IMF Staff Papers
Vol. 51, No. 2
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# Higher or Basic Education? The Composition of Human Capital and Economic Development

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*No country has achieved sustained economic development without investment in education. But do all types of human capital affect growth identically? And which types of schooling—secondary or tertiary—should public policy promote? This paper develops an analytical framework to address these questions. It shows how the composition of human capital stock determines a country's development. Hence, promoting the "wrong" type of schooling can have little effect on development. In addition, the paper helps in understanding why empirical studies have failed to find a significant relationship between schooling and growth.* [JEL O11, O41, I20]

No country has achieved sustained economic development without substantial investment in human capital. Motivated in part by this observation, an extensive theoretical literature has evolved to analyze the channels through which human capital can affect growth (surveys include Barro and Sala-i-Martin, 1995; and Temple, 1999). Much of this literature has emphasized the complementary relationship between human and physical capital, noting how imbalances in these two stocks, as well as human capital externalities, can affect economic growth. However, because human capital is typically treated as a homogeneous concept, very little is understood about how different types of education—tertiary, secondary, and so forth—shape the overall development process.

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Yet both the case study (World Bank, 1998) and the more formal econometric evidence suggest that important complementarities do exist between various types of human capital. And as the evidence from the Green Revolution[1] in Asia suggests, these apparent complementarities can greatly affect development. The highly educated, such as scientists and technicians, appear to have a comparative advantage in understanding and adapting new or existing ideas into production processes. Meanwhile, some minimum level of education is required to follow the production template and successfully execute the production steps (Nelson and Phelps, 1966; Bartel and Lichtenberg, 1987; and Deolalikar and Evenson, 1994). Thus, do all types of human capital affect growth identically? Does the impact of a particular type of human capital on growth depend on the presence of other types of human capital? What are the characteristics of an optimal education policy?

To address these questions, this paper develops a simple analytic framework that emphasizes the role of the composition of the human capital stock. The framework relies on two key assumptions. First, it assumes that each skill type performs a specific but complementary function within the production process in the skilled sector. Moreover, the ideas developed by the highly skilled are assumed to be nonrival but excludable, creating demand linkages between the education types that are external to the firm. And thus, the rate of return for either skill input depends on the educational composition of the entire workforce.

Second, the paper studies these demand factors within the context of endogenous schooling costs. In many countries, the lack of access to schools and the limited supply of teachers negatively affect the schooling investment decision (Mookherjee and Ray, 2000). The argument assumes that previous enrollments—the current stock of educated labor—engender improvements in the educational infrastructure: more potential teachers, more schools, and more suitable curricula, which in turn diminish the sunk cost associated with human capital investment and outwardly shift the supply curve for skilled labor (Foster and Rosenzweig, 1996).

Using this framework, the paper argues that the confluence of demand and supply forces creates a pattern of circularity between educational investment across the various skill categories and demonstrates how the composition—not the level—of the human capital stock determines the long-run steady-state level of development. For instance, consider the case of an economy with a limited number of secondary-educated labor. The inability of the economy to adequately use

[1]While advances in biotechnology, pioneered in the developed countries, made the development of the high-yielding variety (HYV) seeds in such staples as rice and wheat possible, local scientists and agronomists were necessary to adapt these HYV seeds to the local climatic conditions. Once developed, the use of these seed strains are nonrival but can be excluded. However, using HYV seeds requires a greater attention to fertilizer quantity, irrigation, and soil conditions. Thus, as indicated by Foster and Rosenzweig (1996) in the case of India, educated farmers adopted the more technologically advanced seed strains more rapidly than those without sufficient schooling. Furthermore, the authors found that the returns to education increased in those areas where adoption had the highest potential gains. In addition, the expansion of schooling and the many agricultural extension projects designed to facilitate adoption increased the rate of return to research and development in seed technology through the market size effect, as well as through the fact that the feedback from more educated farmers was more useful in developing better seed varieties.

technology within the skilled sector because of the limited supply of secondary-educated labor reduces the productivity of tertiary-educated workers and dampens the overall incentives for education investment. Moreover, the low returns to education may not justify the fixed cost required to invest in schooling, resulting in little human capital accumulation. The model also illustrates that in this case, even large investments in tertiary schooling will have little effect on long-run development, as the extra tertiary-educated skilled labor may not sufficiently raise the return to secondary education to create a self-sustaining investment cycle toward a higher steady state.

The analysis is able to isolate two important characteristics of an optimal education policy. First, education investment is ongoing over time along an optimal path, but its rate of increase diminishes. Thus, the first generation experiences the biggest increase in schooling investment. But each subsequent generation becomes better educated than its predecessor, with the difference in attainment across generations declining with time. Naturally, the cost of education increases as the enrollment level increases—the flow of investment. Also, because of diminishing marginal productivity into the unskilled sector, the shadow cost of moving labor in the skilled sector increases with attainment. Therefore, it is cost minimizing to incur the largest flow of investment initially, when the shadow cost of secondary-schooling investment is at its minimum. Second, the analysis argues that because the social marginal product of labor in the skilled sector depends on the level of the complementary input, the expansion in schooling should occur across both types of schooling simultaneously.

The decentralized model also helps explain the failure of many empirical studies to observe the expected strong correlation between economic growth and human capital accumulation.[2] Much of this research uses the average years of schooling within the population as the sole measure of educational attainment. This methodology implicitly treats each year of schooling as identical, assumes that workers of each education category are perfect substitutes for workers of other education categories, and assumes that the marginal productivity of an additional year of schooling is the same given every level of schooling attainment (Mulligan and Sala-i-Martin, 1995). But as the model indicates, the average years of schooling can mask fundamental differences in the composition of the human capital stock; examples in the paper show that countries with identical average years of schooling can converge to very different development steady states. The model used to develop these arguments is related to the literature on costly investment across multiple sectors (Matsuyama, 1991; Krugman, 1991; and Carrington, Detragiache, and Vishwanath, 1996), as well as to literature that explores the relationship between human capital and development: Lucas (1988), Azariadis and Drazen (1990), and Romer (1990).

[2]Islam (1995), Benhabib and Spiegel (1994), Dasgupta and Weale (1992), Pritchett (1996), Barro and Sala-i-Martin (1995), and Lau, Jamison, and Louat (1991) all find an insignificant or negative correlation between various measures of educational attainment and economic growth. Recently Krueger and Lindahl (1998) have questioned the accuracy of these results. They argue that measurement error in the education data negatively biases the human capital coefficient.

# I. Model

## External Demand Linkages

There are three labor categories: unskilled or unschooled ($U$), low skilled ($L$)—those with only basic education such as secondary schooling—and the high skilled ($H$) or tertiary educated. The terminology tertiary and secondary is used solely to organize the discussion; these linkages can potentially exist across various types of human capital. The economy produces a single consumption good. Production of this good occurs both in the unskilled sector, where only unskilled labor is used, and in the skilled sector, where both low-skilled workers and tertiary-educated managers are complementary inputs. I assume that some of the ideas developed by the high-skilled agents spill over across firm boundaries and improve the productivity of all secondary-educated workers within the skilled sector (Romer, 1990). This externality ensures that low-skilled labor productivity is in part a function of the *total* employment of high-skilled labor. Furthermore, some of the ideas generated by the tertiary-educated labor within a firm become proprietary and are licensed for use by other firms within the sector. Thus, the reward to tertiary investment depends on the number of low-skilled workers—this is the market size effect (Acemoglu, 1998). Using a standard Cobb-Douglas framework, I describe the production structure of a representative firm in the skilled sector:

$$y_i^s = A\left[(g(H)l_i)^{\alpha}(f(L)h_i)^{1-\alpha}\right], \text{ where } g(0) = f(0) = 0, \frac{dg}{dH} > 0, \frac{df}{dL} > 0 \text{ and } \alpha \in (0,1). \tag{1}$$

The functions $g(H)$ and $f(L)$ denote the external effects of aggregate high- and low-skilled labor at the firm level, respectively. To simplify the analysis further, let $g(H) = H$ and $f(L) = L$. Production at the firm level occurs using a constant returns to scale technology, but the external demand linkages between secondary- and tertiary-educated labor generate increasing returns to scale at the sector level (Matsuyama, 1991). Output in the unskilled sector relies solely on unskilled labor and uses a standard Cobb-Douglas technology subject to diminishing marginal productivity. All factor prices are determined by the marginal productivity of the factor. The wages of the unskilled, low skilled, and high skilled are given respectively by

$$w^u = B\alpha U^{\alpha-1}, u^l = A\alpha H, w^H = A(1-\alpha)L, \tag{2}$$

where $B > 0$. I also assume that the population is constant and without loss of generality normalized to a constant $p$:

$$H + L + U = p. \tag{3}$$

To better understand the nature of the transition from unskilled to skilled in the production process and the role of sunk costs, assume that education investment is irreversible.[3] The investment process is sequential, and agents incur a unique fixed cost at each step in the educational ladder. The size of this sunk cost depends on an agent's personal characteristics, such as preferences, family background, and intrinsic ability, as well as policy variables, such as the development of the education infrastructure: distance from home to school, the quality of instruction, and the nature of the curriculum. I assume that these factors are uncorrelated with future productivity. These characteristics are summarized by a cost index $\theta \in \Theta$ and $q(\theta)$ denotes the fraction of the population of type less than or equal to $\theta$; this function is strictly increasing, continuous, and differentiable. Let $c^H(H(t), \theta)$ denote the private cost of tertiary schooling for an agent of type $\theta$, given the stock of tertiary educated workers $H(t)$ at time $t$. In keeping with the idea that the size of the sunk cost diminishes with the stock of educated agents, I assume that $c_1^H(\cdot, \cdot) < 0$ while $c_2^H(\cdot, \cdot) > 0$. The private cost of secondary schooling, $c^L(H(t) + L(t), \theta)$, is similarly defined, and for any $\theta$ it naturally follows that $c^L(H(t) + L(t), \theta) < c^H(H(t), \theta)$. For simplicity, I also assume that agents are endowed with perfect foresight and investment in education is irreversible. Individuals maximize the present discounted value of their income stream by choosing the optimal dates on which to invest in education.[4]

## Investment Decision

Define $V^H(L(\tau_2))$ to be the value of tertiary education at some date $\tau_2$. Since educational investment is irreversible, the value of tertiary education is the present discounted value of its income stream from date $\tau_2$ onward:

$$V^H(L(\tau_2)) = \int_{\tau_2}^{\infty} A(1-\alpha)L(t)e^{-r(t-\tau_2)}dt \tag{4}$$

where $r > 0$ is the constant and exogenously given discount rate, and the high-skilled wage at time $t$ is $W^H(t) = A(1-\alpha)L(t)$. Let $V^L(H(\tau_1), L(\tau_1)|\theta)$ denote the value of secondary education for an agent of type $\theta$ at a date $\tau_1 < \tau_2$. A secondary-educated agent must choose the optimal date on which to incur the sunk cost and invest in tertiary education. This problem can be written as

$$V^L(H(\tau_1), L(\tau_1)|\theta) = \max_{\tau_2}\left\{\int_{\tau_1}^{\tau_2} A\alpha H(t)e^{-r(t-\tau_1)}dt + e^{-r(\tau_2-\tau_1)}\left[V^H(L(\tau_2)) - c(H(\tau_2), \theta)\right]\right\}, \tag{5}$$

where the low-skilled wage at time $t$ is $w^L(H(t)) = A\alpha H(t)$. The structure of the investment problem facing an unskilled agent is similar to the one described above.

[3]This is only a simplifying assumption; in equilibrium, the skill premia are always positive (Lemma 1 in the Appendix)

[4]One interpretation of the idea that individuals are infinitely lived and wait until the optimal date to invest in education is that generations or families pass on their existing level of education to their children. Given the cost of schooling and the demand for (skilled) labor in the current period, these children then decide whether to invest in schooling and add to their family's capital stock or delay and pass on only the existing level to future generations. In this way, if the educational infrastructure rapidly expands, then families and, by extension, society quickly become educated; otherwise it takes a longer time. See Galor and Tsiddon (1997) for an overlapping generations model with some of these characteristics.

For some date $\tau_0$, where $\tau_0 < \tau_1 < \tau_2$, let $V^U(H(\tau_0), U(\tau_0), L(\tau_0)|\theta)$ denote the value of the unskilled state for an individual of type $\theta$. An unskilled individual then selects the optimal date on which to invest in secondary schooling:

$$V^U(H(\tau_0), U(\tau_0), L(\tau_0)|\theta) = \max_{\tau_1} \left\{ \begin{array}{l} \int_{\tau_0}^{\tau_1} w^H(U(t)) e^{-r(\tau-\tau_0)} dt + \\ e^{-r(\tau_1-\tau_0)} \left[ V^L(H(\tau_1), L(\tau_1)|\theta) - c(H(\tau_1) + L(\tau_1), \theta) \right] \end{array} \right\}. \quad (6)$$

Using the condition $L = p - H - U$, let $\gamma^H(H(t), U(t))$ denote the premium induced by tertiary education relative to secondary schooling in period $t$. Similarly, $\gamma^L(H(t), U(t))$ represents the premium to secondary education relative to the unskilled state. Lemma A.1 in the Appendix shows that these respective skill premia are always positive; the irreversibility assumption used in simplifying the Bellman equations does not impose any dynamic inconsistency within the model. As a result, the following result can easily be derived.

**Result 1:** *The behavior of educational attainment along a perfect foresight equilibrium path is described by*

$$U = \frac{w^U(U(t)) - \left[ w^L(H(t)) - rc^L(p - U(t)) \right]}{c_U^L(p - U(t))} \quad (7)$$

$$H = \frac{w^H(L(t)) - w^L(H(t), U(t)) - rc^H(H(t))}{-c_H^H(H(t))}. \quad (8)$$

*There is no investment in tertiary education if*

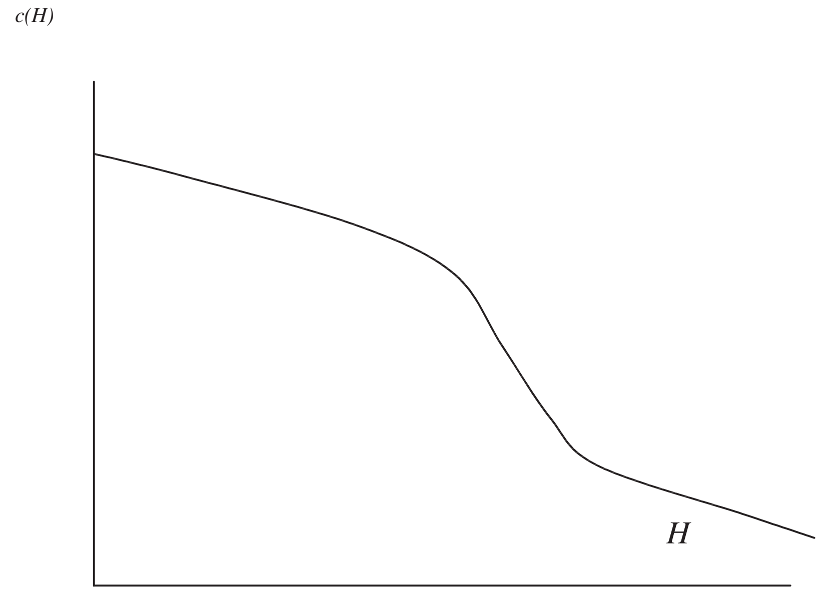
$$rc^H(H(t)) \geq \gamma^H(H(t), U(t)). \quad (9)$$

*There is no investment in secondary education if*

$$rc^L(p - U(t)) \geq \gamma^L(H(t), U(t)). \quad (10)$$

Equations (7) and (8) define a planar dynamical system in ($H$, $U$) space that describes the aggregate behavior of educational attainment. Investment in education is ongoing only if its rate of return is positive. The size of the externality, $c_1(\cdot)$, determines the sensitivity of aggregate behavior toward the capital gains rate. And unless the externality is zero, convergence toward the equilibrium stock of education is gradual, implying that educational attainment occurs slowly over time. Some agents find it optimal to wait for the cost of schooling to diminish rather than invest in education given the current incentives. And the bigger the reduction in schooling costs over time, the more attractive waiting becomes. A stationary state occurs when the present discounted value of the skill premium is less than or equal to the cost of schooling. It should be reiterated that whenever the cost of schooling exceeds the skill premium there is no *new* investment in schooling. But

Figure 1. Nonlinear Externality




this does not imply that those already educated wish to reverse their educational decision; skill premia are always positive along an equilibrium path.

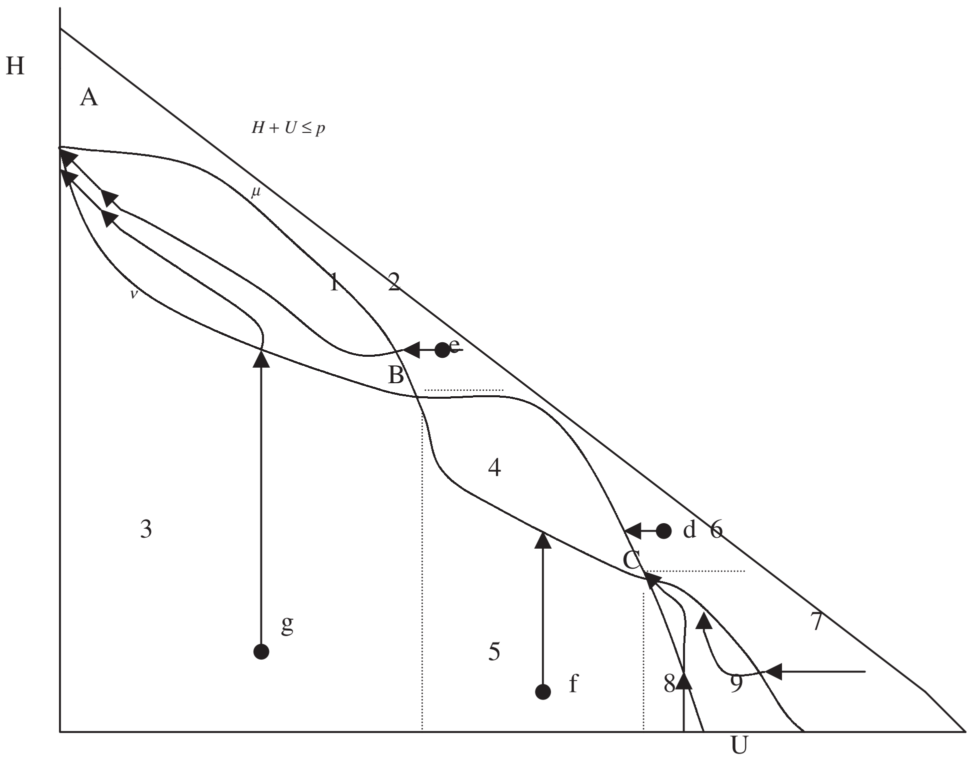
## Equilibrium and Dynamics

Building on the idea of threshold externalities (Azariadis and Drazen, 1990), this subsection uses a phase diagram approach to characterize the equilibrium behavior described in Result 1. In particular, I consider a nonlinear schooling externality, one that captures the idea that over some range of attainment the educational infrastructure may be slow to develop but then improves at a more rapid pace beyond some level of attainment. Figure 1 qualitatively depicts this idea, where the slope of the cost function indicates the size of the externality. This cost structure can produce multiple stable steady states, so that the composition of the human capital stock determines both the pattern of educational attainment and the long-run steady-state level of educational attainment.

The level curves corresponding to equations (7) and (8), $\mu = M^{-1}(0)$, $v = N^{-1}(0)$, are depicted in Figure 2. I assume that $\mu$ and $v$ intersect at three locations. Hence, the "threshold" nature of the externality produces multiple stable steady states. In an economy with a small stock of educational attainment, the large and slowly declining schooling costs are unable to offset the initially weak demand for skilled labor. Therefore, the economy converges to a low steady-state level of attainment. As drawn, vertices $A$ and $C$ are the only asymptotically stable equilibria.[5]

[5]This is shown formally in the Appendix, Result A.2.

Figure 2. Decentralized Dynamics
*(Nonlinear externality)*



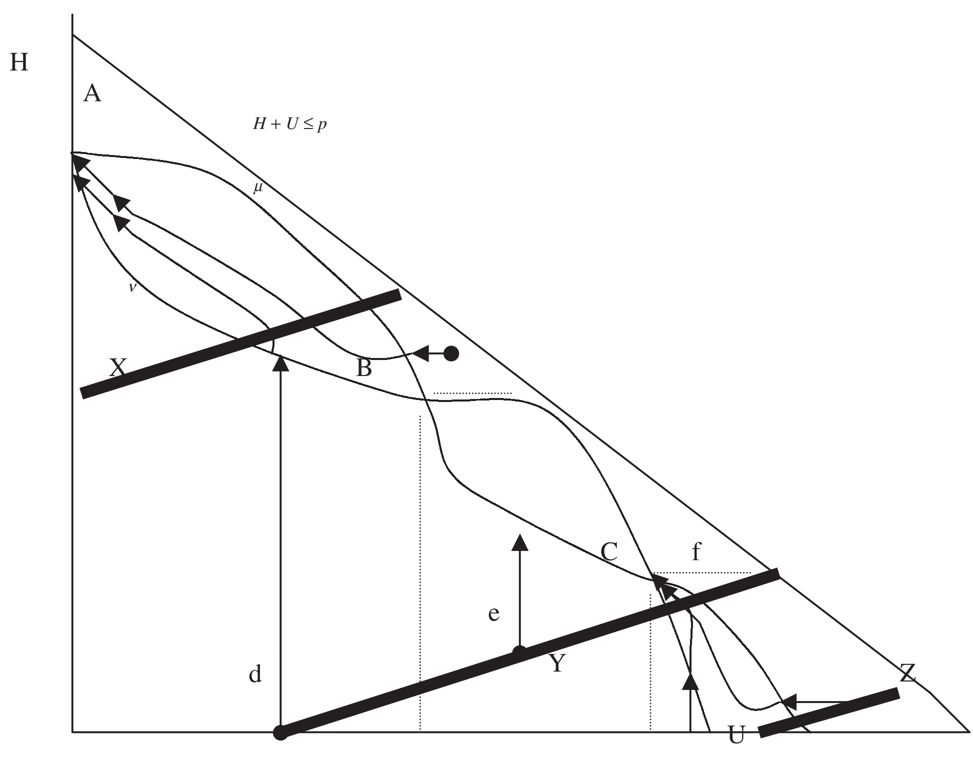

The composition of the human capital stock determines not only the dynamic pattern of educational investment, but also the steady-state level of educational attainment. In Figure 2, production is predominantly undertaken in the skilled sector at point *A*. The equilibrium defined by point *C* characterizes a backward economy with a largely uneducated workforce. To illustrate the pattern of education attainment, consider the set defined by region 4. In this region the rate of return to both types of schooling are negative: $R^H(H, U) < 0$, $R^U(H, U) > 0$ and there is no investment in schooling. Although educational attainment maybe initially ongoing, trajectories that enter region 4 never leave, and educational investment ceases. Consider an initial point *d* in region 6. Investment in secondary schooling is ongoing $\dot{U} < 0$, while the rate of return to tertiary schooling is negative but increasing as the size of the low-skilled labor force grows: $\frac{\partial R^H}{\partial U} < 0$. When the trajectory reaches the boundary of region 4, the rate of return to tertiary schooling is still negative. But rising marginal productivity in the unskilled sector means that the rate of return to secondary schooling is now zero, and educational investment ceases. Economies with endowments in the interior of region 4 never experience economic growth.

In contrast, economies with initial conditions located in regions 1, 2, and 3 converge to equilibrium point *A*. For example, consider an economy defined by point *e* in region 2. In both economies *d* and *e,* the rate of return to tertiary-educated labor is negative, and only secondary investment is ongoing. However, since $H^e > H^d$, then $|R^U(H^e, U^e)| > |R^U(H^d, U^d)|$, which leads to greater investment in secondary education in economy *e.* Because of the external demand linkages, the growing secondary-educated labor force coupled with the lower initial cost of tertiary schooling spark investment in tertiary schooling in economy *e.* This occurs when the trajectory crosses the level curve *u.* Therefore, while economy *d* converges to a steady state on the boundary of region 4, with no increase in its initial stock of tertiary-educated labor, economy *e* converges to the high attainment steady state *A*.

These ideas imply that the average years of schooling can be unrelated to growth. Consider the lines labeled X, Y, and Z in Figure 3. These are iso-average lines, where the stocks of *H* and *U* vary so that the average years of schooling is held constant. That is, if the years of schooling required for performing high-skilled, low-skilled, and unskilled tasks are $n^H$, $n^L$, $n^U$, where $n^H > n^L > n^U$, then the average years of schooling in the population is

$$\bar{n} = \frac{n^H H + n^L L + n^U U}{p}. \tag{11}$$

Figure 3. Decentralized Dynamics
*(Empirical implications)*



The composition of the human capital stock can vary dramatically along an iso-average line. For example, an increase in the number of unskilled workers reduces the total years of schooling in the population. In order to offset this decline and hold the average years constant, there must be a rise in the number of high-skilled workers, where the magnitude of this change depends on the number of years of schooling required for each type of education.

In Figure 3, I draw three such iso-average lines: X, Y, and Z, where $\bar{n}^x > \bar{n}^y > \bar{n}^z$. All endowments located along the X iso-average line converge to the high steady state, while all endowments along the Z iso-average line converge to the low steady state. In the former case, the average years of schooling imply a skilled sector large enough to be self-sustaining. In the latter case, the skilled sector is too small to produce a dramatic shift in the production process, and the economy converges to a low steady state. But because average years of schooling can mask fundamental differences in the composition of the human capital stock, sharp differences in development can still be observed even *along* an iso-average line. For example, consider the endowments *d, e,* and *f* located on the iso-average line Y. Although the average years of schooling remain constant, as we move from point *d* to point *f,* the size of the skilled sector shrinks and so do the private incentives for educational investment. As a result, while economy *d* converges to the high steady state, *f* approaches the low steady state.

## Policy Implications

The previous section demonstrated how large and slowly decreasing private education costs coupled with demand linkages can lead to multiple equilibria. Within this context, unless government policy is carefully chosen, it may ultimately have little or no impact on the economy's long-run steady state. To be more precise, consider the case of a social planner who chooses the level of tertiary and secondary enrollments to maximize the present discounted value of output net of education costs:

$$\max_{\dot{U},\dot{H}} \int_0^\infty e^{-rt}\left[F(H,U) - G\left(H,\dot{H}\right) - J\left(U,\dot{U}\right)\right]dt$$

subject to

$$H(0) = H_0, U(0) = U_0, H + U \leq p, \dot{H} \geq 0, \dot{U} \leq 0$$

where the government internalizes the demand linkages:

$$F(H,U) = AHL + BU^{\alpha} \tag{12}$$

and the total cost of tertiary schooling is given by

$$G = G\left(H,\dot{H}\right), G_H < 0, G_{\dot{H}} > 0 \text{ and } G_{\dot{H}\dot{H}} > 0. \tag{13}$$

The first argument in the cost function reflects the idea that the existing level of tertiary attainment lowers the private cost of tertiary investment. However, a rise in the level of schooling enrollments requires increased expenditures on infrastructure and other education inputs. Thus, I assume that at any instant the total cost of tertiary schooling is an increasing and convex function of the flow of current investment in tertiary schooling: the enrollment level. The cost of secondary schooling is similarly defined:

$$J = J\left(U, \dot{U}\right), J_U > 0, J_{\dot{U}} < 0 \text{ and } J_{\dot{U}\dot{U}} > 0. \tag{14}$$

By making the simplifying assumptions that the marginal impact of attainment on the private cost of schooling is independent of the current enrollment levels, and that the marginal impact of attainment decreases in the level of attainment, the first-order conditions for an optimal policy are both necessary and sufficient. Along an optimal path, the social planner chooses the level of tertiary investment such that the cost difference of endowing the marginal agent with tertiary education at time $t$ rather than at $t + \Delta t$ is just offset by the net social marginal product contributed by that agent over the interval $[t, t + \Delta t]$:

$$\int_{t}^{t+\Delta t} e^{-rs}[F_H - G_H]ds = e^{-rt}G_{\dot{H}(t)} - e^{-r(t+\Delta t)}G_{\dot{H}(t+\Delta t)}. \tag{15}$$

The intuition for secondary investment is similar.

From the first-order conditions (Appendix, Result A.3) along an optimal path the change in the flow of human capital investment diminishes over time. Therefore, while attainment increases over time, the initial change in the level of enrollment in both secondary and tertiary education should be the greatest. That is, along an optimal path the first generations experience the biggest increase in schooling investment. The simultaneous expansion of both kinds of schooling follows from the fact that the social marginal product of labor in the skilled sector depends on the level of the complementary input. Thus, by investing in both types of education, the social planner increases the social marginal product of each unit of labor in the skilled sector.

A combination of two factors leads to the result that the change in enrollment levels should diminish over time. First, an optimal education policy postpones investment in secondary education for the marginal agent from the current to a later date if the net present value of the marginal agent's contribution in the current instant is less than the difference in the marginal costs over the interval. Because the marginal cost of investment in secondary schooling increases in the flow of investment, at each instant policymakers face an upward-sloping supply curve for new skilled labor. Second, over time diminishing marginal productivity in the unskilled sector reduces the net marginal benefit of adding to the secondary skilled capital stock and expanding the skilled sector. Therefore, along an optimal path, the net marginal benefit of skilled labor is at its greatest initially. Hence, it is optimal for the policymaker to increase enrollments over time, with the biggest increases occurring early in the development process. Smaller increases occur later as the shadow cost of skilled labor—the marginal product of unskilled labor—grows, making it profitable to slow the rate of educational investment. In contrast, if the cost of educating the marginal agent was constant (a flat supply curve), then there would be no incentive to postpone investments to lower current marginal costs, and the social planner solves a simple static problem. Note that since the social planner internalizes the demand linkages, as well as the private cost of schooling, the steady-state level of attainment exceeds the decentralized equilibrium.

## II. Conclusion

This paper has argued that the composition of the educational stock plays an important role in shaping the incentives for investment in education. And unless carefully chosen, education policy can prove wasteful, leaving the potential long-run development steady state unchanged. To avoid this outcome, the paper argues that the initial investments in both types of schooling should be the heaviest, and that investments should occur in both education types. The model is helpful in interpreting the empirical literature. The many empirical studies that have failed to detect a positive correlation between the growth in average years of schooling and economic growth is unsurprising. The average years of schooling can mask potentially important differences in the composition. Examples in the text highlight this empirical difficulty.

However, the argument is quite sensitive to both the posited relationship between the two types of schooling and to the modeling of schooling costs. In the case of the former, the complementarity between high- and low-skilled workers may be quite weak as openness to trade and the flow of ideas from abroad may be more critical factors in the determination of educational investment and technical change. Hence, it may well be that developing economies need only invest in secondary schooling, importing high-skilled education embodied in the foreign goods. Separately, the analysis has mechanically modeled schooling costs. But the nature of the cost itself may have implications for the argument. For example, the literature (Benabou, 1993; Bond, Wang, and Yip, 1996; and Hanushek, Leung, and Yilmaz, 2001, 2002) explores different mechanisms through which schooling costs influenced the incentives for schooling investment. The related issue of credit constraints and its impact on education investment has also been subsumed into the paper's general treatment of schooling costs.

There is still an active empirical debate over the role of schooling in growth, with Bils and Klenow (2000) suggesting the converse: growth leading to schooling. Meanwhile, by adjusting for quality differences across country, Hanushek and Kimko (2000) find an improvement in the explanatory power of education on growth. Also, education, fertility, and growth are intimately linked. Analyzing these relationships would offer a better understanding of how the composition of human capital affects development. It is hoped that future work will address some of these issues.

## APPENDIX

### Mathematical Details

**Lemma 1:** *Along an equilibrium path* $\gamma^H(H(t), L(t)) \geq 0$ *and* $\gamma^L(H(t), L(t)) \geq 0$ *for all t.*

Proof: The income stream of a high-skilled agent who earns the high-skilled wage from date $s$ onward is

$$V^H(L(s)) = \int_s^\infty L(t)e^{r(s-t)}dt.$$

Suppose a high-skilled individual finds it optimal to switch to the low-skilled income stream in period $t^*$ and reenters the high-skilled sector at some later date $t^* + \Delta t$. The income profile for such a strategy beginning on any date $s$ is

$$\int_s^{t^*} L(t)e^{r(s-t)}dt + \int_{t^*}^{t^*+\Delta t} H(t)e^{r(t^*-t)}dt + \int_{t^*+\Delta t}^{\infty} L(t)e^{r(t^*+\Delta t-t)}dt.$$

If I chose $s$ to be arbitrarily close to $t^*$, then I can approximate the first two integrals by

$$\frac{(t^*-s)L(s)}{1+(t^*-s)r} + \frac{\Delta t H(t^*)}{1+\Delta t r}.$$

For this income profile to be optimal, it must be the case that $L(s) > H(t)$ and $H(t^*) > L(t^*)$. However, since the individual operated as a high-skilled in instant $s$ but switched to the low-skilled sector in period $t^*$, $H(s) > H(t^*)$ and $L(s) < L(t^*)$. This implies $L(s) > H(s) > H(t^*) > L(t^*) > L(s)$: a contradiction. Therefore, because the tertiary-schooling wage differential is always positive, it implies that $\dot{H} \geq 0$. A similar argument shows that $\gamma^L(H(t), L(t)) \geq 0$.

To conserve notation, and without loss of generality, I assume throughout that $s + u = a$; therefore, let $\gamma(s(t)) = w(s(t)) - w(a - s(t))$ denote the skill premium.

**Result 1:** *The behavior of educational attainment along a perfect foresight equilibrium path is described by*

$$U = \frac{w^U(U(t)) - \left[w^L(H(t)) - rc^L(p - U(t))\right]}{c_U^L(p - U(t))}$$

$$H = \frac{w^H(L(t)) - w^L(H(t), U(t)) - rc^H(H(t))}{-c_H^H(H(t))}.$$

*There is no investment in tertiary education if*

$$rc^H(H(t)) \geq \gamma^H(H(t), U(t)).$$

*There is no investment in secondary education if*

$$rc^L(p - U(t)) \geq \gamma^L(H(t), U(t)).$$

Proof: The derivation for Result 1 depends on the following lemma. Moreover, since the argument is identical for both skill types, generic notation $s$ is used.

The secondary-educated agent indifferent between investing in tertiary education at time $t$ is implicitly defined by the condition

$$\int_t^\infty \gamma(s(z))\, e^{-r(z-t)} dz = c(s(t), \theta^{**}).$$

Since $q(\theta)$, the fraction of the population of type less than or equal to $\theta$, is monotonic, it can be inverted:

$$\theta^{**} = q^{-1}(s(t)) = M(s(t)).$$

For notational simplicity, I express the cost of tertiary education as $c(s(t))$, suppressing $M(s(t))$, and I assume that $c_s(s(t)) < 0$. The following lemma specifies some characteristics of the optimal investment date $\tau_2$ and $\tau_1$ for tertiary and secondary education, respectively.

**Lemma A.1:** *On the optimal investment date* $\tau$ *the following conditions are satisfied:*

$$c(s(\tau)) \leq \int_\tau^\infty \gamma(s(t)) e^{-r(\tau-t)} dt.$$

There does not exist a $\tau' > \tau$ such that

$$c(s(\tau)) - c(s(\tau'))e^{-rt} > \int_\tau^{\tau'} \gamma(s(t)) e^{-r(\tau-t)} dt.$$

Proof: The first condition is obvious. For example, suppose the optimal investment date did not satisfy the inequality, then

$$c(s(\tau)) > \int_\tau^\infty \gamma(s(t)) e^{-r(\tau-t)} dt,$$

the cost of investing in education exceeds the present discounted value of the earnings stream.

To prove the second condition, define the net value of investing on date $\tau_2$ as

$$V(\tau) = \int_\tau^\infty \gamma(s(t)) e^{-r(\tau-t)} dt - c(s(\tau)).$$

This is the present discounted value of the skill premium minus the cost of investing. If date $\tau$ is the optimal investment date, then $V(\tau) \geq V(t)\ \forall t$. If there exists $\tau'$ that satisfies the inequality, then

$$c(s(\tau)) - c(s(\tau'))e^{-rt} > \int_\tau^{\tau'} \gamma(s(t)) e^{-r(\tau-t)} dt,$$

which implies

$$c(s(\tau)) - c(s(\tau'))e^{-rt} > \int_{\tau}^{\infty} \gamma(s(t))e^{-r(\tau-t)}dt - \int_{\tau'}^{\infty} \gamma(s(t))e^{-r(\tau'-t)}dt,$$

rearranging

$$\int_{\tau'}^{\infty} \gamma(s(t))e^{-r(\tau'-t)}dt - c(s(\tau'))e^{-rt} > \int_{\tau}^{\infty} \gamma(s(t))e^{-r(\tau-t)}dt - c(s(\tau))$$

$V(\tau') > V(t)$, a contradiction.

Now using Lemma A.1 we know that for any $\tau' = \tau + \Delta t$

$$c(s(\tau)) - \frac{c(s(\tau + \Delta t))}{1 + \Delta tr} \leq \frac{\Delta t \gamma(s(t))}{1 + \Delta tr} \text{ for } \Delta t > 0$$

while

$$c(s(\tau)) - \frac{c(s(\tau + \Delta t))}{1 + \Delta tr} \geq \frac{\Delta t \gamma(s(t))}{1 + \Delta tr} \text{ for } \Delta t < 0.$$

Therefore, rearranging

$$\frac{c(s(\tau)) - c(s(\tau + \Delta t))}{\Delta t} \leq \gamma(s(\tau)) - rc(s(t))$$

and taking the limit as $\Delta t \to 0$ yields

$$s = \frac{\gamma(s(t)) - rc(s(t))}{-c_1(s(t))}$$

the derivation.

**Result A.2:** *Vertices A and C are the only asymptotically stable equilibria.*

The dynamical system corresponding to this setup is

$$\dot{H} = \frac{A(1-\alpha)p - H(t) - A(1-\alpha)U(t) - rc(H(t))}{-c_H} = -M(U(t), H(t))[c_H]^{-1}$$

$$\dot{U} = \frac{\alpha BU(t)^{\alpha-1} - AH(t) + rk + rc(U(t))}{c_U} = N(U(t), H(t))[c_U]^{-1}$$

**Assumption 1:** $A > \left|\frac{dc}{dH}\right| \forall H < p$ **Assumption 2:** $\left|w_{UU}^{U}\right| > c_U^L \forall U < p.$

The Implicit Function Theorem, the above assumptions, and the behavior of $c(\cdot)$ are used to depict the level curves $H = 0$ and $U = 0$ in Figure 2. An examination of the Jacobean matrix at a candidate equilibrium point reveals that asymptotic stability is obtained if the absolute value of the slope of $U = 0$ is greater than the slope of the $H = 0$ level curve:

$$J(H,U) = \begin{pmatrix} B\alpha(\alpha-1)U^{\alpha-2} + rc_U^L & -A \\ -A & \dfrac{2A + rc_H^H}{c_H^H} + \left[c_{HH}^H\right]\left[A(1-2H-U) - rc^H(H)\right] \end{pmatrix}.$$

**Result A.3:** *If* $\dot{H} \geq 0, \dot{U} \leq 0, G_{\dot{H}\dot{H}} > 0$ *and* $J_{\dot{U}\dot{U}} > 0,$ *then* $\ddot{H} < 0, \ddot{U} > 0.$

Proof: Using familiar techniques from the calculus of variations, the first-order conditions imply

$\dfrac{F_H - rG_{\dot{H}}}{-G_{\dot{H}\dot{H}}} = \ddot{H} < 0.$ A similar argument holds for $\ddot{U} = 0$.

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