Trade and Financial Sector Reforms: Interactions and Spillovers

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Abstract

The allocation of production across firms is a potentially important explanation of the productivity gap between rich and poor economies. Reforms to trade policy and the domestic financial sector are often both key elements of policy packages aimed at reducing productive distortions. However, the impact of each reform in reallocating production within an economy is usually analysed independently. This paper asks how do such general equilibrium effects of trade and domestic financial sector reforms interact in terms of their effects on productivity, wages and utility. Motivated by recent firm-level studies, I add two-way linkages between firms’ production and exporting decisions and their financial constraints to a general equilibrium heterogeneous firm trade model. The interaction effects between reforms appear qualitatively important. Trade and domestic financial sector reforms have complementary effects on the average productivity and size of domestic producers. However, if much reallocative work has already been done through a well-functioning financial sector, the marginal benefits of trade liberalisation for wages and household utility are reduced. Improvements in the ability to use exports as pledgeable collateral enhance both the wage and productivity effects of trade reforms. The model also highlights the potential for financial sector reforms in one economy to be exported via the trade channel, affecting decisions to produce or export in the foreign economy and putting downward pressure on foreign real wages.

1 Introduction

The efficient allocation of production across firms matters for aggregate total factor productivity (TFP). Distortions which shift resources from more to less productive firms can have a sizeable impact on TFP and hence on average output per worker and welfare. For example, it is estimated that removing such resource misallocations could increase TFP in China by 25-40% and in India by 50-60% (Hsieh & Klenow, 2007). Improvements of such magnitudes are equivalent to substantial reductions in the relative productivity gaps compared to the US, moving relative TFP in both countries from around 40% to 60% of the US level (based on the 2004 aggregate TFP estimates of Jorgenson & Vu, 2007). Many of the wide-ranging structural reforms across developing and transition economies in recent decades have been focused on reducing distortions to the allocation of production. Whilst reforms to trade and domestic financial sectors are often both key elements of such policy packages, the respective related empirical and theoretical literatures have generally abstracted from the potentially important reallocative effects of contemporaneous reforms in the other sector. Thus, a fundamental policy question of how reforms interact in terms of their impact on aggregate productivity, wages and welfare cannot be addressed.

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1These estimates are derived from a movement to ‘US efficiency’ based on the distribution of marginal products of capital for plants within sectors. In turn, in a model calibrated to US data, Restuccia & Rogerson (2007) find similar order aggregate TFP effects of distortions to prices faced by individual plants.
The objective of this paper is to develop a theoretical framework to analyse this issue through adding
financial constraints to a two-country (non-symmetric) heterogeneous firm trade model. In order to do
so I add a number of novel, empirically relevant, features to a baseline heterogeneous firm trade model
adapted from Melitz (2003). The first is to consider intermediate production as owned and operated
by heterogeneous entrepreneurs whose borrowing is subject to credit constraints due to agency prob-
lems. The second is to allow these agency problems to vary with the composition of the entrepreneur’s
production between domestic output and exports. This introduces a two-way linkage between a firm’s
export decision and its financial constraints as suggested by firm-level survey data. I then examine the
steady state general equilibrium interactions of the reallocative impacts of trade and domestic financial
sector reforms (considered as reductions in variable trade costs and relaxations in credit constraints
respectively).

The interaction effects between trade reforms and domestic financial sector reforms appear qualita-
tively important. On the one hand, trade and domestic financial sector reforms can have complementary
effects in increasing the average productivity and size of producing entrepreneurs. If entrepreneurs face
less restrictive credit constraints as a result of financial sector reforms then investment can increase more
in response to a lowering of variable export costs. With fixed factor supplies, a greater reallocation
from low to high productivity entrepreneurs is thus required in order to maintain factor market equi-
librium. As a result, the positive effects of trade liberalisation on average productivity and producer
size are enhanced if domestic financial sector reforms are more advanced. On the other hand, in such a
case the marginal gains for wages and household utility as a result of trade liberalisation are reduced.
If credit constraints are less restrictive then effective borrowing costs are lower, intermediate prices
are reduced and real wages are higher. Thus, the marginal benefits of trade liberalisation in lowering
prices and increasing real wages are reduced if much reallocative work has already been done through a
well-functioning domestic financial sector. In terms of the potential linkage between exports and credit
constraints, improvements in the relative ability to pledge exports to creditors amplify the benefits of
trade liberalisation. A further insight of the paper is that even in financial autarky the financial devel-
opment of not just the domestic economy but also its trading partner can play a role in determining the
real wages and the efficiency of domestic production. In particular, domestic financial sector reforms in
one economy can be exported via the trade channel putting downward pressure on foreign real wages.

The value added of the approach adopted below is to provide a framework which allows analysis
of the macroeconomic implications of reforms in one area, for example the domestic financial sector,
conditional on other policy variables, for example the degree of access to international goods markets.
Using this modeling approach to assess whether the interactions between these different reform measures
have quantitatively important empirical effects is an important next step. In the rest of the paper,
Section 2 first discusses the motivation and related literature. Section 3 then provides an overview of
the modeling approach focusing on the credit constraints that entrepreneurs face. Section 4 provides
details of the model with Section 5 analysing the steady state in trade and financial autarky to emphasise
the key mechanisms in the model. Section 6 then opens up the economies to international goods trade.
Section 7 concludes including a discussion of further research and empirical implications.

2 Motivation and related literature

2.1 Motivation

The modeling approach which I adopt is driven by two main features: first, the importance of addressing
firm heterogeneity in any analysis of trade and domestic financial sector reforms and, second, the im-
portance of analysing the interactions of these two reforms. As surveyed in, for example, Bernard et al.
(2007a), recent heterogeneous firm models of international trade are motivated by the observation that
exporters tend to differ from non-exporters in important ways, for example being larger and more pro-
ductive. Surveys indicate that only a limited subset of plants export, for example around 20% in the
1991 US Census of Manufactures (Bernard et al., 2003). There is also empirical support for the self-
selection of higher productivity firms into exporting rather than a causality running from exporting to
higher productivity, eg Clerides et al. (1998). At the same time, financial constraints vary across firms
and recent firm-level data suggests that the degree of financial constraints faced by firms depends on
their export status. For example, using Spanish data Campa & Shaver (2002) find exporters to be less
liquidity constrained whilst for UK data Greenaway et al. (2007) show that being an exporter improves a firm’s liquidity and lowers its leverage. This linkage from export status to a firm’s financial constraints is in addition to the growing empirical evidence on the effects of financial constraints on export levels and patterns. Indeed, the analysis of Greenaway et al. (2007) suggests that the stronger financial health of exporters can be seen as an outcome rather than a determinant of entry into exporting.

The importance of analysing the interaction of trade and domestic financial sector reforms is clearly illustrated by the components of the policy packages implemented by developing and emerging economies. As an example of the association between the two reforms, Figure 1 plots five-year averages for the trade and banking sector transition indicators for central and eastern European countries compiled by the European Bank for Reconstruction and Development (EBRD). The transition indicators represent an assessment of a country’s policies and institutions in each area against specific criteria with scores assigned from 1 to 4.33 such that higher values indicate improvements in the policy areas towards the levels in advanced economies. As expected, the levels of the two policy indicators are positively associated with the second panel highlighting the often contemporaneous progress on both trade and financial sector reforms. In addition, reforms in one sector can take place at a range of initial values for policies in the other sector. These broad patterns hold if one looks at alternative policy measures or outturn indicators of the policy stance, such as trade openness or private credit to GDP. Summary indicators of policy stance are clearly not perfect, for example due to the lack of information on enforcement or their aggregate nature. However, they serve to highlight the empirical importance of analysing the potential trade-offs and complementarities in the effects of trade and financial sector reforms. Such analysis can provide insights into the appropriate design of reform packages in individual countries and can also shed light on the political economy implications of different combinations of reforms.

Figure 1: EBRD transition indicators, 5 year averages: levels and changes

(a) Levels

(b) Change compared to previous period


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2 Also, using World Bank survey data from developing and emerging economies Beck et al. (2006) find that the problem of bank corruption as a constraint to finance is less of a problem for exporters.

3 See, for example, the sector-level analysis in Manova (2006).

4 See, for example, the chronology provided by Henry (2000) of reforms across 12 major emerging markets in the 1980s and early 1990s or the analysis of IMF programme conditionalities provided in IMF (2001).

5 For example, the pattern is similar using average tariff rates and the financial liberalisation indicator of Abiad & Mody (2005) which covers both developed and developing economies.
2.2 Related literature

This work is related to three main strands of literature. The first concerns the reallocative effects of trade reforms in the presence of heterogeneous productivity firms as analysed in a growing theoretical literature (see, for example, the theoretical models of Melitz, 2003; Baldwin & Forslid, 2006). In such models trade liberalisation increases investment and the demand for labour. With a fixed labour supply this leads to a rise in the minimum productivity of producers, shifts production towards higher productivity firms and results in an increase in average productivity. This reallocative process is supported in empirical studies such as Bernard et al. (2006) in relation to the US, Pavcnik (2002) for Chile and Fernandes (2007) for Colombia.

The second strand of related literature concerns the reallocative effects of financial sector reforms. Theoretical models, such as Almeida & Wolfenzon (2005) and Caselli & Gennaioli (2006), illustrate how domestic financial development (for example, contract enforcement or investor protection) can promote aggregate productivity through reallocating resources to more productive firms. From an empirical perspective the beneficial impacts of financial sector reforms on allocative efficiency are supported in studies such as Wurgler (2000), Galindo et al. (2007) and Abiad et al. (2007). For example, the latter paper finds domestic financial sector liberalization to be associated with an improvement in the efficiency of the allocation of production (indicated by a fall in the dispersion of Tobin’s q) in a sample of emerging economies.

The third strand of related literature concerns the impact of financial frictions on trade. Recent papers using sector-level data, such as Hur et al. (2006) and Manova (2006), support the view that financial frictions are an important determinant of trade patterns. This relation has been examined within the heterogeneous firm model of Melitz (2003) by Chaney (2005) and Manova (2006). However, these papers focus on the impact of credit constraints on trade patterns. In doing so they abstract from the potential impact of trade on credit constraints mentioned above, the effects of credit constraints on domestic activities and, importantly, from general equilibrium macroeconomic effects. Related work by Hsu (2006) examines the productivity effects of trade liberalisation in the presence of financial frictions but with symmetric economies and within a different modeling framework based on the transfer of ownership across dynasties. However, the symmetric case abstracts from the more realistic and policy-relevant case of differential financial development across countries which is analysed in this paper.

3 Overview of modeling approach

There are two economies in the model, home and foreign, who may trade intermediate goods. The primary feature of the model is the set of heterogeneous entrepreneurs in each economy which produce, with varying productivities, intermediate goods using labour inputs. As in the standard set-up of Melitz (2003), depending on her productivity an entrepreneur may choose to produce output for the domestic market or to pay additional costs to access also export markets. However, as the entrepreneur’s investment is subject to financial constraints, her production decisions link domestic financial conditions and international trade.

In order to capture the effects of financial frictions on the productive sector I consider intermediate producers as entrepreneurs who maximise their own utility and operate and own their own projects. In doing so, I abstract from entry and exit decisions which can be analysed within the standard Melitz model through the assumption of perfect competition in the productive sector and a fixed cost of entry. Agency problems, such as inalienability of human capital or ex post moral hazard etc., limit a firm’s access to finance. The extent of these borrowing constraints may differ with a creditor’s nationality. This may be due to different legal systems or different informational and transaction costs. The degree of borrowing constraints may also vary with the type of pledgeable output, namely output for domestic or export markets. This might reflect the differential ability of lenders to recover export output or to

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7 Also, in the context of international financial sector reforms, Aoki et al. (2006) emphasise that the impact of capital account liberalization in shifting production across high and low productivity entrepreneurs is dependent on the development of the domestic financial sector.

8 Abiad et al. (2007) also find that trade openness improves the allocation of production across firms but the issue of the interaction of the two reforms is not addressed.

9 It is straightforward to extend the approach to encompass capital or other factor inputs but for simplicity I focus on the labour input case.
monitor exporting activities relative to domestic output or activities respectively.

Agents in each country can access domestic credit markets consisting of one-period risk-free debt contracts (with the underlying contracting problem assumed to yield zero default in equilibrium). Borrowing of one unit of the home consumption good from home creditors at time \( t \) requires a gross repayment of \( R_{t+1} \) units of the time \( t+1 \) home consumption good. Similarly, borrowing a unit of time \( t \) foreign consumption from foreign creditors requires a gross repayment of \( R_{*t+1} \) of the time \( t+1 \) foreign consumption good. I now turn to the form of the borrowing constraints.

### 3.1 Domestic borrowing constraint

Domestic creditors face costs relative to the entrepreneur in recovering output which has been pledged to them. This may be due to the inalienability of human capital as emphasised in Hart & Moore (1994) or could reflect costs (in terms of output) of recovery when there is ex post moral hazard such as in Aghion et al. (1999) or costs of expropriation in the case of ex ante moral hazard. As in, for example, Kiyotaki (1998) and Aoki et al. (2006), these financial frictions restrict an entrepreneur’s borrowing so that debt repayments do not exceed the value to the creditor of pledged output.\(^{10}\) Creditor recovery costs may differ with the creditor’s nationality and the type of pledged receivables:

**Assumption 1** Creditors in the home economy can recover a fraction \( 0 < \theta \leq 1 \) of the domestic intermediate output which has been pledged to them by a home entrepreneur. The corresponding fraction for domestic intermediate output pledged by foreign entrepreneurs to foreign creditors is \( 0 < \theta^* \leq 1 \).

The differential ability of home and foreign lenders to recover domestic outputs pledged by entrepreneurs in their respective economies can be thought to represent both the legal or judicial features of the economy and efficiency of the respective financial systems.

**Assumption 2** The recovery rate on export output pledged to a domestic creditor is a fraction \( \mu \) times the recovery rate on domestic output in home and \( \mu^* \) in foreign with \( \mu \leq 1 \) and \( \mu^* \leq 1 \).

Assumption 2 is motivated by the firm-level empirical evidence from emerging and developing economies of the linkage between exporting and credit constraints.\(^{11}\) On the one hand any additional organisational or informational problems in recovering exports may reduce the relative ability of a creditor to recover exports compared to domestic output. On the other hand, certain institutional features, such as requirements to repatriate export revenues, could work in the opposite direction increasing the relative recovery rate on exports. Thus I allow \( \mu \) to take values greater than or less than one subject to the restriction that the export recovery rate \( \theta \mu \) is bounded in the unit interval (and similarly for \( \theta^* \mu^* \)).

The creditor will only lend an amount such that gross repayments are less than or equal to the recovery value of output pledged to them. Thus the domestic credit constraint for a home entrepreneur indexed by \( \varphi \) can be expressed as:

\[
R_{t+1} b_{t+1} (\varphi) \leq rev_{d,t+1} (\theta y_{d,t+1} (\varphi)) + rev_{x,t+1} (\mu \theta y_{x,t+1} (\varphi))
\]

where \( R_{t+1} b_{t+1} (\varphi) \) are gross repayments made by the entrepreneur in time \( t+1 \) domestic consumption goods for borrowing \( b_{t+1} (\varphi) \) at time \( t \). The creditor can recover a fraction \( \theta \) of next period’s domestic output by the entrepreneur \( y_{d,t+1} (\varphi) \). This provides revenues of \( rev_{d,t+1} (\theta y_{d,t+1} (\varphi)) \) in units of the home consumption good. Similarly \( rev_{x,t+1} (\mu \theta y_{x,t+1} (\varphi)) \) represents the maximum value to the creditor, in units of the home consumption good, of the pledged export output \( y_{x,t+1} (\varphi) \).\(^{12}\) Thus exporting increases the potential pledgeable output to a creditor but, as detailed below, requires additional investment. In this set-up all investment is subject to the borrowing constraint. In contrast, in Chaney (2005) and Manova (2006) borrowing constraints only affect the financing of exporting costs. However, domestic

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\(^{10}\)Extending the model to add tangible assets, for example, capital or land, to the production function would provide an additional form of collateral that could be pledged to creditors. The creditor’s recovery rates on these assets would then affect the optimal ratio of factor inputs. However, if creditors were unable to recover pledged output and could only recover pledged capital or land then financial constraints would only affect relative export to domestic prices if domestic and exporting activities had different production functions.

\(^{11}\)The requirement to pay the additional fixed exporting cost could also, in a situation of hidden information over entrepreneurial productivities, act as a signalling device of higher productivity which could facilitate lending.

\(^{12}\)The subscripts \( d \) and \( x \) will be used throughout to denote domestic and exporting variables respectively.
production costs may also be subject to constraints and, due to the fungibility of funds, applying different constraints to the two types of costs may not be appropriate.\footnote{In Antras & Caballero (2007) financial frictions (which apply in one sector) also apply to all production costs although the nature of revenues does not affect the credit constraint (which limits total investment to a fraction of capital).}

4 Model

Each economy is populated by two sets of infinitely-lived agents, households and entrepreneurs, of mass $L$ and $M$ respectively ($L^*$ and $M^*$ in foreign).\footnote{Foreign variables are denoted throughout by $^*$.} Both sets of agents maximise expected utility over a final consumption good which is produced using intermediate goods. These intermediate goods are themselves produced by entrepreneurs using entrepreneurial and household labour inputs. As mentioned above, agents are able to access domestic credit markets only. In the subsections below I first introduce the final goods and household sectors which provide the context for the subsequent analysis of entrepreneurs’ optimal decisions.

4.1 Final goods sector

The non-traded final consumption good in each country is produced using a constant elasticity of substitution (CES) combination of domestic and imported intermediate goods which are produced by home and foreign entrepreneurs respectively. For example, home final good production, $Y_t$, is given by:

$$Y_t = \left[ \int_{\omega \in \Omega_{d,t}} y_{d,t}^{\omega} \frac{\sigma}{\sigma-1} d\omega + \int_{\omega \in \Omega_{x,t}^*} y_{x,t}^* \frac{\sigma}{\sigma-1} d\omega \right]^{\sigma/(\sigma-1)} \quad (2)$$

where $y_{d,t}(\omega)$ is the home final goods sector demand for an intermediate variety $\omega$ produced by a home entrepreneur; $y_{x,t}^*(\omega)$ is the demand from the home final goods sector for a variety $\omega$ produced by a foreign entrepreneur and exported to home; $\Omega_{d,t}$ denotes the set of available domestically produced intermediate goods in home at time $t$; and, $\Omega_{x,t}^*$ is the set of available intermediate goods in home which were produced by foreign entrepreneurs and exported to home. The elasticity of substitution in production between different intermediate good varieties is $\sigma > 1$. For simplicity I assume that the elasticity of substitution between individual intermediates does not vary between foreign or home intermediates.\footnote{Relaxing this assumption does not change the results substantively if the elasticity between home and foreign produced intermediates is above unity and less than the elasticity between varieties from the same country. Similarly, I abstract from any bias in final goods production towards home-produced intermediates.} A corresponding expression holds in the foreign economy with common substitution elasticities in both countries. Due to selection effects into exporting, the set of home-produced inputs available in home, $\Omega_{d,t}$, may differ from the set of home-produced intermediate inputs available in foreign, $\Omega_{x,t}$ (and similarly for the set of foreign produced intermediate inputs available in foreign and home, $\Omega_{d,t}^*$ and $\Omega_{x,t}^*$ respectively).

The final goods technology is open to all agents and takes place under perfect competition. Intermediate goods are purchased to maximise profits given the final consumption good price ($P_t$ at home and $P_t^*$ in foreign) and the prices of individual intermediate goods. Profit maximisation yields the standard domestic and export demand schedules for each intermediate good $\omega$ produced in home:

Domestic demand: $y_{d,t}(\omega) = \left( \frac{p_{d,t}(\omega)}{P_t} \right)^{-\sigma} Y_t$ \quad (3)

Export demand: $y_{x,t}(\omega) = \left( \frac{p_{x,t}(\omega)}{P_t^*} \right)^{-\sigma} Y_t^*$ \quad (4)

where $p_{d,t}(\omega)$ is the domestic price of variety $\omega$ produced by the home entrepreneur with $p_{x,t}(\omega)$ the price charged for that variety when exported to the foreign country. With zero profits earned on final goods production, the aggregate price indices in home and foreign are given respectively by:
$$P_t = \left[ \int_{\omega \in \Omega_{d,t}} p_{d,t}(\omega)^{1-\sigma} \, d\omega + \int_{\omega \in \Omega_{d,t}^*} p_{d,t}^*(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}} \tag{5}$$

$$P_t^* = \left[ \int_{\omega \in \Omega_{d,t}^*} p_{d,t}^*(\omega)^{1-\sigma} \, d\omega + \int_{\omega \in \Omega_{d,t}} p_{d,t}(\omega)^{1-\sigma} \, d\omega \right]^{\frac{1}{1-\sigma}}$$

With no money in the model the real exchange rate is equal to the ratio of aggregate prices indices in the foreign and home country, $RER_t = P_t^*/P_t$.

### 4.2 Household sector

The representative household in each economy gains utility from consumption of the non-traded final consumption good. In each economy the household labour endowment ($L$ in home and $L^*$ in foreign) is supplied inelastically in the domestic economy and receives a per unit household real wage in terms of the respective country consumption good ($w_{l,t}$ in home and $w_{l,t}^*$ in foreign). Domestic borrowing by the household (denoted by $b_{l,t+1}$ for the home household) is chosen to maximise expected utility given wage income and debt repayments. Household sector borrowing is assumed to be unconstrained. Whilst this is clearly an abstraction it greatly simplifies the analysis since the steady state interest rates are then determined from the household euler equation.

The preferences of the home household are given by:

$$U_{l,t} = \sum_{s=t}^\infty \beta^{s-t} \ln (c_{l,t}^s)$$

where $\beta$ is the household discount rate (common to home and foreign households) and $c_{l,t}^s$ is the household’s consumption of the home final good at time $t$. Consumption and gross debt repayments ($Rtb_{l,t}$ in units of home consumption good) are funded from wage income and new borrowings giving the household sector flow of funds as:

$$c_{l,t}^t + Rtb_{l,t}^t = w_{l,t}^t L + b_{l,t+1}$$

The first order conditions with respect to domestic borrowing give standard unconstrained Euler equations (with corresponding equations for the foreign household). With log utility the household consumes a fraction $(1-\beta)$ of wealth in each period (ie the return on last period savings plus the discounted present value of future wage income).

### 4.3 Entrepreneurial sector

Similar to households, entrepreneurs maximise expected utility over consumption and supply one unit of entrepreneurial labour in each period for which they receive an entrepreneurial real wage ($w_{e,t}^*$ in home and $w_{e,t}^*$ in foreign). An entrepreneur’s labour can be supplied to any producer, ie it is not restricted to use in the operation of the entrepreneur’s own production project. However, a number of crucial characteristics distinguish entrepreneurs from households. First, they have access to an investment project by which they can produce intermediate goods for domestic and export markets. Second, entrepreneurs are heterogeneous in the productivity with which they produce intermediate goods indexed by $\varphi$. The productivity distribution is invariant with cumulative distribution function $G(\varphi)$ and density function $g(\varphi)$. Third, as discussed in Section 3, entrepreneurs face credit constraints which are affected by their choice over investment projects. Finally, entrepreneurs are more impatient than households.

16Entrepreneurial labour income is required so that those entrepreneurs who do not produce in the steady state and are constrained to zero borrowing have positive consumption levels.
4.3.1 Investment projects

In each period the entrepreneur can choose to invest in a variety of projects. First, the entrepreneur may choose not to produce and simply save through domestic savings yielding a gross rate of return of \( R_t \). Second the entrepreneur can invest in the production of intermediate goods for the domestic market. Third, the entrepreneur may choose to invest in production for both the domestic and export markets (given the cost structure detailed below an entrepreneur will never produce for domestic markets only).

Similar to the multiple factor version of Melitz (2003) developed in Bernard et al. (2007b), intermediate goods production by home entrepreneurs requires a composite labour input of entrepreneurial and household labour which incorporates both a per period fixed cost, \( f \), and a variable cost equal to \( 1/\varphi \) per unit of next period output. There is a one period lag in production with next period’s output requiring composite labour input today, \( l_t(\varphi) \):

\[
l_t(\varphi) = \frac{y_{d,t+1}(\varphi)}{\varphi} + f
\]

where \( y_{d,t+1}(\varphi) \) is next period’s production. The composite labour input is a constant returns-to-scale, Cobb-Douglas aggregate over household and entrepreneurial labour inputs, \( l_{t}^{h}(\varphi) \) and \( l_{t}^{e}(\varphi) \) respectively:

\[
l_t(\varphi) = l_t^h(\varphi)^\zeta l_t^e(\varphi)^{1-\zeta}
\]

where \( 0 < \zeta < 1 \). Cost minimisation gives a composite wage \( w_t \) which represents the cost per unit of composite labour input, \( w_t = (w_t^h/\zeta)^\zeta (w_t^e/(1-\zeta))^{1-\zeta} \).

Producers of the final consumption good value a variety of intermediate inputs and so, given the fixed cost, each entrepreneur produces a differentiated intermediate good under monopolistic competition. If the entrepreneur wishes to export she must pay an additional fixed composite labour cost of \( f_x \) per period. Exporting also incurs a variable iceberg transportation cost \( \tau > 1 \) per unit exported. Since the fixed cost of production \( f \) is incurred whether or not the firm produces for the domestic market then the firm will always be better off producing for both the domestic and export markets than for the export market alone. In the former case an entrepreneur’s total composite labour demand is given by:

\[
l_t(\varphi) = \frac{y_{d,t+1}(\varphi)}{\varphi} + \frac{\tau y_{x,t+1}(\varphi)}{\varphi} + f + f_x
\]

4.3.2 Entrepreneurial equity investment

The entrepreneur must choose how to allocate her net worth state variable \( a_t \) between consumption \( c_t(\varphi) \) and investment to maximise discounted utility:

\[
u_t(\varphi) = \sum_{s=t}^{\infty} \delta^{s-t-1} \ln (c_t(\varphi))
\]

subject to the flow of funds and the transition of net worth. The entrepreneurial discount rate, \( \delta \), is assumed common across countries with entrepreneurs assumed to be more impatient than households ie \( \delta > \beta \) where \( \beta \) is the household discount rate.\(^{17}\)

An entrepreneur’s net worth entering time \( t+1 \), \( a_{t+1}(\varphi) \), is composed of the gross returns on the equity investment, \( a_t(\varphi) - c_t(\varphi) \), made at time \( t \). I define the gross rate of return on this investment realised at time \( t+1 \) in units of home consumption good as \( F_{t+1}(\varphi) \). This return varies with the entrepreneur’s investment choice which will depend on aggregate variables plus the entrepreneur’s productivity and level of net worth at time \( t \). Thus the transition of net worth is given by:

\[
a_{t+1}(\varphi) = F_{t+1}(\varphi)(a_t(\varphi) - c_t(\varphi))
\]

Consider the case where the entrepreneur chooses to produce for domestic and export markets. In this case the gross return represents total real revenues from production minus gross debt repayments:

\(^{17}\)A lower effective discount factor for entrepreneurs can equivalently be rationalised by assuming a death probability \( \pi \) such that \( \delta = \pi/\beta \) and that a new generation of entrepreneurs are born each period such that the total population of entrepreneurs remains constant. The assumption of a lower discount rate for entrepreneurs is widely adopted in the related literature on financial constraints to ensure that the entrepreneur can never fully self-finance her investment.
\[ F_{t+1} (\varphi) (a_t (\varphi) - c_t (\varphi)) = y_{d,t+1} (\varphi) \overline{p}_{d,t+1} (\varphi) + y_{x,t+1} (\varphi) \overline{p}_{x,t+1} (\varphi) - R_{t+1} b_{t+1} (\varphi) \] (13)

where the relative domestic and export prices compared with the aggregate price level in the producer’s country of residence are given by \( \overline{p}_{d,t} (\varphi) = p_{d,t} (\varphi) / P_t \) and \( \overline{p}_{x,t} (\varphi) = p_{x,t} (\varphi) / P_t \) respectively.

At time \( t \), the entrepreneur’s equity investment plus entrepreneurial wage received and new domestic borrowings will finance any labour costs incurred in production. Thus the entrepreneur’s flow of funds is:

\[ a_t (\varphi) - c_t (\varphi) + w_{t,t+1}^c + b_{t+1} (\varphi) = w_t l_t (\varphi) \] (14)

where \( w_t l_t (\varphi) \) is the cost of composite labour investment in production projects. Given the flow of funds and net worth transition, the first order condition with respect to consumption choice is:

\[ \frac{1}{c_t (\varphi)} = \left( \frac{\delta F_{t+1} (\varphi)}{c_{t+1} (\varphi)} \right) \] (15)

Combined with the net worth transition equation this yields the standard results with log utility that current consumption is a fixed fraction \((1 - \delta)\) of current wealth (defined as net worth plus the value of future entrepreneurial labour income discounted by gross project returns). Using the Euler equation (15) and net worth transition (12), the entrepreneur’s utility maximising investment project choice is that with the highest return \( F_{t+1} (\varphi) \).

### 4.3.3 Optimal investment project choice

The returns from the different investment projects are derived from the entrepreneur’s optimal choices over borrowing levels and production levels. This maximisation is subject to the flow of funds (14), the net worth transition (12) and definition of the gross equity investment return (13), domestic borrowing constraints (1), the demand from domestic and overseas final goods producers (3 and 4 respectively).

For the home entrepreneur, the first order condition with respect to domestic borrowing \( b_{t+1} (\varphi) \) is:

\[ 1/c_t (\varphi) = \left( \frac{\delta}{c_{t+1} (\varphi)} + \lambda_1^d (\varphi) \right) R_{t+1} \] (16)

where \( \lambda_1^d (\varphi) \) is the multiplier on the domestic borrowing constraint (with corresponding complementary slackness condition holding).

The first order conditions with respect to domestic production, \( y_{d,t+1} (\varphi) \), and export production, \( y_{x,t+1} (\varphi) \), give the following relative pricing conditions:18

\[ \overline{p}_{d,t+1} (\varphi) = \frac{w_t \sigma}{\phi (\sigma - 1) \delta / c_{t+1} (\varphi) + \theta \frac{\rho}{1 - \rho} \lambda_1^d (\varphi)} \] (17)

\[ \frac{\overline{p}_{x,t+1} (\varphi)}{\overline{p}_{d,t+1} (\varphi)} = \frac{1 - \rho}{\delta / c_{t+1} (\varphi) + (\theta \mu) \frac{\rho}{1 - \rho} \lambda_1^d (\varphi)} \] (18)

How do prices differ from the case with no credit constraints? First, credit constraints introduce an additional pricing wedge \( w_{t+1} (\varphi) > 1 \) over the standard unconstrained marginal cost plus fixed mark-up pricing rule. For example, for domestic production, the unconstrained relative price of a home entrepreneur \( \overline{p}_{d,t+1} (\varphi) = \frac{w_t R_{t+1} \sigma}{\phi (\sigma - 1) \delta} \). The additional wedge added to domestic prices due to credit constraints is increasing in the cost of finance and the extent to which the constraint binds and is decreasing with the credit multiplier:

---

18In the absence of aggregate uncertainty and given the CES demand functions which the entrepreneur faces, the choice of investment in the intermediate output project is equivalent to a decision on relative prices.
\[ v_{t+1}(\varphi) = \left( 1 - R_{t+1} \left( 1 - \theta^{\omega-1} \right) c_t(\varphi) \lambda_{t+1}^2(\varphi) \right)^{-1} \] (19)

A similar additional pricing wedge applies to export prices whose value also depends on the relative recovery rate on export output \( \mu \).

Second, compared to the case without credit constraints, export prices differ from domestic prices due not only to the variable trade costs but also the differences in their respective pricing wedges. In turn these pricing wedges depend on the different treatment of export revenues to domestic revenues by creditors.

Given the above optimal decisions of the entrepreneur it is possible to define the different possible values of the gross return \( F_{t+1}(\varphi) \) associated with the different investment options. Comparison of these returns yields the optimal entrepreneurial choice between no production (yielding return \( F_{t+1}^1(\varphi) = R_{t+1} \)); domestic only production (yielding return \( F_{t+1}^2(\varphi) \)); and; domestic and export production (yielding return \( F_{t+1}^3(\varphi) \)). Let the set of entrepreneurs in home and foreign be denoted \( \Omega \) and \( \Omega^* \) respectively.

The set of home entrepreneurs who produce goods for domestic final goods producers, \( \Omega_{d,t+1} \) (where \( \Omega_{d,t+1} \subseteq \Omega \)), are those for whom \( F_{t+1}^2(\varphi) \geq F_{t+1}^1(\varphi) \) and the set who also export to foreign final goods producers, \( \Omega_{x,t+1} \) (where \( \Omega_{x,t+1} \subseteq \Omega_{d,t+1} \)), are those for whom \( F_{t+1}^3(\varphi) \geq F_{t+1}^2(\varphi) > R_{t+1} \). The investment decisions of foreign entrepreneurs can be characterised in a similar manner.

### 4.4 Aggregate conditions

In equilibrium, in each period market clearing conditions must hold in both economies for domestic entrepreneurial and household labour markets, credit markets, intermediate and final consumption goods markets. Funds market clearing implies that total borrowings within each economy are equal to zero \( (B_{t+1} = B_{x,t+1}^* = 0) \). For each intermediate good, total demand from final goods producers must equal entrepreneurial production. Final goods consumption also must equal final goods output in each country, ie \( C_t = Y_t^* \) and \( C_t^* = Y_t^* \) and the value of final goods output equals the value of inputs (by the zero profit condition for final goods producers). In addition to these market clearing conditions, the home and foreign aggregate pricing equations (Equation 5) must be satisfied. Finally, to close the model the balance of payments must be in equilibrium, ie with financial autarky net exports must equal zero:

\[ NX_t \equiv \int_{\Omega_{x,t}} (\mathcal{F}_t^* (\varphi) y_{x,t}^* (\varphi) d\varphi - RER_t \int_{\Omega_{x,t}} (\mathcal{F}_t^* (\varphi) y_{x,t}^* (\varphi) d\varphi = 0 \] (20)

### 4.5 Equilibrium definition

To summarise, given household and entrepreneurial debt repayments and production levels entering period \( t \), an equilibrium is defined by a path of aggregate relative prices \{ \( RER_t, w_t, w_{t+1}^*, R_{t+1}, R_{x,t+1} \} \), aggregate quantities \{ \( Y_t, Y_t^*, C_t, C_t^*, B_{t+1}, B_{x,t+1}^* \) \}, individual entrepreneurial investment choices as reflected in intermediate goods relative prices \{ \( \mathcal{F}_{d,t+1}^* (\varphi), \mathcal{F}_{t,x,t+1}^* (\varphi), \mathcal{F}_{d,t+1}^* (\varphi), \mathcal{F}_{x,t+1}^* (\varphi) \} \), entrepreneurial credit constraint multipliers \{ \( \lambda_t^1(\varphi), \lambda_t^2(\varphi) \) \}, final goods producer intermediate input demands \{ \( y_{d,t+1}^* (\varphi), y_{x,t+1} (\varphi), y_{d,t+1}^* (\varphi), y_{x,t+1} (\varphi) \) \}, household and entrepreneurial consumption levels \{ \( c_t^l, c_t^r, c_t^l, c_t^r \) \} and borrowing by home and foreign households and entrepreneurs \{ \( b_{t+1}^l, b_{t+1}^r, b_{x,t+1}^l, b_{x,t+1}^r \) \} which are consistent with the optimal choices of households, entrepreneurs and final goods producers described above and which satisfy the above aggregate equilibrium conditions. In the sections below I focus on the properties of the steady state equilibrium given my interest in the long-run impact of the interactions between financial sector and trade reforms.

### 5 Domestic financial development with trade autarky

In order to illustrate the channels through which financial development affects aggregate productivity I first analyse the properties of the steady state equilibrium with financial and trade autarky. Each entrepreneur’s productivity is invariant and so, with stable aggregate variables, the entrepreneurial project choice decision (and hence aggregate value of production) will be constant provided each entrepreneur’s
net worth is stable. In this case, each entrepreneur’s consumption is also stable and so aggregate consumption will be stable if household consumption is unchanging. This gives the familiar condition from the household Euler equations \( R = R^* = 1/\beta \). Given the assumption that entrepreneurs are more impatient than households then from the entrepreneurial first order conditions all entrepreneurs will be constrained by their respective domestic borrowing constraints. I can now pin down entrepreneurial patience than households then from the entrepreneurial first order conditions all entrepreneurs will be constrained by their respective domestic borrowing constraints. I can now pin down entrepreneurial patience than households then from the entrepreneurial first order conditions all entrepreneurs will be constrained by their respective domestic borrowing constraints.

5.1 Entrepreneurial production decisions

Substituting the steady state multiplier on the borrowing constraint \( \lambda^1 (\varphi) = (1/R - \delta)/c (\varphi) > 0 \) into the optimal relative prices expressions (17 and 18) gives steady state entrepreneurial relative prices in home (if an entrepreneur produces) of:

\[
\bar{p}_d (\varphi) = \frac{w}{\rho \varphi \Theta_d}
\]

(21)

where \( \Theta_d = \delta + (\beta - \delta) \varphi \sigma^{-1} \) and \( \rho = (\sigma - 1)/\sigma \). The term \( \Theta_d \) can be thought as the reciprocal of the effective borrowing rate faced by entrepreneurs (as credit constraints fall the effective borrowing rate falls to the unconstrained rate of \( 1/\beta \)). Thus, with a common \( \theta \), in the steady state the pricing wedges are constant across entrepreneurs. Using the intermediate good demand functions these prices determine entrepreneurial production levels and revenues.

The comparison between an entrepreneur’s gross returns from saving and from production of intermediates for domestic sale gives the familiar condition that the entrepreneur will only produce if her revenues exceed the borrowing costs associated with production. This is the case provided that the entrepreneur’s productivity is sufficiently high, \( \varphi > \bar{\varphi} \) where \( \bar{\varphi} \) is defined by:

\[
\bar{p}_d (\bar{\varphi}) y_d (\bar{\varphi}) = \frac{f w}{(\beta - \rho \Theta_d)} \Leftrightarrow \bar{\varphi}^{-\sigma} = \frac{w^\sigma f (\rho \Theta_d)^{1-\sigma}}{Y (\beta - \rho \Theta_d)}
\]

(22)

Entrepreneurs who have a productivity lower than \( \bar{\varphi} \) do not produce and, given that their credit constraints still bind, end up in the steady state just consuming their entrepreneurial wage each period. The mass of producing entrepreneurs is given by \( M (1 - G (\bar{\varphi})) \). Following Melitz (2003), aggregate productivity, \( \tilde{\varphi} (\bar{\varphi}) \), can be defined as a weighted average of the productivities of producing entrepreneurs:

\[
\tilde{\varphi} (\bar{\varphi}) = \left[ \frac{1}{1 - G (\bar{\varphi})} \int_{\bar{\varphi}}^{\infty} \varphi^{-\sigma} g (\varphi) d\varphi \right]^{1/(\sigma - 1)}
\]

(23)

A tightening of the credit multiplier on domestic revenues (ie fall in \( \theta \)) has two partial equilibrium effects on the cutoff through raising the entrepreneur’s cost of production and hence price. On the one hand higher price charged by the entrepreneur reduces revenues, increasing the term \( (\rho \Theta_d)^{1-\sigma} \). On the other hand higher prices are reflected in reduced investment needs and hence repayments (increasing the term \( (\beta - \rho \Theta_d) \)). Whilst the former would tend to increase the productivity cutoff the latter may reduce the cutoff. In partial equilibrium, the net effect of these two forces is such that the former effect dominates and the partial derivative of the domestic productivity cutoff with respect to the domestic financial development \( \theta \) is negative:

\[
\frac{\partial \bar{\varphi}_d}{\partial \theta} = - \frac{\bar{\varphi}_d (1 - \theta) (\beta - \delta)^2}{\Theta_d (\beta - \rho \Theta_d)} < 0
\]

Note that these comparative statics ignore the general equilibrium effect of financial development on the cutoff via aggregate prices and quantities. This is an important omission since, as shown below, the general equilibrium effect will tend to mean that higher financial development leads to a higher domestic production productivity cutoff.

\[^{19}\text{As detailed in Melitz (2003), } \tilde{\varphi} (\bar{\varphi}) \text{ is the weighted harmonic mean of producing entrepreneurs’ productivities with the weights given by their relative output shares.}\]
5.2 Properties of the steady state autarky equilibrium

Using the properties of the intermediate demand functions, the autarky equilibrium can be defined by a system of five equations in the real wage, domestic interest rate, domestic production cutoff, aggregate borrowing and final goods output. The corresponding equations are the household euler equation, the productivity cutoff definition, labour and credit market clearing conditions and the aggregate pricing equation.

Proposition 1 In financial and trade autarky, there exists a unique steady state equilibrium in home referenced by relative prices \( \{ w, R \} \), the domestic production cutoff \( \{ \varphi_d \} \) and aggregate quantities \( \{ Y, B \} \) from which all other endogenous variables can be derived. As domestic financial development improves (ie the credit multiplier \( \theta \) increases) then the steady state comparative statics are as follows:

- Average productivity of intermediate output rises through a reallocatioon of production towards more productive firms as the domestic production productivity cutoff, \( \varphi_d \), increases.
- Real wages increase.
- Aggregate consumption of the final good rises.

Proof. See Appendix B.1. ■

Thus by adding a simple credit constraint within this heterogeneous firm model, financial sector reforms increase real wages and average productivity through a reallocative process very similar to that emphasised in relation to trade liberalisation. With households unconstrained, all the reallocative work is done by the wage rate and labour markets.\(^{20}\) If the credit multiplier is relaxed then, for a given wage, entrepreneurs lower their prices, increasing revenues and profits. The rise in \( \theta \) has an intensive margin effect increasing investment of existing producers. Ceteris paribus, in partial equilibrium there would also be an extensive margin effect since, with profits higher, more firms are induced to produce (see Figure 2). However, with lower prices and more varieties the real wage rate rises. This reduces revenues and raises costs. With profits reduced, lower productivity firms exit production. As illustrated in Figure 2, the overall effect is an increase in the productivity required for entrepreneurs to make positive profits from domestic production. Equivalently for labour market clearing to hold the mass of producing firms must fall if the rise in \( \theta \) induces greater labour demand across firms. Although the variety of intermediate inputs falls and the wage rises, the beneficial effects on unit costs of higher \( \theta \) and higher average productivity lead to an overall rise in final goods production and consumption.

6 Domestic financial development and trade

The two economies are now open to trade in goods and entrepreneurs can choose whether to export intermediates in addition to selling them to domestic final goods producers.

6.1 Entrepreneurial production decisions

When the economies are open to trade the relative price of exports to domestic output for a given entrepreneur is defined as follows:

\[
\frac{\bar{p}_x(\varphi)}{\bar{p}_d(\varphi)} = \frac{\tau \Theta_d}{\Theta_x}
\]  

(24)

where \( \Theta_x = \delta + (\beta - \delta)(\mu \theta)^{\frac{\sigma - 1}{\sigma}} \) is the reciprocal of the effective borrowing rate when export output is pledged as collateral. Compared to the standard model, the relative price of exports now depends not just on the trade costs but how export and domestic pledged output are treated by domestic creditors. If the relative pledgeability of export output, \( \mu \), rises then the relative price of export to domestic sales falls. As the overall domestic financial development rises the relative price of exports rises if \( \mu < 1 \), i.e. if the fall in the pricing wedge is greater for domestic production, and falls if \( \mu > 1 \).

\(^{20}\) The comparative static results for average productivity and real wages with respect to the credit multiplier are similar to those in Aoki et al. (2006) when the credit constraint binds. However, in their model, with households also constrained, as the credit multiplier rises there is adjustment of the real interest rate as well as the real wage.
Figure 2: Impact of credit multiplier $\theta$ on entrepreneurial production in trade autarky

Again the comparison between an entrepreneur’s gross returns to domestic production versus those for domestic and export production gives a productivity cutoff $\phi$ above which the additional revenues from exporting as well as producing for domestic sales exceed the additional costs. The exporting productivity cutoff is defined by:

$$p_x(\phi_x y_x(\phi_x) = \frac{f_x w}{(\beta - \rho \Theta_x)} \Leftrightarrow \frac{\phi_x^{\sigma - 1}}{\phi_x^{\sigma - 1}} = \frac{w^{\sigma} RER^{\frac{\sigma - \sigma}{\sigma}} f_x (\tau/\rho \Theta_x)^{\sigma - 1}}{Y^* \left(\beta - \rho \Theta_x\right)}$$

The partial equilibrium effect of greater overall financial development $\theta$ and the ability to lend against export output, $\mu$, is to reduce $\phi_x$, ie increase the propensity of intermediate producers to export. The ratio of the exporting to domestic only productivity cutoff is given by:

$$\left(\frac{\phi_x}{\phi_d}\right)^{\sigma - 1} = \frac{f_s w}{f_x Y^*} \left(\frac{\Theta_d}{\Theta_x}\right)^{\sigma - 1} \left(\frac{\beta - \rho \Theta_d}{\beta - \rho \Theta_x}\right)$$

The relative magnitudes of the exporting to domestic only productivity cutoff (and hence the likelihood of exporting given domestic production) can be decomposed into three terms. First, as variable and relative fixed trade costs increase, ie as $\tau^{\sigma - 1} f_s / f$ rises, the relative level of the exporting cutoff increases. Second, the terms $RER^{\frac{\sigma - \sigma}{\sigma}}$ reflect the relative scale of demand from foreign compared to domestic final goods producers. The higher relative foreign demand the lower the cutoff ratio $\phi_x / \phi_d$. The third term, reflecting financial constraints in the pledging of both domestic and export output, is the novel feature compared to the related literature.

Focusing on this third effect, if the relative ability of creditors to recover exports compared to domestic output, $\mu$, rises then the partial equilibrium effect is to increase propensity of producers to export. However, the impact of changes in overall financial development, $\theta$, is ambiguous. This is because, in contrast to Chaney (2005) and Manova (2006), domestic production is also subject to credit constraints. Thus, a rise in $\theta$ will reduce both the domestic and export production cutoffs in partial equilibrium. For $\mu$ less than one, a rise in overall financial development from a low level initially will tend to increase the relative exporting cutoff since it has a greater effect on the domestic production cutoff. As financial development rises the elasticity of the relative cutoff with respect to $\theta$ falls. If the relative recovery rate $\mu$ is greater than one then the effects are in the opposite directions.

In line with firm-level empirical evidence which suggests a partitioning of producing firms into exporters and non-exporters I focus on situations where $\phi_x / \phi_d > 1$. Given this ordering of the production cutoffs, home entrepreneurs can be partitioned into those who do not produce ($\phi < \phi_d$) who have mass $MG(\phi_d)$, those who produce for domestic markets only ($\phi_d \leq \phi < \phi_d$) who have mass
two countries’ aggregate pricing equations. The balance of payments condition (i.e., balanced trade given financial autarky) and the equations, the definitions of the four productivity cutoffs, labor and credit market clearing conditions at home and abroad, the balance of payments condition (i.e., balanced trade given financial autarky) and the countries’ aggregate pricing equations can be derived. The corresponding equations are the home and foreign household Euler costs outlined in [Bernanke et al.](2003)

Before considering trade and financial reform interactions, I first illustrate how home financial sector reforms affect both the home and the foreign economy for given trade costs. This aids understanding of the mechanisms through which the different reform policies interact. I compare the steady state equilibria of labor unit in both countries. Labor endowments are identical across countries with the masses of labour and entrepreneurs which rescale the results set at \( \sigma = 3.8 \) and those who produce for export as well as domestic markets (\( \varphi_x \leq \varphi \)) who have mass \( M \left( 1 - G \left( \varphi_x \right) \right) \). Similar expressions can be derived in foreign for \( \varphi^*_d \) and \( \varphi^*_x \).

### 6.2 Properties of non-symmetric financial autarky steady state equilibrium

The symmetric case can be shown to be unique without the need for specific distributional assumptions. However, given that financial reforms in particular are undertaken in a domestic economy without the need for corresponding reforms in trading partners, it is of more interest to focus on the non-symmetric equilibrium. Following the related literature, I adopt the empirically plausible Pareto distribution for the numerical solutions where \( G(\varphi) = 1 - \left( \varphi_{\min} / \varphi \right)^{a} \) with \( \varphi_{\min} > 0 \) the lower-bound productivity and \( a \) is the distribution shape parameter.\(^{21}\) Under this distributional assumption, the non-symmetric steady state equilibrium under financial autarky is uniquely defined.

**Proposition 2** In financial autarky with entrepreneurial productivity following a Pareto distribution, there exists a unique steady state equilibrium with costly goods trade which can be uniquely defined by a system of thirteen equations in relative prices \( \{w, w^*, R, R^*, \text{RER} \} \), domestic market production and exporting cutoffs \( \{ \varphi^*_d, \varphi_x, \varphi^*_d, \varphi^*_x \} \) and aggregate quantities \( \{ Y^*, B, B^* \} \) from which all other endogenous variables can be derived. The corresponding equations are the home and foreign household Euler equations, the definitions of the four productivity cutoffs, labor and credit market clearing conditions at home and abroad, the balance of payments condition (i.e., balanced trade given financial autarky) and the two countries’ aggregate pricing equations.

**Proof.** See Appendix B.2. \( \blacksquare \)

### 6.2.1 Numerical parameter values

In the numerical examples below, I focus solely on the impact of differences in credit multipliers between countries with all other parameters identical. For the production and productivity distribution parameters, I follow [Bernard et al.](2007b) in setting the elasticity of substitution between different varieties \( \sigma = 3.8 \) based on the estimates from US data of [Bernard et al.](2003), the shape parameter of the Pareto distribution at \( a = 3.4 \) and the minimum productivity \( \varphi_{\min} = 0.2 \). The discount factors of households and entrepreneurs are set at \( \beta = 0.96 \) and \( \delta = 0.92 \) respectively (yielding a steady state annual real interest rate of 4%). The share of household labour costs in total labour costs is set at \( \zeta = 0.985 \) following [Bernanke et al.](1999) since entrepreneurial production rather than their labour income is the focus of the model. For simplicity, the fixed costs of domestic and export production are set equal to one composite labour unit in both countries. Labour endowments are identical across countries with the masses of labour and entrepreneurs which rescale the results set at \( L = L^* = 100 \) and \( M = M^* = 20 \) giving an export propensity of producers of around 30% when variable trade costs \( \tau = 1.4 \).\(^{22}\) This compares, for example, to estimates of around 21% of plants in the 1991 US Census of Manufactures [Bernard et al.](2003). In developing economies, [Aitken et al.](1997) find that around 27% of a 1986 and 1989 sample of Mexican manufacturing plants were exporters whilst the samples of [Clerides et al.](1998) have 35% of Moroccan plants as exporters (1984-1991), 9.5% of Colombian plants (1981-1991) and a comparable number of 23% for Mexican plants (1986-1990).

### 6.2.2 Impact of reforms to home financial sector development

Before considering trade and financial reform interactions, I first illustrate how home financial sector reforms affect both the home and the foreign economy for given trade costs. This aids understanding of the mechanisms through which the different reform policies interact. I compare the steady state equilibria with goods trade (subject to symmetric iceberg trade costs \( \tau = \tau^* = 1.4 \)) under different values of home financial development (i.e., varying the credit multipliers \( \theta \) and \( \mu \)) with foreign credit multipliers constant at \( \theta^* = 0.5 \) and \( \mu^* = 1 \).

\(^{21}\)For simplicity, the shape parameter \( a \) is common across countries (although the lower bound productivity can be different across countries). [Helpman et al.](2004) illustrate the empirical fit of the Pareto distribution and note that the assumption that \( a > \sigma - 1 \) is required to ensure finite variances of revenues.

\(^{22}\)Such trade costs are of similar order to the 44% estimate of border-related trade barriers in the representative trade costs outlined in [Anderson & van Wincoop](2004).
Entrepreneurial propensity to produce and export  As shown in Figure 3, as the home credit multipliers rise the domestic production productivity cutoff in home rises via the same channels as in autarky (ie increasing investment via the intensive margin with the resulting real wage rise increasing costs and causing lower productivity firms to drop out of production). The higher wage costs also cause the export production productivity cutoff in home to rise. In the foreign economy the impact of changes in $\theta$ all take place via the trade channel. In particular in this general equilibrium set-up home financial development affects not just home export propensity (which has been emphasised in, for example, Chaney, 2005; Manova, 2006) but also the export propensities of foreign firms.

In the home economy the rise in $\theta$ affects the export propensity (ie the ratio $\overline{\sigma}_d/\overline{\sigma}_d$) through two channels. The first is an ambiguously signed partial equilibrium effect via the real exchange rate in reducing the relative demand for foreign relative to home intermediates. In the home economy the net effect is that the propensity to export increases with home financial development if the relative pledgeability of export output $\mu$ is low but falls with $\theta$ for low $\mu$. In the foreign economy it is only the general equilibrium effect that is at work with a rise in home financial development causing a fall in relative demand for foreign intermediates resulting in a reduction in the foreign entrepreneurs’ propensity to export. This reduces overall labour demand and the domestic production cutoff in foreign falls slightly to ensure market clearing. Whilst these results are illustrative they indicate a potential channel through which the financial development in one economy can spill over to the production and export patterns of trading partners.

Real wages and real exchange rate  As in the trade autarky case the home real wage rises with the home credit multiplier (Figure 4), ie the home final good price falls. This reflects a combination of lower mark-ups for existing producers and a rise in the average productivity of domestic and imported inputs which more than offset the reduction in varieties of inputs. In the foreign economy, the direct mark-up effect is not present with the result being that the lower variety of imports contributes to a fall in the foreign real wage as the home credit multiplier increases. Thus the real exchange rate rises with the home credit multipliers as home and foreign final goods prices fall and rise respectively.

Intermediate output productivity and firm size  The aggregate productivity of intermediate production with trade can be defined as a weighted average of productivities in a similar manner to the autarky case taking into account the relative mass of exporters to total producers and the relative level of export production (including iceberg trade costs) to domestic production for a given entrepreneur. As the home credit multiplier rises lower productivity domestic producers and exporters exit in the home economy. This increases the average home productivity of intermediate goods production and the average size of producing entrepreneurs (Figure 5). In the foreign economy there is a slight fall in the aggregate intermediate productivity and firm size reflecting the slight decrease in the domestic production cutoff.

Exports  From the properties of the demand functions for intermediate inputs, the relative value of exports to total intermediate sales (both in units of domestic final consumption good) at the entrepreneurial level is a function of the exporting and domestic production cutoffs and is identical across entrepreneurs. For a home entrepreneur this ratio is given by:

$$exrat = \left( 1 + \frac{\int (\beta - \rho \Theta_d) (\overline{\sigma}_d) \sigma^{-1}}{\int_x (\beta - \rho \Theta_d) (\overline{\sigma}_d) \sigma^{-1}} \right)^{-1}$$  \hspace{1cm} (26)

The impact of changes in $\theta$ on a home entrepreneur’s export ratio combines two effects. First there is the impact on $\overline{\sigma}_d/\overline{\sigma}_d$ which, as detailed in Equation 25, embodies relative demand from home and foreign final goods producers, financing differences between exports and domestic revenues and trade costs. Second, there is the impact on relative net profits per variable unit sold between export and domestic sales (ie $(\beta - \rho \Theta_d)/(\beta - \rho \Theta_d)$). The overall effect depends on the relative ability of entrepreneurs to pledge exports compared to domestic output to creditors. In the home economy, the illustrations suggest that an entrepreneur’s export ratio increases with $\theta$ if $\mu$ is high enough but has an inverted

23The varying lengths of the plots against $\theta$ for different values of $\mu$ reflects the restriction that $\mu \theta \leq 1$ as detailed in Assumption 2.
Figure 3: Impact of home overall financial development on entrepreneurial production cutoffs and propensity to export

(a) Domestic sales cutoff

(b) Exporting cutoff

(c) Entrepreneurial conditional export propensity

Note: Home and foreign identical except credit multipliers. Foreign credit multipliers constant at θ = 0.5 and μ = 1. Variable iceberg trade cost τ = τ = 1.4.

Figure 4: Impact of home overall financial development on composite real wages and real exchange rate under financial autarky

(a) Real wages

(b) Real exchange rate

Note: Home and foreign identical except credit multipliers. Foreign credit multipliers constant at θ = 0.5 and μ = 1. Variable iceberg trade cost τ = τ = 1.4.
U-shaped response if $\mu$ is relatively low as illustrated in Figure 6. Changes in home $\theta$ only affect a foreign entrepreneur’s export ratio in financial autarky via the impact on the propensity to export, i.e. via the direct trade channel. With the foreign propensity to export falling (i.e. $\tau^{\ast}_f / \tau^{\ast}_d$ rising) with $\theta$ this implies a decrease in a foreign entrepreneur’s export ratio as its trading partner’s financial development improves.

In the home economy the positive intensive margin effect on exports of improved home financial development combines with a generally positive extensive margin effect (for all but high levels of relative pledgeability of exports $\mu$) to give an overall increase in the ratio of aggregate intermediate exports to total sales. In the foreign economy, the ratio of aggregate intermediate exports to total sales falls as $\theta$. This reflects both an intensive margin effect (through lower relative demand from home final goods producers for foreign intermediate exports) and an extensive margin effect through the rise in the exporting production cutoff.

**Household and entrepreneurial utility** Household steady state consumption is dependent upon household wage income and the net return on steady state household savings (which, from domestic funds market clearing, are defined by aggregate entrepreneurial borrowing). Thus the effects of changes in home financial development on household utility can be split into a household wage effect and entrepreneurial borrowing effects. The latter can be categorised as a direct pledging effect whereby a rise in $\theta$ for given revenues will increase entrepreneurial borrowing capacity, an intensive margin effect on output via the wage and an extensive margin effect on the mass of producing and exporting entrepreneurs who are able to borrow. In the home economy a rise in $\theta$ leads to a rise in wages and a positive direct pledging effect which more than offsets the extensive margin effects of higher production cutoffs and thus increases household utility (Figure 7). In the foreign economy, with financial autarky, there is no direct pledging effect from changes in $\theta$ and the wage effect via the trade channel leads to a fall in household utility.

Turning to entrepreneurs, the steady state consumption of non-producing entrepreneurs, who are constrained and hence cannot borrow, is solely their entrepreneurial wage income. The consumption of producing entrepreneurs depends on both their entrepreneurial wage income and their profits from total sales (which are increasing in entrepreneurial productivity). Focusing on the home economy, as $\theta$ rises wages increase which unambiguously increases the utility of non-producing entrepreneurs. However, for given productivity higher $\theta$ implies lower profits from production as wages and production costs rise. Thus, for low productivity entrepreneurs utility rises with $\theta$ but for higher productivity producing entrepreneurs the negative profit effect more than outweighs the positive income effect and utility falls with $\theta$ (see Figure 7). This feature can be viewed as similar in nature to the interest group theory.
Figure 6: Impact of home overall financial development on firm- and aggregate-level export to total intermediate revenue ratios under financial autarky

(a) Entrepreneurial export ratio

(b) Aggregate export ratio

Note: Revenues in units of domestic consumption good. Home and foreign identical except credit multipliers. Foreign credit multipliers constant at $\theta^* = 0.5$ and $\mu^* = 1$. Variable iceberg trade cost $\tau = \tau^* = 1.4$.

proposed by Rajan & Zingales (2003) whereby incumbent producers may oppose financial sector reforms because it reduces their profits.

6.2.3 Interaction of trade and domestic financial sector reforms

Having illustrated how changes in home financial development affect both the home and foreign economy in the presence of trade I now turn to the interaction of financial sector and trade reforms. In particular I consider the level and marginal effects on the home economy of symmetric changes in the iceberg variable trade costs and how these effects vary with the home credit multipliers (with the foreign credit multipliers set at $\theta^* = 0.5$ and $\mu^* = 1$). In doing so it is convenient to define a trade freeness measure $TF = \tau^{1-\sigma}$ which lies between zero and one with $TF = 0$ as $\tau$ tends to infinity and $TF = 1$ for $\tau = 1$.

Home entrepreneurial propensity to produce and export: trade liberalisation For given credit multipliers, the impact of trade liberalisation between two economies who are already open to trade follows a similar pattern to that in the original Melitz (2003) setup. A rise in trade freeness leads to a rise in the domestic production cutoff, a fall in the exporting cutoff and rise in the propensity of producers to export in both countries. However, as illustrated in Figure 8, the magnitude of the marginal impact on the domestic production cutoff of these changes in trade freeness appears to increase with home financial development. When home financial development is relatively high the intensive and extensive margin effects of trade liberalisation on labour demand are amplified leading to a greater required rise in the domestic production cutoff to ensure labour market clearing. Whilst the marginal fall in export prices through trade liberalisation is reduced at higher levels of financial development the marginal impact on overall labour demand is higher due to the looser borrowing constraint. Although not illustrated, as in the standard model greater trade freeness increases export propensity and aggregate- and firm-level export ratios. The impact of the level of the credit multiplier is of unclear direction and, for these illustrations, appear to be of a second order compared to these direct effects.

Home real wages and real exchange rate: trade liberalisation Trade liberalisation increases the variety of imported inputs used in final goods production. Whilst the rise in the domestic production cutoff reduces the variety of domestic inputs they are of a higher productivity. The net effect is a fall in the aggregate price level and increase in home real wages. The real wage gains from trade opening

\footnote{Although asymmetric changes in trade costs can be analysed within the model I focus on symmetric changes in trade costs to reflect, for example, the implementation of bilateral or multilateral trade agreements.}
Figure 7: Impact of home overall financial development on steady state utility under financial autarky

(a) Household utility

(b) Home entrepreneurial utility

Note: Home and foreign identical except credit multipliers. Foreign credit multipliers constant at $\theta^* = 0.5$ and $\mu^* = 1$. Variable iceberg trade cost $\tau = \tau^* = 1.4$.

Figure 8: Impact of symmetric changes in variable trade costs on home entrepreneurial production cutoffs

(a) Domestic sales cutoff

(b) Exporting cutoff

Note: Home and foreign identical except overall credit multipliers $\theta$ and $\theta^*$. Foreign credit multipliers constant at $\theta^* = 0.5$ and $\mu^* = 1$. 
appear to be greater if overall financial sector development is lower and, for given \( \theta \), if the relative pledgeability of export output is higher (Figure 9). The intuition is that with \( \theta \) high the aggregate price level at home is relatively low and thus the less the impact of any additional fall in prices due to the trade liberalisation. The rise in trade freeness also increases the real wage in the foreign economy with the marginal impact also greater if home financial development is relatively low. The interplay of these two relative price changes suggests that if home development is lower than that in foreign then the real exchange rate is less than one and is falling with greater trade freeness.

**Figure 9**: Impact of symmetric changes in variable trade costs on home composite real wages and the real exchange rate

![Real Wages and Real Exchange Rate](image)

Note: Home and foreign identical except overall credit multipliers \( \theta \) and \( \theta^* \). Foreign credit multipliers constant at \( \theta^* = 0.5 \) and \( \mu^* = 1 \).

**Home intermediate output productivity and firm size: trade liberalisation** Additional labour demand stimulated by increasing investment in export production as trade freeness rises leads to a rise in the domestic production productivity cutoff. This contributes to a rise in intermediate productivity and average size of producing entrepreneurs (Figure 10). Reflecting the corresponding greater marginal effect of trade liberalisation on the domestic production productivity cutoff these effects are enhanced at higher levels of financial development (and at higher levels of relative export pledgeability for given \( \theta \)). Thus, in contrast with their impact on real wages trade and financial sector reforms appear to have complementary effects on aggregate productivity and firm-size.

**Home household and entrepreneurial utility: trade liberalisation** In terms of household utility, lower trade costs increase wage income. They also affect household saving (ie constrained entrepreneurial borrowing) via intensive and extensive margin effects on total pledgeable output for exporters and domestic producers. As \( \theta \) rises the real wage gains fall whilst the extensive margin effects increase (as the change in cutoffs rises). Overall, these effect combine to reduce the benefits to household utility from trade opening as \( \theta \) rises for given \( \mu \) (see Figure 11). This result is intuitive - using one policy tool, ie trade opening, to improve the allocation of production leads to less utility gain for consumer households if the resource allocation is more efficient in the first place through stronger financial development. However, the utility gains from trade liberalisation are enhanced if, for given \( \theta \), the relative ability to pledge export output compared to domestic output is increased.

Figure 11 provides an illustration of entrepreneurial utility levels for different degrees of trade openness with fixed financial development. For lower productivity entrepreneurs the real wage gains with higher trade freeness lead to higher utility. However, for higher productivity exporting firms greater trade openness may reduce utility if the impact of real wage rises on production costs more than offsets the greater revenues and lower iceberg costs. Although not illustrated, for lower productivity entrepreneurs for whom wage income is particularly important the interaction of trade and financial reforms has similar
Figure 10: Impact of symmetric changes in variable trade costs on home intermediate productivity and average size of producing entrepreneurs

(a) Intermediate output average productivity

(b) Intermediate producer average size

Note: Home and foreign identical except overall credit multipliers \( \theta \) and \( \theta^* \). Foreign credit multipliers constant at \( \theta^* = 0.5 \) and \( \mu^* = 1 \).

Figure 11: Impact of symmetric changes in variable trade costs on steady state home utility under financial autarky

(a) Household utility

(b) Home entrepreneurial utility

Note: Home and foreign identical except overall credit multipliers \( \theta \) and \( \theta^* \). In panel (a) foreign credit multipliers constant at \( \theta^* = 0.5 \) and \( \mu^* = 1 \). In panel (b) entrepreneurial utility illustrated with \( \mu = 1 \) for both economies and \( \theta = 0.25 \).
effects to that on household utility. For other entrepreneurs the picture is more complex combining the relative effects on both wages and intensive and extensive productive margins.

7 Conclusions

This paper provides a tractable extension of a two-economy heterogeneous firm model to incorporate financial constraints affecting both domestic and export production. In doing so it enables analysis of trade and domestic financial sector policy changes across countries in a non-symmetric setting and provides some important insights into the mechanisms through which the two reforms can reallocate production within the two economies, their potential similarities and their interactions. In particular, numerical illustrations suggest that on the one hand domestic financial sector development, in facilitating greater investment, can enhance the marginal effects of trade liberalisation in increasing average productivity and firm size. On the other hand, at higher levels of financial development the marginal benefits of trade liberalisation in reducing aggregate prices and raising real wages and household utility may be reduced. The numerical illustrations also highlight the role of exports as collateral in amplifying the benefits of trade liberalisation and the channels through which domestic financial sector reforms may be transmitted to trading partners.

The analysis of the macroeconomic impact of the interactions of trade and domestic financial sector reforms within a general equilibrium heterogeneous firm setting provides a novel addition to the existing theoretical literature. It also highlights the need for additional empirical research on their quantitative significance since, as noted by Banerjee & Duflo (2005), the question of whether there are potential gains from adding multiple sources of inefficiency to models to explain productivity differences across countries is dependent upon their empirical relevance. The evidence from existing studies on these interactions is limited. At the macro level, Chang et al. (2005) provide support for the complementary effects of financial development and trade openness on growth. At the firm-level, Topalova (2004), for example, considers whether the impact on productivity of trade reforms in India varies across states by financial depth and finds that although the point estimates are similar in states with high or low credit to GDP they are only significant in the former. Further empirical analysis on the interaction of the reallocative effects of trade and financial reforms and their joint impact on aggregate productivity levels is of particular interest. A focus for future work is thus to extend the model to a multi-sector set-up to derive empirically testable predictions on the effects of reform interactions to be applied to sectoral data on average firm-size and labour productivity. In addition to these important empirical steps there is the potential to modify the model to address other policy relevant issues. For example, to extend the analysis beyond the steady state in order to examine the transitional impact of reforms or to incorporate the potential reallocative effects of international financial liberalisation emphasised by Aoki et al. (2006).
A EBRD transition indicators

Full details of the EBRD transition indicator score methodology are available at www.ebrd.com. The transition indicators cover up to 29 Eastern European and former Soviet Union economies from 1989 to 2007: Albania, Armenia, Azerbaijan, Belarus, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, FYR Macedonia, Georgia, Hungary, Kazakhstan, Kyrgyz Republic, Latvia, Lithuania, Moldova, Mongolia, Montenegro, Poland, Romania, Russian Federation, Serbia, Slovak Republic, Slovenia, Tajikistan, Turkmenistan, Ukraine, and Uzbekistan.

The transition indicator scores are based on a judgement of an economy’s progress relative to defined classifications. Scores range from 1 (little change from centrally planned economy) through to 4+ (defined as 4.33) which represents norms of advanced industrial economies. Trade measure also includes restrictions to current account convertibility. In addition to the banking sector measure indicated in the figures below there is also an indicator for non-bank financial institutions and securities markets.

The trade and foreign exchange (FX) indicator ranges from a value of 1 through to 4+ (graded by the EBRD as 4.33) with “1: Widespread import and/or export controls or very limited legitimate access to foreign exchange; 2: Some liberalisation of import and/or export controls; almost full current account convertibility in principle, but with a foreign exchange regime that is not fully transparent (possibly with multiple exchange rates); 3: Removal of almost all quantitative and administrative import and export restrictions; almost full current account convertibility; 4: Removal of all quantitative and administrative import and export restrictions (apart from agriculture) and all significant export tariffs; insignificant direct involvement in exports and imports by ministries and state-owned trading companies; no major non-uniformity of customs duties for non-agricultural goods and services; full and current account convertibility; 4+: Standards and performance norms of advanced industrial economies: removal of most tariff barriers; membership in WTO.”

The corresponding definitions for the banking reform and interest rate liberalisation transition indicator are “1: Little progress beyond establishment of a two-tier system; 2: Significant liberalisation of interest rates and credit allocation; limited use of directed credit or interest rate ceilings; 3: Substantial progress in establishment of bank solvency and of a framework for prudential supervision and regulation; full interest rate liberalisation with little preferential access to cheap refinancing; significant lending to private enterprises and significant presence of private banks; 4: Significant movement of banking laws and regulations towards BIS standards; well-functioning banking competition and effective prudential supervision; significant term lending to private enterprises and significant presence of private banks; 4+: Standards and performance norms of advanced industrial economies: full convergence of banking laws and regulations with BIS standards; provision of full set of competitive banking services.”

B Proofs

B.1 Steady state equilibrium with trade and financial autarky - Proof of existence and uniqueness

The steady state equilibrium in home can be defined by the following system of five equations in relative prices \( \{w, R\} \), domestic market production cutoff \( \{\varphi_d\} \) and aggregate quantities \( \{Y, B\} \) from which all other endogenous variables can be derived:

- Household Euler equation: \( R = 1/\beta \)
- Domestic production cutoff: \( \varphi_d^{-1} = \frac{\sigma^{-1}}{\rho} \frac{f(\rho \varphi_d)^{1-\sigma}}{\beta - \rho \varphi_d} \)
- Funds market clearing: \( B = 0 \)
- Labour market clearing: \( L^\epsilon M^{-\zeta}/f = H(\Theta_d, \varphi_d) \) where \( H(\Theta_d, \varphi_d) = \frac{\rho \varphi_d}{\beta - \rho \varphi_d} A(\varphi_d) + 1 - G(\varphi_d) \) and
  \[ A(\varphi_d) = \frac{1}{\varphi_d} \int_{\varphi_d}^{\infty} (\varphi/\varphi_d)^{\sigma-1} g(\varphi) \, d\varphi. \]
  \( H(\Theta_d, \varphi_d) \) is increasing in \( \Theta_d \) and decreasing in \( \varphi_d \).

Aggregate Pricing equation: \( w^{\sigma-1} = M(\rho \Theta_d)^{\sigma-1} \int_{\varphi_d}^{\infty} \varphi^{\sigma-1} g(\varphi) \, d\varphi \)

To prove existence and uniqueness of the equilibrium first consider the labour market clearing condition. \( H(\Theta_d, \varphi_d) \) is continuously differentiable (provided \( g'(\varphi_d) \) is defined) and tends to zero as \( \varphi_d \to \infty \)
and tends to infinity as \( \varphi_d \to 0 \). There is thus a unique value of \( \varphi_d \) for which the labour market clearing condition holds. Substituting the equilibrium value of \( \varphi_d \) into the aggregate pricing equation uniquely defines the real wage. The domestic production cutoff then uniquely defines final goods output \( Y \). The real interest rate and aggregate borrowing are defined by the Euler equation and funds market clearing respectively.

Comparative statics with respect to \( \theta \):

- \( \frac{\partial \varphi_d}{\partial \theta} > 0 \): From the labour market clearing condition, using the implicit function theorem the equilibrium cutoff value can be shown to be increasing in \( \theta \).
- \( \frac{\partial w}{\partial \theta} > 0 \): Using the definition of the real wage from the aggregate pricing equation and the expression for \( \frac{\partial \varphi_d}{\partial \theta} \) it can be shown that the real wage is increasing in \( \theta \).
- \( \frac{\partial Y}{\partial \theta} > 0 \): In the productivity cutoff equation substitute in for wages from the aggregate pricing equation. Using the labour market clearing condition with some manipulation one obtains final goods output as a function of \( \varphi_d \) independent of \( \theta \) giving \( \frac{\partial Y}{\partial \theta} \propto \frac{\partial \varphi_d}{\partial \theta} > 0 \).

### B.2 Non-symmetric steady state equilibrium with financial autarky - Proof of existence and uniqueness

Using the fact that \( \bar{p}_d(\varphi_d) \equiv (\varphi_d)^{\sigma-1} \) plus the domestic production productivity cutoff then

\[
\bar{p}_d(\varphi_d) y_d(\varphi_d) = \left( \frac{w}{\varphi_d} \right)^{\sigma-1}
\]

Similar expressions can be used to define exporting revenues in relation to \( \varphi_x \) and also for variable labour demand. The steady state equilibrium can be defined by the following system of thirteen equations in relative prices \( \{w, w^*, R, R^*, \text{RER}\} \), domestic market production and exporting cutoffs \( \{\varphi_d, \varphi_x, \varphi_d^*, \varphi_x^*\} \) and aggregate quantities \( \{Y, Y^*, B, B^*\} \) from which all other endogenous variables can be derived:

**Household Euler equations:**

\[
R = 1/\beta; \quad R^* = 1/\beta
\]  

(27)

**Domestic production cutoff definitions:**

\[
\varphi_d^{\sigma-1} = \frac{w^f (\rho \Theta_d)^{1-\sigma} M k \varphi_d^{\sigma-1}}{\beta - \rho \Theta_d}
\]  

(28)

\[
\varphi_d^{\sigma-1} = \frac{w^* f^*(\rho \Theta_d)^{1-\sigma} M^* k \varphi_d^{\sigma-1}}{\beta - \rho \Theta_d}
\]  

(29)

where \( k = a/(a-\sigma+1) \).

Relative cutoffs \( \varphi_x/\varphi_d \) and \( \varphi_x^*/\varphi_d \) (which cancel out foreign and domestic final goods production):

\[
\left( \frac{\varphi_x}{\varphi_d} \right)^{\sigma-1} = \left( \frac{\text{RER} w^*}{w} \right)^{-\sigma} \Upsilon
\]  

(30)

\[
\left( \frac{\varphi_x^*}{\varphi_d} \right)^{\sigma-1} = \left( \frac{\text{RER} w^*}{w} \right)^{\sigma} \Upsilon^*
\]  

(31)

where:

\[
\Upsilon \equiv \frac{f_x}{f^*} \left( \frac{\tau^*_d}{\Theta_x} \right)^{\sigma-1} \left( \frac{\beta - \rho \Theta_d^*}{\beta - \rho \Theta_x^*} \right)
\]

\[
\Upsilon^* \equiv \frac{f_x}{f^*} \left( \frac{\tau^*_d}{\Theta_x} \right)^{\sigma-1} \left( \frac{\beta - \rho \Theta_d}{\beta - \rho \Theta_x} \right)
\]

**Composite labour input clearing conditions:**

\[
\varphi_d^{-a} = \Omega_1 - \Omega_2 \varphi_x^{-a}
\]  

(32)

\[
\varphi_d^{a*} = \Omega_1 - \Omega_2 \varphi_x^{a*}
\]  

(33)
implies that parameter values must satisfy $\Upsilon \Upsilon$ can be solved numerically using standard techniques.

output from the domestic production productivity cutoffs. From these variables all other individual equilibrium values of $\phi$ gives a downward sloping curve which can be shown to intersect (36).

The relative cutoff equations define a system in the two exporting cuts: as mentioned I focus on the case for which $\phi > 1$ and $\phi_1 > 1$ which implies that parameter values must satisfy $\Upsilon \Upsilon > 1$.

For the equilibrium values of $\phi_x$ and $\phi_x^*$ the wages can be uniquely obtained from the aggregate pricing equations (again once the real exchange rate has been substituted in):

$$w^{* -1} = (\Omega_1 - \Omega_2 \phi_x^{* -1})^{1-\sigma} \psi_1 + (\frac{\phi_x^{*}}{\phi_x})^{\sigma} \phi_x^{\sigma-1} \psi^*_2$$

$$w^{\sigma -1} = (\Omega_1 - \Omega_2 \phi_x)\psi_1 + (\frac{\phi_x}{\phi_x})^{\sigma} \phi_x^{-\sigma} \psi^*_2$$

Domestic production cutoffs can be obtained from the labour market clearing conditions given the equilibrium values of $\phi_x$ and $\phi_x^*$ which combined with equilibrium wages then determine equilibrium output from the domestic production productivity cutoffs. From these variables all other individual steady state choice variables can be determined. Given the existence of a unique equilibrium the model can be solved numerically using standard techniques.

$$B = B^* = 0$$

Balance of trade conditions:

$$RER = \frac{w}{w^*} \left( \frac{\phi_x}{\phi_x^*} \right)^{\alpha}$$

where $\Xi \equiv \frac{M_0}{M_0 f_0} \left( \frac{\phi_{min}}{\phi_{min}} \right)^{\alpha}$.

Aggregate pricing equations in home and foreign respectively:

$$1 = w^{1-\sigma} \phi_d^{1-\sigma} \psi_1 + (RER w^* \phi_x^{1-\sigma} \psi^*_1$$

$$1 = w^{\sigma-1} \phi_d \phi_x^{\sigma-1} \psi^*_1 + (RER/ w) \phi_x^{1-\sigma} \psi^*_2$$

where:

$$\psi_1 \equiv (\rho \theta_d) \phi_{min}^*; \psi_2 \equiv (\rho \theta_d^*/\tau^*) \phi_{min}^*; \psi^*_1 \equiv (\rho \theta_d) \phi_{min}; \psi^*_2 \equiv (\rho \theta_d^*/\tau^*) \phi_{min}^*$$

Noting that the labour market clearing conditions define the domestic productivity cutoffs as functions of exporting cutoffs in each country and using the balance of trade condition to substitute out for $RER$ the relative cutoff equations define a system in the two exporting cuts:

$$\left( \frac{\phi_x}{\phi_x^*} \right)^{\sigma} = \left( \frac{\phi_x^*}{\phi_x} \right)^{-\sigma} \Xi$$

$$\left( \frac{\phi_x}{\phi_x^*} \right)^{\sigma} = \left( \frac{\phi_x^*}{\phi_x} \right)^{-\sigma} \Xi^*$$

Each of these equations describe an upward sloping locus in $\{\phi_x, \phi_x^*\}$. Dividing the two equations gives a downward sloping curve which can be shown to intersect (38) uniquely at positive values of the two exporting cutoffs. As mentioned I focus on the case for which $\phi_x/\phi > 1$ and $\phi_x^*/\phi^* > 1$ which implies that parameter values must satisfy $\Upsilon \Upsilon > 1$.
References


