A New Perspective on "The New Rule"of the Current Account[1]

Cedric Tille
Graduate Institute of International and Development Studies, Geneva
CEPR

Eric van Wincoop
University of Virginia
NBER

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Abstract

In an influential contribution that predates the recent renewed interest in portfolio choice models of international capital flows, Kraay and Ventura (2000) offer a "new rule" for the current account that puts portfolio choice at the center of the analysis. The "new rule" states that in response to a change in saving, the change in the current account is equal to the change in saving times the ratio of net foreign assets to wealth. We show that while the focus on portfolio choice is well placed, the inference in terms of the international allocation of saving is misleading. Using both a small country partial equilibrium model and a two-country general equilibrium model with portfolio choice, we show that the "new rule" does not hold as most of an increase in a country's saving is invested abroad. We also show that the empirical evidence presented in Kraay and Ventura (2000) is consistent with an expression for the current account that holds in the steady state of almost any model. The "new rule" does not follow as an implication.

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1 Introduction

In an influential article Aart Kraay and Jaume Ventura (2000, hereinafter KV) develop a "new rule" for the current account. It states that following a transitory income shock "the current account response equals the saving generated by the shock times the country's share of foreign assets in total assets." This stands in sharp contrast to the standard inference that a small country invests most of the additional saving abroad. Their work emphasizes the need to view international capital flows from a portfolio choice perspective. They present a simple model that can account for the new rule, along with supporting empirical evidence. Their contribution is several years ahead of the recent renewed interest in portfolio choice models in open economy macroeconomics and in analyzing international capital flows from a portfolio choice perspective.

In this paper we offer a different interpretation for the empirical findings in KV, which is fundamentally different from the "new rule". We emphasize the need to distinguish the long-run relation between a country's current account and saving from the short-run dynamic response to transitory shocks. Using a two-country general equilibrium model, we find that while the empirical evidence in KV is consistent with the model, the new rule does not follow. Specifically, the steady state current account of a country is equal to its saving times the ratio of net foreign assets to the country's wealth, in line with the empirical evidence presented in KV.[1] By contrast, the extra saving from a temporary income shock is mostly invested abroad, in sharp contrast to the prediction by the new rule.

Before further developing these insights it is useful to first be more precise about the empirical exercise in KV, as well as the follow-up paper Kraay and Ventura (2003). In both papers the current account is regressed on a term equal to the share of net foreign assets in total wealth times saving. In a panel regression the regression coefficient is close to 1. They find that this result is primarily due to the cross-section aspect of the data, for which the empirical fit is very accurate with an R^2 of about 0.85. In contrast, the equation has very little explanatory power along the time series dimension. When subtracting country-specific long-term-averages from both sides of the regression, the R^2 falls to around to 0.02. Kraay and Ventura (2003) acknowledge that "the new rule explains essentially none of

[1] This long-run relationship between saving and the current account is also emphasized in van Wincoop (2003) and Jin and Guo (2008).

the year-to-year within country differences in current accounts."

KV interpret the cross-section regression evidence as being consistent with the new rule. The logic is as follows. Assume that a change in saving does not lead to a change in expected returns, leaving the portfolio allocation across alternative assets unchanged. The resulting capital outflow is then equal to the change in saving times the share of net foreign assets in total wealth. KV develop a model where a change in saving has little impact on portfolio shares, as long as asset return risk is large and diminishing returns to capital are weak. Kraay and Ventura (2003) address the limitations of the new rule in the short-run by introducing adjustment costs to investment. These costs imply that most of an increase in saving is invested abroad in the short-run, while the new rule holds in the long-run.

Our interpretation of the evidence is different. First, we interpret the cross-section evidence as reflecting a steady state allocation. In steady state the ratio of a country's net foreign asset position to its total wealth must be equal to its equivalent in terms of financial flows, namely the ratio of the current account to saving. Therefore the current account is equal to saving times the ratio of net foreign assets to wealth. Second, we show that the time-series evidence is a natural implication of a model with international portfolio choice, even in the absence of adjustment costs.

We first show that in the context of a partial equilibrium small open economy model along the line of KV, an increase in saving is entirely invested abroad to a first-order. One needs an unrealistic high degree of asset return risk to get a result resembling the new rule. We then show that the partial equilibrium nature of this model limits its usefulness for the analysis of capital flows. It ignores some key aggregate accounting identities. In addition, it leads to an inaccurate interpretation of net foreign asset holdings relative to wealth as a portfolio share chosen by domestic investors. With two-way capital flows, this ratio depends on portfolio shares chosen by investors from both countries as well as the relative wealth of the two countries. Moving to a more realistic two-country general equilibrium setup, we find that the model is even further apart from the new rule.

The paper is organized as follows. Section 2 considers a partial equilibrium small open economy model, offering a close parallel to the framework used by KV. The analysis is extended to a full general equilibrium model in section 3. Section 4 concludes.

2 A Small Open Economy Model

We consider a small open economy where investors can buy claims on both domestic capital and a foreign asset whose return is exogenous to the small economy. In parallel with KV, we assume that foreign investors cannot buy claims on the small country's capital. This small open economy setup is essentially the one considered in KV.

2.1 Production and Investment

There is one good available for consumption and investment. Production in the small country uses a constant returns to scale technology combining labor and capital:

$$Y_t = A_t K_t^{1-\omega} N_t^{\omega}$$

where Y is output, A is an exogenous stochastic productivity term, K is the capital input and N the labor input. Productivity follows a simple i.i.d. process:

$$A_t = 1 + \varepsilon_t \tag{1}$$

where ε_t has a $N(0, \sigma_a^2)$ distribution.[2] The assumption that productivity shocks are not persistent is made for the sake of clarity in comparison to KV. As in KV, we focus on the impact on international capital flows of changes in saving that stem from transitory productivity shocks. Allowing for persistence would generate movements in investment that are independent from the impact of the shock on saving, thereby obscuring the analysis.

The labor input is fixed and normalized to unity. The wage, W_t, is equal to the marginal product of labor:

$$W_t = \omega A_t K_t^{1-\omega} \tag{2}$$

The dynamics of the capital stock reflects investment, I_t, and depreciation at a rate δ:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{3}$$

We abstract from adjustment costs in investment, so that consumption and capital goods are identical and the relative price of capital is unity. As explained

[2] While our setup implies that productivity could be negative, our results carry to a setup where productivity shocks are log-normal and the issue does not arise.

in the introduction, Kraay and Ventura (2003) rely on adjustment costs in order to explain why the new rule does not hold in the short-run in time series data. By contrast, we show that the new rule does not hold in the short run even without adjustment costs.

2.2 Two Assets

The gross return on domestic capital is

$$R_{t+1} = 1 - \delta + (1 - \omega) A_{t+1} K_{t+1}^{-\omega} \tag{4}$$

The last term on the right hand side is the marginal product of capital. The return on the foreign asset is written in a similar form:

$$R_{t+1}^{*} = 1 - \delta + D^{*} + \epsilon_{t+1}^{D} \tag{5}$$

where D^{*} is a constant and ϵ_{t+1}^{D} is an i.i.d innovation that has a $N(0, \sigma_d^2)$ distribution, so $1 - \delta + D^{*}$ is the predictable component of the return.

A central aspect in the KV setup is the presence of home bias in portfolios, so that a rise in saving is mostly invested domestically. We generate portfolio home bias by assuming that investors in the small country receive only a fraction $1 - \tau$ of the return (5), where τ is a second-order constant (i.e. proportional to the variance of model innovations) that captures the hurdles of investing abroad. This iceberg cost does not generate a loss in resources, and is instead a fee paid to a broker in the small country, who consumes it in the same period. The portfolio return for investors in the small country between periods t and $t + 1$ is then

$$R_{t+1}^{p} = z_t R_{t+1} + (1 - z_t)(1 - \tau) R_{t+1}^{*} \tag{6}$$

where z_t is the fraction of wealth invested in domestic capital.

2.3 Consumption and Portfolio Choice

We adopt a simple overlapping generation structure with agents living for two periods to ensure a well-defined steady state wealth distribution. Agents borne in period t supply one unit of labor and earn the wage W_t. They consume some of their income when young and invest the rest to finance their consumption when old in period $t + 1$. Denoting the consumption of a young agent in period t by

$C_{y,t}$, and that of an old agent in period $t+1$ by $C_{o,t+1}$, the agent borne in period t maximizes:

$$\frac{(C_{y,t})^{1-\gamma}}{1-\gamma} + \beta E_t \frac{(C_{o,t+1})^{1-\gamma}}{1-\gamma}$$

subject to:

$$C_{o,t+1} = (W_t - C_{y,t}) R^p_{t+1} \tag{7}$$

and the portfolio return (6). The first order conditions with respect to $C_{y,t}$ and z_t are:

$$(C_{y,t})^{-\gamma} = \beta E_t \left(C_{o,t+1}\right)^{-\gamma} R^p_{t+1} \tag{8}$$

$$E_t \left(R^p_{t+1}\right)^{-\gamma} \left(R_{t+1} - (1-\tau) R^*_{t+1}\right) = 0 \tag{9}$$

The Euler equation (9) is a standard arbitrage condition for portfolio choice, which says that the expected product of the asset pricing kernel and asset return is the same across all assets.

As the small country investors are the only ones who can purchase claims on domestic capital, the asset market clearing condition is

$$K_{t+1} = (W_t - C_{y,t}) z_t \tag{10}$$

2.4 Saving and the current account

Aggregate saving in the small economy is the sum of saving by young and old agents. The saving of the former is simply the difference between their wage income and their consumption. The consumption of old agents is given by (7). In line with national accounting, we consider that (net) income of old agents consists of the dividend streams on both assets, net of depreciation and of the fee paid to brokers:

$$\left[z_{t-1}\left(R_t - 1\right) + \left(1 - z_{t-1}\right)\left(R^*_t - 1 - \tau R^*_t\right)\right]\left(W_{t-1} - C_{y,t-1}\right)$$

Recalling that brokers consume their entire income, aggregate savings are:

$$S_t = (W_t - C_{y,t}) - (W_{t-1} - C_{y,t-1}) \tag{11}$$

The current account is saving net of investment: $CA_t = S_t - I^{net}_t$, where I^{net}_t is investment net of depreciation.

Turning to the stocks of financial assets, the overall wealth is simply the saving of young agents, as old agents have exited asset markets. The net foreign asset position is a share $1 - z_t$ of wealth.

2.5 Solution Method

In general, the solution of models with endogenous portfolio choice entails substantial technical difficulties due to the presence of heterogeneous investors, as explained in Devereux and Sutherland (2006) and Tille and van Wincoop (2008). An advantage of our small economy setup is that we do not face this issue as we only consider the portfolio choice of investors in the small country and abstract from the choice by investors in the rest of the world. The portfolio share z_t follows directly from (10).

The solution uses a standard local approximation method. We write a variable x_t as the sum of its components of various orders: $x_t = x(0) + x_t(1) + x_t(2) + ...$ The zero-order component, $x(0)$, is the level of a variable when standard deviations approach zero (deterministic steady state). The first-order component, $x_t(1)$, is proportional to standard deviations of model innovations, or to the innovations themselves. The second-order component, $x_t(2)$, is proportional to the variance of model innovations (or the product of model innovations), and so on.

The zero-order values of the various variables are simply inferred from the zero-order components of the model's equations. The first-order components of the variables is computed from first-order expansions of the equations around the zero-order allocation. It will turn out to be useful to obtain further precision by also computing the second and third-order components of some variables, in particular to contrast our results with KV. All algebraic details can be found in Appendix A and a Technical Appendix available on request.

2.6 Impact of a Temporary Income Shock

2.6.1 Saving and investment

The first-order component of saving is driven by current and lagged productivity shocks:

$$S_t(1) = (1 - \bar{c})(\varepsilon_t - \varepsilon_{t-1})$$

where $\bar{c} = C_y(0)/W(0)$ is the steady-state propensity to consume of young agents. A positive income shock at time t raises saving in order to smooth consumption. Saving in the following period is reduced as the old agents consume the principal value of their increased wealth from the previous period.

The response of the capital stock in period $t+1$, and therefore investment, is inferred from

$$E_t R_{t+1}(1) = E_t R^*_{t+1}(1) = 0 \tag{12}$$

The first equality in (12) is implied by the first-order component of the Euler equation (9) for optimal portfolio choice. The first-order components of expected returns must be equalized across all assets as otherwise optimal portfolio shares would explode for small levels of risk (expected return differences would be large relative to risk). The second equality in (12) follows from our assumption that innovations in the foreign return (5) are i.i.d., so that it's predictable component is fully captured by the zero-order term.

As productivity is i.i.d, A_{t+1} is expected to be unity, hence $E_t A_{t+1}(1) = 0$. Taking the expectation of (4) then implies that to a first-order the domestic capital stock and investment (gross and net) are not affected by the temporary income shock: $K_{t+1}(1) = I_t(1) = I_t^{net}(1) = 0$. Intuitively, the exogenous return on the foreign asset ties down the expected return on domestic capital, which in turn ties down the capital stock to the first-order. The first-order component of the current account then immediately follows as $CA_t(1) = S_t(1) - I_t^{net}(1) = S_t(1)$. The increase in saving is therefore entirely invested abroad and the new rule clearly does not hold. To sum up:

Result 1 *In a small open economy model with one-way capital flows from the small to the large country, a first-order increase in saving in the small country is entirely invested abroad, leading to a corresponding first-order increase in the current account.*

2.6.2 Comparison to Kraay and Ventura

It is useful to compare this result to that of the small open economy model in KV. While the model in KV is different (and in continuous time), the key element that matters for the results above is the same. Just like the model discussed above, KV consider a partial equilibrium small open economy framework. Agents can invest in domestic capital, a risk-free foreign asset and a risky foreign asset (called foreign capital). KV consider the impact of a temporary income shock that raises wealth, denoted by a, and saving. Their key equation (7) describes the impact of the shock on investment:

$$di = \frac{(1-\eta^2)\sigma^2}{(1-\eta^2)\sigma^2 - \frac{\partial \pi}{\partial k} a} \frac{k}{a} da = \zeta ds \tag{13}$$

where k is the capital stock, i is investment, s is saving, π is the net return on domestic capital (corresponding to $R_{t+1} - 1$ in our notation), η is the correlation between the return on domestic capital and foreign capital, and σ^2 is the standard deviation of the return on domestic capital. (13) shows that an increase in saving boosts domestic investment, in sharp contrast to our result above.

While at first (13) appears inconsistent with our Result 1, it is in fact consistent as the change in investment in (13) is very small, i.e. of order higher than one in our framework. Specifically, the term ζ in (13) is of order two and higher as $\partial \pi / \partial k$ has a well-defined zero-order component.[3] This implies that a first-order change in saving is associated with a change in investment of order three and above. This is consistent with Result 1, which says that to the first-order investment does not change. In the Technical Appendix that is available on request we solve the second and third-order components of model variables, from respectively the second and third-order components of model equations. Consistent with (13), we find that while the first and second-order components of investment are zero, its third-order component is positive.

The intuition behind the third-order increase in investment is as follows. As shown above, the entire increase in saving is invested abroad to a first-order. As the domestic capital stock is unchanged to a first-order, the asset market clearing condition (10) implies a first-order drop in the fraction z_t invested at home. This reflects the supply side of domestic capital. The demand side can be obtained from the third-order component of the portfolio Euler equation (9), which shows that the first-order component of the portfolio share z_t depends on the third-order component of the expected excess return on home capital (the difference between (4) and (5)), divided by the variance of the excess return. Therefore a relatively small drop in the third-order component of the excess return generates a first-order drop in the portfolio share z_t, bringing the first-order demand for domestic capital in line with the unchanged supply. The third-order drop in the expected excess return implies a third-order rise in the capital stock and therefore investment, as seen from (4).

[3] (13) implies that the zero and first-order components of ζ are zero, and its second-order component is $\zeta(2) = -(1-\eta^2)\left(\frac{\partial \pi}{\partial k} a\right)^{-1}(k/a)\,\sigma^2$.

Result 2 *In a small open economy model with one-way capital flows from the small to the large country, a first-order increase in saving in the small country leads to a third-order increase in investment.*

The only way that (13) can lead to an inference that domestic investment responds substantially to a savings shock is when asset return risk very high. When σ^2 goes to infinity, ζ converges to k/a. The increase in investment is then equal to the increase in saving times the fraction of wealth invested at home. This implies that the current account is equal to the change in saving times the fraction of wealth invested abroad, which is the new rule.

We can quantify ζ to assess whether σ is large enough to let this third-order term become large. For illustrative purposes we will assume that the capital to wealth ratio, k/a, is 1, so that the new rule implies that all of the increase in saving is entirely invested at home. We adopt our expression for the return on domestic capital (4), which follows from a standard Cobb-Douglas production function. It implies that the zero order component of $(\partial\pi/\partial k)a$ is equal to $-\omega(\pi + \delta)$. We calibrate ζ using equity returns, which if anything overstates the magnitude of third and higher-order components since equity claims are residual claims whose return volatility is considerably higher than that of total claims on a country's capital stock. We rely on stock return data for 13 countries since the 1920's as reported in Jorion and Goetzmann (1999)[4] and set π equal to the average real annual return of 0.033. We set σ equal to the average standard deviation of 0.174. It can be shown that $(1 - \eta^2)\sigma^2$ is equal to σ^2 minus the variance of the world return. Jorion and Goetzmann (1999) report a standard deviation of the global return of 0.121. Finally, we set the annual depreciation rate δ equal to 0.1, which is a standard assumption in calibrations, and the labor share ω equal to 0.7.

These parameters imply that the coefficient ζ in (13) is equal to 0.14, so that 86% of the increase in saving is invested abroad. This stands in sharp contrast to the new rule, which says that 100% is invested at home and nothing is invested abroad. As pointed out above, ζ would be even much smaller if we had used a measure of the overall return on domestic capital, which would have much lower risk than the 17.4% average standard deviation of stock returns.

[4]The countries are the United States, Canada, Austria, Belgium, Denmark, France, Italy, Netherlands, Norway, Spain, Sweden, Switzerland and United Kingdom.

2.7 Limitations of partial equilibrium approach

A limitation of the partial equilibrium small open economy setting discussed so far is that it ignores relevant aggregate accounting identities. The new rule holds if and only if a change in saving does not lead to a change in the ratio of net foreign assets to total wealth. This cannot be the case in general equilibrium. Consider that there are two countries, Home and Foreign, so that the net foreign asset position of Home and Foreign add up to zero. Therefore

$$\frac{NFA^H}{Wealth^H}Wealth^H + \frac{NFA^F}{Wealth^F}Wealth^F = 0$$

Assume that net foreign asset positions are non-zero. Then a change in relative wealth of the two countries due to higher Home saving must lead to a change in the ratio of net foreign assets to wealth in at least one of the countries. The new rule then cannot hold for both countries.

This problem manifests itself in several other ways. Assume that $NFA^H > 0$. If a shock increases Home saving, without any impact on Foreign saving, the new rule implies that the current account rises in Home and does not change in Foreign. This is a clear violation of the aggregate identity that the current accounts sum to zero.

Closely related to this inconsistency of the new rule with accounting identities is the fact that the ratio of net foreign assets to total wealth in a country can generally not be interpreted as a portfolio share chosen by domestic investors. Such an interpretation is correct only in the small open economy model with one-way asset trade discussed above. This however simply reflects the absence of any distinction between *gross* and *net* foreign assets in the model. Portfolio shares are well defined only in terms of gross assets. Allowing for two-way asset holdings, the Home country's gross liabilities reflect the portfolio allocation of Foreign investors.

The ratio of Home net foreign assets to Home wealth is

$$\frac{NFA^H}{Wealth^H} = \frac{Assets^H}{Wealth^H} - \frac{Liabilities^H}{Wealth^F}\frac{Wealth^F}{Wealth^H}$$

The first term on the right-hand side is the share of Foreign assets in the Home investors' portfolio. The second term is the product of the share of Home assets in the Foreign investors' portfolio and the ratio of Foreign wealth to Home wealth. Even when portfolio shares are constant, a change in saving in Home affects the ratio of net foreign assets to wealth as it affects relative wealth. We can summarize these results as follows.

Result 3 *The new rule is inconsistent with aggregate accounting identities that current accounts and net foreign asset positions sum to zero. Closely related to that, the ratio of net foreign assets to wealth is not a portfolio share chosen by domestic investors. It depends on portfolio shares chosen by both Home and Foreign investors as well as the international wealth distribution.*

2.8 Two-Way Asset Trade

These concerns call for an analysis of the new rule in the context of a full general equilibrium model with two-way asset trade. We will do so in the next section. As a preview, in this subsection we will maintain the small open economy framework but allow for two-way asset trade. Agents in the rest of the world can now invest in domestic capital, subject to the same iceberg cost τ that the domestic agents face when investing abroad.

Denoting the asset pricing kernel of the foreign investors by m^*_{t+1}, the equivalent of the Euler equation (9) for the foreign investor is:

$$E_t m^*_{t+1}(1-\tau)R_{t+1} = E_t m^*_{t+1} R^*_{t+1} \tag{14}$$

Since the right-hand side of (14) is entirely exogenous from the perspective of the small open economy, (14) implies that shocks in the small country cannot affect $E_t m^*_{t+1} R_{t+1}$. As m^*_{t+1} is set in the rest of the world, shocks in the small country have no impact on the return on domestic capital, R_{t+1}, and (4) then shows that domestic capital and investment cannot change. Note that this is true to any order, including third and higher order. Intuitively, arbitrage by investors from the infinitely large country completely ties down the expected return in the small country.

In this case the entire increase in saving in the small country will be invested abroad. This is now the case to *any* order of approximation. Allowing for two-way asset trade therefore breaks down the new rule further as it does not even matter how large the standard deviation of asset returns is.

Result 4 *In a small open economy model with two-way capital flows between the small and the large country, a rise in saving in the small country does not affect* *investment in the small country to any order. The additional saving is therefore fully invested abroad.*

Since expected returns do not change to any order, small country investors do not change their portfolio allocation in response to an increase in saving. Gross capital outflows are then equal to the increase in saving times the steady state fraction invested abroad. Its first-order component is $(1-z(0))S_t(1)$. If that were the end of the story, the new rule would hold exactly if $1-z(0)$ were the net foreign asset position relative to wealth. But this is no longer not the case. $1-z(0)$ now captures gross external assets relative to wealth, *not* the net foreign asset position relative to wealth. Moreover, $(1-z(0))S_t(1)$ is equal to gross capital outflows, not net capital outflows.

Net capital flows are also determined by capital inflows that result from portfolio choices by foreign investors. As there is no change in investment, the first-order component of net capital outflows must be $S_t(1)$. Therefore the first-order component of capital inflows must be equal to $-z(0)S_t(1)$. Foreign investors shift money from the small to the large country. Intuitively, foreign investors are indifferent between domestic and foreign assets when the return on domestic capital does not change. They are therefore happy to sell claims on the small country's capital, which is needed to clear the asset market.[5]

3 Two-Country General Equilibrium Model

We now turn to a full two-country general equilibrium model. We extend the partial equilibrium model of the previous section in four directions. First, agents from both countries can buy claims on the capital of the other country. Second, the relative size of the two countries is a free parameter. Third, we allow for an exogenous positive growth rate of the population in both countries in order to allow steady state saving rates to be non-zero. Finally, we allow for different time-discount rates across countries. This leads to different steady state saving rates across the two countries and non-zero steady state net foreign asset positions.

[5] From the perspective of individual investors in the large country this involves an infinitesimally small portfolio reallocation towards the large country as the mass of investors is infinitely larger than in the small country.

3.1 Production and Investment

We call the two countries Home and Foreign, denoted with superscripts H and F. They produce the same good, with the production function in country i $(= H, F)$ given by:

$$Y_t^i = A_t^i (K_t^i)^{1-\omega} (N_t^i)^{\omega}$$

The labor force N_t^i is taken as exogenous. The relative size of the Home labor force is n. Both grow at a constant rate g:

$$N_t^H = n(1+g)^t \qquad\qquad N_t^F = (1-n)(1+g)^t$$

We denote the ratio of a variable relative to the young population in a country by lower case letters. For example: $k_t^i = K_t^i / N_t^i$.

Productivity in both countries follows a simple i.i.d. process:

$$A_t^i = 1 + \varepsilon_t^i \tag{15}$$

where ε_t^i has a $N(0, \sigma_a^2)$ distribution. Without loss of generality we assume that productivity shocks are uncorrelated across the two countries. Wages are equal to the marginal product of labor:

$$W_t^i = \omega A_t^i (k_t^i)^{1-\omega} \tag{16}$$

The dynamics of the capital stock reflects investment and depreciation:

$$K_{t+1}^i = (1-\delta) K_t^i + I_t^i$$

3.2 Two Assets

Investors trade claims on the capital stock of both countries. The gross return on country i capital is

$$R_{t+1}^i = 1 - \delta + (1-\omega) A_{t+1}^i \left(k_{t+1}^i\right)^{-\omega} \tag{17}$$

As in the previous section, investors receive only a fraction $1-\tau$ of the return on assets invested abroad, where τ is again a second-order constant iceberg cost. In period t investors in country i invest a fraction z_t^i of their wealth in Home capital. The portfolio returns from t to $t+1$ of investors in the two countries are then

$$R_{t+1}^{p,H} = z_t^H R_{t+1}^H + (1-z_t^H)(1-\tau) R_{t+1}^F \tag{18}$$

$$R_{t+1}^{p,F} = z_t^F (1-\tau) R_{t+1}^H + (1-z_t^F) R_{t+1}^F \tag{19}$$

3.3 Consumption and Portfolio Choice

We adopt the same overlapping generation structure as in the small open economy model. We allow the time-discount rate to differ across the two countries in order to generate different saving rates in steady state. Specifically, a young agent in country i at time t maximizes

$$\frac{\left(C_{y,t}^{i}\right)^{1-\gamma}}{1-\gamma}+\beta^{i}E_{t}\frac{\left(C_{o,t+1}^{i}\right)^{1-\gamma}}{1-\gamma}$$

subject to:

$$C_{o,t+1}^{i}=(W_{t}^{i}-C_{y,t}^{i})R_{t+1}^{p,i} \tag{20}$$

and the portfolio return (18) or (19). The first order conditions with respect to $C_{y,t}^{i}$ and z_t^i are

$$\left(C_{y,t}^{i}\right)^{-\gamma}=\beta^{i}E_{t}\left(C_{o,t+1}^{i}\right)^{-\gamma}R_{t+1}^{p,i}\quad i=H,F \tag{21}$$

$$E_{t}\left(R_{t+1}^{p,H}\right)^{-\gamma}\left(R_{t+1}^{H}-(1-\tau)R_{t+1}^{F}\right)=0 \tag{22}$$

$$E_{t}\left(R_{t+1}^{p,F}\right)^{-\gamma}\left((1-\tau)R_{t+1}^{H}-R_{t+1}^{F}\right)=0 \tag{23}$$

(22) and (23) are the arbitrage conditions for portfolio choice by Home and Foreign investors, respectively. Both conditions show that the expected product of the asset pricing kernel and asset return are equalized across all assets.

The asset market clearing conditions in the two countries are

$$K_{t+1}^{H}=(W_{t}^{H}-C_{y,t}^{H})N_{t}^{H}z_{t}^{H}+(W_{t}^{F}-C_{y,t}^{F})N_{t}^{F}z_{t}^{F} \tag{24}$$

$$K_{t+1}^{F}=(W_{t}^{H}-C_{y,t}^{H})N_{t}^{H}(1-z_{t}^{H})+(W_{t}^{F}-C_{y,t}^{F})N_{t}^{F}(1-z_{t}^{F}) \tag{25}$$

The budget constraints together with asset market clearing conditions imply that the world goods market equilibrium condition is satisfied as well.

3.4 Solution Method

The solution of the general equilibrium model entails an additional degree of technical complexity compared to the small open economy model. Specifically, it includes portfolio choice by heterogenous investors in the Home and the Foreign country. It is useful to define the worldwide average of portfolio shares, z_t^A, and the cross country difference, z_t^D, as:

$$z_{t}^{A}=\eta z_{t}^{H}+(1-\eta)\,z_{t}^{F}\qquad\qquad z_{t}^{D}=z_{t}^{H}-z_{t}^{F}$$

where η is the share of the Home country in the world wealth in the steady state, with the exact expression detailed below.

The specific problem of such setups is that the zero-order components of the difference of portfolio shares between the two investors, $z^D(0)$, cannot simply be inferred from the zero-order components of the model equations. Intuitively, the asset market clearing conditions (24)-(25) only determine the average share $z^A(0)$ that needs to be invested in Home equity to clear asset markets. They however shed no light on the distribution of the holdings between Home and Foreign investors.

Solving the general equilibrium model requires an extension of standard first and second-order solution methods, as recently developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2008). The zero-order component of the portfolio share difference, $z^D(0)$, reflects the pattern of risk in the economy, which is a second-order dimension. Therefore, it clearly cannot be computed using the zero or first-order components of the model equations alone. The solution method calls for jointly solving the zero-order component of the portfolio share difference, $z^D(0)$, and the first-order component of all other variables. The latter includes the first-order component of the average portfolio share, $z_t^A(1)$. This relies on the second-order component of the difference between the arbitrage conditions for the two investors (22) and (23), as well as the first-order components of all other equations.

The first-order dynamics of the portfolio shares, $z_t^H(1)$ and $z_t^F(1)$, play a substantial role in driving capital flows as they reflect the reallocation by investors across assets following a shock. If we are interested only in *net* capital flows between the two countries, only the average portfolio share $z_t^A(1)$ matters. It fully summarizes the extent to which investors somewhere in the world reallocate their portfolio between Home and Foreign equity. The algebraic details of the solution are presented in Appendix B.

If we are interested in the *gross* capital flows between the two countries, we need to determine exactly which investors reallocate their portfolio by solving for the first-order component of the portfolio share difference, $z_t^D(1)$. This is driven by fluctuations in the variances and covariances in response to changes in the state variables, which is a third-order aspect. We jointly solve for $z_t^D(1)$ and the second-order component of all other variables by combining the third-order component of the difference between (22) and (23) and the second-order components of all other equations. This entails a fair degree of complexity. The detailed steps are described in a Technical Appendix available on request.

Even though the method developed by Devereux and Sutherland (2006) and Tille and van Wincoop (2008) can be quite complex in general, various simplifying assumptions in the model nonetheless allow for a tractable analytical solution. Specifically, the assumption of a unique good removes real exchange rate risk. Moreover, the two-period life cycle allows us to abstract from a portfolio hedge against changes in future expected portfolio returns.

3.5 The current account and saving in steady state

We assume without loss of generality that $\beta^H > \beta^F$, so that the agents in the Home country are relatively patient (give more weight to future consumption). The zero-order components of the variables can be interpreted as their steady state values, as discussed in Devereux and Sutherland (2006). Appendix B shows that the steady state level of consumption by young agents in country i is

$$C_y^i(0) = \frac{R(0)}{R(0) + (\beta^i)^{1/\gamma} R(0)^{1/\gamma}} W(0) = \bar{c}^i W(0)$$

where $W(0)$ and $R(0)$ are the zero-order components of wages and asset returns, which are the same for both countries. Consumption by the young generation is clearly lower in the Home country: $\bar{c}^H < \bar{c}^F$. The zero-order component of net national saving (net of depreciation), scaled by the young population, is

$$s^i(0) = \frac{g}{1+g} \left(1 - \bar{c}^i\right) W(0)$$

The portfolio Euler equations imply that the zero-order components of asset returns must be the same across countries. This implies that the zero-order component of the capital stock is the same across countries, and so is investment. The ratio of investment (net of depreciation) and the labor force is given by $gk(0)$. As the current account is the difference between saving and investment, the patient Home country runs a current account surplus in steady state:

$$ca^H(0) = \frac{g}{1+g}(1-n)\left(\bar{c}^F - \bar{c}^H\right) W(0) > 0 \tag{26}$$

The steady state current account surplus in the Home country leads to a positive net foreign asset position. Scaling variables by the labor supply we write:

$$nfa^H(0) = (1-n)\left(\bar{c}^F - \bar{c}^H\right) W(0) \tag{27}$$

(27) shows that the patient country is a net creditor, while (26) shows that in the presence of a growth trend it runs a current account surplus.

The steady state wealth of country i, per unit of labor, is simply the saving of the young generation: $(1-\bar{c}^i)\,W\,(0)$. The relative share of Home wealth is then

$$\eta = \frac{n(1-\bar{c}^H)}{n(1-\bar{c}^H) + (1-n)\,(1-\bar{c}^F)}$$

Define $x^i(0)$ as the steady state ratio of net foreign assets to wealth. It is easily seen from (26)-(27) that

$$ca^i(0) = x^i(0)s^i(0)$$

Multiplying both sides by $(N/Y)^i(0)$, this can also be expressed as

$$(CA/Y)^i(0) = x^i(0)(S/Y)^i(0) \tag{28}$$

To sum up, the steady state analysis shows that:

Result 5 *In the steady state of the general equilibrium model the current account is equal to saving times the ratio of net foreign assets to wealth.*

In their cross-sectional empirical work KV regress $(CA/Y)^i$ on x^i times $(S/Y)^i$, with all variables averaged over 23 years for 13 OECD countries. They find a coefficient close to one and an excellent fit with an R^2 of 0.68. Kraay and Ventura (2003) use an even longer sample of 32 years for 21 OECD countries and obtain an R^2 of 0.85. By averaging over such a long span of data, the results should be reasonably close to the steady state. These results are not surprising as (28) shows that the relationship holds exactly in the steady state.

This result is much more general than the specifics of our model. Consider any set of economies that grow at a constant steady state rate g, implying:

$$\frac{d(Wealth)}{Wealth} = g \quad ; \quad \frac{d(NFA)}{NFA} = g$$

where $Wealth$ is national wealth. Using that $d(Wealth) = S$ is national saving and $d(NFA) = CA$, it follows that

$$CA = \frac{NFA}{Wealth}S = xS$$

This simply holds as a matter of identity as long as there is asymmetry across countries that leads to cross-sectional variation in steady state saving rates and net foreign asset positions.

KV interpret a unitary coefficient of a regression of CA^i on $x^i S^i$ as consistent with the new rule, an interpretation that is fundamentally different from ours. The logic behind their interpretation is as follows. As $NFA = x * Wealth$, the current account reflects saving and changes in the ratio of net foreign asset to wealth:

$$CA = d(NFA) = x * d(Wealth) + Wealth * d(x) = x * S + Wealth * d(x) \quad (29)$$

The new rule is defined in the context of the current account response to a temporary income shock: "the current account response is equal to the saving generated by the shock multiplied by the country's share of foreign assets in total assets". This last ratio is what we have called net foreign assets relative to wealth, x. KV argue that the new rule holds when a change in saving does not lead to a change in x. It is indeed immediate from (29) that when changes in x are orthogonal to saving, a regression of the current account on $x * S$ should have a coefficient of 1.

We fully agree with this reasoning. However, the empirical evidence in favor of the new rule in KV relates to a cross-section regression, and cannot be interpreted in terms of time series. A cross-section regression of $(CA/Y)^i$ on the product of x^i times $(S/Y)^i$ compares the average levels of saving rates and current accounts across different countries. The new rule is instead about the dynamic response of the current account to a change in saving.

To clarify matters further, consider that a country experiences a shock that permanently raises its saving rate. In our model this is simply done by lowering the time discount rate. Comparing the old and new steady states, x increases. Therefore the new rule does not hold during the adjustment phase as $dx > 0$. Once the economy reaches the new steady state, $dx = 0$ and it follows from (29) that once again $CA = xS$. A cross-section regression reflects to a large extent the steady state, after the adjustment to shocks has already taken place. This is especially the case when taking averages over several decades as KV and Kraay and Ventura (2003) do. To summarize:

Result 6 *Kraay and Ventura find that a cross-country regression of the current account on the product of saving and the ratio of net foreign assets to wealth gives a coefficient close to 1 and has a very good fit. The steady state of the general equilibrium model implies that this simply reflects national accounting identities and should not be interpreted in terms of the dynamic response of the current account to fluctuations in saving.*

3.6 The impact of a transitory income shock

Having described the steady state relation between saving and the current account, we now consider the first-order response to a temporary income shock in the Home country in the form of a rise in ε_t^H. We focus on the main elements of the model, with further details presented in Appendix B.

The first-order components of (22) and (23) imply that the expected returns are equalized across Home and Foreign equity: $E_t R_{t+1}^H(1) = E_t R_{t+1}^F(1) = E_t R_{t+1}(1)$. Taking expectations of (17) then implies that the response of the capital stock is the same in both countries:

$$k_{t+1}^H(1) = k_{t+1}^F(1) = k_{t+1}(1) = \delta_k^k \left[k_t(1) + \frac{k(0)}{1-\omega} \varepsilon_t^A(1) \right] \tag{30}$$

where $0 < \delta_k^k < 1$ and $\varepsilon_t^A(1) = \eta \varepsilon_t^H(1) + (1-\eta) \varepsilon_t^F(1)$. The first-order component of net investment is then the same in both countries as well and equal to $i_t(1) = (1+g) k_{t+1}(1) - k_t(1)$. The capital stock and investment are the same in the two countries because expected returns must be equalized to the first-order. Since investment only depends on world productivity shocks, an income shock in a small country will have little effect on domestic investment.

Result 7 *To the first-order investment is the same across countries and only reflects world productivity shocks. Investment in a small country therefore changes very little in response to an income shock in that country.*

Turning to saving, the first-order component in a country i reflects the dynamics of income and consumption by the young:

$$s_t^i(1) = \tilde{\Delta} W_t^i(1) - \tilde{\Delta} C_{y,t}^i(1) \tag{31}$$

where the tilde is defined for a variable a as

$$\tilde{\Delta} a_t(1) = a_t(1) - \frac{1}{1+g} a_{t-1}(1) \tag{32}$$

Intuitively, aggregate saving reflects the savings by the young generation as well as the dis-saving by the old generation.

The first-order components of wages and consumption by the young are

$$\begin{aligned} W_t^i(1) &= W(0) \left(\varepsilon_t^i + (1-\omega) \frac{k_t(1)}{k(0)} \right) \\ C_{y,t}^i(1) &= \bar{c}^i W_t^i(1) - \frac{1-\gamma}{\gamma} \bar{c}^i (1-\bar{c}^i) W(0) \frac{E_t R_{t+1}(1)}{R(0)} \end{aligned}$$

In equilibrium, world saving is equal to world investment:

$$ns_t^H(1) + (1-n)s_t^F(1) = i_t(1) \tag{33}$$

This implies

$$ca_t^H(1) = s_t^H(1) - i_t(1) = (1-n)(s_t^H(1) - s_t^F(1)) \tag{34}$$

We assess the validity of the new rule by taking the ratio between the Home country's current account and its saving:

$$\frac{ca_t^H(1)}{s_t^H(1)} = (1-n)\left(1 - \frac{s_t^F(1)}{s_t^H(1)}\right) \tag{35}$$

(35) clearly shows that the new rule does not hold. The new rule says that the change in the current account is equal to the change in saving times the ratio x^H of the net foreign asset position relative to wealth. The zero-order component of this ratio reflects the difference in time discount rates across the two countries:

$$x^H(0) = (1-n)\frac{\bar{c}^F - \bar{c}^H}{1 - \bar{c}^H} \tag{36}$$

In the data this ratio is very small, so that the new rule implies that most of an increase in saving is invested at Home.

The new rule then implies that to the first-order $ca_t^H(1) = x^H(0)s_t^H(1)$. This is clearly inconsistent with (35). The impact of the model is most clearly illustrated by considering a log utility of consumption ($\gamma = 1$), as agents then always save a constant share of income. When in addition we abstract from past shocks, so that $k_t(1) = 0$, then $s_t^H(1) = (1 - \bar{c}^H)W(0)\epsilon_t^H > 0$ and $s_t^F(1) = 0$. Therefore $ca_t^H(1) = (1-n)s_t^H(1)$. The impact of a change in saving on the current account depends only on the size of the country. As long as $n < 0.5$, most of the increase in saving is invested abroad. This is independent of the time-discount rates and in sharp contrast to the new rule. When $n \to 0$, all of the increase in Home saving is invested abroad, consistent with the results in section 2.8. We can summarize these findings as follows.

Result 8 *Under a log utility of consumption, the fraction of a first-order increase in saving that is invested abroad only reflects the size of the country. This fraction differs from the steady-state ratio of net foreign assets to wealth, which reflects international asymmetries in the time discount rate.*

While our results are easily illustrated in the case of a log utility, they do not qualitatively change when we allow saving to depend on the expected excess return ($\gamma \neq 1$).

3.7 Capital Flows and Portfolio Allocation

The new rule is about net capital flows. So far we have discussed the impact of a temporary income shock on net capital flows by analyzing the impact on both saving and investment. An important contribution of KV is to think about capital flows in terms of portfolio choice. They particularly shed light on the distinction between portfolio growth and portfolio reallocation. Capital flows associated with portfolio growth are a result of investment of saving at steady state portfolio shares. Capital flows associated with portfolio reallocation result from changes in portfolio shares.

The model in KV suggests that the new rule represents portfolio growth. But this view relies on interpreting the ratio of net foreign assets to wealth as a portfolio share. But as previously discussed, it is better to define portfolio shares in terms of gross portfolios, namely the fractions that Home and Foreign investors invest in Home. These are actual portfolio shares chosen by investors. Portfolio growth then represents capital flows resulting from investment of Home and Foreign saving at steady state portfolio shares by respectively Home and Foreign investors.

To clearly illustrate our point, the rest of this section considers that both countries share the same rate of time preference. Under this parameterization the steady state net foreign asset position is zero, as can be seen from (36). The new rule then implies that a rise in saving is entirely invested domestically and does not lead to any capital flows at all. Our model however shows that the net capital flows associated with portfolio growth are equal to $\left(1 - z^D(0)\right)\left[s_t^H(1) - s_t^F(1)\right]$. The new rule therefore does not hold even in the absence of portfolio reallocation. The portfolio growth component of net capital flows depends on both Home and Foreign saving.

Result 9 *Capital flows reflect both a portfolio growth and a portfolio reallocation component. Portfolio growth reflects the flows that occur when extra saving is allocated at steady state portfolio shares, while portfolio reallocation reflects capital flows associated with time-variation in portfolio shares. Even in the absence of portfolio reallocation the new rule does not hold.*

One can see from (29) that the new rule will hold when changes in the ratio of net foreign assets to wealth are uncorrelated with saving. The ratio of net foreign assets to wealth is

$$x_t^H = \frac{n\left(1 - z_t^H\right)\left(W_t^H - C_{y,t}^H\right) - (1-n)\, z_t^F \left(W_t^F - C_{y,t}^F\right)}{n(W_t^H - C_{y,t}^H)} \tag{37}$$

While we have already made the point that the ratio (37) is not simply a portfolio share, it is still of interest to understand what drives its dynamics and to understand whether and why it is correlated with saving. The dynamics of the first-order component of (37) are:

$$\tilde{\Delta} x_t^H(1) = -\frac{1-n}{n}\left[\frac{\tilde{\Delta} z_t^A(1)}{1-n} + \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\left(s_t^F(1) - s_t^H(1)\right)\right] \tag{38}$$

where $\tilde{\Delta}$ is again defined in (32). A shift of investors anywhere in the world towards Home equity ($\tilde{\Delta} z_t^A(1) > 0$) reduces the Home country gross external assets, if the investors are Home residents, or increases its gross liabilities, if the investors are Foreign residents. Both changes lower the net foreign asset position of Home. High Foreign saving ($s_t^F(1) > 0$) also leads to increased external liabilities as some of the Foreign saving is invested in Home. Similarly, high Home saving ($s_t^H(1) > 0$) leads to increase external assets as some of it is invested abroad. An increase in Foreign relative to Home saving therefore reduces the net foreign asset position.

The first-order component of the average portfolio share is

$$z_t^A(1) = -n(1-n)\, z^D(0)\, \varepsilon_t^D(1) \tag{39}$$

As shown in the Appendix, the zero-order component of the difference in portfolio shares depends positively on the friction τ: $z^D(0) > 0$. The expression for the average portfolio share is intuitive. An increase in Home relative to Foreign productivity boosts the income of Home investors relative to Foreign investors. Because of domestic bias in portfolio holdings, this raises the demand for Home equity relative to Foreign equity. Asset market clearing then requires a reduction in the world demand for Home equity through a reallocation towards Foreign equity ($z_t^A(1) < 0$). This is achieved through a third-order drop in the expected excess return on Home equity.[6]

[6] See Tille and van Wincoop (2008) for a more detailed discussion of the driving forces behind average portfolio shares and the role of expected returns.

As discussed in section 3.6, Home saving depends positively on the Home productivity shock ε_t^H. The ratio of net foreign assets to wealth (38) is then positively correlated with Home saving through two channels. First, higher Home saving directly boost net foreign assets as discussed above. Second, a positive Home productivity shock lowers the average portfolio share invested in Home equity, again increasing Home's net Foreign asset position. As Home saving is clearly positively correlated with changes in the ratio (37), there is a positive correlation between the current account and changes in saving in (29) even though $x^H(0) = 0$.

There are two reasons why KV did not anticipate the correlation between saving and the change in the net foreign asset to wealth ratio. First, by adopting a partial equilibrium framework they missed the point that x_t^H is not a portfolio share and is affected by changes in relative wealth. Second, they did not anticipate that very small changes in expected returns (third-order) lead to first-order portfolio reallocation.

Result 10 *The new rule does not hold because the level of saving is positively correlated with changes in the ratio of net foreign assets to wealth. The latter is driven both by relative saving of Home and Foreign investors and changes in average portfolio shares.*

We should finally point out that there is an additional source of portfolio reallocation that does not affect net capital flows. Changes in the difference z_t^D in portfolio shares only affect gross flows, not net flows. Intuitively, an increase in portfolio home bias leads to increased capital inflows and outflows. When holding z_t^A constant, the difference between inflows and outflows does not change. We show in the Technical Appendix that $z_t^D(1)$ depends on time-varying second moments, which are shown to depend on average productivity shocks.

4 Conclusion

In recent years there has been renewed interest in the role of portfolio choice as a determinant of international capital flows. The contribution by KV was pioneering in that line of research, drawing inferences that stand in sharp contrast to the standard model without portfolio choice. The portfolio choice model of international capital flows that they develop is insightful and the empirical evidence on the relation between saving and the current account is intriguing. However, we do not believe that either the theory or the data imply a "new rule" for the current account. We stress the need to distinguish between the connection between a country's current account and its saving at different horizons. In the long run, the current account is equal to saving times the steady state ratio of net foreign assets to wealth. This is consistent with the empirical evidence in KV. But the new rule is about the dynamic response of the current account to a transitory change in saving. We find that the theory is inconsistent with the new rule. Specifically, the additional saving is invested abroad to an extent that exceeds the steady state share of net foreign assets to wealth.

We show that while this point emerges in a small open economy setting similar to KV, it is even sharper in a general equilibrium framework. The limitation of a partial equilibrium portfolio choice setting is that it ignores some key aggregate accounting identities. Moreover, it is misleading to interpret the ratio of net foreign assets to wealth as a portfolio share chosen by domestic investors. With two-way capital flows, this ratio depends on portfolio shares chosen by investors from both countries as well as the relative wealth of the two countries. We find that in a more realistic two-country general equilibrium setup the model is even further apart from the new rule.

Appendix

A Solution Small Country Model

In this Appendix we first describe the zero-order components of variables (steady state solution), and then use linear expansions of all equations around these zero-order components to compute the first-order solution.

Zero-order component of variables

The model is driven by nine relations. The exogenous processes are domestic productivity (1) and the return on the foreign asset (5). These are completed by the wage (2), the dynamics of domestic capital (3), the return on domestic capital (4), the portfolio return (6), the Euler equation for consumption (8), the optimal portfolio condition (9) and the asset market clearing condition (10).

The zero-order solution is computed by letting the standard deviation of shocks approach zero. (1) implies that $A(0) = 1$. From (9) and (6):

$$R^p(0) = R(0) = R^*(0) = 1 - \delta + D^*$$

The level of domestic capital then follows from (4), the investment from (3), and the wage from (2):

$$K(0) = \left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad I(0) = \delta\left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad W(0) = \omega\left[\frac{1-\omega}{D^*}\right]^{\frac{1-\omega}{\omega}}$$

(8) gives the consumption of young agents, and the portfolio share is obtained from (10):

$$\begin{aligned} C_y(0) &= \frac{\omega}{1+\beta^{\frac{1}{\gamma}}\left[1-\delta+D^*\right]^{\frac{1-\gamma}{\gamma}}}\left[\frac{1-\omega}{D^*}\right]^{\frac{1-\omega}{\omega}} \\ z(0) &= \frac{1-\omega}{\omega}\frac{1}{D^*}\frac{1+\beta^{\frac{1}{\gamma}}\left[1-\delta+D^*\right]^{\frac{1-\gamma}{\gamma}}}{\beta^{\frac{1}{\gamma}}\left[1-\delta+D^*\right]^{\frac{1-\gamma}{\gamma}}} \end{aligned}$$

A useful measure is the ratio between the zero-order component of young consumption and the zero-order component of the wage:

$$\bar{c} = \frac{C_y(0)}{W(0)} = \frac{1}{1+\beta^{\frac{1}{\gamma}}\left[1-\delta+D^*\right]^{\frac{1-\gamma}{\gamma}}}$$

Finally, the zero-order component of the budget constraint (7) gives:

$$C_o(0) = (1 - \bar{c})R(0)W(0)$$

First-Order Solution

We now take first-order Taylor expansions of the nine equations of the model around the zero-order allocation. The domestic productivity (1) and the return on the foreign asset (5) only have first order components:

$$A_t(1) = \varepsilon_t \qquad\qquad R^*_{t+1}(1) = \epsilon^D_{t+1}$$

The first-order component of the equations (5), (9) and (6) immediately imply that the expected first-order component of the return on the domestic and foreign capital, as well as on the portfolio, are zero:

$$E_t R_{t+1}(1) = E_t R^p_{t+1}(1) = E_t R^*_{t+1}(1) = 0$$

Taking an expectation of the first-order component of (4) we have:

$$E_t R_{t+1}(1) = -\omega(1-\omega)K(0)^{-\omega-1}K_{t+1}(1)$$

Since $E_t R_{t+1}(1) = 0$, it follows that $K_{t+1}(1) = 0$.

As the domestic capital stock never changes, the realized wage is obtained from (2):

$$W_t(1) = W(0)\,\varepsilon_t$$

Taking the expectation of the first-order component of the budget constraint (7), we have:

$$E_t C_{o,t+1} = (W_t(1) - C_{y,t}(1))R(0) \tag{40}$$

The first-order component of the consumption Euler equation (8) is:

$$-\gamma \frac{C_{y,t}(1)}{C_y(0)} = -\gamma E_t \frac{C_{o,t+1}(1)}{C_0(0)} + E_t \frac{R^p_{t+1}(1)}{R(0)}$$

Using (40) and $E_t R^p_{t+1}(1) = 0$, we write this as:

$$C_{yt}(1) = \bar{c}W_t(1) = \bar{c}W(0)\varepsilon_t$$

Finally, the first-order component of the asset market clearing condition (10) gives

$$z_t(1) = -z(0)\varepsilon_t$$

The first-order component of savings (11) is:

$$S_t(1) = (1-\bar{c})W(0)(\varepsilon_t - \varepsilon_{t-1})$$

Third-Order Returns

The third-order component of (9) can be written as:

$$\begin{aligned} 0 &= E_t \frac{R_{t+1}(3)}{R(0)} - \gamma E_t \left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)} \right) \frac{R^p_{t+1}(2)}{R(0)} \\ &\quad + \frac{1}{2}(1+\gamma)\gamma E_t \left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)} \right) \left(\frac{R^p_{t+1}(1)}{R(0)} \right)^2 \end{aligned}$$

Following steps detailed in the Technical appendix, the third-order expected excess return on the return of domestic capital is:

$$E_t \frac{R_{t+1}(3)}{R(0)} = -\gamma z(0) \left[\left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 + \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \varepsilon_t$$

A transitory increase in productivity reduces the third-order expected return on domestic capital, leading to a reduction in $z_t(1)$. The third order component of (4) also implies a third-order increase in the capital stock.

B Solution General Equilibrium Model

This Appendix presents the zero-order solution and the first-order expansions of the variables around their zero order values for the general equilibrium model.

Zero-order component of variables

The zero-order components of all equations other than the difference across countries in portfolio Euler equations gives is the zero-order component of all variables other than the zero-order component of the difference across countries in portfolio shares. Variables that grow at rate g are scaled by the country's labor supply.

From (15) we have $A^i(0) = 1$ for $i = H, F$. From the portfolio Euler equations (22)-(23) we have

$$R^H(0) = R^F(0) \equiv R(0)$$

It then follows from (17) and (16) that

$$k^H(0) = k^F(0) \equiv k(0)$$
$$R(0) = 1 - \delta + (1-\omega)k(0)^{-\omega}$$
$$W_H(0) = W_F(0) = \omega k(0)^{1-\omega}$$

Gross investment follows from the capital accumulation:

$$i^H(0) = i^F(0) = (g+\delta)k(0)$$

(20) and (22) are written as:

$$C_o^i(0) = (W(0) - C_y^i(0))R(0)$$
$$C_o^i(0) = C_y^i(0)(\beta^i)^{1/\gamma}R(0)^{1/\gamma}$$

The solution for the young' consumption then follows as:

$$C_y^i(0) = \frac{W(0)R(0)}{R(0) + (\beta^i)^{1/\gamma}R(0)^{1/\gamma}} \equiv \bar{c}^i W(0)$$

The sum of the asset market clearing conditions (24)-(25) gives

$$(1+g)k(0) = n(W(0) - C_y^H(0)) + (1-n)(W(0) - C_y^F(0)) \tag{41}$$

Substituting the expressions for $W(0)$, $C_y^H(0)$ and $C_y^F(0)$ above then yields an implicit solution for $k(0)$. Finally, from the Home asset market clearing condition (24) we have

$$(1+g)k(0) = (W(0) - C^H(0))z^H(0) + (W(0) - C^F(0))z^F(0)\frac{1-n}{n} \tag{42}$$

This gives a solution for a weighted average of portfolio shares, namely $z^A(0) = n$.

We now turn to the implied zero-order components of saving and the current account. Saving is equal to income minus consumption. Aggregate consumption at time t in the Home country is

$$N_t^H C_{y,t}^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H R_t^{p,H} + \tau R_t^F (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H(1 - z_{t-1}^H)$$

The three components are consumption by the young, the old and the brokers that immediately consume the revenues from the fee τ on foreign returns. Aggregate income (in net terms) is

$$W_t^H N_t^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H \left(z_{t-1}^H(R_t - 1 + \delta) + (1 - z_{t-1}^H)(R_t^F - 1 + \delta) - \delta\right)$$

Therefore national saving, which is income minus consumption, is

$$S_t^H = N_t^H(W_t^H - C_{y,t}^H) - N_{t-1}^H(W_{t-1}^H - C_{y,t-1}^H)$$

Dividing by N_t^H and taking the zero-order component, we have

$$s^H(0) = \frac{g}{1+g}(W(0) - C_y^H(0))$$

Since the current account is saving minus investment (net of depreciation), the previous results imply

$$ca^H(0) = \frac{g}{1+g}(W(0) - C_y^H(0)) - gk(0)$$

Substituting (41), this becomes

$$ca^H(0) = \frac{g}{1+g}(1-n)\left(C_y^F(0) - C_y^H(0)\right)$$

Finally we compute the steady state net foreign asset position of the Home country. We have

$$NFA_t^H = (W_t^H - C_{y,t}^H)N_t^H(1 - z_t^H) - (W_t^F - C_{y,t}^F)N_t^F z_t^F$$

Dividing by N_t^H and taking the zero-order component, we have

$$nfa^H(0) = (W(0) - C_y^H(0))(1 - z^H(0)) - (W(0) - C_y^F(0))z^F(0)\frac{1-n}{n}$$

Substituting (42), this becomes

$$nfa^H(0) = (W(0) - C_y^H(0)) - (1+g)k(0)$$

which together with (41) becomes

$$nfa^H(0) = (1-n)\left(C_y^F(0) - C_y^H(0)\right)$$

Total wealth per unit of the labor force is $W(0) - C_y^H(0)$, so that the ratio of net foreign assets to wealth is:

$$\frac{nfa^H(0)}{W(0) - C_y^H(0)} = (1-n)\frac{C_y^F(0) - C_y^H(0)}{W(0) - C_y^H(0)} = x^H(0)$$

First-Order Solution of the other variables

From (15) we have $A_t^i(1) = \varepsilon_t^i$ for $i = H, F$. From a first-order expansion of the portfolio Euler equations, and the definition of the portfolio return, we have

$$E_t R_{t+1}^H(1) = E_t R_{t+1}^F(1) = E_t R_{t+1}^{p,i}(1) \equiv E_t R_{t+1}(1)$$

It then follows from (17) and (16) that

$$\begin{aligned} k_{t+1}^H(1) &= k_{t+1}^F(1) \equiv k_{t+1}(1) \\ E_t R_{t+1}(1) &= -\omega(1-\omega)k(0)^{-\omega-1}k_{t+1}(1) \\ W_t^i(1) &= W(0)\varepsilon_t^i + (1-\omega)\frac{W(0)}{k(0)}k_t(1) \end{aligned}$$

The sum of the first-order components of (24) and (25) implies:

$$k_{t+1}(1)(1+g) = n\left(W_t^H(1) - C_{y,t}^H(1)\right) + (1-n)\left(W_t^F(1) - C_{y,t}^F(1)\right)$$

Using (21) and the expected value of (20), the consumption of young agents is:

$$C_{y,t}^i(1) = \bar{c}^i W_t^i(1) - \frac{1-\gamma}{\gamma}\bar{c}^i(1-\bar{c}^i)W(0)\frac{E_t R_{t+1}(1)}{R(0)}$$

Combining our results, we solve for the first-order capital stock as:

$$k_{t+1}(1) = \delta_k^k\left[k_t(1) + \frac{k(0)}{1-\omega}\varepsilon_t^A(1)\right] \tag{43}$$

where $\varepsilon_t^A(1) = \eta\varepsilon_t^H(1) + (1-\eta)\varepsilon_t^F(1)$, and:

$$\begin{aligned} \delta_k^k &= \frac{n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)}{\Omega_k}\frac{W(0)}{k(0)}(1-\omega) \\ \Omega_k &= 1 + g + \frac{1-\gamma}{\gamma}\left[n\bar{c}^H(1-\bar{c}^H) + (1-n)\bar{c}^F(1-\bar{c}^F)\right]\omega\frac{W(0)}{k(0)}\frac{R(0)-(1-\delta)}{R(0)} \end{aligned}$$

We now have the dynamics of all states variables, namely $k_t(1)$, $\varepsilon_t^A(1)$, and $\varepsilon_t^D(1) = \varepsilon_t^H(1) - \varepsilon_t^F(1)$.

The consumption of young agents in the Home country is computed as:

$$C_{y,t}^H(1) = \delta_k^{cy,H}k_t(1) + \delta_{eA}^{cy,H}\varepsilon_t^A(1) + \delta_{eD}^{cy,H}\varepsilon_t^D(1)$$

where:

$$\begin{aligned}
\delta_{k}^{cy,H} &= \bar{c}^{H}\left(1-\omega\right)\frac{W\left(0\right)}{k\left(0\right)}\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{H})\frac{\omega k\left(0\right)^{-\omega}}{R\left(0\right)}\delta_{k}^{k}\right] \\
\delta_{eA}^{cy,H} &= \bar{c}^{H}W(0)\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{H})\frac{\omega k\left(0\right)^{-\omega}}{R\left(0\right)}\delta_{k}^{k}\right] \\
\delta_{eD}^{cy,H} &= \bar{c}^{H}W(0)\left(1-\eta\right)
\end{aligned}$$

Similarly, the consumption in the foreign country is:

$$C_{y,t}^{F}\left(1\right)=\delta_{k}^{cy,F}k_{t}\left(1\right)+\delta_{eA}^{cy,F}\varepsilon_{t}^{A}\left(1\right)+\delta_{eD}^{cy,F}\varepsilon_{t}^{D}\left(1\right)$$

where:

$$\begin{aligned}
\delta_{k}^{cy,F} &= \bar{c}^{F}\left(1-\omega\right)\frac{W\left(0\right)}{k\left(0\right)}\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{F})\frac{\omega k\left(0\right)^{-\omega}}{R\left(0\right)}\delta_{k}^{k}\right] \\
\delta_{eA}^{cy,F} &= \bar{c}^{F}W(0)\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{F})\frac{\omega k\left(0\right)^{-\omega}}{R\left(0\right)}\delta_{k}^{k}\right] \\
\delta_{eD}^{cy,F} &= -\bar{c}^{F}W(0)\eta
\end{aligned}$$

The average portfolio shares is computed from the clearing of asset markets:

$$z_{t}^{A}\left(1\right)=-\eta\left(1-\eta\right)z^{D}\left(0\right)\left[\varepsilon_{t}^{D}\left(1\right)-\left(\bar{c}^{H}-\bar{c}^{F}\right)\frac{1-\gamma}{\gamma}\frac{\omega(1-\omega)k(0)^{-\omega-1}}{R\left(0\right)}k_{t+1}\left(1\right)\right]$$

The savings of country i are computed as:

$$\begin{aligned}
s_{t}^{i}\left(1\right) &= \left(W_{t}^{i}(1)-C_{y,t}^{i}(1)\right)-\frac{1}{1+g}(W_{t-1}^{i}(1)-C_{y,t-1}^{i}(1)) \\
&= (1-\bar{c}^{i})\tilde{\Delta}W_{t}^{i}\left(1\right)-\frac{1-\gamma}{\gamma}\bar{c}^{i}(1-\bar{c}^{i})\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta}k_{t+1}\left(1\right)
\end{aligned}$$

where:

$$\tilde{\Delta}x_{t}=x_{t}-\frac{1}{1+g}x_{t-1}$$

The current account represents the gap between savings and investment net of depreciation:

$$\begin{aligned}
ca_{t}^{H}\left(1\right) &= (1-\bar{c}^{H})\tilde{\Delta}W_{t}^{H}\left(1\right) \\
&\quad -\left[1+g+\frac{1-\gamma}{\gamma}\bar{c}^{H}(1-\bar{c}^{H})\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\right]\tilde{\Delta}k_{t+1}\left(1\right)
\end{aligned}$$

The ratio of the first-order current account to the first-order savings is:

$$\begin{aligned}\frac{ca_t^H(1)}{s_t^H(1)} &= (1-n)\frac{s_t^H(1)-s_t^F(1)}{s_t^H(1)} \\ &= (1-n)\left[1-\frac{(1-\bar{c}^F)\tilde{\Delta}W_t^F(1)-\frac{1-\gamma}{\gamma}\bar{c}^F(1-\bar{c}^F)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta}k_{t+1}(1)}{(1-\bar{c}^H)\tilde{\Delta}W_t^H(1)-\frac{1-\gamma}{\gamma}\bar{c}^H(1-\bar{c}^H)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta}k_{t+1}(1)}\right]\end{aligned}$$

Difference in portfolio shares

The zero-order difference in portfolio shares, $z^D(0)$, is computed by taking a difference between the second-order components of (22) and (23):

$$0 = 2\tau - \gamma\left(\frac{R_{t+1}^H(1)}{R(0)} - \frac{R_{t+1}^F(1)}{R(0)}\right)\left(\frac{R_{t+1}^{p,H}(1)}{R(0)} - \frac{R_{t+1}^{p,F}(1)}{R(0)}\right)$$

Using the first order solution for the returns on portfolio, this leads to:

$$z^D(0) = \frac{\tau}{\gamma\sigma_a^2}\left[\frac{R(0)}{(1-\omega)k(0)^{-\omega}}\right]^2 \tag{44}$$

The first-order difference in portfolio shares, $z_t^D(1)$, is computed by taking a difference between the third-order components of (22) and (23). Following steps detailed in the technical Appendix, this implies:

$$\begin{aligned}0 &= -\gamma z_t^D(1)\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2 2\sigma_a^2 \\ &\quad -2\gamma z^D(0)\frac{R(0)-(1-\delta)}{R(0)}2\sigma_a^2 E_tR_{t+1}(1)\frac{1}{R(0)} \\ &\quad +2(1-\gamma)\tau\frac{E_tR_{t+1}(1)}{R(0)} \\ &\quad +\gamma(1+\gamma)z^D(0)\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2 2\sigma_a^2\frac{E_tR_{t+1}(1)}{R(0)}\end{aligned}$$

Using our earlier results, the portfolio share is written as:

$$z_t^D(1) = \frac{2\omega(1-\delta)}{R(0)k(0)}z^D(0)\,k_{t+1}(1)$$

Drivers of capital flows

The gross capital outflows and inflows from the perspective of the Home country are given by:

$$\begin{aligned}
out_t^H(1) &= s_t^H(1)\left(1 - z^H(0)\right) - W(0)(1-\bar{c}^H)\tilde{\Delta} z_t^H(1) \\
in_t^H(1) &= \frac{1-n}{n}\left[s_t^F(1)\,z^F(0) + W(0)(1-\bar{c}^F)\tilde{\Delta} z_t^F(1)\right]
\end{aligned}$$

The first term corresponds to the portfolio growth component, while the second term is the portfolio reallocation component. The net capital flows are:

$$\begin{aligned}
net_t^H(1) &= (1-n)\left(1 - z^D(0)\right)\left[s_t^H(1) - s_t^F(1)\right] \\
&\quad + \frac{(1-n)(\bar{c}^F - \bar{c}^H) z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[n s_t^H(1) + (1-n)\, s_t^F(1)\right] \\
&\quad - \frac{1}{n} W(0)\left[n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)\right]\tilde{\Delta} z_t^A(1)
\end{aligned}$$

References

[1] Devereux, Michael B. and Alan Sutherland (2006), "Solving for Country Portfolios in Open Economy Macro Models, " working paper, University of British Columbia.

[2] Jin, Keyu, and Kai Guo (2008), "Composition and Growth Effects of the Current Account: A Synthesized Portfolio View," mimeo, Harvard University.

[3] Jorion, Philippe, and Will Goetzmann (1999), " Global Stock Markets in the Twentieth Century," Journal of Finance 54, 953-980.

[4] Kraay, Aart and Jaume Ventura (2000), "Current Accounts in Debtor and Creditor Countries," *Quarterly Journal of Economics* XCV, 1137-1166.

[5] Kraay, Aart and Jaume Ventura (2003), "Current Accounts in the Long and Short Run," *NBER Macro Annual 2002*.

[6] Tille, Cedric and van Wincoop (2008), "International Capital Flows", CEPR Discussion Paper 6705, February.

[7] van Wincoop, Eric (2003), comment on "Current Accounts in the Long and Short Run" by Aart Kray and Jaume Ventura, *NBER Macro Annual 2002*, 105-110.

Technical Appendix for A New Perspective on "The New Rule" of the Current Account[1]

Cedric Tille
Graduate Institute of International and Development Studies, Geneva
CEPR

Eric van Wincoop
University of Virginia
NBER

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1 A small open economy model

1.1 Production and Investment

Production of the world good in the small country uses a constant returns to scale technology:

$$Y_t = A_t K_t^{1-\omega} N_t^{\omega}$$

where Y is output, K is the capital input and N the labor input. A is an exogenous stochastic productivity term, that follows a simple i.i.d. process:

$$A_t = 1 + \varepsilon_t \tag{1}$$

where ε_t has a $N(0, \sigma_a^2)$ distribution.

The labor input is set to unity, and the the real wage is equal to the margnal product of labor:

$$W_t = \omega A_t K_t^{1-\omega} \tag{2}$$

The capital stock can change over time with investment, without adjustment costs:

$$K_{t+1} = (1-\delta) K_t + I_t \tag{3}$$

where δ is the depreciation rate.

1.2 Consumption and Portfolio Choice

Agents in the small open economy can buy claims on both domestic and foreign capital, that both share the same depreciation rate. Foreign investors do not invest in the domestic capital. The gross returns on domestic and foreign capital are:

$$R_{t+1} = 1 - \delta + (1-\omega) A_{t+1} K_{t+1}^{-\omega} \tag{4}$$

$$R_{t+1}^{*} = 1 - \delta + D^{*} + \epsilon_{t+1}^{D} \tag{5}$$

where ϵ_{t+1}^{D} is a i.i.d. process that follows a $N(0, \sigma_d^2)$ distribution and D^* is a constant.

Investment in the foreign asset does entail a iceberg cost τ that is a second-order constant (proportional to the variance of model innovations). The cost is a fee rebated to domestic brokers who consume it right away. The cost τ can generate portfolio home bias. The portfolio return from t to $t+1$ of small country investors is

$$R^p_{t+1} = z_t R_{t+1} + (1 - z_t)(1 - \tau) R^*_{t+1} \tag{6}$$

where z_t is the fraction of wealth invested in domestic capital.

The economy is characterized by a simple OLG structure with agents living two periods. They earn the wage W_t in (2) when young. They consume in both periods, with the consumption in the second period financed by the return on the portfolio. We denote the consumption of young and old agents at time t by $C_{y,t}$ and $C_{o,t}$ respectively. The optimization problem faced by a young agent at time t is to choose her initial consumtpion $C_{y,t}$ and portfolio allocation z_t to maximize:

$$\frac{(C_{y,t})^{1-\gamma}}{1-\gamma} + \beta E_t \frac{(C_{o,t+1})^{1-\gamma}}{1-\gamma}$$

subject to:

$$C_{o,t+1} = (W_t - C_{y,t}) R^p_{t+1} \tag{7}$$

and (6). The first-order conditions are an Euler relation for the dynamics of consumption and a portfolio arbitrage equation:

$$(C_{y,t})^{-\gamma} = \beta E_t \left(C_{o,t+1}\right)^{-\gamma} R^p_{t+1} \tag{8}$$

$$E_t \left(R^p_{t+1}\right)^{-\gamma} \left(R_{t+1} - (1-\tau) R^*_{t+1}\right) = 0 \tag{9}$$

As the small country investors are the only ones who can purchase claims on domestic capital, the small country asset market clearing condition is:[1]

$$K_{t+1} = (W_t - C_{y,t}) z_t \tag{10}$$

1.3 Some national accouting

Aggregate saving is the sum of saving by the young and the old. Saving by the young is $W_t - C_{y,t}$. The old agents consume $C_{o,t} = (W_{t-1} - C_{y,t-1}) R^p_t$. Recall that the return on any asset is the value of the residual capital plus the dividend stream. In line with national accounting, we consider that (net) income consists

[1] Unlike in the general equilibrium model outlined below, we do not need to specify a good market clearing condition.

of the dividend stream net of depreciation. We also need to subtract the fee paid to brokers. The resulting net income of the old is then

$$\left[z_{t-1}(1-\omega)A_t K_t^{-\omega} + (1-z_{t-1})\left(\bar{D}^* + \kappa_t^* - \tau R_t^*\right) - \delta\right](W_{t-1} - C_{y,t-1})$$

As the income of brokers is consumed right away it plays no role in savings.

National saving is then

$$\begin{aligned} S_t &= W_t - C_{y,t} \\ &\quad + \left[z_{t-1}(1-\omega)A_t K_t^{-\omega} + (1-z_{t-1})\left(\bar{D}^* + \kappa_t^* - \tau R_t^*\right) - R_t^p - \delta\right](W_{t-1} - C_{y,t-1}) \\ &= W_t - C_{y,t} - (W_{t-1} - C_{y,t-1}) \end{aligned}$$

The savings investment gap is simply $S_t - I_t^{net}$ where investment is net of depreciation.

1.4 Zero-order component of variables

The model is driven by ten relations. The exogenous processes are domestic productivity (1) and the return on the foreign asset (5). These are completed by the wage (2), the dynamics of domestic capital (3), the return on domestic capital (4), the portfolio return (6), the consumption of old agents (7), the Euler equation for consumption (8), the optimal portfolio condition (9) and the asset market clearing condition (10).

The zero-order solution is computed by holding all variables constant. (1) implies that $A(0) = 1$. From (9) and (6) the return on the domestic capital and the portfolio is equal to that on the foreign capital:

$$R(0) = R^p(0) = 1 - \delta + D^*$$

The level of domestic capital then follows from (4), the investment from (3), and the wage from (2):

$$K(0) = \left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad I(0) = \delta\left[\frac{1-\omega}{D^*}\right]^{\frac{1}{\omega}} \qquad W(0) = \omega\left[\frac{1-\omega}{D^*}\right]^{\frac{1-\omega}{\omega}}$$

(7) and (8) gives the consumption of young and old agents, and the portfolio share

is obtained from (10):

$$
\begin{aligned}
C_y(0) &= \frac{1}{1+\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}W(0) \\
C_o(0) &= \frac{\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1}{\gamma}}}{1+\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}W(0) \\
z(0) &= \frac{1-\omega}{\omega}\frac{1}{\bar{D}^*}\frac{1+\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}{\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}
\end{aligned}
$$

A useful measure is the ratio between the zero-order component of young consumption and the zero-order component of the wage:

$$
\bar{c} = \frac{C_y(0)}{W(0)} = \frac{1}{1+\beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}} \Rightarrow \beta^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}} = \frac{1-\bar{c}}{\bar{c}}
$$

which implies:

$$
C_o(0) = (1-\bar{c})\,R(0)\,W(0)
$$

1.5 Taylor expansions

We now take Taylor expansions of the nine equations of the model around the zero-order allocation. The domestic productivity (1) and the return on the foreign asset (5) only have first order components:

$$
A_t(1) = \varepsilon_t \qquad\qquad R^*_{t+1}(1) = \epsilon^D_{t+1} \tag{11}
$$

Next, we take quadratic expansions of the various relations. From here on all variables are defined in deviation from their zero-order components.

(2) and (3) become:

$$
\frac{W_t}{W(0)} = A_t + (1-\omega)\frac{K_t}{K(0)} + \frac{1-\omega}{2}\left[2A_t\frac{K_t}{K(0)} - \omega\left(\frac{K_t}{K(0)}\right)^2\right] \tag{12}
$$

$$
K_{t+1} = (1-\delta)K_t + I_t \tag{13}
$$

The domestic and portfolio returns (4) and (6) are:

$$
\frac{R_{t+1}}{R(0)}\frac{R(0)}{\bar{D}^*} = A_{t+1} - \omega\frac{K_{t+1}}{K(0)} + \frac{\omega}{2}\left[-2A_{t+1}\frac{K_{t+1}}{K(0)} + (1+\omega)\left(\frac{K_{t+1}}{K(0)}\right)^2\right] \tag{14}
$$

$$
\begin{aligned}
\frac{R^p_{t+1}}{R(0)} &= z(0)\frac{R_{t+1}}{R(0)} + (1-z(0))\frac{R^*_{t+1}}{R(0)} \\
&\quad -(1-z(0))\tau + z_t\left(\frac{R_{t+1}}{R(0)} - \frac{R^*_{t+1}}{R(0)}\right)
\end{aligned} \tag{15}
$$

The consumption of old agents (7) and the Euler equation for consumption (8) are written as:

$$\begin{aligned}
\frac{C_{o,t+1}}{C_o(0)} &= \frac{1}{1-\bar{c}}\frac{W_t}{W(0)} - \frac{\bar{c}}{1-\bar{c}}\frac{C_{y,t}}{C_y(0)} + \frac{R^p_{t+1}}{R(0)} \\
&\quad + \left[\frac{1}{1-\bar{c}}\frac{W_t}{W(0)} - \frac{\bar{c}}{1-\bar{c}}\frac{C_{y,t}}{C_y(0)}\right]\frac{R^p_{t+1}}{R(0)}
\end{aligned} \tag{16}$$

$$\begin{aligned}
\frac{C_{y,t}}{C_y(0)} &= E_t\frac{C_{o,t+1}}{C_o(0)} - \frac{1}{\gamma}E_t\frac{R^p_{t+1}}{R(0)} + \frac{1+\gamma}{2}\left(\frac{C_{y,t}}{C_y(0)}\right)^2 \\
&\quad + E_t\frac{C_{o,t+1}}{C_o(0)}\frac{R^p_{t+1}}{R(0)} - \frac{1+\gamma}{2}E_t\left(\frac{C_{o,t+1}}{C_o(0)}\right)^2
\end{aligned} \tag{17}$$

The asset market clearing condition (10) is:

$$\frac{K_{t+1}}{K(0)}\frac{K(0)}{W(0)} = \left[\frac{W_t}{W(0)} - \bar{c}\frac{C_{y,t}}{C_y(0)}\right]z(0) + (1-\bar{c})\,z_t + \left[\frac{W_t}{W(0)} - \bar{c}\frac{C_{y,t}}{C_y(0)}\right]z_t \tag{18}$$

Finally, we take a cubic expansion of the optimal portfolio condition (9):

$$\begin{aligned}
0 &= E_t\left(\frac{R_{t+1}}{R(0)} - \frac{R^*_{t+1}}{R(0)}\right) + \tau \\
&\quad -\gamma E_t\left(\frac{R_{t+1}}{R(0)} - \frac{R^*_{t+1}}{R(0)}\right)\frac{R^p_{t+1}}{R(0)} - \gamma\tau E_t\frac{R^p_{t+1}}{R(0)} + \tau E_t\frac{R^*_{t+1}}{R(0)} \\
&\quad +\frac{1}{2}(1+\gamma)\,\gamma E_t\left(\frac{R_{t+1}}{R(0)} - \frac{R^*_{t+1}}{R(0)}\right)\left(\frac{R^p_{t+1}}{R(0)}\right)^2
\end{aligned} \tag{19}$$

1.6 First-order component of variables

We now compute the first-order component of the equations. (11), (19) and (15) immediately imply that the first-order component of the expected return on the domestic and foreign capital, as well as on the portfolio, are zero:

$$E_tR_{t+1}(1) = E_tR^p_{t+1}(1) = E_tR^*_{t+1}(1) = 0$$

Taking an expectation of (14), $E_tR_{t+1}(1) = 0$ implies that the first-order component of the capital stock is zero as well:

$$\frac{K_{t+1}}{K(0)} = \frac{E_tK_{t+1}(1)}{K(0)} = \frac{1}{\omega}E_tA_{t+1}(1) - \frac{1}{\omega}\frac{R(0)}{D^*}\frac{E_tR_{t+1}(1)}{R(0)} = 0$$

Hence the realized return on domestic capital, as well as on the portfolio, are:

$$\begin{aligned}
\frac{R_{t+1}(1)}{R(0)} &= \frac{D^*}{R(0)}\varepsilon_{t+1} \\
\frac{R^p_{t+1}(1)}{R(0)} &= z(0)\frac{D^*}{R(0)}\varepsilon_{t+1} + (1 - z(0))\frac{1}{R(0)}\epsilon^D_{t+1}
\end{aligned}$$

As the domestic capital stock never changes, the realized wage is obtained from (12):

$$\frac{W_t(1)}{W(0)} = \varepsilon_t$$

The consumption of young and old agents immediately follows from (16) and (17):

$$\begin{aligned}
\frac{C_{y,t}(1)}{C_y(0)} &= \varepsilon_t \\
\frac{C_{o,t+1}(1)}{C_o(0)} &= \varepsilon_t + z(0)\frac{D^*}{R(0)}\varepsilon_{t+1} + (1 - z(0))\frac{1}{R(0)}\epsilon^D_{t+1} \\
&= \frac{W_t(1)}{W(0)} + \frac{R^p_{t+1}(1)}{R(0)}
\end{aligned}$$

Using the previous results, the first-order component of the asset market clearing condition (18) implies

$$z_t(1) = -z(0)\,\varepsilon_t$$

A temporary income shock boosts wages and consumption of the young in equal proportion, hence the savings of the young also increase. The portfolio allocation is shifted away from the domestic asset as the supply of domestic capital remains constant.

Finally, the first-order component of the savings is:

$$\begin{aligned}
\frac{S_t(1)}{W(0)} &= \left[\frac{W_t}{W(0)} - \bar{c}\frac{C_{y,t}}{C_y(0)}\right] - \left[\frac{W_{t-1}}{W(0)} - \bar{c}\frac{C_{y,t-1}}{C_y(0)}\right] \\
&= (1 - \bar{c})(\varepsilon_t - \varepsilon_{t-1})
\end{aligned}$$

1.7 Second-order component of variables

We now turn to the second-order component of model equations. Note that all cross-products entail only first-order terms that we have solved for.

We start with the second-order component of (19), recalling that $R^*_{t+1}(2) = 0$:

$$\begin{aligned} E_t \frac{R_{t+1}(2)}{R(0)} &= -\tau + \gamma E_t \left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)} \right) \frac{R^p_{t+1}(1)}{R(0)} \\ &= -\tau + \gamma \left[z(0) \left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 - (1 - z(0)) \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \end{aligned}$$

where we assume that ε_{t+1} and ϵ^D_{t+1} are orthogonal for simplicity. Notice that no state variable enters, hence: $E_t R_{t+1}(2) = R(2)$ at any period. Second order expected returns are time invariant.

Next take the second order component of (14), using our result that the first-order component of the capital is zero:

$$\frac{R_{t+1}(2)}{R(0)} \frac{R(0)}{D^*} = -\omega \frac{K_{t+1}(2)}{K(0)}$$

As $K_{t+1}(2) = E_t K_{t+1}(2)$, we take expectations on both sides and write:

$$\frac{K_{t+1}(2)}{K(0)} = \frac{R(0)}{\omega D^*}\tau - \gamma \frac{R(0)}{\omega D^*} \left[z(0) \left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 - (1 - z(0)) \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right]$$

The international financial friction τ cost boosts domestic capital, as resources that are not invested abroad have to be invested into domestic capital. Investment abroad is encouraged by volatile domestic shocks, and discouraged by volatile foreign shock. Notice that no state variable enters, hence: $K_{t+1}(2) = K(2)$ at any period. This also implies that $R_{t+1}(2) = E_t R_{t+1}(2)$. (15) then implies:

$$\begin{aligned} \frac{R^p_{t+1}(2)}{R(0)} &= z(0) \frac{R_{t+1}(2)}{R(0)} - (1 - z(0))\tau + z_t(1) \left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)} \right) \\ &= -\tau + z(0)\gamma \left[z(0) \left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 - (1 - z(0)) \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \\ &\quad - z(0)\varepsilon_t \left(\frac{D^*}{R(0)} \varepsilon_{t+1} - \frac{1}{R(0)} \epsilon^D_{t+1} \right) \end{aligned}$$

This second-order component of the portfolio return is time variant due to movements in $z_t(1)$.

The second-order component of (12) immediately implies that at any period:

$$\begin{aligned} \frac{W_t(2)}{W(0)} &= (1 - \omega) \frac{K_t(2)}{K(0)} \\ &= \frac{1 - \omega}{\omega} \frac{R(0)}{D^*}\tau - \gamma \frac{1 - \omega}{\omega} \frac{R(0)}{D^*} \left[z(0) \left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 - (1 - z(0)) \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \end{aligned}$$

The elements that favor domestic investment (high friction, low domestic volatility, high foreign volatility), boost the domestic wage.

Using our first-order results, the second-order components of (16) and (17) are:

$$\begin{aligned}
\frac{C_{o,t+1}(2)}{C_o(0)} &= \frac{1}{1-\bar{c}}\frac{W(2)}{W(0)} - \frac{\bar{c}}{1-\bar{c}}\frac{C_{y,t}(2)}{C_y(0)} + \frac{R^p_{t+1}(2)}{R(0)} \\
&\quad + \varepsilon_t\left[z(0)\frac{D^*}{R(0)}\varepsilon_{t+1} + (1-z(0))\frac{1}{R(0)}\epsilon^D_{t+1}\right] \\
\frac{C_{y,t}(2)}{C_y(0)} &= E_t\frac{C_{o,t+1}(2)}{C_o(0)} - \frac{1}{\gamma}E_t\frac{R^p_{t+1}(2)}{R(0)} \\
&\quad + \frac{1-\gamma}{2}\left[(z(0))^2\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 + (1-z(0))^2\left(\frac{1}{R(0)}\right)^2\sigma_d^2\right]
\end{aligned}$$

Combining these relations, we write:

$$\begin{aligned}
\frac{C_{y,t}(2)}{C_y(0)} &= \frac{W(2)}{W(0)} + (1-\bar{c})\frac{1-\gamma}{\gamma}\tau - (1-\bar{c})\frac{1-\gamma}{2}(z(0))^2\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 \\
&\quad + (1-\bar{c})\frac{1-\gamma}{2}(1+z(0))(1-z(0))\left(\frac{1}{R(0)}\right)^2\sigma_d^2
\end{aligned}$$

and:

$$\begin{aligned}
\frac{C_{o,t+1}(2)}{C_o(0)} &= \frac{W(2)}{W(0)} - \left(1+\bar{c}\frac{1-\gamma}{\gamma}\right)\tau \\
&\quad + \left(1+\bar{c}\frac{1-\gamma}{\gamma}\right)z(0)\gamma\left[z(0)\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 - (1-z(0))\left(\frac{1}{R(0)}\right)^2\sigma_d^2\right] \\
&\quad - \bar{c}\frac{1-\gamma}{2}\left[(z(0))^2\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 + (1-z(0))^2\left(\frac{1}{R(0)}\right)^2\sigma_d^2\right] \\
&\quad + \frac{1}{R(0)}\varepsilon_t\epsilon^D_{t+1}
\end{aligned}$$

$C_{y,t}(2)$ is not time variant, but $C_{o,t+1}(2)$ is through its last term. The elements that favor domestic investment (high friction, low domestic volatility, high foreign volatility, boost the consumption of the young, but have an ambiguous impact for the old.

From the second-order component of (10) we get the second-order portfolio share:

$$\frac{K(2)}{K(0)}\frac{K(0)}{W(0)} = \left[\frac{W(2)}{W(0)} - \bar{c}\frac{C_{y,t}(2)}{C_y(0)}\right]z(0) + (1-\bar{c})z_t(2) + (1-\bar{c})\varepsilon_t z_t(1)$$

which is solved as:

$$
\begin{aligned}
z_t(2) &= \left[\frac{1}{1-\bar{c}}\frac{1-\omega}{\omega D^*} - (1-\omega)\,z(0)\right]\frac{K(2)}{K(0)} + z(0)\,(\varepsilon_t)^2 \\
&+ \left[\begin{array}{c} \bar{c}\frac{1-\gamma}{\gamma}\tau - \bar{c}\frac{1-\gamma}{2}\,(z(0))^2\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 \\ +\bar{c}\frac{1-\gamma}{2}\,(1+z(0))\,(1-z(0))\left(\frac{1}{R(0)}\right)^2\sigma_d^2 \end{array}\right] z(0)
\end{aligned}
$$

While $\varepsilon_t > 0$ leads to a first order decrease in z, the second order component increases.

1.8 Expected excess returns

The first-order component of the expected excess return is zero. To higher orders, the excess return is simply given by the domestic return. We already solved for $E_t R_{t+1}(2)$ from the second-order component of (19).

Now take the third-order component of (19), using the fact that $R_{t+1}(2)$ is not time variant and $E_t R_{t+1}(1) = E_t R^p_{t+1}(1) = E_t R^*_{t+1}(1) = 0$:

$$
\begin{aligned}
0 &= E_t\frac{R_{t+1}(3)}{R(0)} - \gamma E_t\left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)}\right)\frac{R^p_{t+1}(2)}{R(0)} \\
&+\frac{1}{2}(1+\gamma)\,\gamma E_t\left(\frac{R_{t+1}(1)}{R(0)} - \frac{R^*_{t+1}(1)}{R(0)}\right)\left(\frac{R^p_{t+1}(1)}{R(0)}\right)^2
\end{aligned}
$$

Recall that:

$$
\begin{aligned}
\frac{R_{t+1}(1)}{R(0)} &= \frac{D^*}{R(0)}\varepsilon_{t+1} \\
\frac{R^p_{t+1}(1)}{R(0)} &= z(0)\,\frac{D^*}{R(0)}\varepsilon_{t+1} + (1-z(0))\frac{1}{R(0)}\epsilon^D_{t+1}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{R_{t+1}(2)}{R(0)} &= -\tau + \gamma\left[z(0)\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 - (1-z(0))\left(\frac{1}{R(0)}\right)^2\sigma_d^2\right] \\
\frac{R^p_{t+1}(2)}{R(0)} &= -\tau + z(0)\,\gamma\left[z(0)\left(\frac{D^*}{R(0)}\right)^2\sigma_a^2 - (1-z(0))\left(\frac{1}{R(0)}\right)^2\sigma_d^2\right] \\
&\quad - z(0)\,\varepsilon_t\left(\frac{D^*}{R(0)}\varepsilon_{t+1} - \frac{1}{R(0)}\epsilon^D_{t+1}\right)
\end{aligned}
$$

Using all these results, the third-order component of (19) becomes:

$$E_t \frac{R_{t+1}(3)}{R(0)} = -\gamma z(0) \left[\left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 + \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \varepsilon_t$$
$$-\frac{1}{2}(1+\gamma)\gamma E_t \left(\begin{array}{c} (z(0))^2 \left(\frac{d^*}{R(0)} \varepsilon_{t+1} \right)^3 - (1-z(0))^2 \left(\frac{1}{R(0)} \epsilon_{t+1}^D \right)^3 \\ + (1-3z(0))(1-z(0)) \left(\frac{D^*}{R(0)} \varepsilon_{t+1} \right) \left(\frac{1}{R(0)} \epsilon_{t+1}^D \right)^2 \\ +(2-3z(0))z(0) \left(\frac{D^*}{R(0)} \varepsilon_{t+1} \right)^2 \left(\frac{1}{R(0)} \epsilon_{t+1}^D \right) \end{array} \right)$$

As the shocks are orthogonal, and $E_t(\varepsilon_{t+1})^3 = E_t\left(\epsilon_{t+1}^D\right)^3 = 0$, this becomes:

$$E_t \frac{R_{t+1}(3)}{R(0)} = -\gamma z(0) \left[\left(\frac{D^*}{R(0)} \right)^2 \sigma_a^2 + \left(\frac{1}{R(0)} \right)^2 \sigma_d^2 \right] \varepsilon_t$$

This shows that an increase in productivity lowers the expected excess return to the third-order. This implies an increase in the capital stock and investment that is third-order as well.

2 A two-country general equilibrium model

2.1 Production and Investment

The relative size of the two countries is a free parameter. In order to generate non-zero steady state saving, we allow for exogenous population growth in both countries, along with different time-discount rates across countries.

We will call the countries Home and Foreign, denoted with superscripts H and F. They produce the same good using the technology

$$Y_t^i = A_t^i (K_t^i)^{1-\omega} (N_t^i)^{\omega} \qquad i = H, F$$

A fraction n of the world population lives in the Home country and a fraction $1-n$ in the Foreign country. Population grows at a rate g in both countries. The young cohorts are then:

$$N_t^H = n(1+g)^t \qquad N_t^F = (1-n)(1+g)^t \tag{20}$$

We use lower case letters to denote the ratio of a variable relative to the young population in the country. For example, $k_t^i = K_t^i / N_t^i$. We express the model in terms of these scaled variables.

Productivity in both countries follows a simple i.i.d. process

$$A_t^i = 1 + \varepsilon_t^i \tag{21}$$

where ε_t^i has a $N(0, \sigma_a^2)$ distribution, and productivity shocks are uncorrelated across the two countries. Wages are equal to the marginal product of labor:

$$W_t^i = \omega A_t^i (k_t^i)^{1-\omega} \tag{22}$$

The capital stocks evolves according to

$$(1+g)k_{t+1}^i = (1-\delta)\, k_t^i + i_t^i \tag{23}$$

where δ is the depreciation rate.

2.2 Consumption and Portfolio Choice

Trade in assets is not restricted and agents from both countries can buy claims on the capital of the other country. The gross return on country i capital is:

$$R_{t+1}^i = 1 - \delta + (1-\omega) A_{t+1}^i \left(k_{t+1}^i\right)^{-\omega} \tag{24}$$

When investing abroad each country receives the gross return times $1-\tau$, where τ is again a second-order constant iceberg cost that captures the hurdles of investing abroad. It is a fee paid to a domestic broker who immediately consumes the revenues. Country i invests a fraction z_t^i in Home capital. The portfolio returns from t to $t+1$ of investors from both countries is then

$$R_{t+1}^{p,H} = z_t^H R_{t+1}^H + (1 - z_t^H)(1-\tau) R_{t+1}^F \tag{25}$$
$$R_{t+1}^{p,F} = z_t^F (1-\tau) R_{t+1}^H + (1 - z_t^F) R_{t+1}^F \tag{26}$$

The OLG structure is as in the small open economy model, leading to a well-defined steady state wealth distribution. We allow the time-discount rates in the two countries to be different. A young agent in country i at time t maximizes:

$$\frac{\left(C_{y,t}^i\right)^{1-\gamma}}{1-\gamma} + \beta^i E_t \frac{\left(C_{o,t+1}^i\right)^{1-\gamma}}{1-\gamma}$$

subject to:

$$C_{o,t+1}^i = (W_t^i - C_{y,t}^i) R_{t+1}^{p,i} \tag{27}$$

and the portfolio return (25) or (26).

The first order conditions with respect to $C^i_{y,t}$ and z^i_t are:

$$\left(C^i_{y,t}\right)^{-\gamma} = \beta^i E_t \left(C^i_{o,t+1}\right)^{-\gamma} R^{p,i}_{t+1} \tag{28}$$

$$E_t \left(R^{p,H}_{t+1}\right)^{-\gamma} \left(R^H_{t+1} - (1-\tau) R^F_{t+1}\right) = 0 \tag{29}$$

$$E_t \left(R^{p,F}_{t+1}\right)^{-\gamma} \left((1-\tau) R^H_{t+1} - R^F_{t+1}\right) = 0 \tag{30}$$

The asset market clearing conditions are

$$k^H_{t+1}(1+g) = \left(W^H_t - C^H_{y,t}\right) z^H_t + \left(W^F_t - C^F_{y,t}\right) \frac{1-n}{n} z^F_t \tag{31}$$

$$k^F_{t+1}(1+g) = \left(W^H_t - C^H_{y,t}\right) \frac{n}{1-n}(1 - z^H_t) + \left(W^F_t - C^F_{y,t}\right)(1 - z^F_t) \tag{32}$$

These two relations can be written in a more compact way. First, we define the average portfolio share invested into Home equity:

$$z^A_t = \eta z^H_t + (1-\eta) z^F_t \tag{33}$$

where η is the share of the Home investors in world wealth in the steady state ($\bar{c}^i$ is the steady state ratio of consumption by the young to wage in country i):

$$\eta = \frac{n(1-\bar{c}^H)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)} \tag{34}$$

We also define the difference in portfolio shares $z^D_t = z^H_t - z^F_t$. We can then write:

$$z^H_t = z^A_t + (1-\eta) z^D_t \qquad\qquad z^F_t = z^A_t - \eta z^D_t$$

(31)-(32) are then:

$$\begin{aligned} k^H_{t+1}(1+g) &= \left[\left(W^H_t - C^H_{y,t}\right) + \left(W^F_t - C^F_{y,t}\right) \frac{1-n}{n}\right] z^A_t \\ &\quad + \left[\left(W^H_t - C^H_{y,t}\right)(1-\eta) - \eta \left(W^F_t - C^F_{y,t}\right) \frac{1-n}{n}\right] z^D_t \end{aligned} \tag{35}$$

$$\begin{aligned} k^F_{t+1}(1+g) &= \left[\left(W^H_t - C^H_{y,t}\right) \frac{n}{1-n} + \left(W^F_t - C^F_{y,t}\right)\right] (1 - z^A_t) \\ &\quad - \left[\left(W^H_t - C^H_{y,t}\right) \frac{n}{1-n} (1-\eta) - \left(W^F_t - C^F_{y,t}\right) \eta\right] z^D_t \end{aligned} \tag{36}$$

It is useful to take the sum of (35), times n, and (36), times $1-n$:

$$\left[n k^H_{t+1} + (1-n) k^F_{t+1}\right](1+g) = n \left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right) \tag{37}$$

2.3 Saving and current account

Saving is equal to income minus consumption. Aggregate consumption at time t in the Home country is

$$N_t^H C_{y,t}^H + (W_{t-1}^H - C_{y,t-1}^H) N_{t-1}^H R_t^{p,H} + \tau R_t^F (W_{t-1}^H - C_{y,t-1}^H) N_{t-1}^H (1 - z_{t-1}^H)$$

The three components are consumption by the young, the old and the brokers that immediately consume the revenues from the fee τ on foreign returns. Aggregate income net of depreciation is

$$W_t^H N_t^H + (W_{t-1}^H - C_{y,t-1}^H) N_{t-1}^H \left(z_{t-1}^H (R_t^H - 1) + (1 - z_{t-1}^H)(R_t^F - 1) \right)$$

Therefore national saving, which is income minus consumption, is

$$S_t^H = N_t^H (W_t^H - C_{y,t}^H) - N_{t-1}^H (W_{t-1}^H - C_{y,t-1}^H) \tag{38}$$

Dividing by N_t^H we have

$$s_t^H = (W_t^H - C_{y,t}^H) - \frac{1}{1+g}(W_{t-1}^H - C_{y,t-1}^H)$$

An analogous derivation applies to the Foreign country, so that for $i = H, F$

$$s_t^i = \left(W_t^i - C_{y,t}^i \right) - \frac{1}{1+g}(W_{t-1}^i - C_{y,t-1}^i) \tag{39}$$

The current account is (using a net measure of investment):.

$$ca_t^i = s_t^i - i_t^{net,i} \tag{40}$$

Aggregate saving is equal to aggregate investment, so that

$$N_t^H s_t^H + N_t^F s_t^F = N_t^H i_t^{n,H} + N_t^F i_t^{n,F}$$
$$\Rightarrow ca_t^F = -\frac{n}{1-n} ca_t^H$$

In terms of asset stocks, we compute the net foreign asset position of the Home country, and scale it by the size of the Home country:

$$\begin{aligned} NFA_t^H &= N_t^H (W_t^H - C_{y,t}^H)(1 - z_t^H) - N_t^F (W_t^F - C_{y,t}^F) z_t^F \\ nfa_t^H &= (W_t^H - C_{y,t}^H)(1 - z_t^H) - \frac{1-n}{n}(W_t^F - C_{y,t}^F) z_t^F \end{aligned} \tag{41}$$

The wealth of country i is the size of its labor force times $W_t^i - C_{y,t}^i$.

The ratio between the Home country net foreign assets and its wealth is:

$$nfaw_t^H = \frac{NFA_t^H}{N_t^H (W_t^H - C_{y,t}^H)} = (1 - z_t^H) - z_t^F \frac{1-n}{n} \frac{W_t^F - C_{y,t}^F}{W_t^H - C_{y,t}^H} \tag{42}$$

2.4 Zero-order component of variables

The model is driven by 18 relations: two exogenous productivity processes (21), two wages (22), two dynamics of capital (23), two returns on capital (24), two returns on portfolio (25)-(26), two budget constraints (27), two consumption Euler equations (28), two portfolio Euler equations (29)-(30) and two asset market clearing conditions (35)-(36). All these equations are written in terms of stationary variables.

The zero-order solution is computed by holding all variables constant. (21) implies that $A(0) = 1$ in both countries. From (29)-(30) and (25)-(26) all rates of return are equalized. (24) and (22) then implies that the capital / labor ratio $k(0)$ and the wages $W(0)$ do not differ across countries, and the investment rate follows from (23):

$$i(0) = (g + \delta) k(0)$$

From (24) and (22) the return and wages are expressed in terms of the capital intensity:

$$\begin{aligned} W(0) &= \omega (k(0))^{1-\omega} \\ R(0) &= 1 - \delta + (1-\omega) k(0)^{-\omega} \end{aligned}$$

(27) and (28) imply that consumption is tilted towards youth in the country with the smaller β:

$$\frac{C_y^i(0)}{C_o^i(0)} = \left[\beta^i\right]^{-\frac{1}{\gamma}} [R(0)]^{-\frac{1}{\gamma}}$$

In terms of consumption levels:

$$\begin{aligned} C_y^i(0) &= \frac{1}{1 + \left[\beta^i\right]^{\frac{1}{\gamma}} [R(0)]^{\frac{1-\gamma}{\gamma}}} W(0) \\ C_o^i(0) &= \frac{\left[\beta^i\right]^{\frac{1}{\gamma}} [R(0)]^{\frac{1}{\gamma}}}{1 + \left[\beta^i\right]^{\frac{1}{\gamma}} [R(0)]^{\frac{1-\gamma}{\gamma}}} W(0) \end{aligned}$$

For convenience, we define:

$$\bar{c}^i = \frac{C_y^i(0)}{W(0)} = \frac{1}{1 + \left[\beta^i\right]^{\frac{1}{\gamma}} [R(0)]^{\frac{1-\gamma}{\gamma}}}$$

The smaller β^i (the more impatient the country), the larger $\bar{c}^i$.

The zero order component of (37) is:

$$k(0)(1+g) = W(0)\left[n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)\right] \tag{43}$$

As the returns, wages and consumptions are functions of the capital intensity, (43) gives an implicit solution for $k(0)$, from which we get the full zero-order solution.

Using (43), along with the fact that $n\left(1-\bar{c}^H\right)(1-\eta) - \eta\left(1-\bar{c}^F\right)(1-n) = 0$ from (34), the zero order component of (35) implies:

$$z^A(0) = n \tag{44}$$

Savings (39) in country i is

$$s^i(0) = \frac{g}{1+g}\left(W(0) - C_y^i(0)\right) = \frac{g}{1+g}\frac{\left[\beta^i\right]^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}{1+\left[\beta^i\right]^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}W(0)$$

Using (43) the current account of country i is:

$$\begin{aligned} ca^i(0) &= \frac{g}{1+g}\left[W(0) - C_y^i(0) - (1+g)k(0)\right] \\ &= -\frac{g}{1+g}\left[\bar{c}^i - \left[n\bar{c}^H + (1-n)\bar{c}^F\right]\right]W(0) \end{aligned}$$

The country runs a deficit when the consumption of its young exceeds the world average young consumption, which occurs when its β^i is low (i.e. it is impatient). for instance, setting $i = H$, the Home country runs a current account deficit when $\beta^H < \beta^F$:

$$\begin{aligned} ca^H(0) &= -(1-n)\frac{g}{1+g}\left[\bar{c}^H - \bar{c}^F\right]W(0) \\ &= (1-n)\frac{g}{1+g}\frac{\left[\beta^H\right]^{\frac{1}{\gamma}} - \left[\beta^F\right]^{\frac{1}{\gamma}}}{1+\left[\beta^H\right]^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}}\frac{[R(0)]^{\frac{1-\gamma}{\gamma}}W(0)}{1+\left[\beta^F\right]^{\frac{1}{\gamma}}[R(0)]^{\frac{1-\gamma}{\gamma}}} \end{aligned} \tag{45}$$

In terms of net foreign assets (41) becomes

$$\begin{aligned} nfa^H(0) &= \left[\left(1-\bar{c}^H\right)\left(1-z^H(0)\right) - \frac{1-n}{n}\left(1-\bar{c}^F\right)z^F(0)\right]W(0) \\ &= \left(1-\bar{c}^H\right)W(0) - \frac{1}{n}\left[n\left(1-\bar{c}^H\right)z^H(0)) + (1-n)\left(1-\bar{c}^F\right)z^F(0)\right]W(0) \\ &= \left(1-\bar{c}^H\right)W(0) - \frac{n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)}{n}z^A(0)W(0) \\ &= \left(1-\bar{c}^H\right)W(0) - k(0)(1+g) \end{aligned}$$

where we used (43) and (44).

Using (43) the Home country is a net debtor when when $\beta^H < \beta^F$:

$$\begin{aligned} nfa^H(0) &= -(1-n)\left[\bar{c}^H - \bar{c}^F\right] W(0) \qquad (46) \\ &= -(1-n)\frac{\left[\beta^H\right]^{\frac{1}{\gamma}} - \left[\beta^F\right]^{\frac{1}{\gamma}}}{1+\left[\beta^H\right]^{\frac{1}{\gamma}}\left[R(0)\right]^{\frac{1-\gamma}{\gamma}}} \frac{\left[R(0)\right]^{\frac{1-\gamma}{\gamma}} W(0)}{1+\left[\beta^F\right]^{\frac{1}{\gamma}}\left[R(0)\right]^{\frac{1-\gamma}{\gamma}}} \end{aligned}$$

We then have

$$x^H(0) = \frac{nfa^H(0)}{we^H(0)} = \frac{ca^H(0)}{s^H(0)} \Rightarrow ca^H(0) = x^H(0)\, s^H(0)$$

where $we^H(0) = \left(1-\bar{c}^H\right) W(0)$ is the wealth and $x^H(0)$ is the steady state ratio of the net foreign asset position to wealth. Using (41) we write:

$$x^H(0) = nfaw^H(0) = -(1-n)\frac{\bar{c}^H - \bar{c}^F}{1-\bar{c}^H} = \frac{\eta - z^A(0)}{\eta} = \frac{\eta - n}{\eta}$$

2.5 Taylor expansions

We now take Taylor expansions of the equations of the model around the zero-order allocation. the productivity processes (21) only have a first order component:

$$A_t^i(1) = \varepsilon_t^i \qquad (47)$$

The wages (22), and dynamics of capital (23) are:

$$\begin{aligned} \frac{W_t^i}{W(0)} &= A_t^i + (1-\omega)\frac{k_t^i}{k(0)} + \frac{1-\omega}{2}\left[2A_t^i\frac{k_t^i}{k(0)} - \omega\left(\frac{k_t^i}{k(0)}\right)^2\right] \qquad (48) \\ (1+g)\,k_{t+1}^i &= (1-\delta)\,k_t^i + i_t^i \qquad (49) \end{aligned}$$

The returns on capital (24) and on portfolio (25)-(26) are:

$$\begin{aligned} \frac{R_{t+1}^i}{R(0)}\frac{R(0)}{R(0)-(1-\delta)} &= A_{t+1}^i - \omega\frac{k_{t+1}^i}{k(0)} - \omega A_{t+1}^i\frac{k_{t+1}^i}{k(0)} + \frac{\omega(1+\omega)}{2}\left[\frac{k_{t+1}^i}{k(0)}\right]^2 \qquad (50) \\ \frac{R_{t+1}^{p,H}}{R(0)} &= z^H(0)\frac{R_{t+1}^H}{R(0)} + \left(1-z^H(0)\right)\frac{R_{t+1}^F}{R(0)} \qquad (51) \\ &\quad -\left(1-z^H(0)\right)\tau + z_t^H\left(\frac{R_{t+1}^H}{R(0)} - \frac{R_{t+1}^F}{R(0)}\right) \\ \frac{R_{t+1}^{p,F}}{R(0)} &= z^F(0)\frac{R_{t+1}^H}{R(0)} + \left(1-z^F(0)\right)\frac{R_{t+1}^F}{R(0)} \qquad (52) \\ &\quad -z^F(0)\,\tau + z_t^F\left(\frac{R_{t+1}^H}{R(0)} - \frac{R_{t+1}^F}{R(0)}\right) \end{aligned}$$

The budget constraints (27) and two consumption Euler equations (28) are:

$$\begin{aligned}\frac{C^i_{o,t+1}}{C^i_o(0)} &= \frac{1}{1-\bar{c}^i}\frac{W^i_t}{W(0)} - \frac{\bar{c}^i}{1-\bar{c}^i}\frac{C^i_{y,t}}{C^i_y(0)} + \frac{R^{p,i}_{t+1}}{R(0)} \\ &\quad + \left[\frac{1}{1-\bar{c}^i}\frac{W^i_t}{W(0)} - \frac{\bar{c}^i}{1-\bar{c}^i}\frac{C^i_{y,t}}{C^i_y(0)}\right]\frac{R^{p,i}_{t+1}}{R(0)} \end{aligned} \tag{53}$$

$$\begin{aligned}\frac{C^i_{y,t}}{C^i_y(0)} &= E_t\frac{C^i_{o,t+1}}{C^i_o(0)} - \frac{1}{\gamma}E_t\frac{R^{p,i}_{t+1}}{R(0)} + \frac{1+\gamma}{2}\left(\frac{C^i_{y,t}}{C^i_y(0)}\right)^2 \\ &\quad + E_t\frac{C^i_{o,t+1}}{C^i_o(0)}\frac{R^{p,i}_{t+1}}{R(0)} - \frac{1+\gamma}{2}E_t\left(\frac{C^i_{o,t+1}}{C^i_o(0)}\right)^2 \end{aligned} \tag{54}$$

where $\bar{c}^i = C^i_y(0)/W(0)$. The asset market clearing conditions (35)-(36) are:

$$\begin{aligned} & nk^H_{t+1}(1+g) \\ = & \left[n\left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right)\right] n \\ & + \left[n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)\right] W(0)\, z^A_t \\ & + \frac{n(1-n)\, z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[\left(W^H_t - C^H_{y,t}\right)(1-\bar{c}^F) - (1-\bar{c}^H)\left(W^F_t - C^F_{y,t}\right)\right] \\ & + \left[n\left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right)\right] z^A_t \\ & + \frac{n(1-n)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[\left(W^H_t - C^H_{y,t}\right)(1-\bar{c}^F) - (1-\bar{c}^H)\left(W^F_t - C^F_{y,t}\right)\right] z^D_t \end{aligned} \tag{55}$$

and:

$$\begin{aligned} & (1-n)\, k^F_{t+1}(1+g) \\ = & \left[n\left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right)\right](1-n) \\ & - \left[n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)\right] W(0)\, z^A_t \\ & - \frac{n(1-n)\, z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[\left(W^H_t - C^H_{y,t}\right)(1-\bar{c}^F) - (1-\bar{c}^H)\left(W^F_t - C^F_{y,t}\right)\right] \\ & - \left[n\left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right)\right] z^A_t \\ & - \frac{n(1-n)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[\left(W^H_t - C^H_{y,t}\right)(1-\bar{c}^F) - (1-\bar{c}^H)\left(W^F_t - C^F_{y,t}\right)\right] z^D_t \end{aligned} \tag{56}$$

Note that the sum of these relations is:

$$\left[nk^H_{t+1} + (1-n)\, k^F_{t+1}\right](1+g) = n\left(W^H_t - C^H_{y,t}\right) + (1-n)\left(W^F_t - C^F_{y,t}\right)$$

Finally, we take a cubic expansion of the portfolio Euler equations (29)-(30):

$$\begin{aligned} 0 &= E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right) + \tau \\ &\quad -\gamma E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right)\frac{R^{p,H}_{t+1}}{R(0)} + \tau\left(\frac{E_t R^F_{t+1}}{R(0)} - \gamma\frac{E_t R^{p,H}_{t+1}}{R(0)}\right) \\ &\quad + \frac{\gamma(1+\gamma)}{2}E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right)\left(\frac{R^{p,H}_{t+1}}{R(0)}\right)^2 \end{aligned} \tag{57}$$

and:

$$\begin{aligned} 0 &= E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right) - \tau \\ &\quad -\gamma E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right)\frac{R^{p,F}_{t+1}}{R(0)} - \tau\left(\frac{E_t R^H_{t+1}}{R(0)} - \gamma\frac{E_t R^{p,F}_{t+1}}{R(0)}\right) \\ &\quad + \frac{\gamma(1+\gamma)}{2}E_t\left(\frac{R^H_{t+1}}{R(0)} - \frac{R^F_{t+1}}{R(0)}\right)\left(\frac{R^{p,F}_{t+1}}{R(0)}\right)^2 \end{aligned} \tag{58}$$

2.6 First-order component of variables

We now focus on the first-order component in the linear terms in the Taylor expansion. (57)-(58) and (51)-(52) immediately implies that the expected returns are all the same:

$$E_t R^i_{t+1}(1) = E_t R^{p,i}_{t+1}(1) \equiv E_t R_{t+1}(1) \qquad \forall i$$

(50) then gives the capital (which is the same for both countries), as well as the innovations in returns:

$$k_{t+1}(1) = -\frac{1}{\omega(1-\omega)k(0)^{-\omega-1}}E_t R_{t+1}(1) \tag{59}$$

$$R^i_{t+1}(1) = E_t R_{t+1}(1) + (1-\omega)k(0)^{-\omega}\varepsilon^i_{t+1} \tag{60}$$

We conjecture that

$$k_{t+1}(1) = \delta^k_k k_t(1) + \delta^k_{eA}\varepsilon^A_t(1) + \delta^k_{eD}\varepsilon^D_t(1) \tag{61}$$

where $\varepsilon^A_t(1) = \eta\varepsilon^H_t(1) + (1-\eta)\varepsilon^F_t(1)$, and $\varepsilon^D_t(1) = \varepsilon^H_t(1) - \varepsilon^F_t(1)$.

From (48) we get the solution for wages:

$$\frac{W_t^i(1)}{W(0)} = \varepsilon_t^i + (1-\omega)\frac{k_t(1)}{k(0)}$$

(49) gives the investment:

$$i_t(1) = \left[(1+g)\,\delta_k^k - (1-\delta)\right] k_t(1) + (1+g)\left[\delta_{eA}^k \varepsilon_t^A(1) + \delta_{eD}^k \varepsilon_t^D(1)\right]$$

The key is to solve for the coefficients in (61). The sum of (55) and (56) implies:

$$k_{t+1}(1)(1+g) = n\left(W_t^H(1) - C_{y,t}^H(1)\right) + (1-n)\left(W_t^F(1) - C_{y,t}^F(1)\right) \tag{62}$$

(54) and the expected value of (53) are written as:

$$\begin{aligned} E_t C_{o,t+1}^i(1) &= R(0)\left[W_t^i(1) - C_{y,t}^i(1)\right] + (1-\bar{c}^i)W(0)\,E_t R_{t+1}(1) \\ \frac{(1-\bar{c}^i)R(0)}{\bar{c}^i} C_{y,t}^i(1) &= E_t C_{o,t+1}^i(1) - \frac{1}{\gamma}(1-\bar{c}^i)W(0)E_t R_{t+1}(1) \end{aligned}$$

Combining these relations we get:

$$C_{y,t}^i(1) = \bar{c}^i W_t^i(1) - \frac{1-\gamma}{\gamma}\bar{c}^i(1-\bar{c}^i)W(0)\frac{E_t R_{t+1}(1)}{R(0)} \tag{63}$$

(62), the solution for wages, and (59) then lead to the coefficients in (61):

$$\begin{aligned} \delta_k^k &= \frac{n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)}{\Omega_k}\frac{W(0)}{k(0)}(1-\omega) \\ \delta_{eA}^k &= \frac{n\left(1-\bar{c}^H\right) + (1-n)\left(1-\bar{c}^F\right)}{\Omega_k} W(0) = \delta_k^k \frac{k(0)}{1-\omega} \\ \delta_{eD}^k &= 0 \end{aligned}$$

where

$$\Omega_k = 1 + g + \frac{1-\gamma}{\gamma}\left[n\bar{c}^H(1-\bar{c}^H) + (1-n)\,\bar{c}^F(1-\bar{c}^F)\right]\omega\frac{W(0)}{k(0)}\frac{R(0)-(1-\delta)}{R(0)}$$

We now have solved for the dynamics of the first-order component of state variables. The expected first order returns follow from (59). The unexpected returns are given by (60) and (51)-(52):

$$\begin{aligned} R_{t+1}^{p,H}(1) &= E_t R_{t+1}(1) + (1-\omega)k(0)^{-\omega}\left[z^H(0)\,\varepsilon_{t+1}^H + \left(1-z^H(0)\right)\varepsilon_{t+1}^F\right] \\ R_{t+1}^{p,F}(1) &= E_t R_{t+1}(1) + (1-\omega)k(0)^{-\omega}\left[z^F(0)\,\varepsilon_{t+1}^H + \left(1-z^F(0)\right)\varepsilon_{t+1}^F\right] \end{aligned}$$

The first-order component of consumption of young agents is derived from (63). In the Home country we get:

$$C_{y,t}^{H}(1)=\delta_{k}^{cy,H}k_{t}(1)+\delta_{eA}^{cy,H}\varepsilon_{t}^{A}(1)+\delta_{eD}^{cy,H}\varepsilon_{t}^{D}(1)$$

where:

$$\begin{aligned}
\delta_{k}^{cy,H} &= \bar{c}^{H}(1-\omega)\frac{W(0)}{k(0)}\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{H})\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eA}^{cy,H} &= \bar{c}^{H}W(0)\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{H})\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eD}^{cy,H} &= \bar{c}^{H}W(0)(1-\eta)
\end{aligned}$$

While in the Foreign country we get:

$$C_{y,t}^{F}(1)=\delta_{k}^{cy,F}k_{t}(1)+\delta_{eA}^{cy,F}\varepsilon_{t}^{A}(1)+\delta_{eD}^{cy,F}\varepsilon_{t}^{D}(1)$$

where:

$$\begin{aligned}
\delta_{k}^{cy,F} &= \bar{c}^{F}(1-\omega)\frac{W(0)}{k(0)}\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{F})\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eA}^{cy,F} &= \bar{c}^{F}W(0)\left[1+\frac{1-\gamma}{\gamma}(1-\bar{c}^{F})\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eD}^{cy,F} &= -\bar{c}^{F}W(0)\eta
\end{aligned}$$

The first-order component of consumption of old agents is derived from (53). In the home country, consumption depends on the lagged state variables and the current shocks:

$$C_{o,t+1}^{H}(1)=\delta_{k}^{co,H}k_{t}(1)+\delta_{eA}^{co,H}\varepsilon_{t}^{A}(1)+\delta_{eD}^{co,H}\varepsilon_{t}^{D}(1)+\delta_{eA2}^{co,H}\varepsilon_{t+1}^{A}(1)+\delta_{eD2}^{co,H}\varepsilon_{t+1}^{D}(1)$$

where:

$$\begin{aligned}
\delta_{k}^{co,H} &= (1-\bar{c}^{H})(1-\omega)\frac{R(0)W(0)}{k(0)}\left[1-\left(1+\bar{c}^{H}\frac{1-\gamma}{\gamma}\right)\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eA}^{co,H} &= R(0)W(0)(1-\bar{c}^{H})\left[1-\left(1+\bar{c}^{H}\frac{1-\gamma}{\gamma}\right)\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right]\\
\delta_{eD}^{co,H} &= R(0)W(0)(1-\eta)(1-\bar{c}^{H})\\
\delta_{eA2}^{co,H} &= \left(1-\bar{c}^{H}\right)W(0)\left[R(0)-(1-\delta)\right]\\
\delta_{eD2}^{co,H} &= \left(1-\bar{c}^{H}\right)W(0)\left[R(0)-(1-\delta)\right]\left(n-\eta+(1-\eta)z^{D}(0)\right)
\end{aligned}$$

In the Foreign country we get:

$$C_{o,t+1}^{F}(1)=\delta_{k}^{co,F}k_{t}(1)+\delta_{eA}^{co,F}\varepsilon_{t}^{A}(1)+\delta_{eD}^{co,F}\varepsilon_{t}^{D}(1)+\delta_{eA2}^{co,F}\varepsilon_{t+1}^{A}(1)+\delta_{eD2}^{co,F}\varepsilon_{t+1}^{D}(1)$$

where:

$$\begin{aligned}
\delta_{k}^{co,F} &= \left(1-\bar{c}^{F}\right)(1-\omega)\frac{R(0)\,W(0)}{k(0)}\left[1-\left(1+\bar{c}^{F}\frac{1-\gamma}{\gamma}\right)\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right] \\
\delta_{eA}^{co,F} &= \left(1-\bar{c}^{F}\right)R(0)\,W(0)\left[1-\left(1+\bar{c}^{F}\frac{1-\gamma}{\gamma}\right)\frac{\omega k(0)^{-\omega}}{R(0)}\delta_{k}^{k}\right] \\
\delta_{eD}^{co,F} &= -\left(1-\bar{c}^{F}\right)R(0)\,W(0)\eta \\
\delta_{eA2}^{co,F} &= \left(1-\bar{c}^{F}\right)W(0)\left[R(0)-(1-\delta)\right] \\
\delta_{eD2}^{co,F} &= \left(1-\bar{c}^{F}\right)W(0)\left[R(0)-(1-\delta)\right]\left(n-\eta-\eta z^{D}(0)\right)
\end{aligned}$$

2.6.1 Portfolio shares

We multiply the first-order component of (55) by $1-n$, and the first order component of (56) by n, and take the difference to show that:

$$\begin{aligned}
z_{t}^{A}(1) &= -\frac{n(1-n)\,z^{D}(0)}{\left[n(1-\bar{c}^{H})+(1-n)(1-\bar{c}^{F})\right]^{2}}\frac{1}{W(0)} \\
&\quad\times\left[\left(W_{t}^{H}(1)-C_{y,t}^{H}(1)\right)(1-\bar{c}^{F})-(1-\bar{c}^{H})\left(W_{t}^{F}(1)-C_{y,t}^{F}(1)\right)\right]
\end{aligned} \tag{64}$$

After further algebra, this becomes:

$$z_{t}^{A}(1)=-\eta(1-\eta)\,z^{D}(0)\left[\left(\varepsilon_{t}^{H}-\varepsilon_{t}^{F}\right)-\left(\bar{c}^{H}-\bar{c}^{F}\right)\frac{1-\gamma}{\gamma}\frac{\omega(1-\omega)k(0)^{-\omega-1}}{R(0)}k_{t+1}(1)\right]$$

When the countries are symmetric ($\bar{c}^{H}=\bar{c}^{F}$) a productivity improvement in the Home country boosts its wage and savings. It also leads to a portfolio shift away from Home equity ($z_{t}^{A}(1)<0$) when there is home bias ($z^{D}(0)$).

When the countries are assymetric, world shocks also matter. Consider that the Home country is more impatient, so $\bar{c}^{H}>\bar{c}^{F}$. (61) shows that a productivity shock anywhere boosts future capital ($k_{t+1}(1)$). This leads to a portfolio shift towards Home equity ($z_{t}^{A}(1)>0$). The intuition is as follows. The increase in investment is equal in the two countries. However, the Home agents save too little, so the investment in the Home country has to be funded partially by Foreign agents who dominate world saving. If there is no portfolio bias, the Foreign agents send enough money to the Home country. With portfolio home bias, the Foreign investors would tend to invest too much in their country. Financing the increase in Home capital (with impatient Home agents and Foreign investors having home bias) requires a portfolio shift towards Home equity.

Pinning down the $z^D(0)$ requires the linear and quadratic terms in (57)-(58):

$$\begin{aligned}0 &= E_t\left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)}\right) + \tau - \gamma E_t\left(\frac{R^H_{t+1}(1)}{R(0)} - \frac{R^F_{t+1}(1)}{R(0)}\right)\frac{R^{p,H}_{t+1}(1)}{R(0)} \\ 0 &= E_t\left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)}\right) - \tau - \gamma E_t\left(\frac{R^H_{t+1}(1)}{R(0)} - \frac{R^F_{t+1}(1)}{R(0)}\right)\frac{R^{p,F}_{t+1}(1)}{R(0)}\end{aligned}$$

We take the difference between these two relations and use our results for the first-order components of the various returns:

$$\begin{aligned}0 &= 2\tau - \gamma\left(\frac{R^H_{t+1}(1)}{R(0)} - \frac{R^F_{t+1}(1)}{R(0)}\right)\left(\frac{R^{p,H}_{t+1}(1)}{R(0)} - \frac{R^{p,F}_{t+1}(1)}{R(0)}\right) \\ &= \frac{1}{R(0)^2}\left[2(R(0))^2\tau - \gamma z^D(0)\left[(1-\omega)k(0)^{-\omega}\right]^2 E_t\left(\varepsilon^H_{t+1} - \varepsilon^F_{t+1}\right)^2\right]\end{aligned}$$

$$\Rightarrow z^D(0) = \frac{\tau}{\gamma\sigma_a^2}\left[\frac{R(0)}{(1-\omega)k(0)^{-\omega}}\right]^2 \tag{65}$$

2.7 Second-order excess returns

We start with the second-order expected excess return, and write (57) and (58) as:

$$\begin{aligned}0 &= E_t\left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)}\right) + \tau - \gamma\left(2z^H(0) - 1\right)\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2\sigma_a^2 \\ 0 &= E_t\left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)}\right) - \tau - \gamma\left(2z^F(0) - 1\right)\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2\sigma_a^2\end{aligned}$$

Multiply the first relation by η and the second by $1-\eta$, and add them using:

$$z^H(0) = n + (1-\eta)z^D(0) \qquad\qquad z^F(0) = n - \eta z^D(0)$$

to get:

$$\begin{aligned}&E_t\left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)}\right) \\ &= -(2\eta-1)\tau + (2n-1)\gamma\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2\sigma_a^2\end{aligned} \tag{66}$$

(66) shows that the second-order expected excess returns reflects assymetries in size, n, or shares in steady-state wealth, η. If Home investors are impatient ($\eta < 0.5 \Rightarrow 2\eta - 1 < 0$), the second order friction leads to a second order excess return on the Home equity: $E_t \left(R^H_{t+1}(2) - R^F_{t+1}(2) \right) > 0$. Intuitively, Home agents do not save enough, so Foreign investors need to finance investment in the Home country. Inducing them to hold Home equity requires a second order expected excess return.

Now take the second-order components of (50), using the fact that $k^H_{t+1}(1) = k^F_{t+1}(1)$:

$$\begin{aligned} & \left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)} \right) \frac{R(0)}{R(0) - (1-\delta)} \\ = \quad & -\omega \frac{k^H_{t+1}(2) - k^F_{t+1}(2)}{k(0)} - \omega \left(\varepsilon^H_{t+1} - \varepsilon^F_{t+1} \right) \frac{k_{t+1}(1)}{k(0)} \end{aligned} \tag{67}$$

Taking expectations and using (66) we write:

$$\frac{k^H_{t+1}(2) - k^F_{t+1}(2)}{k(0)} = \frac{1}{\omega} \left((2\eta - 1)\tau - (2n-1)\gamma \left[\frac{R(0) - (1-\delta)}{R(0)} \right]^2 \sigma^2_a \right) \frac{R(0)}{R(0) - (1-\delta)}$$

Using our first order results, (67) becomes:

$$\begin{aligned} & \frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)} \\ = \quad & -(2\eta - 1)\tau + (2n-1)\gamma \left[\frac{R(0) - (1-\delta)}{R(0)} \right]^2 \sigma^2_a \\ & -\omega \left(\varepsilon^H_{t+1} - \varepsilon^F_{t+1} \right) \frac{1}{k(0)} \frac{R(0) - (1-\delta)}{R(0)} \left[\delta^k_k k_t(1) + \delta^k_H \varepsilon^H_t + \delta^k_F \varepsilon^F_t \right] \end{aligned} \tag{68}$$

2.8 First-order portfolio difference

Take the third-order terms terms in (57)-(58):

$$\begin{aligned} 0 \quad = \quad & E_t \left(\frac{R^H_{t+1}(3)}{R(0)} - \frac{R^F_{t+1}(3)}{R(0)} \right) \\ & -\gamma E_t \left(\frac{R^H_{t+1}(1)}{R(0)} - \frac{R^F_{t+1}(1)}{R(0)} \right) \frac{R^{p,H}_{t+1}(2)}{R(0)} - \gamma E_t \left(\frac{R^H_{t+1}(2)}{R(0)} - \frac{R^F_{t+1}(2)}{R(0)} \right) \frac{R^{p,H}_{t+1}(1)}{R(0)} \\ & +\tau \left(\frac{E_t R^F_{t+1}(1)}{R(0)} - \gamma \frac{E_t R^{p,H}_{t+1}(1)}{R(0)} \right) + \frac{\gamma(1+\gamma)}{2} E_t \left(\frac{R^H_{t+1}(1)}{R(0)} - \frac{R^F_{t+1}(1)}{R(0)} \right) \left[\frac{R^{p,H}_{t+1}(1)}{R(0)} \right]^2 \end{aligned} \tag{69}$$

and:

$$\begin{aligned}
0 &= E_t\left(\frac{R^H_{t+1}(3)}{R(0)}-\frac{R^F_{t+1}(3)}{R(0)}\right) \\
&\quad -\gamma E_t\left(\frac{R^H_{t+1}(1)}{R(0)}-\frac{R^F_{t+1}(1)}{R(0)}\right)\frac{R^{p,F}_{t+1}(2)}{R(0)}-\gamma E_t\left(\frac{R^H_{t+1}(2)}{R(0)}-\frac{R^F_{t+1}(2)}{R(0)}\right)\frac{R^{p,F}_{t+1}(1)}{R(0)} \qquad (70)\\
&\quad -\tau\left(\frac{E_tR^H_{t+1}(1)}{R(0)}-\gamma\frac{E_tR^{p,F}_{t+1}(1)}{R(0)}\right)+\frac{\gamma(1+\gamma)}{2}E_t\left(\frac{R^H_{t+1}(1)}{R(0)}-\frac{R^F_{t+1}(1)}{R(0)}\right)\left[\frac{R^{p,F}_{t+1}(1)}{R(0)}\right]^2
\end{aligned}$$

From (51)-(52) we write:

$$\begin{aligned}
\frac{R^{p,H}_{t+1}(1)}{R(0)} &= n\frac{R^H_{t+1}(1)}{R(0)}+(1-n)\frac{R^F_{t+1}(1)}{R(0)}+(1-\eta)z^D(0)\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)} \\
\frac{R^{p,F}_{t+1}(1)}{R(0)} &= n\frac{R^H_{t+1}(1)}{R(0)}+(1-n)\frac{R^F_{t+1}(1)}{R(0)}-\eta z^D(0)\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}
\end{aligned}$$

and:

$$\begin{aligned}
\frac{R^{p,H}_{t+1}(2)}{R(0)} &= \left[n+(1-\eta)z^D(0)\right]\frac{R^H_{t+1}(2)}{R(0)}+\left(1-n-(1-\eta)z^D(0)\right)\frac{R^F_{t+1}(2)}{R(0)} \\
&\quad -\left(1-n-(1-\eta)z^D(0)\right)\tau+\left[z^A_t(1)+(1-\eta)z^D_t(1)\right]\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)} \\
\frac{R^{p,F}_{t+1}(2)}{R(0)} &= \left[n-\eta z^D(0)\right]\frac{R^H_{t+1}(2)}{R(0)}+\left(1-n+\eta z^D(0)\right)\frac{R^F_{t+1}(2)}{R(0)} \\
&\quad -\left(1-n+\eta z^D(0)\right)\tau+\left[z^A_t(1)-\eta z^D_t(1)\right]\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}
\end{aligned}$$

Now take the difference between (69) and (70):

$$\begin{aligned}
0 &= -\gamma E_t\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}\frac{R^{p,H}_{t+1}(2)-R^{p,F}_{t+1}(2)}{R(0)} \\
&\quad -\gamma E_t\frac{R^H_{t+1}(2)-R^F_{t+1}(2)}{R(0)}\frac{R^{p,H}_{t+1}(1)-R^{p,F}_{t+1}(1)}{R(0)} \\
&\quad +\tau E_t\left(\frac{R^H_{t+1}(1)+R^F_{t+1}(1)}{R(0)}-\gamma\frac{R^{p,H}_{t+1}(1)+R^{p,F}_{t+1}(1)}{R(0)}\right) \\
&\quad +\frac{\gamma(1+\gamma)}{2}E_t\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}\left(\left[\frac{R^{p,H}_{t+1}(1)}{R(0)}\right]^2-\left[\frac{R^{p,F}_{t+1}(1)}{R(0)}\right]^2\right)
\end{aligned}$$

which becomes, using the fact that all expected first order returns are equal to $E_t R_{t+1}(1)$:

$$\begin{aligned}
0 &= -\gamma z_t^D(1) E_t \left(\frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)}\right)^2 \\
&\quad -2\gamma z^D(0) E_t \frac{R_{t+1}^H(2) - R_{t+1}^F(2)}{R(0)} \frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)} \\
&\quad +2(1-\gamma)\tau \frac{E_t R_{t+1}(1)}{R(0)} \qquad (71) \\
&\quad +\gamma(1+\gamma) z^D(0) E_t \left(\frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)}\right)^2 \left[n\frac{R_{t+1}^H(1)}{R(0)} + (1-n)\frac{R_{t+1}^F(1)}{R(0)}\right] \\
&\quad +(1-2\eta) z^D(0) z^D(0) \frac{\gamma(1+\gamma)}{2} E_t \left(\frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)}\right)^3
\end{aligned}$$

From the shock processes we write:

$$\begin{aligned}
E_t\left(\varepsilon_{t+1}^H - \varepsilon_{t+1}^F\right)^3 &= E_t\left(\varepsilon_{t+1}^H\right)^3 - E_t\left(\varepsilon_{t+1}^F\right)^3 + 3E_t\left[\left(\varepsilon_{t+1}^F\right)^2 \varepsilon_{t+1}^H - \left(\varepsilon_{t+1}^H\right)^2 \varepsilon_{t+1}^F\right] \\
&= E_t\left(\varepsilon_{t+1}^H\right)^3 - E_t\left(\varepsilon_{t+1}^F\right)^3 \\
&= 0
\end{aligned}$$

and (using the fact that for a normal distribution $E_t(\varepsilon_{t+1})^3 = 0$:

$$\begin{aligned}
& E_t\left(\varepsilon_{t+1}^H - \varepsilon_{t+1}^F\right)^2\left(n\varepsilon_{t+1}^H + (1-n)\varepsilon_{t+1}^F\right) \\
&= nE_t\left(\varepsilon_{t+1}^H\right)^3 + (1-n)E_t\left(\varepsilon_{t+1}^F\right)^3 \\
&= 0
\end{aligned}$$

Using (60) we write:

$$\begin{aligned}
E_t\left(\frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)}\right)^2 &= \left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2 2\sigma_a^2 \\
E_t\left(\frac{R_{t+1}^H(1) - R_{t+1}^F(1)}{R(0)}\right)^3 &= 0
\end{aligned}$$

and:

$$
\begin{aligned}
& E_t\left(\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}\right)^2\left[n\frac{R^H_{t+1}(1)}{R(0)}+(1-n)\frac{R^F_{t+1}(1)}{R(0)}\right] \\
= \; & E_t\left(\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}\right)^2\left[\frac{E_tR_{t+1}(1)}{R(0)}+\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\left(n\varepsilon^H_{t+1}+(1-n)\varepsilon^F_{t+1}\right)\right] \\
= \; & \left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2 2\sigma^2_a\frac{E_tR_{t+1}(1)}{R(0)} \\
& +\left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^3 E_t\left(\varepsilon^H_{t+1}-\varepsilon^F_{t+1}\right)^2\left(n\varepsilon^H_{t+1}+(1-n)\varepsilon^F_{t+1}\right) \\
= \; & \left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)}\right]^2 2\sigma^2_a\frac{E_tR_{t+1}(1)}{R(0)}
\end{aligned}
$$

Finally, using (68) we write:

$$
\begin{aligned}
& E_t\frac{R^H_{t+1}(2)-R^F_{t+1}(2)}{R(0)}\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)} \\
= \; & E_t\left(\frac{R^H_{t+1}(2)-R^F_{t+1}(2)}{R(0)}\right)E_t\left(\frac{R^H_{t+1}(1)-R^F_{t+1}(1)}{R(0)}\right) \\
& -\omega(1-\omega)k(0)^{-\omega-1}\frac{R(0)-(1-\delta)}{R(0)}\left[\delta^k_k k_t(1)+\delta^k_H\varepsilon^H_t+\delta^k_F\varepsilon^F_t\right]E_t\left(\varepsilon^H_{t+1}-\varepsilon^F_{t+1}\right)^2\frac{1}{R(0)} \\
= \; & \frac{R(0)-(1-\delta)}{R(0)}2\sigma^2_aE_tR_{t+1}(1)\frac{1}{R(0)}
\end{aligned}
$$

Using all these results, (71) becomes:

$$
\begin{aligned}
0 \; = \; & -\gamma z^D_t(1)\left[\frac{R(0)-(1-\delta)}{R(0)}\right]^2 2\sigma^2_a \\
& -2\gamma z^D(0)\frac{R(0)-(1-\delta)}{R(0)}2\sigma^2_a\frac{E_tR_{t+1}(1)}{R(0)} \\
& +2(1-\gamma)\tau\frac{E_tR_{t+1}(1)}{R(0)} \\
& +\gamma(1+\gamma)z^D(0)\left[\frac{R(0)-(1-\delta)}{R(0)}\right]^2 2\sigma^2_a\frac{E_tR_{t+1}(1)}{R(0)}
\end{aligned}
$$

Using (65) this further simplifies to:

$$\begin{aligned}
z_t^D(1) &= \frac{2}{\gamma}\left[\frac{R(0)}{R(0)-(1-\delta)}\right]^2 \frac{\tau}{\sigma_a^2}\left(1-\left[\frac{R(0)}{R(0)-(1-\delta)}\right]^2 \frac{R(0)-(1-\delta)}{R(0)}\right)\frac{E_t R_{t+1}(1)}{R(0)} \\
&= -\frac{2}{\gamma}\left[\frac{R(0)}{R(0)-(1-\delta)}\right]^2 \frac{\tau}{\sigma_a^2}\frac{(1-\delta)}{R(0)-(1-\delta)}\frac{E_t R_{t+1}(1)}{R(0)} \\
&= \frac{2\omega}{\gamma}\left[\frac{R(0)}{R(0)-(1-\delta)}\right]^2 \frac{\tau}{\sigma_a^2}\frac{1-\delta}{k(0)}\delta_k^k\left[k_t(1)+\frac{k(0)}{1-\omega}\varepsilon_t^A(1)\right]\frac{1}{R(0)} \\
&= \frac{2\omega(1-\delta)}{R(0)k(0)} z^D(0)\, k_{t+1}(1) \qquad (72)
\end{aligned}$$

This shows that a productivity improvement anywhere in the world leads to a retrenchment ($z_t^D(1) > 0$).

2.9 First-order component of balance of payments

2.9.1 Savings and current account

Turning to the balance of payments, the first-order component of (39) is:

$$\begin{aligned}
s_t^i(1) &= \left(W_t^i(1) - C_{y,t}^i(1)\right) - \frac{1}{1+g}(W_{t-1}^i(1) - C_{y,t-1}^i(1)) \\
&= \left(\tilde{\Delta} W_t^i(1) - \tilde{\Delta} C_{y,t}^i(1)\right) \\
&= (1-\bar{c}^i)\tilde{\Delta} W_t^i(1) - \frac{1-\gamma}{\gamma}\bar{c}^i(1-\bar{c}^i)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta} k_{t+1}(1)
\end{aligned}$$

where we defined:

$$\tilde{\Delta} x_t = x_t - \frac{1}{1+g} x_{t-1}$$

The first-order savings-investment gap for the Home country is:

$$\begin{aligned}
ca_t^H(1) &= (1-\bar{c}^H)\tilde{\Delta} W_t^H(1) \qquad (73) \\
&\quad - \left[1+g+\frac{1-\gamma}{\gamma}\bar{c}^H(1-\bar{c}^H)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\right]\tilde{\Delta} k_{t+1}(1)
\end{aligned}$$

The ratio of the first-order current account to the first-order savings is:

$$\begin{aligned}
\frac{ca_t^H(1)}{s_t^H(1)} &= (1-n)\frac{s_t^H(1) - s_t^F(1)}{s_t^H(1)} \\
&= (1-n)\left[1 - \frac{(1-\bar{c}^F)\tilde{\Delta} W_t^F(1) - \frac{1-\gamma}{\gamma}\bar{c}^F(1-\bar{c}^F)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta} k_{t+1}(1)}{(1-\bar{c}^H)\tilde{\Delta} W_t^H(1) - \frac{1-\gamma}{\gamma}\bar{c}^H(1-\bar{c}^H)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta} k_{t+1}(1)}\right]
\end{aligned}$$

2.9.2 Gross and net assets

The first order Home gross assets are (in Home per capita terms):

$$
\begin{aligned}
gfa_t^H(1) &= (W_t^H(1)(1) - C_{y,t}^H(1)(1))(1 - z^H(0)) - W(0)(1-\bar{c}^H) z_t^H(1) \\
&= (W_t^H(1)(1) - C_{y,t}^H(1)(1)) \left(1 - n - (1-\eta) z^D(0)\right) \\
&\quad - W(0)(1-\bar{c}^H) \left[z_t^A(1) + (1-\eta) z_t^D(1)\right]
\end{aligned}
$$

Similarly, gross liabililities are (in Home per capita terms):

$$
\begin{aligned}
gfl_t^H(1) &= \frac{1-n}{n}(W_t^F(1) - C_{y,t}^F(1)) z^F(0) + \frac{1-n}{n} W(0)(1-\bar{c}^F) z_t^F(1) \\
&= \frac{1-n}{n}(W_t^F(1) - C_{y,t}^F(1)) \left(n - \eta z^D(0)\right) \\
&\quad + \frac{1-n}{n} W(0)(1-\bar{c}^F) \left[z_t^A(1) - \eta z_t^D(1)\right]
\end{aligned}
$$

The first order net asset position is then:

$$
\begin{aligned}
& nfa_t^H(1) = (1-n)\left[(W_t^H(1) - C_{y,t}^H(1)) - (W_t^F(1) - C_{y,t}^F(1))\right] \\
& -\frac{1}{n}\frac{n(1-n) z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)} \left[\left(W_t^H(1) - C_{y,t}^H(1)\right)(1-\bar{c}^F) - (1-\bar{c}^H)\left(W_t^F(1) - C_{y,t}^F(1)\right)\right] \\
& -\frac{1}{n} W(0)\left[n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)\right] z_t^A(1)
\end{aligned}
$$

We can check that the current account corresponds to the dynamics of the net foreign asset position. Using (55) the previous equation becomes first write:

$$
nfa_t^H(1) = W_t^H(1) - C_{y,t}^H(1) - k_{t+1}(1+g)(1)
$$

Using the definition of savings, this becomes:

$$
\begin{aligned}
nfa_t^H(1) &= s_t^H(1) + \frac{1}{1+g}(W_{t-1}^H(1) - C_{y,t-1}^H(1)) - k_{t+1}(1+g)(1) \\
&= s_t^H(1) - k_{t+1}(1)(1+g) + k_t(1) + \frac{1}{1+g} nfa_{t-1}^H(1) \\
&= s_t^H(1) - i_t^{net}(1) + \frac{1}{1+g} nfa_{t-1}^H(1)
\end{aligned}
$$

$$
\Rightarrow ca_t^H(1) = \tilde{\Delta} nfa_t^H(1)
$$

The first-order component of the ratio of net foreign asset to welath (42) is:

$$\begin{aligned}
nfaw_t^H(1) &= -z_t^H(1) - \frac{1-n}{n}\frac{1-\bar{c}^F}{1-\bar{c}^H} z_t^F(1) \\
&\quad -\frac{z^F(0)}{W(0)(1-\bar{c}^H)}\frac{1-n}{n}\left[\left[W_t^F(1) - C_{y,t}^F(1)\right] - \frac{1-\bar{c}^F}{1-\bar{c}^H}\left[W_t^H(1) - C_{y,t}^H(1)\right]\right] \\
&= -\frac{1}{\eta} z_t^A(1) - \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\frac{1-n}{n}\left[\left[W_t^F(1) - C_{y,t}^F(1)\right] - \frac{1-\bar{c}^F}{1-\bar{c}^H}\left[W_t^H(1) - C_{y,t}^H(1)\right]\right]
\end{aligned}$$

2.9.3 Financial flows

As there are no valuation gains, the current account corresponds to net financial flows. The gross outflows are:

$$\begin{aligned}
\tilde{\Delta} gfa_t^H(1) &= (\tilde{\Delta} W_t^H(1) - \tilde{\Delta} C_{y,t}^H(1))\left(1 - n - (1-\eta) z^D(0)\right) \\
&\quad -W(0)(1-\bar{c}^H)\left[\tilde{\Delta} z_t^A(1) + (1-\eta)\tilde{\Delta} z_t^D(1)\right] \\
&= s_t^H(1)\left(1 - z^H(0)\right) - W(0)(1-\bar{c}^H)\tilde{\Delta} z_t^H(1)
\end{aligned}$$

The first row corresponds to the portfolio growth component, while the second row is the portfolio reallocation component. Similarly:

$$\begin{aligned}
\tilde{\Delta} gfl_t^H(1) &= \frac{1-n}{n}(\tilde{\Delta} W_t^F(1) - \tilde{\Delta} C_{y,t}^F(1))\left(n - \eta z^D(0)\right) \\
&\quad + \frac{1-n}{n} W(0)(1-\bar{c}^F)\left[\tilde{\Delta} z_t^A(1) - \eta \tilde{\Delta} z_t^D(1)\right] \\
&= \frac{1-n}{n}\left[s_t^F(1) z^F(0) + W(0)(1-\bar{c}^F)\tilde{\Delta} z_t^F(1)\right]
\end{aligned}$$

and:

$$\begin{aligned}
\tilde{\Delta} nfa_t^H(1) &= (1-n)\left[(\tilde{\Delta} W_t^H(1) - \tilde{\Delta} C_{y,t}^H(1)) - (\tilde{\Delta} W_t^F(1) - \tilde{\Delta} C_{y,t}^F(1))\right] \\
&\quad - \frac{(1-n) z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[\begin{array}{c} (\tilde{\Delta} W_t^H(1) - \tilde{\Delta} C_{y,t}^H(1))(1-\bar{c}^F) \\ -(1-\bar{c}^H)(\tilde{\Delta} W_t^F(1) - \tilde{\Delta} C_{y,t}^F(1)) \end{array}\right] \\
&\quad - \frac{1}{n} W(0)\left[n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)\right]\tilde{\Delta} z_t^A(1) \\
&= (1-n)\left(1 - z^D(0)\right)\left[s_t^H(1) - s_t^F(1)\right] \\
&\quad + \frac{(1-n)(\bar{c}^F - \bar{c}^H) z^D(0)}{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)}\left[n s_t^H(1) + (1-n) s_t^F(1)\right] \\
&\quad - \frac{1}{n} W(0)\left[n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F)\right]\tilde{\Delta} z_t^A(1)
\end{aligned}$$

The dynamics in the ratio between Home net foreign assets and wealth are:

$$\begin{aligned}
\tilde{\Delta} nfaw_t^H(1) &= -\frac{1}{\eta}\tilde{\Delta} z_t^A(1) - \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\frac{1-n}{n}\left[s_t^H(1) - \frac{1-\bar{c}^F}{1-\bar{c}^H}s_t^F(1)\right] \\
&= -\frac{1}{\eta}\tilde{\Delta} z_t^A(1) - \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\frac{1-n}{n}\left[s_t^H(1) + \frac{n}{1-n}\frac{1-\bar{c}^F}{1-\bar{c}^H}s_t^H(1)\right] \\
&\quad + \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\frac{1-\bar{c}^F}{1-\bar{c}^H}\frac{1}{n}i_t^{net}(1) \\
&= -\frac{1}{\eta}\tilde{\Delta} z_t^A(1) - \frac{z^F(0)}{W(0)(1-\bar{c}^H)}\left[\left(\frac{1-n}{n} + \frac{1-\bar{c}^F}{1-\bar{c}^H}\right)s_t^H(1) - \frac{1-\bar{c}^F}{1-\bar{c}^H}\frac{1}{n}i_t^{net}(1)\right]
\end{aligned}$$

Shocks to savings clearly affect the ration between net asset and wealth. Consider the case where $\bar{c}^H = \bar{c}^H = \bar{c}$ and n goes to zero:

$$n\left[\tilde{\Delta} nfaw_t^H(1)\right] = -\tilde{\Delta} z_t^F(1) - \frac{z^F(0)}{W(0)(1-\bar{c})}\left[s_t^H(1) - i_t^{net}(1)\right]$$

Focus on a productivity shock in the Home country. As the country is small, the shock has no impact on capital and investment, and we get:

$$\begin{aligned}
\tilde{\Delta} z_t^F(1) &= \tilde{\Delta} z_t^A(1) = -n(1-n)z^D(0)\tilde{\Delta}\varepsilon_t^H(1) \\
s_t^H(1) &= (1-\bar{c})\tilde{\Delta} W_t^H(1) = (1-\bar{c})W(0)\tilde{\Delta}\varepsilon_t^H(1)
\end{aligned}$$

which implies:

$$\begin{aligned}
n\left[\tilde{\Delta} nfaw_t^H(1)\right] &= -\tilde{\Delta} z_t^F(1) - \frac{n - nz^D(0)}{W(0)(1-\bar{c})}s_t^H(1) \\
&= n\left[(1-n)z^D(0) - \left(1 - z^D(0)\right)\right]\tilde{\Delta}\varepsilon_t^H(1)
\end{aligned}$$