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An Application of the "Fan-Chart Approach" to Debt Sustainability in Post-HIPC Low-Income Countries

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A "Fan-Chart Approach" to Debt Sustainability in Post-HIPC Low-Income Countries

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Abstract

We analyse the debt dynamics in countries that benefited from the HIPC/MDRI debt relief initiatives with a view to applying a probabilistic approach to estimating future debt paths for those countries. We extend the probabilistic approach to public debt sustainability analysis (DSA) proposed by Celasun et al. (2006). This required addressing the twin challenges of a the time period that is too short to conduct country-by-country estimations and the presence, suggested by econometric evidence, of a break–point around 2006 in the dynamics of debt accumulation. To overcome the data limitations, we pool the data and estimate a panel VAR, thus taking advantage of the large cross–section. To account for the break–point, while applying a probabilistic approach to forecasting debt paths, we use the post–break–point information so as not to bias the forecasts of debt paths. As an illustration of the approach we apply the methodology to eight countries with different debt profiles.

JEL Classification Numbers:

Keywords: HIPC, MDRI, fan chart, debt sustainablity.

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I. Introduction

1. In 1996, the IMF and the World Bank jointly launched the Highly Indebted Poor Countries (HIPC) Initiative to help relieve the external debt burden which had become unsustainable in a number of low-income countries (LICs), mostly in Africa. The HIPC Initiative involved coordinated action by the international financial community, including multilateral institutions, to reduce the external debt burden of these countries. The HIPC Initiative complemented traditional debt-relief mechanisms, concessional financing, and the pursuit of sound economic policies.

2. The HIPC Initiative marked a significant step up from traditional debt–relief mechanisms for LICs and introduced innovations in the treatment of LICs' debt, such as a systematic treatment of multilateral debt and a focus on poverty reduction. This Initiative was enhanced in 1999 to provide deeper, broader, and faster debt relief to eligible members.[2] The enhancements also aimed to strengthen the links between debt relief and poverty reduction, particularly through social policies. In 2005 the HIPC Initiative was supplemented by the Multilateral Debt Relief Initiative (MDRI). The MDRI added a possibility of debt relief from multilateral institutions—the IMF, the World Bank, and the African Development Fund (AfDF)—for countries completing the HIPC Initiative process.[3]

3. Both initiatives were broadly successful, with the debt levels of participating countries having declined to sustainable levels, and only a few eligible countries are yet to reach their completion point. That said some renewed concerns about the debt accumulation by the LICs have been made. For example, Leo (2009) worried about the pace of new lending by the IFIs in the context of generally favorable debt sustainability assessments, which could result in unwitting re-accumulation of unsustainable debt stocks. He suggested "assertive and corrective action" from the international community lest it be faced with "the prospect of a HIPC IV agreement in not too distant future."

4. The IMF and World Bank developed the Debt Sustainability Analysis (tool), as part of the Debt Sustainability Framework (DSF) to assess the debt situation and outlook of low-income countries (LICs). Because of its forward-looking nature and built-in robustness checks, the DSA may serve as an early warning signal for a potentially unsustainable buildup of external and public debt. The Fund/Bank's DSF (and DSA) has undergone a number of modifications over the years and was last reviewed in 2012 (see IMF 2003 and 2012)This review concluded that the experience with the DSF to date suggested that it had performed relatively well. Nevertheless, the framework also has its critics, which is due in part to the uncertainty that policy frameworks and economic environments experience

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[2] Further information on HIPC is available via Internet on IMF.org at the following link: http://www.imf.org/external/np/exr/facts/hipc.htm.

[3] Further information on MDRI is available via Internet on IMF.org at the following link: http://www.imf.org/external/np/exr/facts/mdri.htm.

through common shocks and other unpredictable events. For example, some analysts have qualms about the quality of inputs entering the DSAs (Baduel and Price (2012)) or prefer applying a systematic probabilistic approach (for example Garcia and Rigobon (2005), Celasun, et al. (2006)). Others stressed unavoidable imprecision of DSAs owing to inherent uncertainty DSAs try to model (Panizza (2008), Wyplosz (2007, 2011).

5. This paper contributes to the literature through analysis of debt dynamics in countries that were beneficiaries of the HIPC and the MDRI. The starting point is to apply the probabilistic approach to the DSA proposed by Celasun, et al. (2006) and the insights from Hevia (2012). Our econometric analysis suggests that the implementation of HIPC/MDRI may have coincided, or marked, a structural break in the dynamics of debt accumulation for HIPC/MDRI countries. For the purposes of this study it causes the available sample to be considerably shorter, as not accounting for this break would bias the coefficient estimates that are used to forecast the debt paths. In line with the literature, our analysis includes a role in the dynamics of public debt stemming from real GDP growth, real interest rates, capital gains/losses on debt denominated in foreign currency owing to exchange rate changes, primary fiscal deficits, and debt relief. We refer to the first three variables as the "deep debt determinants" because they affect the endogenous non-fiscal debt dynamics. Our specification also includes exogenous variables that affect HIPCs. In a first step, we estimate the relationship between the "deep" determinants and debt by applying a vector autoregressive (VAR) model. The data's short time dimension (2001–11) and fairly large cross-section (33 countries) poses a challenge,[4] which we resolve by pooling the data and estimating a panel VAR. Next, we estimate a generic fiscal reaction function to account for fiscal behavior, as in Celasun et. al (2006). Finally, we simulate sets of random shocks for each of the determinants of debt and the primary deficit and use these to forecast trajectories of the deep determinants, the primary balance, and debt. To illustrate this approach, the probabilistic forecasts of debt paths are arranged in "fan" charts for eight countries with different debt profiles (Bolivia, Burkina Faso, Ethiopia, Ghana, Guyana, Honduras, Nicaragua, and Senegal).[5]

6. The remainder of the paper is organized as follows. Section II describes the data and the evolution of public debt for 33 HIPCs between 2001 and 2011. Section III explains the panel data estimation methodology. Section IV discusses the estimation results. Section V puts it all together through presentation and analysis of fan chart forecasts. Section VI concludes.

[4] Our dataset with data on the primary fiscal balance, composition of public debt (domestic and foreign), and interest payments starts only from 2001.

[5] See Arizala et al. (2008) for an introduction to the application of fan charts to debt sustainability analysis.

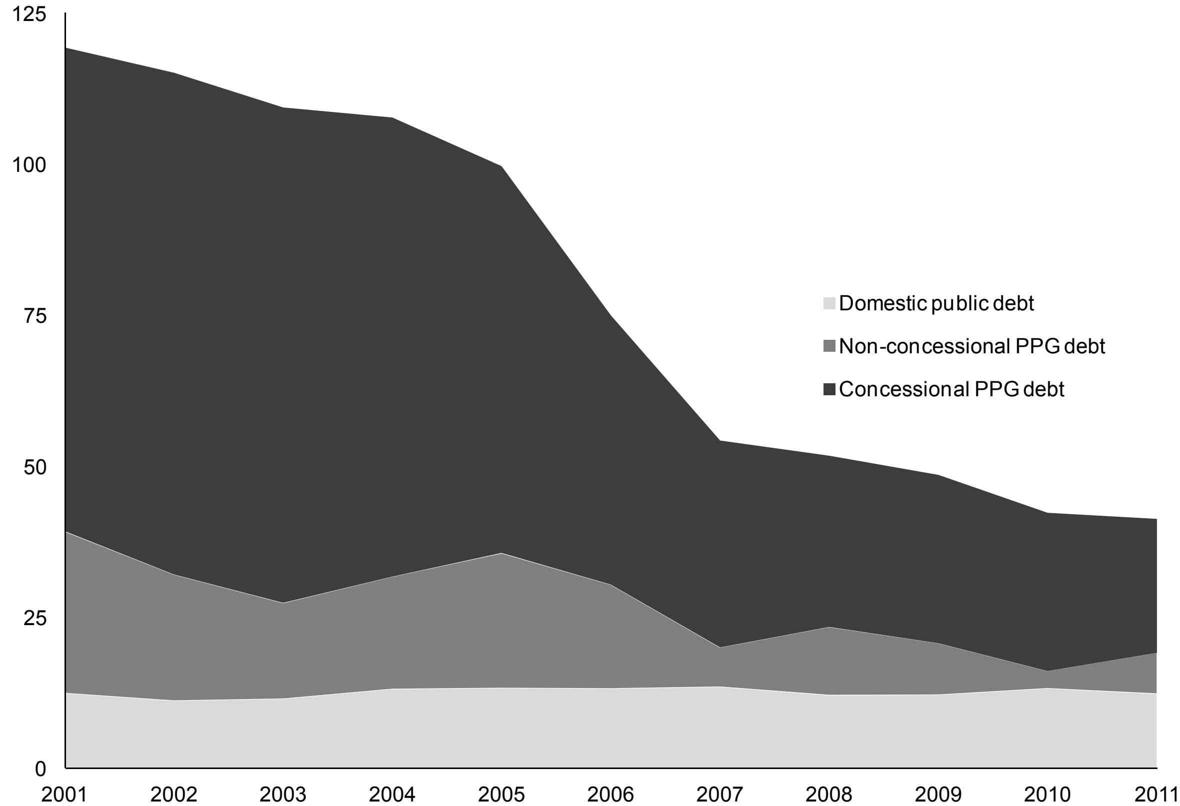
II. Some Stylized facts: Trends and composition of Public debt in HIPCs

7. Figure 1 shows the evolution of average public debt for 33 countries that were HIPC beneficiaries. One observes a striking difference in the debt levels pre-2005 and post-2007. The average public debt ratio decreased from 120 percent of GDP in 2001 to 54 percent in 2007, and then to 41 percent in 2011, the final year of our data set. The sharp decline in the debt levels is thanks in large part to debt relief provided by the HIPC initiative and the MDRI. Indeed, by 2006, 21 out of the 33 countries in our sample (accounting for more than three-fourths of the total debt in our dataset) had reached their completion points and had qualified for 100 percent relief on eligible debt from the IMF, the International Development Association of the World Bank (IDA), and the African Development Fund (AfDF). Another eight HIPIC/MDRI beneficiaries reached their completion points between 2007 and 2010, and by the end of 2011, only Comoros, Côte d'Ivoire, Guinea, and Chad had not yet reached their completion points.[6]

[6] Comoros reached the completion point in December 2012, Côte d'Ivoire in June 2012, Guinea in September 2012.

Figure 1. Average Public Debt for Countries Eligible for HIPC Initiative Relief

(In percent of GDP)



Source: IMF, Strategy, Policy and Review Department DSA Database and own calculations.

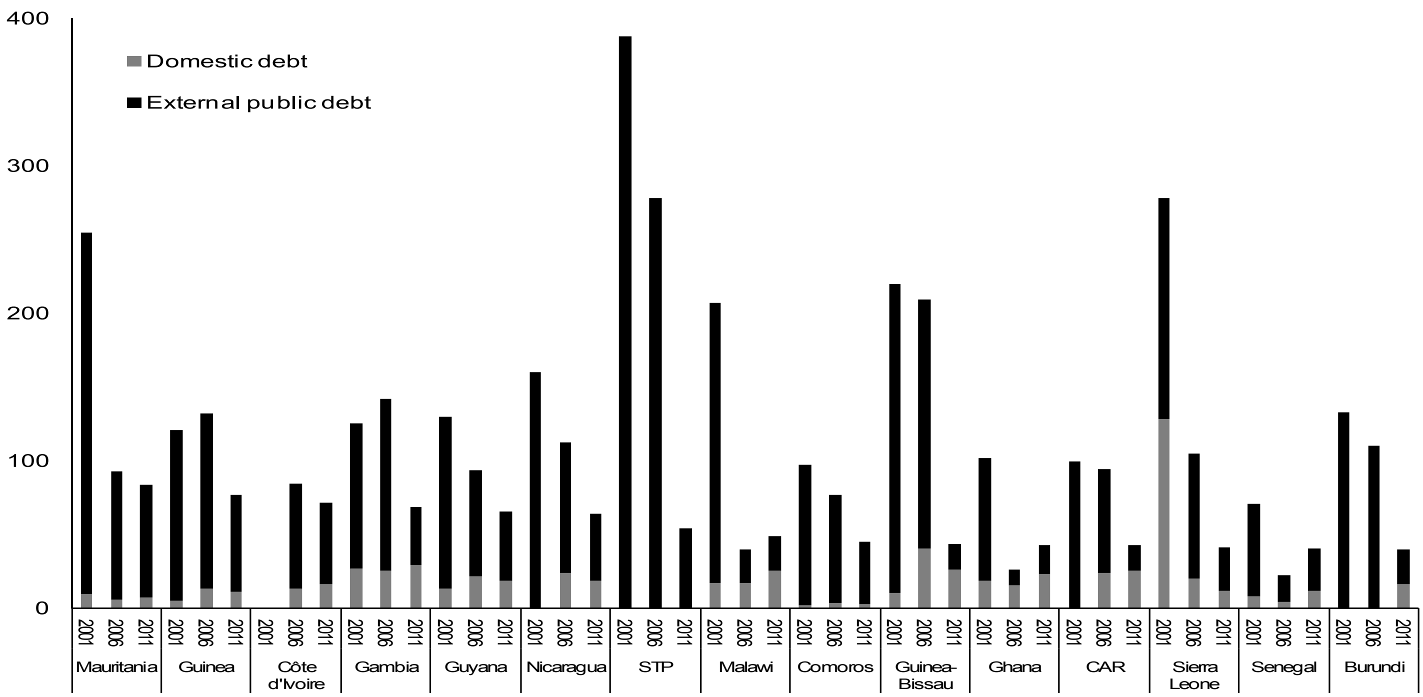
Note: Data available for 33 countries. Afghanistan, The Republic of the Congo, and Liberia are excluded due to limited data.

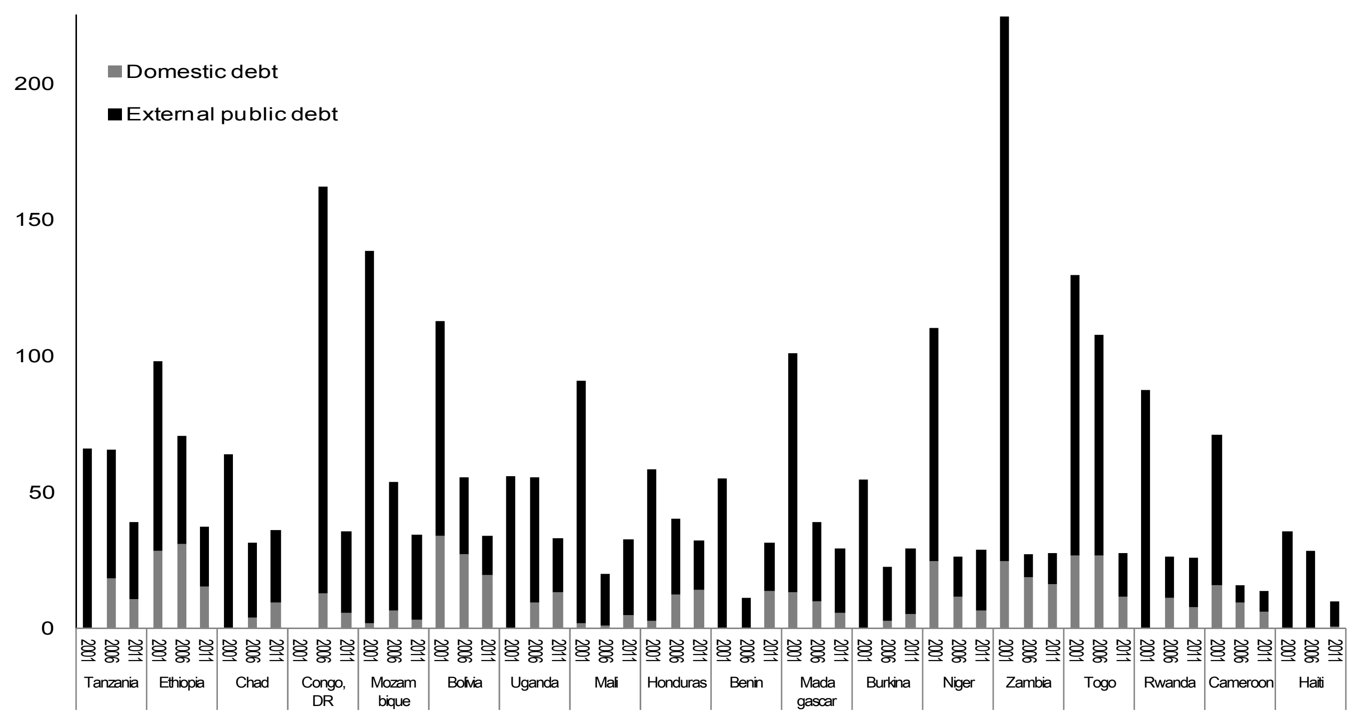
8. Regarding the composition of public debt, the average share of concessional public and publicly guaranteed (PPG) debt, which accounted for 67 percent of the total public debt of the 33 HIPCs in 2001, dropped to 63 percent in 2007,and further to 53 percent in 2011. The share of non-concessional debt declined as well, while the average share of the public domestic debt grew from about 10 to 30 percent of total during that time.

9. While public external debt in percent of GDP declined in almost all countries included in the sample (as a result of debt relief), domestic public debt did not decline in step (Figure 2). In 23 out of 33 countries, domestic public debt increased, from an average of 5 percent of GDP in 2001 to an average of 14 percent of GDP in 2011. In the remaining countries the average public debt ratio decreased from 34 to 10 percent of GDP.

Figure 2. Public Debt in Countries Eligible for HIPC Initiative Relief: Comparison of 2001, 2006, and 2011

(In percent of GDP)



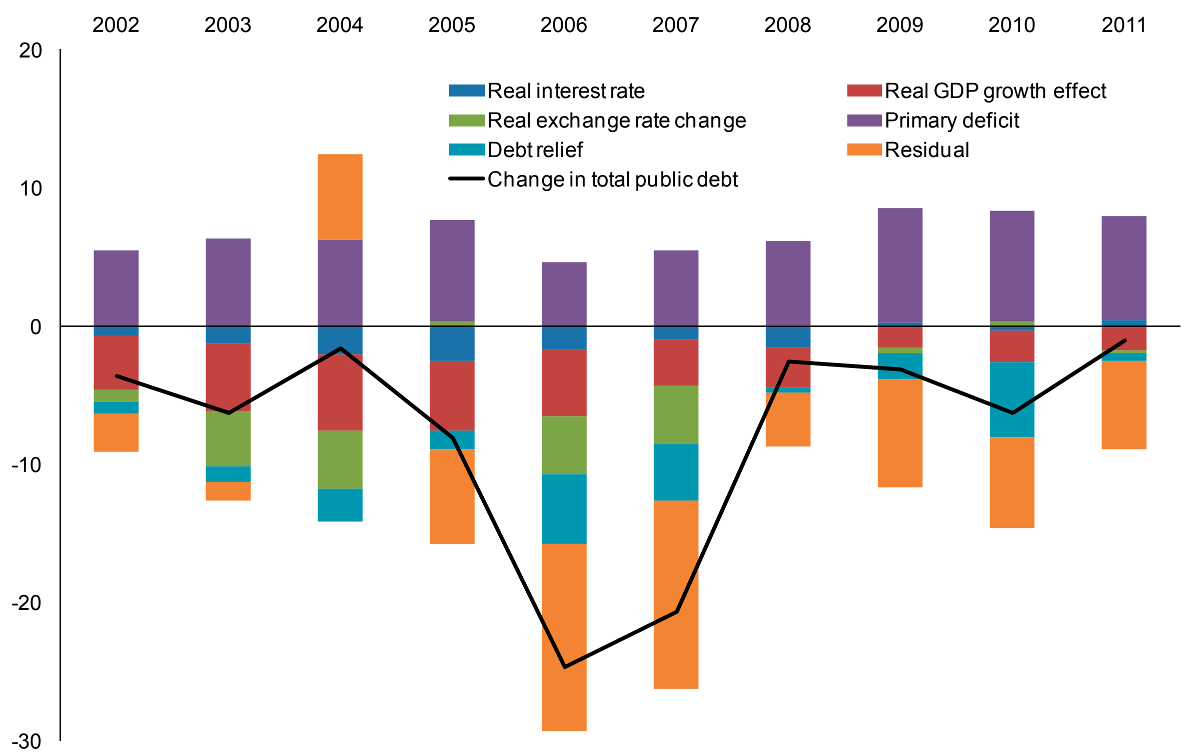


Source: IMF, Strategy, Policy and Review Department DSA Database and own calculations.

10. The determinants of the *dynamics* of public debt ratios are illustrated in Figure 3, which shows the decomposition of the ratio of public debt to GDP for the 33 HIPCs, again covering the period 2001–11. The decomposition indicates: (i) real GDP growth, real exchange rates, and real interest rates reduced the public debt-to-GDP ratio; (ii) primary fiscal deficits increased that ratio; and (iii) the contribution of debt relief increased in 2006 and 2007, the years following the launch of the MDRI. Turning to some details, average real GDP growth was 5 percent and accounts for a significant portion of the decline in the debt ratio in every year. Real interest rates were negative through 2008, mostly owing to low nominal interest rate on the concessional debt, which accounted for the majority of public debt. Real effective exchange rates appreciated through 2010, thus contributing to a decrease in the debt-to-GDP ratio. Finally, we note the significant contribution of the residual to debt reduction in 2006 and 2007, which comes as a surprise. While the exact reason is not clear, one possible explanation could be a mis-recording of debt relief after the HIPCs reached their completion points (e.g., of flow relief versus stock relief).

Figure 3. Average Change in total Public Debt for 33 HIPCs

(In percent)



Source: IMF, Strategy, Policy and Review Department DSA Database and own calculation.

III. Overview of the Methodology

11. This section introduces the methodology to: (i) determine whether there is a structural break in the dynamics of debt accumulation for HIPC/MDRI countries and (ii) apply the probabilistic approach to forecast debt paths of HIPC/MDRI beneficiaries.

12. Recall that the evolution of the nominal public debt follows the equation:

$$D_t = D_{t-1} + \left(i_t^d \frac{D_{t-1}^d}{D_{t-1}} + i_t^f \frac{D_{t-1}^f}{D_{t-1}} \right) D_{t-1} + \Delta e_t \left(1 + i_t^f\right) D_{t-1}^f - PB_t - DR_t + O_t$$

In this equation, D_t is the total nominal public debt in local currency at time t, D_t^d and D_t^f are the domestic and external public debt respectively, i_t^d is the (implicit) nominal interest rate on domestic public debt,[7] i_t^f is the (implicit) nominal interest rate on external public debt, Δe_t is the rate of depreciation of the nominal exchange rate, PB_t is the primary balance, and O_t captures other debt-creating flows such as recapitalization costs, privatization receipts, contingents liabilities, treasury float, debt relief, and so on.

Dividing both sides of the equation by nominal GDP, and denoting shares of GDP by lower case letters, the expression becomes:

$$d_t = \frac{1}{1 + g_t} \left[d_{t-1} + \left(r_t^d \frac{d_{t-1}^d}{d_{t-1}} + r_t^f \frac{d_{t-1}^f}{d_{t-1}} \right) d_{t-1} + \Delta \epsilon_t \left(1 + r_t^f\right) d_{t-1}^f \right] - pb_t + o_t$$

where g_t is the real GDP growth, nominal interest rates are replaced by real interest rates (r_t^d and r_t^f), and depreciation of the nominal exchange rate (Δe_t) is replaced by depreciation of the real exchange ($\Delta \epsilon_t$).

13. While it is apparent that the HIPC and MDRI Initiatives significantly reduced debt–to–GDP ratios levels in HIPC-eligible countries, it is possible that these initiatives may also have had an impact on the relationship among the "deep" determinants (g_t, r_t^d, r_t^f, and $\Delta \epsilon_t$), the primary balance, and the debt levels, respectively. Indeed, the policy reforms surrounding the provision of debt relief aimed to create the conditions for sustained higher growth and reduction in poverty that would also discourage a renewed non-sustainable accumulation of debt in the future. We leave the nature of such changes among macroeconomic variables from the HIPC/MDRI era for others to explore. For the purposes of this paper, we look for

[7] Implicit interest rates are computed using the formula $i_t = \frac{I_t}{D_{t-1}}$, where I_t is the total amount of interest payments on initial-period debt.

early evidence of that changing relationship by testing for an existence of a break–point around the granting of the debt relief.[8]

14. An obvious way to test for such a break-point would be to estimate a VAR model for the "deep" determinants in each country and then conduct the test around each country's completion point. Unfortunately, this is not possible owing to data limitations—with only 11 annual observations for each country, and these countries having reached their completion points at different points in time. To resolve these difficulties, we adopt a pragmatic solution by estimating a panel VAR model, as such a common approach allows us to compensate for the small time series problem and take advantage of the cross-section of 33 countries.[9] The estimation of a panel VAR is the first step of our procedure; it will be followed by testing for a possible structural break and estimating a fiscal reaction function.

15. The first step is to formulate the following panel VAR specification with exogenous regressors (PVARX):

$$Y_{it} = \alpha_i + \Theta Y_{it-1} + \Phi X_{it} + \varepsilon_{it}, i = 1, \ldots, n; t = 1, \ldots, T \quad (1)$$

with $Y_{it} = (g_{it}, r_{it}^d, r_{it}^f, \Delta\epsilon_{it})'$, $X_{it} = (g_t^w, r_t^w)$ where, g_{it} is the real GDP growth, r_{it}^d is the real interest rate on domestic debt, r_{it}^f is the real interest rate on foreign debt, $\Delta\epsilon_t$ is the change in real exchange rate, α_i is the country-specific fixed effect, g_t^w denotes the world real GDP growth rate, and r_t^w denotes the world real interest rate. X_{it} contains exogenous variables that are likely to affect the endogenous variables (Y_{it}). The error terms ε_{it} are i.i.d. shocks satisfying $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}\varepsilon_{it}') = \Omega$ for all i and t, $E(\varepsilon_{it}\varepsilon_{is}') = 0$ for all i and $t \neq s$, and $E(\varepsilon_{it}\varepsilon_{js}') = 0$ for all s, t and $i \neq j$. The slope coefficients Θ and Φ and the covariance matrix Ω are common across countries.This use of a PVARX specification is novel, as the literature on probabilistic approach to the public DSA commonly has so far used only fully endogenous VAR regressions. In addition, when estimating PVARX we implement a version of the bootstrap bias correction algorithm originally proposed by Pesaran and Zhao (1999) and later extended by Everaert and Pozzi (2007). This allows us to address the well-known complication of the presence of fixed effects for the estimation and inference in linear dynamic panel data models, as is the case in equation (1). Indeed, the presence of fixed effects generally causes the ordinary least square (OLS) fixed-effects estimator to be inconsistent when the time dimension of the panel is short.

[8] A related question would be why (or how) the debt relief initiatives structurally modified the way debt is accumulated in LICs, for example, possibly through a modified debt-growth relationship, a change in the debt dynamics, or through modification of other relationships. Depetris and Kraay (2005), Presbitero (2009), and Johansson (2010) do not find evidence that debt relief has been associated with much stronger growth; Marcelino and Hakobyan (2014) report results that debt relief did associate with higher growth.

[9] As noted earlier, Afghanistan, The Republic of Congo, and Liberia are excluded due to data limitations.

16. The second step of the methodology is to test for a structural break in the dynamics of the determinants of debt accumulation in HIPCs, specifically, around the time of the MDRI. While the PVARX specification helps to overcome the limitations posed by the short time series dimension of data, it does not by itself resolve the issue of countries reaching their completion points at different times. Our approach is to test for a common break–point, and we chose 2006 as a potential candidate for the common break–point year for our sample. Some 21 out of the 33 HIPCs in the sample had reached their completion points before 2006 and the dynamics of public debt shown in Figures 1–3 intuitively indicate that 2006 would be a good candidate year for testing for the presence of a break–point in debt dynamics of HIPCs.

17. While we are not aware of any established procedure to test for a break–point in a PVARX specification, nonetheless, a Chow test for a VAR model can be easily generalized and extended to the PVARX model. Specifically, we use the "split sample" (SS) test, which assumes that the covariance matrix of the error term is unchanged over the sample period. Intuitively, this seems reasonable, as the possible shocks that HIPCs have faced since the postulated break–point may be considered broadly unchanged even if the debt accumulation may follow different dynamics post–2006. Nevertheless, we do consider the "break–point" (BP) test, as it accounts for the probability that the covariance matrix of the error term may have changed over the sample period. The results of SS and BP tests are presented in Table 3 in the Appendix. Both tests had rejected the null hypothesis of constant parameters over the whole period (2001–11) for the 33 HIPCs and for the 21 HIPCs that reached their completion points before or during 2006.

18. When interpreting the results of the SS or BP tests, one needs to keep in mind that the tests only confirm there was a significant change in the dynamics of the "deep" determinants of the public debt. It is of course possible that factors other than HIPC/MDRI could have caused the changes in the dynamics. This is an interesting area for further study, but one which we do not pursue in this paper. In any case, the evidence suggests that the post–2006 dynamics of debt accumulation for HIPC/MDRI beneficiaries have changed and, therefore, forecasts of debt accumulation for these countries need to account for that.

19. The third and the final step in the methodology is the estimation of a fiscal reaction function, which ties the primary balance with public debt. The relevance of a fiscal reaction function for the dynamics in a public DSA stems from the fact that governments concerned with solvency can be expected to raise the primary balance in response to an increase in the public debt-to-GDP ratio. We estimate the fiscal reaction function for 2007–11 to capture the relationship between the post–break debt dynamics and the fiscal position.[10]

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[10] In other words, we make sure that we are not capturing the reaction of primary balances to changes in debt–to–GDP ratios coming from debt relief.

20. Following Celasun et al. (2006), we estimate the general specification given by:[11]

$$pb_{it} = \gamma_0 + \rho d_{it-1} + \delta\, ygap_{it} + cpia_{it}\beta + \mu_i + \varepsilon_{it}, i = 1, \dots, n; t = 1, \dots, T \qquad (2)$$

where pb_{it} is the ratio of the primary balance to GDP in country i and year t, d_{it-1} is the public debt to GDP ratio observed at the end of year $t-1$, $ygap_{it}$ is demeaned real GDP growth (used to approximate an output gap), μ_i is an unobserved, country-specific fixed effect, and $cpia_{it}$ is the World Bank's CPIA index used as a measure of the countries' policy and institutional quality. The error terms ε_{it} are i.i.d. shocks satisfying $E(\varepsilon_{it}) = 0$, $E({\varepsilon_{it}}^2) = \Omega$ for all i and t, $E(\varepsilon_{it}\varepsilon_{is}) = 0$ for all i and $t \neq s$, and $E(\varepsilon_{it}\varepsilon_{js}) = 0$ for all s, t and $i \neq j$. The estimation of equation (2) needs to take into account two sources of endogeneity. First, d_{it-1} might be correlated with the country-specific fixed effect (μ_i) since countries with higher fiscal surpluses will, on average, tend to have a lower level of public debt. And second, the output gap may be correlated with the contemporaneous fiscal policy shocks ε_{it}. To correct these potential endogeneity problems we use the system GMM estimator of Arellano and Bover (1995)/Blundell and Bond (1998) estimator.

We also considered a nonlinear fiscal reaction function which allows for a differentiated response to debt accumulation conditioned on the debt to GDP ratio exceeding 50 percent of GDP, and a differentiated effect of the output gap depending on the latter being pro– or counter–cyclical.[12] That specification is as follows:

$$pb_{it} = \gamma_0 + \rho d_{it-1} + \bar{\rho}[\![D_{it-1}(d]\!]_{it-1} - 50) + \delta\, ygap_{it} + \theta\, P_{it} ygap_{it} + cpia_{it}\beta + \mu_i + \varepsilon_{it}, \quad (3)$$

where D_{it-1} is a dummy variable that equals one if the debt in $\mathrm{t}-1$ exceeds 50 percent of GDP and P_{it} is a dummy variable that equals one if the output gap is positive (an expansion). Note that in this specification, ρ is interpreted as the change of a primary balance in response to a change in a lagged debt of less than 50 percent of the GDP, whereas $\rho + \bar{\rho}$ is interpreted as the change of a primary balance in response to a change in a lagged debt greater than 50 percent of the GDP. A similar interpretation applies to the coefficients on the output gap where we test whether fiscal policy reacts differently to different phases of the business cycle. This is captured by a "positive output gap" dummy variable that assumes a value of one when the output gap is positive.

[11] Celasun et al. (2006) provide details of specification and discuss the practical challenges of estimation of a fiscal reaction function.

[12] The 50 percent threshold corresponds to the DSF's threshold for Debt/GDP for countries with strong policy component http://www.imf.org/external/np/exr/facts/jdsf.htm

IV. Estimation results

A. PVARX estimation

21. Tables 1 and 2 in the Appendix report results for the panel VARX estimation, both for the full sample of 33 HIPCs and for the 21 HIPCs that reached their completion points before or during 2006. As discussed, these results describe the dynamics of determinants of public debt stemming from real GDP growth, real interest rates, capital gains/losses on debt denominated in foreign currency owing to exchange rate changes, primary fiscal deficits, and debt relief. We refer to the first three variables as the "deep debt determinants" because they affect the endogenous non-fiscal debt dynamics. Our specification also includes exogenous variables that affect HIPCs. The coefficients are estimated over 2007–11. The restriction of the estimation period to 2007–11 is owing to the results of SS and BP tests presented in Table 3 in the Appendix. The SS test had rejected the null hypothesis of constant parameters over the whole period (2001–11) for the 33 HIPCs and for the 21 HIPCs that reached their completion points before or during 2006. The BP test also supports this conclusion.

B. Fiscal reaction function estimation

22. Table 1 reports the estimation of the fiscal reaction function for the OLS, Least-Squares Dummy Variable (LSDV), and system GMM estimators. Recall, however, that the OLS and LSDV estimators are biased owing to the endogeneity problems mentioned in paragraph 20. All regressions contain a full set of time dummies to account for cross–sectional correlation across time. The first five specifications are estimated for 2001–11, while the final specification is estimated for 2007–11.

Table 1. Results of the estimation of the fiscal reaction function

Dependent variable: Primary Fiscal Balance						
	OLS	LSDV	System GMM (1)	System GMM (2)	System GMM (3)	System GMM (4)
Lagged debt	-0.037*** (0.006)	-0.004 (0.007)	-0.038* (0.020)	0.348** (0.163)	0.379** (0.191)	0.285** (0.138)
Output gap	-0.031 (0.112)	-0.038 (0.087)	-0.735** (0.319)	-1.080** (0.540)	-1.262* (0.736)	-1.156** (0.588)
Institutions	-0.010*** (0.004)	-0.011 (0.012)	0.002 (0.013)	0.001 (0.022)	0.007 (0.033)	0.015 (0.030)
Debt Spline (50 percent)				-0.424** (0.180)	-0.464** (0.208)	-0.306* (0.172)
Positive output gap					0.872 (1.603)	
Constant	-0.036** (0.016)	-0.030 (0.031)	-0.050 (0.038)	-0.190** (0.081)	-0.230* (0.125)	-.227** (0.107)
Country dummies	No	Yes	No	No	No	No
Time dummies	Yes	Yes	Yes	Yes	Yes	Yes
Hansen test (P-value)			0.53	0.39	0.39	0.85
AR(1) test (P-value)			0.22	0.21	0.23	0.23
AR(2) test (P-value)			0.74	0.97	0.90	0.77
Number of observations	330	330	231	231	231	198
Number of instruments			31	31	31	26

Note: * = significant at 10%; ** at 5%; *** at 1%. P-values for the test statistics are reported for the tests of over-identifying restrictions and serial correlation in the residuals of the difference equation in the System GMM regression. The Blundell-Bond (1998) System GMM regressions use the third lag of debt and the second lag of the output gap (GMM-style), in addition to the world GDP growth and the second-fourth lags of the terms of trade (IV-style).[13] The System GMM equation (2) does not include the dummy variable for the positive output gap. Regression (4) is estimated for 2007–11, others for 2001–11.

[13] In all regressions, the number of instruments is always less than the number cross-section units.

23. As we had already noted, our preferred specification in Table 1is equation (4). We arrive at this specification in several steps. We begin with correcting for endogeneity by applying a system GMM estimator and estimating system GMM (1) specification. The negative sign on lagged debt in system GMM (1) specification is puzzling: this would imply that, even in a period of intense debt relief coupled with macroeconomic adjustment, fiscal policy would be imprudent and exacerbate debt problems. Even as such fiscal expansions might occur in short time spans (e.g., the accommodative fiscal policy in response to the global financial crisis—an accommodation made possible in part by debt relief), a negative sign on lagged debt would not seem plausible over a long period of time. In addition, such unusual dynamics would likely apply only to countries with the levels of debt not exceeding certain "safe" thresholds — for example 50 percent of GDP. To test this conjecture, we introduce a dummy variable for countries with debt to GDP exceeding 50 percent of GDP (debt spline in Table 1).[14] The results are in the specification (2) in Table 1. As expected the sign on lagged debt is positive for countries with "low" levels of debt (<50 percent of GDP) and negative for those with "high" levels of debt (>50 percent of GDP).

24. Next, the system GMM specification (3) in Table 1 tests whether fiscal policy reacts differently to different phases of the business cycle. This is captured by a "positive output gap" dummy variable. A positive coefficient on this dummy variable would indicate that the HIPCs improve their fiscal position during booms. Because the estimated coefficient is not statistically significant we do not retain that variable in a set of explanatory variables.

25. Finally, we arrive at the results of our preferred specification in system GMM specification (4). The signs of the estimated coefficients in GMM system (4) are as in GMM system (2). Specifically, the coefficient on lagged debt is positive and the one on a "high" debt dummy is negative. The coefficient on the institutional quality is positive but insignificant which may be due to the limited variability in the variable.[15] Nonetheless, the sign of the coefficient suggests that countries with better institutional quality are able to foster a stronger fiscal position.

V. PUTTING IT ALL TOGETHER

A. Fan Charts Forecasts

26. This section presents the results of application of the methodology laid out above. In line with the steps outlined above, we produce "fan" charts of forecasts as follows:

(i) Use PVARX estimates to predict the trajectories of the "deep" determinants assuming the common distribution and co-movements among the variables in combination with a country – specific fixed effect;

[14] We used the formula $D_{it-1}.(d_{it-1} - 50)$ instead of $D_{it-1}.d_{it-1}$ because of multicollinearity problems.

[15] Information on CPIA is available via Internet at: http://go.worldbank.org/EEAIU81ZG0.

(ii) Predict primary balances using the System GMM equation (4) from Table 1. The fiscal reaction function predicts primary balances according to:

$$\widehat{pb}_{it+\tau} = \hat{\gamma}_0 + \hat{\rho} d_{it+\tau-1} + \hat{\delta}\, ygap_{it+\tau} + \hat{\theta} P_{it+\tau} ygap_{it+\tau} + \varphi_{it+\tau}, \text{ for } \tau = 1, \ldots, 5,$$

with $\varphi_{it+\tau}$ denoting a policy shock drawn from a mean-zero normal distribution with variance equal to the country specific variance of residuals;[16]

(iii) Put together the first two steps and forecast large number of dept paths combined in "fan" charts.

27. For illustrative purposes, Figure 4 shows fan charts of debt to GDP trajectories for eight HIPCs that reached their completion points in 2006 or before: Bolivia, Burkina Faso, Ethiopia, Ghana, Guyana, Honduras, Nicaragua, and Senegal. In the charts, the middle area marks the 20 percent confidence interval around the median projection (which is shown as an orange line for 2012–16). The fan "cone" represents a confidence interval of 80 percent around the median projection. We superimpose the IMF/WB's baseline DSA forecasts of debt-to-GDP ratios for the countries mentioned above on their fan charts. The IMF/WB baseline DSA forecasts are shown as red lines.

28. The fan chart forecasts may serve as a broad benchmark, or "confidence interval", for the DSA forecast. As seen in Figure 4, the baseline DSA forecasts for the countries in the illustrative example fall within the plausible ranges depicted in the fan charts. In addition, the fan charts forecasts may be interpreted as pointing out negative or positive developments when they cross a threshold that could be country- or policy-specific. For example if a ratio of 50 percent of debt to GDP were chosen as a policy threshold, then only Guyana, Nicaragua, and Senegal debt outlooks would be considered negative.

29. Additionally, the debt outlooks for "fan" chart forecasts may be used to get a broad sense of whether the DSA forecasts seem plausible. For example, in the case of Bolivia, Burkina Faso, and Ghana, the baseline DSA forecasts fall within the 20 percent confidence interval around the median fan chart forecast.[17] For those cases, the DSA and fan chart forecasts suggest overlapping median expectations of the debt sustainability. Where DSA and fan chart forecasts diverge, further information about the projected path may be warranted, as it could well be that the DSA forecasts use information not captured by the fan chart forecasts. In such cases, the comparison of the DSA forecast against the fan chart bands

[16] This specification is similar to the "constant" policy scenario of Celasun et al. (2006) and as (4) in Table 1 accounts for a nonlinear specification on debt.

[17] The country applications are included only for illustrative purposes. The reader should keep in mind that estimations use actual data only through 2011 and the baseline DSA estimates based on the same data available at the time of writing. The purpose is to illustrate and contrast the deterministic and the probabilistic approaches to debt sustainability analysis; these should not be mistaken for projections based on current and revised data that would currently be available.

could provide an additional frame of reference to consider the assumptions and policy expectations behind the DSA forecasts. For example, in the example of Guyana, the DSA forecast indicates a steady improvement in the debt sustainability outlook, while in the case of Honduras it is a steady deterioration.

30. One needs to keep in mind the limitations of probabilistic forecasts owing to the relatively short time for which the PVARX and the fiscal reaction function are estimated. Nevertheless similar limitations, although in less transparent way, constrain the deterministic forecasts. We also note that none of the approaches discussed here represent a fully consistent analytical framework to create projections. For example, they approaches do not explicitly link the public investments, which may be financed by debt, and the resulting growth which would allow for debt servicing capacity. For an alternative approach that uses a model-based framework incorporating such links, see Buffie et al. (2012).

B. Probabilistic Debt Sustainability Index

31. Another application of probabilistic forecasts is to interpret those as probabilities that specified events will occur. For example, the probabilistic forecasts can be used to calculate the proportion of forecasts exceeding a certain threshold in a specific year—resulting in the probability that a debt ratio will be higher/lower than the threshold in that year. Conversely, one can use the forecasts to calculate probabilities that a change in debt will not exceed a certain threshold, i.e., calculate probabilities of a non–accelerating debt accumulation or de–cumulation.

32. Celasun et al. (2006) calculate such probabilities and combine them in an index using the formula below:

$$\Pr(d_{t+\tau} < d_t) * [1 - \Pr(d_{t+\tau} > d_t + x)],$$

where x is a debt increase considered acceptable within the projection period compared to the starting period (in our case, public debt at the end of 2011). The first part of the formula calculates the probability of public debt to GDP falling below its initial level, τ periods ahead. The second part calculates the probability that the ratio of public debt to GDP would not exceed the starting ratio by more than x percentage points in the τ periods ahead (in the application x is set at 10 percent). These two values may be multiplied to form an index. Notice that the index itself has no probabilistic interpretation—it is simply used to pull together informational content of the probabilistic forecasts. Intuitively, higher values of an index indicate more desirable positions: the higher value combines a desirably high probability that the debt declines below the initial threshold with a desirably high probability that the increase in debt will not exceed the specified cap.

33. We follow Celasun et al. (2006) and calculate an index by multiplying the calculated values for the two probabilistic forecasts of debt developments. Table 2 reports values of the probabilistic debt sustainability index for our selection of post–HIPCs. Even though the index does not have any intrinsic meaning, it allows for relative comparisons and ranking of

countries. Table 2 shows that Bolivia, Ethiopia, Ghana, and Honduras would breach a threshold in the final year of the projection period, when a threshold of 0.4 is chosen for illustrative purposes.

Figure 4. Illustrative Fan Charts of Public Debt-to-GDP Ratio in Selected HIPCs

(In percent of GDP)

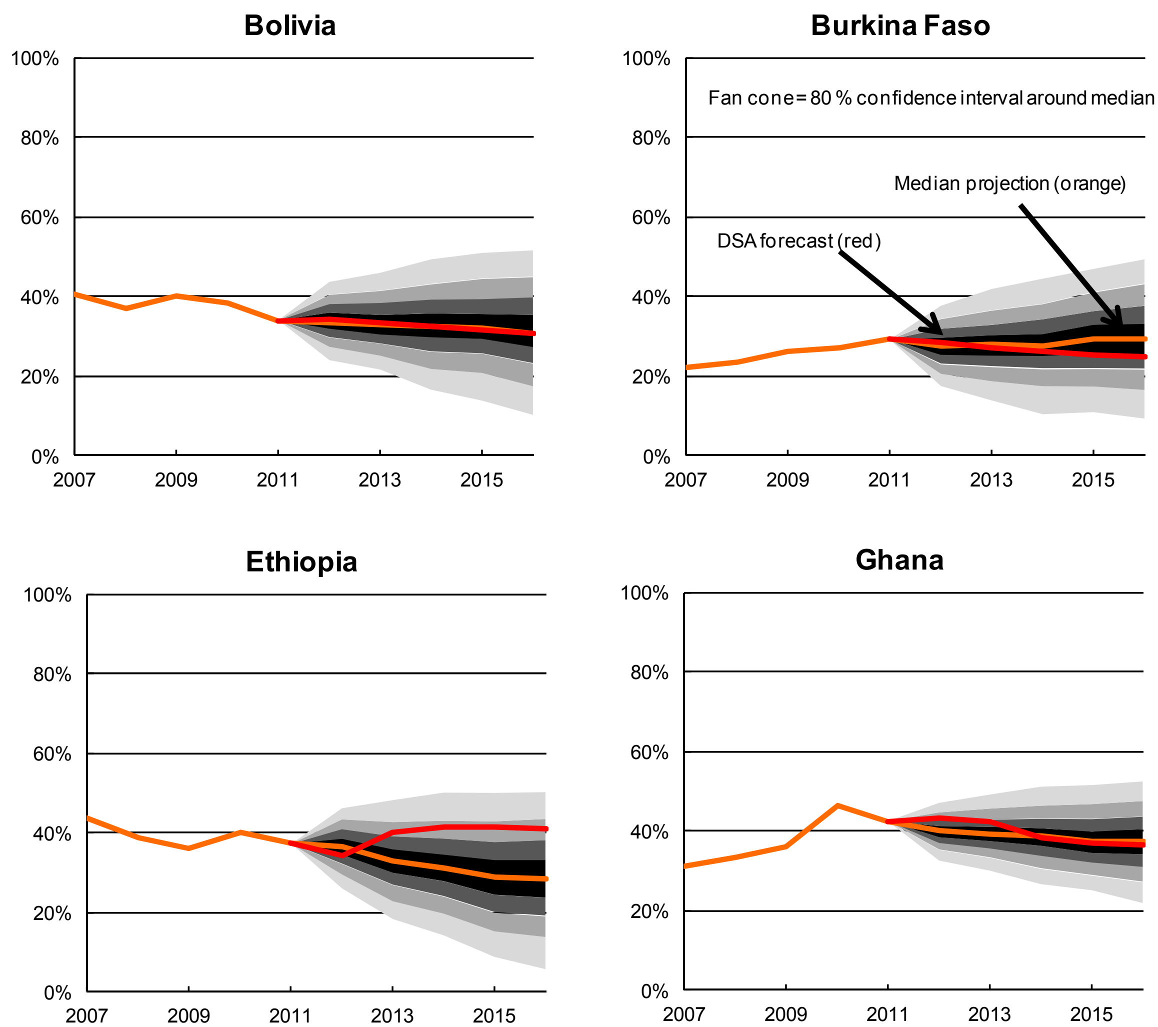
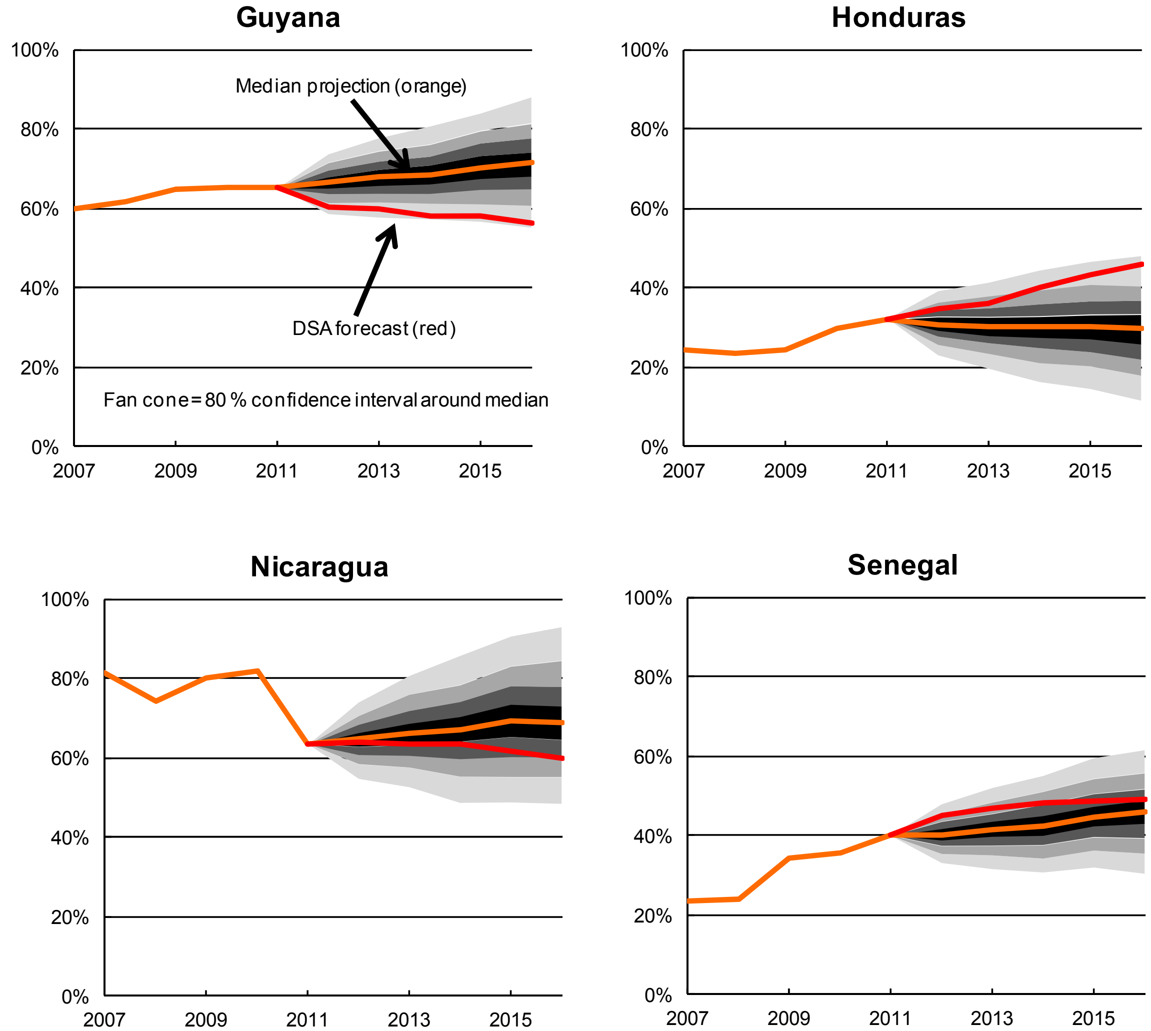


Figure 4. (concluded).



Source: IMF staff calculations. 2012 is the first year of projection period.

Table 2. Probabilistic Debt Sustainability Assessment—Illustration

	2012	2013	2014	2015	2016
	Debt ratio lower than in 2011				
Bolivia	0.51	0.54	0.55	0.54	0.57
Burkina-Faso	0.58	0.56	0.55	0.49	0.49
Ethiopia	0.54	0.64	0.68	0.69	0.68
Ghana	0.69	0.68	0.68	0.68	0.66
Guyana	0.40	0.37	0.36	0.31	0.31
Honduras	0.58	0.57	0.58	0.57	0.57
Nicaragua	0.44	0.41	0.39	0.37	0.38
Senegal	0.48	0.41	0.40	0.31	0.33
	Debt ratio more than 10 percent of GDP higher than in 2011				
Bolivia	0.09	0.14	0.18	0.21	0.22
Burkina-Faso	0.07	0.14	0.18	0.24	0.27
Ethiopia	0.08	0.12	0.13	0.13	0.15
Ghana	0.02	0.06	0.07	0.09	0.10
Guyana	0.06	0.18	0.23	0.35	0.37
Honduras	0.06	0.08	0.13	0.18	0.17
Nicaragua	0.12	0.26	0.32	0.40	0.39
Senegal	0.05	0.14	0.22	0.31	0.35
	Sustainability index 1/				
Bolivia	0.47	0.47	0.45	0.43	0.45
Burkina-Faso	0.54	0.48	0.45	0.38	0.36
Ethiopia	0.49	0.57	0.59	0.60	0.58
Ghana	0.68	0.64	0.63	0.62	0.60
Guyana	0.38	0.31	0.28	0.20	0.19
Honduras	0.55	0.53	0.50	0.47	0.47
Nicaragua	0.39	0.30	0.27	0.22	0.23
Senegal	0.46	0.35	0.31	0.22	0.21

1/ The index is calculated as follows: $\Pr(d_{t+\tau} < d_t) * [1 - \Pr(d_{t+\tau} > d_t + x)]$, where x is a debt increase considered to be acceptable within the projection period (see above).

VI. Conclusions

34. We set out to analyze the debt dynamics in countries that benefited from HIPC/MDRI debt relief with a view to apply a probabilistic approach to estimation of future debt paths of LICs that had benefited from HIPC/MDRI debt relief. One challenge is that the time period is too short to conduct country-by-country estimations. Another challenge is that the econometric evidence suggests the presence, around 2006, of a break–point in the dynamics of debt accumulation. As a result, a probabilistic approach to forecasting debt paths for those countries must use the post–break–point information, as including the full data would bias the coefficient estimates and therefore also the forecasts of debt paths.

35. We then proceeded to use the econometric results in dynamic simulations of the probabilistic approach to the public DSAs, and illustrated this approach for a number of countries. We estimated the relationship between the determinants and debt, and then also a generic fiscal reaction function to account for fiscal behavior. We simulated sets of random shocks for each of the determinants of debt and the primary deficit and plugged those in the estimated equations to construct future trajectories of the determinants, the primary balance, and the debt ratio. We used the results to show that the probabilistic forecasts can be used as illustrative “confidence bands” to assess the forecasts from the DSA, and applied this approach to a set of countries. Finally, following in the footsteps of Celasun et al (2006), we illustrated the use of the probabilistic forecasts for the debt sustainability analysis with construction of a probabilistic debt sustainability index.

APPENDIX

Table 1. PVARX Estimates for 33 HIPCs; 2007-2011

	Lagged								Exogenous	
	GDP Growth		Domestic Interest Rate		Foreign Interest Rate		Real Effective Exchange Rate (change)		World GDP Growth	World Interest Rate
GDP Growth	-0.18	*	0.29	***	-0.24	***	-0.14	***	0.13	0.03
	(0.10)		(0.05)		(0.05)		(0.03)		(0.32)	(0.37)
Domestic Interest Rate	-0.18		0.29	***	-0.24	**	-0.14		0.13	0.03
	(0.22)		(0.11)		(0.12)		(0.07)		(0.71)	(0.82)
Foreign Interest Rate	0.22		-0.13	*	-0.07	**	0.18	***	-1.49	-0.02
	(0.16)		(0.08)		(0.09)		(0.05)		(0.52)	(0.60)
Real Effective Exchange Rate (change)	-0.49	***	0.43	***	0.06		-0.41	***	0.97	0.90
	(0.28)		(0.14)		(0.15)		(0.09)		(0.91)	(1.05)

Note: standard errors in parentheses; * = significant at 10%; ** at 5%; *** at 1%.
Source: IMF, Strategy, Policy and Review Department DSA Database and own calculations.

Table 2. PVARX Estimates for 21 HIPCs with Completion Points pre-2006 and in 2006; 2007-2011

	Lagged								Exogenous	
	GDP Growth		Domestic Interest Rate		Foreign Interest Rate		Real Effective Exchange Rate (change)		World GDP Growth	World Interest Rate
GDP Growth	0.03		0.26	***	-0.18	**	-0.23	***	0.26	-0.32
	(0.10)		(0.05)		(0.06)		(0.03)		(0.17)	(0.18)
Domestic Interest Rate	0.21		-0.17	***	-0.24		-0.03	***	-1.51	-1.39
	(0.19)		(0.09)		(0.11)		(0.06)		(0.33)	(0.34)
Foreign Interest Rate	0.13		-0.19		-0.51		-0.06	***	-1.23	-0.68
	(0.16)		(0.08)		(0.10)		(0.05)		(0.28)	(0.30)
Real Effective Exchange Rate (change)	-0.20	***	0.38	***	0.31	*	-0.26	***	-0.39	-0.41
	(0.31)		(0.15)		(0.18)		(0.10)		(0.55)	(0.57)

Note: standard errors in parentheses; * = significant at 10%; ** at 5%; *** at 1%.
Source: IMF, Strategy, Policy and Review Department DSA Database and own calculations.

Table 3. Chow Tests

	H_o: No structural break in 2006			
	33 HIPCs (full sample)		21 HIPCs with Completion Points pre-2006 and in 2006	
	Sample-Split (SS)	Break-Point (BP)	Sample-Split (SS)	Break-Point (BP)
Distribution Under H_o	Chi-sq $^{2}{}_{156}$	Chi-sq $^{2}{}_{172}$	Chi-sq $^{2}{}_{120}$	Chi-sq $^{2}{}_{136}$
95% Quantile	186.1	203.6	133.2	151.0
Statistics	227.8	363.4	188.6	266.8
P-values 1/	0.000	0.000	0.000	0.000

1/ H_o of no structural break in 2006 is rejected.
Source: IMF, Strategy, Policy and Review Department DSA Database and own calculations.

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