INTERNATIONAL MONETARY FUND

Staff Guidance Note on Debt Sustainability Analysis for Market Access Countries

Prepared by the Policy Development and Review Department

Approved by Mark Allen

July 3, 2008

Contents **Page**

I. INTRODUCTION

Since the endorsement by the Executive Board in June 2002 of the paper on "Assessing Sustainability" (May 28, 2002), the Fund has followed a more systematic approach to assessing public and external debt sustainability in the context of both program design and Article IV surveillance. The framework—further refined in a subsequent paper, "Sustainability Assessments—Review of Application and Methodological Refinements" (June 10, 2003)—attempts to bring a greater degree of consistency and discipline to sustainability analyses, including by laying bare the basis on which baseline projections are made and subjecting them systematically to sensitivity tests. As part of continuing efforts to improve debt sustainability analysis (DSA) and responding to the concerns of some of the shareholders as well the experience with the framework thus far, further modifications were made in July 2005 (Information Note On Modifications To The Fund's Debt Sustainability Assessment Framework For Market Access Countries, July 1, 2005).

This note provides information on the use of the latest version of the DSA template for market access countries, that is, for countries not included in the low-income (PRGF-eligible) country group. Key requirements for country teams are laid out in Box 1.

The framework consists of two templates: one covering the public sector (FiscalSustainTable_standard.xls) and one covering the external sector (ExternalSustainTable_standard.xls). While the underlying logic of the templates is similar, there are important differences between them, as described in subsequent sections of this note. Both templates consist of a baseline scenario, usually the set of macroeconomic projections that form the basis for understandings on a Fund-supported program or the articulation of the authorities' intended policies as discussed with the staff in a surveillance context. Together with a detailed presentation of the baseline scenarios, the templates also facilitate assessments of sensitivity of debt dynamics to a number of assumptions, essentially providing a tool to stress test the baseline.

Output tables, which are produced separately for public and external debt sustainability exercises, contain a decomposition of baseline projections and the outcome of a historical scenario (and a "no policy change" scenario for the public debt DSA). In addition, a panel chart is produced for each of the stress tests, presenting results graphically.

It is important to underscore that the templates are merely one of the available tools for assessing debt sustainability. While an analysis of the results of the templates should form an integral part of briefing papers and staff reports, they often need to be complemented by other tools, including country-tailored scenarios focusing on the specific vulnerabilities faced by each country. Moreover, other techniques—such as full stochastic simulations of debt dynamics—can also usefully be used (see below).

Box 1. Key Requirements for Country Teams

Industrialized Countries:

- Public debt DSA should preferably be included in briefing paper for all Article IV consultations. The need for an external DSA would be discussed at the pre-brief meeting.
- If there are significant risks, DSAs should be discussed in the staff report; otherwise, the reports could simply indicate that the analysis was conducted but there were no significant changes from the previous year. The templates should be reported at least every other year.
- Alternative frameworks (i.e., not based on the standard framework) may be used instead of the templates if they satisfy minimum standards (outlined in Section VI) of this note.

Emerging Market Countries:

- DSAs should be prepared for briefing papers and staff reports for Article IV consultations, for requests for UFR in the GRA, and in program reviews if developments in the outlook for external or public debt dynamics since the previous review could warrant a change in policies.
- Pre-brief meeting will discuss alternative scenario(s) to be explored, if necessary. Country teams will then develop the relevant scenarios and examine their impact on debt sustainability; both the baseline and alternative scenario(s) will be presented in the staff report.
- Staff is encouraged to discuss the results of the DSA templates, and the alternative scenarios, with the authorities.
- Two DSA tables (one each for public and external debt sustainability) and two charts should be included in staff reports. Results can be discussed in an appendix, but the implications should be brought into the main text, insofar as they affect the medium-term outlook.
- The baseline and alternative scenario should be discussed in the main text of the report.
- Alternative frameworks (i.e., not based on the standard framework) may be used instead of the templates if they satisfy minimum standards (outlined in Section VI of this note).

Low Income (PRGF-Eligible) Countries:

- Use the templates developed for low-income countries, which explicitly take into account the net present value of debt.
- Concepts are discussed in Operational Framework For Debt Sustainability Assessments In Low-Income Countries—Further Considerations (March 28,2005).

The use of the external and/or fiscal sustainability framework is not mandated in cases where debt ratios are very low and unlikely to rise significantly under plausible scenarios; however, staff should make clear the reasons in the brief and staff report.

This note is organized as follows: Section II discusses the analytical underpinnings of the DSA approach; Section III and IV describe the debt equations in the fiscal and external templates, highlighting formulations of different concepts presented in the framework. Section V then pays particular attention to the country-tailored scenarios introduced to the framework. Section VI discusses implementation of the framework and country coverage, including standards that alternative frameworks need to meet to be acceptable. Section VII concludes with a brief overview of the operation of the template.

II. ANALYTICAL UNDERPINNINGS

This section lays out some of the basic underpinnings of debt sustainability analysis. It explains why it is important to look at debt ratio dynamics in assessing debt sustainability, highlights the limitations of this approach, and discusses the relevance of the stress tests featured in the templates.[1]

Usefulness of examining debt ratio dynamics

An agent (or a sector of an economy, or a country as a whole) is solvent if the net present value (NPV) of the income stream (excluding interest payments) is at least as large as the NPV of expenditure plus any initial debt. This means that the agent is meeting his intertemporal budget constraint, or that he meets his so-called "no-Ponzi game" condition.[2] This implies:

- **For a government**, that the NPV of future primary balances must be equal to or greater than the NPV of the **public debt** stock (or simply equal to or greater than the public debt stock, if the latter is issued at market rates).[3]
- **For a country**, that the NPV of the non-interest current account balances is equal to or exceeds the NPV of its **external debt**.

The relation between this condition and the debt-to-GDP ratio, the focus of DSA templates, can be easily established. It can be shown that if the debt-to-GDP ratio is on a non-explosive path, i.e., it is either stable or declining, the solvency condition is automatically met.[4] This provides a strong rationale for evaluating solvency by looking at the projected behavior of debt ratios.[5] If the debt-to-GDP ratio is projected to decline, the entity (government or country) is solvent—hence, the focus of the DSA template on the debt-to-GDP projections.

---

[1] Assessing Sustainability provides further discussion of these issues, defining sustainability and highlighting differences between solvency and liquidity considerations.

[2] In a "Ponzi game" scheme, agents borrow to service their debt, which eventually becomes unsustainable.

[3] This assumes that the government will not service its future debt by printing money, i.e., through seignorage. Alternatively, one needs to include seignorage as part of the primary surplus, as central bank profits are typically transferred to the budget. Another alternative is to exclude debt to the central bank from the definition of debt stock, yielding essentially the consolidated position of the government and the central bank.

[4] See, for example, Chapter 2 of Blanchard, O., and S. Fischer, 1989, *Lectures on Macroeconomics*, MIT Press, Cambridge, MA. See also the discussion in Barro, R., and X. Sala-i-Martin, 1995, *Economic Growth*, McGraw-Hill, New York, NY.

[5] However, even if the debt ratio is declining, it is worth examining whether this is the result of a continued primary deficit but an assumed GDP growth (g) in excess of the interest rate (r). While $g > r$ is possible over the short run, it would be imprudent to assume that this condition holds over the long run. Conversely, a rising debt ratio may not necessarily signify unsustainability, as it may reflect purely temporary factors. However, such developments should be flagged and discussed.

Beyond solvency, the entity may also face liquidity risk—that is, a situation where available financing and liquid assets are insufficient to meet maturing obligations. For this reason, the templates also track gross financing needs.

Limitations of the approach

This basic statement needs to be qualified in a number of respects.

First, what is being assessed is not the sustainability of debt but rather whether certain policies, summarized in a path for the primary balance for public debt assessments or in a path for the non-interest current balance for external debt assessments, involve a sustainable debt path. Note that no statement is made regarding the sustainability of these policies, including political sustainability. This approach can be used to calculate the size of a primary surplus required to service a given a level of debt, but it cannot ascertain whether it is feasible, or appropriate, to run primary surpluses of that size. Thus, this approach, in isolation, cannot provide a specific answer to the question of whether a debt stock is sustainable, or whether some form of debt restructuring may be necessary.

Second, and relatedly, this approach focuses mainly on debt dynamics—rather than the debt level—with a certain debt path regarded as sustainable as long as it declines. While this is, in principle, correct (in terms of meeting the intertemporal budget constraint), problems may still arise if the debt ratio is stabilized at a high level. In such a case, it will be necessary to run large primary surpluses forever, which may well be politically unsustainable. Furthermore, the country may be more exposed to the vagaries of financial markets, affecting the level of its interest rate. Sustainability Assessments - Review Of Application And Methodological Refinements (Section VI) takes a closer look at how these debt ratios can be interpreted, reporting other studies on debt thresholds as well as a probit analysis showing a marked rise in probability of a crisis when the total external debt ratio is above 40-60 percent of GDP (with the probability of a crisis being higher the more closed the country is trade). It is also worth noting that the templates model circumstances that could lead to a crisis—including as a result of shocks to the baseline scenario—but not the dynamics of the crisis itself.[6]

Third, the approach may not be adequate in examining roll-over risks—the risk that maturing debt cannot be re-financed. As noted earlier, liquidity risk may arise even in the presence of solvency, and while the DSA templates also provide some information on roll-over risks (as summarized in the gross financing needs derived in the templates), a full evaluation of these risks requires a more disaggregated—and higher frequency—data on the debt stock.

[6] Figure 1 of Sustainability Assessments - Review Of Application And Methodological Refinements shows pre-crisis trends in certain key parameters using in the DSA. The combined shock used in the templates is intended to capture some of patterns thus observed.

The role of the stress tests

The DSA templates start from a detailed analysis of a baseline projection for debt ratios; subsequent sections discuss the key parameters that drive the behavior of these debt ratios. An integral part of the DSA templates is to assess the effect that changes in these parameters would have on the debt ratio dynamics. To this end, the templates run a battery of stress tests, which can be interpreted in two ways.

In a strictly probabilistic sense, the debt ratio calibrated from a stress test could be interpreted as a probabilistic upper bound of the confidence interval for the baseline projection if the debt ratio and the underlying parameters of debt dynamics are considered to be random variables. In the current templates, the (one-sided) confidence level is set at 70-85 percent for the most extreme stress test, implying a *p*-value of the debt ratio of 15-30 percent. (For details on this approach, see Box 2.)

This probabilistic interpretation assumes that the baseline reflects an unbiased projection around which it is legitimate to compute confidence intervals. Past experience, however, has suggested that staff projections often exhibit an optimistic bias (with projected debt dynamics following a "willow-tree" pattern, as discussed in Assessing Sustainability). Thus, outside of a strict probabilistic interpretation, the debt projections provided by the DSA templates can also be interpreted simply as a tool to evaluate debt paths under less optimistic assumptions than those assumed in the baseline.

It is important to emphasize that the results are not full-fledged scenarios, as there is no interaction among variables. More specifically, the stress tests are built by replacing in the baseline projections the value of one variable (for instance, the interest rate), leaving all other variables unchanged, with only a few exceptions: (i) interest payments on debt are endogenous, i.e., they increase in scenarios where interest rates are higher; (ii) public sector revenues are endogenous with respect to GDP: shocks in the latter also affect revenue; and, of course, (iii) the debt stocks are endogenous. But the remaining parameters are left mostly unchanged: for example, in the stress tests featuring a shock to GDP, the current account remains unchanged.[7] This implies the need to interpret the stress tests with a grain of salt.

[7] Of course, if various relevant elasticities can be computed, staff is encouraged to modify the framework accordingly to allow more complex dynamics.

Box 2. Determining the Size of Permanent Shocks: Stochastic Simulation Approach

Determining the appropriate size of permanent shocks requires balancing two concerns: shocks should be large enough to realistically capture the medium-term risk in debt dynamics, but small enough to generate debt ratios with a reasonable likelihood of occurrence to be of policy relevance (since it would not be sensible to undertake extreme adjustment against the possibility of a large but very unlikely shock).

Stochastic simulations were conducted to provide a basis for judging the likelihood of alternative scenarios and calibrating the size of shocks accordingly. If the true probability distribution of debt ratio is known, the likelihood of the debt path resulting from the standard stress tests can be measured directly. By experimenting with different sizes of shocks, the appropriate magnitude for the stress test shocks can then be chosen, balancing the considerations listed above.

In reality, the true probability distribution of debt ratio is unknown. Stochastic simulation fills this gap by simulating the probability distribution of the debt ratio from historical data. In the simulation, key parameters that affect debt dynamics are treated as random variables, and are drawn from a *multivariate* normal distribution whose mean and variance-covariance matrix are estimated from historical data.* Stochastic simulation thus allows for interaction among the parameters of debt dynamics in constructing empirically the probability distribution of the path of the debt ratio.

Given the history-dependence, and thus country-specificity, of the simulated probability distribution, the likelihood of the resulting debt ratio for a given size of stress test shock differs across countries. Furthermore, the standard templates involve multiple stress tests that generate different debt ratios over the projection horizon. For these reasons, the calibration of the stress test shocks is based on the *average likelihood of the worst-case (highest) debt ratio in the fifth year of the projection horizon* where the average is taken across countries in the sample.

Stochastic simulation exercises conducted for a sample of emerging market countries indicate that **when a half-standard deviation permanent shock is used for the standard stress tests, the probability that the debt ratio will be higher than that implied by the most extreme stress test is around 15-30 percent**, which seems to be a reasonable balance between capturing the medium-term risks to debt dynamics without being so extreme as to be irrelevant for policy discussions.**

---

* For technical details regarding simulation of the probability distribution of debt ratio from historical data, see Appendix I in Sustainability Assessments - Review Of Application And Methodological Refinements. Simulations using boot-strapping techniques (i.e., without the need to assume a joint normal distribution) yield similar results.

** Policy Review Division in PDR can make available, upon request, GAUSS programs on stochastic simulation of debt ratios based on country's historical data (as described in Sustainability Assessments - Review Of Application And Methodological Refinements).

III. THE PUBLIC SECTOR DEBT TEMPLATE

The baseline

The fiscal sustainability template analyzes the behavior of the debt-to-GDP ratio, *with all variables expressed in domestic currency*. In formulating the baseline projection, several issues need to be considered, including the definition of debt, the coverage of the public sector, and the treatment of contingent liabilities. In general, it is recommended that the definition of debt be based on gross liabilities, and that the coverage of public debt be as broad as possible, including, where feasible, public enterprises as well as local governments.[8]

The public debt ratio can be decomposed as follows:

$$d_{t+1} - d_t = \frac{1}{(1+g+\pi+g\pi)}(\hat{r} - \pi(1+g) - g + \varepsilon\alpha(1+\hat{r}))d_t - pb_{t+1}$$

where *d* is the debt-to-GDP ratio, *pb* is the primary balance, $\hat{r}$ is a weighted average of domestic and foreign interest rates, α is the share of foreign-currency denominated public debt, π is the change in the domestic GDP deflator, and *g* the real GDP growth rate.[9] Changes in the exchange rate (local currency per U.S. dollar) are denoted by ε, with $\varepsilon > 0$ indicating a depreciation of the local currency. (See Annex I for steps to derive the debt equation.)

Based on the above equation, the fiscal template identifies different channels that contribute to the evolution of the debt-to-GDP ratio: the primary deficit (line 4 of sheet "Table" or "Table_SR") and the endogenous/automatic factors (line 7), which include the real interest rate, real GDP growth, and exchange rate movements. The contribution of the real interest rate to the evolution of the debt ratio is defined in the template as $\frac{\hat{r} - \pi(1+g)}{(1+g+\pi+g\pi)} d_t$ (line 9), the contribution of the real growth rate as $-\frac{g}{(1+g+\pi+g\pi)} d_t$ (line 10), and that of an

[8] It is important to ensure, however, that the concept of public sector used for the debt corresponds to the concept used for the deficit.

[9] The fourth term in the numerator would correctly include the interest rate on foreign-currency denominated debt, instead of the weighted average rate. The latter is used in the template for simplicity, since a breakdown between interest payments on domestic and foreign-currency debt is often not available. However, the template allows for a separate treatment of interest payments on foreign and domestic debt, which is recommended when the two interest rates differ considerably to avoid large residuals.

exchange rate depreciation as $\frac{\varepsilon\alpha(1+\hat{r})}{(1+g+\pi+g\pi)}\ d_t$ (line 11).[10] The separation of the different factors allows an assessment of their relative importance for the evolution of the debt ratio. It also serves as the basis for stress tests, which consist of a number of permanent shocks to these variables (isolated and combined; see below). Results of these tests are summarized together with the baseline projections in a separate figure, with the numerical values also presented in the sheet "Table."

The template also includes other debt-creating or debt-reducing flows, e.g., from recognition of contingent liabilities or privatization receipts. Changes in gross debt arising from other below-the-line operations, such as repayment of debt financed by a reduction in financial assets, and cross-currency movements are included in a residual. It is critical to monitor the behavior of this residual, as it may highlight errors in implementing the approach. A large residual may, in particular, signal a breach of the flow-stock identity linking the deficit to changes in debt. The residual should be small unless it can be explained by specific factors.

In addition to the debt-to-GDP ratio (and the implicit path of the debt-to-revenue ratio) the template pays explicit attention to the gross financing needs of the public sector (defined as the public sector deficit, plus all maturing debt) deriving it for the baseline in percent of GDP and in billions of dollars.

The template also calculates the debt-stabilizing primary balance (last column of the table): this is the primary balance required to keep the debt level (as a ratio of GDP) constant if all the relevant variables in the debt dynamics equation stated earlier remained at the level reported in the last year of projection.

The historical and the no-policy-change scenarios

The *historical average scenario* presents an alternative evolution of the debt ratio under the assumption that all key variables are at their respective historical averages throughout the projection period. The template calculates averages over the ten-year period, and uses that information to project debt dynamics five years ahead.[11] This scenario is a rough test of the "realism" of baseline projections; the baseline projections may be argued to be overly optimistic when it differs remarkably from predicted debt evolution if historical patterns were followed. Of course, this scenario may be somewhat misleading if significant changes in

[10] Note that a clear separation between the different channels in the debt equation is not possible. As a result, the terms defined as exchange rate and interest contributions, while dominated by these respective variables, are also affected by some of the other variables. Also, the current template calculates the real interest rate on foreign debt using domestic inflation, which may become an issue in countries with a high proportion of foreign-currency debt, possibly resulting in a rising real interest rates when domestic inflation is falling. This should be borne in mind when interpreting the results of the real interest rate shock.

[11] If country teams wish to change the parameters used in this scenario, changes should be made in the "Table" sheet, and justification provided.

economic policies have taken place in recent history—using the early years of transition, for instance, would bias the "historical" value of certain parameters and thus might result in too high a debt ratio. More generally, circumstances reflecting credible changes in policies (and thus lower baseline debt ratio projections) may look anomalous in comparison. In any case, using this scenario—and justifying any large anomalies—could usefully discipline baseline projections.

The public debt sustainability template also contains a *no-policy-change scenario*. As a default, this scenario is presented as one in which primary balance is kept constant in future years (and equal to the projection for the current year). This could be modified to allow an unchanged cyclically adjusted primary position, or to represent dynamics arising out of current fiscal policies (e.g., if sizable one-off measures will expire), but should be accordingly noted.

The stress tests

Stress tests are used to assess the behavior of the public debt ratio under different assumptions on key parameters. As described earlier, permanent shocks equal to one-half standard deviation are applied to the baseline projections of each of the parameters, and path of debt ratios derived. One-quarter standard deviation shocks are applied in the combined shock test. In the public sector debt template, these shocks are applied to the interest rate, growth, and primary balance. In addition, the template also examines debt trajectory in the case of a 30 percent real depreciation of the local currency and a contingent liabilities shock of 10 percent of GDP. The latter is presented as a rough measurement of an increase in debt-creating flows, given the difficulties in discussing contingent liabilities risk.[12] If better measures are available for a given country, staff is encouraged to use them for the purposes of this stress test.

Results of these stress tests are now presented in a graphical manner, using panel charts that allow immediate comparison with baseline projections. Each panel contains (i) the baseline; (ii) the relevant stress test; (iii) a small box reporting the historical value of the key parameter, together with the average value of the same parameter in the baseline and under the stress test; and (iv) the numerical value of the debt ratio in the baseline and stress test at the end of the forecast horizon. The first (top left) panel compares the baseline to the historical scenario, with the gross financing need (in percent of GDP) under the baseline also reported.

[12] This issue was discussed in detail in Sustainability Assessments - Review Of Application And Methodological Refinements. Section V presents various approaches to evaluating risks arising from contingent liabilities, distinguishing between those explicit and implicit in nature. Appendix III contains cross-country econometric evidence on the cost of banking crises.

IV. THE EXTERNAL SECTOR DEBT TEMPLATE

The baseline

The external sustainability template analyzes debt incurred externally by domestic residents (both the public and the private sector). Consistent with the WEO convention, *the variables in the template are expressed in U.S. dollar terms.* On this basis, the external debt dynamics can be decomposed as follows:

$$d_{t+1} - d_t = \frac{1}{(1+g+\rho+g\rho)}(\hat{r} - g - \rho(1+g) + \varepsilon\alpha(1+\hat{r}))d_t - tb_{t+1}$$

where α is (now) the share of domestic-currency debt in total external debt, ε is the change in the exchange rate, now expressed in U.S. dollars per local currency unit (with $\varepsilon > 0$ indicating a appreciation of the domestic currency), *tb* is the non-interest current account balance in percent of GDP, and ρ is the change in the domestic GDP deflator, expressed in U.S. dollar terms, with $(1+\rho) = (1+\pi)(1+\varepsilon)$ All other variables are defined as in the public sector debt analysis. (See Annex II for steps to derive the debt equation.)

The external template, too, separates the different channels affecting the evolution of the debt ratio. The contribution of the non-interest current account deficit is depicted in line 4 (of sheet "Table" as well as "Table_SR"). The "automatic debt-dynamics" resulting from interest rate, growth, and exchange rate movements are summarized in line 9, and split between the interest rate contribution $\frac{\hat{r}}{(1+g+\rho+g\rho)}\ d_t$ (line 10), the contribution of real GDP growth $-\frac{g}{(1+g+\rho+g\rho)}\ d_t$ (line 11), and that of price and exchange rate changes $\frac{-\rho(1+g)+\varepsilon\alpha(1+\hat{r})}{(1+g+\rho+g\rho)}\ d_t$ (line 12).[13]

As current account deficits are generally financed not only by debt-creating flows, but partly by the equity component of FDI and portfolio investment, the template explicitly subtracts such flows from the changes in the debt ratio. Other factors that affect the debt ratio, such as valuation changes linked to cross-currency movements and changes in foreign assets (that leave net debt unchanged) are captured in the residual.

As in the case of the public debt template, the residual should be checked carefully. A large residual may signal errors, particularly in the stock-flow identity.

[13] Similarly to the public debt analysis, the separation is not "clean," as the last term also includes interest and growth rate effects. Note that there is an interest rate component here as well; this could alternatively have been included in the interest rate contribution.

Once again, the template also calculates debt-stabilizing non-interest current account balance (last column of the table), assuming that all key macroeconomic variables remain at the level reported in the last year of projection, for all scenarios and stress tests. The template also presents the gross financing needs—derived as the sum of the non-interest current account deficit and maturing external debt—in billions of dollars as well as in percent of GDP. These numbers should be carefully monitored to examine signs of liquidity pressures on the economy.

The historical scenario

To examine the robustness of results, a historical scenario is also run, in a manner similar to that in the public debt template. The setup of this scenario, as well as its interpretation, is similar to that discussed in the public debt template. Large deviations from the debt trajectory in the baseline should be explained, as they might arise from either an optimistic bias in baseline projection or through a credible policy reform that can justify delinking past performance from future expectations.

The stress tests

Similar to the public debt template, stress tests are conducted for some key parameters (in addition to the combined test) and results presented in a graphical manner. Key parameters being shocked include the interest rate, growth rate, and current account balance, with the combined shock being applied to all three of these variables at once. In addition, the template also calculates the effect of a 30 percent real depreciation on the path of the debt ratio. The layout of panel charts is virtually identical to that in the fiscal template, with baseline being compared to debt trajectories under shocked scenarios. The first (top left) panel, as before, compares the baseline to the historical scenario, presenting in addition the gross financing needs (as percent of GDP) under the baseline.

V. COUNTRY-TAILORED SCENARIOS

As noted, the DSA templates are one tool to assess debt sustainability. Their results need to be properly interpreted, and discussed in the text of staff reports in combination with additional information. In particular, staff reports are expected to include *country-specific alternative scenarios*, as needed. Country teams should discuss the set-up of these alternative scenarios with functional departments in the course of pre-brief meetings, to encourage analysis that may be most appropriate given the country's specific circumstances. Such shocks may include, to cite a few examples, multiple-currency debt shocks, real domestic and foreign interest rate shocks, and commodity price shocks for resource-rich countries. While these scenarios are not mandatory, they are encouraged, particularly when certain country-specific tests may be desired or when the baseline and various stress tests give rise to questions about sustainability of the debt ratio.

Country-tailored scenarios for country facing major demographic problems, such as many advanced economies, typically focus on the effect on public debt of aging. In this case, the

time horizon of the projections typically covers several decades. Even in these cases, it would be important to stress test baseline projections with respect to key parameters such as productivity growth, employment growth, and per capita health care spending.[14]

VI. Implementation of the Framework and Country Coverage

DSA templates should be included in briefing papers and staff reports for Article IV discussions, for requests for UFR in the GRA, and in program review papers if developments in the outlook for external or public debt dynamics since the previous review could warrant a change in policies. Staff is strongly encouraged to discuss the results of the DSA templates with country authorities during the missions. The results could be presented and discussed in a separate annex to the staff report, but the main conclusions should be reported and discussed in the main text.

Some modifications may be made to make the framework more appropriate given country-specific circumstances: these may include the choice of the period over which the stress-test parameters are calibrated (the template uses averages and standard deviations over the past 10 years), the choice of the period over which projections are made (the template uses five-year projections, consistent with WEO projections), or the use of net, as opposed to gross, debt. Such changes, however, should be properly noted and justified in the staff report.

The DSA templates should be included in the briefs and staff reports for all countries under circumstances listed above, with the following exceptions:

- The use of the external and/or fiscal sustainability framework is not mandated in countries where debt ratios are very low and are unlikely to rise significantly under plausible scenarios; however, staff should make clear the reasons in the brief and staff report.[15]

- For advanced countries, a DSA for public debt dynamics should preferably be included in the briefing paper for all Article IV consultations.[16] The need for an external DSA would depend on various factors, including having a sufficiently large net international investment position, and would need to be discussed in pre-brief meetings. If the results indicate significant risks to debt sustainability—or material changes from previous year's assessment—then the templates should be included in the staff report. Otherwise, the

[14] See, for example, the 2004 Article IV Staff Report for Italy (February 10, 2005) and the 2005 Article IV Staff Report for Canada (March 29, 2005).

[15] On the other hand, in countries for which staff would not normally present an analysis of external sustainability (e.g., because market access is insignificant), country teams are nonetheless encouraged to include the assessment if they feel that it would be useful.

[16] The 2003 paper (Sustainability Assessments - Review Of Application And Methodological Refinements) made a case for greater flexibility in industrialized countries.

reports could simply indicate that the analysis was conducted but that there were no significant changes from the previous year. In any case, the templates should be reported at least every other year.

- For all countries, the templates do not need to be reported if alternative frameworks are presented that meet the following standards:[17]

  (i) A baseline scenario with at least a five-year horizon, with assumptions clearly spelled out;
  (ii) A "historical" scenario, showing debt dynamics under the country's historical performance;
  (iii) A "no policy change" scenario, showing debt dynamics under the country's current policies;
  (iv) Systematic stress tests that are comparable in magnitude to those in the standard framework (or a full-fledged stochastic simulation); and
  (v) A projection of gross financing needs along the baseline path.

VII. Instructions on Operation

To use either template:

1. Go to the relevant Worksheet ("Input_fiscal" for fiscal, "Input_external" for external). Make sure that the year marked in red (column S for fiscal, column R for external) is the first year for which projections are being made.
2. Check if the coverage is appropriate (as most of the input in the spreadsheet is based on WEO data). If yes, change country code to that of your country in column "Series_Code," refresh by selecting EDSS refresh button. Check if values are correct.
3. Insert those figures not refreshed via WEO (highlighted).
4. If coverage is inadequate and needs to be tailored to specific circumstances of your country, hardcopy the relevant data into the spreadsheet from your database.
5. If WEO projections are outdated (or the "program" baseline projection differs from the WEO projection), then hardcopy all relevant figures from your database/program projections. (Do not refresh the sheet afterwards to avoid results being overwritten; alternatively, delete WEO in column database to prevent data from being refreshed).
6. On Worksheet "Table," change the year in Column S (in both fiscal and external templates) to be the first year projected. [This should be the same year as inputted in 1 above]

Once "Input_fiscal" or "Input_external" is filled in correctly, all the scenarios and stress test results are automatically calibrated and summarized in the output table in sheet "Table." As

[17] These standards were laid out in Sustainability Assessments - Review Of Application And Methodological Refinements and endorsed by the Board.

countries may find exchange rate projections to be market-sensitive, we report the results to be published in “Table_SR.” As mentioned earlier, this contains results of baseline decomposition, the historical scenario (and no policy change scenario in the case of public DSA), and gross financing needs under the baseline. Results of the stress test are now reported in a panel chart (in sheet “PanelChart”). Briefing papers and staff reports should, unless exempted by factors discussed in section III of this note, contain “Table_SR” and “PanelChart” from both fiscal and external DSA templates.

ANNEX I: Public Sector Debt

The underlying equation for the evolution of public debt is:

$$D_{t+1} = [(1+\varepsilon)(1+r_f)DF_t] + (1+r_d)DD_t - PB_{t+1}$$

where D_{t+1} is the total stock of debt at time *t+1*, and *PB* is the primary balance. The debt stock is composed of debts denominated in both domestic as well as foreign currencies. Domestic-currency debt (DD_t) evolves according to the interest rate in the market (r_d), while the evolution of the foreign-currency debt (DF_t), expressed in domestic currency, is affected not just by the foreign interest rate (r_f) but also by changes in the exchange rate ($\varepsilon = \frac{e_{t+1} - e_t}{e_t}$, with *e* defined as *units of local currency per U.S. Dollar*).[18] A depreciation of the local currency ($\varepsilon > 0$) leads to an increase in foreign currency debt, expressed in local currency terms. In the template all foreign-currency debt is assumed to be in US dollars. However, if this is not an appropriate assumption in a particular country, one could feasibly derive ε as a weighted average, or express it in terms of the dominant currency in the debt stock, if this is not the U.S. dollar.

The analysis looks at debt stocks relative to GDP. Therefore, defining lower-case variables as upper-case variables expressed as a proportion of GDP (e.g., $d_{t+1} = \frac{D_{t+1}}{Y_{t+1}}$), the above equation can be expressed, in percent of GDP, as:

$$d_{t+1} = \left[\frac{(1+\varepsilon)(1+r_f)}{(1+g)(1+\pi)} df_t\right] + \frac{(1+r_d)}{(1+g)(1+\pi)} dd_t - pb_{t+1}$$

with π representing the change in the domestic GDP deflator, and *g* the real GDP growth rate. Simple algebra yields:

$$d_{t+1}(1+g+\pi+g\pi) = (1+\varepsilon)(1+r_f)df_t + (1+r_d)dd_t - (1+g+\pi+g\pi)pb_{t+1}$$

---

[18] Foreign-currency debt at the end of time *t+1* is equal to: $DF^{\$}_{t+1} = (1+r_f)DF^{\$}_t$. Using the respective end-of-period exchange rates at *t* and *t+1*, this equation can be expressed as: $\frac{DF^{LC}_{t+1}}{e_{t+1}} = (1+r_f)\frac{DF^{LC}_t}{e_t}$, where *LC* refers to Local Currency. Solving for $DF^{LC}_{t+1} = \frac{e_{t+1}}{e_t}(1+r_f)DF^{LC}_t$ leads to the term inside the square brackets in the equation.

Expanding terms and rearranging, with $d_t = df_t + dd_t$, we get:

$$d_{t+1}(1+g+\pi+g\pi) = d_t + \varepsilon(1+r_f)df_t + (r_f df_t + r_d dd_t) - (1+g+\pi+g\pi)pb_{t+1}$$

Since data on domestic and foreign interest rates may not be consistently available, the equation is further simplified. Letting α represent the *share of total public sector debt that is incurred in foreign currency* ($df_t = \alpha d_t$), the third term on the right-hand side of the equation can be rewritten as $\hat{r}d_t$, where $\hat{r}$ is a weighted average of domestic and foreign interest rates ($\hat{r} = \alpha r_f + (1-\alpha)r_d$)

Adding and subtracting $(g+\pi+g\pi)d_t$ to the right-hand side, allowing r_f to be approximately equal to $\hat{r}$, and rearranging the equation leads to the following equation, which forms the basis for the fiscal template: [19]

$$d_{t+1} - d_t = \frac{1}{(1+g+\pi+g\pi)}(\hat{r} - \pi(1+g) - g + \varepsilon\alpha(1+\hat{r}))d_t - pb_{t+1}$$

[19] The template allows for a separation of interest rates on domestic- and foreign-currency debt, if available, which is recommended if the two rates differ significantly (see also footnote 2 of the main text).

ANNEX II: External Sector Debt

External debt evolves according to:

$$D_{t+1} = (1+r_f)DF_t + [(1+r_d)(1+\varepsilon)DD_t] - TB_{t+1}$$

with *TB* denoting the non-interest current account balance. The change in the exchange rate ε is now defined in terms of U.S. dollars per local currency, with $\varepsilon > 0$ representing an appreciation of the domestic currency.[20] All other variables are defined as before.

As before, defining lower case variables as the upper case variables as a proportion of GDP, debt evolution can be written as:

$$d_{t+1} = \frac{(1+r_f)}{(1+g)(1+\rho)}df_t + \frac{(1+r_d)(1+\varepsilon)}{(1+g)(1+\rho)}dd_t - tb_{t+1}$$

where ρ is the change in the domestic GDP deflator (π) expressed in U.S. dollar terms. It is derived as $\rho = (1+\pi)(1+\varepsilon)-1$. Once again, further algebra yields:

$$(1+g+\rho+g\rho)d_{t+1} = (1+r_f)df_t + (1+r_d)(1+\varepsilon)dd_t - (1+g+\rho+g\rho)tb_{t+1}$$

Letting $d_t = df_t + dd_t$, and rearranging terms:

$$(1+g+\rho+g\rho)d_{t+1} = d_t + \varepsilon(1+r_d)dd_t + (r_d dd_t + r_f df_t) - (1+g+\rho+g\rho)tb_{t+1}$$

Letting α now represent the *share of domestic currency-denominated debt in total external debt* (i.e., $dd_t = \alpha d_t$), and setting $\hat{r}$ to be the weighted average of domestic and foreign interest rates, we get:

$$(1+g+\rho+g\rho)d_{t+1} = d_t + \varepsilon(1+r_d)dd_t + \hat{r}d_t - (1+g+\rho+g\rho)tb_{t+1}$$

In a manner analogous to the fiscal template, adding and subtracting $(g+\rho+g\rho)d_t$ to the right-hand side, allowing r_d to be approximately equal to $\hat{r}$, and rearranging leads to the following debt equation, which is used in the external debt sustainability template:

$$d_{t+1} - d_t = \frac{1}{(1+g+\rho+g\rho)}(\hat{r} - g - \rho(1+g) + \varepsilon\alpha(1+\hat{r}))d_t - tb_{t+1}$$

[20] While it would be correct to use end-of-period exchange rates, the template is based on average rates, which are used at the same time to convert the denominator into U.S. dollars.