}$ will shift $X_{t,S}$ downwards and, simultaneously, increases the conditional volatility of the process, a most likely event in a scenario of financial distress. Since $F_{\eta}^{-1}(\tau|\mathbf{\Omega}_t)$ is expected to be negative for left-tail probabilities τ , the loss-related CoVaR parameter $\delta^-(\tau)$ is positive under these conditions. As a result, a shock leading to negative values of $X_{t,i}$ not only will increase the VaR of the individual bank, but may also feed into the $Q_{X_{t,S}}(\tau|\mathbf{\Omega}_t)$ function leading the conditional VaR of the system to a greater level in absolute terms.

Equation (10) acknowledges nonlinear patterns in $Q_{X_{t,S}}(\tau|\mathbf{\Omega}_t)$ as a function of $X_{t,i}$, but it is still linear in parameters. Hence, under standard regularity conditions, $\boldsymbol{\theta}(\tau)$ can be estimated consistently by the linear quantile regression methodology proposed by Koenker and Bassett (1978). In particular, the quantile-regression estimator of $\boldsymbol{\theta}(\tau)$ given $\mathbf{\Omega}_t$, denoted $\widehat{\boldsymbol{\theta}}(\tau)$, is defined

as

$$\arg \min_{\mathbf{b} \in \mathbb{R}^n} \sum_{t=1}^T \rho_{\tau} (X_{t,S} - \mathbf{\Omega}'_t \mathbf{b}) \quad (11)$$

at a fixed τ , with $\rho_{\tau}(z) = z(\tau - \mathbb{I}_{(z < 0)})$ and n denoting the number of parameters to be estimated. Define the signed processes $VaR_{\tau,t}^{i-} = VaR_{\tau,t}^i \times \mathbb{I}_{(VaR_{\tau,t}^i < 0)}$ and $VaR_{\tau,t}^{i+} = VaR_{\tau,t}^i \times \mathbb{I}_{(VaR_{\tau,t}^i \geq 0)}$. Then, given the estimates of (11), forecasts of the CoVaR process based on the asymmetric model are generally generated according to

$$\widehat{CoVaR}_t^{S^i}(\tau, \tau^*) = \widehat{\alpha}(\tau) + \mathbf{M}'_{t-1} \widehat{\gamma} + \mathbf{N}'_{t-1} \widehat{\xi}(\tau) + \widehat{\delta}^-(\tau) \widetilde{VaR}_{\tau^*,t}^{i-} + \widehat{\delta}^+(\tau) \widetilde{VaR}_{\tau^*,t}^{i+} \quad (12)$$

given previous estimates of the signed VaR processes. Setting $\tau^* = \tau$, and recalling (2), we then can write

$$\begin{aligned} \Delta \widehat{CoVaR}_t(\tau) &= \widehat{CoVaR}_t^{S^i}(\tau) - \widehat{CoVaR}_t^{S^i}(\tau, 0.5) \\ &= \widehat{\delta}^-(\tau) \left[\widetilde{VaR}_{\tau,t}^{i-} - \widetilde{VaR}_{0.5,t}^{i-} \right] + \widehat{\delta}^+(\tau) \left[\widetilde{VaR}_{\tau,t}^{i+} - \widetilde{VaR}_{0.5,t}^{i+} \right] \end{aligned} \quad (13)$$

which generalizes (5) in an obvious way.

Under fairly general conditions, the estimator $\widehat{\boldsymbol{\theta}}(\tau)$ in (11) is consistent and asymptotically normal such that $\sqrt{T} \left(\widehat{\boldsymbol{\theta}}(\tau) - \boldsymbol{\theta}(\tau) \right) \xrightarrow{d} \mathcal{N}(0, \mathbf{V}_{\tau})$ as the sample size T diverges, with \mathbf{V}_{τ} denoting a finite covariance matrix. The model analyzed in AB arises as a particular case in this setting by considering the same state variables in the conditional mean and variance, $\mathbf{N}_t = \mathbf{M}_t$, which only implies a trivial rearrangement of parameters in our specification, and imposing the restrictions $\delta_{1,i} = \delta_{2,i}$, $\sigma_{1,i} = \sigma_{2,i}$, and $\sigma_{0,i} = 0$ in the corresponding equations. Therefore, the suitability of the asymmetric CoVaR model against a simpler symmetric specification can directly be addressed testing the restriction $H_0 : \delta^-(\tau) = \delta^+(\tau)$ through the Wald test statistic

$$T \left[\mathbf{R}' \widehat{\boldsymbol{\theta}}(\tau) \right] \left[\mathbf{R}' \mathbf{V}_{\tau} \mathbf{R} \right]^{-1} \left[\mathbf{R}' \widehat{\boldsymbol{\theta}}(\tau) \right] \xrightarrow{d} \chi_{(1)}^2 \quad (14)$$

where $\chi_{(1)}^2$ denotes a Chi-squared distribution with one degree of freedom, and

$\mathbf{R} = (0, \dots, 0, -1, 1)'$ is a $(n \times 1)$ selection vector with non-zero entries in the last two rows, corresponding to the position of the $\delta^-(\tau)$ and $\delta(\tau)$ parameters. The asymptotic covariance matrix \mathbf{V}_{τ} can be estimated consistently by different procedures, as surveyed in Koenker (2005); see Section 5 for details in our empirical analysis.

IV. Data

Our dataset is obtained from different sources. We gathered quarterly balance sheet data of individual banks in the U.S. financial industry over the period Q1 1990 through Q4 2010 from the Federal Reserve Bank of Chicago Bank Regulatory Database. This includes accounting data from the required regulatory forms filed for supervising purposes by regulated depository financial

institutions. More specifically, we collected data for the class of publicly traded institutions focusing on Bank Holding Companies (BHC) and Commercial Banks (CB). The total sample includes 32,204 panel-data observations grouped into 791 BHC and 65 CB. For any of these companies, we collected data of different accounting items such as total assets, liabilities, equity, and leverage ratio.

In order to implement the quantile regression methodology, we apply a selection filter that requires banks to be traded over at least 500 weeks on the stock market. This somewhat arbitrary choice seeks to obtain a good compromise between the number of time-series observations that ensure valid inference in the quantile-regression analysis and the total number of firms included in the filtered sample that ensures a meaningful cross-sectional analysis. The resulting subsample is composed of 340 BHC and 25 CB, totaling 21,786 panel-data observations. It should be noted that, while the CoVaR methodology is implemented on each bank in the filtered sample, we use different criteria to define the returns of the financial system as a whole, among which we construct a single index composed of all the returns of all individual banks in the total sample. In this approach, all the available information is used to define the dependent variable in the quantile-regression analysis; see Section 5 for further details.

Table 1 reports standard descriptive statistics of the main accounting indicators used in our analysis, distinguishing between total and filtered sample, and between BHC and CB. A most distinctive feature of the banking industry is a high degree of leverage (high total assets to capital ratios), which provides little room to soak losses and justifies regulatory concerns. The high level of dispersion in the total size of the banking institutions is also remarkable, since the sample includes firms with total assets (or liabilities) that range from a fairly small book value to over USD 2 trillion. For instance, the book value of Bank of America's total assets reached USD 2.2 trillion in 2010, representing over 15 percent of U.S. GDP. Clearly, BHC companies tend to be larger than CB. Comparing the accounting variables in the total and the filtered samples we observe that the distribution of banks in the filtered sample is shifted to the right with respect to the distribution of the total sample. Firms that do not fulfill the minimum length requirement are essentially small and medium-size banks which are unlikely to represent a serious concern to the system. Hence, the main interpretation of the qualitative evidence obtained from the analysis on the performance of the asymmetric model in relation to its symmetric counterpart is not affected by the design of the sample. Logically, the estimates from both models reflect features that are related to the predominance of large firms in the sample. We shall discuss this issue in greater detail in Section 5.

Following AB, we use a range of economic and financial state variables that typically capture the time-varying dynamics of expected returns and/or the conditional volatility. As shown in Cenesizoglu and Timmerman (2008), these variables may be useful predictors of the tail dynamics of the conditional distribution of returns. In particular, we use the return of the market portfolio (proxied by the S&P500 index), different term structure measures capturing the level and slope of

the yield curve, and a measure of credit risk. Specifically, we include as state variables changes in the U.S. Treasury bill secondary market 3-month rate, the yield spread between the U.S. Treasury benchmark bond 10-year and the U.S. 3-month T-bill, and the credit spread between the 10-year Moody's seasoned Baa corporate bond and the 10-year U.S. Treasury bond. We also include the CBOE index of implied volatility (VIX) to capture market equity risk as a predictor of market volatility. The data is extracted from the Chicago Board Options Exchange, the Federal Reserve Board's H.15 Release, and Datastream. Table 2 reports descriptive statistics.

Finally, the period of analysis is long enough to include different cycles of economic activity. Economic recessions are usually identified exogenously after a period of two consecutive quarters of negative GDP growth. Similarly, the total sample includes the greatest financial turmoil in the U.S. since the Great Depression. Although the predictive variables in our analysis are strongly tied to the economic cycle, we additionally control for potential structural-break effects through dummy variables related to bearish market conditions. Thus, we define an Economic Recessions dummy variable taking the value equal to one in the periods identified as a macroeconomic recessions by the NBER (July 1990- March 1991, March 2001- November 2001 and December 2007-June 2009) and zero otherwise. Similarly, we define a Financial Recession indicator taking the value equal to one from August 2007 through March 2009. This period matches the timing of maximum disruption in money markets caused by financial uncertainty and counterparty credit risk. In this period, the spread between the 3-month LIBOR and the 3-month OIS rates, widely taken as a market indicator of counterparty risk premium, reached historical maximums and remained well above the average pre-crisis level.⁷

V. Downside Comovement in the U.S. Banking Industry

For each bank in the sample, we construct a weekly return time-series, $X_{t,i}$, defined as the simple growth rate of the market-valued total assets, $A_{t,i}$, held by the i -th bank. These series are computed as the product of the leverage ratio (total assets to book equity) and the market value of equity. Whereas market equity data are available at a weekly frequency, balance sheet data for U.S. financial institutions are disclosed on a quarterly basis. To circumvent the sampling frequency mismatch, we smoothed weekly the quarterly leverage ratio using cubic spline interpolation, a well-known technique in applied finance. It should be mentioned that the results in the subsequent analysis reported below are not particularly sensitive to this consideration. We also applied a constant leverage ratio approach (*i.e.*, applying a constant weekly value equal to that in the quarter) as in AB noting no qualitative difference.⁸

⁷The average value of the LIBOR-OIS spread over August 2007 through March 2009 reached 981 basis points, well above the pre-crisis average value of 104 basis points.

⁸In our view, using smoothing techniques may be preferable because it allows us to avoid seasonal effects related to the timing in which accounting information is updated.

Given these series, we adopt two different procedures to define representative portfolios of the banking system. First, given the observations in the total sample, we determine a single banking industry index. The value of this portfolio is simply a value-weighted average of the market-valued total assets held by all the banks in the sample at a certain time. Hence, the returns of this portfolio, $X_{t,S}$, are defined as $X_{t,S} = \sum_{j=1}^{N_t} \omega_{t,j} X_{t,j}$, where following AB we set $\omega_{t,j} = A_{t-1,j} / \sum_{s=1}^{N_t} A_{t-1,s}$, with N_t denoting the number of banks at time t . Therefore, although we shall compute CoVaR measures for a subset of $N = 365$ banks that fulfill minimum length requirements, all the available information is used to generate the returns of the system in this approach. We shall refer to this procedure as the ‘single-index system’ approach in the sequel.

Alternatively, and considering exclusively the filtered sample constituted by $N = 365$ firms, for each one of these banks we construct a portfolio representative of the surrounding system. Thus, for each firm, we define a bank-specific system portfolio with returns given by $X_{t,S} = \sum_{j=1}^N \omega_{t,j}^* X_{t,j}$, where the weights are now computed according to $\omega_{t,j}^* = 0$ if $j = i$ and $\omega_{t,j}^* = A_{t-1,j} / \sum_{s=1, j \neq s}^N A_{t-1,s}$ otherwise, *i.e.*, a weighted average of all the individual returns in the filtered sample except that of the bank under analysis. This procedure ensures a small-sample adjustment that prevents a mechanical correlation effect (*i.e.*, a spurious interdependence) between the individual bank and the system when the total number of institutions N is not particularly large and/or when a single institution has a significant weight in relation to the rest of the financial system. Because the bank under analysis is not included, the analysis of tail comovements is more rigorous and necessarily rules out the possibility of spurious interrelations stemming from the simultaneous presence of the same firm in both portfolios. Note that, although the bank is not explicitly related to the system under this approach, it may be implicitly related if the bank’s balance is interconnected with the remaining banks’ balances. This approach has been implemented in López-Espinosa *et al.* (2012) to address systemic interrelations in the global banking industry. We shall refer to this procedure as the ‘multiple-index system’ approach in the sequel.

Both the individual VaR and the CoVaR functions are estimated at the weekly frequency using a vector of economic and financial variables as potential predictors. This vector, denoted \mathbf{Z}_t in the sequel, includes a constant (Constant), the CBOE implicit volatility index (VIX), the return of the S&P500 index (Market Return), changes in the U.S. Treasury bill secondary market 3-month rate (Δ T-bill), yield spread between the U.S. Treasury benchmark bond 10-year and the U.S. 3-month T-bill (Yield Slope), credit spread between the 10-year Moody’s seasoned Baa corporate bond and the 10-year U.S. Treasury bond (Default Premium) and two dummy variables related to economic recessions (NBER recessions) and the 2007-2009 financial recession (Financial Recession).

A. Main Empirical Results

In this section, we discuss the suitability of different specifications of the CoVaR equation at the usual shortfall probabilities $\tau \in \{0.01, 0.05\}$ in downside risk modelling. The parameters that characterize the symmetric CoVaR proposed by AB can generally be estimated from the linear quantile regression model

$$X_{t,S} = \mathbf{Z}'_{t-1} \boldsymbol{\gamma}_i(\tau) + \delta_i(\tau) X_{t,i} + u_{\tau,t} \quad (15)$$

with $i = 1, \dots, N$, $t = 1, \dots, T_i$, and $u_{\tau,t}$ denoting an error term satisfying fairly general, standard conditions. Note that the number of time-series observations available for each firm in the filtered sample ranges from a minimum length of 500 weeks to a maximum of 1,096 weeks in the period.⁹ The dependent variable, $X_{t,S}$, is determined according to either the single-index or the multiple-index system approach described previously. Similarly, the asymmetric CoVaR model can be estimated from the linear quantile regression model

$$X_{t,S} = \mathbf{Z}'_{t-1} \boldsymbol{\gamma}_i(\tau) + \delta_i^-(\tau) X_{t,i}^- + \delta_i^+(\tau) X_{t,i}^+ + u_{\tau,t} \quad (16)$$

as discussed in Section 3.

To generate exogenous forecasts of the individual VaR process of each bank and compute time-varying Δ CoVaR downside risk contributions, we estimate the linear quantile regression model

$$X_{t,i} = \mathbf{Z}'_{t-1} \boldsymbol{\beta}_i(\tau) + \varepsilon_{\tau,t} \quad (17)$$

at the τ -th quantile, using the same set of predictive variables as in the CoVaR model. Quantile regressions to model VaR dynamics have been used, among others, in Taylor (1999), Chernozhukov and Umantsev (2001), and Cenesizoglu and Timmerman (2008). Given the resulting estimates, $\tilde{\boldsymbol{\beta}}_i(\tau)$, the bank-specific VaR forecasts are simply determined by $\widetilde{VaR}_{\tau,t}^i = \mathbf{Z}'_{t-1} \tilde{\boldsymbol{\beta}}_i(\tau)$. We then generate weekly $\Delta \widehat{CoVaR}_t^{S|i}(\tau)$ measures as a function of the quantile-regression estimates of the symmetric and asymmetric CoVaR models and the individual VaR forecasts, as discussed previously.

The asymptotic covariance matrix of the parameter estimates is inferred by combining kernel-density estimation with a heteroscedasticity-consistent covariance matrix estimation; see Koenker (2005). In particular, define the outer-product matrix $\mathbf{A}_{\tau,T} = \tau(1-\tau) \sum_{t=1}^T \boldsymbol{\Omega}_t \boldsymbol{\Omega}'_t / T$, and let be $\mathbf{D}_{\tau,T} = (Th_T)^{-1} \sum_{t=1}^T \mathcal{K}(\hat{u}_{\tau,t}/h_T) \boldsymbol{\Omega}_t \boldsymbol{\Omega}'_t$, where $\mathcal{K}(\cdot)$ is a kernel function, h_T is a bandwidth parameter, $\hat{u}_{\tau,t}$ denotes the estimated residuals from the quantile regression, and $\boldsymbol{\Omega}_t$ denotes the set of variables involved, *e.g.*, $\boldsymbol{\Omega}_t = \left(\mathbf{Z}'_{t-1}, X_{t,i}^-, X_{t,i}^+ \right)'$ in the more general case treated here. Then, a consistent estimate of \mathbf{V}_{τ} is given by the ‘sandwich-type’ estimator $\mathbf{D}_{\tau,T}^{-1} \mathbf{A}_{\tau,T} \mathbf{D}_{\tau,T}^{-1}$.

⁹The average number of time-series observations is 781. The distribution of the number of available observations in our sample does not seem to be related to firm-specific characteristics. For instance, the cross-sectional correlation between available observations and average size over the period is 4.96 percent.

We implemented the Gaussian kernel in this approach and selected h_T optimally according to Silverman's rule, *i.e.*, $h_T = 0.9 \times \min \{\hat{\sigma}_u, IQR_{\hat{u}}\} \times T^{-1/5}$, where $\hat{\sigma}_u$ and $IQR_{\hat{u}}$ denote the sample standard deviation and the sample interquartile range of $\hat{u}_{\tau,t}$.

Table 3 shows the median of the parameter estimates of equations (15) and (16) across banks in the filtered sample, the median of the respective robust t -statistics for individual significance, and the median of the pseudo- \mathcal{R}^2 in the quantile regressions given the single- and multiple-index system approaches. Table 3 also reports the sample rejection frequencies of several meaningful test statistics at the 95 percent confidence level involving the main CoVaR parameters. In particular, we are interested in the null hypothesis that $X_{t,S}$ is not affected by $X_{t,i}$, which implies $H_{0,Ind} : \delta_i(\tau) = 0$ in (15) and $H_{0,Ind} : \delta_i^-(\tau) = \delta_i^+(\tau) = 0$ in (16). These hypotheses can be tested through Wald-type tests statistics asymptotically distributed as a $\chi_{(1)}^2$ and $\chi_{(2)}^2$, respectively. Note that, given the estimates of the respective risk models, this is a formal test for the existence of tail interdependence or interconnectedness. Focusing on the asymmetric CoVaR model (16), we additionally address the individual significance of the loss-related CoVaR coefficient, namely $H_{0,Loss} : \delta_i^-(\tau) = 0$, and the suitability of the symmetric restriction that gives rise to (15), namely $H_{0,Sym} : \delta_i^-(\tau) = \delta_i^+(\tau)$. These hypotheses can be tested through Wald-type test statistics asymptotically distributed as $\chi_{(1)}^2$.

The estimates from the different CoVaR models in Table 3 reveal several common features. First, considering a single-index or a multiple-index approach to construct returns of the banking system does not cause major differences. Results are fairly robust to the definition of the system and we reach similar conclusions. Second, among the different state variables used in our analysis, the implicit volatility index is the most effective predictor of the tail of the conditional distribution. This result is not particularly striking, since downside risk dynamics on high-frequency data are mainly driven by market volatility. As the τ -th quantile decreases (*i.e.*, as we focus on more extreme events), the influence of volatility becomes more important and other lagged state variables lose predictive power. This is consistent with the widely agreed fact that most extreme events are triggered by unpredictable, irregular events that feature market volatility, and which are characterized in quantitative models through a jump process. Among the remaining variables, the yield slope and the default premium tend to exhibit tail forecasting ability as well. These variables are closely related to market uncertainty and the risk of a market crash. The structural-break dummy related to the financial recession is strongly significant, showing that the banking business went through an adverse regime characterized by greater volatility and market uncertainty. The NBER indicator of economic recessions does not seem to add incremental information over the remaining variables.

Turning our attention to the CoVaR parameters, the estimates of the $\delta(\tau)$ coefficient in the symmetric model are, as expected, mostly positive, revealing significant tail-interrelations between the system and the individual banks. Since there are little or no qualitative differences between the different definitions given to the banking system, for ease of exposition we discuss results for

the multiple-index system approach in what follows. The median of the $\delta(\tau)$ estimates across banks ranges from 0.086 (at $\tau = 0.01$) to 0.094 (at $\tau = 0.05$). The null hypothesis $H_{0,Ind} : \delta(\tau) = 0$, which excludes systemic interrelations, cannot be rejected at the usual 95 percent confidence level for 43 percent ($\tau = 0.01$) and 57 percent ($\tau = 0.05$) of the banks in this sample. Obviously, parameter estimates and sample rejection ratios reflect unconditional measures in this analysis. Attending to certain firm-specific characteristics, particularly size, we can observe meaningful conditional trends that shall be discussed in greater detail later on.

The analysis of the outcomes from the asymmetric CoVaR confirms the existence of a more sophisticated pattern characterizing tail-interdependences. The cross-sectional medians of the $\delta^-(\tau)$ and $\delta^+(\tau)$ estimates are positive and show clear evidence of asymmetric responses characterizing the left tail of the conditional distribution of system's returns. For instance, at $\tau = 0.01$, the median of $\widehat{\delta}^-(\tau)$ is 0.372, while the median of $\widehat{\delta}^+(\tau)$ takes on a much more conservative value of 0.030. The estimates of the loss-related coefficient $\delta^-(\tau)$ are sizeable and highly significant in most cases, whereas the estimates of $\delta^+(\tau)$ tend to be relatively small and not statistically significant in many cases. Not surprisingly, therefore, the assumption $H_{0,Sym} : \delta^-(\tau) = \delta^+(\tau)$ that gives rise to the symmetric model considered in AB is largely rejected for most of the firms analyzed in our sample in favour of the asymmetric extension. In particular, the (unconditional) rejection ratio of the symmetric restriction is around 70 percent but we remark that there is strong evidence of size-related trends. Independently of the definition of the system, the analysis on the sample rejection ratios of $H_{0,Loss} : \delta^-(\tau) = 0$ and $H_{0,Ind} : \delta^-(\tau) = \delta^+(\tau) = 0$ reveal massive rejections. This shows strong evidence of tail-interdependence and interconnectedness in the sample in which the existence of downside comoments, captured by the loss-related coefficient $\delta^-(\tau)$, play a major role. In contrast, the misspecified symmetric CoVaR model shows a much more conservative picture which may lead to misleading conclusions.

It is of particular relevance for policy considerations to characterize conditional patterns in the cross-section analysis of the CoVaR estimates as a function of variables that proxy the systemic importance of a firm. International banking regulations have considered that the contribution to systemic risk of an individual bank is reflected in the size of its liabilities and the impact that its failure may have on markets and the real economy.¹⁰ Also, the failure of a large financial institution is more likely to generate shockwaves throughout the financial system and harm the real economy. In the absence of further information, therefore, firm's size is the most natural indicator of the systemic importance of a firm. Conditioning on variables that proxy for this magnitude allows us to explore the existence of systematic biases in the estimation of CoVaR models which may be particularly relevant to address systemic importance. To this end, we proxy

¹⁰The BCBS Bank for International Settlements (2011) has developed an indicator-based measurement approach to identify and measure the systemic importance of banks. There are five indicators, namely, bank size, interconnectedness, substitutability, cross-border activity, and complexity. Size, measure by total exposures, account for 20 percent of the overall index.

the size of each individual bank using either the time-series median of its total assets over the period or the corresponding median value of its liabilities.

Table 4 presents cross-sectional medians of the main estimates from the symmetric and asymmetric CoVaR models across size-sorted deciles, with size being proxied by total assets. The table also reports the conditional rejection ratios of the set of hypotheses $H_{0,Ind}$, $H_{0,Loss}$, and $H_{0,Sym}$ as well as cross-sectional medians of the $\Delta CoVaR$ downside risk measure. Table 5 shows the same analysis with size being proxied by liabilities. For the sake of saving space, we only report the results for the main CoVaR parameters that characterize tail comovements and the $\Delta CoVaR$ measure and focusing on the multiple-index system approach. Similar results were obtained for the single-index approach and hence omitted. Summarizing tables with complete results are available upon request.

The overall qualitative evidence that arises for both proxies of size is similar and, therefore, we discuss the results from total assets in Table 4 for ease of exposition. We first focus on the results from the symmetric CoVaR model. The median of $\hat{\delta}(\tau)$ in the top decile, formed by firms with largest capitalization, is 0.150 ($\tau = 0.01$) and 0.276 ($\tau = 0.05$), respectively, whereas the corresponding values at the bottom decile are considerably smaller, ranging from 0.060 ($\tau = 0.01$) to 0.041 ($\tau = 0.05$). According to these estimates, therefore, the financial system is, on average, more sensitive to shocks in the class of large-scale firms, which is consistent with the too-big-to-fail hypothesis. The median values of the estimated $\Delta CoVaR$ measures reflect this feature and tend to increase in absolute terms with firm size, showing that the relative contribution of an individual bank to the risk of the whole system tends to be greater depending on the size of the firm. Similarly, the rejection ratio of $H_{0,Ind} : \delta(\tau) = 0$ in the symmetric CoVaR model increases on size, although we note remarkable differences across the target probability, τ . At $\tau = 0.05$, the rejection ratio of $H_{0,Ind}$ in the top-size decile (88.89 percent) is much greater than the corresponding value at the bottom decile (29.73 percent) and we observe an almost monotonic path across deciles. At $\tau = 0.01$, however, the rejection ratio of this hypothesis dramatically collapses at the top deciles and approaches the values observed at the bottom decile (38 percent), so the differences across deciles are less remarked. This feature stems mainly from the fact that $\delta(\tau)$ is on average smaller at $\tau = 0.01$, particularly, at the top deciles. This particularly counterintuitive result may arise from the conjunction of the characteristic small-sample biases that occur in the quantile-regression analysis at probabilities close to the boundary limits (Chernozhukov, 2005), and the biases stemming from the misspecification of the functional form of the conditional quantile function in the symmetric model.

Turning our attention to the asymmetric CoVaR model, we observe stronger evidence of size-dependent conditional patterns. For instance, at $\tau = 0.01$, the cross-sectional median of the loss-related coefficient $\delta^-(\tau)$ ranges from 0.816 in the top-size decile to 0.135 in the bottom size-decile. The median values of $\delta^-(\tau)$ and $\delta^+(\tau)$ show almost monotonically increasing paths along deciles. Similarly, the rejection ratios of the test statistics related to $H_{0,Ind}$, $H_{0,Loss}$, and

$H_{0,Sym}$ show larger probabilities of rejections as size increases. In particular, a significant link between the system and an individual bank, characterized by an asymmetric response, is more likely to be observed as the size of the firm increases. This feature reveals the existence of important systematic biases stemming from the misspecification of the quantile function. In sharp contrast to the estimates from the symmetric model, the estimates of $\delta^-(\tau)$ tend to be larger at the 1 percentile than at the 5 percentile. This is consistent with most of the empirical findings in the literature suggesting that the degree of correlation between financial assets and markets become stronger in downside market movements; see, for instance, Longin and Solnik (2001). Acknowledging the existence of asymmetric responses featuring the tail distribution leads to a more clear and consistent picture of tail-interdependence.

Finally, as in the symmetric CoVaR model, the average estimates of ΔCoVaR downside risk measure show an increasing pattern across deciles in absolute terms which is consistent with the too-big-to-fail hypothesis. However, we observe sheer differences between the average magnitude of such estimates that tend to be magnified as size increases: the estimates from the symmetric model are considerable smaller than those from the asymmetric generalization. According to the statistical analysis, the asymmetric model outperforms its symmetric counterpart. Consequently, it must be concluded that the symmetric model is prone to generate ΔCoVaR estimates that largely underestimate, in absolute terms, the risk contribution of an individual bank to the system as a whole.

B. Discussion

The analysis in the previous section reveals that the practical consequences of the misspecification involved in the symmetric CoVaR can be particularly severe. First, this model tends to generate downward biased estimates of the empirical link that ties the tail of the conditional distribution of the system to the returns of an individual bank. The estimates of $\delta(\tau)$ reflect, by construction, an average response to positive and negative shocks alike. Consequently, the symmetric CoVaR model is prone to underestimate the magnitude of tail-dependence in an adverse scenario characterized by large negative returns. In contrast, the tail of the system is directly related to individual losses in the asymmetric CoVaR model, yielding downside risk estimates of a higher magnitude. To appraise the size of the parameter biases involved, note that, from the results reported in Table 3, the average value of $\widehat{\delta}^-(\tau)$ ranges from 0.372 ($\tau = 0.01$) to 0.337 ($\tau = 0.05$), which is roughly 4 times bigger than the corresponding estimates of $\delta(\tau)$ in the symmetric model.

Second, and as a direct consequence of these biases, the symmetric CoVaR model produces conservative inference about the significance of the parameters that capture conditional comovements. Hence, statistical inference intended to formally detect systemic interrelations on the estimates of the risk model is likely to lead to misleading conclusions. According to Table 3, whereas the rejection ratio of $H_{0,Ind}$ is around 90 percent in the estimates of the asymmetric

CoVaR model, this magnitude dramatically collapses in the symmetric model showing values not greater than 50 percent. In our view, the statistical evidence presented by the asymmetric CoVaR, suggesting that most of the banks in the sample are interconnected, is more likely to be close to the real picture. The sample analyzed in this paper is mostly formed by BHC. These banks are closely intertwined financially through lending to and borrowing from their peers, holding balance deposits with each other, and showing similar risk exposures. Of course, the systemic importance of these banks is not expected to be the same as this may vary depending on a number of factors, but the estimates of the econometric risk model should reflect a high proportion of interconnectedness in this sample. While the symmetric model offers conservative figures, the formal test based on the estimates of the asymmetric model does acknowledge this.

Third, the symmetric CoVaR model produces estimates of the ΔCoVaR downside risk measure which are biased downwards in absolute terms. Once more, this is the direct consequence of the downward bias in the estimation of the main parameters that characterize the conditional quantile function. For many banks in our sample, particularly those belonging to top size deciles, the symmetric CoVaR model largely diminish the contribution of an individual bank to the total risk of the system. For instance, the bank with the largest capitalization in our sample during the period analyzed is Citigroup. In November 2008, this BHC received a massive stimulus package from the U.S. Government Treasury including a \$20 billion capital injection in the form of preferred stocks. In November 2011, this firm was included in the list of 29 systemically important banks released by the Financial Stability Board (FSB) that would have to raise their core Tier I capital ratios under the provisions of the Basel III regulatory framework. According to the asymmetric CoVaR methodology, the median of the weekly ΔCoVaR estimates of Citigroup at the 1 percentile over the sample period is -0.0565 . In sharp contrast, the symmetric CoVaR model yields an extraordinarily conservative average estimate of -0.008 over the same period, dwarfing by a factor of seven times its impact on the left tail asset return of the financial system.

In our view, the most adverse consequence of the misspecification involved in the symmetric CoVaR is that biases do not occur randomly, but are systemically related to a firm's characteristics, such as size. According to our analysis, the downward-biased distortions caused by the model misspecification are more likely to occur and are more pronounced in large-scale firms. For instance, the cross-sectional median of the $\delta^-(\tau)/\delta(\tau)$ ratio for firms in the bottom-size at $\tau = 0.01$ is 1.31, showing mild biases for small companies; however, the median value of this ratio for firms belonging to the top-size decile is 5.08. Since the inferred dynamics of the ΔCoVaR process are fundamentally driven by these parameters, the misspecified symmetric CoVaR model can lead to severely biased estimates of the systemic importance of a firm as measured by ΔCoVaR . Indeed, according to the estimates reported in Table 4, the cross-sectional median of the average ΔCoVaR process in the asymmetric model at $\tau = 0.01$ is just 1.26 times larger than that in the symmetric model for firms in the bottom-size decile. In contrast, the median of the asymmetric ΔCoVaR process is 4.98 times bigger than its symmetric counterpart in banks in the top-size decile. It is not a coincidence that these ratios take values on

average similar to those of $\delta^-(\tau)/\delta(\tau)$.

To provide a sense of the systematic biases implied by the symmetric model, Figure 1 presents the individual estimates of the main parameters in the CoVaR models as well as the respective average values of ΔCoVaR estimates. In particular, the figure on the left-hand side confronts the estimates $\widehat{\delta}_i(\tau)$ and $\widehat{\delta}_i^-(\tau)$ against the time-series median of size (in logs) of the total assets held by an individual bank. Similarly, the figure on the right-hand side shows the time-series median (in absolute value) of the estimates of ΔCoVaR for each bank over the period against its log-size value.

As size increases, both $\delta(\tau)$ and $\delta^-(\tau)$ increase, as discussed previously, and so do the associated values of the average ΔCoVaR estimates. This feature is largely consistent with the too-big-to-fail hypothesis and motivates the concerns of regulators and supervisors over large-scale firms. However, the estimates of the asymmetric coefficient $\delta^-(\tau)$ tend to dominate $\delta(\tau)$ almost uniformly as size becomes larger and, hence, the same pattern arises in the estimated ΔCoVaR dynamics. The differences may not be important for firms with small capitalization, as shown in Figure 1, but they tend to spread out sharply and rapidly as the size of the firm increases. Note that the rate of change in the CoVaR estimates is much greater under the asymmetric case, showing that biases occur systematically, and that these are more relevant for large-scale banks.

In summary, the overall picture that emerges from this analysis suggests that imposing a symmetric restriction in the CoVaR model can generally lead to potentially large biases in the resulting estimates. These biases are systematic and are more likely to occur precisely in the class of financial institutions with largest capitalization, *i.e.*, firms which are more likely to pose a systemic threat to the financial system and the real economy.

C. Robustness Checks

In this section, we report the main outcomes from several analyses intended to check the robustness and generality of the previous results. To save space, we discuss the main evidence based on the multiple-index system approach, noting that complete results are available upon request.

Bank holding companies and commercial banks

The sample analyzed in this paper is mainly composed of BHC, *i.e.*, financial institutions that own and/or control at least one U.S. bank. The group of CB, on the other hand, is much more reduced, since the number of traditional banks which are large enough to be traded on the stock market is considerably smaller. There are sharp differences in terms of business profile, regulation, and supervision between these two classes of firms which are likely to affect the

systemic importance of individual banks. In relative terms, CB typically exhibit smaller size than BHC; see Table 1 for details. Furthermore, these banks are subject to the Glass-Steagall Act of 1933 and, hence, can only engage in traditional banking activities. On the other hand, BHC tend to be larger companies which, by virtue of the Gramm-Leach-Bliley Act of 1999, can engage in a number of authorized activities beyond traditional lending. These companies increasingly resorted to diversification activities before the onset of the financial crisis in order to offset declining traditional bank-related profits. Although the reduced cross-sectional number of observations of CB in our sample calls for some caution to interpret results, it is of general interest to comment the main differences in the estimation of the CoVaR models for firms grouped into BHC and CB, respectively. The main results from this analysis are reported in Table 6.

Broadly speaking, we observe the same characteristic patterns discussed previously and which generally advise for the use of the asymmetric CoVaR model. For both BHC and CB the symmetric CoVaR model tends to be rejected in favor of its asymmetric generalization, although the rejection ratio of $H_{0,Sym}$ is smaller in companies belonging to the CB group. This is consistent with the findings that asymmetries are, on average, more likely to occur in large-scale firms. More interestingly and, as expected, there are meaningful differences between the systemic importance of BHC and CB according to the CoVaR estimates. First, the rejection ratio of $H_{0,Ind}$ testing interconnectedness is considerably smaller in the CB group, particularly, under the asymmetric model. Not surprisingly, therefore, the average systemic importance of CB, as measured by the cross-sectional median of the time-series medians of the estimates of ΔCoVaR , is considerably smaller than that of BHC. Both the symmetric and the asymmetric models capture this phenomenon, although once more the difference is greater, in absolute terms, when using the asymmetric CoVaR. We note that this evidence does not necessarily arise as a consequence of having a smaller average size, but also from their different business model. For instance, BHC used intensively whole-sale short-term funding to finance their operations before the crisis, whereas CB relied more heavily on traditional deposit funding. This feature makes these banks more resilient to the propagation of shocks originated in financial markets and, reciprocally, the banking system is less vulnerable to the failure of these banks.

Nonlinear models

We have proposed a direct nonlinear generalization of the symmetric model in AB. There are both economic and econometric reasons that suggest that this simple extension may outperform the restricted model, particularly, when the firm involved is a large-scale bank. The empirical findings discussed previously support the suitability of the asymmetric model. More generally, it could be possible to extend this structure by allowing further nonlinear patterns as a function of a number of regimes. For instance, the asymmetric model may be embedded into the class of

piecewise linear models for which we may consider a general specification of the form

$$X_{t,S} = \mathbf{Z}'_{t-1} \boldsymbol{\gamma}_i(\tau) + \sum_{j=1}^{k+1} \delta_{i,j}(\tau) X_{t,i} \times \mathbb{I}_{(X_{t,i} \in \mathcal{C}_j(\mathcal{T}))} + u_{\tau,t} \quad (18)$$

where $\mathcal{C}_j(\mathcal{T})$, $j = 1, \dots, k+1$, $k \geq 1$, denotes a collection of statistical classes defined as a function of k threshold values \mathcal{T} such that $\cup_{j=1}^{k+1} \mathcal{C}_j(\mathcal{T}) = \mathbb{R}$ and $\mathcal{C}_l(\mathcal{T}) \cap \mathcal{C}_s(\mathcal{T}) = \emptyset$ for all $l \neq s$.

Intuitively, the real line is partitioned into $k+1$ disjoint sets given k threshold values \mathcal{T} such that the tail-response of $X_{t,S}$, as measured by the $\delta_{i,j}(\tau)$ coefficient, may vary along the distribution of $X_{t,i}$. Our setting is embedded in this general structure, since we consider a threshold value set to zero, and two classes with positive and negative values of $X_{t,i}$.

Several procedures in the existing literature may be used along these lines, however, at a cost of a higher complexity. For instance, the class of threshold models proposed by Hansen (2000) may be useful to determine endogenously different threshold values. A serious attempt to estimate these models is beyond the scope of this paper, but it is interesting to analyze intuitively whether the asymmetric specification provides a fair enough representation relative to more complex models. To this end, we estimated a piecewise linear CoVaR model with bank-specific threshold values arbitrarily set to the empirical deciles of the unconditional distribution of $X_{t,i}$, *i.e.*, we estimate the quantile model $X_{t,S} = \mathbf{Z}'_{t-1} \boldsymbol{\gamma}_i(\tau) + \sum_{j=1}^{10} \delta_{i,j}(\tau) X_{t,i} \times \mathbb{I}_{(X_{t,i} \in \mathcal{C}_j(\mathcal{T}_i))} + u_{\tau,t}$, where the $\delta_{i,j}(\tau)$ coefficient captures the link between the τ -th conditional quantile of $X_{t,S}$ and the j -th unconditional decile of $X_{t,i}$.

Figure 2 shows the cross-section median of the estimates of the decile-related CoVaR parameters $\{\delta_{i,j}(\tau)\}$ in this model at $\tau = 0.05$. These are depicted in grey bars. To compare the magnitude of these estimates with the previous estimates from the symmetric and asymmetric CoVaR models, we superpose the unconditional medians of $\bar{\delta}(\tau) = 0.086$ (blue dashed-dotted line), $\bar{\delta}^-(\tau) = 0.372$ (red dashed line), and $\bar{\delta}^+(\tau) = 0.048$ (green dotted line). Those values have been reported in Table 3. Our main aim is to determine if the asymmetric CoVaR model provides a reasonable representation against further extensions. Figure 2 reveals that, on average, the most important effect for the modeling of downside tail comovements is to acknowledge the existence of asymmetric responses. The empirical estimates related to deciles above the median, corresponding to positive returns, hardly contribute to explain left-tail comovements and have a small coefficient associated to. In sharp contrast, observations of $X_{t,i}$ below the median, corresponding to negative returns, trigger large comovements. The average differences across the different negative deciles are far less important, and the asymmetric CoVaR model seems to make a fair job in capturing the average comovement with the system. Therefore, although nonlinear models are likely to introduce further refinements at a cost of higher complexity, the asymmetric model presents a good balance between explanatory power and model tractability.

Figure 2 makes clear the large downward bias that estimates from the (misspecified) symmetric model are generally exposed to. The average magnitude of $\delta(\tau)$ pools the information related to

negative and positive shocks and, as a result, it substantially diminishes the importance of negative shocks to explain left-tail downside comovements. While generalized nonlinear pattern may provide further refinements, it is clear that allowing for asymmetries provides first-order gains in CoVaR modeling.

Returns of different representative portfolios and other considerations

We performed a number of additional checks to analyze the robustness of the results against the definition given to the returns of the individual bank and the system portfolios. The qualitative evidence is the same as that discussed previously and, therefore, we omit presenting summarizing tables to save space, but note that these are available under request. First, we computed the CoVaR models on the returns of a representative portfolio formed by equity. Considering stock returns instead of balance-sheet related returns led to the same conclusions about the performance of the asymmetric model and the patterns of systemic importance discussed previously. Second, we used liabilities as a weighting variable to define the returns of the system instead of total assets. Liabilities may reflect de-leveraging pressures more accurately than market-valued total assets, as discussed in López-Espinosa *et al.* (2012). Again, using this weighting scheme did not lead to significant changes in relation to those discussed previously. Finally, and as commented before, we used linear interpolation (rather than cubic splines) to smooth weekly quarterly accounting data, with no major changes in the analysis.

VI. Concluding Remarks

In this paper, we have discussed the suitability of the CoVaR procedure recently proposed by Adrian and Brunnermeier (2011). This valuable approach helps understand the drivers of systemic risk in the banking industry. Implementing this procedure in practice requires specifying the unobservable functional form that relates the dynamics of the conditional tail of system's returns to the returns of an individual bank. Adrian and Brunnermeier (2011) build on a model that assumes a simple linear representation, such that returns are proportional.

We show that this approach may provide a reasonable approximation for small-sized banks. However, in more general terms, and particularly for large-scale banks, the linear assumption leads to a severe underestimation of the conditional comovement in a downward market and, hence, their systemic importance may be perceived to be lower than their actual contribution to systemic risk. Yet, how to measure and buttress effectively the resilience of the financial system to losses crystallizing in a stress scenario is the main concern of policy makers, regulatory authorities, and financial markets alike. Witness the rally on U.S. equities and dollar on March 14, 2012 after the regulator announced favorable bank stress test results for the largest nineteen U.S. bank holding companies.

The reason is that the symmetric model implicitly assumes that positive and negative individual returns are equally strong to capture downside risk comovement. Our empirical results however, provide robust evidence that negative shocks to individual returns generate a much larger impact on the financial system than positive disturbances. For a median-sized bank, the relative impact ratio is sevenfold. We contend that this non-linear pattern should be acknowledged in the econometric modeling of systemic risk to avoid a serious misappraisal of risk. Moreover, our analysis suggests that the symmetric specification introduces systematic biases in risk assessment as a function of bank size. Specifically, the distortion caused by a linear model misspecification is more pronounced for larger banks, which are also more systemic on average. Our results show that tail interdependence between the financial system and a bottom-size decile bank which is contracting its balance sheet is 2.2 times larger than its average comovement. More strikingly, this ratio reaches 5.4 for the top-size decile bank. This result is in line with the too-big-to-fail hypothesis and lends support to recent recommendations put forth by the Financial Stability Board to require higher loss absorbency capacity on large banks.¹¹ Likewise, it is consistent with the resolution plan required by the Dodd-Frank Act for bank holding companies and non-bank financial institutions with \$50 billion or more in total assets. Submitting periodically a plan for rapid and orderly resolution in the event of financial distress or failure will enable the FDIC to evaluate potential loss severity and minimize the disruption that a failure may have in the rest of the system, thus performing its resolution functions more efficiently. This measure will also help alleviate moral hazard concerns associated with systemic institutions and strengthen the stability of the overall financial system.

To capture the asymmetric pattern on tail comovement, we propose a simple yet effective extension of the original CoVaR model. This extension preserves the tractability of the original model and its suitability can formally be tested formally through a simple Wald-type test, given the estimates of the model. We show that this simple extension is robust to more general specifications capturing non-linear patterns in returns, though at the expense of losing tractability.

The refinement of the CoVaR statistical measure presented in the paper aims at quantifying asymmetric spillover effects when strains in banks' balance sheets are elevated, and thus contributes a step towards strengthening systemic risk monitoring in stress scenarios. Furthermore, its focus on tail comovement originated from negative perturbations in the growth rate of market-valued banks' balance sheets, may yield insights into the impact on the financial system from large-scale deleveraging by banks seeking to rebuild their capital cushions. This particular concern has been recently rekindled by the continued spikes in volatility in euro area financial markets. It has been estimated that, following pressures on the European banking system as banks cope with sovereign stress, European banks may shrink their combined balance

¹¹In October 2010, the Financial Stability Board recommended that all jurisdictions should put in place a policy framework to reduce risks and externalities associated with domestic and global systemically important financial institutions, combining higher capital buffers, an efficient resolution mechanism, and more intensive supervisory oversight.

sheet significantly with the potential of unleashing shockwaves to emerging economies hurting their financial stability (IMF, 2012). The estimation of the impact on the real economy from aggregate weakness of the financial sector, and the design of optimal macroprudential policies to arrest systemic risk when tail interdependencies feed non-linearly into the financial system, are left for future research.

Figure 1. Comparison of median estimates from the symmetric and asymmetric CoVaR models. The figure on the left shows the point estimates of $\widehat{\delta}^-(\tau)$ (Asymmetric) and $\widehat{\delta}(\tau)$ (Symmetric) for each bank. The figure on the right shows the absolute-valued medians of the ΔCoVaR estimates.

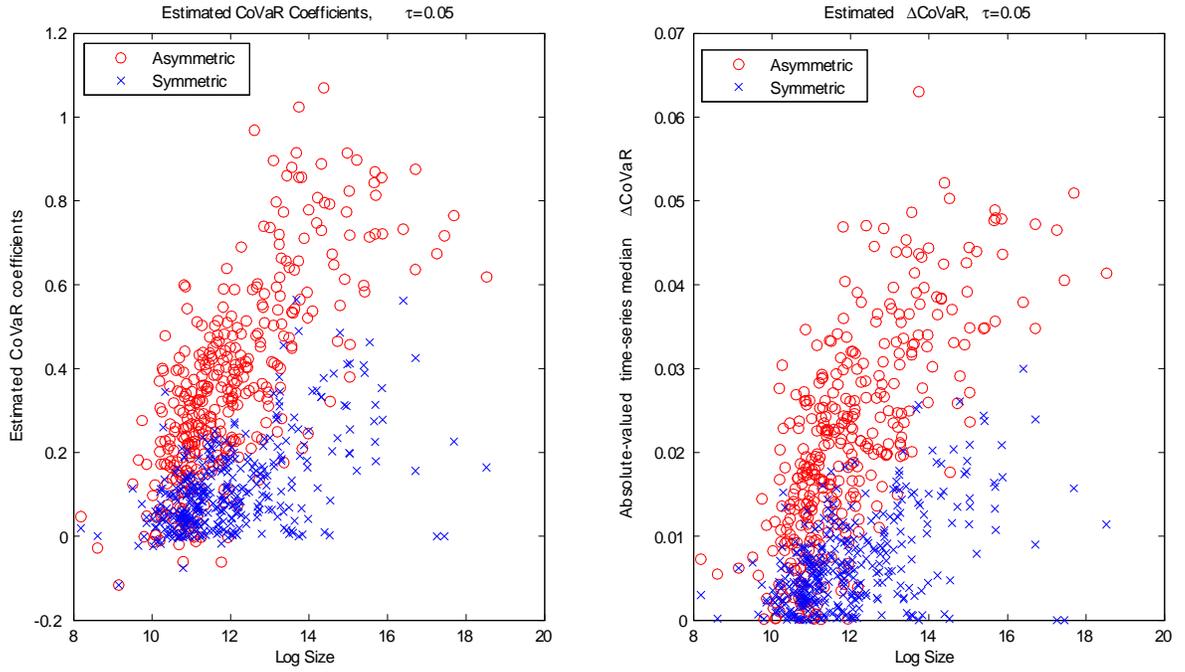


Figure 2. Cross-sectional median estimates of the decile-based coefficients $\{\delta_{i,j}(\tau)\}$ from the piecewise linear CoVaR (grey bars), median estimates of the symmetric CoVaR $\delta(\tau)$ coefficient (blue dashed-dotted line), and median of the asymmetric CoVaR coefficients $\delta^-(\tau)$ and $\delta^+(\tau)$ (red line and green dotted line, respectively) at $\tau = 0.05$.

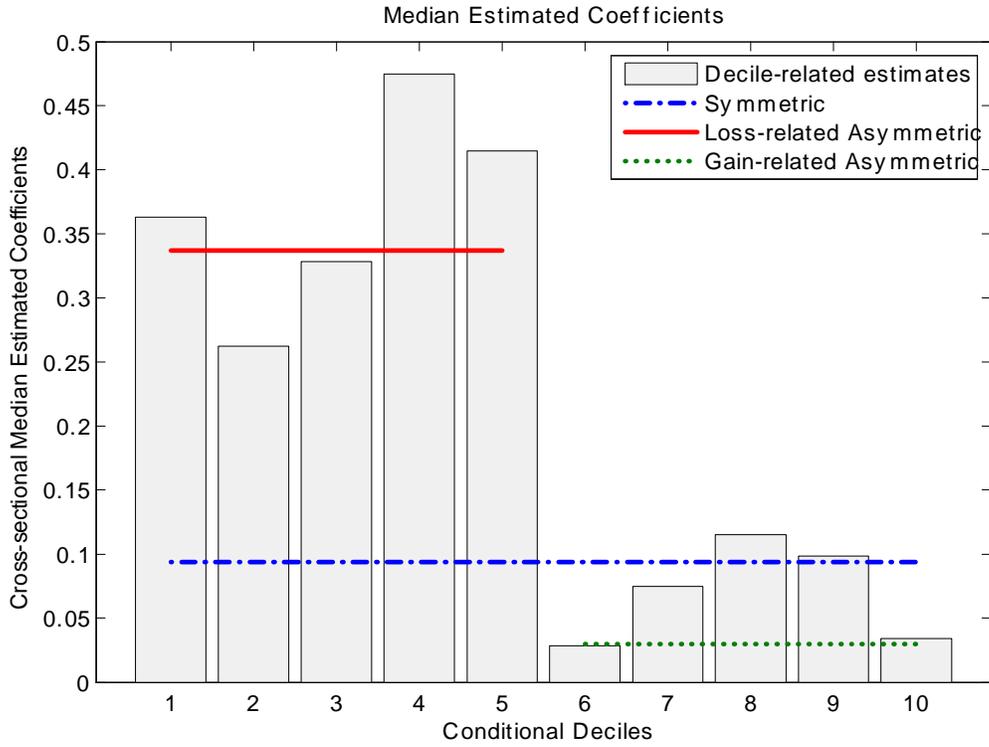


Table 1: Sample descriptives (sample mean, median, standard deviation, 99% and 1% percentiles) of accounting variables for the total sample, and the filtered sample after excluding firms with less than 500 observations. Leverage is measured as the ratio Total Assets to Equity. BCH and CB stands for bank holding companies and commercial banks, respectively. Accounting variables are denominated in \$ thousands.

	Total Sample					Filtered Sample				
	Total Assets	Liabilities	Equity	Market Value	Leverage	Total Assets	Liabilities	Equity	Market Value	Leverage
<u>All Banks</u>										
Mean	15,443,019	14,178,811	1,264,208	2,211,305	12.13	20,374,562	18,698,136	1,676,426	2,962,483	11.86
Median	1,114,213	1,018,717	96,350	146,790	11.74	1,610,327	1,475,962	131,441	220,165	11.62
St. Dev.	100,825,099	92,767,851	8,217,582	12,244,563	24.45	121,613,611	111,889,724	9,908,967	14,702,676	29.08
99% Perct.	260,159,000	238,823,000	20,780,000	41,743,444	23.76	366,574,000	343,674,000	28,757,074	58,506,420	22.06
1% Perct.	112,224	100,229	9,156	6,006	5.65	152,800	135,470	13,275	8,856	5.69
<u>BHC</u>										
Mean	16,480,418	15,131,847	1,348,571	2,360,246	12.14	21,196,452	19,453,048	1,743,405	3,082,061	11.87
Median	1,231,930	1,128,742	104,957	162,026	11.77	1,688,911	1,550,991	138,469	232,312	11.63
St. Dev.	104,480,668	96,131,711	8,515,355	12,683,596	25.29	124,402,300	114,455,869	10,135,956	15,034,947	29.75
99% Perct.	269,013,000	249,459,509	21,563,000	43,657,467	23.59	396,045,000	367,511,000	30,358,000	61,366,280	22.02
1% Perct.	168,129	151,257	12,196	7,605	5.78	191,200	169,369	16,593	13,178	5.83
<u>CB</u>										
Mean	1,723,666	1,575,149	148,517	241,595	12.02	3,128,126	2,857,181	270,945	453,285	11.81
Median	405,464	371,356	33,895	44,914	11.36	530,789	473,585	45,640	66,428	11.23
St. Dev.	6,187,887	5,645,432	546,061	1,012,994	6.48	9,099,863	8,300,549	804,542	1,498,117	3.91
99% Perct.	39,603,076	35,827,592	3,737,268	6,622,792	27.00	50,519,153	46,127,538	4,480,191	8,749,758	22.75
1% Perct.	37,532	32,573	2,588	1,649	4.33	29,355	23,745	2,061	1,530	4.22

Table 2. Descriptive statistics of the economic and financial state variables used as predictive variables in the CoVaR and VaR modelling. The variables are the implied volatility index (VIX), the annualized market return of the S&P500, the change in the U.S. Treasury bill secondary market 3-month rate (ΔT -bill), the yield spread between the U.S. Treasury benchmark bond 10-year and the U.S. 3-month T-bill (Yield Slope), and the credit spreads between the 10-year Moody's seasoned Baa corporate bond and the 10-year U.S. Treasury bond (Default Premium). The descriptive statistics show the number of observations (Obs), mean, standard deviation (St.Dev.), minimum, maximum, first quartile (Q1), median, third quartile (Q3), and the first-order autocorrelation parameter (ρ_1).

	Obs.	Mean	St.Dev.	Min.	Max.	Q1	Median	Q3	ρ_1
VIX	1,096	20.554	8.318	9.700	80.060	14.560	19.255	24.295	0.939
Market Return (Annualized)	1,096	7.630	17.904	-13.852	13.827	-1.073	0.221	1.485	-0.116
ΔT -bill	1,096	-0.006	0.129	-1.530	1.420	-0.040	0.000	0.040	-0.193
Yield Slope	1,096	1.835	1.190	-0.625	3.920	0.775	1.792	2.842	0.991
Default Premium	1,096	2.271	0.827	1.112	6.402	1.687	2.050	2.654	0.986

Table 3: This table shows the median of parameter estimates across banks of the quantile regression models $X_{t,S} = Z'_{t-1} \gamma_i(\tau) + \delta_i(\tau) X_{t,i} + u_{\tau,t}$ (Symmetric CoVaR) and $X_{t,S} = Z'_{t-1} \gamma_i(\tau) + \delta_i^-(\tau) X_{t,i}^- + \delta_i^+(\tau) X_{t,i}^+ + u_{\tau,t}$ (Asymmetric CoVaR) at the 1% and 5% quantiles, the median of the robust t -statistics for individual significance (in parenthesis), and the median of the pseudo- \mathcal{R}^2 , with $X_{t,S}$ determined according to the multiple- or single-index system approach, and given the predictive variables in Table 2. The bottom part of the table shows the sample frequency of rejections at the 95% confidence level of the Wald tests for $H_{0,Ind} : \delta(\tau) = 0$ and $H_{0,Ind} : \delta^-(\tau) = \delta^+(\tau) = 0$, in the symmetric and asymmetric models, respectively, and those of $H_{0,Loss} : \delta^-(\tau) = 0$ and $H_{0,Sym} : \delta^-(\tau) = \delta^+(\tau)$ in the asymmetric CoVaR model.

	Multiple-index System						Single-index System					
	Symmetric CoVaR			Asymmetric CoVaR			Symmetric CoVaR			Asymmetric CoVaR		
	1%	5%	()	1%	5%	()	1%	5%	()	1%	5%	()
Constant	0.028 (3.90)	0.020 (3.16)	0.023 (2.82)	0.014 (2.50)	0.028 (3.64)	0.020 (3.16)	0.023 (2.76)	0.014 (2.50)	0.023 (2.76)	0.014 (2.50)	0.023 (2.76)	0.014 (2.50)
VIX	-0.026 (-6.26)	-0.002 (-6.57)	-0.002 (-5.79)	-0.002 (-5.82)	-0.003 (-6.16)	-0.002 (-6.54)	-0.002 (-5.65)	-0.002 (-5.78)	-0.002 (-5.65)	-0.002 (-5.78)	-0.002 (-5.65)	-0.002 (-5.78)
Market Return	0.007 (0.11)	0.141 (2.44)	-0.006 (-0.06)	0.114 (1.99)	0.007 (0.10)	0.140 (2.47)	-0.008 (-0.08)	0.113 (2.06)	-0.008 (-0.08)	0.113 (2.06)	-0.008 (-0.08)	0.113 (2.06)
ΔT -Bill	-0.003 (-0.28)	0.015 (1.37)	0.002 (0.21)	0.014 (1.27)	-0.004 (-0.27)	0.014 (1.36)	0.003 (0.20)	0.014 (1.29)	0.003 (0.20)	0.014 (1.29)	0.003 (0.20)	0.014 (1.29)
Yield Slope	0.002 (1.04)	0.002 (1.74)	0.003 (1.15)	0.003 (1.78)	0.002 (1.02)	0.003 (1.70)	0.002 (1.12)	0.003 (1.79)	0.002 (1.12)	0.003 (1.79)	0.002 (1.12)	0.003 (1.79)
Default Premium	-0.018 (-3.86)	-0.010 (-2.28)	-0.014 (-3.03)	-0.008 (-1.85)	-0.018 (-3.89)	-0.010 (-2.30)	-0.014 (-2.79)	-0.008 (-1.83)	-0.014 (-2.79)	-0.008 (-1.83)	-0.014 (-2.79)	-0.008 (-1.83)
NBER Recessions	0.017 (1.38)	0.011 (1.62)	0.017 (1.58)	0.009 (1.36)	0.017 (1.36)	0.011 (1.60)	0.017 (1.51)	0.009 (1.38)	0.017 (1.51)	0.009 (1.38)	0.017 (1.51)	0.009 (1.38)
Financial Recession	-0.014 (-2.32)	-0.029 (-5.68)	-0.013 (-1.66)	-0.025 (-4.66)	-0.015 (-2.22)	-0.030 (-5.53)	-0.013 (-1.61)	-0.025 (-4.64)	-0.013 (-1.61)	-0.025 (-4.64)	-0.013 (-1.61)	-0.025 (-4.64)
$X_{t,i}$	0.086 (1.34)	0.0941 (2.35)	-	-	0.089 (1.33)	0.094 (2.40)	-	-	0.089 (1.33)	0.094 (2.40)	-	-
$X_{t,i}^-$	-	-	0.372 (5.64)	0.337 (6.98)	-	-	0.374 (5.58)	0.341 (7.17)	-	-	0.374 (5.58)	0.341 (7.17)
$X_{t,i}^+$	-	-	0.048 (0.55)	0.030 (0.56)	-	-	0.050 (0.49)	0.031 (0.72)	-	-	0.050 (0.49)	0.031 (0.72)
Pseudo- \mathcal{R}^2	43.42%	28.95%	46.15%	30.57%	43.52%	29.02%	46.25%	30.70%	46.25%	30.70%	46.25%	30.70%
Frequency of rejections Wald Test												
Freq. Rej. $H_{0,Ind}$	42.47%	57.53%	88.49%	90.68%	42.74%	58.63%	87.67%	87.95%	42.74%	58.63%	87.67%	87.95%
Freq. Rej. $H_{0,Loss}$	-	-	82.47%	88.49%	-	-	81.10%	90.41%	-	-	81.10%	90.41%
Freq. Rej. $H_{0,Sym}$	-	-	66.19%	73.42%	-	-	61.10%	74.79%	-	-	61.10%	74.79%

Table 4: This table shows the median of parameter estimates across size-sorted deciles from the quantile regression models

$\bar{X}_{t,S} = Z'_{t-1}\gamma_i(\tau) + \delta_i(\tau) \bar{X}_{t,i} + u_{\tau,t}$ (Symmetric CoVaR) and $\bar{X}_{t,S} = Z'_{t-1}\gamma_i(\tau) + \delta_i^-(\tau) \bar{X}_{t,i} + \delta_i^+(\tau) \bar{X}_{t,i} + u_{\tau,t}$ (Asymmetric CoVaR) at the 1% and 5% quantiles, with $\bar{X}_{t,S}$ determined according to the multiple-index system approach. The table also reports the sample frequency of rejections at the 95% confidence level of the Wald tests for $H_{0,Ind} : \delta(\tau) = 0$ and $H_{0,Ind} : \delta^-(\tau) = \delta^+(\tau) = 0$, in the symmetric and asymmetric models, respectively, and $H_{0,Loss} : \delta^-(\tau) = 0$ and $H_{0,Sym} : \delta^-(\tau) = \delta^+(\tau)$ in the asymmetric CoVaR model. Finally, \mathcal{R}^2 denotes the median of the pseudo- \mathcal{R}^2 in these models, and $\Delta CoVaR$ denotes the cross-sectional median of the weekly medians of the $\Delta CoVaR$ estimates over the period at the corresponding quantiles.

		1% Quantile									
Size-sorted Deciles		Symmetric CoVaR					Asymmetric CoVaR				
	$\delta(\tau)$	$H_{0,Ind}$	\mathcal{R}^2	$\Delta CoVaR$	$\delta^-(\tau)$	$\delta^+(\tau)$	$H_{0,Ind}$	$H_{0,Loss}$	$H_{0,Sym}$	\mathcal{R}^2	$\Delta CoVaR$
Top	0.150	38.88%	39.80%	-0.013	0.816	0.095	100%	100%	80.56%	45.61%	-0.071
9th decile	0.129	38.89%	40.50%	-0.012	0.595	0.066	88.89%	91.67%	61.11%	42.26%	-0.050
5th decile	0.125	50.00%	48.48%	-0.013	0.409	0.057	86.84%	86.84%	73.68%	49.70%	-0.042
2nd decile	0.071	43.24%	48.72%	-0.007	0.164	0.018	83.78%	62.16%	67.57%	48.85%	-0.017
Bottom	0.060	37.84%	28.63%	-0.007	0.135	0.002	78.38%	70.27%	64.86%	30.92%	-0.023
Total sample	0.086	42.47%	43.42%	-0.011	0.372	0.048	88.49%	82.47%	66.19%	46.15%	-0.043

		5% Quantile									
Size-sorted Deciles		Symmetric CoVaR					Asymmetric CoVaR				
	$\delta(\tau)$	$H_{0,Ind}$	\mathcal{R}^2	$\Delta CoVaR$	$\delta^-(\tau)$	$\delta^+(\tau)$	$H_{0,Ind}$	$H_{0,Loss}$	$H_{0,Sym}$	\mathcal{R}^2	$\Delta CoVaR$
Top	0.276	88.89%	29.66%	-0.015	0.726	0.075	100%	100%	97.22%	36.94%	-0.039
9th decile	0.181	64.86%	27.29%	-0.011	0.574	0.051	100%	100%	97.30%	31.08%	-0.033
5th decile	0.114	67.57%	29.02%	-0.008	0.352	0.030	100%	97.30%	86.49%	30.68%	-0.022
2nd decile	0.044	32.43%	29.39%	-0.003	0.165	0.005	70.27%	64.86%	64.86%	29.51%	-0.010
Bottom	0.041	29.73%	17.02%	-0.003	0.126	0.012	67.57%	62.16%	56.76%	17.21%	-0.008
Total sample	0.094	57.53%	28.95%	-0.006	0.337	0.030	90.68%	88.49%	73.42%	30.57%	-0.020

Table 5: This table shows the median of parameter estimates across liabilities-sorted deciles from the quantile regression models

$X_{t,S} = Z'_{t-1} \gamma_i(\tau) + \delta_i(\tau) X_{t,i} + u_{\tau,t}$ (Symmetric CoVaR) and $X_{t,S} = Z'_{t-1} \gamma_i(\tau) + \delta_i^-(\tau) X_{t,i} + \delta_i^+(\tau) X_{t,i} + u_{\tau,t}$ (Asymmetric CoVaR) at the 1% and 5% quantiles, with $X_{t,S}$ determined according to the multiple-index system approach. The table also reports the sample frequency of rejections at the 95% confidence level of the Wald tests for $H_{0,Ind} : \delta(\tau) = 0$ and $H_{0,Ind} : \delta^-(\tau) = \delta^+(\tau) = 0$, in the symmetric and asymmetric models, respectively, and $H_{0,Loss} : \delta^-(\tau) = 0$ and $H_{0,Sym} : \delta^-(\tau) = \delta^+(\tau)$ in the asymmetric CoVaR model. Finally, \mathcal{R}^2 denotes the median of the pseudo- \mathcal{R}^2 in these models, and $\Delta CoVaR$ denotes the cross-sectional median of the weekly medians of the $\Delta CoVaR$ estimates over the period at the corresponding quantiles.

Liabilities-sorted Deciles	1% Quantile											
	Symmetric CoVaR						Asymmetric CoVaR					
	$\delta(\tau)$	$H_{0,Ind}$	\mathcal{R}^2	$\Delta CoVaR$	$\delta^-(\tau)$	$\delta^+(\tau)$	$H_{0,Ind}$	$H_{0,Loss}$	$H_{0,Sym}$	\mathcal{R}^2	$\Delta CoVaR$	
Top	0.136	35.14%	38.38%	-0.012	0.827	0.088	100%	100%	81.08%	45.19%	-0.071	
9th decile	0.150	40.28%	40.54%	-0.014	0.622	0.069	94.59%	94.59%	72.97%	41.79%	-0.056	
5th decile	0.073	32.43%	43.35%	-0.010	0.353	0.028	86.49%	83.78%	62.16%	44.39%	-0.037	
2nd decile	0.063	50.00%	49.05%	-0.007	0.190	0.011	80.00%	65.00%	62.50%	49.91%	-0.019	
Bottom	0.067	41.18%	28.88%	-0.008	0.183	0.019	79.41%	67.65%	61.76%	31.55%	-0.024	
Total sample	0.086	42.47%	43.42%	-0.011	0.372	0.048	88.49%	82.47%	66.19%	46.15%	-0.043	

Liabilities-sorted Deciles	5% Quantile											
	Symmetric CoVaR						Asymmetric CoVaR					
	$\delta(\tau)$	$H_{0,Ind}$	\mathcal{R}^2	$\Delta CoVaR$	$\delta^-(\tau)$	$\delta^+(\tau)$	$H_{0,Ind}$	$H_{0,Loss}$	$H_{0,Sym}$	\mathcal{R}^2	$\Delta CoVaR$	
Top	0.255	89.19%	28.48%	-0.015	0.721	0.062	100%	100%	97.30%	35.25%	-0.038	
9th decile	0.180	72.97%	27.22%	-0.011	0.552	0.067	100%	100%	94.59%	30.05%	-0.033	
5th decile	0.085	62.16%	28.83%	-0.005	0.245	0.029	97.30%	91.89%	70.27%	30.40%	-0.016	
2nd decile	0.044	43.24%	29.90%	-0.003	0.163	0.017	72.97%	67.57%	64.86%	30.46%	-0.011	
Bottom	0.039	32.43%	17.83%	-0.002	0.172	0.017	70.27%	67.57%	43.24%	18.26%	-0.008	
Total sample	0.094	57.53%	28.95%	-0.006	0.337	0.030	90.68%	88.49%	73.42%	30.57%	-0.020	

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