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Germany: Technical Note on Stress Testing

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GERMANY

STRESS TESTING

TECHNICAL NOTE

JULY 2011

INTERNATIONAL MONETARY FUND
MONETARY AND CAPITAL MARKETS DEPARTMENT

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GLOSSARY

BCBS	Basel Committee on Banking Supervision
bp	Basis point
CEBS	Committee for European Banking Supervision
CCA	Contingent claims analysis
CDS	Credit default swap
ECB	European Central Bank
ES	Expected shortfall
EU	European Union
ICAAP	Internal Capital Adequacy Assessment Process
IMF	International Monetary Fund
IRB	Internal Ratings-Based
LCR	Liquidity Coverage Ratio
LRS	Linear Combinations of Ratios of Spacings
MGEV	Multivariate extreme value
NiGEM	National Institute of Global Econometric Model
NSFR	Net Stable Funding Ratio
PD	Probability of default
PP	Percentage point
QIS	Quantitative Impact Study
RNDP	Risk neutral default probabilities
ROC	Regulatory capital
ROE	Return on Equity
ROC	Return on Capital
RWA	Risk-Weighted Assets
SPD	State price density
UK	United Kingdom
US	United States
VaR	Value-at-Risk
WEO	World Economic Outlook
YTM	Yield-to-maturity

EXECUTIVE SUMMARY

This note summarizes the stress tests undertaken for the German banking system as part of the Financial Sector Assessment Program (FSAP) Update. Solvency tests for the German banking system assessed medium-term vulnerabilities under two adverse macroeconomic scenarios. The tests considered a variety of measures of soundness, and took into account funding costs, sovereign risk, upcoming changes in the regulatory rules, and behavioral changes of banks. The liquidity tests simulated a sudden withdrawal of funding sources, and the maturity mismatch of banks. Tables 1–3 provide an overview of the key elements of the stress tests. Both the solvency and liquidity stress tests were undertaken in close cooperation with the authorities, and using a framework that facilitates comparison with peer countries.

The solvency stress tests simulated the impact of a double-dip recession scenario (with a spike in short-term interest rates) and a slow growth scenario on the solvency of the vast majority of the German banking system for the period from 2011–2015. The analysis was based on (i) a bank-by-bank balance sheet approach; and (ii) a market-based system-wide (or systemic) approach. The two methods complement each other, allowing for a comprehensive coverage of the German banking system on the one hand, and the incorporation of spillover effects/contagion on the other. Supplementary analysis based on publicly available data sought to shed light on other potential sources of vulnerabilities, namely the effect of (a) more severe macroeconomic shock scenarios; (b) higher hurdle rates and/or testing against core Tier 1 capitalization; and (c) stress resulting from marking-to-market all peripheral debt securities held by banks.

The liquidity tests were top-down tests to assess potential vulnerabilities to short-term and, to some extent, also to medium-term liquidity shocks; (i) a reverse test to determine the relative vulnerability of different banking groups to losses of funding based on supervisory data; and (ii) the computation of proxies for the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) for the majority of German banks based on publicly available data.

The test results revealed that German banks are robust against many shocks, but also that important vulnerabilities remain. Measured against current and future supervisory standards, German banks would need limited amounts of additional capital even under the adverse scenarios, with some exceptions if measured against core Tier 1 capitalization.[1] Nevertheless, some banks suffer from balance sheet fragilities, and widespread low profitability will make it challenging for many to raise the level and quality of their capitalization, as required under the new Basel III regime and by tougher market conditions. Part of the small private banks will face challenges ahead and more consolidation can be

[1] Recent capital measures by large German banks focus on those exceptions.

expected in the Sparkassen and cooperative sectors. Although exposures abroad in aggregate are well diversified, particular institutions have more concentrated and potentially worrisome exposures, including to vulnerable European countries. Moreover, some institutions such as certain Landesbanken are especially exposed to a spike in funding costs. The analysis also suggests that larger banks and some private banks are vulnerable to liquidity risk owing to their heavy reliance on wholesale funding.

The larger banks are more susceptible to funding cost risk, which could be a challenge going forward. Given persisting uncertainties in funding markets, it is possible that banks with relatively low levels of capital will experience a sharp increase in funding costs, making it more difficult to recover even if the economic environment improves. However, the relatively strong fiscal stance of Germany has, so far, mitigated funding problems. Furthermore, certain large banks display a greater reliance on volatile income components, such as trading income, and cross-border activities.

Despite the resilience of bank solvency, the secular effects of low profitability are pervasive. Across pillars, low profitability implies that most banks would earn only low returns on equity (ROE)—and would thus be constrained in paying out dividends to attract capital—even in a relatively benign macroeconomic environment. Factors that reduce profitability, such as a flat or inverted yield curve, amplify these effects.[2]

These results appear broadly similar when tested for sensitivity. Basel III effects (such as adjustments to risk-weights) add to potential strains, but do not appear to be key potential triggers of distress given the long transition periods unless markets anticipate the rules.[3] If one includes stress on sovereign debt holdings in the banking book as well as bank debt securities (using publicly available data[4]), uses hurdle rates higher than the regulatory minimum to anticipate Basel III effects (e.g., 2 percentage points (PPs) more), and accounts for potential even more severe macroeconomic stress, capital needs would of course be more substantial, but not unmanageable for the system as a whole. Capital needs based on the core Tier 1 ratios appear higher, suggesting that banks should intensify efforts to further improve the quality of their capital.

Results obtained using a market-based, systemic approach corroborate those presented above, and suggest that general vulnerabilities to the banking sector have eased from

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[2] The key role of interest rate income has been documented in Deutsche Bundesbank's 2010 Financial Stability Review.

[3] According to Bundesbank's latest Financial Stability Review, German banks would need to raise about €50 billion to adjust to the impending economic and regulatory conditions.

[4] The analysis used bank-by-bank data published by CEBS in July 2010 and aggregate information for the German banking system as of end-2010.

recent highs, although systemic tail risk remains. A very small number of banks generate the bulk of systemic effects, which have increased sharply at the onset of the credit crisis and have spilled over to the rest of the banking sector. The magnitudes of the estimated contribution to systemic risk of the largest German banks is comparable to that estimated for large United States (US) banks in the recent U.S. FSAP, but rather smaller than that for large United Kingdom (UK) banks; thus, results are plausible given differences in the degree of concentration. Large banks still pose significant tail risk to the system, albeit at lower levels than observed during the height of the credit crisis.

The liquidity stress tests found that most banks are able to cope with large liquidity shocks. The large banks and various private banks exhibit some vulnerability toward a sudden withdrawal of wholesale funding, which is also reflected in a less stable funding profile for longer maturities. Smaller German banks and especially the Sparkassen and cooperative banks benefit from their broad deposit base. Due to limitations on the availability of data, it was not possible to consider liquidity positions (or risk to funding costs) by currency.

Solvency Test: Overview of Assumptions

Table 1. Macroeconomic Variables under the Scenarios used for the Solvency Tests

	2010	2011	2012	2013	2014	2015
Real GDP Growth						
Actual/Baseline	3.3	2.0	2.0	1.8	1.7	1.3
Double Dip & spike in short-term interest rates	NA	0.5	-1.0	2.7	2.3	1.5
Slow Growth	NA	1.1	1.2	1.0	0.9	0.6
Short-term Interest Rate						
Actual/Baseline	0.8	1.0	1.3	1.8	2.2	2.5
Double Dip & spike in short-term interest rates	NA	2.0	2.2	1.3	1.2	1.4
Slow Growth	NA	1.1	1.2	1.6	2.3	2.7
Long-term Interest Rates						
Actual/Baseline	2.9	3.0	3.1	3.3	3.4	3.4
Double Dip & spike in short-term interest rates	NA	2.1	2.2	2.2	2.3	2.3
Slow Growth	NA	3.2	3.4	3.6	3.8	3.9

Source: Staff estimates based on analysis by Deutsche Bundesbank.

Table 2. Overview of Other Assumptions used for the Solvency Tests

Domain	Element	Specific Rules/Assumptions
Scenarios	(i) Baseline; (ii) Double Dip (1.5 times CEBS) with a spike in short-term interest rates; (iii) Slow Growth	• Macroeconomic/financial variables (GDP, interest rates) conditional on scenario based on the National Institute Global Econometric Model (NiGEM). • Aim to ensure broad consistency with other European FSAPs (and with European stress tests).
(Risk) factors assessed	Loss rates, Profit, Credit Growth, Sovereign Debt, Funding Costs, Taxes, RWAs	• **Credit losses** based on satellite models depending on scenario (overall pattern: decrease of loss rates compared to 2009). • **Profit** (interest income, interest expenses, net fee and commission income, and operating expenses) based on satellite models conditional on scenario. Net profit before tax adjusted for nonrecurring income in order to avoid misleading results (in one case also for losses that occurred during 2009 due to structural changes), informed by/benchmarked with 2010 profits. • **Trading income** based on statistical matching of trading income and GDP growth using a parametric fit of their historical distribution over the last 15 years (i.e., a decline in GDP growth is assumed to result in a deterioration of trading income). • **Sovereign debt**: Haircut on sovereign debt holdings in the trading book based on market expectations during 2010 (haircuts were lower than in the European stress tests for AAA-rated countries and approximately in line with the Committee for European Banking Supervisors (CEBS) for vulnerable European countries); CEBS exposures data from end-2009/early 2010 was used for the test, complemented by country-level data published by Bundesbank. • **Funding costs** based on satellite model, including a nonlinear effect (11 bps of total assets) once **Tier 1 ratios drops below 6 percent**. • **Tax assumption**: 25 percent in case of positive net income, zero otherwise. • **Changes in Risk-Weighted Assets (RWAs) under stress:** For credit risk change of RWAs based on the Internal Ratings Based framework (large banks only, based on work by Schmieder and others (2011)): increase of RWAs conditional on change of credit quality under stress, reduction of RWAs by 2.5 times of average risk-weight for defaulted credit exposure. For the smaller banks proportional reduction of RWAs, i.e., in line with simulated losses, and credit growth, respectively; RWAs for market risk equal to nominal GDP growth (accounting for business volume: applied to all banks); same assumptions for operational risk, but only for large banks;

Stress test assumptions related to Basel III	Hurdle rates in line in line with Basel III minima; Change in RWAs	• **Hurdle rates for Tier I and total capital according to Basel III** (i.e., increasing from 2013 onwards). • **Changes in RWAs resulting from changes in regulation:** Increase of total RWAs by 8.5 percent in 2011 due to Basel III changes for the large banks (Quantitative Impact Study (QIS) 6 result for large banks: 23 percent) and 0.4 percent for the small banks (QIS 6: 4 percent); lower ratios account for (a) the fact that increase in the RWA includes changes in definition of capital and (b) to account for some behavioral changes of banks (as intended by the Basel committee); • **Change in eligible capital:** Capital was subject to phase-out as foreseen by Basel III schedule, using QIS 6 results for large and small banks, respectively.
Behavioral adjustment of banks	Dividend pay-out rules (similar to Basel III minima); credit growth	• **Dividend pay-out depending on capitalization under stress:** 0 percent if total capitalization buffer of a bank is 0.5 PPs or less above 8 percent, and 40 percent in case capitalization buffer is more than 2.5 PPs (which reflects the magnitude of the proposed “capital conservation buffer” under Basel III); the rule is similar to the maximum payout under Basel III, which is, however, based on Tier 1 capitalization. • **Credit growth in line with nominal GDP** for banks with a Tier 1 capitalization buffer of 2.5 PPs above the regulatory minimum; credit growth decreases by 2 PPs for each decrease in Tier 1 ratios by 1 PP conditional on the buffer being less than 2.5 PPs. Hence, credit growth becomes negative once capitalization is slightly above minimum capital adequacy ratio (as the baseline nominal GDP growth is around 3-4 percent for most years). • **Other business strategy considerations**: any interim raising of capital until end-2010 considered in calculations.
Outcome	Template and assumptions	• **Metrics:** Focus on Tier 1 capitalization (and total capital ratios); core Tier 1 capitalization subject to supplementary tests. • **Treatment of Failures:** Banks that fall below the regulatory minimum before 2015 remain in the sample for the outer years; however, capital cannot become negative. • **Output template:** limited to aggregate figures, but includes some dispersion of results. • **Outcome by banking groups** (large banks, Sparkassen, cooperative banks). For smaller private banks, the heterogeneity of the sample data gaps precluded the estimation of reliable satellite models, and therefore they are excluded from the main results. However, the group was subject to additional quantitative analysis.

Source: IMF staff.

Table 3. Liquidity Test: Overview of Assumptions

Type of tests	(i) Implied Cash Flow test (gradual test for five subsequent periods to test sensitivity of banks against outflow of funding) based on supervisory data; (ii) other calculations based on publicly available data for the majority of German banks (Liquidity coverage ratio/LCR; Net Stable Funding Ratio/NSFR)	• **Type of tests:** Top-down tests with uniform assumption for all German banks. Assumption: no policy action/recourse by the European Central Bank (ECB). • Test (i): Simulation of gradual outflow of wholesale funding with/without customer deposits for five consecutive periods, taking into account of “fire sales” of liquid assets by banks. • Tests (ii): Calculation of proxies for the LCR and NSFR. • Data on currency breakdown of liquidity positions unavailable.
Assumptions	Outflow of liabilities; liquidity of assets to be used for fire sales of assets	• Test (i): Assumptions for outflow of customer deposits: 5 percent outflow of customer demand deposits and 3 percent of customer term deposits during each period (both portions of the then remaining deposits); 13.3 percent of banks deposits (on demand) and 10 percent of the time deposits of banks; other wholesale funding is assumed to be withdrawn by 15 percent for each period; Fire sales of liquid assets at haircut (in percent) : cash (0), AAA government bond holding (0); high quality liquid bonds (1); trading assets (20); lower quality bond holdings (30); derivatives (50). • Tests (ii): Minimum outflow ratios for stable and less stable customer deposits (5, 10 percent, respectively). Other assumptions based on expert judgment, also to account for lack of granularity of data.
Outcome	Template and assumptions	• **Outcome by banking groups** (large banks, Sparkassen, cooperative banks, small private banks). • **Test (i) is a reverse test**, i.e., to test the relative vulnerability of banks with respect to a progressively larger liquidity shock (and not to assess how many banks pass a hurdle in a given scenario). **“**Liquidity need” is a proxy, not taking into account possible “flight to safety” or other flows among banks.
Source: IMF staff.		

I. Solvency Stress Tests [5]

A. Overview

1. **The solvency tests covered a period of five years and assessed different levels of stress as well as systemic effects for the bulk of the German banking system, with a focus on broader (structural) tendencies rather than single institutions.** The aim was to gain a comprehensive view on the vulnerabilities in the system and differences across pillars within a medium-term context. Thus, the focus differed from that of other recent stress test exercises (in particular recent and forthcoming European stress tests coordinated by the European Banking Authority jointly with the ECB), which aimed/will aim at analyzing inherent risks within a two-year window and assessing potential capital needs of specific institutions. The analysis carried out as part of the FSAP also included liquidity risk.

2. **The core FSAP stress tests were carried out jointly with Bundesbank based on supervisory data; in addition to that, the mission has run supplementary tests to complement the outcome.** The advantage of this setting is that the vast majority of the system has been tested based on a framework that involved various stakeholders (Bundesbank's economic department, financial supervision, and banking supervision as well as the IMF mission) and used most recent supervisory data. The additional tests were meant to account for recent developments on the one hand and to allow for consistency with other FSAPs and related work.

3. **The vulnerabilities faced by the banking system were assessed under two macroeconomic scenarios:** (a) a sharp "double dip" recession caused by demand and oil price shocks, leading to an inversion of the yield curve (i.e., a spike of interest rates on the short-end and to lower interest rates on the long end); and (b) a prolonged period of very low growth. These scenarios correspond to the main risk for the German financial system identified ex ante, drawing on developments in the broader macrofinancial context (Appendix I–Risk Assessment Matrix). Results under these scenarios, which were prepared using the NiGEM, were benchmarked against those obtained under a *baseline scenario* that is in line with the October 2010 World Economic Outlook (WEO) projections, which itself does not envisage a return to rapid growth.

4. **The macroeconomic scenarios were linked to the developments of financial risks for banks through satellite models.** The link to macroeconomic conditions was established both for credit losses and profit. For the large banks, a risk-based solvency measure was used (i.e., one that links changes in credit quality to changes in RWAs), while smaller banks were assessed based on a statutory capital ratio (i.e., as used for the Standardized Approach).

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[5] The note was prepared by Andreas Jobst and Christian Schmieder.

5. **Close attention was paid to certain specific risk factors, (i) sovereign debt risk, (ii) funding risks as well as (iii) upcoming regulatory reforms.** The assessment in the core stress test of risks originating from banks' holding of sovereign debt explicitly included only the trading book. Funding risks arise from the substantial funding needs of large German banks in the coming years, which could trigger an increase in costs and thus affect their solvency position. Supplementary analysis were carried out to assess the effect of stress to all sovereign exposures (i.e., including in the banking book) and bank debt holdings on banks' solvency and funding costs, based on publicly available data. The assessment of the upcoming reforms in the context of Basel III, both through a change of hurdle rates and changes with respect to the computation of risk positions[6], were also taken into account. The framework accounted for taxes to be paid by banks. Hurdle rates above the regulatory minimum and core Tier 1 capitalization levels were subject to supplementary tests.

6. **Due to the comparatively long period covered by the tests, some behavioral adjustment by banks was modeled.** The framework assumed that banks with low capital buffers would retain all profits while banks would pay out a portion corresponding to the long-term average otherwise (40 percent). The nonlinear relationship assumed for the payout is consistent with the maximum pay-out rules foreseen under Basel III. In addition, credit growth was assumed to grow in line with nominal GDP for banks in "normal" conditions, but allowed for deleveraging by weak banks to cope with stress.

7. **The solvency tests were based on two methodologically distinct but analytically complementary methods,** allowing for a comprehensive coverage of the German banking system on the one hand and the incorporation of spillover effects/contagion on the other:

- *A balance sheet approach*, which has the advantage of near-full coverage of the German banking system. The framework included a variety of building blocks (as outlined above) necessary to allow for a meaningful[7] top-down forecast of potential medium-term developments. The aim was to shed light on the potential evolution of capital adequacy driven by macrofinancial conditions and potential shortages of capital. The computation was based on a stress test framework developed by Schmieder, Puhr, and Hasan (2011).

- *A market-based systemic approach* ("systemic contingent claims analysis, (SCCA)). The use of a SCCA portfolio model type approach based on market data allowed computing *joint* solvency risk given potential spillover effects across larger German banks. To do so, the contribution of banks to systemic risks was assessed. The estimation was based on Gray and Jobst (2010 and forthcoming) as well as Gray, Jobst and Malone (2010).

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[6] Changes in the eligibility of capital under Basel III during the forecast horizon were taken into account based on publicly available data.

[7] As a caveat, it has to be taken into account that confidence levels become wider with longer time horizons.

B. Macroeconomic Scenario

8. **Two macro stress scenarios (double dip with spike in short-term interest rates, slow growth) were used in addition to a baseline scenario to assess stress in the banking system (Table 1).** The two macro shocks reflect the main risks for Germany as identified ex-ante. The baseline scenario is derived from the WEO (October 2010). The macroeconomic evolution was forecasted through 2015 by means of NiGEM, and incorporates cross-border effects and policy reactions. In order to ensure consistency with the other European FSAPs, the GDP path was predetermined.

9. **The rationale for a double-dip scenario (leading to an inversion of the yield curve) originates from the persistent uncertainty of the growth prospects of the world economy.** While Germany has seen a strong rebound of its economy in 2010, its dependency on exports makes the economy vulnerable against changes in the world economy. Hence, in macroeconomic terms (i.e., using NiGEM) an external demand shock of the main trading partners of Germany (European Union (EU) economies, the US and China) has been paired with a strong surge of oil prices.[8] The GDP path was predefined for the first two, and was predicted with the macro model along with the interest rates. Compared to the baseline scenario, there is a cumulative deviation of GDP growth by 3.7 percent over the five-year horizon, with a 5.4 PP fall relative to the baseline in the first two years (equivalent to 1.5 times the shock assumed in the CEBS tests and two standard deviations in historical terms).[9] Short-term and long-term rates tend to be about 100 basis points (bps) lower than in the baseline, except for a spike in short rates during the first two years (where the rates are about 100 bps above baseline) in response to the inflationary effect of the oil price increase.

10. **The slow growth scenario simulates a "malaise" growth path.** Under this scenario, growth constantly deviates by 0.7 to 0.9 PPs from the baseline forecast for Germany's real output, which goes down from 2 percent in 2011 towards 1 percent in 2015. A slow growth scenario in Germany could result from the accumulation of structural rigidities, fiscal burdens, demographic pressure, and current uncertainties both in economic and political circumstances could lead to a prolonged period of very low growth in Europe more generally. The slow growth scenario was simulated through a supply-side shock, i.e., a permanent decrease in the technical progress worldwide, resulting in a flattening of the path of potential output. With lower potential output, inflationary pressures are higher than in a demand-led recession, so interest rates are project to be slightly higher than under the baseline. The cumulative deviation of GDP growth is 4 percent.

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[8] Oil prices have already gone up significantly, and additional geopolitical conflicts in the Middle East could trigger an additional increase in oil prices towards levels of $200, which was the underlying assumption.

[9] In historical terms (i.e., referring to the last 30 years and accounting for the German unification), the simulated shock corresponds approximately to a shock of 2 standard deviations (cumulative deviation of 5 PPs during the first two years compared to a historical average of 2.5 percent (over two years)).

C. Satellite Models

11. **Satellite models have been used to determine credit losses and various components of profit, including funding costs.** The dynamic regression models were computed based on bank level panel data of German banks from the last 15 years, distinguishing between large and small banks. Models generally included a lagged term, GDP growth, and other variables, particularly interest rates. The signs of the coefficients were as expected. The specifications of the satellite models are displayed in Appendix II.

12. **Credit losses were forecasted based on impairments, based on a similar model as used in Bundesbank's Financial Stability Review.** All sectors have been assumed to be subject to the same elasticities.[10] The sector-specific levels for the simulated forecasts (for corporate, retail, public, and financial institutions) were derived from regulatory data for the German Internal Ratings-Based (IRB) banks.

13. **For profits, four separate models were used to forecast interest income,[11] interest expenses, net fee and commission income, and the operational expenses.** All models included a lagged term, GDP growth, interest rates, and other control variables (Appendix II). Income taxes were assumed to be at 25 percent for banks with positive levels of profit and zero otherwise. For the large banks, a model describing net interest income was used, with additional funding costs conditional on the capitalization of banks (the latter was informed by models for the interest rate expenses, see para. 15f).

14. **Funding costs were linked to the Tier 1 capitalization of banks under stress. It was assumed that the increase in funding costs takes effect once the Tier 1 capital ratio drops below 6 percent.** The sensitivity of funding costs to capitalization was included in the satellite model for interest expenses, combined with expert judgment. Accordingly, interest expenses were simulated to increase according to empirical evidence plus some additional factor, reflecting the non-linearity of funding costs with respect to capitalization.

15. **The effects on funding costs are accounted for in the *subsequent* year.** Hence, the effect of deteriorating solvency increasing funding costs contemporaneously has been neglected for simplicity.

16. **The trading income under stress was aligned with GDP growth using historical data for the last 15 years**. On balance, the approach makes trading income relatively sensitive to macroeconomic conditions and is thus relatively conservative. Empirical evidence suggests that there is an only weak relation between the trading result and macroeconomic conditions, but it was assumed that unfavorable trading results tend to

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[10] Specifications with bank-specific portions of exposure to broad sectors (corporate, financial institutions, public sector) showed limited differences across sectors.

[11] For the large banks, a model using net interest income was employed.

coincide with macroeconomic shocks—a situation that many German banks experienced during the recent crisis, for example. To this end, GDP growth under each scenario and year was first linked to the corresponding GDP growth rate during the last 15 years (i.e., the growth rate closest to the projected one). In the second step, past changes of trading results were fitted to a historical distribution (whose median value subsequently matched to the median GDP growth rate). Finally, the corresponding "point estimate" of trading income was chosen at the probability level commensurate to the likelihood of observed change in GDP growth. Under the double-dip scenario in 2012, for example, which is close to the second worst GDP growth rate during the last 15 years, the trading result at the 90th percentile of its historical distribution was estimated for each bank.[12]

17. **Nonrecurring income and expenses, such as extraordinary items, were set to zero for 2010 and beyond.** As such, the simulation of income was based on pre-impairment operating income subject to stress conditional on the scenario.

D. Other Elements of Stress Tests

Dividend pay-out rule and retained income[13]

18. **Dividend payout behavior is modeled explicitly.** Dividend payouts are payable out of the previous year's profit and, thus, cannot result in a drop below any of the minimum capital requirements. Specifically, it was assumed that dividends are paid only by banks that satisfy capital adequacy after having created adequate provisions for impairment of assets and transfer of profits to staff benefits and statutory reserves. Moreover, well-capitalized banks (i.e., banks that meet the minimum capital requirement and generate positive earnings after taxes) are assumed to pay out dividends only if they reported profits.

19. **Banks were assumed to pay out up to a maximum of 40 percent of their profit, in line with empirical evidence for Germany.** Banks that meet the minimum total capital ratio of at least 8.0 percent (after the envisaged dividend payout and, at the same time, exhibit sufficient Tier 1 and core 1 capitalization) but fall below the 10.5 percent threshold are considered capital-constrained and follow a payout schedule as displayed in Table 4—in anticipation of the full adoption of the Capital Conservation Buffer of 2.5 PPs under the Basel III proposal (Figure 2). Banks with a total capital ratio of at least 10.5 percent are limited to a maximum dividend payout of 40 percent, which is in line with past pay-out ratios of German banks.[14] Under Basel III, the maximum pay-out rules are defined based on core

[12] This statistical mapping technique via a high-dimensional parametric fit function to align GDP with trading helped reduce discontinuity of the matched series.

[13] The dividend payout ratio is defined as the percentage of "dividend payable in a year" to "net income during the year."

[14] The pay-out ratio for the Sparkassen is lower, but if one includes social spending then 40 percent becomes a valid benchmark.

Tier 1 capitalization rather than based on total capitalization, but otherwise the rules are similar as displayed in Table 5.

Table 4. Pay-out Ratio Conditional on Capitalization under Stress
(In percent)

Buffer above minimum capital ratio (in percent)	FSAP (dividend pay-out ratio based on total capital ratio)	Basel III (maximum pay-out ratio based on core Tier 1 ratio)
0-0.5	5	0
0.5-1	10	20
1-1.5	15	20
1.5-2	20	40
2-2.5	30	40
>2.5	40	40 to 100

Source: IMF staff.

Banks' balance sheets

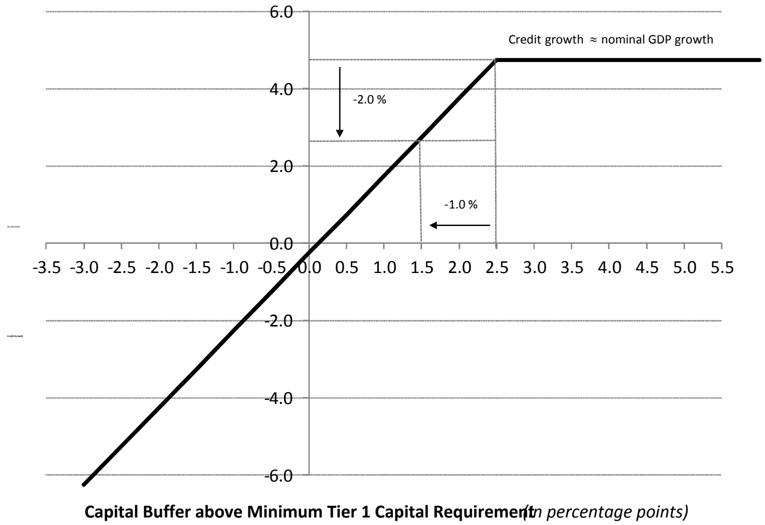
Projected end-2010 balance sheets

20. **The tests have been carried out based on the projected end-2010 balance sheets and early-2010 holdings of sovereign debt.** Profits and losses are forecasted by means of satellite models. The base for 2010 profits (and retained earnings) was 2009 profits excluding nonrecurring income items, i.e., using banks' steady-state earnings capacity. Thus, profit should be understood as net operating income, adjusted for impairments and other income. The projected 2010 profits were benchmarked with the preliminary 2010 figures, and it was found that they were, by and large, in line. The data for capitalization (and risk-weights) were taken from reported end-September 2010 data (in specific cases end-2010 figures were used). In two cases, structural changes (recent increase of capital and change of ownership) were taken into account to avoid misleading results. For the sovereign debt holdings, the exposures disclosed as part of the 2010 European stress tests were used.

Balance sheet growth

21. **Firms' balance sheets are assumed to grow in line with nominal GDP, except for credit growth, where banks affected by stress were assumed to slow growth, including through deleveraging.** For well-capitalized banks, it was assumed that credit grows in line with nominal GDP. Based on empirical evidence and expert judgment, it was assumed that credit growth starts declining once firms' capitalization reaches the threshold of 2.5 PPs above the minimum Tier 1 ratio in each year over the forecasting horizon (as foreseen under Basel III). For each rounded PP below this threshold, credit growth was simulated to decline by 2 PPs (Figure 1). The growth of banks' assets was taken into account in the satellite models.

Figure 1. Credit Growth Conditional on Tier 1 Ratio



Source:IMF staff.

22. **Asset disposals or acquisitions in 2010 were not considered in order to preserve more "granularity" in the results.** As such, the structure of the banking system as of end-2009 was used as a basis for projections and subsequent analyses. In particular, Deutsche Postbank was kept in the sample as a separate legal entity, notwithstanding the fact that it has been majority-owned by Deutsche Bank since November 2010. However, Hypo Real Estate was excluded from the test, for two reasons: (a) the bank is 100 percent state-owned; and (b) the bulk of the toxic assets have been transferred to a bad bank, which not only means that the remainder of the bank is less likely to face additional stress, but also that the remaining bank has become substantially.

Risk-weighted Assets

23. **The tests explicitly simulated changes of RWAs for IRB banks under stress.** To this end, the projected credit losses conditional on the scenario were used as a proxy for the IRB credit parameters. The bank-level specific loss ratio was re-scaled to asset class specific levels based on average IRB credit risk parameters for German banks (using supervisory data).[15] The changes of RWAs, taking also into account credit growth, were computed using approximation formulas (Schmieder and others, 2011). For defaulted exposures, the predefault risk-weight was assumed to be 2.5 times the average risk-weight of nondefaulted credit. For banks reporting under the standardized approach, RWAs were adjusted for losses

[15] In case of a bank-level loss ratio of 1 percent, for example, the expected loss for bank exposure would be set at 0.25 percent and for corporate at 1.5 percent. For a bank with a loss level of 2 percent, the asset class specific expected losses would be twice as high.

and credit growth "only" and not on RWAs, i.e., neglecting potential changes of external ratings, etc.

Basel III

24. **The tests took into account recent and prospective changes of regulatory rules** (Table 5), which include (i) a change in minimum capital requirement ratios (Tier 1 and common/core Tier 1); (ii) an increase in RWAs; and (iii) changes of capital eligibility under Basel III from 2013 onwards. The simulation of the impact of the changes was informed by the QIS-6 (Basel Committee on Banking Supervision (BCBS) 2010b).[16] The phase-out of core Tier 1 eligibility was simulated based on supplementary analysis (i.e., publicly available data), again informed by the QIS-6. The pay-out rule used for the stress test is similar to the maximum pay-out foreseen by Basel III (Table 4).

Table 5. Overview on the Basel III Minimum Capital Requirements

Annex 2: Phase-in arrangements (shading indicates transition periods)
(all dates are as of 1 January)

	2011	2012	2013	2014	2015	2016	2017	2018	As of 1 January 2019
Leverage Ratio	Supervisory monitoring		Parallel run 1 Jan 2013 – 1 Jan 2017 Disclosure starts 1 Jan 2015					Migraton to Pillar 1	
Minimum Common Equity Capital Ratio			3.5%	4.0%	4.5%	4.5%	4.5%	4.5%	4.5%
Capital Conservation Buffer						0.625%	1.25%	1.875%	2.50%
Minimum common equity plus capital conservation buffer			3.5%	4.0%	4.5%	5.125%	5.75%	6.375%	7.0%
Phase-in of deductions from CET1 (including amounts exceeding the limit for DTAs, MSRs and financials)				20%	40%	60%	80%	100%	100%
Minimum Tier 1 Capital			4.5%	5.5%	6.0%	6.0%	6.0%	6.0%	6.0%
Minimum Total Capital			8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%
Minimum Total Capital plus conservation buffer			8.0%	8.0%	8.0%	8.625%	9.25%	9.875%	10.5%
Capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital			Phased out over 10 year horizon beginning 2013						
Liquidity coverage ratio	Observation period begins				Introduce minimum standard				
Net stable funding ratio		Observation period begins						Introduce minimum standard	

Source: Basel Committee on Banking Supervision (BCBS 2010a)

25. **In terms of minimum capital ratios, the tests assessed solvency in accordance with the agreement published by the Basel Committee on Banking Supervision in December 2010** (BCBS 2010a, Figure 2). Accordingly, as of January 1, 2013, banks will need to meet the minimum requirements of 3.5 percent for the common/core Tier 1 ratio, 4.5 percent for Tier 1 ratio in addition to the 8.0 percent total capital/RWAs. The phase-in of

[16] The leverage ratio was not explicitly taken into account, thereby accounting for the fact that it will only come into effect by 2018. See http://www.bis.org/press/p100912.htm.

the capital conservation buffer is scheduled for 2016 to 2019; i.e., does not fall into the period assessed by the tests.

26. **The increase in RWAs due to changes in regulation (Basel III) was informed by the QIS-6 (BCBS 2010b).** The full impact as determined in the QIS was reduced by the portion subject to phase-out of capital and further reduced by half to account for potential behavioral changes by banks.[17] As such, RWAs for the large banks were simulated to increase by 8.5 percent during 2011 and 0.4 percent for the smaller banks.[18]

27. **The phase-out of capital under Basel III was simulated using QIS-6 results (BCBS 2010b).** Total capitalization and Tier 1 was reduced by 10 percent of the portion found to become noneligible in the QIS-6 (about 30 percent for the large banks and 15 percent for the smaller banks) in 2013, 2014, and 2015.[19] The data on core Tier 1 capital[20] (as available by September 2010 in the supervisory systems) was not considered robust enough to be used for the core tests. Fund staffed examined Core Tier I capital ratios based on public information (available by end-2010) as part of supplementary analysis, also spurred by the European stress tests in 2011 which use core Tier 1 ratios.

Treatment of fixed-income holdings

28. **Ongoing fiscal uncertainties were captured through potential adverse changes in asset values in case of stress.** The main concern in this context is the possibility of increasing country-specific risk premia, which are manifest in sovereign yield spreads and CDS rates. These market prices have seen large increases and volatility in the last two years, with two periods of particular stress during 2010 (spring and fall 2010).

29. **The stress tests used a market-based approach to infer "haircuts" on government debt sovereign exposures in the trading book (see Appendix III and Table 6).** The resulting haircuts are broadly comparable to those used in the CEBS exercise. To this end, the future yield-to-maturity (YTM) (and associated haircut) based on changes in the market-implied default probability was computed for each year of the forecast horizon relative to the base period (i.e., 2010) after accounting for changes in forward contracts on the country-specific sovereign credit default swap (CDS) spread. It was assumed that there is

[17] Behavioral adjustments to reduce RWAs do not necessarily denote deleveraging, but will also happen with a reduction of activities that consume a substantial amount of capital (i.e., risky activities), in line with the purpose of Basel III.

[18] For the universe of banks participating in the QIS 6, the increase of RWAs was computed to be 23 percent for large (Group 1 banks) and 4 percent for smaller banks (Group 2 banks).

[19] Hence, total eligible capital was reduced by about 3 percent each for the large banks in 2013, 2014, and 2015.

[20] Basel III envisages the phase out of certain items (such as deferred tax assets and minority interests) from Core Tier I capital (BCBS, 2010a).

no general shift in the yield curve. In order to account for the fact that market expectations were highly volatile in recent months, the haircuts were computed based on average forward prices on one-year CDS spreads during the last 24 months.

30. **The core stress tests (i.e., the ones based on supervisory data) included the effect of such haircuts on fixed-income holdings in the trading book, but not those in the banking book.** Supplementary tests based on publicly available data (aggregate country-specific data published by Bundesbank) were carried out by Fund staff, including also sovereign debt holdings in the banking book as well as bank debt holdings (as sovereign downgrades are likely to lead to bank downgrades) for currently vulnerable European countries (i.e., Greece, Ireland, Italy, Portugal, Spain and Belgium).[21] The latter analysis sought to shed light on the economic solvency profile of banks, which has been subject to close scrutiny by market participants in wholesale funding markets.

31. **This approach to calibrating the haircuts is based on historical data available at the time of the FSAP mission, yet, for some countries, market prices have fallen sharply since then.** The estimated effects of the projected haircuts must, therefore, be viewed as relatively modest and larger losses would be incurred if these risks are realized. To account for the current circumstances some additional sensitivity analysis were run. Ultimately, were turmoil to spread to larger countries that are more closely tied to Germany, the impact through solvency and funding channels might become even larger, but such scenarios are inherently characterized by great uncertainty.

[21] It was assumed that 25 percent of the cross-border bank exposure to the GIIPSB countries (as reported by Bundesbank) are medium and long-term debt securities. Official data on this ratio are not available, but indications are available from banks' annual reports and other public data sources (aggregate data). The assumption is considered conservative on the system level, but for specific banks the portion could be higher.

Table 6. Haircuts on Debt holdings for Core Tests

Sovereign Debt Haircut, *In percent*

	5-year rate									
	Baseline					*Slow Growth and Double Dip Scenarios*				
Year	**2011**	**2012**	**2013**	**2014**	**2015**	**2011**	**2012**	**2013**	**2014**	**2015**
Belgium	1.76	1.88	1.79	1.66	1.55	4.56	4.74	4.61	4.42	4.26
France	0.00	0.04	0.05	0.03	0.01	0.77	1.03	1.09	0.99	0.90
Germany	0.00	0.00	0.02	0.00	0.00	0.30	0.41	0.45	0.43	0.43
Netherlands	0.30	0.34	0.35	0.34	0.34	0.61	0.68	0.71	0.70	0.70
UK	0.10	0.27	0.25	0.10	0.16	1.19	1.39	1.31	1.02	1.03
Greece	12.77	10.85	9.07	8.21	7.28	26.61	23.20	20.00	18.56	16.81
Ireland	6.12	5.55	4.77	4.26	3.78	13.83	12.81	11.39	10.56	9.59
Italy	2.75	2.78	2.72	2.55	2.44	5.66	5.64	5.58	5.36	5.18
Portugal	2.25	1.76	0.94	0.27	0.00	10.72	9.72	8.06	6.77	5.64
Spain	1.90	1.73	1.56	1.37	1.25	6.10	5.74	5.44	5.17	5.02
United States	0.14	0.17	0.20	0.21	0.23	0.36	0.49	0.65	0.75	0.85
Memo item: CEBS haircuts (absolute)										
Belgium	1.40	3.10	—	—	—	3.47	6.51	—	—	—
France	1.50	3.00	—	—	—	2.21	4.93	—	—	—
Germany	0.10	2.30	—	—	—	2.03	3.69	—	—	—
Netherlands	1.10	2.50	—	—	—	1.79	4.83	—	—	—
UK	5.00	6.90	—	—	—	2.56	5.52	—	—	—
Greece	3.90	4.30	—	—	—	22.60	30.22	—	—	—
Ireland	1.60	4.20	—	—	—	7.58	11.11	—	—	—
Italy	1.20	2.90	—	—	—	3.96	7.10	—	—	—
Portugal	2.30	3.70	—	—	—	10.94	15.92	—	—	—
Spain	1.30	4.10	—	—	—	6.55	9.86	—	—	—
US	1.30	4.40	—	—	—	4.41	9.37	—	—	—
Memo item: Difference between new haircuts and CEBS haircuts										
Belgium	0.36	-1.22	—	—	—	1.09	-1.77	—	—	—
France	-1.50	-2.96	—	—	—	-1.44	-3.91	—	—	—
Germany	-0.10	-2.30	—	—	—	-1.73	-3.29	—	—	—
Netherlands	-0.80	-2.16	—	—	—	-1.18	-4.15	—	—	—
UK	-4.90	-6.63	—	—	—	-1.37	-4.14	—	—	—
Greece	8.87	6.55	—	—	—	4.01	-7.03	—	—	—
Ireland	4.52	1.35	—	—	—	6.25	1.70	—	—	—
Italy	1.55	-0.12	—	—	—	1.70	-1.46	—	—	—
Portugal	-0.05	-1.94	—	—	—	-0.22	-6.20	—	—	—
Spain	0.60	-2.37	—	—	—	-0.45	-4.13	—	—	—
US	-1.16	-4.23	—	—	—	-4.05	-8.88	—	—	—

Note: Baseline - 50th percentile of past price changes of forward contracts on credit default swaps (CDS) with maturity terms between one and five years respectively (since end-2008); the 75th percentile is applied for the calculation of haircuts in the double dip and slow growth scenarios.

Source: IMF staff.

E. Balance Sheet-Based Solvency Tests

Introduction

32. **The balance sheet based solvency tests were run separately for 3 banking groups, the 16 largest German banks[22], the Sparkassen, and the other cooperative banks, on an unconsolidated basis.[23]** Grouping banks reveals potential differences across pillars.

33. **The fourth important group of German banks, small private banks (accounting for 11 percent of the banking system's assets), was omitted for the core stress tests**. The main reason was that the heterogeneity of the sample data gaps precluded the estimation of reliable satellite models, and therefore they are excluded from the core tests. However, supplementary single factor tests were carried out.

34. **The core tests (i.e., the tests based on supervisory data) revealed capitalization under stress (with a focus on Tier 1 capital) and capital shortfall with respect to prudential requirements, if applicable, as well as the risk drivers for the outcome.** The dispersion of the results for each peer group was used to allow for a more clear-cut picture of the range of results. It was also used to shed light on the fact that banks are likely to hold more capital than foreseen by regulatory minimum rules, not least to attain a favorable rating, but also to satisfy the demand of markets.

35. **It is important to highlight that it was assumed that banks will neither raise capital[24] nor materially change their balance sheet structure (other than deleveraging) and that the precision of forecast for five years are less precise than for shorter periods of time.** Hence, one has to apply some caveats when it comes to the interpretation of the results.

36. **Supplementary analyses based on publicly available data were carried out by Fund staff to assess additional dimensions considered relevant in the broader context of the FSAP Update.** The supplementary analysis included (a) a variation of the hurdle rates (and calculations for core Tier 1); (b) stress of all sovereign and bank debt securities to the European periphery; and (c) the simulation of an even more severe double dip (one-third larger than the core "double dip" scenario).

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[22] That is, the 14 large banks and the other two Landesbanken.

[23] The group of Sparkassen and cooperative banks does not include the banks that were classified as large banks.

[24] However, all action to increase capital was taken into account, including, for the supplementary tests based on core tier 1 capitalization, that two large banks have increased their capitalization during the EBA tests.

Core Solvency Tests

37. **Banks were found to meet regulatory minimum ratios under baseline conditions, but some banks could face challenges to cope with stress conditions (Table 7 and Figure 2).** Vulnerabilities have been identified in a few large banks, but also some banks in the Sparkassen and cooperative sector.

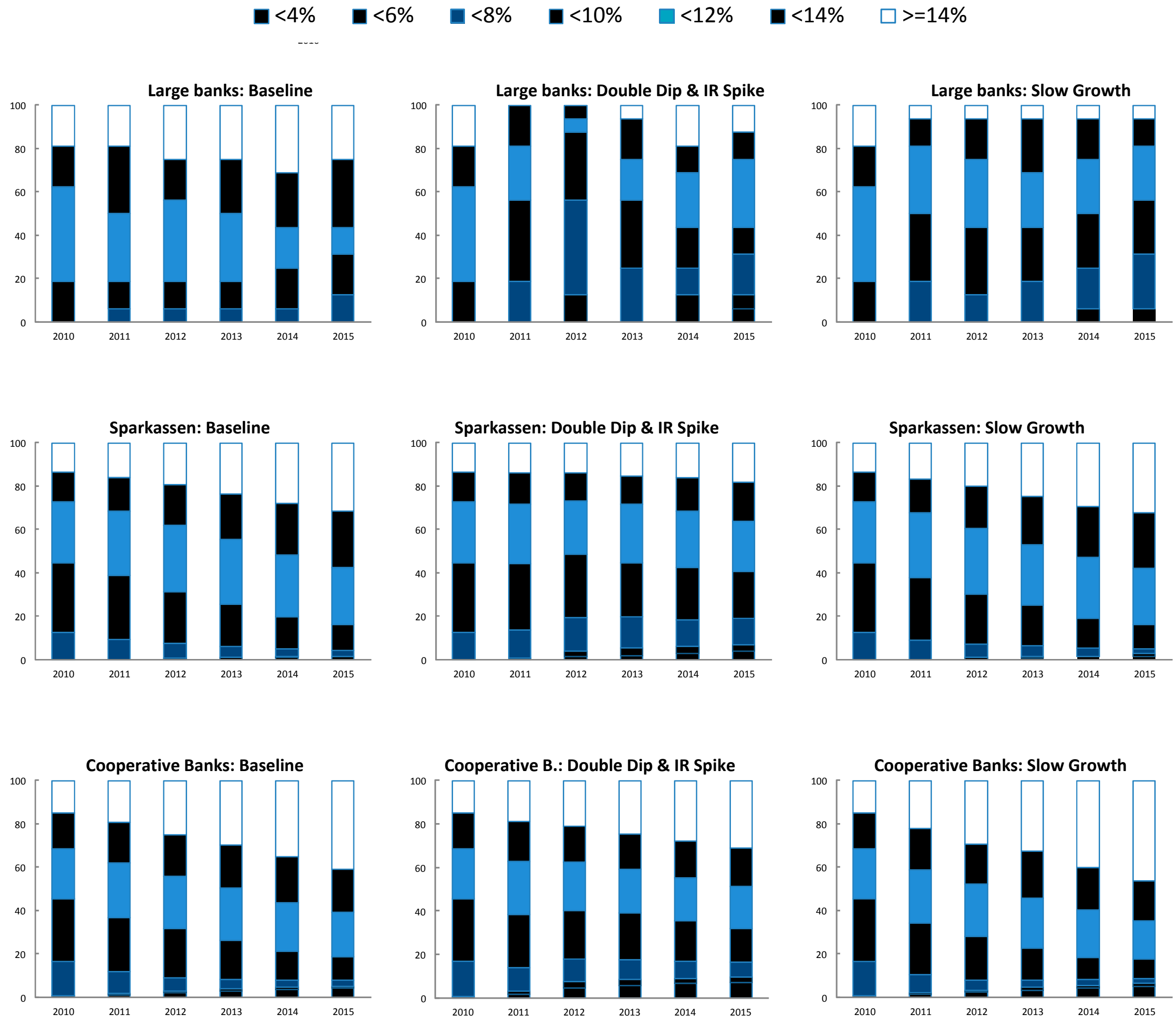
Table 7. Overview of Outcome of Core Solvency Tests by Banking Group

		2010	2011	2012	2013	2014	2015
		Baseline					
Tier 1 Ratio (percent)	Large banks	12.9	13.0	13.6	13.8	13.7	13.4
	Savings banks	10.3	10.8	11.3	11.8	12.3	12.7
	Cooperative Bank	10.0	10.6	11.1	11.5	12.0	12.4
Number of banks failing the tests	Large banks	0	0	0	0	0	0
	Savings banks	0	0	0	4	6	6
	Cooperative Bank	0	13	26	35	50	58
Capital shortfall (euro millions)	Large banks	0	0	0	0	0	0
	Savings banks	0	0	0	115	288	389
	Cooperative Bank	0	135	169	237	391	553
		Double Dip & Interest rate Spike					
Tier 1 Ratio (percent)	Large banks	12.9	10.8	8.6	10.7	11.6	11.1
	Savings banks	10.3	10.4	10.1	10.2	10.5	10.7
	Cooperative Bank	10.0	10.6	10.2	10.4	10.6	10.9
Number of banks failing the tests	Large banks	0	0	0	0	1	2
	Savings banks	0	0	6	13	22	30
	Cooperative Bank	0	19	51	77	94	109
Capital shortfall (euro millions)	Large banks	0	0	0	0	234	2,548
	Savings banks	0	0	188	319	597	857
	Cooperative Bank	0	157	315	506	832	1,145
		Slow Growth					
Tier 1 Ratio (percent)	Large banks	12.9	11.6	11.5	11.1	10.5	9.7
	Savings banks	10.3	10.9	11.4	11.8	12.3	12.7
	Cooperative Bank	10.0	10.9	11.5	11.9	12.4	12.6
Number of banks failing the tests	Large banks	0	0	0	0	1	1
	Savings banks	0	0	2	5	6	12
	Cooperative Bank	0	15	29	41	57	75
Capital shortfall (euro millions)	Large banks	0	0	0	0	57	1,690
	Savings banks	0	0	22	185	332	426
	Cooperative Bank	0	138	174	290	475	669

Source: Deutsche Bundesbank and IMF staff.

Note: The number of failed banks and the capital needs are to be understood in "cumulative-like terms" (it is not cumulative as some banks could recover and leave the red zone); the situation by 2012, for example, shows that the Tier 1 ratio of the large banks was 8.6 percent, and then recovered to 11.1 percent by 2015. By 2012, no bank fell under the then minimum Basel Tier 1 capital ratio.

Figure 2. Outcome of Core Solvency Tests—Dispersion by Bank Group



Source: Deutsche Bundesbank and IMF staff.

38. **The large banks identified as being vulnerable were among those subject to ongoing post-crisis talks.** If one sets the hurdle rates at the regulatory minimum, capital needs by 2015 were estimated at around EUR 2.5 billion under the double dip scenario and EUR 1.7 billion under the slow growth scenario. The core tests also show that banks would face challenges only in the outer years of the stress horizon, i.e., 2014–2015. Supplementary tests based on publicly available data indicate that the use of core Tier 1 as yardstick raises capitalization capital needs by about EUR 10 billion under double dip conditions (Figure 4).

39. **The Sparkassen have been found to be in a position to resist stress as defined in the core tests, with some exceptions.** For the Sparkassen sector, overall capital needs have been found to be at below EUR 1 billion. The outcome suggests that there will be some additional consolidation to ensure that banks meet regulatory changes and cope with potential stress, triggered by the interest income, but also through potential credit losses at specific institutions. However, the situation is manageable for the Sparkassen in overall terms.

40. **The cooperative sector is robust, but some additional consolidation can be expected.** The situation under the core stress tests is similar as for the Sparkassen sector, namely that the bulk of institutions is well-capitalized and in a position to cope with regulatory changes, but some institutions are likely to face challenges in case of an interest rate shock combined with credit losses, with capital needs amounting to EUR 1.1 billion under the double dip scenario. The fact that some banks fall short of the Basel III minimum under the baseline illustrates that some institutions will have to strengthen their solvency profile over time, for example by means of mergers.

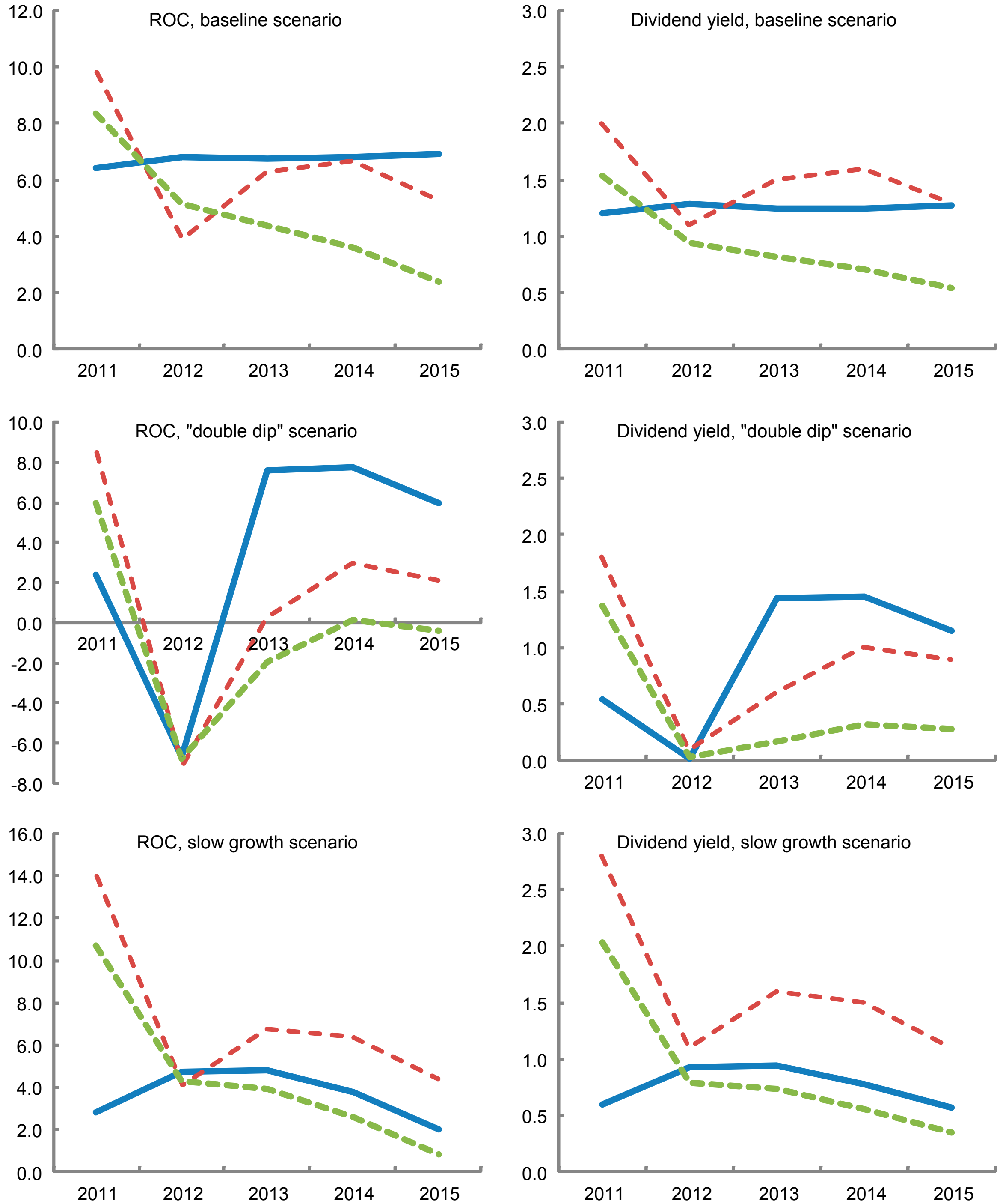
Bank profitability

41. **Supplemental stress testing based on public data allowed the projections of bank profitability.** Figure 3 shows the return on regulatory capital (ROC) and dividends relative to regulatory capital under the baseline scenario. Due to data limitations, it was not possible to show ROE. Return on assets is more difficult to compare across banks because of their heterogeneous business models.

42. **Even under a baseline scenario of continued steady growth, ROC will be modest on average.** Dividend payouts from many banks will need to be very low to meet capital requirements. For the smaller banks, the trend is determined largely by the evolution of the net interest income.

Figure 3. Projected Bank ROC and Dividend Payout Yield

(percent)



Source: Staff estimates based on publicly available data.

Small private banks

43. **When tested against credit and concentration risk, the small private banks appear heterogeneous—some banks appear robust while others show some weaknesses (Table 8).** The sensitivity tests were carried out based on end-September 2010 capitalization and simulated an increase in loss rates compared to 2009, i.e., an additional increase from elevated levels by 20 percent and 50 percent in relative terms, respectively. Of the 148 small private banks included in the test, most are able to withstand such shocks, and median capitalization drops by about 0.6 percent in the more severe case. A default of the three largest borrowers, which constitutes a highly adverse scenario, has a comparatively limited impact. Nevertheless, the weak private banks should be subject to increased scrutiny, particularly the ones that show vulnerabilities, and are not backed by a strong parent bank. At least part of this group of banks will be challenged by Basel III, both in terms of solvency and liquidity and capital will have to be built up through retained earnings in many cases. Also, the competition with the Sparkassen and cooperative sectors restricts their business to niche markets.

Table 8. Sensitivity Analysis for Small Private Banks

Sensitivity Test		Before Test (Q3 2010)			After Test				
		Q25	Median	Q75	Q25	Median	Q75	Number of Banks Below Regulatory Minimum	Capital Needs (EUR million)
Increase of Loss Rates by 20 Percent	Total CAR	11.8	15.6	22.0	11.4	15.3	21.9	2	80
	Tier 1 Ratio	9.7	13.8	21.4	9.5	13.8	21.1		
	RoE	0.0	2.2	8.4	-2.4	0.8	7.3		
Increase of Loss Rates by 50 Percent	Total CAR	11.8	15.6	22.0	11.1	14.9	21.7	4	230
	Tier 1 Ratio	9.7	13.8	21.4	9.3	13.2	20.6		
	RoE	0.0	2.2	8.4	-4.9	0.0	6.0		
Default of 3 largest borrowers	Total CAR	11.8	15.6	22.0	10.7	14.5	21.7	5	150
	Tier 1 Ratio	9.7	13.8	21.4	8.6	12.5	20.0		
	RoE	0.0	2.2	8.4	-9.8	-3.0	1.5		

Source: Deutsche Bundesbank.

Supplemental solvency tests

44. **Supplementary tests for the large banks based on publicly available data assessed the impact of (i) higher hurdle rates and core Tier 1 ratios; (ii) stress to assets vis-à-vis the European periphery held in the banking book; and (iii) even more adverse macroeconomic conditions.**[25] The supplementary tests simulate the anticipation of Basel III rules, and account for enforced market scrutiny in wholesale funding markets, persisting stress in peripheral Europe and for the global economy more generally as well as geopolitical uncertainties. As parts of the German banking system were in the focus of attention during

[25] The scenario doubles the macroeconomic severity of the CEBS stress test, i.e., 2.6 standard deviations in historical terms.

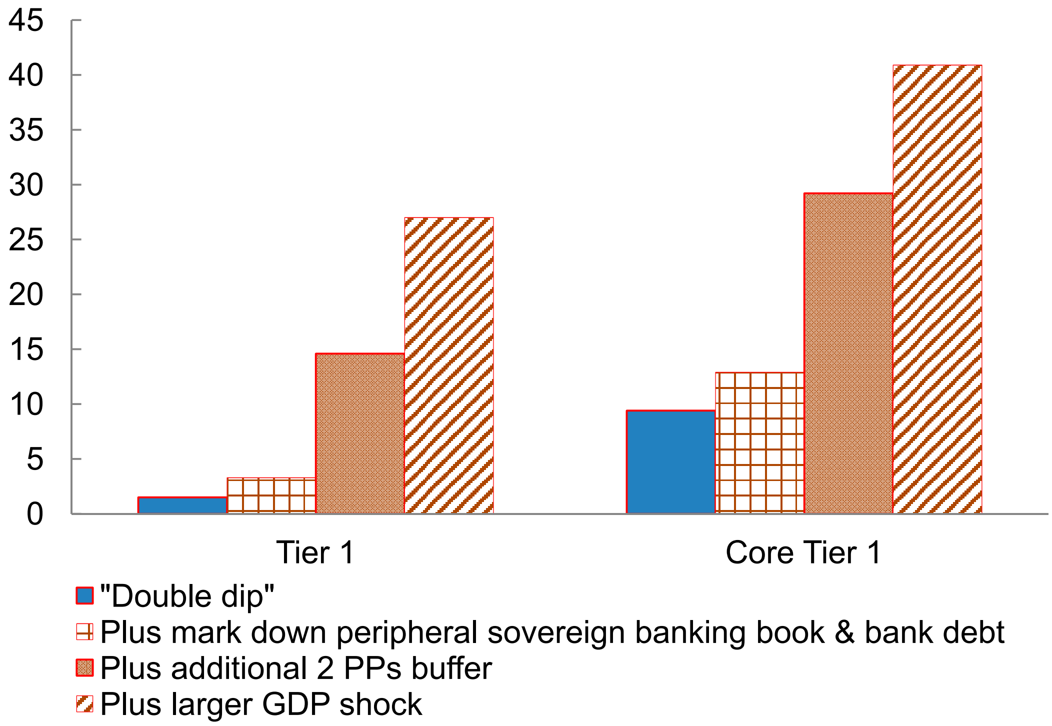
the financial crisis, the analysis intends to assess how more extreme circumstances could impact structural trends.

45. **The supplementary tests suggest, furthermore, that the capital shortfall may turn out higher than those computed under the core tests, but it should remain contained in aggregate unless there is a generalized intense crisis in financial markets.** The simulations focus on the double-dip scenario and the large bankss. As displayed in Figure 4, the analysis carried out based on publicly available data reveals results similar to those obtained from supervisory data, which are reported in Table 7 and Figure 2 (the estimated capital shortfall is slightly lower, at €1.5 billion, than when supervisory data are used). If all claims on the most vulnerable sovereigns (Greece, Ireland, Italy, Portugal, Spain, Belgium) and related claims on banks are subject to a haircut inferred from market prices, these banks would suffer gross losses of €23 billion in 2011; accounting for positive valuation effects in the outer years, losses would be €17 billion at end-2015.[26] However, many banks can absorb the losses in capital buffers and profits; in a few cases, these losses would result in an additional capital shortfall totaling €1.8 billion for Tier 1 capital. However, since the time when the stress-testing exercise was carried out, market perception of risk for some vulnerable countries have deteriorated further; larger losses would be incurred if these risks are realized.[27] Ultimately, were turmoil to spread to larger countries that are more closely tied to Germany, the impact through solvency and funding channels might become much larger, but such scenarios are inherently characterized by great uncertainty. If one adds an additional capital buffer of 2 PPs and a more extreme macroeconomic scenario, capital needs rise further up to EUR 27 billion, but still appear manageable for the system as a whole. As for the core tests, capital needs would be higher if measured against core Tier 1, which confirms that strengthening the quality of capital should be a priority, especially for the weak ends of the systems. Some action is already underway to address this need.

[26] Sovereign CDS rates for these countries can be used to derive estimates of market expectations of sovereign "haircuts." The "haircuts" used here are set at the 75th percentile of the distribution of these market expectations, starting from the 2010Q4 average CDS rates.

[27] To illustrate the sensitivity of results, a hypothetical severe write down of 60 percent of sovereign claims on Greece, Ireland, and Portugal is estimated to cost these banks €42 billion in 2011, and €36 billion by 2015; the hypothetical additional Tier 1 capital shortfall would be €14 billion.

Figure 4. Supplementary Tests for the Large Banks



Source: Staff estimates based on publicly available data.

1/ Estimates are based on publicly available data are carried for the double dip and interest rate spike scenario. The outcome shows the capital needs by 2015 if one progressively adds up to 3 elements of extra stress: a) stress of sovereign debt holdings in the banking book as well as related bank debt securities; b) an additional Tier 1 or Core Tier 1 capital buffer of 2 percentage points of risk-weighted assets (RWAs) above the regulatory Basel II/III minimum in each year; and c) a more severe macroeconomic scenario corresponding to 2.6 standard deviations of the historical GDP series (1980–2010) after allowing for German reunification.

F. Market-based (Systemic) Solvency Test

Introduction

46. **The tests based on the Systemic CCA framework account for the dependence between single banks to estimate the joint market-implied expected losses under systemic distress assumptions.**[28] Under this approach, the magnitude of systemic risk depends on banks' size and interconnectedness in a multivariate framework. The banking sector is viewed as a portfolio of individual expected losses, specified as implicit put options (with individual risk parameters), whose joint exposure to common risk factors can be accounted for by including their dependence structure (since conventional bivariate correlation is ill-suited for systemic risk analysis when extreme events occur jointly and in a

[28] See Gray and Jobst (2010 and forthcoming) as well as Gray and others (2010).

nonlinear fashion). This aggregation technique also helps quantify the magnitude of potential risk transfer to the public sector.

47. **In order to understand individual risk exposures (and associated public sector contingent liabilities) in times of stress, first, the CCA is applied to construct risk-adjusted (economic) balance sheets of financial institutions (Appendix IV).** In its basic concept, the CCA quantifies default risk on the assumption that owners of corporate equity in leveraged firms hold a call option on the firm value after outstanding liabilities have been paid off.[29] More specifically, CCA determines the risk-adjusted balance sheet of firms where assets are stochastic and may be above or below promised payments on debt. When there is a chance of default, the repayment of debt is considered "risky" – to the extent that it is not guaranteed in the event of default. Higher uncertainty about changes in future asset value, relative to the default barrier, increases default risk which occurs when assets decline below the barrier.

48. **In this framework, market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option in the form of a credit spread above the risk-free rate that compensates investors for holding risky debt.** The put option value is determined by the duration of the total debt claim, the leverage of the firm, and the volatility of its asset value.[30] The specification used was adopted to achieve robust and reliable estimation results in light of shortcomings of the traditional Merton (1974) model.[31][32]

49. **The implicit put option value calculated for each institution from equity market and balance sheet information can be combined with information from CDS markets to approximate the contingent liabilities to the public sector.** If there are explicit or implicit government financial guarantees, they benefit the bank's debt holders (but do not affect

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[29] Shareholders also have the option to default if their firm's asset value ("reference asset") falls below the present value of the notional amount of outstanding debt ("strike price") owed to bondholders at maturity. So, corporate bond holders effectively write a European put option to equity owners, who hold a residual claim on the firm's asset value in non-default states of the world. Bond holders receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky corporate debt (and bearing the potential loss) due to the limited liability of equity owners.

[30] The value of the put option is subject three principles: (i) the values of liabilities (equity and debt) are derived from assets; (ii) liabilities have different priority (i.e., senior and junior claims); and (iii) assets follow a stochastic process.

[31] The Merton model has shown to consistently underpredict spreads (Jones and others, 1984; Ogden, 1987; Lyden and Saranti, 2000), with more recent studies pointing to considerable pricing errors due to its simplistic nature (Eom and others, 2004).

[32] This approach here is an alternative to other proposed extensions aimed at imposing more realistic assumptions, such as the introduction of stationary leverage ratios (Collin-Dufresne and Goldstein, 2001) and stochastic interest rates (Longstaff and Schwartz, 1995). Incorporating early default (Black and Cox, 1976) does not represent a useful extension in this context given the short estimation and forecasting time window used for the CCA analysis.

equity values in a major way) when banks are close to distress.[33] Thus, CDS spreads should capture only the expected loss *retained* by the financial institution—and borne by unsecured senior creditors. Hence, the scale of market-implied government guarantees can be heuristically approximated by the difference between the total expected losses derived from equity and CDS prices. As the likelihood of credit events is reduced, resulting in lower CDS spreads, the time pattern of contingent liabilities can be measured (Appendix V).

50. **While this definition of market-implied contingent liabilities provides a useful indication of possible sovereign risk transfer, its estimation depends on a variety of assumptions.** The extent to which the put option values (and associated expected losses) differ from the ones implied by CDS spreads might reflect distortions stemming from the modeling choice (and the breakdown of efficient asset pricing in situations of illiquidity, especially at times of extreme stress), changes in market conditions, disparate investor behavior, and the capital structure impact of crisis interventions beyond the influence of explicit or implicit guarantees, such as equity dilution in the wake of capital injections by the government.

51. **The CCA-generated, market-implied expected losses of individual firms can be generalized to generate estimates of average and extreme system-wide solvency risk ("joint tail risk").** These estimates are based on the multivariate density of each bank's individual marginal distribution of market-implied expected losses and their dependence structure within a system of financial institutions—the Systemic CCA approach (see Appendix VI).

52. **This approach can also be used to quantify the contribution of specific institutions to systemic risk (at different levels of statistical confidence) and how policy measures may influence the size and allocation of this systemic risk over time.[34]** Yet, since equity market information, the essential data input to this calculation, is not available for most German banks, a hybrid approach for the estimation of expected losses and contingent liabilities is adopted. For the two largest banks, sufficient market data from equity and credit markets were available to assess the historical magnitude of contingent, and generate forecasts of joint market-implied expected losses using implied put option value from equity prices under different macroeconomic scenarios. The stress testing framework

[33] Note that a bank's CDS spread captures only the expected loss *retained* by the bank after accounting for the implicit government guarantee.

[34] The contribution to systemic risk is derived as the partial derivative of the multivariate density relative to changes in the relative weight of the univariate marginal distribution of individual expected losses at the specified percentile. More specifically, the total expected shortfall can be written as a linear combination of expected shortfalls of individual banks, whose relative weights (in the weighted sum) are given by the second order cross-partial derivatives of the inverse of the joint probability density function to changes in both the dependence function and individual expected losses. Since point estimates of systemic risk are derived from a time-varying multivariate distribution, it is more comprehensive than the current exposition of both CoVaR (Adrian and Brunnermeier, 2008) and Marginal Expected Shortfall (Acharya and others, 2009) (as well as extensions thereof, such as Huang and others, 2009).

for these two banks is based on a dynamic factor model of market-implied expected losses conditional on forecasted changes of macroeconomic variables using the same satellite models as in the balance sheet approach, where profitability and losses determine a change in implied assets and asset volatility. For all others banks in the sample, the analysis is limited to the estimation of individual contributions to systemic using the bank's CDS spread (and not equity market information) for the estimation of expected losses, and, thus, captures only the residual risk of each bank.

53. **In addition, the Systemic CCA framework can be used for stress testing.** By modeling how macroeconomic conditions and bank-specific income and loss elements (net interest income, fee income, trading income, operating expenses, and credit losses) have influenced the changes in the financial institution's market-implied expected losses (as measured by monthly implicit put option values), it is possible to link a particular macroeconomic path to financial sector performance in the future.

Results

54. **Most of the systemic risk is attributable to the two largest commercial banks (Table 9).** The contributions of different groups of institutions to tail-risk of expected losses are consistent with the balance sheet approach in the previous section, and the utilization of support government measures after the collapse of Lehman Brothers. A closer examination of the percentage share of systemic tail risk in different historical periods suggests that the largest commercial banks contributed about two-thirds, which increases to more than 74 percent from September 15 to December 31, 2008. Landesbanken have a disproportionate impact on systemic tail risk; since the second half of 2009, they have persistently contributed more than half to the joint distribution of market-implied losses of sample banks at the 95th percentile.

Table 9. Individual Contributions of Large Banks to Systemic Risk from Market-Implied Expected Losses

Germany: Banking Sector (SIFIs) - Individual Contribution to Systemic Risk
(average over each time period, in percent)

	Pre-Crisis: end-June, 2005—end-June, 2007	Subprime Crisis: July 1, 2007—Sept. 14, 2008	Crisis Period 1: Sept. 15—end-Dec., 2008	Crisis Period 2: Jan. 1—end-Sept., 2009	Crisis Period 3: Oct. 1, 2009—end-Feb., 2010	Sovereign Crisis: March 1—end-June, 2010	Sovereign Crisis: July 1—end-Dec., 2010	*Average*
				***Expected* systemic risk** (median) 1/				
Min	0.0	0.2	0.1	0.0	0.0	0.0	0.0	*0.1*
25th percentile	0.8	1.2	0.4	0.2	1.1	0.7	0.3	*0.7*
Median	**1.9**	**2.6**	**1.2**	**0.9**	**2.5**	**1.6**	**0.7**	***1.7***
75th percentile	5.0	6.4	3.8	5.2	5.2	3.2	3.8	*5.1*
Max	64.1	48.7	75.8	67.7	64.7	66.1	75.6	*63.3*
Top commercial banks (2)	66.0	62.5	76.8	70.9	70.5	70.7	80.0	*68.4*
Landesbanken (7)	22.9	22.3	14.2	16.4	21.4	24.9	14.3	*20.5*
of which: crisis supported (4)	13.2	16.5	8.6	5.2	16.3	11.9	7.1	*11.7*
Other banks (4)	11.2	15.2	9.0	12.7	8.0	4.4	5.6	*11.1*
				***Extreme* systemic risk** (95^{th} percentile) 1/				
Min	0.1	0.1	0.1	0.0	0.1	0.1	0.0	*0.1*
25th percentile	1.9	1.2	1.2	0.6	1.5	1.7	0.6	*1.3*
Median	**3.9**	**2.8**	**3.3**	**1.9**	**5.5**	**4.6**	**2.1**	***3.2***
75th percentile	8.3	9.0	12.4	9.3	8.8	7.9	8.8	*8.9*
Max	33.3	40.8	28.1	39.5	29.5	38.0	35.2	*36.1*
Top commercial banks (2)	18.2	30.2	21.8	26.1	31.8	21.8	25.1	*23.6*
Landesbanken (7)	57.3	41.0	48.3	46.7	54.5	64.7	54.0	*51.8*
of which: crisis supported (4)	29.1	21.4	22.8	8.1	22.7	23.8	25.9	*22.8*
Other banks (4)	24.5	28.9	29.9	27.1	13.7	13.5	20.9	*24.7*

1/ Each bank's percentage share is based on its time-varying contribution to the multivariate density of expected losses at the 50th and the 95th percentile. The multivariate probability distribution is generated from univariate marginals (based on generalized extreme value assumption) and a time-varying dependence structure.

55. **These market-based estimates can also be used to calculate a "fair value" price of a systemic risk surcharge (Table 10).** The fair value (in basis points) of a risk-based surcharge that would compensate for expected losses arising from systemic solvency risk (see Appendix IV, Box 2) would amount to about 3 basis points on uncovered liabilities on average, which is in line with the German bank levy, with took into effect on January 1, 2011, as part of the Bank Restructuring Act to finance the restructuring and orderly resolution of financial institutions in distress and reduce the public sector cost of future bail-outs.[35] According to the latest draft proposal, annual payments under the new levy could reach up to 4 basis points of bank liabilities, excluding insured deposits and regulatory capital instruments, with small and mid-sized institutions with liabilities under EUR 10 billion being charged the minimum of 2 basis points.[36] However, the systemic risk contribution of the two large commercial banks would require a far higher insurance

[35] This charge is intended to support the reorganization of systemically important activities of distressed institutions, but not to absorb losses from activities subject to ordinary insolvency proceedings. After considering the time-variation of expected losses (and their distributional behavior), it would be possible to devise a counter-cyclical surcharge by combining estimates at different percentile levels of statistical confidence.

[36] The levy is capped at 15 percent of unconsolidated income over the assessment period (i.e., the previous year), which materially reduces the payment amount for the largest banks and mitigates the procyclical impact in times of low profitability.

premium of more than 20 basis points according to model estimates. If the statistical confidence of these estimates were increased to also capture tail risks, i.e., extreme cases of joint distress, the average annual "through-the-cycle" systemic surcharge for systemically important financial institutions would exceed 30 basis points.

Table 10. Fair Value Insurance Premium for Individual Contributions of Large Banks to Systemic Risk

Germany: Banking Sector (SIFIs) - Fair Value Insurance Premium for Individual Contribution to Systemic Risk
(average over each time period, in basis points)

	Pre-Crisis: end-June, 2005—end-June, 2007	Subprime Crisis: July 1, 2007—Sept. 14, 2008	Crisis Period 1: Sept. 15—end-Dec., 2008	Crisis Period 2: Jan. 1—end-Sept., 2009	Crisis Period 3: Oct. 1, 2009—end-Feb., 2010	Sovereign Crisis: March 1—end-June, 2010	Sovereign Crisis: July 1—end-Dec., 2010	*Average*
				***Expected* systemic risk** (median) 1/				
Min	0.0	0.1	0.6	0.3	0.1	0.2	0.0	*0.2*
25th percentile	0.3	1.2	1.6	2.6	3.3	2.7	1.3	*1.9*
Median	**0.4**	**2.7**	**2.6**	**5.7**	**5.0**	**3.9**	**2.3**	***3.2***
75th percentile	0.6	4.9	6.9	24.1	8.0	5.8	8.8	*8.4*
Max	2.5	11.0	23.1	62.8	16.8	26.3	41.9	*26.3*
Top commercial banks (2)	1.7	8.4	17.9	47.2	13.5	20.0	28.3	*19.6*
Landesbanken (7)	0.8	3.3	5.5	20.1	6.9	9.4	7.8	*7.7*
of which: crisis supported (4)	0.9	4.6	5.6	8.0	9.0	7.7	6.3	*6.0*
Other banks (4)	0.4	3.3	5.1	17.4	3.7	2.6	4.2	*5.2*
				***Extreme* systemic risk** (95th percentile) 1/				
Min	0.1	3.7	3.8	1.8	4.6	1.0	0.4	*2.2*
25th percentile	1.3	15.4	20.4	19.4	29.9	20.3	10.2	*16.7*
Median	**1.8**	**22.0**	**36.4**	**39.4**	**72.5**	**38.2**	**19.7**	***32.8***
75th percentile	2.4	36.4	100.1	149.4	91.0	49.8	63.8	*70.4*
Max	3.2	35.1	38.9	98.0	52.6	43.1	57.7	*46.9*
Top commercial banks (2)	1.0	14.5	24.6	44.3	41.9	20.8	26.6	*24.8*
Landesbanken (7)	4.5	42.4	81.3	155.6	118.4	76.2	85.8	*80.6*
of which: crisis supported (4)	4.2	38.6	59.3	48.3	83.0	56.8	72.9	*51.9*
Other banks (4)	2.2	35.5	85.0	127.8	43.3	27.4	45.6	*52.4*

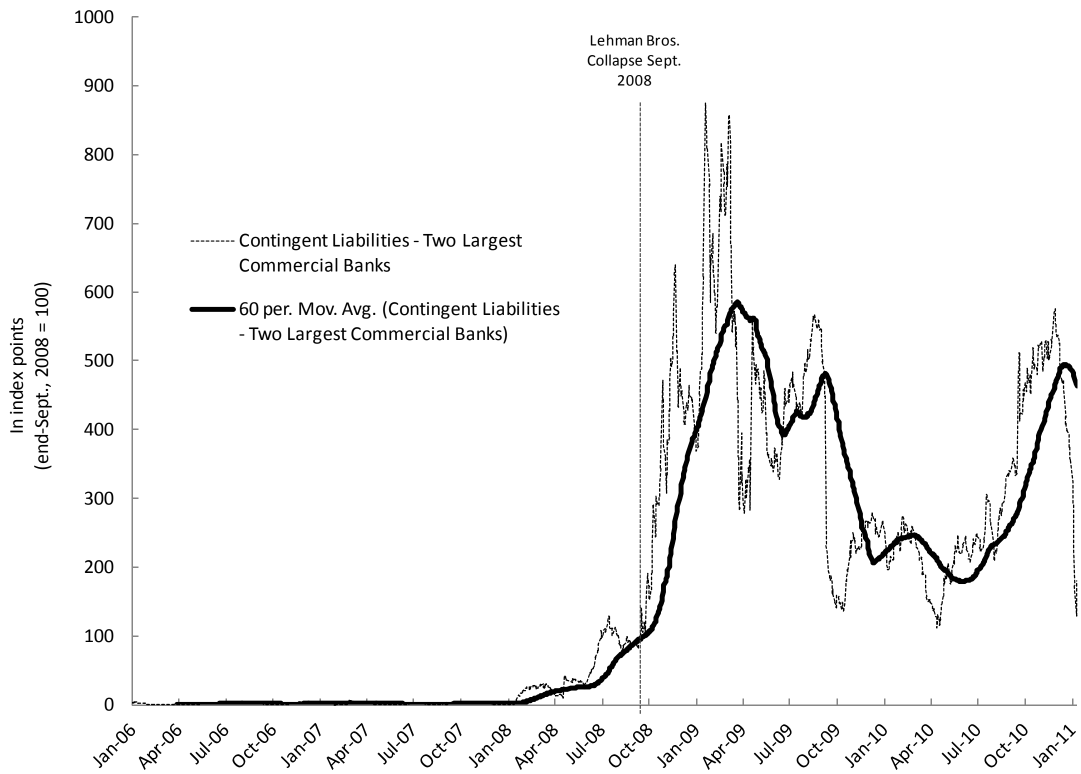
1/ The insurance premium for the systemic risk contribution are derived conditional on the relationship between the marginal contribution to systemic risk (see Table 8 above) and the level of debt service obligations over the estimation horizon.

Source: Staff estimates.

56. **Market-implied contingent liabilities of the two largest commercial banks, which are taken as indicative of strains affecting larger banks, rose precipitously during the crisis.** The combined tail risk of contingent liabilities for the two largest banks, as measured as the 95th percentile expected losses (and indexed to the amount estimated around the time of the Lehman collapse at end-September 2008; see Appendix V), shows spikes in the first and third quarter of 2009, indicating a high government exposure to financial sector distress. The magnitude of public sector cost to financial distress of either bank is still elevated (Figure 5).

Figure 5. Market-Implied Historical Contingent Liabilities of Large Commercial Banks

(in index points, end-Sept. 2008=100)

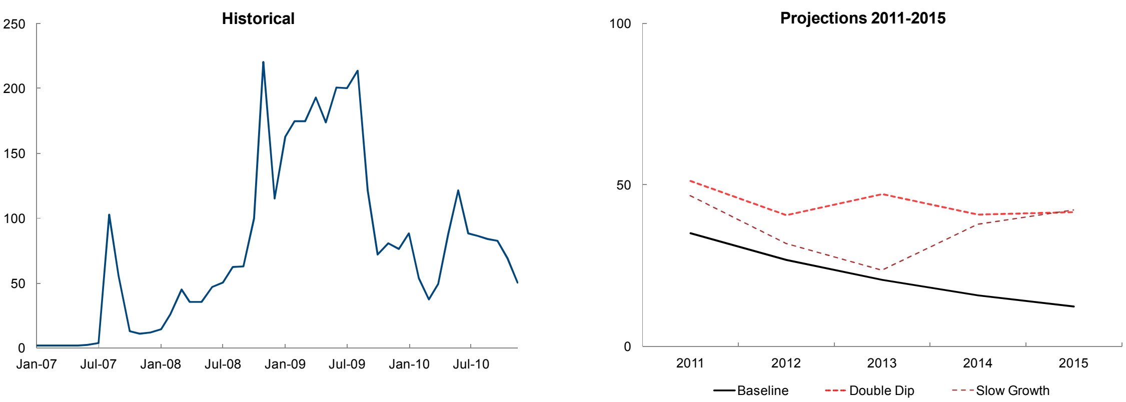


Source: Market data and staff estimates.

57. **The stress test on joint market-implied expected losses confirms the persistence of some medium-term vulnerabilities (Figure 7).**[37] Based on the same specification of macrofinancial linkages to bank performance, and using the market-implied put option value derived from CDS spreads only (rather than equity prices), which excludes contingent liabilities and reflects only the residual risk of bank balance sheets, there is some reduction in vulnerability. However, under both adverse scenarios, market-implied expected losses increase again, and indeed tend to rise over the forecast horizon. Lower profitability in tandem with weaker performance in credit risk and fee-based income conspire to heightened tail risk to levels close to those seen during the intense phase of the crisis. Nonetheless, even tail risks under the severe macroeconomic stresses remains far below the levels observed during the recent financial crisis. While the projections are based on a narrow sample of banks, they are plausible and corroborate the results from the balance sheet-based approach.

Figure 6. Forecast Market-Implied Expected Losses

(Residual risk based on 95th percentile based on multivariate density of expected losses; index points, end-Sept. 2008=100)



Source: Market data and staff estimates.

II. Liquidity Tests for Bank

Specification of the tests

58. **The core liquidity tests focused on the short-term resilience of banks with respect to a sudden, sizeable withdrawal of funding; supplementary tests assessed upcoming changes of the regulatory framework.** To this end, the tests provide an overview of the degree of resilience of the system and individual banks. The core tests were carried out based on supervisory data and included all German banks, while the supplementary tests were carried out based on publicly available data for the bulk of banks.

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[37] The analysis was based on daily data from January 1, 2005, to end-January 2011. Key inputs used were the daily market capitalization of the two largest firms (from Bloomberg), the default barrier (Appendix IV) estimated for each firm based on quarterly financial accounts (for all sample firms), and the one-year CDS spreads from *Markit*.

A key element of the tests was to distinguish between different banking groups, which also included the small private banks.

59. **The core test simulated a gradual outflow of funding during five consecutive periods, to test banks' ability to cope with stress in a reverse test setting.** First, a shock to withdrawal funding was simulated, i.e., customer deposits were assumed to be not affected (Test a). [38] This shock accounts for the fact that (unsecured) wholesale funding proved to be a vulnerable link in the financial system since the onset of the crisis. In the next step, the outflow of customer deposits was added (Test b). In empirical terms, the first two to three "periods" would constitute shocks that were observed during the crisis for about a dozen of banks that were hit hard, while period four and five can be considered extreme conditions.[39] The parameters are displayed in the Table 10.

Table 11. Specification of the Liquidity Tests

Implied Cash Flow Test (5 periods)	Liability side	Asset side	Other assumptions
Test a: outflow of banks deposits and wholesale funding "only" Test b: outflow of all funding as displayed	Consecutive outflow of liabilities for 5 periods: • Customer deposits: 5 percent of demand deposits, 3 percent of term deposits • Bank deposits: 15 percent for demand deposits; 10 percent for term deposits • Wholesale funding: 15 percent for secured funding, 20 percent of unsecured funding	Asset that remain liquid under stress and haircuts: • Cash and cash-like positions (Haircut: 0 percent) • Government debt holdings (0) • High-quality liquid assets (1) • Trading assets (20) • Derivatives (50)	10 percent of the liquid assets (e.g., government bonds, high-grade investment/trading securities, and derivatives) are encumbered, i.e., used as a collateral to receive funding

Source: Staff estimates.

60. **Supplementary analysis sought to shed light on how banks cope with upcoming Basel III rules.** As an important caveat, it has to be highlighted that these analysis required several assumptions (e.g., with respect to contractual maturities and the credit "quality" of securities), as the full granularity of data was not available. As such, the outcome is meant to

[38] The level of outflows of liabilities and the liquidity of assets under stress was set in accordance with empirical evidence, assumptions used in other FSAPs and upcoming regulatory changes.

[39] For further information see Schmieder and others (forthcoming).

inform insights on broad tendencies and differences across banking groups. For the LCR, the minimum parameters for the outflow of stable and less stable deposits have been chosen.[40] For the NSFR, the parametrization of the tests has been chosen in line with the final Basel III publication as of December 2010 (BCBS 2010c). The computed Basel III metrics were cross-checked against the outcome of the recent quantitative impact study (QIS 6), which included 68 German banks (out of 230 European banks). The QIS did not reveal country-specific results, but showed that European banks are more vulnerable to liquidity shocks than are U.S. banks. The aggregate outcome for European banks has been used as a proxy.

61. **Separate data to assess liquidity risks for U.S. dollar funding were not available for the analysis.** Given the importance of accounting for currency mismatches in funding markets it is highly relevant to carry out analysis for each key currency—as also documented in the Basel III rules. It is recommended, therefore, that supervisory data to carry out liquidity tests in different currencies be added on a timely basis, together with a more general overhaul of regular liquidity reporting.

Results

62. **The liquidity stress tests found that most banks are able to cope with large liquidity shocks; large banks and private banks appear more vulnerable to liquidity shocks, and especially wholesale funding market shocks, than do smaller retail banks (Figure 8).** The large banks and particularly some of the small private banks exhibit some vulnerability toward a sudden withdrawal of wholesale funding. For the larger banks, a potential liquidity shortfall occurs only in the "extreme shock area," while smaller private banks appear more vulnerable in general. The Sparkassen and cooperative banks benefit from their broad deposit base. Given the relevance of customer deposits under Basel III it can be expected that there will be more intense competition among banks, ultimately reducing profit, which is shown in recent data on deposit rates.[41]

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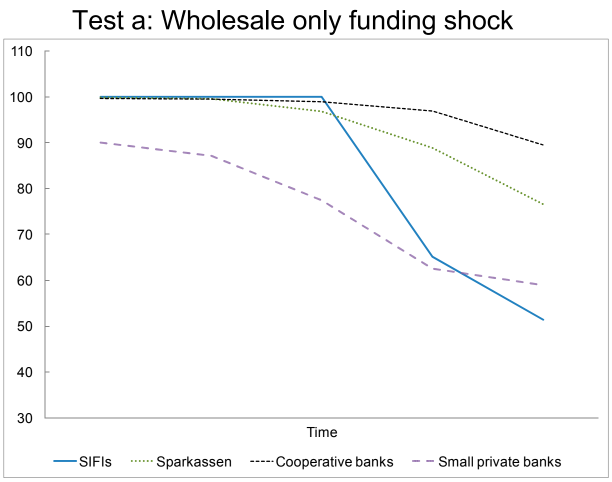
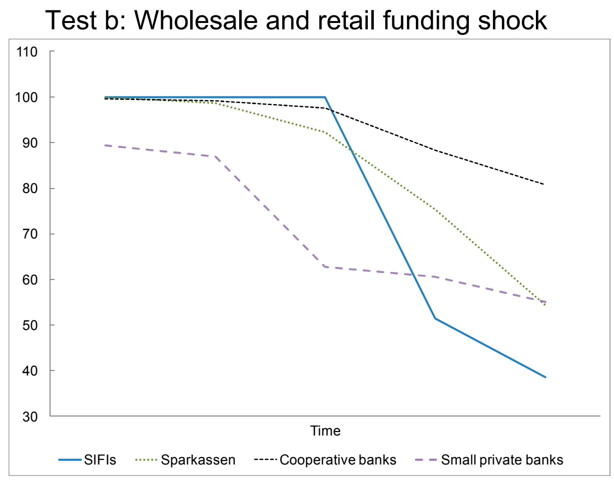
[40] Other assumptions were (i) share of high quality liquid assets needed to satisfy margin calls: 10 percent; (ii) Market value change of derivatives (20); (iii) share of asset-backed securities maturing within the next 30 days (10); (iv) share of undrawn but committed liabilities that are drawn (50); and (v) share of assets reinvested (80).

[41] Recent (unpublished) work by Bundesbank shows that the margins of deposits rates relative to the riskless level has dropped by 75 bps since the onset of the financial crisis (i.e., 2009).

Figure 7. Liquidity Stress Test Results
(percent of banks liquid)



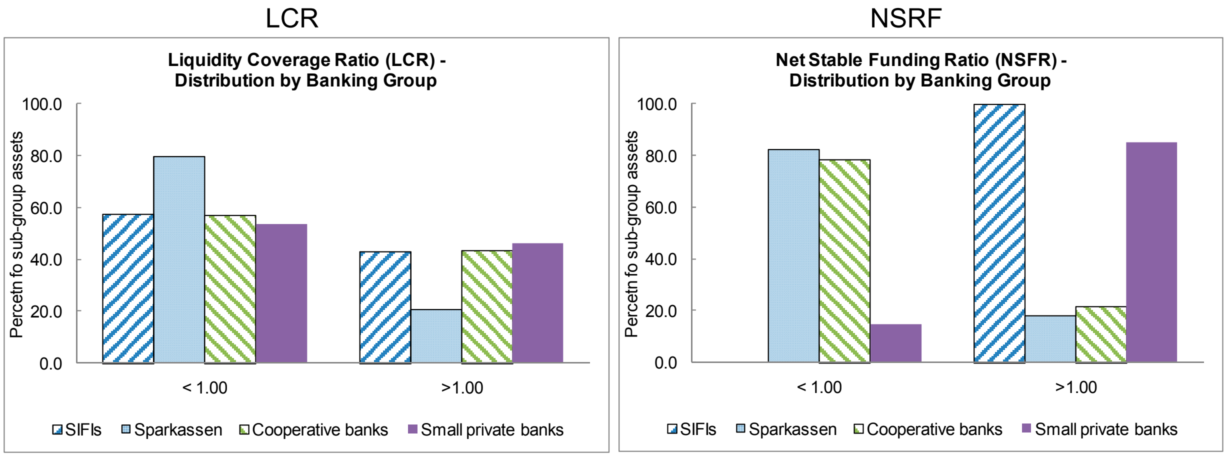


Source: Deutsche Bundesbank and IMF staff.

63. **As an important caveat, it has to be taken into account that the core test, a ”reverse” stress tests, was used to estimate how large a shock is required to cause severe distress, rather than projections under scenarios.** Also, although it is convenient to refer to “periods,” these units should be interpreted as gradations in the severity of shock in funding and asset markets.

64. **German banks appear in heterogeneous positions with respect to Basel III liquidity rules, in line with the outcome of the QIS-6 (Figure 8).** Taking into account the caveat with respect to the data, the LCR appears to be an issue for half of the banks. The smaller German banks (especially Sparkassen and cooperative banks) have ample customer deposits, but lesser liquid assets than the larger banks. For the NSFR, the maturity gap of the smaller banks’ business appears to account for their lower NSFR ratios, while the large banks and the small private banks appear to be in a better position. The outcomes show the value of doing meaningful liquidity analysis—looking from different angles and containing vulnerabilities that appear most striking.

Figure 8. Proxies for LCR and NSFR



Source: Staff calculation based on publicly available data.

III. Stress Tests Carried out by the German Authorities

65. **The stress tests run by Deutsche Bundesbank on a regular basis comprise top-down and bottom-up tests assessing solvency risk, spillover risks and liquidity risks.** The tests focus on the domestic financial system in general and the systemically important banks in particular. Stress tests include single risk analysis and macro shocks. Liquidity tests are run by banks based on scenarios considered relevant for each bank as agreed with the authorities. Stress tests have also been carried out as part of the European stress tests and as part of banks' internal risk management purposes (Pillar 2 and the Internal Capital Adequacy Assessment Process).

66. **The stress tests are constantly revised, accounting for upcoming risks on the one hand and evolutions on methods on the other.** The stress tests are used to inform the authorities about risks, and further discussed with banks. The tests are published in the Financial Stability Review, for example.

67. **Experience from the crisis and the prospect of Basel III suggest that certain enhancements would be worthwhile (which applies also to most peer countries), including:**

- More complete coverage of the banking sector for the more integrated tests, including "second tier" and third tier banks, such as the smaller private banks and potentially the banks that are outside of the core supervisory focus (public banks, specialized banks).
- A longer time horizon to identify potential structural vulnerabilities.
- Calculation of a range of metrics, including Core Tier I and profitability measures.
- Better modeling of funding cost risk, including those of U.S. dollar funding.

- Further improving liquidity stress tests assessing liquidity in the euro and the U.S. dollar, and concentration risk.

68. **Stress tests are also run for the insurance sector.** Bottom-up tests, which are under the supervision of BaFin, include all German insurers on a regular basis based on predefined scenarios. Also, top-down tests and scenarios for insurers which are under the supervision of BaFin have been run to assess specific risks for the system, such as persistent low interest rates for life insurers. These tests will need to evolve with the introduction of Solvency II requirements, and in recognition of the specific insurance risks (such as the interaction of low interest rates and longevity).

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APPENDIX I. RISK ASSESSMENT MATRIX

Threat	Likelihood Considerations	Impact Considerations
Sharp "double dip" recession	**Low** It is possible that in particular, the U.S. and European economies may suffer another sharp contraction in output. Recovery from the last recession is far from over: there is substantial excess capacity in many industries. Hence, the Germany economy would be very exposed to a drop in demand for capital goods and consumer durables, and also a relapse of consumer confidence in Germany. Moreover, the economies of main partners where German financial institutions are active, such as Central, Eastern, and Southern Europe, might suffer a severe contraction. Restricted supply of financing could generate a negative feedback loop in Germany and in partner countries. A rise in energy and commodity prices could induce monetary policy tightening in Europe, raising short-term rates and inverting the yield curve. However, double-dip recession in the US might be associated with monetary easing there, resulting in the appreciation of the euro, which would harm European firms.	**Medium** German banks' credit quality would be directly affected by the shock to the German export and household sectors, and also to borrowers abroad. They would also suffer market losses on other exposures, for example, to commercial real estate in the US. An inversion of the yield curve would adversely affect bank profitability, especially for the retail banks. Nonbank financial institutions would be affected mainly by market losses on securities holdings. Life insurers, German *Pensionskassen* and German *Pensionsfonds* would face a prolonged period of very low long-term interest rates, driving up their liabilities. However, exposures to the "toxic assets" and mispricing that were at the core of the recent global financial crisis seem to have been reduced, and some of the weakest institutions have already been "weeded out." Hence, a sharp but short recession may not generate much systemic risk for Germany.
Very slow growth in Europe and low interest rates	**Medium** The accumulation of structural rigidities, fiscal burdens, demographic pressure, and uncertainty could lead to a prolonged period of very low growth in Europe. Unemployment would remain high and rising, investment would be weak, and fiscal crowding out would remain unrelieved. Even some deflation is possible. Restricted supply of financing could generate a negative feedback loop in Germany	**High** German banks' traditionally low profitability would be further reduced. Negative feedback to loan supply is possible, while competition for good borrowers would drive down spreads. Banks' ability to meet higher capital requirements would be put in question. Life insurers, German *Pensionskassen* and German *Pensionsfonds* would face a

Threat	Likelihood Considerations	Impact Considerations
	and in partner countries.	prolonged period of very low interest rates, driving up their liabilities. A flat or negatively sloped yield curve would significantly reduce the profitability of retail banks, which, however, have traditionally been less affected by other conjunctural factors such as GDP growth.
Sustained high sovereign risk	**Medium** The recent rise in spreads on "peripheral" sovereign debt may become entrenched and spread to a wide class of advanced and emerging market countries. Corporate spreads and sovereign-linked assets (such as variable rate mortgages) would be forced up, leading to a deterioration in loan performance. It is also possible that the solvency of subnational units of government in some countries may come under greater strain.	**Medium** German financial institutions hold substantial amounts of foreign sovereign, sovereign-linked, and subnational government claims. Even without a "credit event," substantial losses would have to be acknowledged. Note that this scenario could well be combined with others.
Regulatory uncertainty and regulatory burden	**High** Uncertainty about the final form and calibration of new regulations is likely to persist for some time. It is possible that regulations are introduced that turn out to have flaws, requiring another round of amendments.	**Low** The money-market banks and large financial groups are most likely to be affected by likely regulatory changes, both directly and through regulatory competition with other jurisdictions. Several groups will need to increase core capital and decrease leverage. Changes in capital requirements may force changes in the ownership structure of some vertically-linked institutions. The EU has advanced relatively rapidly in resolving regulatory uncertainty, for example, through the process of amending financial sector directives. International efforts coordinated largely by the Financial Stability Board, the Basel Committee, and the Fund now seem likely to yield a compromise: some measures such as on medium-term funding will be moderated, and a long phase-in period will be designed to allow the industry to adapt.

Threat	Likelihood Considerations	Impact Considerations
		Regulatory burden could be a challenge for other financial sectors, such as insurance where Solvency II capital requirements are being introduced, but the effects are more likely to be secular than acute.
Sustained dislocation in funding markets	**Medium** Continued uncertainties about economies in general and the situation of institutions may lead to renewed illiquidity or high premia in funding markets.	**Medium** Certain German banks that are heavily reliant on market funding, including through interbank borrowing, securitization, and the issuance of covered bonds would be most affected, especially if the disruption were sustained. Banks may resort to increased competition for retail deposits, squeezing profitability further. Possibly, U.S. dollar funding would be more problematic for German banks than would euro funding, which ultimately can be provided by the ECB. Banks with a funding surplus may also suffer lower returns on excess funds placed in “safe havens.” Note that this scenario could well be combined with others.

APPENDIX II. SATELLITE MODELS

Large Banks

Dependent	Lagged Term	10 year Bund	3 month interest rate	Real GDP growth	Size (logarithm of total assets, lagged)	Equity to total assets in %, lagged	Non performing loans to customer loans in %, lagged	Funding Gap (Difference between customer loans and deposits in percent of total assets, lagged)	Customer loans to total assets in %, lagged	Constant	R^2
Net interest income to total assets in %	0.8864*** (.018)	0.0146* (.008)	-0.0080** (.003)	0.0029** (.001)						0.0476** (.025)	
Interest expenses to total assets in %	-0.1*(Tier 1 Capitalization after stress minus 6 percent) [i.e., reduction of interest income by 10 basis points for a each percentage point drop of capitalization below 6 percent]										
Net fee and commission income to total assets in %	0.7131*** (0.052)	-	-	0.0027* (0.001)	-0.0208** (0.009)	0.0110* (0.006)	-	-	-	0.5487** (0.230)	0.712
Operating expenses to total assets in %	0.7585*** (0.033)	-	-	-0.0095** (0.003)	-0.0358* (0.020)	-	-	-	-	1.0579* (0.512)	0.725
LLP (impairments on all assets in % of total assets)	1.8735*** (.430)	1.0243* (.621)	-0.2334* (0.138)	-7.7910** (3.95)		-17.5960** (8.855)		-0.7529* (0.400)	1.5981* (.941)	-2.6562 (1.808)	

Small and Medium-Sized Banks

Dependent	Lagged Term	10 year Bund	3 month interest rate	Real GDP growth	Size (logarithm of total assets, lagged)	Equity to total assets in %, lagged	Non performing loans to customer loans in %, lagged	Funding Gap /1	Customer loans to total assets in %, lagged	Constant	R^2
Interest Income to total assets in %	0.4725*** (0.036)	0.3306*** (0.031)	0.0324*** (0.011)	0.0298*** (0.003)	0.1651*** (0.047)	-	-0.0093*** (0.002)	-	0.0026** (0.001)	-2.2434** (0.939)	0.612
Interest expenses to total assets in %	0.5239*** (0.025)	0.1324*** (0.015)	0.1613*** (0.004)	0.0173*** (0.002)	0.0271 (0.025)	-0.0265*** (0.004)	-	0.0014*** (0.000)	-	-0.2538 (0.510)	0.681
Net fee and commission income to total assets in %	0.3473*** (0.094)	-	-	0.0134*** (0.003)	0.0808** (0.038)	0.0046 (0.011)	-	-	-	-1.1031 (0.708)	0.143
Operating Expenses in %	0.3357*** (0.080)	-	-	0.0053* (0.003)	-0.2222*** (0.070)	-	-	-	-	6.0244*** (1.517)	0.166
LLP (In of write-downs in lending business in % of customer loans)	0.5508*** (0.047)	-	-	-0.0940*** (0.020)	-0.0619 (0.072)	-	-	-	-	1.4017 (1.803)	0.357
In of write-ups in lending business in % of customer loans	0.4469*** (0.050)	-	-	-0.0294** (0.012)	-0.0808 (0.112)	-	-	-	-	1.3480 (2.796)	0.262

1/ Difference between customer loans and deposits in percent of total assets, lagged

APPENDIX III. TREATMENT OF FIXED-INCOME SECURITIES

69. **The calculation of haircuts on fixed income holdings under different macro scenarios is based on an IMF-developed model for the valuation of sovereign debt using information from CDS markets.** Sovereign bond prices for each year under each scenario are calculated using market expectations of default risk as reflected in forward rates on five-year sovereign CDS contracts. Five-year bonds are assumed to be representative of the maturities of banks' bond holdings. Bonds for which market quotes from Bloomberg were available were selected consistent with the approach taken by CEBS for the 2010 Europe-wide stress test by choosing maturities between 4.5 and 6.5 years.

70. **The standard pricing formula for coupon-bearing bonds is reconciled with the zero-coupon bond pricing formula** $\exp((-rT)(1-LGD\times PD(T)))$ with the cumulative probability of default (PD) and loss given default (LGD), in order to project bond prices contingent on changes in idiosyncratic risk (irrespective of changes in the term structure of yields). Since the sample bonds carry regular coupon payments, the cash flow pricing formula

$$P_{b,T-t}=\prod_{k=1}^{T-t}\frac{c}{(1+r_t)^{(T-t)/n}}+\frac{f}{(1+r_t)^{T-t}}$$

of the bond b in year t and time to maturity T-t is stripped of coupon payments c (with payout frequency n) and set equal to the quasi-zero coupon price at the last observable sample date, after controlling for changes in market valuation over the course of 2010 in excess of baseline expectations of each country-specific yield-to-maturity according to CEBS. Thus, one can write

$$P_{b,T-t}=\frac{f}{(1+r_t)^{T-t}}=\exp\left(\left(-r_f t+\left(YTM_{end-2010,baseline}-YTM_{end-2010,actual}\right)+\frac{s_{CDS}}{10,000}\right)(T-t)\right),$$

where r_t is the yield in each year, f is the face value and the five-year cash CDS spread $s_{CDS,j}=-\ln\left(1-LGD\times PD\left(t\right)\right)/T$ of country j, which replaces the term for default risk in $\exp((-rT)(1-LGD\times PD(T)))$.[42] Note that the actual end-2010 YTM and the five-year cash CDS spread refer to values observed at end-2010. The equation above is then solved for the risk-free rate r_f (before the first forecast year) by maximum likelihood.

[42] Note that this is a simplified CDS pricing formula, which does not take into account the valuation effects of credit events between (quarterly/semiannual) CDS premium payment dates.

71. **For all bonds of each sample country, the future bond prices $P_{b,t,j}$ (up to five years) are calculated by applying the forward five-year sovereign CDS spread $F\left(s_{CDS,j}\right)_t$ to the modified zero-coupon pricing formula**

$$P_{b,t,j} = \exp\left(\left(-r_t T + F\left(s_{CDS,j}\right)_t \big/ 10,000\right)t\right)$$

in order to inform estimates of default risk (and its impact on future bond values relative to end-2010) for each year of the forecast horizon based on the previously estimated risk-free rate r_t. This is done for several bonds of each sample country (with a residual maturity T of about five years).

72. **More specifically, the past dynamics of expected default risk are used to determine parametric estimates of future haircuts.** The monthly variations of forward rates on CDS spreads $F\left(s_{CDS,j}\right)_t$ between January 2009 and December 2010 are parametrically calibrated as a generalized extreme value distribution with point estimates $\hat{x}_{t,a} = \hat{\mu}_j + \hat{\sigma}_j \big/ \hat{\xi}_j \left(\left(-\ln\left(a\right)\right)^{-\hat{\xi}_j} - 1\right)$, where $1+\xi_j\left(x-\mu_j\right)\big/\sigma_j > 0$, scale parameter $\sigma_j > 0$, location parameter μ_j, and shape parameter $\xi_j = 0.5$.[43] The higher the absolute value of shape parameter, the larger the weight of the tail and the slower the speed at which the tail approaches its limit.[44] For the baseline scenario, the median (50th percentile) for $\hat{x}_{t,a=0.5}$ is chosen. Since haircuts under the adverse scenario should reflect the volatility of market expectations, country-specific shocks to $F\left(s_{CDS,j}\right)_t$ are assumed at the 75th percentile (for the mild "double dip" scenario and the slow growth scenario ("adverse 1"), and 90th percentile (for the severe "double dip" scenario ("adverse 2") of the probability distribution. Thus, for each year over the forecast horizon, there are three "stressed" bond prices $\left\{P_{b,t,j_{baseline}}; P_{b,t,j_{adverse1}}; P_{b,t,j_{adverse2}}\right\}$ based on three different forward CDS rates $\left\{F\left(s_{CDS,j}\right)_{t_{baseline}}; F\left(s_{CDS,j}\right)_{t_{adverse1}}; F\left(s_{CDS,j}\right)_{t_{adverse2}}\right\}$.

73. **Corresponding haircuts were calculated for each bond from changes in bond prices relative to the base year 2010, using the following specification**

$$\Delta P_{b,t,j} = \left(P_{b,t,j} \big/ P_{b,0,j} - 1\right) \times 100,$$

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[43] The upper tails of most (conventional) limit distributions (weakly) converge to this parametric specification of asymptotic behavior, irrespective of the original distribution of observed maxima (unlike parametric value-at-risk (VaR) models).

[44] All raw moments are estimated by means of the *Linear Combinations of Ratios of Spacings* (LRS) estimator.

where $P_{b,0}$ is the bond price in the base year.[45]

74. **The haircut *h* for each sovereign *j* is calculated as an issuance size-weighted average of individual projected haircuts applied to a *k*-number of bonds outstanding,**[46] so that

$$h_{t,j} = \max\left(\sum_{b=1}^{k} \Delta P_{b,t,j} \times Amt_{b,j}\left(\sum_{b=1}^{k} Amt_{b,j}\right)^{-1}, 0\right),$$

where $\Delta P_{b,t,j}$ is the haircut on bond *b*, and Amt_b is the outstanding amount of bond *b* issued by country *j*. These haircuts should then be applied to banks' sovereign bond exposures to countries[47] $j \in J$ held in both the banking and trading books as of end-2010. The sovereign bond losses or changes in valuation in each year *t* over the forecast horizon are calculated as $\sum_{j}^{J} h_{t,j} \times \text{exposure}_{0,j}$, based on a bank's total exposure to country *j* at end-2010. Sovereign exposure gains, should they materialize, are ignored for stress test purposes.

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[45] Note that the haircut estimation is not fully accurate, because in each year over the projected time horizon, the projected YTM is imposed on an unchanged set of bonds. This implies no new government issuance (and time-invariant coupon), which overstates the actual haircut (unlike in cases when the sample of bonds changes and the remaining maturity is kept constant over the projected time period).

[46] Haircuts cannot take negative values when price appreciation occurs between years (e.g., in response to "safe haven flows").

[47] Austria, Belgium, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal, Spain, Sweden, Switzerland, the UK, and the US.

APPENDIX IV. THE CONTINGENT CLAIMS ANALYSIS APPROACH—STANDARD DEFINITION

75. **The CCA is used to construct risk-adjusted balance sheets, based on three principles.** The principles are: (i) the values of liabilities (equity and debt) are derived from assets; (ii) liabilities have different priority (i.e., senior and junior claims); and (iii) assets follow a stochastic process. Assets (present value of income flows, proceeds from assets sales, etc.) are stochastic and over a horizon period may be above or below promised payments on debt which constitute a default barrier. Uncertain changes in future asset value, relative to the default barrier, are the driver of default risk which occurs when assets decline below the barrier. When there is a chance of default, the repayment of debt is considered "risky," to the extent that it is not guaranteed in the event of default (risky debt = risk-free debt minus guarantee against default). The guarantee can be held by the debt holder, in which case it can be thought of as the expected loss from possible default or by a third party guarantor, such as the government.

76. **In the first structural specification, commonly referred to as the Black-Scholes-Merton framework (or in short, the "Merton model") of capital structure-based *option pricing theory* (Black and Scholes, 1973; Merton, 1973 and 1974), total value of firm assets follows a stochastic process may fall below the value of outstanding liabilities.** Thus, the asset value *A(t)* at time *t* describes a continuous asset process so that the physical probability distribution of the end-of-period value is

$$A(T-t) \sim A(t)\exp\left\{\left(r_A+\sigma_A^2/2\right)(T-t)+\sigma_A\sqrt{T-t}z\right\},$$

for time to maturity *T-t*. More specifically, *A(t)* is equal to the sum of its equity market value, *E(t)*, and its risky debt, *D(t)*, so that $A(t)=E(t)+D(t)$. Default occurs if *A(t)* is insufficient to meet the amount of debt owed to creditors at maturity, which constitute the bankruptcy level ("default threshold" or "distress barrier"). The equity value *E(t)* is the value of an implicit call option on the assets, with an exercise price equal to default barrier. It can be computed as the value of a call option $E(t)=A(t)\Phi(d_1)-Be^{-r(T-t)}\Phi(d_2)$, with $d_1=\left[\ln\left(A(t)/B\right)+\left(r+\sigma_A^2/2\right)(T-t)\right]\left(\sigma_A\sqrt{T-t}\right)^{-1}$, $d_2=d_1-\sigma_A\sqrt{T-t}$, asset return volatility σ_A, and the cumulative probability $\Phi(.)$ of the standard normal density function. Both the asset, *A(t)*, and asset volatility, σ_A, are valued *after* the dividend payouts. The value of risky debt is equal to default-free debt minus the present value of expected loss due to default,

$$D(t) = Be^{-r(T-t)} - P_E(t)\ .$$

77. **Thus, the present value of market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option, which is calculated with the default threshold *B* as strike price on the asset value *A(t)* of each institution.** Thus, the present value of market implied expected loss can be computed as

$$P_E(t) = Be^{-r(T-t)}\Phi(-d_2) - A(t)\Phi(-d_1),$$

over time horizon $T-t$ at risk-free discount rate r, subject to the duration of debt claims, the leverage of the firm, and asset volatility. Note that the above option pricing method for $P_E(t)$ does not incorporate skewness, kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices. Since the implicit put option $P_E(t)$ can be decomposed into the PD and the LGD,

$$P_E = \Phi(-d_2)\left(1 - \frac{\Phi(-d_1)}{\Phi(-d_2)}\frac{A(t)}{Be^{-rT}}\right)Be^{-r(T-t)} = PD \times LGD,$$

there is no need to introduce potential inaccuracy of assuming a certain LGD. As a consequence of the assumptions on the underlying asset price process, this would imply the risk-neutral probability distribution (or *state price density* (SPD)) of $A(t)$ is a log-normal density

$$f_t^*\left(A(T)\right) = e^{-r_{t,T-t}T-t}\frac{\partial^2 E(t)}{\partial B^2}\Big|_{B=A(T)}$$
$$= \frac{1}{A(T)\sqrt{2\pi\sigma^2(T-t)}}\exp\left[-\frac{\left[\ln\left(A(T)/A(t)\right) - \left(r_{t,T-t} - \sigma^2/2\right)(T-t)\right]^2}{2\sigma^2(T-t)}\right]$$

with mean $\left(r - \sigma_A^2/2\right)(T-t)$ and variance $\sigma_A^2(T-t)$ for $\ln\left(A(T)/A(t)\right)$, where $r_{t,T-t}$ and $f^*(.)$ denote the risk-free interest rate and the risk-neutral probability density function (or state price density (SPD)) at time *t*, with risk measures

$$\Delta \stackrel{def}{=} \frac{\partial E(t)}{\partial A(t)} = \Phi(d_1) \text{ and } \Gamma \stackrel{def}{=} \frac{\partial^2 E(t)}{\partial A(t)^2} = \frac{\Phi(d_1)}{A(t)\sigma\sqrt{T-t}}.$$

78. **In this analysis, the Merton model is refined without altering the analytical form by means of the closed-form Gram-Charlier (GC) model of Backus and others (2004), which allows for kurtosis and skewness in returns and does not require market option prices to implement, but is constructed using the same diffusion process for asset prices.**[48] The above option pricing method, however, does not incorporate skewness and

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[48] Further refinements of this model would include various simulation approaches at the expense of losing analytical tractability. The ad hoc model of Dumas, Fleming, and Whaley (1998) is designed to accommodate the implied volatility smile and is easy to implement, but requires a large number of market option prices. The

(continued)

kurtosis, which can account for implied volatility smiles of equity prices. Thus, the Merton model above is enhanced – without altering the analytical form by means of the closed-form GC model of Backus and others (2004), which allows for kurtosis and skewness in returns based on the same diffusion process for asset prices, and does not require option prices in its calibration. The model is constructed around the Gram-Charlier expansion of the density function of asset changes defined by a standard normal random variable in the Merton model. For default threshold *B* as strike price on the asset value *A(t)* of each institution, the price of a European put option can be written as

$$P_E\left(t\right)=Be^{-r(T-t)}\Phi\left(-d_2\right)-A\left(t\right)\Phi\left(-d_1\right)$$
$$-A\left(t\right)\Phi\left(d_1\right)\sigma_A\left[\frac{\gamma\left(t\right)_1}{3!}\left(2\sigma_A-d_1\right)-\frac{\gamma\left(t\right)_2}{4!}\left(1-d_1{}^2+3d\sigma_A-3\sigma_A^2\right)\right]$$

79. over time horizon $T-t$ at risk-free discount rate *r*, subject to the duration of debt claims, the leverage of the firm, the sensitivity $d_1=\left(\ln\left(A\left(t\right)/B\right)+\left(r_A+\sigma_A^2/2\right)\left(T-t\right)\right)\Big/\sigma\sqrt{T-t}$ of the option price to changes in *A(t)*, $d_2=d_1-\sigma_A\sqrt{T-t}$, and asset volatility σ_A, and the Gram-Charlier correction for *t*-period skewness γ_1 and kurtosis γ_2 in returns based on the same diffusion process for asset prices.[49]

80. **Since the Merton model also contains empirical irregularities that can influence the estimation of implied assets (which also affects the calibration of implied asset volatility), we estimate the SPD of implied asset values using equity option prices without any assumptions on the underlying diffusion process (Box 1 below).** Using equity option prices, we can derive the risk-neutral probability distribution of the underlying asset price at the maturity date of the options. We determine the implied asset value as the expectation over the empirical SPD by adapting the Breeden and Litzenberger (1978) method, together with a semi-parametric specification of the Black-Scholes option pricing formula (Aït-Sahalia and Lo, 1998). More specifically, this approach uses the second derivative of the call pricing function (on European options) with respect to the strike price

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pricing models by Heston (1993) and Heston and Nandi (2000) allow for stochastic volatility, but the parameters driving these models can be difficult to estimate. Many other models have been proposed, to incorporate stochastic volatility, jumps, and stochastic interest rates. Bakshi and others (1997), however, suggest that most of the improvement in pricing comes from introducing stochastic volatility. Introducing jumps in asset prices leads to small improvements in the accuracy of option prices. Other option pricing models include those based on copulas, Levy processes, neural networks, Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models, and nonparametric methods. Finally, the binomial tree proposed by Cox, Ross and Rubinstein (1979) spurned the development of lattices, which are discrete-time models that can be used to price any type of option—European or American, plain-vanilla or exotic.

[49] The advantage of the GC model is that it is only slightly more complicated to implement than the Merton model because only two additional parameters—skewness and kurtosis—need to be estimated. The disadvantage is that it is assumes that these parameters are constant.

(rather than option prices as identifying conditions). Estimates are based on option contracts with identical time to maturity, assuming a continuum of strike prices. Since available strike prices are always discretely spaced on a finite range around the actual price of the underlying asset, interpolation of the call pricing function inside this range and extrapolation outside this range are performed by means nonparametric (local polynomial) regression of the implied volatility surface (Rookley, 1997).

Box 1. Estimation of the Empirical SPD

Breeden and Litzenberger (1978) show Arrow-Debreu prices can be replicated via the concept of the *butterfly spread* on European call options. This spread entails selling two call options at strike price *K* and buying two call options with adjacent strike prices $K^- = K - \Delta K$ and $K^+ = K + \Delta K$ respectively, with the stepsize ΔK between the two call strikes. If the terminal underlying asset value $A(T) = K$ then the payoff $Z(.)$ of $1/\Delta K$ of such butterfly spreads at time $T - \tau$ (and time to maturity τ) is defined as

$$Z\left(A(T), K; \Delta K\right) = Price\left(A(T-\tau), \tau, K; \Delta K\right)\Big|_{\tau=0} = \frac{u_1 - u_2}{\Delta K}\Big|_{A(T)=K, \tau=0} = 1,$$

with

$$u_1 = C\left(A(T-\tau), \tau, K + \Delta K\right) - C\left(A(T-\tau), \tau, K\right)$$

and

$$u_2 = C\left(A(T-\tau), \tau, K\right) - C\left(A(T-\tau), \tau, K - \Delta K\right).$$

$C(A, \tau, K)$ denotes the price of a European call option with an underlying asset price *A*, a time to maturity τ and a strike price *K*. As $\Delta K \to 0$, $Price\left(A(T-\tau), \tau, K; \Delta K\right)$ of the position value of the butterfly spread becomes an Arrow-Debreu security paying 1 if $A(T) = K$ and zero in other states. If $A(T) \in \mathbb{R}^+$ is continuous, however, we obtain a security price

$$\lim_{\Delta K \to 0}\left(\frac{Price\left(A(t), \tau, K; \Delta K\right)}{\Delta K}\right)\Bigg|_{K=A(T)} = f^*\left(A(T)\right)e^{-r_{t,\tau}},$$

where $r_{t,\tau}$ and $f^*(.)$ denote the risk-free interest rate and the risk-neutral probability density function (or state price density (SPD)) at time *t*. On a continuum of states *K* at infinitely small ΔK a complete state pricing function can be defined. Moreover, as $\Delta K \to 0$, this price

$$\lim_{\Delta K \to 0}\left(\frac{Price\left(A(t), \tau, K; \Delta K\right)}{\Delta K}\right) = \lim_{\Delta K \to 0}\frac{u_1 - u_2}{(\Delta K)^2} = \frac{\partial^2 C_t(.)}{\partial K^2}$$

will tend to the second derivative of the call pricing function with respect to the strike price evaluated at *K*, provided that $C(.)$ is twice differentiable. Thus, we can write

$$\frac{\partial^2 C_t(.)}{\partial K^2}\Big|_{K=A(T)} = f_t^*\left(A(T)\right)e^{-r_{t,\tau}\tau}$$

across all states, which yields the SPD

$$f_t^*\left(A(T)\right) = e^{-r_{t,\tau}\tau}\frac{\partial^2 C_t(.)}{\partial K^2}\Big|_{K=A(T)}$$

under no-arbitrage conditions and without assumptions on the underlying asset dynamics. Preferences are not restricted since no-arbitrage conditions only assume risk-neutrality with respect to the underlying asset. The only requirements for this method are that markets are perfect, i.e., there are no transactions costs or restrictions on sales, and agents are able to borrow at the risk-free interest rate.

81. **The implied asset value is estimated directly from option prices (in tandem with an option pricing approach that takes into account higher moments of the underlying asset diffusion process).** This avoids the calibration error of using two-equations-two unknowns in the traditional Merton model in solving both implied asset value and asset volatility simultaneously. Thus, asset volatility can be derived from:

$$\sigma_A\left[1+\frac{\gamma(t)_1}{3!}\left(2\sigma_A-d_1\right)-\frac{\gamma(t)_2}{4!}\left(1-d_1^{\,2}+3d\sigma_A-3\sigma_A^2\right)\right]=\frac{E(t)}{A(t)\Phi(d_1)}\sigma_E.$$ [50]

[50] The two-equations-two-unknowns approach is based on Jones and others (1984), which was subsequently extended by Ronn and Verma (1986) to a single equation to solve two simultaneous equations for asset value and volatility as two unknowns. Duan (1994), however, shows that the volatility relationship between implied assets and equity is redundant if equity volatility is stochastic. An alternative estimation technique for asset volatility introduces a maximum likelihood approach (Ericsson and Reneby, 2004 and 2005), which generates good prediction results.

Box 2. Using Systemic CCA to Calculate a Possible Systemic Risk Surcharge

There is an ongoing debate about the advantages and disadvantages of systemic risk capital add-ons or premium-based fees, whether such levies should be charged ex-post or ex-ante, and whether proceeds would go to special funds or to general government revenue.

In the context of presented stability analysis, the Systemic CCA model-derived expected losses can be used to calculate a "fair value" price of a systemic risk surcharge for all sample banks. To illustrate this, the fair value (in basis points) of a risk-based surcharge that would compensate for expected losses arising from systemic solvency risk on an actuarial basis can be written as

$$-\frac{1}{T}\ln\left(1-\frac{\frac{1}{T}G^{-1}_{\mu,\sigma,\xi}\left(a;P_{p,T}\right)}{\sum_{j}^{p}B_{j}e^{-rT}}\right)\times 10,000,$$

where B represents the aggregate default barrier of all p-institutions in the sample, r is the risk-free rate, T is time horizon of the surcharge, and $G^{-1}_{\mu,\sigma,\xi}(.)$ is the multivariate density function (with location, scale and shape parameters μ,σ and ξ of individual expected losses $P_{p,T}$ (equity put option).

As an illustration, using the results obtained from the Systemic CCA analysis above, the estimated average annual "through-the-cycle" systemic surcharge for systemically important financial institutions would at least be 3 basis points (see Table 9), which seems to be in line with the German bank levy, with took into effect on January 1, 2011, as part of the newly adopted Bank Restructuring Act. This charge would be on debt liabilities excluding insured deposits and regulatory capital instruments.[51] A reasonable systemic surcharge for systemically important financial institutions during stress periods would be about 30 basis points per year if estimations are based on observations during the recent financial crisis following the collapse of Lehman Brothers in September 2008.

[51] After considering the time-variation of expected losses (and their distributional behavior), it would be possible to devise a counter-cyclical surcharge by combining estimates at different percentile levels of statistical confidence.

APPENDIX V. HEURISTIC APPROXIMATION OF CONTINGENT LIABILITIES FROM THE FINANCIAL SECTOR

82. **The market-implied expected losses calculated for each financial institution from equity market and balance sheet information using the CCA can be combined with information from CDS markets to estimate the government's contingent liabilities.** The put option value $P_{CDS}(t)$ using CDS prices reflects the expected losses associated with default net of any financial guarantees, i.e., residual default risk on unsecured senior debt and can be written as

$$P_{CDS}(t) = \left(1 - \exp\left(-\left(s_{CDS}(t)/10{,}000\right)\left(B/D(t)-1\right)(T-t)\right)\right)Be^{-r(T-t)}.$$

The linear adjustment $\left(B/D(t)-1\right)$ is needed if outstanding debt B trades either above (below) par value D, which decreases (increases) the CDS spread $s_{CDS}(t)$ (in bps) due to an implicit recovery rate of the CDS contract at notional value and below (above) the recovery rate implied by the market price *D(t)*. This negative (positive) difference ("basis") between the CDS spread and the corresponding bond spread represents the ratio between recovery *at face value*, which underpins the CDS spread calculation, and recovery *at market value*, which applies to the commensurate bond spread.[52]

$P_{CDS}(t)$ above is derived by rearranging the specification of the CDS spread

$$s_{CDS}(t) = -(T-t)^{-1}\ln\left(1 - P_{CDS}(t)/Be^{-r(T-t)}\right)\times\left(B/D(t)-1\right)\times 10{,}000$$

under the risk-neutral measure, assuming a survival probability

$$1-\overline{p} = \exp\left(-\int_0^t h(u)\,du\right) = \exp(-ht)$$

at time t with cumulative default rate p, and a constant hazard rate $s(t)_{CDS} \approx h$. Then $P_{CDS}(t)$ can be used to determine the fraction

$$\alpha(t) = 1 - P_{CDS}(t)/P_E(t)$$

[52] We approximate the change in recovery value based on the stochastic difference between the standardized values of the fair value CDS (FVCDS) spread and the fair value option adjusted spread (FVOAS) reported by *Moody's KMV* (MKMV). Both FVOAS (FVCDS) are credit spreads (in bps) over the London Interbank Offered Rate for the bond (CDS) of a particular company, calculated by MKMV's valuation model based on duration (term) of t years (where t=1 to 10 in one-year increments). Both spreads imply an LGD determined by the industry category. In practice, this adjustment factor is very close to unity for most of the cases, with a few cases where the factor is within a 10 percent range (0.9 to 1.1).

of total potential loss due to default, $P_E(t)$, covered by implicit guarantees that depress the CDS spread below the level that would otherwise be warranted for the option-implied default risk.[53] In other words, $\alpha(t)P_E(t)$ is the fraction of default risk covered by the government (i.e., its contingent liability) and $(1-\alpha(t))P_E(t)$ is the risk retained by an institution and reflected in the CDS spreads. Thus, the time pattern of the government's contingent liability and the retained risk in the financial sector can be measured.

83. **While this definition of market-implied contingent liabilities provides a useful indication of possible sovereign risk transfer, the estimation of the alpha-value depends on a variety of assumptions that influence the assessment of the likelihood of government support, especially at times of extreme stress during the credit crisis.** Some caveats regarding the estimation of expected losses (and contingent liabilities) are in order:

- *Equity prices might not only reflect fundamental values due to both shareholder dilution and trading behavior that obfuscate proper economic interpretation.* During the credit crisis, rapid declines in market capitalization of financial firms were not only a signal about future solvency risk, but also reflected a "flight to quality" motive that was largely unrelated to expectations about future firm earnings or profitability. Assuming that CDS pricing was efficient, the definition of the alpha-value would erroneously flag implicit government support due to extremely low equity valuations but not as a result of depressed CDS spreads (in expectation of possible guarantee to short-term creditors). However, empirical evidence does not concur with such a "denominator effect" of equity prices on the alpha-value. For the given sample, a high cointegration and weaker negative correlation between equity prices and CDS spreads during stress periods suggest consistent co-movement but lower sensitivity of CDS spreads to changes in default risk over time amid rapidly declining levels of market capitalization.

- *The equality condition of default probabilities derived from equity prices and from CDS spreads eliminate the possibility of positive alpha-values.* Carr and Wu (2007) and Zou (2003) show that for many corporations the put option values from equity options and CDS are closely related.[54] Arbitrage trading between both price shows in the synthetic replication of credit protection on guaranteed bonds using equity (i.e., a long position in an equity option "straddle" combined with a short CDS position). However, in stress

[53] Note that the estimation assumes a European put option, which does not recognize the possibility of premature execution. This might overstate the actual expected losses inferred from put option values in comparison with the put option derived from CDS spreads.

[54] Carr and Wu (2007) find that equity options used in a modified CCA seem to produce risk-neutral default probabilities (RNDP) matching fairly closely RNDPs derived from CDS (sometimes higher, sometimes lower, and differences seem to predict future movements in both markets). Zou (2003) finds that divergences of default probabilities derived from equity options used in CCA model and CDS disappear or revert driven by capital arbitrage relationships and trading impacts. The paper by Yu (2006) uses a less sophisticated model based on *CreditGrades*, which contains some simplifying assumption that are currently being revised by *RiskMetrics*.

situations, the implicit put options from equity markets and CDS spreads differ in their price sensitivity to the impact of changes in the underlying capital structure on the implicit default probability and, thus, should be priced differently.

Appendix VI. The Systemic CCA Methodology—Calculating the Systemic Worst-Case Scenario Using Multivariate Generalized Extreme Value

84. **The Systemic CCA framework (Gray and Jobst, 2010; Gray and Jobst, forthcoming) is predicated on the quantification of the systemic financial sector risk.** It is applied in this context to generate a multivariate extreme value distribution (MGEV) that formally captures the potential of tail realizations of market-implied joint expected losses. The analysis of dependence is completed independently from the analysis of marginal distributions, and, thus, differs from the classical approach, where multivariate analysis is performed jointly for marginal distributions and their dependence structure by considering the complete variance-covariance matrix, such as the multivariate MGARCH approach. We first define a nonparametric dependence function of individual expected losses. We then combine this dependence measure with the marginal distributions of these individual expected losses, which are assumed to be generalized extreme value (GEV). These marginal distributions estimated via the LRS method, which identifies possible limiting laws of asymptotic tail behavior of normalized extremes (Coles and others, 1999; Poon and others, 2003; Stephenson, 2003; Jobst, 2007). The dependence function is estimated iteratively on a unit simplex that optimizes the coincidence of multiple series of cross-classified random variables – similar to a Chi-statistic that measures the statistical likelihood of observed values to differ from their expected distribution.

85. **More specifically, we first specify the asymptotic tail behavior of the vector-valued series $\mathbf{X}_{i,j} \equiv \mathbf{P}_{i,j} = \left(P_1^n, ..., P_m^n\right)$ of expected losses** (i.e., put option values) of an *m* number of financial sector entities *j* as the limiting law of an *n*-sequence of normalized maxima (over rolling window estimation period of τ=120 days and daily updating), so that the j^{th} univariate marginal

$$y_j = y_j(x) = \left(1 + \xi_j\left(x - \mu_j\right)/\sigma_j\right)_+^{-1/\xi_j} \quad (\text{for } j = 1, ..., m)$$

lies in the domain of attraction of the generalized extreme value (GEV) distribution, where $1+\xi_j\left(x-\mu_j\right)/\sigma_j > 0$, scale parameter $\sigma_j > 0$, location parameter μ_j, and shape parameter ξ_j. The higher the absolute value of shape parameter, the larger the weight of the tail and the slower the speed at which the tail approaches its limit.

86. **Second, the multivariate dependence structure of joint tail risk of expected losses is derived nonparametrically as the convex function**

$$A(\omega) = \min\left(1, \max\left\{n\left\{\sum_{i=1}^{n} \bigwedge\nolimits_{j=1}^{m} \frac{y_{i,j}/\hat{y}_{\bullet j}}{\omega_j}\right\}^{-1}, \omega, 1-\omega\right\}\right),$$

over the same estimation window, where $\hat{y}_{\bullet j} = \sum_{i=1}^{n} y_{i,j} / n$ and $0 \leq \max\left(\omega_1,...,\omega_{m-1}\right) \leq A\left(\omega_j\right) \leq 1$ for all $0 \leq \omega_j \leq 1$, subject to the optimization of the (*m*-1)-dimensional unit simplex

$$S_m = \left\{ \left(\omega_1,...,\omega_{m-1}\right) \in \mathbb{R}_+^n : \omega_j \geq 0, 1 \leq j \leq m-1; \sum_{j=1}^{m-1} \omega_j \leq 1 \text{ and } \omega_m = 1 - \sum_{j=1}^{m-1} \omega_j \right\}.$$

87. **Finally, after estimation of the marginal distributions and the dependence structure** over the a rolling window of τ number of days, we obtain the multivariate distribution

$$G_{t,\hat{\xi},\hat{\mu},\hat{\sigma}}\left(x\right) = \exp\left\{ -\left(\sum_{j=1}^{m} y_j\right) A\left(\omega\right) \right\}$$

at time $t = \tau + 1$, using the maximum likelihood estimation $\hat{\theta}_{MLE} = \underset{\theta}{\arg\max} \prod_{i=1}^{n} g\left(x;\theta\right)$.

88. **We then obtain the *Expected Shortfall* (ES) (or conditional VaR) as the probability-weighted residual density beyond a prespecified statistical confidence level** (say, *a*=0.95) of maximum losses, where point estimate of joint expected losses is defined as[55]

$$\hat{x}_{t,a} = G^{-1}_{t,\hat{\xi},\hat{\mu},\hat{\sigma}}\left(a\right) = \hat{\mu}_j + \hat{\sigma}_j / \hat{\xi}_j \left(\left(-\frac{\ln\left(a\right)}{A\left(\omega\right)} \right)^{-\hat{\xi}_j} - 1 \right).$$

ES defines the average estimated value *z* of the aggregate expected losses over estimation days τ in excess of the statistical confidence limit. Thus, we can write ES at time *t* as

$$ES_{t,\tau,a} = -\mathrm{E}\left[z_\tau \middle| z_\tau \geq G_t^{-1}\left(a\right) = VaR_{t,a} \right]$$

at a threshold quantile value

$$VaR_{t,\tau,a} = \sup\left\{ G_t^{-1}\left(\bullet\right) \middle| \Pr\left[z\tau > G_t^{-1}\left(\bullet\right) \right] \geq a = 0.95 \right\}.$$

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[55] ES is an improvement over VaR, which, in addition to being a pure frequency measure, is "incoherent"; i.e., it violates several axioms of convexity, homogeneity, and subadditivity found in coherent risk measures. For example, subadditivity, which is a mathematical way to say that diversification leads to less risk, is not satisfied by VaR.

ES can also be written as a linear combination of individual ES values, where the relative weights (in the weighted sum) are given by the second order cross-partial derivatives of the inverse of the joint probability density function $G_t^{-1}(a)$ to changes in both the dependence function and the individual marginal severity of expected loses. Thus, by re-writing $ES_{t,\tau,a}$ above, we obtain the sample ES

$$ES_{t,\tau,a} = -\sum_{j}^{m} MC_{j,\tau,a} \mathrm{E}\left[z_{j,\tau} \middle| z_{m,\tau} \geq G_t^{-1}(a) = VaR_{t,q_a} \right],$$

where the relative weight of institution *j* is defined as the marginal contribution

$$MC_{j,\tau,a} = \frac{\partial^2 G_t^{-1}(a)}{\partial y_{j,a} \partial A^{-1}(\omega)} \quad s.t. \sum_{i}^{m} MC_{j,\tau,a} = 1 \text{ and } MC_{j,\tau,a} z_{i,\tau} \leq z_{m,\tau}$$

to expected shortfall

$$ES_{t,\ell,a} = -\sum_{j}^{m} \psi_{j,\ell,a} \mathrm{E}\left[z_{j,\ell} \middle| z_{m,\ell} \geq G_t^{-1}(a) = VaR_{t,q_a} \right],$$

attributable to the joint effect of both the marginal distribution $y_{j,a}$ and the change of the dependence function $A(.)$ absent institution *j*.