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Policy Cooperation, Incomplete Markets and Risk Sharing

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In the past few years, there has been increasing concern about the effects of monetary policy followed by the U.S. and other advanced countries on emerging markets. There are increasing calls for some sort of policy coordination or cooperation. At the very least, policymakers in advanced countries have been asked to consider the effects of their policies on the rest of the world. Raghuram Rajan (2014), the governor of the Central Bank of India says, "In its strong form, I propose that large country central banks, both in advanced countries and emerging markets, internalize more of the spillovers from their policies in their mandates, and are forced by new conventions on the 'rules of the game' to avoid unconventional policies with large adverse spillovers and questionable domestic benefits." From a more technical perspective, Ostry and Ghosh (2013) note that "Taken as a whole, the various structural models and econometric studies suggest substantial cross-border spillovers operating through both direct and indirect effects. These may be especially large during times of crisis, but even in more normal times, they are sufficient to justify greater coordination of macroeconomic policies."

This paper compares policy under coordination versus self-oriented strategies. There is a large academic literature that has studied the nature of cooperative versus non-cooperative monetary policy in open economies, which is surveyed briefly below, but no brief survey can do justice to the great volume of work done over the years. This study aims at making a contribution to the nature of strategic versus coordinated policy in the context of the New Keynesian literature.

The New Keynesian approach to monetary policy has been successful and influential in a number of ways. Some of the key contributions are built on rich dynamic models that are solved numerically and provide insights into the goals and tradeoffs for policy decisions. The New Keynesian literature has also contributed to our understanding of monetary policy even with models that are very simple, by developing intuition for what variables monetary policy should target and what tradeoffs face monetary policymakers.

For example, one of the early insights in the closed-economy New Keynesian macroeconomic literature was that under staggered price setting, inflation is distortionary. When firms do not all adjust prices at the same time, a general inflation will lead to misalignment of relative goods prices. Prices that have not been adjusted for some time will be too low – leading to inefficiently high demand for the products with these low prices. In this early literature, the "divine coincidence" emerged – the policy that drove inflation to zero also achieved the goal of full employment. That coincidence arose in the simple models because, in the absence of inflation, the macroeconomy was undistorted.[1] If nominal prices were held constant by monetary policy, then there would be no need for price adjustment, so sticky nominal prices would not lead to any misallocations. Subsequent developments included other important macroeconomic distortions in the model, in which case the optimal policy potentially involved tradeoffs between the goals of inflation and full employment, for example.

The New Keynesian monetary policy literature has developed tools that have been very useful in clarifying the objectives of monetary policy. In essence, this literature treats monetary policy like a traditional public finance problem. The approach examines equilibrium macroeconomic models where the market outcome deviates from the efficient outcome due to one or more distortions, such as sticky nominal prices. It identifies distortions in the economy, and assesses the trade-offs for policy in abating those distortions. One of the key achievements of this literature has been to show how the objectives of the policymaker can be expressed in terms of a "targeting rule" for policy. The policymaker's goal is assumed to be maximizing the welfare of households. The targeting rule is derived from the first-order conditions for that optimization problem. In general, policymakers cannot achieve the first-best outcome that is possible in an undistorted economy. The targeting rule shows how policymakers should trade off deviations from the efficient outcome. For example, in general, simultaneously achieving the goals of an efficient level of output and a non-distortionary level of inflation is not possible. The targeting rule shows how the deviations from these efficient outcomes should be traded off – for example, how much inflation should the policymaker accept in order to drive output closer to the full-employment level? The targeting rule does not have a unique representation, but often it can be expressed in simple and intuitive ways. For example, simple closed-economy New Keynesian models express the targeting rule in terms of a log-linear function of the output gap (the difference between actual output and the efficient level of output) and the deviation of inflation from the targeted rate.

An analogous literature developed for the open economy. Some simple models imply that the tradeoffs facing the policymaker in open economies are very similar or even identical to

[1] That is, undistorted assuming that there was an appropriate constant subsidy to output to alleviate the underproduction that arises in the model from the monopoly power of producers.

those for the closed economy. For example, the simple model of Clarida et. al. (2002) derives the optimal tradeoff between the objectives of reducing inflation and the output gap. It is the same targeting rule that holds for the corresponding closed economy. That does not mean that open economy considerations are unimportant. For example, the exchange rate is a key variable that affects the adjustment of inflation and output.

However, subsequent literature has introduced other distortions into the economy which have implications for the optimal targeting rule. Engel (2011), building on a substantial earlier literature, shows that when there is local-currency pricing – producers set prices for goods in the currency of consumers – the optimal targeting rule involves tradeoffs among the goals of zero inflation, zero output gap, and zero exchange rate misalignments. The latter are defined as deviations from the law of one price will arise under local currency pricing when the nominal exchange rate fluctuates in response to shocks. Corsetti et. al. (2010) show that when asset markets are incomplete, a particular deviation from allocations under optimal risk sharing may also be part of the targeting rule.

These latter two studies examine policy assuming cooperation between policymakers in different countries. However, the literature has not characterized the targeting rule for policymakers when there is not cooperation, except in some special cases. It is important to understand what the objectives of policy are under non-cooperation in order to gauge what would change if the world moved toward a regime of more policy coordination. The literature has demonstrated that, under non-cooperation, policymakers have an incentive to improve their own country's welfare relative to the rest of the world, and, as in the optimal tariff literature, that policy can manifest itself through manipulation of the country's terms of trade.

Our goal is to construct a targeting rule that expresses policy tradeoffs in terms of deviations from the undistorted economy. In essence, this requires us to first characterize optimal policy in the case of complete markets, but this is a difficult problem. One of the contributions of the paper is, indeed, to see how optimal policy is set when financial markets are complete. "Complete" means that households in the model can trade in a complete set of state-contingent claims. Obviously, the real world does not have such markets, so what do we gain from calculating optimal policy in such a world? First, it is a benchmark – it helps us to understand what is lost to the economy when markets are incomplete. Second, in fact in simple models, sometimes the outcomes of the economy under complete markets can be replicated when only a small number of assets such as stocks and bonds are traded. The insight we get from the complete markets model is that, under non-cooperation, policymakers try not only to influence their terms of trade, but in essence also try to influence asset prices to favor their own residents.

In the end, even the very simple model of this paper does not yield a very simple targeting rule for optimal monetary policy. But the development of the optimal rule helps us to understand the nature of policy when there is not cooperation among policymakers. We can interpret recent discussions of the objectives of monetary policymakers at the Federal Reserve Board and the European Central Bank in light of this targeting rule.

Besides characterizing the targeting rule for optimal policy under non-cooperation and incomplete markets, we can solve for the equilibrium of the model under cooperation and non-cooperation when markets are complete or incomplete. While the model is extremely simple, such comparisons help us to understand how cooperative policy might reduce spillovers. In particular, we find that spillovers, per se, are not something to be avoided. Spillovers are not evidence of a distorted world economy so, even under cooperative policy, there are spillovers of idiosyncratic shocks from one country to another.

The literature on optimal macroeconomic policy when policymakers act strategically goes back at least to the pioneering work of Hamada (1974, 1976, 1979.) Cooper (1985) surveys some of the early work in this area. As dynamic macroeconomic models advanced, so did the study of non-cooperative policy. Canzoneri and Henderson (1991) and Persson and Tabellini (1995) summarize the advances of the literature in the 1980s and 1990s.

This study contributes most directly to the literature on optimal policy under cooperation or non-cooperation in micro-founded New Keynesian models. The key early contributions of this literature are Obstfeld and Rogoff (1995, 1996, 2000, 2002a), Clarida et. al. (2002) and Gali and Monacelli (2005).

Benigno (2002) is one of the first to couch the non-cooperative policy game in a New Keynesian setting. That paper shows how the policymaker tries to exploit its monopoly power in the world market for the good it exports to improve the welfare of its residents. Obsteld and Rogoff (2002b) is a seminal work. That paper argues that in practice, the gains from monetary policy cooperation are small. Benigno and Benigno (2003), Devereux and Engel (2003) and Corsetti and Pesenti (2005) further examine whether there are gains from cooperation under some generalizations of the Obstfeld and Rogoff model. Other contributions in the New

Keynesian literature to the debate on the gains from cooperation include Pappa (2004) and Canzoneri, et. al. (2005). Corsetti, et. al. (2010) survey the literature on optimal monetary policy in open economies, and consider the differences between non-cooperative and cooperative policy.

This study does not attempt to address the question of whether there are gains from cooperation. Indeed, the model is far too simple to make a useful contribution to that debate. Instead, the aim is to give insights into the objectives of policy when there are strategic considerations, and to compare those to the case of cooperation, in a more general setting than the previous literature, where asset market considerations matter.

1. The Model

The model laid out here is very simple. It is possibly the simplest two-country model that is still general enough to demonstrate some of the principles behind optimal policy when markets are incomplete. In an international setting, when all households have identical Cobb-Douglas preferences over consumption of Home and Foreign goods, or have log utility, then markets are effectively complete even when there is no asset trade because terms of trade movements provide complete insurance. This result is well known in the literature from the work of Cole and Obstfeld (1991). This model deviates from that case by assuming preferences have constant relative risk aversion but not necessarily logarithmic, and with Cobb-Douglas utility but home bias in preferences.

There are two countries, Home and Foreign. Each produces a single good in competitive markets, using labor as an input. Households in each country consume the output of both countries. There is home bias in preferences of goods consumption. Households in each country get utility from consumption but lose utility from work. There is a representative household in each country. The model is static.

The model laid out here is one in which prices are flexible, and markets are competitive. The optimal policies to deal with market incompleteness can be supported with a flexible taxation policy. However, as we explain below, the crucial equations of the model can also be derived in a sticky-price setting with monopolistic producers, with only some slight modifications. Hence, the policy rules we derive apply in this standard (albeit static) New Keynesian setting.

Here we present the equations describing the Home country. The Foreign country is symmetric. Then we present equilibrium conditions.

In the notation here, a subscript on a variable means the variable is a function of the state denoted by that subscript. For example, consumption in state *j*, C_j, can be interpreted to mean $C\left(\nabla_j\right)$, where ∇_j denotes state *j*. Before the realization of the random variables (which are productivity levels in the Home and Foreign countries) the probability of state *j* is π_j. It is necessary to carefully designate consumption, output and prices in each state, and the probability of each state, because under one specification of the model, state-contingent claims are traded before the realization of the state. In the other specification, international asset markets are exogenously incomplete – we assume trade in goods is balanced, and there is no asset trade.

Households:

The objective of Home households is to maximize:

(1) $$U = \frac{1}{1-\sigma}\sum \pi_j C_j^{1-\sigma} - \sum \pi_j N_j \ .$$

Here, C_j denotes the consumption aggregate, which is defined by:

$$C_j = C_{H,j}^{\nu/2} C_{F,j}^{(2-\nu)/2} \ ,$$

where $C_{H,j}$ is the consumption of the good produced in Home, and $C_{F,j}$ is consumption produced in Foreign. There is home-bias in consumption, so $1 < \nu \le 2$. (The Foreign consumer puts a weight of $\nu/2$ on the Foreign good.) N_j is labor input.

We can derive the standard demand curves.:

(2) $$C_{H,j} = \frac{\nu}{2} P_{H,j}^{-1} P_j C_j \qquad \text{and} \qquad C_{F,j} = \frac{2-\nu}{2} P_{F,j}^{-1} P_j C_j ,$$

where $P_j \equiv \left(\nu/2\right)^{-\nu/2}\left(\left(2-\nu\right)/2\right)^{(\nu-2)/2} P_{H,j}^{\nu/2} P_{F,j}^{(2-\nu)/2}$.

The analogous conditions for the Foreign household are:

(3) $$C_{F,j}^{*} = \frac{\nu}{2} P_{F,j}^{*-1} P_j^{*} C_j^{*} \qquad \text{and} \qquad C_{H,j}^{*} = \frac{2-\nu}{2} P_{H,j}^{*-1} P_j^{*} C_j^{*} ,$$

where $P_j^* = (\nu/2)^{-\nu/2}\left((2-\nu)/2\right)^{(\nu-2)/2} P_{F,j}^{*\nu/2} P_{H,j}^{*(2-\nu)/2}$

When markets are complete, prior to the realization of the state, the household can buy or sell a state-contingent claim that pays off one unit in nominal terms if the state is realized.[2]

In any given state *j*, when the state is realized, the household faces the budget constraint:

(4) $$P_j C_j = W_j N_j - T_j + D_j \quad .$$

Here, W_j is the wage, T_j is a lump-sum tax imposed on the household (or subsidy if T_j is negative), and D_j is the amount of state-contingent bonds purchased for state *j*, which could be negative.

Before the realization of the state, the household chooses its quantity of state-contingent bonds, and its consumption and labor supply for each state, to maximize (1) subject to:

$$\sum Z_j P_j C_j = \sum Z_j \left(W_j N_j - T_j\right) + \sum Z_j D_j$$

Here, Z_j is the price of a claim on state *j*. The last term in this expression must be zero – the value of consumption at state-contingent prices must equal the value of net income, so

$$\sum Z_j D_j = 0 .$$

It is useful to write out the Lagrangian for the Home household's problem:

$$\mathcal{L} = \frac{1}{1-\sigma}\sum \pi_j C_j^{1-\sigma} - (1+\phi)\sum \pi_j N_j + \lambda\left(\sum Z_j \left(W_j N_j - T_j\right) - \sum Z_j P_j C_j\right) .$$

First-order conditions are:

$$\pi_j C_j^{-\sigma} = \lambda P_j Z_j$$

$$\pi_j = \lambda W_j Z_j ,$$

which can be summarized as:

(5) $$\frac{C_j^{-\sigma}}{\lambda P_j} = \frac{1}{\lambda W_j} = \frac{Z_j}{\pi_j} \quad .$$

Similarly, for Foreign households, we can conclude:

(6) $$\frac{C_j^{*-\sigma}}{\lambda^* E_j P_j^*} = \frac{1}{\lambda^* E_j W_j^*} = \frac{Z_j}{\pi_j} .$$

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[2] The numeraire for state contingent claims is irrelevant. We express things in nominal terms here so that the model can be easily generalized to the sticky nominal price case.

(Note that we have expressed the Foreign nominal price level, P_j^*, and the Foreign nominal wage, W_j^*, in units of Foreign currency. Then E_j is the nominal exchange rate, expressed as Foreign currency price of Home currency. Note that in this paper, E_j denotes the exchange rate, while E will denote expectations. Z_j, the price of a contingent claim in units of Home currency, is the same for Foreign and Home households.)

Firms

A representative competitive firm produces output according to the production function:

$$Y_j = A_j N_j$$

Y_j is output of the Home good, and A_j is productivity for Home production which is an exogenously given random variable. (Foreign output is denoted Y_j^* and Foreign productivity is A_j^*.)

The firm is a price taker and sells output to Home and Foreign households. We assume the law of one price holds, $P_{H,j} = E_j P_{H,j}^*$. The firm chooses output to maximize:

$$P_{H,j}Y_j - (1-\tau_j)W_jY_j / A_j \quad , \tag{7}$$

where τ_j is a government subsidy to employment.

The first-order condition is given by:

$$P_{H,j} = (1-\tau_j)W_j / A_j \quad . \tag{8}$$

The law of one price also holds for the Foreign firm, and the analogous first-order condition is:

$$P_{F,j}^* = (1-\tau_j^*)W_j^* / A_j^* \quad . \tag{9}$$

Equilibrium

First-order conditions (2)-(9) hold.

Define the Home country’s terms of trade as the price of its imports relative to its exports:

$$S_j = P_{F,j} / P_{H,j} \quad .$$

With this definition and the definition of the consumer price levels in each country, we note that equations (5) and (6) imply

(10) $C_j^{\sigma}\lambda = S_j^{\nu-1}C_j^{*\sigma}\lambda^*$.

Solving out for wages from equations (5)-(9), we can write:

(11) $A_j S_j^{(\nu-2)/2} = (1-\tau_j)\left(Y_j / A_j\right)^{\phi} C_j^{\sigma}$, and

In addition, we have goods market clearing:

(12) $Y_j = C_{H,j} + C_{H,j}^*$, and $Y_j^* = C_{F,j} + C_{F,j}^*$.

Then, using (2) and (3), these equations may be rewritten as:

(13) $Y_j = \frac{\nu}{2} S_j^{(2-\nu)/2} C_j + \frac{2-\nu}{2} S_j^{\nu/2} C_j^*$

and

(14) $Y_j^* = \frac{\nu}{2} S_j^{(\nu-2)/2} C_j^* + \frac{2-\nu}{2} S_j^{-\nu/2} C_j$.

An important step is to solve for the ratio of Lagrange multipliers, λ / λ^*, in equation (10).[3] From the zero profit condition under free entry in competitive markets and the budget constraint (4), using the definition of profits (7), we have:

$$P_j C_j = P_{H,j} Y_j + D_j \, .$$

Then using the equilibrium condition (12) and the demand equations (2) and (3), we can write this equation as:

$$P_j C_j = \frac{\nu}{2} P_j C_j + \frac{2-\nu}{2} E_j P_j^* C_j^* + D_j$$

Multiply by the price of contingent claims, sum up over states, and recall $\sum Z_j D_j = 0$, to get:

$$\sum Z_j P_j C_j = \sum Z_j E_j P_j^* C_j^* \, .$$

Then use the first-order conditions (5) and (6) to write

$$Z_j = \frac{\pi_j C_j^{-\sigma}}{\lambda P_j} \quad \text{and} \quad Z_j = \frac{\pi_j C_j^{*-\sigma}}{\lambda^* E_j P_j^*} \, .$$

Substituting into the previous expression, we find:

[3] These steps draw on the Appendix of Devereux and Engel (2003).

$$\frac{1}{\lambda}\sum \pi_j C_j^{1-\sigma} = \frac{1}{\lambda^*}\sum \pi_j C_j^{*1-\sigma} \text{, or}$$

(15) $$\frac{\lambda}{\lambda^*} = \frac{\sum \pi_j C_j^{1-\sigma}}{\sum \pi_j C_j^{*1-\sigma}} = \frac{\mathrm{E}\left(C^{1-\sigma}\right)}{\mathrm{E}\left(C^{*1-\sigma}\right)}.$$

We can use (10) and (15) to solve for the terms of trade as:

(16) $$S_j = C_j^{\sigma/(\nu-1))} C_j^{*-\sigma/(\nu-1))} \left[\mathrm{E}\left(C^{1-\sigma}\right)\right]^{1/(\nu-1)} \left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]^{-1/(\nu-1)} .$$

Incomplete markets

Our goal is to compare cooperative and non-cooperative policy under incomplete markets with the optimal cooperative and non-cooperative policies when markets are complete. The case of incomplete markets we consider here is one in which there is no asset trade at all. In this case, there are no state-contingent claims payoffs, so the state-by-state budget constraint for Home households becomes simply:

$$P_j C_j = W_j N_j - T_j .$$

Households choose consumption and labor input state-by-state subject to this constraint. The demand equations (2) and (3) still hold. The consumption-leisure tradeoffs also still hold:

(17) $$\frac{C_j^{-\sigma}}{P_j} = \frac{1}{W_j} \quad \text{and} \quad \frac{C_j^{*-\sigma}}{P_j^*} = \frac{1}{W_j^*} .$$

Firms' decisions are the same as under complete markets.

The goods market clearing equation (12) still holds. The asset-market equilibrium condition (10) is replaced by the balanced trade equation:

(18) $$C_j = S_j^{\nu-1} C_j^*$$

Discussion of the approach to the optimal policy problems

Policymakers in the Home country commit to a policy before state contingent claims are traded in the complete-markets economy. It is difficult to conceive of the opposite timing – that asset markets open before policymakers commit to a policy. If the policymaker has not committed to a policy, then agents would have to assign probabilities that the policymaker would adopt certain policies. It is not easy to see how that could be modeled.

We assume the Home policymaker chooses C_j for all *j*, taking C_j^* for all *j* as given. It may be more natural, or at least in line with previous literature to have output, Y_j, as the strategy rather than consumption, which is equivalent to making labor supply, $N_j = Y_j / A_j$, the strategy. We opt for taking consumption in each state for the policymaker's strategy because, as the next section shows, we can write utility explicitly as a function of aggregate consumption, which helps to make the problem more transparent. However, in general, the outcome of the model may depend on the strategy – that is, if output were the strategy variable, we may derive a different targeting rule and find a different equilibrium under non-cooperation.

There are two cases in which it would be unwise to use consumption as the strategy. As is well known in the literature, if either there is no home bias in preferences ($\nu = 1$) or preferences are logarithmic in aggregate consumption ($\sigma = 1$), then under complete markets, $C_j = C_j^*$ in all states. In that case, choosing C_j taking C_j^* as given is a trivial problem, but clearly anything that satisfies the budget constraint would qualify as a Nash equilibrium. The first of these two cases is not interesting from our point of view anyway, because in this case even if there is no asset trade at all and trade is balanced, we have $C_j = C_j^*$ in all states, so markets are effectively complete under balanced trade. There is no difference, in other words, between the complete markets world and the balanced trade world in that case.

The Home policymaker in the non-cooperative case wants to influence the terms of trade in order to increase the value of the Home household's income. When markets are complete, the terms of trade for state *j* are influenced not just by the level of output in state *j*. The policy rule aims to increase the Home household's wealth when the market for state-contingent claims open, which can be seen as influencing λ / λ^* given by $\mathrm{E}\left(C^{1-\sigma}\right) / \mathrm{E}\left(C^{*1-\sigma}\right)$, as shown in equations (15) and (16). In essence, the policymaker is trying to influence asset prices, so as to make his own residents wealthier.

The policy strategies in each country under non-cooperation involve committing to a consumption level in each state. However, that does not describe the implementation of the policy. The policy is implemented by choosing the employment subsidy in each country. In the Home country that is given by τ_j in equation (7). Once optimal policies have been determined for each state in equilibrium, then the output level and terms of trade are determined by equations (13), (14), and (16) under complete markets or (18) under incomplete markets. Given the equilibrium values of C_j, Y_j, and S_j, the subsidy that supports the equilibrium can be derived directly from equation (11). We will not explicitly take that last step since we have no explicit interest in characterizing the "instrument rule" that supports optimal policy.

Keynesian version of the model

We have set up the model with perfectly competitive firms and flexible prices, where the policy instrument is a time-varying subsidy. However, the model can easily be converted to a New Keynesian model with sticky nominal prices. The budget constraints and objectives of the firm have been written in nominal terms. Nominal prices are not determined in the flexible price model set out here. But if prices were sticky, these same equations could be used to characterize optimal monetary policy. Sticky prices here means that nominal prices are set in advance, before the realization of the state. They are set simultaneously with the purchase and sale of state-contingent claims – that is, after the policy commitment and before the realization of the state. To transform the model to a Keynesian sticky price model, we would need to assume that each good is produced by a monopolist. The price of home goods, $P_{H,j}$, would be a price index over an infinite variety of goods produced by monopolists, with a constant elasticity of substitution (greater than one.) This is the set-up, for example, in Devereux and Engel (2003) and Obstfeld and Rogoff (2004). Because the model is static, these prices are set and are independent of the realization of the state. Under producer currency pricing (which means that each firm sets a single price in units of its own currency), $P_{H,j}$ and $P^*_{F,j}$ are fixed and do not depend on the state j.

While introducing monopolistic producers also introduces another distortion into the model – inefficient levels of production due to the monopoly power of firms – this is not a distortion that monetary policy under commitment is able to address. If monetary policy tried to introduce an expansionary bias in order to offset the underproduction arising from the monopoly distortion, firms would simply set higher ex ante nominal prices. We will assume that the policymaker do not have access to an employment subsidy, τ_j, that can vary across states, but we will assume that they set a state-independent subsidy, τ, that would lead to an efficient level of output in the absence of productivity shocks. These assumptions follow common ones in the

New Keynesian literature. On the one hand, it is usually assumed that fiscal policy is not as flexible as monetary policy, so while monetary policy can be changed state by state, fiscal policy cannot. On the other hand, state-independent fiscal policy is feasible. The assumption that it is efficient in the case of no shocks is one of convenience.

Given these modifications, it is easy to see how our model can be interpreted as one for optimal monetary policy. Written in nominal terms, $P_{H,j}$ and $P^{*}_{F,j}$ are each set equal to one, and the relative price, $E_j P^{*}_{F,j} / P_{H,j}$ is just equal to the nominal exchange rate E_j. That is essentially equivalent to a model in which $P_{H,j}$ and $P^{*}_{F,j}$ are each set in advance. The average level of output in such a model would be different than that in the competitive model because of the monopoly distortion, but the state-dependent parts of the model would be identical.

We also do not derive an "instrument" rule for the Keynesian case. In fact, we have not even stated what the instrument of monetary policy would be in this model. In a dynamic model, much of the literature assumes that the nominal interest rate is the instrument. There is no interest rate in this model because it is static. Perhaps a static analog to the nominal interest rate would be the level of nominal consumption, $P_j C_j$. That is, we might posit that our policymakers directly control the level of nominal expenditure in the economy. As in much of the New Keynesian literature, we do not introduce explicitly the markets through which the policymaker controls its instrument. We assume that the policymaker can control nominal spending just as the standard literature assumes the policymaker directly controls the nominal interest rate.

There is an important subtle distinction between the monetary policy interpretation and the competitive, tax policy interpretation of the model. Below we approximate the model in a non-stochastic steady state. We assume optimal policy is followed in the steady state. In the tax model, that assumption is straightforward – the optimal tax rule is simply applied in the steady state case. However, monetary policy only is effective in targeting fluctuations, and will have no effect on the steady state. So, under the monetary policy interpretation, we still must assume that policymakers can impose a non-state contingent tax that gives the efficient outcome in the non-stochastic steady state.

2. Objective Function of Policymakers

In this section, we display the objective function for policymakers under complete and incomplete markets, both under non-cooperation and cooperation.

a. *Non-cooperative policy*

When markets are complete, substitute the expression (16) for the terms of trade into the goods market equilibrium condition (13):

$$(19)\quad \begin{aligned} Y_j &= \frac{\nu}{2} C_j^{[\sigma(2-\nu)+2(\nu-1)]/2(\nu-1)} C_j^{*-\sigma(2-\nu)/2(\nu-1)} \left[\mathrm{E}\left(C^{1-\sigma}\right)\right]^{(2-\nu)/2(\nu-1)} \left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]^{-(2-\nu)/2(\nu-1)} \\ &+ \frac{2-\nu}{2} C_j^{\sigma\nu/2(\nu-1)} C_j^{*-[\sigma\nu-2(\nu-1)]/2(\nu-1)} \left[\mathrm{E}\left(C^{1-\sigma}\right)\right]^{\nu/2(\nu-1)} \left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]^{-\nu/2(\nu-1)} \end{aligned} .$$

The objective of the Home policymaker under complete markets can then be written as:

$$(20)\quad \begin{aligned} &\frac{1}{1-\sigma}\mathrm{E}\left(C^{1-\sigma}\right) \\ &-\frac{\nu}{2}\mathrm{E}\left(C^{[\sigma(2-\nu)+2(\nu-1)]/2(\nu-1)} C^{*-\sigma(2-\nu)/2(\nu-1)} A^{-1}\right)\left[\mathrm{E}\left(C^{1-\sigma}\right)\right]^{(2-\nu)/2(\nu-1)} \left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]^{-(2-\nu)/2(\nu-1)} \\ &-\frac{2-\nu}{2}\mathrm{E}\left(C^{\sigma\nu/2(\nu-1)} C^{*-[\sigma\nu-2(\nu-1)]/2(\nu-1)} A^{-1}\right)\left[\mathrm{E}\left(C^{1-\sigma}\right)\right]^{\nu/2(\nu-1)} \left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]^{-\nu/2(\nu-1)} \end{aligned} .$$

Under balanced trade, from equation (18), we have:

$$(21)\quad S_j = C_j^{1/(\nu-1)} C_j^{*-1/(\nu-1))} \ .$$

Substituting this expression into the goods market equilibrium condition (13), we find:

$$(22)\quad Y_j = C_j^{\nu/2(\nu-1)} C_j^{*-(2-\nu)/2(\nu-1)} \ .$$

In this case, the objective of the Home policymaker is given by:

$$(23)\quad \frac{1}{1-\sigma}\mathrm{E}\left(C^{1-\sigma}\right) - \mathrm{E}\left(A^{-1} C^{\nu/2(\nu-1)} C^{*-(2-\nu)/2(\nu-1)}\right) .$$

At this point, it is helpful to compare the policymaker's incentive to control the terms of trade when markets are incomplete compared to the complete markets case.

If the policymaker had no influence on the terms of trade and had to accept the terms of trade as given, he would maximize

$$(24)\quad \frac{1}{1-\sigma}\mathrm{E}\left(C^{1-\sigma}\right) - \mathrm{E}\left(A^{-1}\left(\frac{\nu}{2} S^{(2-\nu)/2} C + \frac{2-\nu}{2} S^{\nu/2} C^*\right)\right),$$

whether markets are complete or incomplete. This equation is derived by simply substituting the demand for output, given by equation (13) into the objective function. In choosing consumption, the Home policymaker trades off the additional utility from consumption with the additional work effort required to produce the consumption goods that arises because an increase in Home consumption increases demand for the Home good.

The policymaker does not, however, take the terms of trade as given. Consider how the Home policymaker chooses consumption to respond to a Home productivity shock, A_j. It wants Home consumption to increase, of course, and will trade off the benefit of increased consumption with the increased work effort for a given level of productivity. In addition, when Home productivity rises, so supply of the Home good increases, there will tend to be a drop in the relative price of Home output (an increase in S_j.) This induces even more demand for the Home good, from both Home and Foreign residents. There is an incentive for the Home policymaker to exercise its monopoly power on world markets for the Home good by limiting the decline in the price of its export. In order to do this, it must restrict the increase in supply of the good. With less an increase in output and therefore a smaller increase in S_j, there must be a smaller increase in Home consumption (holding Foreign consumption constant), assuming there is home bias in demand ($\nu > 1$).

Consider the tradeoff that this implies in the New Keynesian version of the model. An increase in Home productivity raises potential output, so absent any reaction by the policymaker, current output falls short of potential. Monetary policy must be expansionary, as that will raise demand for output and push output up toward potential. This is a common feature of optimal policy in New Keynesian models – that monetary policy is accommodating in response to a supply shock. One channel through which demand for the Home good increases is the exchange-rate channel. The expansionary policy causes a depreciation of the Home currency, which results in a drop in the relative price of the Home good.

However, because the policymaker exploits its monopoly power in the market for its own good, it will try to modify the depreciation of the currency so that the price of its good does not fall so much. The strategic element of policy involves limiting exchange rate movements.

When trade is balanced, a decline in the price of the Home good allows the Home country to have higher aggregate consumption. We can write the trade balance condition (18) as:

$$P_{Hj}^{\nu-1} C_j = \left(E_j P_{Fj}^*\right)^{\nu-1} C_j^* \ .$$

When Home goods become cheaper, and Home goods are overweighted in Home utility, then Home aggregate consumption can rise relative to Foreign consumption. Conversely, when the policymaker wants to limit the drop in the price of Home goods, then the trade balance condition requires that the increase in Home consumption be limited. Now compare the trade balance condition with the equilibrium condition when markets are complete, given by equation (10), which can be written as:

$$P_{Hj}^{\nu-1} C_j^{\sigma} \lambda = \left(E_j P_{Fj}^*\right)^{\nu-1} C_j^{*\sigma} \lambda^* .$$

For now, hold constant λ and λ^*, the Lagrange multipliers. Compared to the previous condition under trade balance, we can see that under complete markets when $\sigma > 1$, the adjustment in consumption for a given change in the price of Home goods is smaller.

When markets are complete, households do not rely just on the value of sales of current output to support consumption. There are also the payoffs from state contingent claims, which could be positive or negative. One factor that influences dividends from these claims is the prices of goods. When the price of Home consumption is low, Home households would like to receive a dividend so they can buy more when prices are cheap. When σ is large, the gain in utility from only a small transfer is large. When $\sigma > 1$, a price decrease leads to a smaller increase in consumption under complete markets than under balanced trade.

Now, in fact, policy cannot treat λ and λ^* as constants. λ is the marginal utility of wealth of Home consumers. Any change in productivity or policy that affects consumption in state *j* also affects the Home consumer's overall wealth. Recall from equation (15) that $\lambda / \lambda^* = \mathrm{E}\left(C^{1-\sigma}\right) / \mathrm{E}\left(C^{*1-\sigma}\right)$. When current consumption rises for Home, it has a small effect in pushing down Home's marginal utility of wealth when $\sigma > 1$ because it increases $\mathrm{E}\left(C^{1-\sigma}\right)$. Put more simply, the Home agent's wealth increases.

We can rewrite the expression for the terms of trade under complete markets, given by (16) as

$$S_j = S_j^{BT} \left\{C_j^{\sigma-1}\left[\mathrm{E}\left(C^{1-\sigma}\right)\right]\right\}^{1/(\nu-1)} \left\{C_j^{*\sigma-1}\left[\mathrm{E}\left(C^{*1-\sigma}\right)\right]\right\}^{-1/(\nu-1)} .$$

Here, S_j^{BT} refers to the terms of trade that would prevail under balanced trade, given by the right-hand-side of equation (21). When $\sigma > 1$, we can see that the required change in C_j for a given change in the terms of trade is smaller under complete markets. The term in the {} brackets, $C_j^{\sigma-1}\left[\mathrm{E}\left(C^{1-\sigma}\right)\right]$, captures the difference between the consumption restriction needed under complete markets and balanced trade to achieve a given terms of trade effect. Monetary policy works on both parts of this expression: current consumption, C_j and the marginal utility of wealth which influences $\mathrm{E}\left(C^{1-\sigma}\right)$.

b. Cooperative policy

The cooperative policy problem under complete markets can be greatly simplified by noticing that a policymaker that wishes to maximize the sum of the utility of Home and Foreign households has no incentive to influence λ / λ^*. The standard practice in the literature, which is completely correct, is to set $\lambda / \lambda^* = 1$, in which case equation (16) simplifies to

$$S_j = C_j^{\sigma/(\nu-1)} C_j^{*-\sigma/(\nu-1)}.$$

The objective function can be written as:

$$\begin{aligned} (25) \quad & \frac{1}{1-\sigma}\mathrm{E}\left(C^{1-\sigma}\right) + \frac{1}{1-\sigma}\mathrm{E}\left(C^{*1-\sigma}\right) \\ & -\mathrm{E}\left\{\frac{\nu}{2}C^{\left[\sigma(2-\nu)+2(\nu-1)\right]/2(\nu-1)}C^{*-\sigma(2-\nu)/2(\nu-1)}A^{-1} + \frac{2-\nu}{2}C^{\sigma\nu/2(\nu-1)}C^{*-\left[\sigma\nu-2(\nu-1)\right]/2(\nu-1)}A^{-1}\right\} \\ & -\mathrm{E}\left\{\frac{\nu}{2}C^{*\left[\sigma(2-\nu)+2(\nu-1)\right]/2(\nu-1)}C^{-\sigma(2-\nu)/2(\nu-1)}A^{*-1} + \frac{2-\nu}{2}C^{*\sigma\nu/2(\nu-1)}C^{-\left[\sigma\nu-2(\nu-1)\right]/2(\nu-1)}A^{*-1}\right\} \end{aligned}.$$

Under balanced trade, we can use equation (23), and symmetry, to write the objective of the cooperative policymaker as:

$$(26) \quad \frac{1}{1-\sigma}\mathrm{E}\left(C_j^{1-\sigma} + C_j^{*1-\sigma}\right) - \mathrm{E}\left\{\left(A^{-1}C_j^{\nu/2(\nu-1)}C_j^{*-(2-\nu)/2(\nu-1)}\right) + \left(A^{*-1}C_j^{*\nu/2(\nu-1)}C_j^{-(2-\nu)/2(\nu-1)}\right)\right\}.$$

Under cooperation, there is no incentive to manipulate the terms of trade whether markets are complete or not.

3. Targeting Rules

In this section, we characterize the targeting rule for policymakers. The targeting rule shows in a simple way how the policymaker trades off competing targets for monetary policy – how much it should weigh competing objectives. We compare the targeting rule under cooperation versus under strategic behavior by individual country policymakers. How are the objectives different? What does the global policymaker care about compared to the self-oriented national policymaker?

Targeting rules for this simple model are easily derived from the welfare functions of the previous section. The policymaker sets consumption to maximize utility, and the objective functions are expressed in terms of consumption, so the first-order condition from the unconstrained optimization problem gives us the targeting rules.

Cooperative case

We can begin by deriving the targeting rule of the global policymaker. As shown in the Appendix, log-linear approximations to the first-order conditions for maximizing global welfare (that is, maximizing (26)) when there is no trade in state-contingent claims and trade is balanced are given by:

$$\sigma(\nu-1)^2 c_j^R + \nu(2-\nu)c_j^R - (\nu-1)a_j^R = 0$$

$$\sigma c_j^W - a_j^W = 0 .$$

In the notation introduced here, for any variable x_j, we define an average "world" variable as $x_j^W \equiv \frac{x_j + x_j^*}{2}$, and an average "relative" variable as $x_j^R \equiv \frac{x_j - x_j^*}{2}$. Lower case letters denote the log of the deviation of the corresponding upper case variable from the log of the variable under the non-stochastic "steady state" (which is the outcome if productivity were constant and equal to one in all states in both countries.)

To understand the objectives of the global policymaker, it is helpful to look at some special cases. If there is no home bias in consumption (if $\nu = 1$), then the policymaker would set Home and Foreign consumption equal. We would have $c_j^R = 0$, and $c_j = c_j^* = c_j^W = a_j^W / \sigma$. The policymaker would insure that the two countries shared the global shock to productivity. At the other extreme, if there were complete home bias in consumption ($\nu = 2$), each country's consumption would respond only to its own changes in productivity: $c_j = a_j / \sigma$, $c_j^* = a_j^* / \sigma$. The intermediate case involves some but not complete spillovers from the Foreign country to Home. This is explored in more depth in section 4.

Under complete markets, the corresponding first-order conditions are given by:

$$\sigma \bar{c}_j^R - (\nu - 1) a_j^R = 0$$

$$\sigma \bar{c}_j^W - a_j^W = 0 .$$

Here we use an overbar on the consumption variables to denote the value of consumption under complete markets. The condition for sharing of the world shock is the same as under incomplete markets, but the effect of relative shocks on relative consumption is different under complete markets.

One can express the objectives of the policymaker under incomplete markets in terms of distortions – the difference between the target under incomplete markets and under complete markets:

$$\sigma \tilde{c}_j^R - (\sigma - 1) \nu (2 - \nu) c_j^R = 0$$

$$\tilde{c}_j^W = 0 \ .$$

Here, for any variable x_j, we define $\tilde{x}_j$ as the deviation of the variable under incomplete markets from the value it takes under complete markets: $\tilde{x}_j = x_j - \bar{x}_j$. In this case of a global policymaker, $\tilde{c}_j$ is the difference between the outcomes for the log of consumption under incomplete markets and complete markets for given realizations of the Home and Foreign productivity shocks.[4]

The second of these "targeting rules" is simple – the policymaker will set world consumption equal to the value it takes when markets are complete. The first, however, reveals a tradeoff. The policymaker cannot set relative consumption at the same levels as are achievable under complete markets. The policymaker would like to be able to do that and achieve the globally efficient outcome that can be achieved under complete markets (and cooperative policy.) But markets are not complete and the policymaker is constrained by the balanced trade

[4] In order to take these differences, it is important that the log linearizations in both cases of incomplete and complete markets are around the same steady state.

condition. He could set $\tilde{c}_j^R$ equal to zero, but then would not be able to reach the target of $\tilde{c}_j^W = 0$. The optimality condition says that (assuming $\sigma > 1$) the policymaker will set $c_j^R > \overline{c}_j^R$ whenever $c_j^R > 0$.

To understand this tradeoff, we introduce the variable M_j, the deviation from risk sharing that occurs under complete markets. Specifically, the Foreign marginal utility of a unit of expenditure is given by $C_j^{*-\sigma} / P_j^*$ and the Home marginal utility of a unit of expenditure is given by $C_j^{-\sigma} / P_j$. M_j is defined as the ratio of $C_j^{*-\sigma} / P_j^*$ to $C_j^{-\sigma} / P_j$:

$$M_j \equiv \left[C_j^{*-\sigma} / P_j^* \right] \Big/ \left[C_j^{-\sigma} / P_j \right] = C_j^{\sigma} C_j^{*-\sigma} S_j^{\nu-1}.$$

In the complete markets cooperative equilibrium, ex ante the global policymaker treats the two countries equally, so $\mathrm{E}\left(C^{1-\sigma}\right) = \mathrm{E}\left(C^{*1-\sigma}\right)$. From (10) and using (15), we have $M_j = 1$ in the complete markets equilibrium. However, under the assumption of balanced trade, $M_j = C_j^{\sigma-1} C_j^{*-(\sigma-1)}$. Hence, m_j is the log of the deviation from the outcome from complete markets risk sharing. It is the risk-sharing distortion, as defined in Corsetti et. al. (2010).

We can rewrite the first targeting rule as:

$$\sigma \tilde{c}_j^R + 2\nu\left(2-\nu\right) m_j = 0.$$

The policymaker under complete markets faces a tradeoff between the objective of achieving the relative consumption that is desired under complete markets and the objective of minimizing the consumption distortion introduced by market incompleteness. Under complete markets, policymakers set:

$$\overline{c}_j^R = \frac{\nu - 1}{\sigma} a_j^R.$$

Under incomplete markets,

$$c_j^R = \frac{\nu - 1}{\sigma\left(\nu - 1\right)^2 + \nu\left(2-\nu\right)} a_j^R.$$

When $\sigma > 1$, c_j^R responds more to relative productivity shocks under incomplete markets than under complete markets. The policymaker must allow for more responsiveness of relative consumption in order to help alleviate the risk sharing distortion.

We can recast these targeting rules in terms of output gaps instead of consumption gaps. Taking approximations to the goods market clearing conditions, we find:

$$\hat{c}_j^R = \frac{\nu-1}{D}\hat{y}_j^R + \frac{\nu(2-\nu)}{2D}m_j \text{, and} \qquad \hat{c}_j^W = \hat{y}_j^W .$$

Under cooperation, the targeting rules can then be written as:

$$\sigma\hat{y}_j^R + \frac{(\sigma-1)(\nu-1)\nu(2-\nu)}{2}m_j = 0$$

$$\hat{y}_j^W = 0$$

The second condition simply says the global policymaker sets the world output gap to zero – it sets global output at the same level as would be achieved under complete markets. But the first condition demonstrates a tradeoff between setting the relative output gap to zero and minimizing the risk-sharing distortion.

Non-cooperative equilibrium

How does the targeting rule for the non-cooperative policymaker compare to that of the cooperative policymaker? Does the strategic policymaker care about deviations from risk sharing? How are strategic elements reflected in the targeting rule?

Targeting rules under non-cooperation are, in essence, reaction functions. The Home policymaker's targeting rule is derived from the first-order condition for choosing optimal consumption taking the productivity shocks in each country as given *and* taking Foreign consumption as given.

As the Appendix shows, a log-linear approximation to the first-order condition for the Home policymaker under incomplete markets is given by:

$$\left(2\sigma(\nu-1)+2-\nu\right)c_j - (2-\nu)c_j^* - 2(\nu-1)a_j = 0 .$$

It is helpful to rewrite this term, using the market-clearing condition for Home output and the solution for the terms of trade under balanced trade. That is, taking a log-linear approximations to (13) and (21), we have

$$c_j = y_j - \left(\frac{2-\nu}{2}\right)s_j \quad \text{and} \quad c_j^* = y_j - \frac{\nu}{2}s_j .$$

Substituting into the approximation of the first-order condition above, we have:

(27) $\sigma y_j - \frac{(\sigma-1)(2-\nu)}{2} s_j - a_j = 0$.

This representation of the targeting rule neatly captures the open-economy aspects of the policymaker's objectives. If the country were not open to the rest of the world – for example, if there were complete home bias in consumption, $\nu = 2$ – the policymaker trades off the disutility of work with the utility of an additional unit of consumption. Since consumption would equal output in the closed economy, the targeting rule would be $\sigma y_j - a_j = 0$. But the policymaker in the open economy also cares about the terms of trade. It acts as a monopolist to diminish terms of trade fluctuations. So, when a_j increases, the country's output will fall, and the terms of trade will worsen (s_j will increase.) But the optimal policy rule has the policymaker limiting those terms of trade fluctuations. For a given change in output, it reduces s_j when a_j increases.

Under complete markets, the first-order condition for the Home policymaker is given by:

$$
\begin{aligned}
&-\left(2\sigma(\nu-1)+2-\nu\right)\overline{c}_j + (2-\nu)c_j^* + 2(\nu-1)a_j \\
&-(\sigma-1)(2-\nu)\left(1+\sigma(2\nu-1)\right)\left(\overline{c}_j - \mathrm{E}\overline{c}\right) + \left(\sigma^2-1\right)(2-\nu)\left(c_j^* - \mathrm{E}c^*\right) + 2\sigma(2-\nu)(\nu-1)\left(a_j - \mathrm{E}a\right) = 0
\end{aligned}
$$

It is helpful in interpreting this first-order condition to recall from the discussion in section 2 above that $C_j^{\sigma-1}\left[\mathrm{E}\left(C^{1-\sigma}\right)\right]$ measures the difference between the consumption restriction needed under complete markets and balanced trade to achieve a given terms of trade effect. In log terms, that is given by $\mu_j \equiv (\sigma-1)\left(\overline{c}_j - \mathrm{E}\overline{c}\right)$. For the sake of symmetry, it is also helpful to define $\mu_j^* \equiv (\sigma-1)\left(\overline{c}_j^* - \mathrm{E}\overline{c}^*\right)$, though the Home policymaker takes μ_j^* as given in choosing his optimal consumption level. We can then simplify the first-order condition under complete markets to:

$$
\begin{aligned}
&-\left(2\sigma(\nu-1)+2-\nu\right)\overline{c}_j + (2-\nu)c_j^* + 2(\nu-1)a_j - (2-\nu)\left(1+\sigma(2\nu-1)\right)\mu_j \\
&+(\sigma+1)(2-\nu)\mu_j^* + 2\sigma(2-\nu)(\nu-1)\left(a_j - \mathrm{E}a\right) = 0
\end{aligned}
.
$$

As in the case of incomplete markets, we can rewrite this expression in terms of output and the terms of trade. We use the fact that under complete markets

$$\overline{s}_j = \frac{1}{\nu-1}\left(\overline{c}_j - \mu_j - \left(c_j^* - \mu_j^*\right)\right) .$$

Together with the market clearing condition for Home goods, we find:

$$\overline{c}_j = \overline{y}_j - \left(\frac{2-\nu}{2}\right)\left(s_j - \mu_j + \mu_j^*\right) \qquad \text{and} \qquad \overline{c}_j^* = \overline{y}_j - \left(\frac{\nu}{2}\right)\left(s_j + \mu_j - \mu_j^*\right).$$

Substituting into the first-order condition, we find:

$$\begin{aligned}(28)\qquad &\sigma\overline{y}_j - \frac{(\sigma-1)(2-\nu)}{2}s_j - a_j \\ &+\frac{2-\nu}{2(\nu-1)}\left(\sigma\nu\mu_j - \sigma(2-\nu)\mu_j^* - 2(\sigma-1)(\nu-1)\left(a_j - \mathrm{E}a\right)\right) = 0\end{aligned}.$$

The targeting rule (28) has been written in a way to highlight the differences between the objectives under complete markets and those under incomplete markets, given in (27).

Rewrite the targeting rule under incomplete markets as:

$$(29)\qquad \sigma\hat{y}_j - \frac{(\sigma-1)(2-\nu)}{2}\hat{s}_j + \frac{\sigma\nu(2-\nu)}{2(\nu-1)}\hat{\mu}_j - (\sigma-1)(2-\nu)\left(\sigma y_j - a_j - (\sigma\mathrm{E}y - \mathrm{E}a)\right) = 0\,.$$

Compared to the case of the closed economy, $\nu = 2$, we can see there are two additional objectives for the non-cooperative policymaker under complete markets. On the one hand, he would like to manipulate the terms of trade to exploit the country's monopoly power. This does not simply entail, however, setting the terms of trade equal to the value it would take under complete markets, so $\hat{s}_j = 0$. Because the amount of consumption required to achieve a given terms of trade change is different under complete markets and incomplete markets, the tradeoff between output and the terms of trade is different, which accounts for the term involving $\hat{\mu}_j$. In addition, the policymaker in the open economy has the opportunity to exploit international markets to achieve more efficient allocations across states. In a closed economy, the (approximate) log of the marginal rate of substitution between consumption and leisure is given by $\sigma y_j - a_j$. Greater levels of welfare could be reached if that could be smoothed across states, but in a closed economy with homogeneous households, there is no opportunity for this type of insurance. The term $\sigma y_j - a_j - (\sigma\mathrm{E}y - \mathrm{E}a)$ indicates an objective to achieve smooth risk in the global economy.

So the non-cooperative policymaker would like to minimize the output gap, but in addition has the goal of manipulating the terms of trade as a monopolist would, and of using international markets to achieve some risk sharing.

Discussion

There has been a widespread discussion concerning the spillovers of Federal Reserve policy, and to a lesser extent, European Central Bank policy, in recent years. The flashpoint of that discussion was the comment in September 2010 by Guido Mantega, the Brazilian finance minister, that "we are in the middle of a currency war"[5], referring to the depreciation of the major currencies against those of Brazil and other emerging markets. In April 2014, Raghuram Rajan, the governor of the Reserve Bank of India, noted that the major countries were not sufficiently considering the effects of spillovers of their monetary policies, noting the "initiation of unconventional policy as well as an exit whose pace is driven solely by conditions in the source country," specifically aiming his remarks at monetary policy in the U.S. and other industrial countries that "hold interest rates near zero for long, as well as balance sheet policies such as quantitative easing or exchange intervention, that involve altering central bank balance sheets in order to affect certain market prices."[6] Olivier Blanchard, Jonathan Ostry and Atish Ghosh of the International Monetary Fund observe "if large players in the global economy are responsible for significant adverse spillovers across a swath of smaller countries, this needs to be acknowledged as well, and feasible remedies considered."[7]

The response by the Federal Reserve has been to note that their legal mandate is to keep prices stable and unemployment low, while recognizing the potential for spillovers to the global economy. Ben Bernanke (2012) notes at the outset of his address that "All of the Federal Reserve's monetary policy decisions are guided by our dual mandate to promote maximum employment and stable prices." He goes on to argue that nonetheless " it is not at all clear that accommodative policies in advanced economies impose net costs on emerging market economies, for several reasons." He argues that "by boosting U.S. spending and growth, it has the effect of helping support the global economy as well."

[5] See Financial Times, 2010.
[6] See Rajan, 2014.
[7] Blanchard, Ostry and Ghosh, 2013.

Stanley Fischer (2014) states "the Federal Reserve's objectives are given by its dual mandate to pursue maximum sustainable employment and price stability, and our policy decisions are targeted to achieve these dual objectives." He goes on to observe "The state of the U.S. economy is significantly affected by the state of the world economy. A wide range of foreign shocks affect U.S. domestic spending, production, prices, and financial conditions. To anticipate how these shocks affect the U.S. economy, the Federal Reserve devotes significant resources to monitoring developments in foreign economies, including emerging market economies (EMEs), which account for an increasingly important share of global growth. The most recent available data show 47 percent of total U.S. exports going to EME destinations. And of course, actions taken by the Federal Reserve influence economic conditions abroad. Because these international effects in turn spill back on the evolution of the U.S. economy, we cannot make sensible monetary policy choices without taking them into account."

Neither Bernanke nor Fischer claim that U.S. policy is cooperative. They both note that the Federal Reserve tries to achieve its internal objectives. It does not ignore the rest of the world, but it does not alter its policies to take into account their effect on the rest of the world, except insofar as global repercussions can then feed back onto the U.S. economy. Non-cooperative policy does not mean that the policymaker is actively trying to take advantage of other economies. It means exactly that it is concerned with only its own objectives when setting policy rules. In that sense, U.S. monetary policy fits into the non-cooperative framework.

However, note that the Federal Reserve policy is aimed at "employment and price stability." We have seen that the targeting rule in the global economy involves trading off the employment (or output) objective with a terms of trade objective. (Inflation is not present in our static model, but in a dynamic version of this model with staggered pricing, the targeting rule would surely trade off inflation as well.) One interpretation of the remarks by Bernanke and Fischer is that while policy is not coordinated, the Fed also ignores the terms of trade objective. It does not attempt to exploit its monopoly power in export markets.

The exploitation of that monopoly power involves limiting the terms of trade fluctuations, at least in response to the productivity changes that are the exogenous shocks in this model. When output is short of potential, monetary policy needs to be expansionary but the strategic central bank will dampen the potential depreciation needed to reach full employment. In other words, Mantega is unlikely to be satisfied with a U.S. monetary policy that focuses on employment and inflation, because it is still the case that the expansionary effects of U.S. monetary policy during times of employment shortfalls may have negative spillovers on Brazil through the exchange rate channel.

In the next section, we compare the extent of spillovers under cooperative policy versus non-cooperative policy.

4. Spillovers

In this section, we solve the model under complete markets and incomplete markets, with cooperation and without cooperation, in order to understand the nature of spillovers. Our focus will be on the spillovers to consumption, but similar analysis would apply to output.

It is important to note that our focus is on spillovers – the elasticity of consumption in one country with respect to a shock in the other country – but not on the overall level of consumption. As is well known, the level of consumption is lower under non-cooperation. Our analysis is couched in terms of the effects of shocks on the deviation of the log of consumption from its steady-state value, but the steady-state value of consumption is lower under non-cooperation compared to cooperation. In particular, under cooperation, the steady state value of Home or Foreign consumption, *C*, is equal to one (when productivity levels in both countries take on a value of one.) But under non-cooperation, $C = \left(\frac{2(\nu - 1)}{\nu} \right)^{1/\sigma}$ which is strictly less than one, as long as there is not complete home bias.

We begin our analysis by noting that, by definition, $c_j = c_j^R + c_j^W$. In all cases – complete markets, incomplete, cooperation or non-cooperation – we find

$$c_j^W = a_j^W / \sigma .$$

Whether there is cooperation or not, and independently of the completeness of financial markets, under optimal policy, the elasticity of consumption with respect to the "world" shock is given by $1/\sigma$. Any differences in the spillovers of Foreign productivity shocks to Home consumption comes through differences in the elasticity of c_j^R with respect to a_j^R.

It is useful to begin by considering the solution for Home consumption under complete markets with cooperation, because allocations are globally efficient in this case. There we find:

$$c_j^R = \frac{\nu - 1}{\sigma} a_j^R .$$

If Home and Foreign households had the same consumption preferences, so $\nu = 1$, there would be no difference in their consumption in equilibrium under the optimal policy: $c_j^R = 0$. Relative shocks have an effect on relative consumption in the efficient allocation because of home bias in preferences. Each country's consumption is more susceptible to its own country's productivity shocks because each country consumes more of its own output.

Under non-cooperation (and complete markets),

$$c_j^R = \frac{(\nu - 1)\left(1 + (\sigma - 1)(2 - \nu)\right)}{\sigma\left(1 + (\sigma - 1)\nu(2 - \nu)\right)} a_j^R .$$

In the case of home bias and in the empirically plausible case of $\sigma > 1$, the elasticity of c_j^R with respect to a_j^R is *smaller* under non-cooperation. In this sense, spillovers are smaller when policymakers follow strategic objectives rather than cooperate.

To understand this, note that in equilibrium, under complete markets, from (16),

$$s_j = \frac{2\sigma}{\nu - 1} c_j^R .$$

Here we have used the equilibrium result $\mathrm{E}c = \mathrm{E}c^* = 0$. As we have discussed above, the terms of trade fluctuations are smaller under non-cooperation compared to cooperation, because the strategic policymaker acts as a monopolist to limit the response of its price to supply shocks.

There is, however, another way to view spillovers. When we take the elasticity of c_j^R with respect to a_j^R as a measure of spillovers, we are holding constant the effect the Foreign shock has on Home consumption through its effect on c_j^W. In general, a shock to a_j^* will have effects on both a_j^R and a_j^W. The latter will be unaffected only when a shock to a_j exactly offsets the a_j^* disturbance. We can represent the solutions for Home consumption under cooperation and non-cooperation as:

$$c_j^{COOP} = b^C a_j^R + \frac{1}{\sigma} a_j^W$$

$$c_j^{NON} = b^N a_j^R + \frac{1}{\sigma} a_j^W \quad ,$$

where b^C and b^N respectively, refer to the coefficients on a_j^R in the solutions for the cooperative and non-cooperative cases above, respectively. We have seen that $b^C > b^N > 0$, so that the relative shock has a larger effect on consumption in the cooperative case. But the key thing to recognize is that the equilibrium terms of trade movements serve to modify the effect of the Foreign shock on Home consumption. If there is a decline in a_j^*, then there will be a global drop in consumption, but the relative price of the Foreign good, s_j, increases. This works to increase Home consumption relative to Foreign consumption, thus modifying the impact of the shock on Home consumption. The overall transmission is positive – a decline in Foreign productivity leads to a decline in both Foreign and Home consumption. This is true under both cooperation, so $\frac{1}{\sigma} - b^C > 0$, and under non-cooperation, $\frac{1}{\sigma} - b^N > 0$. It follows from $b^C > b^N > 0$ that the overall spillovers are smaller under cooperation than under non-cooperation:

$$\frac{1}{\sigma} - b^N > \frac{1}{\sigma} - b^C > 0 .$$

Another way to see this is to consider a structure for the productivity shocks in which there is a common, global component, a_j^G, and idiosyncratic Home and Foreign components, a_j^H and a_j^F, respectively. These three components are assumed to be mutually uncorrelated. This structure for productivity shocks is not general. For example, it precludes the possibility of negative correlation between Home and Foreign productivity. We can assume:

$$a_j = a_j^G + a_j^H$$

$$a_j^* = a_j^G + a_j^F$$

From this structure, we have that the "relative" shock is simply determined by the idiosyncratic components:

$$a_j^R = \frac{a_j^H - a_j^F}{2} \quad .$$

The "world" shock is the sum of the global component and the average of the idiosyncratic components:

$$a_j^W = a_j^G + \frac{a_j^H + a_j^F}{2} \ .$$

It may be natural to think about spillovers in terms of the effects of an idiosyncratic Foreign shock on Home consumption. The effect is given by $\frac{1}{2}\left(\frac{1}{\sigma} - b^j\right), \ j = C, N$, so we could conclude that spillovers are smaller under cooperation.

The same analysis holds for incomplete markets. There we find:

$$b^N = \frac{\nu - 1}{\sigma - (\sigma - 1)(2 - \nu)}$$

$$b^C = \frac{\nu - 1}{\sigma - (\sigma - 1)\nu(2 - \nu)} .$$

The spillovers from relative shocks are greater under cooperation: $b^C > b^N > 0$. But the spillovers from idiosyncratic shocks are smaller under cooperation: $\frac{1}{\sigma} - b^N > \frac{1}{\sigma} - b^C > 0$.

So there is some subtlety required in thinking about whether cooperation reduces international spillovers. Under non-cooperation, the terms of trade movements in response to supply shocks are smaller than under cooperation because of the duopolistic behavior of strategic policymakers. However, since terms of trade fluctuations serve to modify the transmission of shocks that comes through their effects on global productivity, the net spillovers are smaller under cooperation.

It is important to emphasize that this is not a welfare analysis. Here we are only addressing the question of whether we can reasonably expect cooperation to reduce spillovers, not whether the outcome is desirable.

5. Conclusions

The central objective of this study was to derive a targeting rule for monetary policy in a non-cooperative, strategic environment, and to compare it with the targeting rule under cooperation. In order to do so, we built a very simple static model. The targeting rule is based on a comparison of optimal policy under incomplete markets and complete financial markets. The model is far too stylized to draw realistic conclusions about the conduct of monetary policy. It is, perhaps, a building block for richer models.

Engel (2014) surveys some of the challenges for the academic literature on non-cooperative monetary policy. Here, briefly, we make note of three.

Policymakers appear to be very concerned about the effects of monetary policy on the stability of capital flows. Rajan (2014), for example, states “ideally, recipient countries would wish for stable capital inflows, and not flows pushed in by unconventional policy.” Benoit Coueré of the Executive Board of the European Central Bank asserts “The volatility in capital flows to and from emerging economies over the last year was also mainly related to expectations of US monetary policy.” Fischer (2014) observes “EME critics argued that U.S. policy accommodation contributed to a surge of capital inflows and excessive credit growth in their economies, creating risks of financial instability. But, as time wore on, most EMEs seemed glad to receive those flows.” Most modern open-economy macroeconomic models do not accord a very prominent role for capital flows. There may be rebalancing of portfolios of assets in response to shocks, but typically in monetary policy analysis those shifts are kept in the background. In any case, it is not clear from the perspective of these models why capital flows present a problem. The models apparently do not capture the concerns of policymakers.

Second, the literature, including this paper, treats monetary policy coordination as if a single monetary policymaker would choose policy to maximize the joint welfare of all cooperating countries. In practice, it is more likely that further coordination would be reached as a result of meetings among monetary policymakers, where each policymaker enters the room hoping to improve his own country’s welfare but recognizing that there are gains to be made from cooperation. Perhaps a more fruitful model of such a process would be a bargaining game. Such a model could explore the roles of the smaller economies and the potential gains from forming coalitions.

Third is the possibility of political influences in setting monetary policy. Coueré points to this: “The first factor, which is often ignored in the academic literature, is the political economy of coordination. In the real world, the ability of central banks to deviate from a pre-determined path of action differs across countries. Central banks operate in different economic structures and institutional set-ups, notably in terms of mandates, time horizons and objectives. There are also differences in accountability arrangements and in economic and political cycles. All of this affects domestic policy incentives.” In the literature on international trade policies, the policymaker is modeled as facing a tradeoff between domestic welfare objectives and political constraints. That approach may be fruitful in understanding monetary policy as well.

Fischer (2014) observes "My teacher Charles Kindleberger argued that stability of the international financial system could best be supported by the leadership of a financial hegemon or a global central bank. But I should be clear that the U.S. Federal Reserve System is not that bank. Our mandate, like that of virtually all central banks, focuses on domestic objectives. As I have described, to meet those domestic objectives, we must recognize the effect of our actions abroad, and, by meeting those domestic objectives, we best minimize the negative spillovers we have to the global economy." It is clear that non-cooperative policy entails focusing on domestic objectives. Does the Fed, in fact, pursue those objectives to the fullest, including exploiting monopoly power in the terms of trade? Or, conversely, do central banks implicitly cooperate in their policy, recognizing the dangers of pursuing purely strategic policy? These are surely questions for further theoretical and empirical research.

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Appendix

Non-cooperation

The first-order condition for choosing C_j under non-cooperation and incomplete markets, from maximizing (23) is given by:

(30) $$C_j^{-\sigma} - \left(\frac{\nu}{2(\nu-1)}\right) A_j^{-1} C_j^{(\nu-2(\nu-1))/2(\nu-1)} C_j^{*-(2-\nu)/2(\nu-1)} = 0 .$$

We will log-linearize this around the symmetric non-stochastic steady state in which $A=1$, $C=C^*$. We have in the steady state

(31) $$C^{-\sigma} = \frac{\nu}{2(\nu-1)} .$$

Let lower case letters be the deviation of the log of the upper case letters from the log of the steady state values of the upper case variables. A log-linear approximation to (30) is:

(32) $$-\left(2\sigma(\nu-1)+2-\nu\right)c_j + (2-\nu)c_j^* + 2(\nu-1)a_j = 0 .$$

Under complete markets, deriving the log approximation of the first-order condition is more complicated. The first order condition for choosing C_j to maximize (20) is given by:

$$\frac{\partial U}{\partial C_j} = \pi_j C_j^{-\sigma}$$

$$-\pi_j \left\{ \left(\frac{\nu}{2}\right)\left(\frac{\sigma(2-\nu)+2(\nu-1)}{2(\nu-1)}\right) C_j^{\sigma(2-\nu)/2(\nu-1)} C_j^{*-\sigma(2-\nu)/2(\nu-1)} A_j^{-1} \left[\sum \pi_i C_i^{1-\sigma}\right]^{(2-\nu)/2(\nu-1)} \left[\sum \pi_i C_i^{*1-\sigma}\right]^{-(2-\nu)/2(\nu-1)} \right\}$$

$$-\pi_j \left\{ \begin{array}{l} (1-\sigma)\left(\frac{2-\nu}{2(\nu-1)}\right)\frac{\nu}{2} C_j^{-\sigma} \left[\sum \pi_i C_i^{1-\sigma}\right]^{[(2-\nu)-2(\nu-1)]/2(\nu-1)} \left[\sum \pi_i C_i^{*1-\sigma}\right]^{-(2-\nu)/2(\nu-1)} \times \\ \sum \pi_i \left(C_i^{[\sigma(2-\nu)+2(\nu-1)]/2(\nu-1)} C_i^{*-\sigma(2-\nu)/2(\nu-1)} A_i^{-1} \right) \end{array} \right\}$$

$$-\pi_j \left\{ \left(\frac{2-\nu}{2}\right)\left(\frac{\sigma\nu}{2(\nu-1)}\right) C_j^{[\sigma\nu-2(\nu-1)]/2(\nu-1)} C_j^{*-[\sigma\nu-2(\nu-1)]/2(\nu-1)} A^{-1} \left[\sum \pi_i C_i^{1-\sigma}\right]^{\nu/2(\nu-1)} \left[\sum \pi_i C_i^{*1-\sigma}\right]^{-\nu/2(\nu-1)} \right\}$$

$$-\pi_j \left\{ \begin{array}{l} (1-\sigma)\left(\frac{\nu}{2(\nu-1)}\right)\left(\frac{2-\nu}{2}\right) C_j^{-\sigma} \left[\sum \pi_i C_i^{1-\sigma}\right]^{[\nu-2(\nu-1)]/2(\nu-1)} \left[\sum \pi_i C_i^{*1-\sigma}\right]^{-\nu/2(\nu-1)} \times \\ \sum \pi_i \left(C_i^{\sigma\nu/2(\nu-1)} C_i^{*-[\sigma\nu-2(\nu-1)]/2(\nu-1)} A_i^{-1} \right) \end{array} \right\}$$

$$= 0$$

Note that $\frac{\partial U}{\partial C_j}$ is a function of $C_1, C_2, \ldots, C_N$; $C_1^*, C_2^*, \ldots, C_N^*$; and, $A_1, A_2, \ldots, A_N$, where N is the number of states. We will do a log linear approximation around the steady state. We interpret the steady state this way: Each state occurs with the given probability, but in all states, $A_j = 1$ and $C_j^* = C_j = C$. Examining the first-order condition above, we see that in steady state, $C^{-\sigma} = \frac{\nu}{2(\nu-1)}$, which is the same as in the case of incomplete markets.

We then have the approximation:

$$\frac{\partial U}{\partial C_j} \approx C \sum_{i=1}^{N} \frac{\partial^2 U}{\partial C_j \partial C_i} \overline{c}_i + C \sum_{i=1}^{N} \frac{\partial^2 U}{\partial C_j \partial C_i^*} c_i^* + \sum_{i=1}^{N} \frac{\partial^2 U}{\partial C_j \partial A_i} a_i = 0 .$$

The – over c_i in this expression is there to indicate this is the complete markets model. That is, $\overline{c}_i$ is the deviation of the log of consumption in state *i* from the log of its steady state value in the complete markets model. The derivatives on the right hand side of this expression are all evaluated at the steady state. In taking these derivatives, we will treat the π_i as parameters. After some tedious algebra, we find:

$$\frac{\partial^2 U}{\partial C_j^2} = -\pi_j C^{-1}\left(\frac{\sigma\nu}{2(\nu-1)} + \frac{\sigma\nu(2-\nu)}{4(\nu-1)^2}\big(\sigma+2(\sigma-1)(\nu-1)\big)\right) + \pi_j\pi_j C^{-1}\left(\frac{(\sigma-1)\nu(2-\nu)}{4(\nu-1)^2}\right)\big(1+\sigma(2\nu-1)\big)$$

$$\frac{\partial^2 U}{\partial C_j \partial C_i} = \pi_i\pi_j C^{-1}\left(\frac{(\sigma-1)\nu(2-\nu)}{4(\nu-1)^2}\right)\big(1+\sigma(2\nu-1)\big) \text{ for } i \neq j \ .$$

$$\frac{\partial^2 U}{\partial C_j \partial C_j^*} = \pi_j C^{-1}\frac{\sigma^2\nu(2-\nu)}{4(\nu-1)^2} - \pi_j\pi_j C^{-1}\frac{(\sigma^2-1)\nu(2-\nu)}{4(\nu-1)^2}$$

$$\frac{\partial^2 U}{\partial C_j \partial C_i^*} = -\pi_i\pi_j C^{-1}\frac{(\sigma^2-1)\nu(2-\nu)}{4(\nu-1)^2} \text{ for } i \neq j \ .$$

$$\frac{\partial^2 U}{\partial C_j \partial A_j} = \pi_j\left(\frac{\sigma\nu(2-\nu)+\nu(\nu-1)}{2(\nu-1)}\right) - \pi_j\pi_j\frac{(\sigma-1)\nu(2-\nu)}{2(\nu-1)}$$

$$\frac{\partial^2 U}{\partial C_j \partial A_i} = -\pi_i\pi_j\frac{(\sigma-1)\nu(2-\nu)}{2(\nu-1)} \text{ for } i \neq j \, .$$

Substituting into the general expression for the approximation, we find:

$$-\pi_j\left(\frac{\sigma\nu}{2(\nu-1)}+\frac{\sigma\nu(2-\nu)}{4(\nu-1)^2}\left(\sigma+2(\sigma-1)(\nu-1)\right)\right)\overline{c}_j$$
$$+\pi_j\left(\frac{(\sigma-1)\nu(2-\nu)}{4(\nu-1)^2}\right)\left(1+\sigma(2\nu-1)\right)\sum_{i=1}^{N}\pi_i\overline{c}_i$$
$$+\pi_j\frac{\sigma^2\nu(2-\nu)}{4(\nu-1)^2}c_j^*-\pi_j\frac{(\sigma^2-1)\nu(2-\nu)}{4(\nu-1)^2}\sum_{i=1}^{N}\pi_i c_i^*$$
$$+\pi_j\left(\frac{\sigma\nu(2-\nu)+\nu(\nu-1)}{2(\nu-1)}\right)a_j-\pi_j\frac{(\sigma-1)\nu(2-\nu)}{2(\nu-1)}\sum_{i=1}^{N}\pi_i a_i=0$$

This can be written more simply as:

$$-\left(\frac{\sigma\nu}{2(\nu-1)}+\frac{\sigma\nu(2-\nu)}{4(\nu-1)^2}\left(\sigma+2(\sigma-1)(\nu-1)\right)\right)\overline{c}_j$$
$$+\left(\frac{(\sigma-1)\nu(2-\nu)}{4(\nu-1)^2}\right)\left(1+\sigma(2\nu-1)\right)\mathrm{E}\overline{c}+\frac{\sigma^2\nu(2-\nu)}{4(\nu-1)^2}c_j^*-\pi_j\frac{(\sigma^2-1)\nu(2-\nu)}{4(\nu-1)^2}\mathrm{E}c^*$$
$$+\left(\frac{\sigma\nu(2-\nu)+\nu(\nu-1)}{2(\nu-1)}\right)a_j-\frac{(\sigma-1)\nu(2-\nu)}{2(\nu-1)}\mathrm{E}a=0$$

With some further rewriting, we can express this relationship as:

$$-\left(2\sigma(\nu-1)+2-\nu\right)\overline{c}_j+(2-\nu)c_j^*+2(\nu-1)a_j$$
$$-(\sigma-1)(2-\nu)\left(1+\sigma(2\nu-1)\right)\left(\overline{c}_j-E\overline{c}\right)+\left(\sigma^2-1\right)(2-\nu)\left(c_j^*-Ec^*\right)+2\sigma(2-\nu)(\nu-1)\left(a_j-Ea\right)=0$$

Cooperation:

The first-order condition for choosing C_j under non-cooperation and incomplete markets, from maximizing (26) is given by:

$$C_j^{-\sigma}-\frac{\nu}{2(\nu-1)}C_j^{(2-\nu)/2(\nu-1)}C_j^{*-(2-\nu)/2(\nu-1)}A_j^{-1}+\frac{2-\nu}{2(\nu-1)}C_j^{*\nu/2(\nu-1)}C_j^{-\nu/2(\nu-1)}A_j^{*-1}=0$$

We will log-linearize this around the symmetric non-stochastic steady state in which $A=1$, $C=C^*$. We have in the steady state $C=1$. This can be approximated as

$$-\sigma c_j - \frac{\nu(2-\nu)}{2(\nu-1)^2}\left(c_j - c_j^*\right) + \frac{\nu}{2(\nu-1)} a_j - \frac{2-\nu}{2(\nu-1)} a_j^* = 0 \ .$$

Symmetrically, we can approximate the first-order condition for choosing C_j^* as:

$$-\sigma c_j^* - \frac{\nu(2-\nu)}{2(\nu-1)^2}\left(c_j^* - c_j\right) + \frac{\nu}{2(\nu-1)} a_j^* - \frac{2-\nu}{2(\nu-1)} a_j = 0$$

The first-order condition for choosing C_j under cooperation and complete markets, from maximizing (25) is given by:

$$\begin{aligned}
&C_j^{-\sigma} - \left(\frac{\nu}{2}\right)\left(\frac{\sigma(2-\nu)+2(\nu-1)}{2(\nu-1)}\right) C_j^{\sigma(2-\nu)/2(\nu-1)} C_j^{*-\sigma(2-\nu)/2(\nu-1)} A_j^{-1} \\
&-\left(\frac{2-\nu}{2}\right)\left(\frac{\sigma\nu}{2(\nu-1)}\right) C_j^{[\sigma\nu-2(\nu-1)]/2(\nu-1)} C_j^{*-[\sigma\nu-2(\nu-1)]/2(\nu-1)} A_j^{-1} \\
&+\left(\frac{\nu}{2}\right)\left(\frac{\sigma(2-\nu)}{2(\nu-1)}\right) C_j^{-[\sigma(2-\nu)+2(\nu-1)]/2(\nu-1)} C_j^{*[\sigma(2-\nu)+2(\nu-1)]/2(\nu-1)} A_j^{*-1} \\
&+\left(\frac{2-\nu}{2}\right)\left(\frac{\sigma\nu-2(\nu-1)}{2(\nu-1)}\right) C_j^{-\sigma\nu/2(\nu-1)} C_j^{*\sigma\nu/2(\nu-1)} A_j^{*-1} = 0
\end{aligned} \ .$$

We will log-linearize this around the symmetric non-stochastic steady state in which $A=1$, $C=C^*$. We have in the steady state $C=1$. This can be approximated as

$$-\sigma\bar{c}_j - \frac{\sigma^2\nu(2-\nu)}{2(\nu-1)^2}\left(\bar{c}_j - \bar{c}_j^*\right) + \left(\frac{\sigma\nu(2-\nu)+\nu(\nu-1)}{2(\nu-1)}\right) a_j - \left(\frac{\sigma\nu(2-\nu)-(2-\nu)(\nu-1)}{2(\nu-1)}\right) a_j^* = 0$$

Symmetrically, we can approximate the first-order condition for choosing C_j^* as:

$$-\sigma\bar{c}_j^* - \frac{\sigma^2\nu(2-\nu)}{2(\nu-1)^2}\left(\bar{c}_j^* - \bar{c}_j\right) + \left(\frac{\sigma\nu(2-\nu)+\nu(\nu-1)}{2(\nu-1)}\right) a_j^* - \left(\frac{\sigma\nu(2-\nu)-(2-\nu)(\nu-1)}{2(\nu-1)}\right) a_j = 0 .$$