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# QE in the Future: The Central Bank's Balance Sheet in a Fiscal Crisis 

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# QE in the future: the central bank's balance sheet in a fiscal crisis* 

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#### Abstract

Studies of quantitative easing (QE) typically focus on the recent past and study its effectiveness during a financial crisis when nominal interest rates are zero. This paper examines instead the usefulness of QE in a future fiscal crisis, modeled as a situation where the fiscal outlook is inconsistent with both stable inflation and no sovereign default. The crisis can lower welfare through two channels, the first via aggregate demand and nominal rigidities, and the second via contractions in credit and disruption in financial markets. Managing the size and composition of the central bank's balance sheet can interfere with each of these channels, stabilizing inflation and economic activity, but there are limits to what QE can achieve.


JEL codes: E44, E58, E63.
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[^0]
## 1 Introduction

At the start of the 2008 financial crisis, central banks engaged in unconventional policies, buying many risky assets and giving credit to a wide variety of private agents, but just a few years later, the balance sheets of the the Bank of England, the Bank of Japan, the ECB, and the Federal Reserve were dominated by only a few items. ${ }^{1}$ The liabilities consisted largely of reserves that pay interest, whereas before the crisis there was mostly currency. The assets included some foreign currency and short-term securities issued directly by the government or backed by it, as before, but now also a large stock of longer-term government securities. These changes were the result of quantitative easing (QE) policies, which both increased the size of the balance sheet of the central bank, by issuing reserves, as well as its composition by extending the maturity of the bonds held. ${ }^{2}$

The motivation for these policies was the combination of a financial crisis and zero nominal interest rates, and the desire to increase liquidity and lower lower long-term yields (Bernanke, 2015). The economic theory to support them relied on models where short-term interest rates are zero (Bernanke and Reinhart, 2004) or where financial frictions during crises prevent arbitrage across asset classes and drive changes in term premia (Vayanos and Vila, 2009; Gertler and Karadi, 2013). Yet, the financial crisis is now several years behind and interest rates are expected to take off from zero soon. When this happens, will QE and the use for it disappear, as did so many of the other unconventional monetary policies?

Looking towards the future, this paper asks whether QE would have an effect on macroeconomic outcomes during a fiscal crisis. The focus on a fiscal crisis is motivated by the current data: public debt in the United Kingdom, Japan, many European countries, and the United States is at historically high levels. It is plausible, perhaps even likely, that the next macroeconomic crisis will be fiscal, as suggested by the recent experience in Greece. At the same time, while the literature has focussed on understanding QE after a financial shock, there is no comparable work on the role of QE when the source of the problems is fiscal.

This paper writes down a simple model of fiscal and monetary policy where, in normal times, QE is neutral. However, during a fiscal crisis, the central bank's management of its balance sheet can have a powerful effect on outcomes through two channels. First, the composition of the central bank's balance sheet alters the composition of the privately-held

[^1]public debt. In turn, this affects the sensitivity of inflation to fiscal shocks. After a fiscal shock that makes anticipated fiscal surpluses fall short of paying for the outstanding debt, the price level increases to lower the real value of the debt. The maturity of the combined government debt and bank reserves held by the public determines the size and time profile of this bout of inflation. This unexpected inflation lowers welfare directly through price dispersion, and indirectly via aggregate demand and output gaps. QE can reduce these welfare losses.

The second channel comes through the costs of default during a fiscal crisis. Banks suffer losses when government bonds are impaired, and with lower net worth, there is a credit crunch that causes a recession. Ex ante, in anticipation of this default, government bonds start paying a risk premium that lowers their ability to serve as safe collateral. Financial markets may freeze, due to an absence of safe assets, lowering output and welfare. QE can take the risk of default out the balance sheet of the banks and into the balance sheet of the central bank, reducing the extent of the credit crunch and increasing the effective supply of safe assets.

After studying these new virtues of QE, I turn to re-examining some of its common criticisms. Isn't QE just fiscal policy that could be done (and undone) by changes in the profile of issuance of bonds by the fiscal authority? Isn't QE stealth monetary financing of the deficit? Doesn't QE just delay needed fiscal reforms? Within the model of this paper, I show that the answer to these three questions is no. QE is not a substitute for Treasury policy, it does not generate a transfer of monetary-generated resources to the government, and it need not weaken the desirability of fiscal surpluses. I further show that these three criticisms would be valid for different monetary policies, but not for properly executed QE.

Finally, I describe some limitations of QE within the model. In particular, I show that QE may put the solvency of the central bank at risk, it induces a redistribution from households to banks, and it may require precise targeting of its purchases. These are serious limitations that hamper the ability of policymakers to use QE, in spite of the potentials described in this paper.

The model I use is of the standard new Keynesian variety, where monetary policy is not neutral, as surveyed in Woodford (2003), Gali (2008) and Mankiw and Reis (2010). Unlike the literature on QE so far, there is no explicit role for forward guidance in the model and there is perfect arbitrage between bonds of different maturities. I introduce financial frictions instead through banks and financial markets following on the footsteps of Gertler and Kiyotaki (2010), Bolton and Jeanne (2011), and Balloch (2015), and I model the
composition of the central bank's balance sheet and its solvency following Reis (2013b), Hall and Reis (2015) and Reis (2015). The response of inflation to a fiscal crisis uses the fiscal theory of the price level, and particularly builds on Cochrane (2001, 2014). ${ }^{3}$ The modelling of sovereign default in a closed economy builds on Uribe (2006).

## 2 A model of monetary policy and its roles

The goal of the model is to highlight economic channels qualitatively, rather than quantify their effect, so I make several stark assumptions that future research can relax. I start by presenting the two government agencies that are at the center of the study, the fiscal and monetary authorities, before discussing the private economy, made up of consumers, firms, and financial institutions.

### 2.1 The fiscal authorities

In reality, governments issue liabilities of many different kinds. However, central banks focus their operations on the more liquid government bonds, in part so that their value can be inferred from market prices, and in part so that they can be sold quickly if needed. Of these liquid assets, the more dominant are nominal bonds of different maturities. While the maturity structure of the outstanding public debt can be complex, in the model I simplify by considering only two types: short-term (1-period) and long-term (2-period) bonds. The face value of bonds outstanding at date $t$ that mature in one period is denoted by $b_{t}$ and they trade for price $q_{t}$. These include both long-term bonds issued at $t-1$ as well as short-term bonds just issued, since they are equivalent from the perspective of date $t$. Long-term bonds issued at date $t$ trade at price $Q_{t}$ and pay $B_{t}$ at $t+2$. These prices are in nominal units, and the price level is $p_{t}$.

The government has real expenditures of an exogenous amount $g_{t}$, and must fund them through two sources of revenue. The first are real dividends from the central bank $d_{t}$. The second are fiscal revenues from taxation, $f_{t}$, which the government can choose. I model the distortionary effects of taxation in a stark way: the fiscal authority can costlessly choose any $f_{t}<\bar{f}_{t}$ as a lump-sum revenue, but it is infinitely costly to generate higher fiscal surpluses. A perspective on the fiscal limit $\bar{f}_{t}$ is that it is the peak of the Laffer curve, the upper bound on what the government can collect from its citizens without leading to widespread tax evasion.

[^2]Combining these ingredients, the government flow budget is:

$$
\begin{equation*}
\delta_{t}\left(b_{t-1}+q_{t} B_{t-1}\right)=p_{t}\left(d_{t}+f_{t}-g_{t}\right)+q_{t} b_{t}+Q_{t} B_{t} . \tag{1}
\end{equation*}
$$

The variable $\delta_{t} \in[0,1]$ is the repayment rate at date $t$ on bonds that were outstanding. When $\delta_{t}=1$, debts are honored. Otherwise, $1-\delta_{t}$ is the haircut suffered by the bondholders.

In turn, the intertemporal budget constraint of the government is:

$$
\begin{equation*}
\left(\frac{\delta_{t}}{p_{t}}\right)\left(b_{t-1}+q_{t} B_{t-1}\right)=\mathbb{E}_{t}\left[\sum_{\tau=0}^{\infty} m_{t, t+\tau}\left(d_{t+\tau}+f_{t+\tau}-g_{t+\tau}\right)\right] . \tag{2}
\end{equation*}
$$

Future uncertain payoffs are priced by a stochastic discount factor $m_{t, t+\tau}$. Another way to write this constraint would be to state that the government debt cannot be a Ponzi scheme. ${ }^{4}$

Fiscal policy consists of picking surpluses $\left\{f_{t}\right\}$ and debt management $\left\{\delta_{t}, b_{t}, B_{t}\right\}$, which includes both whether and how much to repay old debts, as well as how to manage the maturity of outstanding debt.

### 2.2 The central bank

The central bank's main liability, and its main tool in implementing monetary policy in all modern central banks, is the amount of nominal reserves $v_{t}$. Reserves have four crucial features that make it special. First, they are held exclusively by banks, as only them can hold these deposits at the central bank. Second, they are default free, since the central bank commits to exchange them one to one at any time with currency, and the central bank is the sole issuer of currency. Third, they are the unit of account in the economy, so their real value is $1 / p_{t}$, and if the central bank tries to run a Ponzi scheme on reserves the price level will be infinity. And fourth and finally, they pay an interest rate that is set by the central bank at $i_{t}$. These four features of real-world reserves play an important role in the model, and I will come back to them many times.

These reserves finance the central bank's holdings of the two government bonds, $b_{t}^{c}$ and $B_{t}^{c}$, as well as the gap between the dividend that the central bank pays the fiscal authority, $d_{t}$, and the seignorage revenue it earns from issuing currency $h_{t}$ minus central bank expenses. While higher inflation affects seignorage, both by debasing its real value and by affecting the desire to hold currency, Hilscher, Raviv, and Reis (2014b) find empirically that the elasticity

[^3]of seignorage with respect to inflation is quite small. For simplicity, I let the central bank expenses vary with inflation as well, so that take seignorage net of expenses $s_{t}$ is exogenous. ${ }^{5}$

Combining all these ingredients, the flow budget of the central bank is:

$$
\begin{equation*}
v_{t}-v_{t-1}=i_{t-1} v_{t-1}+q_{t} t_{t}^{c}+Q_{t} B_{t}^{c}-\delta_{t}\left(b_{t-1}^{c}+q_{t} B_{t-1}^{c}\right)+p_{t}\left(d_{t}-s_{t}\right) \tag{3}
\end{equation*}
$$

The intertemporal constraint on the central bank that follows from the no-Ponzi scheme on reserves is:

$$
\begin{equation*}
\frac{\left(1+i_{t-1}\right) v_{t-1}-\delta_{t}\left(b_{t-1}^{c}+q_{t} B_{t-1}^{c}\right)}{p_{t}}=\mathbb{E}_{t}\left[\sum_{\tau=0}^{\infty} m_{t, t+\tau}\left(s_{t+\tau}-d_{t+\tau}\right)\right] \tag{4}
\end{equation*}
$$

On top of these resource constraints, the central bank also faces a solvency constraint in the form of a lower bound on the (possibly negative) dividends to the fiscal authorities. This is the case as long as the central bank does not have unlimited fiscal backing from the Treasury, which is the relevant case during a fiscal crisis (Hall and Reis, 2015). There are different possible types of insolvency for a central bank depending on the relationship between the central bank and the fiscal authority (Reis, 2015). Here I assume the weakest of these, intertemporal solvency: $\mathbb{E}_{t}\left(\sum_{\tau=0}^{\infty} m_{t, t+\tau} d_{t+\tau}\right) \geq 0$.

Given these constraints, monetary policy consists of choices of the interest rate paid on reserves $\left\{i_{t}\right\}$ and balance-sheet policies $\left\{v_{t}, h_{t}, b_{t}^{c}, B_{t}^{c}\right\}$. Some of these may follow rules, they do not need to be exogenous. The central bank could choose to issue zero reserves and hold zero bonds, while still setting interest rates and rebating seignorage in full every period. This is the typical case considered in studies of monetary policy (Woodford, 2003).

### 2.3 Households

A representative household has preferences given by:

$$
\begin{equation*}
\mathbb{E}_{t}\left[\sum_{\tau=0}^{\infty} \beta^{\tau}\left[u\left(c_{t+\tau}+g_{t+\tau}-\frac{l_{t+\tau}^{1+\alpha}}{1+\alpha}\right)\right]+\varkappa \ln \left(\frac{h_{t+\tau}}{p_{t+\tau}}\right)\right] \tag{5}
\end{equation*}
$$

where $u():. \mathbb{R}^{+} \rightarrow \mathbb{R}$ with $u^{\prime}() \geq$.0 and $u^{\prime \prime}() \leq$.0 is the household's utility function, $c_{t}$ is aggregate consumption and $l_{t}$ are hours worked. The particular functional form inside the utility function implies that private consumption and public services are perfect substitutes.

[^4]This simplification avoids dealing with how to value these different goods, which are not the subject of this paper, and makes households indifferent as to the size of government.

Another implicit assumption in the specification of preferences above is that there are no income effects on hours worked, and $\alpha$ is the inverse of the wage-elasticity of labor supply, since the optimal labor supply is:

$$
\begin{equation*}
l_{t}^{\alpha}=w_{t} / p_{t} \tag{6}
\end{equation*}
$$

where $w_{t}$ is the nominal wage rate.
Including currency in the utility function is likewise a shortcut way of introducing a demand for this liability of the central bank, since optimal behavior implies that:

$$
\begin{equation*}
\frac{h_{t}}{p_{t}}=\frac{1}{u^{\prime}(.)\left(1-q_{t}\right)} \tag{7}
\end{equation*}
$$

Higher consumption and lower interest rates on bonds both raise the amount of currency that agents want to hold.

Finally, since households can choose to hold the two bonds across periods, the standard no-arbitrage condition holds:

$$
\begin{equation*}
\mathbb{E}_{t}\left(\frac{m_{t, t+1} \delta_{t+1} p_{t}}{q_{t} p_{t+1}}\right)=\mathbb{E}_{t}\left(\frac{m_{t, t+2} \delta_{t+1} \delta_{t+2} p_{t}}{Q_{t} p_{t+2}}\right)=1 . \tag{8}
\end{equation*}
$$

The stochastic discount factor is, as usual, equal to the discounted marginal utility in the future as a ratio of marginal utility today: $m_{t, t+\tau}=\beta^{\tau} u^{\prime}\left(c_{t+\tau}+\ldots\right) / u^{\prime}\left(c_{t}+\ldots\right)$.

### 2.4 Firms

There is a single final good, produced by a competitive firm, that results from aggregating a continuum of varieties of goods:

$$
\begin{equation*}
y_{t}=\left(k_{t}^{\theta} \int_{0}^{k_{t}} y_{t}(j)^{\frac{\sigma-1}{\sigma}} d j\right)^{\frac{\sigma}{\sigma-1}} \tag{9}
\end{equation*}
$$

where $\sigma>1$ is the elasticity of demand for each variety since: $y_{t}(j)=\left(p_{t}(j) / p_{t}\right)^{-\sigma} k_{t}^{\theta \sigma} y_{t}$. The measure of varieties available for production is denoted by $k_{t} \in[0,1]$, with the strength of this "love for variety" or "congestion externality" determined by the parameter $\theta$. The associated static cost-of-living price index is: $p_{t}=\left[k_{t}^{\theta \sigma} \int_{0}^{k_{t}} p_{t}(j)^{1-\sigma} d j\right]^{1 /(1-\sigma)}$.

Each variety can be produced by a single monopolistic firm only if it has one unit of capital. Therefore the number of varieties is the same as the amount of capital employed in the economy, or the measure of firms in operation. ${ }^{6}$ Capital is available from banks for rental at price $1+r_{t}$, and because of free entry, this payment just covers the profits of firms:

$$
\begin{equation*}
1+r_{t}=\max _{p_{t}(j)}\left\{\left(p_{t}(j) y_{t}(j)-w_{t}\right) / p_{t}\right\} . \tag{10}
\end{equation*}
$$

The firm's maximization is subject to the demand for the good and to the technology: $y_{t}(j)=a_{t} l_{t}(j)$, where $l_{t}(j)$ is the labor used and $a_{t}$ is productivity. The labor market clears: $l_{t}=\int l_{t}(j) d j$. Standard calculations show that the desired optimal price is a constant markup over marginal cost. Following the usual assumption in the literature, I assume that there is a constant sales subsidy that offsets the monopoly distortion so that the desired price just equals marginal cost:

$$
\begin{equation*}
p_{t}^{*}=w_{t} . \tag{11}
\end{equation*}
$$

The final constraint to the firm's problem is that only a fraction $\lambda$ of firms can choose their price equal to their desired level. The remainder must choose their prices with oneperiod old information, and set them approximately equal to $\mathbb{E}_{t-1}\left(p_{t}^{*}\right) .^{7}$ Because all firms are ex ante identical, this price dispersion is an inefficiency that leads to under-production. Aggregating across firms:

$$
\begin{align*}
& y_{t} k_{t}^{\frac{1+\sigma \theta}{1-\sigma}} \Delta_{t}=a_{t} l_{t}  \tag{12}\\
& \Delta_{t} \equiv k_{t}^{\frac{1+\sigma \theta}{1-1 / \sigma}}\left(\frac{p_{t}^{*}}{p_{t}}\right)^{-\sigma}\left[\lambda+(1-\lambda)\left(\frac{\mathbb{E}_{t-1}\left(p_{t}^{*}\right)}{p_{t}^{*}}\right)^{-\sigma}\right] \tag{13}
\end{align*}
$$

so that $\Delta_{t} \geq 1$, which is minimized at 1 when $\mathbb{E}_{t-1}\left(w_{t}\right)=w_{t}$ so all prices are the same, is a measure of price dispersion.

[^5]
### 2.5 Capital and the interbank market

Every period, the economy receives an endowment of capital of one unit that fully depreciates within the period. This capital can either be turned one-to-one into consumption, or used to produce varieties of goods. Either way, since there is no storage technology or any way to transfer resources over time, the market clearing condition for goods is:

$$
\begin{equation*}
c_{t}+g_{t}+k_{t}=y_{t}+1 \tag{14}
\end{equation*}
$$

However, capital must make its way to the firms through a financial system with frictions. In period $t-1$, the ownership of next period's capital is split with a fraction $\kappa$ belonging to banks, while the remaining $1-\kappa$ belongs to households. Banks are randomly drawn i.i.d. from the household every period, so that they only effectively operate separately from the household for two periods. ${ }^{8}$

In their first period of life, $t-1$, banks meet each other in an interbank market, similar to the one in Bolton and Jeanne (2011) and Balloch (2015). In particular, only a fraction $\omega$ of the banks will be matched with with firms and productive possibilities next period, while the remaining $1-\omega$ can only trade in financial markets. Banks without opportunities could sell their claim on capital for a reserve at the central bank or for a on-period bond. Alternatively, they could lend to the productive banks in the interbank market, and since they are perfectly competitive, the interest rate in that market is $1+i_{t-1}$. The feasibility constraint on interbank lending is: $x_{t} \leq(1-\omega) \kappa$.

Aside from their ownership of claims on capital, the good banks can also buy government bonds or reserves. In equilibrium, they would not want to do so, since the return on lending the capital out to firms is higher. ${ }^{9}$ Yet, with the amount $x_{t}$ they receive in interbank loans, the good banks can only commit to repay back a share $\xi \leq 1$. Between periods, they can run away with a share $1-\xi$ of the interbank loans, and this action is not verifiable by the other banks, so they cannot stop it. However, running away leads the bank to lose ownership of its marketable assets, which the creditors can seize. Therefore, the incentive constraint for them to repay the interbank loans is:

$$
\begin{equation*}
(1-\xi) x_{t} \leq q_{t-1} b_{t-1}^{p}+v_{t-1} \tag{15}
\end{equation*}
$$

[^6]where $b_{t-1}^{p}$ are the bond holdings by borrowing banks, which can be sold next period. With a large $\xi$, the relevant case, banks hold only little collateral allowing them to borrow large amounts in interbank markets. ${ }^{10}$

### 2.6 Deposits and credit

At the start of the period $t$, all uncertainty is realized, and the productive banks find themselves itself with $\omega \kappa+x_{t}$ claims on capital available. At the same time, the collateral that they held is now worth only $\delta_{t} b_{t-1}^{p}+\left(1+i_{t-1}\right) v_{t-1}$. Because interbank markets are senior to all other claims, the good banks must pay their debt in this market and their net worth after doing so $n_{t}=\omega \kappa_{t}-b_{t-1}^{p}\left(1-\delta_{t}\right)$.

The banks can then raise more funds in the deposit market, where they deal with households, and I model this following Gertler and Kiyotaki (2010). Deposits, $z_{t}$, face a feasibility constraint $z_{t} \leq 1-\kappa$. Since the households can transform their capital into consumption one-for-one and they behave competitively, the return on deposits is $1 .{ }^{11}$

Aside from feasibility though, there is also an incentive constraint. Banks can only pledge a share $\gamma$ of their revenues, but can abscond with the remainder. For them to choose not to do so, the return from paying depositors in full must exceed that from absconding:

$$
\begin{equation*}
\left(1+r_{t}\right)\left(n_{t}+z_{t}\right)-z_{t} \geq(1-\gamma)\left(1+r_{t}\right)\left(n_{t}+z_{t}\right) \tag{16}
\end{equation*}
$$

The combined effect of the two frictions, in interbank and deposit markets, is that the unit of capital available in the economy will end up funding only $k_{t}$ projects, which is the sum of the capital in productive banks, the capital raised in interbank markets, plus the amount raised as deposits net of losses in the collateral portfolio.

### 2.7 Equilibrium

An equilibrium is a collection of outcomes in goods markets $\left\{c_{t}, y_{t}, y_{t}(j), p_{t}, p_{t}(j)\right\}$, in labor markets $\left\{l_{t}, l_{t}(j), w_{t}\right\}$, in the credit, deposit and interbank markets $\left\{r_{t}, k_{t}, x_{t}, z_{t}, b_{t}^{p}\right\}$, and in bond markets $\left\{q_{t}, Q_{t}\right\}$, such that all agents behave optimally and all markets clear, and

[^7]given exogenous processes for $\left\{\bar{f}_{t}, s_{t}\right\}$ together with choices for fiscal policy $\left\{f_{t}, \delta_{t}, b_{t}, B_{t}\right\}$ and monetary policy $\left\{i_{t}, z_{t}, v_{t}, b_{t}^{c}, B_{t}^{c}\right\}$.

If there is no default and the incentive constraints in the capital markets do not bind so that all capital is employed, then this is a standard new Keynesian model. On the side of the private sector, production follows the standard Dixit-Stiglitz setup, and the introduction of nominal rigidities implies that, after log-linearization, there is a Phillips curve linking output to the surprises in the price level $p_{t} / E_{t-1}\left(p_{t}\right)$ and an Euler equation linking output growth to the real interest rate. On the side of the government, the budgets of the fiscal and monetary authorities can be collapsed into one, and Ricardian equivalence holds, so bond holdings or reserves have no effect on equilibrium. With a standard rule for interest rates, like the Taylor rule, together with choosing currency supply to accommodate its demand, and a Ricardian policy of choosing fiscal surpluses to pay for existing debts, then the price level would be pinned down irrespective of the central bank's balance sheet. This a completely conventional economy, and the choices of reserves and bond holdings by the central bank $\left\{v_{t}, b_{t}^{c}, B_{t}^{c}\right\}$ have no effect on equilibrium outcomes.

## 3 Fiscal crisis and quantitative easing

The model of the previous section could be solved quantitatively using numerical tools, and the effect of different balance-sheet policies could be derived in response to shocks to fiscal capacity, as well as to productivity, government spending or monetary shocks. Leaving that for future research, the approach that I will take is instead to simplify the uncertainty in the model to focus on fiscal crises and on a restricted set of policies, QE.

### 3.1 The fiscal crisis

A fiscal crisis is an exogenous contraction in $\overline{f_{t}}$. For simplicity, this the only source of uncertainty in the economy. That is, I assume that productivity, seignorage, and government spending are all know and, without loss generality, they are constant at $a, s, g$, respectively.

However, at date 0 everyone unexpectedly learns that the fiscal limit at date 1 may be lower. At all other dates but 1 , the fiscal limit is equal to a constant $\bar{f}$. But at date 1 , with probability $1-\pi, \bar{f}_{t}=\bar{f}-\phi$, while otherwise $\bar{f}_{t}=\bar{f}$. The two key parameters describing the fiscal crisis are then $\pi$, capturing its likelihood from the perspective of date 0 , and $\phi$ capturing the severity of the crisis at date $1 .{ }^{12}$

[^8]Moreover, the following holds:
Assumption 1: The initial conditions on government liabilities are:

1. $v_{-1} / \beta+b_{-1}-b_{-1}^{c}+\beta\left(B_{-1}-B_{-1}^{c}\right) \leq(\bar{f}+s-g) /(1-\beta)$.
2. $\beta \phi \geq(\bar{f}+s-g) /(1-\beta)-\left[v_{-1} / \beta+b_{-1}-b_{-1}^{c}+\beta\left(B_{-1}-B_{-1}^{c}\right)\right]$.

The first assumption on the initial debt ensures that, bar a fiscal crisis, the government is able to pay its debts.

The second assumption on the size of the fiscal shortfall ensures that in a fiscal crisis, the shortfall in revenue relative to spending is such that it will require a default in public debt or an increase in inflation to restore the intertemporal budget constraint of the government. A fiscal crisis arises when, at date 0 , all learn that there may be a fall in feasible fiscal revenues at date 1 . As long as the size of the crisis is large enough, or $\phi$ is sufficiently large, an outcome when there is no default and price stability is impossible in equilibrium.

### 3.2 Quantitative easing

The focus of this paper are quantitative easing policies, which consist of changes in the balance sheet such that: $\hat{v}_{t}=q_{t} \hat{b}_{t}^{c}+Q_{t} \hat{B}_{t}^{c}$. Quantitative easing policies are of the twin choices by the central bank of how many reserves to issues, and what maturity of government bonds to acquire with them.

With the focus on QE, for the other monetary policy choices, I make conventional assumptions. First, I assume that the central bank would like to choose its interest rate policy to be consistent with a price level target of $p_{t}=1$. It will do so as long as it can, that is as long as this is consistent with the existence of an equilibrium. It is well known that in this class of models, a price-level target is optimal (Reis, 2013a), and setting it to the constant 1 is just a useful simplification. Given this policy, I further assume that the initial conditions for prices and expectations are: $\mathbb{E}_{-1}\left(p_{0}\right)=p_{-1}=1$.

Second, I assume that the central bank chooses the supply of currency $z_{t}$ in order to accommodate the demand for it in equation (7) in a way that is consistent with its price-level target. No modern central bank uses the supply of currency as its operating tool. Section 7 will relax this assumption. Finally, throughout the analysis, I will take the government't choice of debt policy $b_{t}, B_{t}$ as given, and re-examine the role of this assumption in section 7 as well.
dynamics of the fiscal crisis and QE, but here I focus on the qualitative channels.

### 3.3 Final set of assumptions

With risk averse agents, the maturity of the government liabilities held by the private sector could be used to provide insurance against the risk of inflation (e.g. Lustig, Sleet, and Yeltekin, 2008). While this is an interesting channel, it has both been explored in the literature on the optimal maturity of government debt, and it is likely not very relevant to reserves, which in reality are overnight liabilities. To separate the new effects studied in this paper from this provision of insurance, I assume that agents are risk neutral, so the $u($. function is linear.

This assumption also implies that the stochastic discount factor is constant: $m_{t, t+\tau}=\beta^{\tau}$, so that the equilibrium can be solved exactly, without needing to approximate the equilibrium relations because of risk premia. Moreover, linearity implies that welfare is proportional to the amount of output produced in the economy, making it easier and more transparent to solve for optimal policies.

A final set of parameter restrictions makes sure that we focus on equilibria of the model that are interesting for the question in this paper:

Assumption 2: The following parameter restrictions hold:

1. $(a / \sigma)(1-1 / \sigma)^{1 / \alpha} \geq 1$
2. $\theta=(1-1 / \sigma) /(1+1 / \alpha)-1 / \sigma$.

The first assumption guarantees that productivity is high enough so that it is socially optimal to use capital for production rather than consumption.

The second assumption is more peculiar and deserves some explanation. It fixes the love for variety as a precise formula of the elasticities of product demand and labor supply. It is well known that in monopolistic competition models like this one, the profits of an individual firm may increase or decrease as more firms enter the market. On the one hand, as more firms enter, the demand for each variety declines, keeping total spending fixed. On the other hand, total output increases because of an externality, since more varieties produced raise overall welfare and demand for all goods. The parameter $\theta$ controls the strength of this second effect relative to the first. Making this particular assumption on its value results in the return on each firm earned by the bank, $r_{t}$ being constant. This simplifies the analysis considerably, while losing little in terms of the generality of the results. ${ }^{13}$

[^9]
### 3.4 Welfare

The appendix proves the following result:
Lemma 1. Letting $y_{t}^{*}=a^{(1+1 / \alpha)}$ be the first-best level of output, and $k^{*}=1$ be the first best level of capital, welfare in this economy at date $t$ is:

$$
\begin{equation*}
\mathbb{E}_{t} \sum_{\tau=0}^{\infty} \beta^{\tau} y_{t+\tau}^{*}\left[\frac{y_{t+\tau}-k_{t+\tau}+k^{*}}{y_{t+\tau}^{*}}-\left(\frac{y_{t+\tau}}{y_{t+\tau}^{*}}\right)^{1+\alpha} \frac{\Delta_{t+\tau}^{1+\alpha}}{1+\alpha}\right] . \tag{17}
\end{equation*}
$$

The first best is achieved when $k_{t}=1$ and $\Delta_{t}=1$. If prices are flexible, then welfare every period is an increasing linear function of $k_{t}$. If capital markets are efficient, so $k_{t}=1$, then stabilizing prices so that $E_{t-1} w_{t}=w_{t}$, achieves the "divine coincidence" of both eliminating price dispersion $\left(\Delta_{t}=1\right)$ and the output gap $\left(y_{t}=y_{t}^{*}\right)$.

Welfare in this economy depends on the total amount of output that is produced, and this in turn depends on how much capital is available to firms and on well allocated is labor across them. When all capital is employed $\left(k_{t}=1\right)$ and there are no surprises distorting the setting of prices by firms $\left(\mathbb{E}_{t-1}\left(w_{t}\right)=w_{t}\right.$ so $\left.\Delta_{t}=1\right)$, welfare is maximized. Lower interbank lending lowers output by locking capital in banks that do not have access to lending opportunities. Lower deposits lower output by locking capital in households. Both prevent valuable capital from reaching the firms that could put it to productive use. ${ }^{14}$ Unexpected inflation lowers output by making some firms make pricing mistakes, inducing price dispersion and a misallocation of labor.

If there was no collateral constraint in interbank markets, no skin-in-the game constraint in the deposit market, and no price rigidity, then this economy would achieve the first best (subject to the monopolist distortion). That is, if $\lambda=\xi=\gamma=1$, the equilibrium is efficient, regardless of fiscal and monetary policy. Otherwise, after a fiscal shock, welfare may be lower because of the three frictions in the economy: nominal rigidities by firms leading to price dispersion, credit frictions in collecting funds from depositors, and liquidity frictions in the interbank market.

[^10]
## 4 The neutrality of QE

The goal of this paper is to investigate the circumstances under which QE may have an effect. It is useful to start from the opposite perspective, when QE is neutral. Within the model in this paper, there are two benchmarks of neutrality that correspond to the bulk of the literature on central bank balance sheets.

### 4.1 QE in normal times

The case where there is no fiscal crisis corresponds to $\phi=0$, so that the fiscal limit is unchanged in period 1 (and the probability of a crisis, $\pi$, becomes irrelevant). Still, the fiscal authority must choose a combination of fiscal surpluses and debt management $\left\{f_{t}, b_{t}, B_{t}\right\}$, while the monetary authority chooses the size and composition of its balance sheet $\left\{v_{t}, b_{t}^{c}, B_{t}^{c}\right\}$.

Proposition 1. If the fiscal authority chooses $f_{t}$ so that $f_{t}=(1-\beta)\left(v_{-1} / \beta+b_{-1}-b_{-1}^{c}+\right.$ $\left.\beta B_{-1}-\beta B_{-1}^{c}\right)-s+g$ at all dates, and issues enough bonds $\beta b_{t} \geq(1-\xi)(1-\omega) \kappa$ at all dates, then the economy reaches the efficient outcome.

Intuitively, the fiscal authority chooses fiscal surpluses to pay for the debt preventing any default, and issues enough bonds so that the interbank market can function. Since there no shocks, there are no price surprises and no price dispersion. Therefore, the economy reaches the first best.

Noticeably, proposition 1 does not mention anywhere what the QE policy of the central bank is. The equilibrium is independent of $\left\{v_{t}, b_{t}^{c}, B_{t}^{c}\right\}$. QE is neutral in normal times.

Why is this the case? When the central bank buys government bonds with reserves, it is only exchanging one type of government liability for another. Short-term bonds and reserves are perfectly equivalent, as are long-term bonds since they can be traded next period with no risk. Therefore, QE has no effect on the solvency of the government, on its ability to pay for is debts, and therefore on the price level or on the likelihood of default.

Mathematically, this can be seen because the consolidated government budget constraint only includes reserves in a term: $\left(1+i_{t-1}\right) v_{t-1}+b_{t-1}-b_{t-1}^{c}+q_{t}\left(B_{t}-B_{t}^{c}\right)$. Any QE policy consist of exchanging reserves for government bonds. But, the arbitrage condition linking $i_{t}$, $q_{t}$ and $Q_{t}$ ensures that such a purchase would leave this expression unchanged.

In turn, in credit markets, reserves can be equally used as government bonds in the interbank market. If banks have enough short-run bonds to satisfy their needs in interbank markets, then no supply of reserves in exchange for these bonds has an effect on the credit
constraints. Again, the two types of government liabilities are perfect substitutes, so that QE has no effect on the amount of credit in the economy or on the efficiency with which capital is allocated.

### 4.2 Wallace neutrality

Another useful benchmark applies even to circumstances when there is a fiscal crisis. As emphasized by Wallace (1981) and Eggertsson and Woodford (2003), short-term bonds and reserves are just two forms of government liabilities. Each is denominated in nominal terms, and promises a certain nominal return next period. Therefore, one would expect that open market operations, which trade reserves for short-term bonds, have no effect on equilibrium outcomes. In turn, trading reserves for long-term government bonds is likewise neutral since with frictionless markets, arbitrage across maturities ensures that the relative quantities outstanding also do not matter. QE policies and the central bank balance sheets do not matter.

This is no longer the case if the fiscal crisis can lead to default. Short-term government bonds and reserves are not perfect substitutes once one allows for default, and neither are long-term bonds. Central banks never default on reserves. Reserves can only be redeemed for currency, but the central bank can both print currency and expand or retire reserves at will, so there is never a need to default on reserves. Short-term government bonds, on the other hand, can and are defaulted on. Therefore, when default is possible, open market operations will, in general, affect equilibrium, including the choice by the authority to default or not.

If there is no default, so $\delta_{t}=1$ at all dates, then it is easy to check that in all the equilibrium conditions in the model only the sum $v_{t}+q_{t} b_{t}$ appears even in the period of a fiscal crisis. Therefore, exchanging reserves for short-terms bonds is neutral. Yet, as the next section will show, exchanging reserves for long-term government bonds does have an effect, even when there is no default.

## 5 The effect of QE on inflation and aggregate demand

Given a fiscal crisis, the government must choose the level of taxation $f_{t}$. While either default or inflation are inevitable, higher tax collections will lower either of these and raise welfare. Therefore, the fiscal authority optimally chooses $f_{t}=\bar{f}_{t}$, and the intertemporal consolidated
government budget constraint becomes:

$$
\begin{equation*}
\frac{\left(1+i_{t-1}\right) v_{t-1}+\delta_{t}\left[b_{t-1}-b_{t-1}^{c}+q_{t}\left(B_{t-1}-B_{t-1}^{c}\right)\right]}{p_{t}}=\left[\sum_{\tau=0}^{\infty} \beta^{\tau}\left(\bar{f}_{t+\tau}-g+s\right)\right] . \tag{18}
\end{equation*}
$$

Aside from taxation, the fiscal authorities can also opt for one of two regimes (or an in between). Either they commit to never defaulting on government bonds, do $\delta_{t}=1$ at all dates, and the central bank has to choose the nominal interest rate to adjust to the resulting path for the price level. This is sometimes called a fiscal dominance, or non-Ricardian policy regime. Or, the central bank stays committed to the price level target but the fiscal authority defaults on its bonds. In this section, I analyze the former case, while the next section covers the latter. ${ }^{15}$

From period 2 onwards, after the crisis is over, the analysis is straightforward and similar to the case in the precious section. Recalling that $p_{t}=1$ from period 2 onwards, equation (18) at date 2 becomes:

$$
\begin{equation*}
\frac{v_{1}}{\beta}+b_{1}-b_{1}^{c}+\beta\left(B_{1}-B_{1}^{c}\right)=p_{2}\left(\frac{\bar{f}+s}{1-\beta}\right) \tag{19}
\end{equation*}
$$

If the central bank issues one more unit of reserves, it buys with it either $1 / \beta$ more short-term bonds or $1 / \beta^{2}$ more long-term bonds, since $\beta$ and $\beta^{2}$ are the equilibrium prices of short-term and long-term bonds, respectively. Therefore, QE leaves the left-hand side of this equation unchanged and so any QE policy is consistent with the price level target of $p_{2}=1$. Because $v_{1}$ is chosen during the crisis, at date 1 , this neutral QE happens while the economy is experiencing the fiscal shock.

### 5.1 The non-neutrality of QE during the crisis

Let $p_{1}^{\prime}$ denote the price level if there is a crisis and $p_{1}^{\prime \prime}$ if there is no crisis. They are pinned down by the two equations describing the consolidated government budget constraint at date

[^11]1 in the two states of the world:

$$
\begin{align*}
& \frac{\left(1+i_{0}\right) v_{0}+b_{0}-b_{0}^{c}}{p_{1}^{\prime}}+\beta\left(B_{0}-B_{0}^{c}\right)=\frac{\bar{f}+s}{1-\beta}-\phi  \tag{20}\\
& \frac{\left(1+i_{0}\right) v_{0}+b_{0}-b_{0}^{c}}{p_{1}^{\prime \prime}}+\beta\left(B_{0}-B_{0}^{c}\right)=\frac{\bar{f}+s}{1-\beta} \tag{21}
\end{align*}
$$

It follows right away that $p_{1}^{\prime}>p_{1}^{\prime \prime}$. Prices rise if there is a crisis. Intuitively, if the fiscal capacity falls, the government is no longer able to service its debt. Surprise inflation is the only way to lower its nominal value, so prices must rise in that state of the world relative to the alternative.

It also follows from these two equations that a form of QE that issues reserves to buy short-term government bonds will have no effect on the price level. Since, by arbitrage, $1+i_{0}=1 / q_{0}$, these purchases leave the left-hand side of the two equations unchanged. They exchange one short-term government liability for another, and Wallace neutrality applies.

If the purchases are of long-term government bonds, the picture changes. Since we just found that QE in the form of short-term bonds is neutral, let QE take place solely for buying long-term bonds: $\hat{v}_{0}=Q_{0} \hat{B}_{0}^{c}$. Then, subtracting the budget constraints at date 1 for these two states of the world reveals that:

$$
\begin{equation*}
\hat{v}_{0}\left[\frac{1}{p_{1}^{\prime \prime}}-\frac{1}{p_{1}^{\prime}}\right]=\phi \tag{22}
\end{equation*}
$$

The larger is the balance sheet of the central bank, the smaller is the dispersion of inflation across the two states of the world. The composition of QE is crucial though. It is only when the central bank issues reserves to buy long-term bonds that it lowers the dispersion of prices.

### 5.2 QE and expected prices

If instead of subtracting the budget constraints at the two states in period 1 , one multiplies them by their probabilities and adds them:

$$
\begin{equation*}
\left(v_{0}+b_{0}-b_{c}^{0}\right) \mathbb{E}_{0}\left(\frac{1}{p_{1}}\right)+\beta\left(B_{0}-B_{0}^{c}\right)=\frac{\bar{f}+s}{1-\beta}-\phi(1-\pi) . \tag{23}
\end{equation*}
$$

The deeper and more likely the crisis, the larger has to be the increase in expected prices. However, the increase in $\mathbb{E}_{0}\left(1 / p_{1}\right)$ is independent of QE. Because the inverse function is
convex, then larger QE that lowers the dispersion of prices will lower expected inflation as well (but not affect its inverse).

Turning then to the date 0 version of equation (18), it is:

$$
\begin{equation*}
\frac{v_{-1} / \beta+b_{-1}-b_{-1}^{c}}{p_{0}}+\beta\left(B_{-1}-B_{-1}^{c}\right) \mathbb{E}_{0}\left(\frac{1}{p_{1}}\right)=\frac{\bar{f}+s}{1-\beta}-\beta \phi(1-\pi) \tag{24}
\end{equation*}
$$

The prospect of a crisis next period raises prices at date 0 , relative to when no crisis is foreseen $(\pi=1)$. Yet, since $\mathbb{E}_{0}\left(1 / p_{1}\right)$ is independent of QE , so is $p_{0}$. The central bank's policies at date 0 will not affect the price level then. All they do is affect the price level next period, but only in its sensitivity to the fiscal shock occurring.

### 5.3 Intuition and desirability of QE

Why is QE at date 0 not neutral on inflation at date 1? A fiscal crisis is a time when, unable to raise surpluses and unwilling to default, the only path for the fiscal authority it to erode of the the real value of the nominal debt payments coming due via inflation. The maturity of the public debt held by private agents is the key determinant of by how much does surprise inflation lower the real value of the debt (Hilscher, Raviv, and Reis, 2014a). When the central bank buys long-term bonds, it shortens the maturity of the debt held by the public, so it makes more of the debt coming due. Thus, a smaller price increase is necessary to bring the real value of the outstanding debt in line with projected fiscal surpluses. While fiscal policy determines that inflation must happen, monetary policy can affect its time profile via QE.

In terms of welfare, without default, the capital markets function efficiently and all of the existing capital gets allocated to investment. Therefore, we are in the $k_{t}=1$ case of the welfare lemma, and policy should focus on minimizing price surprises in order to keep output closer to its efficient level and minimize the misallocation due to inflation. According to the model, the central bank should then engage in QE by issuing reserves and buying long-term bonds. This will bring on price stability and reduce the output gap, via aggregate demand.

## 6 QE, default and credit freezes

The previous section assumed that the fiscal authority stayed committed to never defaulting. This section takes the other extreme case, where prices stay on target so $p_{t}=1$ at all dates,
but default is inevitable. I start by characterizing the impact of QE on default, and then proceed to analyze its effects on credit and welfare.

### 6.1 QE and the size of default

The consolidated intertemporal budget constraint of the government now pins down the extent of default. Starting with period 2 onwards, for there to not be default, the debt inherited from before, $b_{1}+\beta B_{1}$, must satisfy:

$$
\begin{equation*}
v_{t-1} / \beta+b_{t-1}-b_{t-1}^{c}+\beta\left(B_{t}-B_{t}^{c}\right)=\frac{\bar{f}+s}{1-\beta} \tag{25}
\end{equation*}
$$

Clearly, QE policy from period 1 onwards has no effect, since it leaves the left-hand side if the equation unchanged.

In period 1, we may or may not have a crisis. If there is no crisis, the government would neither like to default that period, nor have too little debt since this would imply a larger default the previous period. Therefore, the government liabilities coming due in period 1 are pinned down by:

$$
\begin{equation*}
\left.v_{0} / q_{0}+b_{0}-b_{0}^{c}+\beta\left(B_{0}-B_{0}^{c}\right)\right]=\frac{\bar{f}+s}{1-\beta} \tag{26}
\end{equation*}
$$

If instead there is a crisis, the same equation for the other state of the world pins down the extent of default in that state of the world:

$$
\begin{equation*}
\delta_{1}=1-\frac{\phi}{\frac{\bar{f}+s}{(1-\beta)}-\frac{v_{0}}{\beta}} . \tag{27}
\end{equation*}
$$

The higher is the fiscal loss, the larger the extent of default.
QE lowers the recovery rate in period 1 , and this is the case whether it comes with buying short-term or long-term government bonds. This is perhaps not surprising. Since central banks do not default on reserves, but fiscal authorities do, substituting one for the other will affect how much default per bond must happen to bring the fiscal situation into balance. More interesting, while QE changes the recovery rates on debt, the size of the transfer from bondholders to the government does not change. The extend of default in real terms is the same but, since there are fewer government bonds in private hands, each must pay back less to its holder. Therefore, QE provides no fiscal relief.

Finally, turning to period 0 , the extent of default is:

$$
\begin{equation*}
\delta_{0}=\min \left\{\frac{\frac{\bar{f}+s}{1-\beta}-\beta(1-\pi) \phi}{v_{-1} / \beta+b_{-1}-b_{-1}^{c}+\beta\left(\pi+(1-\pi) \delta_{1}\right)\left(B_{-1}-B_{-1}^{c}\right)}, 1\right\} \tag{28}
\end{equation*}
$$

QE only affects this quantity through the recovery rate at date 1. Intuitively, QE affects the recovery rate on bonds held in period 1 , which affects their price at date 0 . If the fiscal crisis was unexpected $(\pi=1)$, then there would be no default at date 0 , nor any effect of QE at this date. More generally, the more unlikely the fiscal crisis, the less likely that there will be any default at date 0 . Whereas default at date 1 depends on the size of the crisis, default at date 0 depends on the expectation that there will be a crisis in the future.

To conclude, QE can alter the size of the default per bond, but not its total size. It does not alter the flow of resources that this generates from the private to the public sector.

### 6.2 QE and ex post bank losses

In this subsection, I consider the case where $\pi \rightarrow 1$, so that default only happens at at date 1 and is unexpected. The next subsection will focus on the effects of $\pi<1$.

Unexpected default has two effects on credit. The first one comes because banks held government bonds $b_{0}^{p}$ as collateral in the interbank market. When they repay these loans in period 1 , they suffer a loss of $b_{0}^{p}\left(1-\delta_{1}\right)$, and this amount of capital finds itself in the hands of the unproductive banks unavailable for credit. This is a transfer from the banking sector to the fiscal authority as a whole, which lower the capital available for production. The second effect comes because this loss lowers the net worth of banks. The incentive constraint in the deposit market in equation (16) therefore tightens since banks have less "skin in the game" so they cannot raise as many deposits.

Combining these two effects, the amount of capital invested in the economy is:

$$
\begin{equation*}
k_{1}=\min \left\{\kappa-b_{0}^{p}\left(1-\delta_{1}\right)+\left(\frac{\gamma(1+r)}{1-\gamma(1+r)}\right)\left[\omega \kappa-b_{0}^{p}\left(1-\delta_{1}\right)\right], 1\right\} . \tag{29}
\end{equation*}
$$

If the repayment rate in period 1 is low enough that $k_{1}<1$ then after a default, the higher are the bond holdings of the banks, the lower is the capital invested.

QE can affect this amount. In the model, bonds are only held by banks across periods in order for the interbank market to work. But because bonds and reserves are perfect substitutes as forms of collateral, and only the good banks hold reserves, then when the
supply of reserves increases, the bond holdings by banks decline. Therefore, higher $v_{0}$ lowers $b_{0}^{p}$ one to one, and therefore raises capital invested, output, and welfare. This effect of QE happens regardless of whether QE is executed by buying short-term of long-term government bonds. Only the size of the central bank's balance sheet matters, and the larger it is, the lower the losses by banks.

### 6.3 QE and ex ante freezes due to default

Consider now the effect of $\pi<1$. While in the previous subsection, the binding constraint on lending worked through the deposit market, the focus now is on the interbank market, where loans are constrained by:

$$
\begin{equation*}
x_{1} \leq\left(\frac{\beta}{1-\xi}\right)\left[\left(\pi+(1-\pi) \delta_{1}\right) b_{0}^{p}+v_{0}\right] . \tag{30}
\end{equation*}
$$

The larger is the fiscal crisis (higher $\phi$ ), the lower is $\delta_{1}$ from equation (27) and the lower the right-hand side. The more likely a fiscal crisis (lower $\pi$ ), the lower the right-hand side. Both increase the risk premium on government bonds and make it more likely that this constraint binds, interbank lending is lower than the amount of capital in the hands of unproductive banks, and there is too little investment, output and welfare.

To relax this constraint, banks would have to buy more bonds $b_{0}^{p}$ to pledge as collateral. Through the mechanism discussed in the previous subsection, ex post losses will be larger as a result. At the same time, the amount of bonds that banks can hold is bound above by the supply of government bonds: $b_{0}^{p} \leq b_{0}$. In this case, there are not enough safe assets to serve as the collateral needed to sustain the functioning of the financial system, and the credit market freezes. ${ }^{16}$

QE can make up for the shortfall. While issuing reserves to buy short-term bonds would make no difference, QE that buys long-term bonds relaxes the constraint. More reserves allow for more financial transactions to take place between financial intermediaries, which in turn allows more capital to be allocated to those that have good investment opportunities. This boosts credit, investment, output and welfare. The fact that the composition of QE matters is driven by the assumption that only short-term bonds are used in interbank markets, so only they show up in the incentive constraint above. While in the model maturity is tightly

[^12]linked to safety, all that matters is that the central bank can use QE to buy some assets that are risky, and not those that are equally safe. But, as long as the demand for safe assets by banks exhaust all of the government bonds available that can fulfill this role, and that there are other assets that QE can buy so that it can on net create safe assets by issuing reserves, the same results would hold. ${ }^{17}$

## 7 Common criticisms of QE

So far, this paper has identified two channels, one through aggregate demand and inflation, and the other through the credit market, by which QE can have a positive effect on welfare during a fiscal crisis. Yet, popular discussion of QE often mention three pitfalls to QE, which I now discuss.

### 7.1 QE versus Treasury debt management

A common criticism of QE policies is that their sole effect is to change the time profile of the overall government's obligations towards the public. Since the Treasury could do this itself, by changing the maturity of its bond issuances, QE would be unnecessary. This is supported by(Greenwood, Hanson, Rudolph, and Summers, 2014) documenting that when the Federal Reserve extended the maturity of its bond portfolio after 2008, the U.S. Treasury increased the average maturity of the stock of public debt partly offsetting QE. ${ }^{18}$

In terms of the model in this paper, the fiscal authority chooses $\left\{b_{t}, B_{t}\right\}$. The question in the model is whether this choice allows the Treasury to achieve the same outcomes as the central bank choosing $\left\{i_{t}, v_{t}, b_{t}^{c}, B_{t}^{b}\right\}$ regardless of what the central bank does. The answer is no.

In terms of the effect of the fiscal crisis on inflation, studied in section 5, the Treasury's choices of debt issuance could substitute for the QE policies of the central bank. Fearing a near fiscal crisis, in period 0 , the fiscal authority could change the relative issuance of short-term and long-term bonds to affect price dispersion and inflation. Yet, this cannot be achieved regardless of the central bank. The monetary authority must set an interest rate consistent with the debt issuance policy to achieve this path for inflation. At an extreme

[^13]example, if the monetary authority keeps the nominal interest rate fixed at all dates, say by $i_{t}=\beta$, then after the fiscal crisis, the price level will be permanently higher. In that case, the value of bonds of all maturities falls by exactly the same proportional amount and the maturity of the debt is irrelevant. QE is effective because the central bank can coordinate it with its interest-rate policy, but the fiscal authority cannot do this by itself.

Turning to the effect of the fiscal crisis on credit markets, QE is even more clearly distinct from debt management. Even though reserves and short-term bonds can both be used as collateral, they are different in three key ways that determine their effectiveness. The first is that reserves are default free, unlike government bonds. If the government increases the supply of bonds, the recovery rate per bond will fall, but the overall loss to the private sector stays the same. Therefore, issuing reserves the central bank gives banks a tool with which to shield their net worth, and so credit, from default. Bonds offer no such shield.

Second, because reserves are the unit of account, the nominal price of reserves is always 1 in nominal units. The price of bonds instead falls with an increase in the risk premium due to default. Therefore, as the fiscal crisis becomes more severe, for a given face value of bonds outstanding, their market value contracts, and their usefulness as a collateral diminishes, as I discussed in section 6.3. The ability of reserves to be used as a safe asset, on the other hand, is always constant by definition.

The third property of reserves is that they have to be held by banks, whereas bonds are held by banks and households. Therefore, when the monetary authority increased the supply of reserves, it was sure that these would be held by banks, lowering their exposure to default risk and reducing the fall in net worth and credit after a fiscal shock. Instead, if the fiscal authority increased the supply of short-term government bonds, this would simple lead to more holdings by the households, with no effect on credit or output. As long as there are enough bonds available, the household is the marginal holder of government bonds so changes in bond supply gave no effect on banks.

Of course, if the fiscal authority responds to QE by changing its debt choices, this will offset some of the effects of QE and may even make the joint policies counter-productive. As always, it is the coordination of fiscal and monetary policy that determines macroeconomic outcomes. But, the special properties of reserves and the central bank's control of the nominal interest rate imply that QE and debt management are far from the same policy.

### 7.2 QE and monetary financing of the debt

Another common criticism is that a central bank that buys government bonds during a fiscal crisis must be engaging in monetary financing of the deficit. This argument was particularly in the limelight during the implementation of QE by the ECB in the last two years.

In terms of the model, the answer is again clear: QE does not generate any extra resources for the fiscal authority, independently of consumption and output in the economy. This can be easily seen from the consolidated intertemporal budget constraint of the government in equation (18). The central bank only generates fiscal revenues insofar as it increases the present value of seignorage. But, seignorage is exogenous in the model. Therefore, there is no transfer of resources to the fiscal authority across all the policy experiments in this paper so far.

A related accusation of QE is that it prevents sovereign default and/or is inflationary. Yet, as emphasized in section 6 , QE actually reduces the recovery rate on government bonds in a fiscal crisis. The transfer of resources form the private to the public sector during a default was exactly the same, and by taking bonds away from private hands, the central bank rather makes the default o each bond be more intense. Moreover, in that section inflation was kept at zero throughout, and in section 5 QE could not affect expected inverse inflation. QE was not inflationary.

Two important features of the model drive this stark answer. First, the central bank buys government bonds in the open market, paying market prices for them. QE does not overpay for government liabilities. Second, in the model QE was defined as $\hat{v}_{t}=q_{t} \hat{b}_{t}^{c}+Q_{t} \hat{B}_{t}^{c}$ instead of $\hat{m}_{t}=q_{t} \hat{b}_{t}^{c}+Q_{t} \hat{B}_{t}^{c}$. In words, QE is financed with interest-paying reserves, not currency.

Some of the benefits of QE could be replicated with this alternative policy. Monetary purchases of long-term government bonds would change the maturity of government liabilities held by the public and affect the extent of surprise inflation. If banks could use currency in the interbank market, then monetary purchases would likewise provide the safe assets that the financial market needs to operate in a fiscal crisis.

However, this policy comes with three important differences relative to QE. First, currency is not exclusively held by banks unlike reserves. Therefore, as with the issuance of more short-term government bonds, issuing currency will not prevent the credit crunch that comes with the fall in bank capital after a default, since it is the household, not banks, that holds currency at the margin. Second, issuing currency as an independent policy tool comes with inflation. As usual, given the money demand function in equation(7), an exogenous
increase in $h_{t}$ will come with an increase in the price level, which if unexpected lowers welfare in the model. Third, issuing currency will increase seignorage, $s_{t}$. Unlike reserves, which pay interest, currency does not remunerate its holder, so that when the central bank borrows using currency and invests in interest-earning assets it earns a revenue. As this is transferred to the fiscal authorities, it constitutes proper monetary financing of the deficits.

There is therefore some truth to the claim that purchases of government bonds by the central bank can come with higher inflation and monetary financing of the deficit. But his only happens if these purchases are financed with issuing currency, instead of QE that uses instead interest-paying reserves.

### 7.3 QE and the incentive to raise taxes

A final common criticism of QE is that it allows fiscal authorities to not implement needed reforms. This is an argument often heard in the Euro-area.

In the model, the fiscal authority could choose any $f_{t} \leq \bar{f}_{t}$. Yet, if the fiscal authority wants to maximize social welfare, it was optimal to raise the maximum tax revenue, whether QE was present or not. QE did not affect this incentive. Simply put, a fiscal crisis is socially costly, and if the fiscal authorities can reduce its size, it will be optimal to do so.

Related, imagine that the fiscal authority can choose $g_{t}$, the government purchases that so far I assumed were exogenous. In that case, by cutting spending, the fiscal crisis could be averted. That is, at date 0 , learning of the fall in its fiscal capacity, the government could lower $g_{t}$ in the present or future and prevent default next period. At the same time, this unexpected cut in $g_{t}$ would cause a recession in this standard economy with nominal rigidities where aggregate demand matters. If the central bank set interest rates optimally, it could keep inflation stable and output at the first best. This line of argument is well understood. Crucially, QE does not change any of it. If the cut is possible and implemented, the economy remains in "normal times" and QE is neutral as discussed in section 4. Otherwise, if there is still a fiscal crisis, conditional on its size, then the arguments for QE still apply.

Finally, during fiscal crises, fiscal authorities often engage in financial repression. One simple way to model this is as a tax on the returns from holding government bonds. If so, this tax is isomorphic to default in the model. The desire to do it is equivalent to the desire to default, and QE does not affect its inevitability to balance the government accounts.

## 8 Limitations of QE

So far, this paper has established that QE can have an effect on macroeconomic outcomes in a fiscal, and it can raise welfare, through two separate channels. There are limitations to QE within the model such that it may not be able to achieve the first best.

The first limitation comes from the central bank's solvency. By issuing reserves to buy long-term bonds, the central bank is engaging in maturity transformation, and with it comes risk. In one of the two states of the world in period 1 , the central bank will make losses, and engaging in QE increases these. Likewise, when the central bank issues reserves to buy bonds that then default, it removes those losses from the banks's balance sheets and bring them into its own. Each of these will imply hat the present value of dividends that the central bank can distribute to the fiscal authority will shrink. From the perspective of the consolidated budget of the government this is immaterial, as the losses of the central bank are gains of the fiscal authority. But the constraint that the central bank has to be intertemporally solvent puts an upper bound on QE and reserves. A fiscal authority in a crisis will most likely not be willing to bail out its central bank. The historical experience rather suggests the opposite: during a fiscal crisis, the Treasury tries to extract more resources from the central bank(Sargent, 1982).

A second limitation of QE is that, in the case of default, it involves redistribution of resources within society. As discussed in section 6, the recovery rate falls with QE. As banks get giving access to an exclusive vehicle, reserves, that provides a shield from the costs of default, the non-bank holders of government bonds lose more in default. In fact, the government budget constraint imposes that the private sector must lose precisely the same amount, regardless of the amount of reserves. QE lowers the losses of banks by increasing the losses of households. In the model, this only improves welfare because there is no welfare cost of redistributing resources away from households coupled with a benefit from avoiding banks suffer losses. But, it would be reasonable to include politically-motivated limitations on this redistribution, and these would imply limitations on QE.

A third limitation of QE is that it affects the net supply safe assets in the economy only insofar as the extra reserves can buy risky assets. Insofar as it is hard to measure which assets are safe or not, many forms of QE may turn out to be ineffective, just as purchases of short-term bonds were in the model. Related, there may be a limit to QE if the supply of public debt at different maturities is too low.

## 9 Conclusion and some evidence

This paper put forward arguments for why quantitative easing can be a tool for the future in central banking. While QE may have been neutral in the past, in a future where fiscal crisis are possible, and perhaps long lasting, QE can play two roles. First, it can allow the central bank to stabilize inflation by managing the sensitivity of inflation to fiscal shocks. Second, it can prevent a credit crunch after a fiscal crisis by lowering the losses suffered by banks during a default and providing safe assets that financial markets can use to promote financial stability. Each of these roles is consistent with the traditional objectives of central banking of stabilizing inflation, real activity and financial activity. This paper described how managing the size and composition of the central bank's balance sheet could exploit these channels to achieve better outcomes.

Is there any historical evidence supporting these channels? Starting in 2010, the ECB started purchasing sovereign bonds held by banks as part of its SMP and then OMT programs. Initially these were sterilized, so they came with no increase in reserves, but since 2014 they have allowed reserves to grow. The goal was to remove risky assets from banks in order to spur credit. The logic behind these programs is close to the one in this paper, and the model predicts that sterilizing them or not makes an important difference in their effectiveness.

Also, starting in 2009, the ECB greatly increased the supply of reserves, on great part through its LTRO program. The logic used to justify them was similar to the model of the interbank market in this paper, as it was perceived that banks needed safe assets for collateral so that the financial market would work. Yet, there is little evidence that the program led to an increase in credit. The model would predict this if the incentive constraint was not binding, and provides some support to the more recent targeted LTRO program that requires banks to increase credit in exchange for the reserves.

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[^1]:    ${ }^{1}$ Reis (2009) surveys these unconventional monetary policies.
    ${ }^{2}$ The literature describing these policies and measuring their impact includes event studies on the responses of yields (e.g., Krishnamurthy and Vissing-Jorgensen, 2011), estimated DSGEs on macroeconomic variables (e.g., Chen, Cúrdia, and Ferrero, 2012), and instrumental-variables regressions on loan supply (e.g., Morais, Peydro, and Ruiz, 2015).

[^2]:    ${ }^{3}$ A related paper is Corhay, Kung, and Morales (2014), but they focus on modeling movements in bond premia, and on the effects of policy at the zero lower bound.

[^3]:    ${ }^{4}$ The associated no Ponzi scheme constraint is: $\lim _{T \rightarrow \infty} \mathbb{E}_{t}\left[\delta_{t+T}\left(b_{t+T-1}+q_{t+T} B_{t+T-1}\right) / p_{t+T}\right]=0$.

[^4]:    ${ }^{5}$ That is, letting $\tilde{s}_{t}$ be net central bank expenses, then $s_{t}=\left(h_{t}-h_{t-1}\right) / p_{t}-\tilde{s}_{t}$, so I assume that variations in $\tilde{s}_{t}$ dominate those in seignorage so that net seignorage is exogenous. Section 7 relaxes this assumption.

[^5]:    ${ }^{6}$ This treatment of capital may seem different from what is usual in the literature, where capital is often a variable factor of production. Yet, if $k_{t}=1$, which will be the first best, this setup of production is identical to that in the textbook new Keynesian model without capital. The important assumption here is rather that capital cannot be accumulated over time, unlike in the neoclassical growth model that is often at the basis of DSGE models.
    ${ }^{7}$ Nominal rigidities last only for one period, so the effects of the fiscal crisis and QE on unexpected inflation happen only in one period, instead of spreading over time. This is for simplicity since, beyond propagation, more delayed price adjustments would not add to the economic mechanisms at play. Moreover, this canonical model does not require taking a stand on what is the best model of time-dependent price adjustment.

[^6]:    ${ }^{8}$ This assumption implies that banks do not accumulate net worth over time. This simplifies the analysis by restraining the effect of shocks to bank equity to one period.
    ${ }^{9}$ Because banks can freely choose to hold reserves or short-term bonds, no arbitrage requires that they are indifferent, so reserves must pay the same return as one-period bonds: $1 / q_{t}=1+i_{t}$.

[^7]:    ${ }^{10}$ In reality, this interbank market corresponds more closely to the repo market, where financial institution lend to each other against safe, usually government, collateral.
    ${ }^{11}$ With a return of 1 , the household is indifferent between depositing capital in the bank or simply transforming it into consumption. One should interpret the return on deposits as infinitesimally above 1 to break this indifference and make households want to deposit all their capital in the banking system.

[^8]:    ${ }^{12}$ Extending the shocks to occur in many periods would be important to learn about the quantitative

[^9]:    ${ }^{13}$ Tedious but straightforward algebra shows that if prices are flexible then $r_{t}=r \equiv(A / \sigma)(1-1 / \sigma)^{1 / \alpha}>1$.

[^10]:    ${ }^{14}$ Another way to understand this result is to note that, in a symmetric equilibrium, $y_{t}=a_{t} l_{t} t_{t}^{\frac{1+\sigma \theta}{\sigma-1}}$. The model has increasing returns to scale on aggregate in labor and capital, and so the first best is to set capital at its maximum, 1. Once there, there are constant returns to scale in the variable factor, labor.

[^11]:    ${ }^{15}$ An alternative, in between regime, would be where the fiscal authority exogenously chooses some default rate $\delta_{t}$ not necessarily one. That case would include elements of the two sections.

[^12]:    ${ }^{16}$ Gorton (2010); Caballero and Farhi (2014) argue that this was the case in 2008 as a result of the financial crisis, since increases in uncertainty led to rises in margins. In our model, this would map into a lower $\xi$, which makes it more likely that equation (30) binds.

[^13]:    ${ }^{17}$ Including long-term bonds in the incentive constraint of bankers in the interbank market would make no difference to any of the other results before this sub-section.
    ${ }^{18}$ In the opposite direction, Cochrane (2014) discusses monetary policy and QE as if it controlled perfectly the entire maturity structure of the debt outstanding, so that it becomes synonymous with Treasury debt management.

