III The Steady-State Analogue Model and Mark III Solution Methodology

As noted in Section I, the Mark III version of the model incorporates a major advance in the treatment of the long-run properties of MULTIMOD. Unlike in previous generations, each of the dynamic equations in MULTIMOD Mark III has a steady-state analogue equation. The system of steady-state analogue equations, SSMOD, is maintained separately from the system of dynamic equations, DYNMOD.[36]

The addition of SSMOD strengthens the properties and analytic capabilities of MULTIMOD in several important ways. SSMOD can be used both as an interpretive device for understanding long-run comparative statics and as a vehicle for determining model-consistent terminal conditions for dynamic analysis. Moreover, in any model that embodies forward-looking expectations, such as MULTIMOD, the medium-run responses of macroeconomic variables to exogenous policy changes or other shocks are influenced by the long-run properties of the model; thus, the quality of the dynamic analysis generated by a forward-looking model depends importantly on whether the long-run properties of the model have solid theoretical foundations and on whether the dynamic equations and their steady-state counterparts are specified consistently. Both of these features have been achieved in MULTIMOD Mark III.

To illustrate these points, the next subsection describes two dynamic equations in MULTIMOD and derives the steady-state analogue equations. The remainder of the section then discusses the use of SSMOD as an interpretive device for understanding long-run comparative statics and medium-term dynamics, the manner in which SSMOD is used to generate a control path that extends the baseline scenario from the *World Economic Outlook* and converges to a model-consistent steady state, the advantages of using a steady-state model to obtain terminal conditions, and the Mark III solution methodology.

An Example of Parallel Equations in DYNMOD and SSMOD

Consider the following two dynamic equations:

$$\frac{CA_t}{Y_t} = \frac{TB_t}{Y_t} + r_t \frac{NFA_{t-1}}{Y_t} \tag{1}$$

and

$$\frac{CA_t}{Y_t} = \frac{NFA_t - NFA_{t-1}}{Y_t}. \tag{2}$$

Equation (1) is a balance of payments identity that expresses the current account balance (CA) as the sum of the trade balance (TB) and the interest receipts on net foreign assets (NFA), where r denotes the nominal interest rate, the subscripts denote time periods, and all three terms in the equation have been scaled by the level of nominal GDP (Y). For illustrative purposes, we assume that interest receipts are paid on a short-term financial asset that rolls over each period, with the nominal interest rate in period t applied to the outstanding net claims on foreigners that exist at the end of period $t - 1$.[37] Dividing both sides of the balance of payments identity by Y allows us to provide some illustrative calculations for the usual case in which nominal GDP is expanding over time. Equation (2) is just an identity that equates the current account balance to the change in net foreign assets.

To derive the analogue equations for SSMOD, let g denote the steady-state growth rate of all nominal variables, such that $NFA_{t-1} = NFA_t/(1 + g)$. Substitut-

[36]In moving to a modeling system with parallel dynamic and steady-state equations, Mark III has followed in the footsteps of other national and two-region models that have been designed for policy analysis. For examples of such models and some relevant applications, see Laxton and Tetlow (1992), Black and others (1994), Bryant (1996), Coletti and others (1996), Faruqee, Laxton, and Symansky (1997), Bryant and Zhang (1996a, 1996b), and Black and others (1997).

[37]This simplification is made for expositional purposes. In Mark III, the rate of return on net foreign assets is a blend of the rate of return on a short-term debt instrument and the rate of return on a long-term debt instrument.

Box 4. Government Debt, Net Foreign Liabilities, and the Real Exchange Rate

In the core version of Mark III, real interest rate *differentials* in the long run are assumed to be independent of the stocks of government debt.[1] Thus, government debt in a particular country will influence real interest rates in the long run only if it affects global saving and investment and the *level* of the world real interest rate. For small countries, such global effects will likely be small, but they may be significant for larger countries; Box 2 illustrates the spillover effects of an increase in government debt in the United States. Moreover, while the worldwide effects of the government debt of an individual country may be small, a debt buildup in several industrial countries could have important combined effects on global saving, global investment, and the world real interest rate, as illustrated in Box 9.

The table presents estimates of the long-run effects of changes in the ratio of government debt to GDP for each of the major industrial countries.[2] In each of the two panels, the debt-to-GDP ratio is increased by 10 percentage points for individual countries, one at a time. These increases are achieved through temporary tax cuts; in the long run, tax rates must rise to finance the higher interest burdens that result from higher levels of government debt.[3] The top panel provides estimates derived from the individual country models in isolation, abstracting from any induced effects on, or feedback from, the rest of the world, and thus assuming that the equilibrium world real interest rate is fixed. In the second case, the results are derived from the full multicountry model, with the steady-state world real interest rate adjusting endogenously to equate world saving and investment.

As shown in the top panel, with the world real interest rate fixed, an increase in government debt (in percentage points of GDP) translates into roughly the same increase in net foreign liabilities in the steady state. The increase in debt initially induces a rise in the domestic interest rate and an appreciation of domestic currency, which affects the current account and leads to the buildup of net foreign liabilities over time. To service the resulting higher interest payments to foreigners in the steady state, there must be a larger net flow of goods and nonfactor services from the home country to foreigners (recall equation (5)). And under normal assumptions about intertemporal consumption preferences, this implies that the real exchange rate must depreciate in the long run, more so for relatively closed economies (in particular, Japan and the United States) than for other cases.

For the unconstrained simulations summarized in the lower panel, the endogenous world real interest rate rises by up to 11 basis points, with the extent of the increase depending primarily on the size of the country in which the debt increase occurs. In this case, the expan-

[1]Endogenous country-specific risk premiums are not included in the core version of Mark III because empirical estimates of their behavior were too unappealing. However, users of Mark III can easily incorporate their own assumptions about risk premiums, and the model can be used to compare the macroeconomic implications of alternative assumptions. For examples, see Laxton and Tetlow (1992); Bayoumi and Laxton (1994); Black and others (1994); Macklem, Rose, and Tetlow (1995); and Laxton and Prasad (1997).

[2]The dynamic effects of government debt in a small open economy are discussed in Box 9.

[3]For each country, the basic tax rate on aggregate nominal GDP is reduced by 2 percentage points for five years and is then allowed to rise, while the tax rate on capital income is held constant at its baseline level.

ing this latter condition into equations (1) and (2) provides the steady-state analogue equations.

$$\frac{CA}{Y} = \frac{TB}{Y} + \frac{r}{(1+g)}\frac{NFA}{Y} \tag{3}$$

and

$$\frac{CA}{Y} = \frac{NFA}{Y}[g/(1+g)]. \tag{4}$$

The Steady-State Model as an Interpretive Device

The steady-state model can provide an important interpretive device for understanding long-term comparative statics and medium-term dynamics. Consider, for example, an application of equation (4). If all nominal variables in the economy are growing at 5 percent in the steady state and net foreign assets are equal to 100 percent of GDP, the current account surplus must be a little less than 5 percent of GDP. Conversely, a net debtor country would be running a current account deficit in long-run equilibrium. If we combine equations (3) and (4) we can obtain an equation that links a country's steady-state trade balance to its level of net foreign assets.

$$\frac{TB}{Y} = \left(\frac{g-r}{1+g}\right)\frac{NFA}{Y}. \tag{5}$$

The interpretation of this equation is straightforward. In long-run equilibrium, under the steady-state condition that $r > g$ (see discussion below), when the ratio of a country's net foreign assets (or li-

Long-Run Effects of Temporary Tax Cuts That Permanently Increase Debt-to-GDP Ratios by 10 Percentage Points

	Net Foreign Liabilities[1,2]	Current Account Balance[1,3]	Trade Balance[1,3]	Real Competitiveness Index[2]	Real Interest Rate[4]
Single-country model results					
Canada	10.0	–0.5	0.2	–0.2	—
France	10.8	–0.5	0.3	–0.5	—
Germany	10.9	–0.5	0.3	–0.5	—
Italy	10.1	–0.5	0.2	–0.4	—
Japan	11.5	–0.5	0.3	–1.9	—
United Kingdom	9.9	–0.5	0.2	–0.3	—
United States	10.1	–0.5	0.2	–0.7	—
Full multicountry model results					
Canada	9.9	–0.5	0.2	–0.2	1
France	10.4	–0.5	0.3	–0.5	2
Germany	10.2	–0.5	0.2	–0.4	3
Italy	9.7	–0.5	0.2	–0.4	2
Japan	9.1	–0.4	0.2	–1.2	7
United Kingdom	9.3	–0.4	0.2	–0.3	2
United States	5.6	–0.3	0.2	–0.7	11

[1]As a percent of nominal GDP.
[2]Percent deviation from baseline.
[3]Percentage point deviation from baseline.
[4]Basis point deviation from baseline.

sionary effect of the tax cut partly spills over on the rest of the world, putting upward pressure on foreign interest rates and dampening the initial exchange rate appreciation. Accordingly, the shock has a smaller effect on the current account, and the steady-state level of net foreign liabilities increases less than in the first set of simulations, especially for larger countries such as the United States.

abilities) to GDP is stable, the country will receive (or pay) net income on its international asset (or liability) position that is a constant fraction of its GDP. The payments received by a net creditor country will finance a steady-state trade deficit, with imports exceeding exports. Conversely, payments by a net debtor country to its creditors will require a steady-state trade surplus.

These implications of equation (5) provide important intuition for understanding the sensitivity of the long-run equilibrium level of the real exchange rate to shocks that affect the desired net foreign asset position of the economy. This is illustrated in Box 4, which describes the long-run effects of changes in the ratios of government debt to GDP; the speeds with which different types of variables converge to steady-state values following a shock are illustrated later. In a fully specified intertemporal model that attains a full stock-flow equilibrium, the steady-state levels of the trade balance and the real exchange rate will reflect the desired net foreign asset position of the economy. Thus, an increase in desired net foreign liabilities caused by an increase in government debt will require a larger trade surplus in order to finance the higher interest obligations. And this can be obtained only if the real exchange rate depreciates in the long run.

On Using SSMOD to Construct the Control Solution

In previous generations of MULTIMOD, behavior beyond the *World Economic Outlook* projection hori-

Box 5. Traditional Solution Techniques and the MARK III Methodology

The numerical complexity of solving a small-scale rational expectations model is equivalent to that of solving very large scale backward-looking econometric models. To make such a task manageable, traditional algorithms were designed to break large blocks of simultaneous equations into smaller pieces and then to use an iterative procedure to ensure consistency across blocks until the full system converged.

The traditional Fair-Taylor (1983) algorithm separated the problem into three types of iterations, and a Gauss-Seidel iterative procedure was relied upon to solve each layer of iteration. The problems with this approach are well known; it can be time consuming and is not guaranteed to find a solution even when a well-defined saddle-point stable solution exists; see Armstrong and others (1998). Indeed, practitioners using this technique have frequently been forced to rely heavily on certain tuning parameters (ordering, convergence tolerance limits, damping factors, divergence factors, and so on) to help the algorithms achieve convergence in a reasonable amount of time. At the same time, these difficulties have made it unattractive to rely on certain classes of models for which the problems are particularly severe. In practice, this has meant that model builders until recently have had to either linearize their models or restrict their attention to issues that could be dealt with more easily with traditional algorithms. However, the enormous advance in computer technology over the last few years has made it possible to design and implement more robust methods for solving medium-sized nonlinear models that feature model-consistent expectations; see Armstrong and others (1998) and Juillard and others (1998).

The development of the Mark II version of MULTIMOD was done principally with an extended Fair-Taylor (F-T) algorithm. This algorithm used the Newton-Raphson method to solve for Type I iterations and a Gauss-Seidel iterative scheme to solve for the simultaneity that arises from model-consistent expectations. Juillard and others (1998) show that with the Newton-Raphson-based Laffargue-Boucekkine-Juillard (L-B-J) algorithm, fairly accurate solutions for Mark II can be obtained in two iterations, and that in most cases the model converges in about four or five iterations with extremely accurate solution values. This should not be surprising, however, because the Mark II version did not contain many significant nonlinearities; in such cases, one should expect that a Newton-Raphson-based algorithm would obtain extremely accurate solutions in a few iterations.

The L-B-J approach involves stacking the equations—or combining the Type I and Type II iterations in the F-T algorithm—and then employing a method first proposed by Laffargue (1990), then developed by Boucekkine (1995), Juillard (1996), and Hollinger (1996), to exploit information about the repetitive and sparse structure of the full simultaneous problem. The results in Juillard and others (1998) will be very encouraging to anyone who is interested in building medium-sized macro models designed for policy analysis. They report significant savings of time in comparison with traditional and extended F-T algorithms, even when fairly loose conventional convergence tolerance limits are allowed for F-T. Second, they show that when the F-T convergence tolerance is tightened sufficiently to approximately replicate the L-B-J solutions, the relative time savings of the L-B-J algorithm become enormous.

These results understate the potential benefits of a state-of-the-art algorithm like the L-B-J algorithm because the tests were performed on the Mark II version of MULTIMOD, which was developed with an extended F-T algorithm. The Mark II version is approximately linear in that it generally only takes a few L-B-J iterations to achieve full convergence for a large set of shocks. Juillard and others (1998) discuss a few cases where MULTIMOD development has been impeded because F-T had difficulty finding solutions. Here, the relative comparison becomes more difficult because it depends on how adept the user is at tuning certain parameters to achieve convergence, and this depends on the particular shock and model under consideration.

The solution methodology for Mark III is based on the following two steps. First, terminal conditions are

zon was constrained to be consistent with the imposed assumptions that the primary fiscal and trade balances converge gradually to zero (thus eventually stabilizing the stocks of public and net international debt relative to GDP) and that the real rate of interest converges to the steady-state rate of growth. The latter condition, while extremely convenient for creating a baseline control solution, had the undesirable feature of making fiscal policy a Ponzi game. In particular, governments could issue long-term debt with no apparent real costs, because such debt could always be rolled over at real interest rates that were no greater than the steady-state growth rate.[38]

[38]If the real interest rate was less than the growth rate, there would be no real costs, and indeed there would be potential benefits, from delaying fiscal consolidation. In fact, if the real interest rate were less than the growth rate and independent of fiscal policy, a government could reduce tax rates, issue debt instruments, and allow growth in its tax base to eliminate the debt. For this reason, most theoretical optimizing models used for policy analysis impose a no-Ponzi-game condition; see, for reference, the discussion in Blanchard and Fischer (1989).

computed by solving the steady-state analogue models using a standard Newton-Raphson algorithm that exploits sparsity in the Jacobian. Second, the L-B-J algorithm is used to solve the dynamic model given initial conditions, shocks to the exogenous forcing processes, and estimates of the terminal conditions that are obtained from the steady-state models.

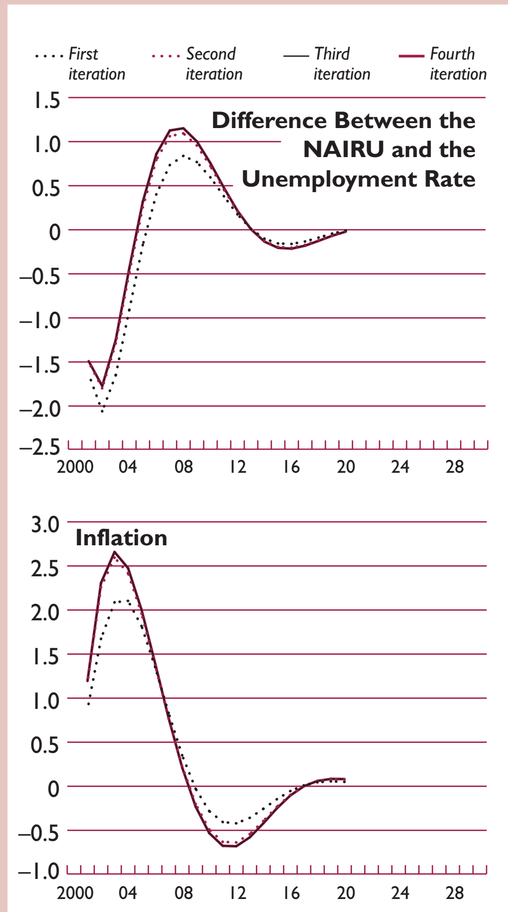
To illustrate the iterative process of the solution method, the figure shows the solution paths for the inflation rate and the unemployment gap (that is, the NAIRU minus the unemployment rate) following a 10 percent increase in the money supply in Germany when the world economy is initially in a position of full stock-flow equilibrium. Because long-run monetary neutrality holds in Mark III, a 10 percent increase in the money supply results in a 10 percent increase in the German price level, and all real variables eventually return to their baseline values.

In the short run, the effects of money supply shocks on the real economy can be quite significant and depend critically on the nature of the Phillips curve. In Mark III, the Phillips curve is convex and includes a binding short-run capacity constraint, so that extremely large expansionary money supply shocks have greater effects on the price level than they do on the unemployment rate. Conversely, negative money shocks have greater contractionary effects on the economy than positive money shocks of the same magnitude; Box 8 provides a discussion of the asymmetric effects of money shocks.

The figure reports the solution values for four iterations. The first iteration is obtained by linearizing the model around the control (or baseline) solution and then solving the linearized version of the model. The second iteration is obtained by linearizing the model around the solution values obtained from the first iteration; the iterative process continues until it converges. In the illustrated example, because the linearized short-run Phillips curve trade-off is flatter in the first iteration than it is in the neighborhood of the true solution, the decline in the unemployment rate in the first year is overestimated and the increase in the inflation rate is underestimated. The process continues in a very orderly manner and converges after a few L-B-J iterations.

Convergence Properties of MARK III

(Deviations from baseline values, in percentage points)



In MULTIMOD Mark III, the extensions of the baseline control solution beyond the projection horizon of the *World Economic Outlook*, and the construction of model-consistent terminal conditions, also rely heavily on assumptions about the steady-state levels of the real interest rate and the rate of growth. These assumptions, however, can be modified. For the core version of Mark III, the baseline level of the steady-state real interest rate on short-term government debt (risk-free assets) is set at an imposed value of 4.25 percent—the average real rate of return that accrued during 1987–96 on the short-term public debts of the Group of Seven countries.[39] Furthermore, as in MULTIMOD Mark II, all countries are assumed to converge to a steady-state growth rate consistent with the *World Economic Outlook*'s projection of potential output growth for the United States.

[39]The real long-term interest rate in the steady state is assumed to be 5.25 percent, reflecting an assumption that the equilibrium term premium is 100 basis points.

Once the steady-state values of the real interest rate and the potential output growth rate are specified, SSMOD and DYNMOD are used to generate a complete baseline control solution (extending beyond the WEO horizon) with model-consistent terminal conditions. But the Mark III generation of MULTIMOD has the scope to explore the implications of alternative sets of terminal conditions—or, more precisely, of modifications in "exogenous assumptions" that lead to alternative sets of terminal conditions.[40] As was the case with the Mark II generation, MULTIMOD also has the scope to simulate and study the individual country SS and DYN models in isolation.[41]

On Using SSMOD to Obtain Terminal Conditions

In addition to strengthening the theoretical foundations of MULTIMOD and the consistency between its dynamic and steady-state behavior, the introduction of a steady-state analogue model has eliminated the scope for potentially large inaccuracies in solving for terminal conditions. The problem of poor terminal conditions has plagued forward-looking nonlinear models in general.

Box 5 discusses the difficulties of solving models with traditional techniques. In the past, users of MULTIMOD have had three choices for dealing with potential inaccuracies caused by poor terminal conditions. First, they could rely upon an iterative approach, such as Fair-Taylor Type III iterations, to eliminate the effects of inaccurate terminal conditions. Second, they could simply choose a simulation horizon that was sufficiently distant that any errors in the end point would have only a small impact on the solution values over the time period of interest.[42] Third, users could guess new terminal conditions based on their understanding of the model. Each of the three approaches could be a time-consuming process, and there was no guarantee that the estimates were precise, especially if users were interested in obtaining estimates of the long-run comparative statics of the model. The introduction of SSMOD and new solution techniques has eliminated these problems.

The Mark III Solution Methodology

The development and use of forward-looking macro models in policymaking institutions have proceeded at a pace much slower than predicted in the early 1980s. An important reason is that researchers have not had access to robust and efficient solution techniques for solving nonlinear forward-looking models. The numerical complexity of solving a forward-looking macro model is considerably more onerous than solving a backward-looking model of the same size. Fortunately, the dramatic reduction in the cost of computer memory has made it possible to design better solution algorithms. Accordingly, MULTIMOD is now solved with a Newton-Raphson algorithm, which for nonlinear forward-looking models tends to converge much more rapidly and accurately than algorithms involving Gauss-Seidel iteration.[43] This technical advance has two related implications. By making Newton-Raphson iteration feasible, it avoids the large errors in convergence that sometimes arise under Gauss-Seidel iteration. In addition, with the introduction of an algorithm that is considerably faster and much less prone to simulation failures than its predecessor algorithms, it has become feasible for builders of nonlinear forward-looking models to increase the scope of their analytic frameworks. See Box 5 for additional discussion of MULTIMOD's new solution algorithm and its implications.

[40] This capability reflects advances in solution methodology.

[41] The TROLL programs used to create SSMOD and DYNMOD for Mark III build up the world models by combining the codes of individual country models. This means that it is straightforward to simulate and study the individual country SS and DYN models in isolation. Obviously, this approach can save considerable time and computer resources in analyzing shocks where there are only second-order feedback effects across countries.

[42] The horizon over which variables converge to steady-state values in the baseline solution of the model may differ from the time it takes to achieve convergence in the shock-minus-control values of the same variables. In the Mark III baseline, real interest rates and some real growth rates reach their steady-state values within 10 or 20 years, while demographics take more than half a century to settle down to a zero rate of population growth in all countries.

[43] See Juillard and Laxton (1996), and Juillard and others (1998) for a comparison of the properties of this algorithm with those of conventional first-order methods.