Demographics and the Behavior of Interest Rates

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Secular Stagnation 2015

Policy Questions (BoE, Theme 5)

Is demographic information relevant for the bond market?

- How might shifts in **demographics** and income distribution affect **equilibrium real interest rates** and the wider economy and financial system?
- To what extent have **secular trends** and/or policies, primarily changes in inequality and **demographics**, affected equilibrium rates of interest?
- Which of these trends has been the **key driver**?
- And are these effects likely to be *permanent* or *temporary*?

Source: http://www.bankofengland.co.uk/research/Pages/onebank/datasets.aspx#4

Outline

- Introduction
  - motivation
  - related Literature
- A simple model of the components of the yield curve
  - motivation for the demographic variable
- Affine term structure model (ATSM) with demographics
- Horse race
  - statistical accuracy and economic value
- Long-term projections
- Alternative permanent components of spot rates
- Robustness
- Conclusions

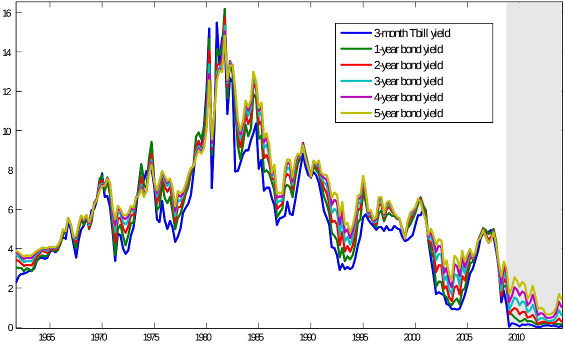
Motivation

- Decomposition of spot rates as the sum of two processes (Fama, 2006; Cieslak and Povala, 2015)
  - a **mean-reverting** component
  - a **persistent** long term expected value
- Traditional term structure models $\Rightarrow$ mean-reverting component using stationary variables
  - yields are highly **persistent**
  - poor **forecasting** performance
- The *secular stagnation* hypothesis (Hansen, 1939; Summers, 2014; Eggertsson and Mehrotra, 2014)
  - demographics (low population growth) $\Rightarrow$ **low real rates**
- The effect of demographics on real rates is **not unequivocal**
  - $\Downarrow$ *fertility*, $\Uparrow$ *life expectancy* $\Rightarrow$ $\Downarrow$ real rates
  - $\Uparrow$ longevity, $\Uparrow$ old population $\Rightarrow$ $\Uparrow$ real rates

Motivation

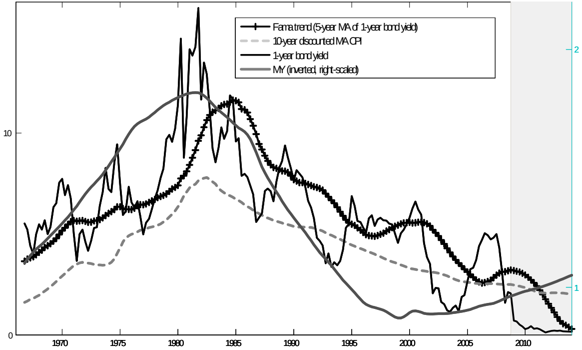
- **Nominal** bond yields

$$y_t^{(n)} = r_t^{(n)} + E_t \pi_{t,t+n}\,, \;\; n = \{1/4, 1, 2, 3, 4, 5\}$$



Motivation

- **Persistent** component
  - *backward* vs. forward looking proxies



Contribution

- Evidence on the link between interest rate **persistence** and **age composition** of U.S. population
- **Demographic** information in the **policy function** of the central bank
- **Trend-cycle composition** in an affine term structure model (ATSM)
- Improved **forecasting** performance of ATSM
- **Economic value** of demographic information
- **Robustness**
  - international evidence
  - uncertainty about the demographic variable
  - Monte Carlo exercise

Related Literature

Persistent Component of Spot Rates

- **Parsimony**: a small number of '*stationary*' factors (Litterman, Scheinkman, 1991)
    - principal components: **level**, slope and curvature
- **Persistence**:
    - structural breaks (Bai and Perron, 2003; Rapah and Wohar, 2005)
    - regime-switching models (Gray, 1996; Ang and Bekaert, 2002)
    - a time-varying long-term expected value (Fama, 2006; Cieslak and Povala; 2015)
- **Predictability**:
    - *bond excess returns*: linear combination of forward rates (Cochrane and Piazzesi, 2005; Cieslak and Povala; 2015)
    - *future spot rates*: less successful (Duffee, 2002; Favero et al., 2012)

Related Literature

Demographic fluctuations and Asset Prices

- Bakshi and Chen (1994): **life-cycle portfolio** hypothesis
  - **composition** of population age structure
- Geanakoplos, Magill and Quinzii (2004, henceforth GMQ) OLG model
  - (real) asset prices and **middle-aged young ratio**: MY_t
  - robust to *monetary* shocks (Gozluklu and Morin, 2015)
- Demographics and nominal interest rates
  - **Real interest rate** and age structure: Brooks (1998); Bergantino (1998); Davis and Li (2003)
  - **Inflation** and age structure: Lindh and Malberg (2000), Juselius and Takats (2015)

Related Literature

Monetary Policy Rules

- **Taylor rule (1993)** models policy rates
    - cyclical fluctuations in (expected) output and inflation
    - long term equilibrium rate: the real rate and (implicit) **inflation target**
        - the long-term equilibrium rate is as a constant - **discount factor**
        - a **time-varying** intercept in the feedback rule (Woodford, 2001)
- **Policy inertia**⇒ persistence (Clarida et al., 2000, Woodford, 2001)
    - current policy rate as a function of past policy rate
    - lack of forecastability
    - persistent shocks (Rudebusch, 2002)⇒ illusion of inertia

Over longer horizons, expectation of nominal and real yields rather than the inflation expectations dominate in the term structure (Evans, 2003)

Related Literature
Term Structure Models

- Affine term structure models (**ATSM**)
  - unobservable and **observable** variables⇒ VAR specification
  - time-varying **risk premium**
  - (**no arbitrage)** restrictions
- **Forecasting** models
  - **yields-only** (Christensen, Diebold and Rudebusch, 2011)
  - **macro-finance models** (Ang and Piazzesi, 2003; Diebold and Rudebusch, 2013)
- **Disappointing** forecasting results (Favero, Niu and Sala, 2012)

A Simple Model

- A **simple** framework:

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t[y_{t+i}^{(1)} \mid I_t] + rpy_t^{(n)}$$

$$y_t^{(1)} = rr_t^* + \pi_t^* + \beta(E_t\pi_{t+k} - \pi^*) + \gamma E_t x_{t+q}$$

- Decomposition of **1-period** policy rates

$$y_t^{(1)} = P_t^{(1)} + C_t^{(1)} = \rho_0 + \rho_1 MY_t + \rho_2 X_t$$

$$P_t^{(1)} = \rho_0 + \rho_1 MY_t = rr_t^* + \pi_t^* \equiv y_t^*$$

$$C_t^{(1)} = \rho_2 X_t = \beta(E_t\pi_{t+k} - \pi^*) + \gamma E_t x_{t+q}$$

π_t : inflation, x_t : output/unemployment gap, MY_t is the ratio of middle-aged (40-49) to young (20-29) U.S. population.

A Simple Model

- The **longer maturity yields** can be written as follows

$$y_t^{(n)} = \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1}\rho_1 MY_{t+i} + b^{(n)}C_t + rpy_t^{(n)}$$

$$y_t^{(n)} = P_t^{(n)} + C_t^{(n)}$$

$$P_t^{(n)} = \rho_0 + \frac{1}{n}\sum_{i=0}^{n-1}\rho_1 MY_{t+i}$$

$$C_t^{(n)} = b^{(n)}X_t + rpy_t^{(n)}$$

MY and the Real Rate

- **Overlapping Generations Model (OLG, GMQ2004)**
  - 3 periods: young, middle-aged, retired
  - No uncertainty, power utility
  - Endowments: w= (w_y,w_m,0), D: aggregate dividends
  - Population structure: **odd**: {N,n,N}, **even**: {n,N,n}
- Agents redistribute **income over time**
  - Consumption / saving decision
  - Bond / Equity: $1 + r^i = \frac{1}{q^i} = \frac{D + q_{eq}^{i+1}}{q_{eq}^i}, i = o, e$
- **Key Prediction**: Prices alternate between odd and even periods
- **Robustness**: Bequests, social security, production, uncertainty (GMQ, 2004,) and monetary shocks (Gozluklu and Morin, 2015)

The GMQ Model

The agent born in an *odd* period then faces the following **budget constraint**

$$c_y^o + q_e c_m^o + q_o q_e c_r^o = w^y + q_e w^m$$

and in an *even* period

$$c_y^e + q_o c_m^e + q_o q_e c_r^e = w^y + q_o w^m$$

Moreover, in equilibrium the following **resource constraint** must be satisfied

$$\begin{aligned} Nc_y^o + nc_m^o + Nc_r^o &= Nw^y + nw^m + D \\ nc_y^e + Nc_m^e + nc_r^e &= nw^y + Nw^m + D \end{aligned}$$

where D is the aggregate dividend for the investment in financial markets.

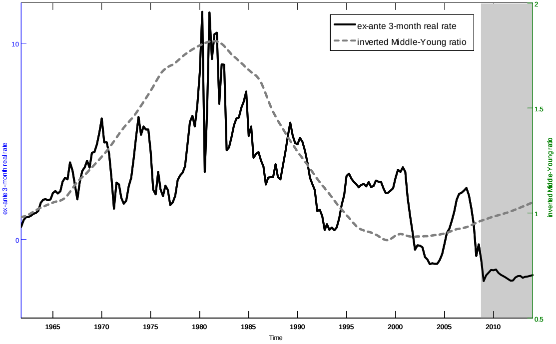
The GMQ Model

- If $q_o = q_e = 0.5$ (discount rate)$\Rightarrow c_y^i = c_m^i = c_r^i$, $i = o, e$
  - **but** calibrated values of wages and aggregate dividend from US $\Rightarrow$ excess demand either for **consumption** or **savings**
- **Odd period**:$\{$N,n,N$\}$ $\Rightarrow$ MY ratio is small$\Rightarrow\Uparrow$ consumption $\Downarrow q_o$
- **Even period**:$\{$n,N,n$\}$ $\Rightarrow$ MY ratio is large$\Rightarrow\Uparrow$ savings $\Uparrow q_e$
- Let q_t^b be the price of the bond at time t, in a *stationary* equilibrium, the following holds

$$q_t^b = q_o \text{ when period odd}$$
$$q_t^b = q_e \text{ when period even}$$
$$\Rightarrow q_o < q_e$$

Ex-ante (Short) Real Rate

- **Ex-ante** real rate obtained using an autoregressive model for inflation
  - robust to alternative specifications and longer sample



- We estimate the following **Demographic ATSM**:

$$
\begin{aligned}
y_t^{(n)} &= -\frac{1}{n}\left(A_n + B_n' X_t + \Gamma_n MY_t^n\right) + \varepsilon_{t,t+1},\ \varepsilon_{t,t+n} \sim N(0, \sigma_n^2) \\
y_t^{(1/4)} &= \delta_0 + \delta_1' X_t + \delta_2 MY_t \\
X_t &= \mu + \Phi X_{t-1} + \nu_t \quad \nu_t \sim i.i.d.N(0, \Omega)
\end{aligned}
$$

where $\Gamma_n = \left[\gamma_0^n, \gamma_1^n \cdots, \gamma_{n-1}^n\right]$, $MY_t^n = [MY_t, MY_{t+1} \cdots, MY_{t+n-1}]'$.

- **State vector**: $X_t = [f_t^o,\ f_t^u]$
  - $f_t^u = \left[f_t^{u,1}, f_t^{u,2}, f_t^{u,3}\right]$ contain **unobservable** factors
  - $f_t^o = [f_t^{\pi}, f_t^{x}]$ are two **observable factors,** inflation and output gap, extracted from large-data sets (Ludvigson and Ng, 2009)

- **Time varying risk premium**
  - **affine** in five state variables $\lambda_0 = \left[\begin{array}{ccccc} \lambda_0^{\pi} & \lambda_0^{x} & \lambda_0^{u,1} & \lambda_0^{u,2} & \lambda_0^{u,3} \end{array} \right]$
  - λ_1 is a diagonal matrix

$$\Lambda_t = \lambda_0 + \lambda_1 X_t$$
$$m_{t+1} = \exp(-y_t^{(1/4)} - \frac{1}{2}\Lambda_t' \Omega \Lambda_t - \Lambda_t \varepsilon_{t+1})$$

  - **no-arbitrage** restrictions

$$\begin{array}{c} A_{n+1} = A_n + B_n' \left(\mu - \Omega \lambda_0 \right) + \frac{1}{2} B_n' \Omega B_n + A_1 \\ B_{n+1}' = B_n' \left(\Phi - \Omega \lambda_1 \right) + B_1' \\ \Gamma_{n+1} = \left[-\delta_2, \Gamma_n \right] \end{array}$$

- **Parsimony**: the effects of demographics on the term structure depend on a single parameter: δ_2.

Full-sample Estimates

- **Maximum likelihood** estimation
  - Chen and Scott (1993) methodology
  - further restrictions, e.g. Ω block diagonal

	Demographic ATSM					Macro ATSM				
Companion form Φ										
	−0.125 (0.082)	0.137 (0.123)	−0.153 (0.140)	−0.253 (0.135)	0.165 (0.111)	−0.133 (0.095)	0.134 (0.104)	0.067 (0.105)	−0.311 (0.132)	0.240 (0.192)
	−0.057 (0.073)	0.348 (0.087)	0.147 (0.090)	0.079 (0.125)	−0.220 (0.112)	−0.054 (0.072)	0.380 (0.104)	−0.092 (0.110)	0.066 (0.168)	−0.279 (0.119)
	−0.028 (0.040)	0.041 (0.026)	0.764 (0.142)	−0.251 (0.040)	0.101 (0.068)	−0.015 (0.009)	0.059 (0.041)	0.981 (0.023)	0.036 (0.120)	−0.087 (0.112)
	−0.017 (0.028)	0.057 (0.021)	−0.178 (0.040)	0.622 (0.174)	0.060 (0.032)	−0.015 (0.039)	0.075 (0.024)	−0.039 (0.060)	0.608 (0.141)	0.172 (0.043)
	−0.002 (0.018)	0.001 (0.021)	0.240 (0.075)	0.189 (0.076)	0.754 (0.095)	0.020 (0.041)	−0.044 (0.052)	0.018 (0.107)	0.305 (0.034)	0.681 (0.127)
Short rate parameters										
δ_1	−0.006 (0.039)	0.157 (0.121)	0.000 (0.000)	0.000 (0.000)	2.739 (0.372)	−0.007 (0.059)	0.263 (0.119)	2.321 (0.588)	0.000 (0.000)	1.544 (0.957)
δ_2	−0.010 (0.004)					0				

Out-of-Sample Forecasts

- Out-of-sample forecasts of our model at **different horizons**
- ATSM models are good at describing the **in-sample** yield data and **explain** bond excess returns
  - but fail to beat the **random walk benchmark**, especially in long horizon forecasts
- In our multi-period ahead forecasts, we follow **iterated forecast** procedure, where
  - multiple step ahead forecasts by iterating the **one-step** model forward

$$\widehat{y}^{(n)}_{t+h|t} = \widehat{a}_n + \widehat{b}_n \hat{X}_{t+h|t} + \widehat{c}_n MY^n_{t+h}$$

$$\hat{X}_{t+h|t} = \sum_{i=0}^{h} \hat{\Phi}^i \widehat{\mu} + \hat{\Phi}^h \hat{X}_t$$

- where $\widehat{a}_n = -\frac{1}{n}\widehat{A}_n$, $\widehat{b}_n = -\frac{1}{n}\widehat{B}_n$ and $\widehat{c}_n = -\frac{1}{n}\widehat{\Gamma}_n$.

Out-of Sample Forecasts

- **Random Walk** Benchmark

Panel A. Random walk Benchmark										
h	4		8		12		16		20	
	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)
$\hat{y}^{(1/4)}_{t+h\|t}$	1.224 (0.016)	0.814 (0.208)	0.941 (0.001)	5.624 (0.000)	0.813 (0.000)	8.118 (0.000)	0.832 (0.000)	7.057 (0.000)	0.932 (0.000)	5.803 (0.000)
$\hat{y}^{(1)}_{t+h\|t}$	1.158 (0.010)	0.338 (0.368)	0.923 (0.006)	5.188 (0.000)	0.821 (0.001)	7.466 (0.000)	0.839 (0.000)	6.359 (0.000)	0.935 (0.001)	5.391 (0.000)
$\hat{y}^{(2)}_{t+h\|t}$	1.158 (0.034)	−0.145 (0.558)	0.951 (0.000)	4.317 (0.000)	0.874 (0.000)	6.088 (0.000)	0.897 (0.001)	5.083 (0.000)	0.991 (0.013)	4.281 (0.000)
$\hat{y}^{(3)}_{t+h\|t}$	1.158 (0.008)	−0.337 (0.632)	0.982 (0.393)	3.649 (0.000)	0.926 (0.001)	4.890 (0.000)	0.948 (0.113)	4.070 (0.000)	1.036 (0.258)	3.341 (0.000)
$\hat{y}^{(4)}_{t+h\|t}$	1.154 (0.000)	−0.397 (0.654)	1.008 (0.065)	3.126 (0.001)	0.969 (0.390)	3.892 (0.000)	0.990 (0.002)	3.286 (0.001)	1.070 (0.090)	2.651 (0.004)
$\hat{y}^{(5)}_{t+h\|t}$	1.147 (0.000)	−0.387 (0.651)	1.027 (0.002)	2.705 (0.003)	1.003 (0.075)	3.076 (0.001)	1.023 (0.172)	2.689 (0.004)	1.096 (0.016)	2.182 (0.015)

Out-of Sample Forecasts

- **Macro ATSM** Benchmark

Panel B. Macro ATSM Benchmark										
h	4		8		12		16		20	
	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)	RMSFE (GW)	CW (p-val)
$\hat{y}^{(1/4)}_{t+h\|t}$	1.060 (0.000)	1.496 (0.067)	0.894 (0.000)	8.410 (0.000)	0.778 (0.000)	9.673 (0.000)	0.747 (0.001)	9.203 (0.000)	0.756 (0.002)	6.962 (0.001)
$\hat{y}^{(1)}_{t+h\|t}$	1.014 (0.002)	2.531 (0.006)	0.859 (0.011)	9.218 (0.000)	0.761 (0.010)	10.689 (0.000)	0.744 (0.005)	9.757 (0.000)	0.760 (0.001)	7.158 (0.000)
$\hat{y}^{(2)}_{t+h\|t}$	0.989 (0.000)	2.967 (0.002)	0.837 (0.001)	9.487 (0.000)	0.752 (0.001)	11.280 (0.000)	0.743 (0.000)	10.080 (0.000)	0.766 (0.001)	7.485 (0.000)
$\hat{y}^{(3)}_{t+h\|t}$	0.975 (0.000)	3.181 (0.001)	0.825 (0.000)	9.596 (0.000)	0.749 (0.000)	11.476 (0.000)	0.745 (0.000)	10.122 (0.000)	0.773 (0.001)	7.687 (0.000)
$\hat{y}^{(4)}_{t+h\|t}$	0.965 (0.000)	3.394 (0.000)	0.817 (0.000)	9.713 (0.000)	0.746 (0.000)	11.491 (0.000)	0.748 (0.000)	10.083 (0.000)	0.778 (0.001)	7.829 (0.000)
$\hat{y}^{(5)}_{t+h\|t}$	0.959 (0.000)	3.598 (0.000)	0.811 (0.000)	9.801 (0.000)	0.745 (0.000)	11.387 (0.000)	0.751 (0.000)	9.980 (0.000)	0.784 (0.001)	7.906 (0.000)

Forecast Usefulness

- The optimal weight κ^* to minimize the expected *out-of-sample loss* of the **combined forecast** (Carriero and Giacomini, 2011)

$$y^{(n),*}_{t+h|t} = \widehat{y}^{(n),RW}_{t+h|t} + (1-\kappa)(\widehat{y}^{(n),DATSM}_{t+h|t} - \widehat{y}^{(n),RW}_{t+h|t})$$

Out-of-Sample Forecast Usefulness

Panel A. Bond Yields - Quadratic Loss

h	4	8	12	16	20
	$\widehat{\kappa}$ $(t^{\kappa=0}),[t^{\kappa=1}]$	$\widehat{\kappa}$ $(t^{\kappa=0}),[t^{\kappa=1}]$	$\widehat{\kappa}$ $(t^{\kappa=0}),[t^{\kappa=1}]$	$\widehat{\kappa}$ $(t^{\kappa=0}),[t^{\kappa=1}]$	$\widehat{\kappa}$ $(t^{\kappa=0}),[t^{\kappa=1}]$
$\widehat{y}^{(1/4)}_{t+h\|t}$	0.816	0.098	−0.238	−0.035	0.307
	(4.60***)	(0.56)	(−1.19)	(−0.15)	(2.02**)
	[−1.04]	[−5.16***]	[−6.17***]	[−4.36***]	[−4.55***]
$\widehat{y}^{(1)}_{t+h\|t}$	0.708	−0.040	−0.232	−0.016	0.316
	(3.61***)	(−0.30)	(−1.17)	(−0.07)	(2.26**)
	[−1.49]	[−7.83***]	[−6.19***]	[−4.59***]	[−4.88***]
$\widehat{y}^{(2)}_{t+h\|t}$	0.726	−0.076	−0.134	0.112	0.413
	(3.17***)	(−0.59)	(−0.73)	(0.59)	(3.51***)
	[−1.20]	[−8.35***]	[−6.18***]	[−4.70***]	[−4.98***]
$\widehat{y}^{(3)}_{t+h\|t}$	0.744	−0.068	−0.019	0.226	0.490
	(2.97***)	(−0.50)	(−0.11)	(1.34)	(4.63***)
	[−1.02]	[−7.74***]	[−6.04***]	[−4.60***]	[−4.81***]
$\widehat{y}^{(4)}_{t+h\|t}$	0.754	−0.035	0.091	0.320	0.548
	(2.83***)	(−0.23)	(0.57)	(2.04**)	(5.48***)
	[−0.92]	[−6.80***]	[−5.68***]	[−4.34***]	[−4.53***]
$\widehat{y}^{(5)}_{t+h\|t}$	0.755	0.011	0.188	0.395	0.590
	(2.71***)	(0.07)	(1.20)	(2.62***)	(6.05***)
	[−0.88]	[−5.93***]	[−5.16***]	[−4.01***]	[−4.20***]

Utility Loss

- The optimal weight w^*(function of κ^*) to minimize **portfolio utility loss** (Carriero and Giacomini, 2011)
  - bond yields vs. bond **excess returns**

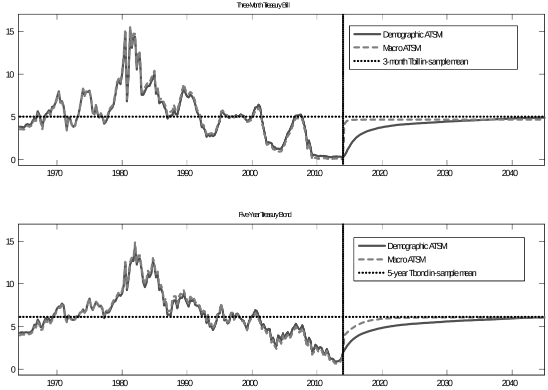
$$\widehat{rx}_{t+1} = -n\ \widehat{y}^{(n)}_{t+1|t} + (n+1)y_t^{(n+1)} - y_t^{(n/4)}$$

$$\widehat{rx}_{t+2} = -n\ \widehat{y}^{(n)}_{t+2|t} + (n+2)y_t^{(n+2)} - y_t^{(1/4)}$$

Panel B. Bond Excess Returns - Portfolio Utility Loss		
holding period	1-year	2-year
	$\widehat{\kappa}$ $(t^{\kappa=0})$,$[t^{\kappa=1}]$	$\widehat{\kappa}$ $(t^{\kappa=0})$,$[t^{\kappa=1}]$
Demographic ATSM	0.595	0.316
	(1.57)	(1.85*)
	[−1.07]	[−4.00***]
Macro ATSM	0.611	0.707
	(2.61***)	(1.94*)
	[−1.67*]	[−0.80]

Long-Term Projections

- **Gradual** recovery to long-run average
  - **secular stagnation?**



Permanent Component of Spot Rates

- Fama (2006): time-varying expected value
  - a **dummy** variable: peak August 1981
  - a backward-looking (5-year) **moving average of spot rates**
- Cieslak and Povala (2015): temporary-permanent decomposition
  - cycle factor
    - time varying risk premium
  - a permanent component - **trend inflation**
    - a discounted moving-average of past realized core inflation

Permanent Component of Spot Rates

- The following models are estimated (Fama, 2006)

$$
\begin{aligned}
y^{(1)}_{t+4x} \text{-} y^{(1)}_{t} &= a^x + b^x D_t + c^x [f^{(1)}_{t,t+4x} \text{-} y^{(1)}_{t}] + d^x [y^{(1)}_{t} \text{-} P^{(1),i}_{t}] + \varepsilon_{t+4x} \\
P^{(1),1}_{t} &= \frac{1}{20} \sum_{i=1}^{20} y^{(1)}_{t-i-1} \\
P^{(1),2}_{t} &= \frac{\sum\limits_{i=1}^{40} v^{i-1} \pi_{t-i-1}}{\sum\limits_{i=1}^{40} v^{i-1}} \\
P^{(1),3}_{t} &= e^x \frac{1}{4} \sum_{i=1}^{4} MY_{t+i-1}
\end{aligned}
$$

Permanent Component of Spot Rates

Empirical Evidence

Predictive Regressions for the 1-year Spot Rate

$$y^{(1)}_{t+4x} - y^{(1)}_t = a^x + b^x D_t + c^x [f^{(1)}_{t,t+4x} - y^{(1)}_t] + d^x [y^{(1)}_t - P^{(1),i}_t] + \varepsilon_{t+4x}$$

$x = 2$	a^x (s.e.)	b^x (s.e.)	c^x (s.e.)	d^x (s.e.)	e^x (s.e.)	R^2
no cycle	−1.99 (0.26)	2.36 (0.134)	1.29 (0.17)			0.28
Fama cycle	−1.88 (0.25)	3.30 (0.38)	0.42 (0.24)	−0.54 (0.12)		0.35
Fama cycle no dummy	−0.74 (0.25)		0.87 (0.28)	−0.01 (0.11)		0.11
CP cycle	0.78 (0.38)		−0.17 (0.27)	−0.63 (0.13)		0.20
MY cycle	6.83 (1.16)		0.11 (0.20)	−0.54 (0.08)	−0.093 (0.009)	0.27

Robustness

- **International** panel
  - 35 countries: significant link between yields and MY
- **Uncertainty** on MY
  - future fertility/ immigration
  - foreign holdings of US debt securities
- **Monte-Carlo** simulation
  - residuals from an autoregressive model
  - not spurious

Conclusions

- We show that demographic variable MY_t captures the **persistent component** of nominal rates
- We propose a **demographics based policy rate** target
- We extend **parsimoniously** the no-arbitrage affine term structure model by incorporating demographic information
- Term structure models with demographics perform better than macro-factors based models both in terms of statistical accuracy and economic value, particularly in the **long-run forecasts**
- We predict a **gradual recovery** (to long-run average) in interest rates over the next decade

International Panel

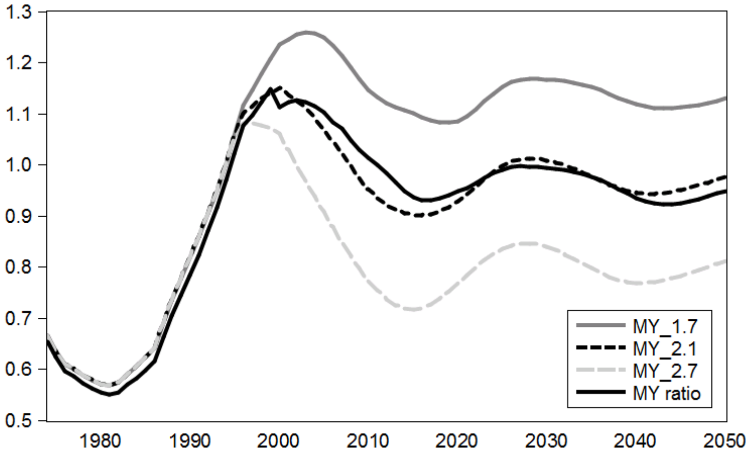
- **International** Panel of 35 countries
  - sample 1960-2011

Table 6
International Panel

Benchmark model:	$R_{lt} = \alpha_0 + \alpha_1 R_{lt-1} + \varepsilon_t$		
Augmented model:	$R_{lt} = \beta_0 + \beta_1 R_{lt-1} + \beta_2 \mathrm{MY}_t + \varepsilon_t$		
Specification	R_{lt-1}	MY_t	$\bar{R}^2$
(1)	0.729 (8.39***)		0.55
(2)	0.676 (7.29***)	−0.044 (−3.78***)	0.58

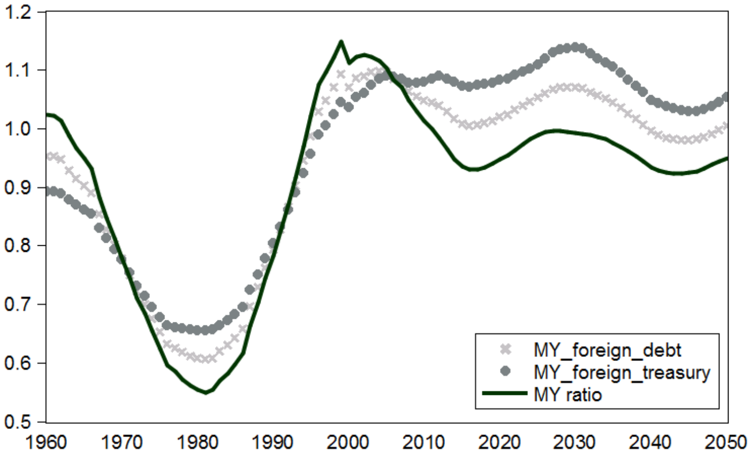
Future Fertility

- Three **different fertility scenarios**
  - based on 1975 population report



Foreign Holdings

- MY ratio **adjusted** for foreign holdings
  - age composition of foreign investors



Monte Carlo Simulation

- Do we observe a **spurious** relation between MY and ex-ante real rate?
  - whole sample: p-value 0.039
  - pre-crisis: p-value 0.018