Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound

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Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound

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Key question

What is the macroeconomic impact of monetary policy at the ZLB?

Conventional approach before ZLB

- VAR with the fed funds rate

But since December 2008, the fed funds rate has been near zero
Challenges of zero lower bound

Challenges

- What framework to study unconventional monetary policy?
  - The fed funds rate has been replaced by large-scale asset purchases and forward guidance as primary policy tools.
- How to describe the yield curve?
  - Gaussian ATSM allows negative interest rates.

Shadow rate term structure model: Black (1995)

- Non-negative short rate: \( r_t = \max(r, s_t) \)
- Analytical solution does not exist in general
## Contributions

This paper

- an analytical approximation for SRTSM
Contributions

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- shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
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- an analytical approximation for SRTSM
- shadow rate has similar dynamic correlations with macro variables as the fed funds rate did previously
- our shadow rate updated monthly by Atlanta Fed

www.frbatlanta.org/cqer/researchcqcq/shadow_rate.cfm
Outline

1. Model
2. Shadow rate
3. Macroeconomic Implications
4. Conclusion
Bond pricing

Risk-neutral factor dynamics:

\[ X_{t+1} = \mu^Q + \rho^Q X_t + \sum \varepsilon^Q_{t+1}, \quad \varepsilon^Q_{t+1} \sim N(0, I). \]

Pricing kernel

Pricing equation

\[ P^n_t = \mathbb{E}^Q_t[\exp(-r_t - r_{t+1} - \ldots - r_{t+n-1})] \]

Yield

\[ y^n_t = -\frac{1}{n} \log(P^n_t) \]

Forward rate

\[ f_{n,n+1,t} = (n + 1)y_{n+1,t} - ny_{nt} \]
SRTSM and GATSM

**SRTSM**

\[
\begin{align*}
r_t &= \max(r, s_t) \\
s_t &= \delta_0 + \delta_1 X_t
\end{align*}
\]

Forward rate

\[
f_{n,n+1,t}^{SRTSM} = r + \sigma_n^Q g \left( \frac{a_n + b'_n X_t - r}{\sigma_n^Q} \right)
\]

where \( g(z) = z\Phi(z) + \phi(z) \)

**GATSM**

\[
\begin{align*}
r_t &= \delta_0 + \delta_1 X_t \\
f_{n,n+1,t}^{GATSM} &= a_n + b'_n X_t.
\end{align*}
\]
Property of $g(.)$

$$y = g(z)$$

$$y = z$$

$$f_{n,n+1,t}^{SRTSM} \begin{cases} 
\approx r, \text{ at the ZLB} \\
\approx a_n + b_n'X_t = f_{n,n+1,t}^{GATSM}, \text{ when interest rates are high}
\end{cases}$$
## Model fit

- **GSW Data**: monthly 1990-2013; maturities: 3m, 6m, 1y, 2y, 5y, 7y, 10y
- **Estimation**: Kalman filters
- **Average absolute approximation error between 1990M1 and 2013M1**

<table>
<thead>
<tr>
<th></th>
<th>3M</th>
<th>6M</th>
<th>1Y</th>
<th>2Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward rate error</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.13</td>
<td>0.69</td>
<td>1.14</td>
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<td>435</td>
<td>551</td>
<td>600</td>
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<tr>
<td>yield error</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.24</td>
<td>0.42</td>
<td>0.78</td>
</tr>
</tbody>
</table>
Model fit

Log likelihood values
- SRTSM: 856; GATSM: 755

Figure: Average forward curve in 2012

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Summary for unconventional monetary policy?
Shadow rate and LSAPs
Monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

\[ s_t^o = \begin{cases} 
\text{effective federal funds rate} & \text{before 2009} \\
\text{shadow rate} & \text{since 2009}
\end{cases} \]
Factor augmented vector autoregression

Replace the fed funds rate with $s_t^o$ in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t^o + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

- $Y_t^m$: 97 economic variables from 1960 to 2013
- $x_t^m$: 3 underlying macro factors

Factor dynamics:

$$\begin{bmatrix} x_t^m \\ s_t^o \end{bmatrix} = \begin{bmatrix} \mu_x \\ \mu_s \end{bmatrix} + \begin{bmatrix} \rho_{xx} & \rho_{xs} \\ \rho_{sx} & \rho_{ss} \end{bmatrix} \begin{bmatrix} X_{t-1}^m \\ S_{t-1}^o \end{bmatrix} + \sum^m \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_{t, MP} \end{bmatrix}, \quad \begin{bmatrix} \varepsilon_t^m \\ \varepsilon_{t, MP} \end{bmatrix} \sim N(0, I)$$

- monthly VAR(13)
- $\sum^m$: Cholesky decomposition
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

Hypothesis I

$H_0: \rho_{xs}(t < \text{Great Recession}) = \rho_{xs}(t > \text{Great Recession})$  

$\Rightarrow p = 0.29$ for so  

$\Rightarrow p = 0.0007$ for EFFR

Hypothesis II

$H_0: \rho_{sx}(t < \text{Great Recession}) = \rho_{sx}(t > \text{Great Recession})$  

$\Rightarrow p = 1$ for so  

$\Rightarrow p = 1$ for EFFR

Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate.
Measures of monetary policy

Can we use shadow rate as similar summary of Fed actions as fed funds rate provided historically?

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Implication: researchers can use shadow rate to update earlier studies that had been based on the historical fed funds rate.
Historical decomposition

What if there had been no monetary policy shocks?

- **realized:** $\varepsilon_{t}^{MP} = \hat{\varepsilon}_{t}^{MP}$
- **counterfactual:** $\varepsilon_{t}^{MP} = 0$ for ZLB

Unconventional monetary policy

- reduced the shadow rate by 0.4% between 2011 and 2013.
**Historical decomposition**

**What if there had been no monetary policy shocks?**

- **realized:** $\varepsilon_t^{MP} = \hat{\varepsilon}_t^{MP}$
- **counterfactual:** $\varepsilon_t^{MP} = 0$ for ZLB

Unconventional monetary policy

- reduced unemployment by 0.13% in Dec 2013.
Counterfactual II

What if the shadow rate had been kept at $r$?

- counterfactual: $\varepsilon^\text{MP}_t$ is such that $s^o_t = r$ at ZLB

Unconventional monetary policy

- reduced unemployment by 1% in December 2013

![Policy rate and Unemployment charts](chart.png)
Impulse response: full sample

A -25bps monetary policy shock
Full sample FAVAR(13) vs. ZLB FAVAR(1)

ZLB with effective federal funds rate

![Graphs showing economic indicators over time]
Full sample FAVAR(13) vs. ZLB FAVAR(1)

ZLB with shadow rate

Policy rate

Industrial production index

Consumer price index

Capacity utilization

Unemployment

Housing starts
Forward guidance

ZLB duration

\[ \tau_t = \inf \{ \tau_t \geq 0 | s_{t+\tau} \geq r \} . \]
Conclusion

Method

- Develop an approximation for bond prices in the SRTSM

Economics

- The shadow rate exhibits similar dynamic correlations with economic variables after the Great Recession as the fed funds rate did earlier in data.
- Unconventional monetary policy lowered the unemployment rate by 0.13% in December 2013.
Source: www.frbatlanta.org/cqer/researchcqq/shadow_rate.cfm
ECB shadow rate

[Graph showing the trend of ECB shadow rate and official bank rate from 1995 to 2015]
Pricing kernel

Factor dynamics:

\[ X_{t+1} = \mu + \rho X_t + \sum \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I). \]

Pricing kernel

\[ m_{t+1} = r_t + \frac{1}{2} \lambda_t' \lambda_t + \lambda_t' \varepsilon_{t+1} \]
\[ \lambda_t = \lambda_0 + \lambda_1 X_t \]

where \( \mu^Q = \mu - \sum \lambda_0 \), and \( \rho^Q = \rho - \sum \lambda_1 \)

Pricing equation

\[ P^n_t = \mathbb{E}_t[\exp(-m_{t+1})P^{n-1}_{t+1}] \]
Bond recursions

\[ a_n = \delta_0 + \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \mu^Q - \frac{1}{2} \delta'_1 \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right) \sum' \left( \sum_{j=0}^{n-1} (\rho^Q)^j \right)' \delta_1, \]

\[ b'_n = \delta'_1 (\rho^Q)^n. \]
Model specification

\( r = 0.25, \) interest rate on reserves

three factors

Normalization: restrict \( Q \) parameters

Repeated eigenvalues

\[
\rho^Q = \begin{bmatrix}
\rho_1^Q & 0 & 0 \\
0 & \rho_2^Q & 1 \\
0 & 0 & \rho_2^Q
\end{bmatrix}.
\]
Kalman filters

State equation

\[ X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I) \]

observation equation for SRTSM ⇒ extended Kalman filter

\[ f_{n,n+1,t}^o = r + \sigma_n^Q g \left( \frac{a_n + b'_n X_t - r}{\sigma_n^Q} \right) + \eta_{nt}, \eta_{nt} \sim N(0, \omega) \]

observation equation for GATSM ⇒ Kalman filter

\[ f_{n,n+1,t}^o = a_n + b'_n X_t + \eta_{nt}, \eta_{nt} \sim N(0, \omega) \]
Approximation error for ZLB

Average absolute approximation error between 2009M1 and 2013M1

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<tbody>
<tr>
<td>forward rate error</td>
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<td>forward rate level</td>
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<td>yield error</td>
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<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.91</td>
<td>1.50</td>
<td>2.37</td>
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Robustness
Robustness

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## Robustness

<table>
<thead>
<tr>
<th>Method</th>
<th>$p$-value for $\rho_{1s}^{xs} = \rho_{3s}^{xs}$</th>
<th>$p$-value for $\rho_{1x}^{sx} = \rho_{3x}^{sx}$</th>
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</thead>
<tbody>
<tr>
<td>Baseline</td>
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<tr>
<td>A1 estimate $r$</td>
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<td>1.00</td>
</tr>
<tr>
<td>A2 2-factor SRTSM</td>
<td>0.13</td>
<td>0.97</td>
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<tr>
<td>A3 Fama-Bliss</td>
<td>0.38</td>
<td>1.00</td>
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<tr>
<td>A4 5-factor FAVAR</td>
<td>0.70</td>
<td>1.00</td>
</tr>
<tr>
<td>A5 6-lag FAVAR</td>
<td>0.09</td>
<td>0.98</td>
</tr>
<tr>
<td>A5 7-lag FAVAR</td>
<td>0.19</td>
<td>0.97</td>
</tr>
<tr>
<td>A5 12-lag FAVAR</td>
<td>0.22</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Historical decomposition

- Policy rate
- Industrial production index
- Consumer price index
- Capacity utilization
- Unemployment
- Housing starts

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Counterfactual II

Policy rate

Industrial production index

Consumer price index

Capacity utilization

Unemployment

Housing starts

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Impulse responses: forward guidance

A monetary policy shock to increase the ZLB by 1 year

- ZLB duration
- Industrial production index
- Consumer price index
- Capacity utilization
- Unemployment
- Housing starts
Unemployment rate decreases by 0.1% with

- a one year increase in the expected ZLB duration
- 15 basis-point decrease in the policy rate

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