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The Global Economy Model: Theoretical Framework

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This paper has two purposes. First, it provides a thorough exposition of the theoretical framework underlying the Global Economy Model (GEM), as the model stands in early 2008. Second, it discusses a number of variants and alternative features considered in the GEM-related literature since Laxton and Pesenti (2003). For an updated survey of GEM and other dynamic, stochastic, general-equilibrium applications at the IMF, the reader is referred to Botman and others (2007). Each section starts with a formal description of the relevant equations, and is followed by a presentation of modeling variants and options. When appropriate, the section provides a more detailed discussion of how the building blocks of GEM relate to the literature. It is worth emphasizing from the very beginning that the paper is meant to be used as a technical reference on GEM and related models, with apologies for the somewhat pedantic attention to details and formulas that stems directly from this premise.
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I. Peeking Inside the Box: Model Structure and Basic Notation

Building on recent theoretical developments in international finance and monetary economics, especially the "new open-economy macroeconomics" literature since the seminal contributions of Maurice Obstfeld and Kenneth Rogoff (1995, 2000, 2002), the Global Economy Model (GEM) aims to

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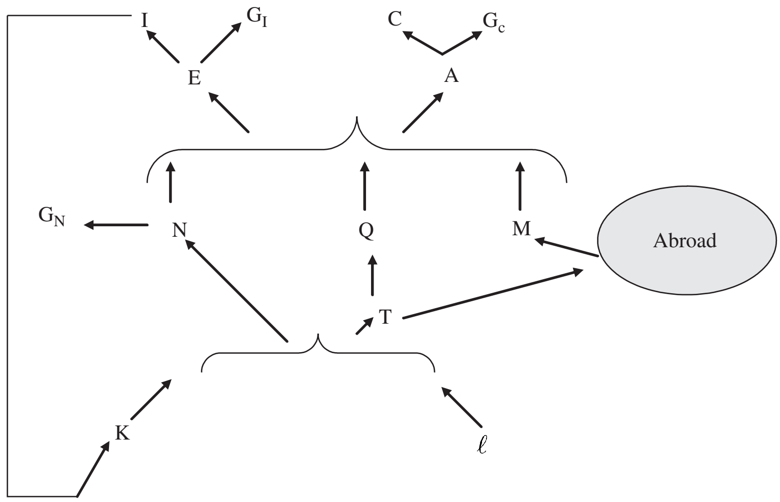
Figure 1. The Structure of the Model



provide an optimizing intertemporal framework capable of addressing basic policy questions involving international transmission of policy and structural shocks, while reproducing key elements of macroeconomic interdependence among countries and regional blocs.

Like other recent dynamic, stochastic, general-equilibrium (DSGE) models, the design of GEM combines the long-run properties of real business cycle models with short-run "Keynesian" dynamics stemming from nominal rigidities and inertia in the inflation process. Although obviously indebted to the classic Mundell-Fleming-Dornbusch tradition, GEM builds on explicit microfoundations allowing for a tightly integrated treatment of positive elements and welfare considerations.

A useful way to approaching GEM is by familiarizing oneself with its broad characteristics and notation with the help of a visual representation. Figure 1 illustrates the key macroeconomic variables in a representative country.

Consider first the *households* sector. Each household consumes a final good (C in Figure 1), and supplies labor (ℓ) to all domestic firms. Some households do not have access to capital markets. They finance their consumption exclusively through disposable labor incomes. The remaining households own the portfolio of domestic firms and the domestic capital stock (K), which they rent to domestic firms. They also buy and sell two bonds: a domestic bond denominated in domestic currency, and an international bond issued in zero net supply worldwide. When households sell or purchase the international bond they pay a premium to financial intermediaries, whose size is a function of the aggregate net asset position of the country. Labor and physical capital are immobile internationally. The market for capital is competitive, and capital accumulation is subject to adjustment costs. In the labor market, wage contracts are subject to nominal rigidities.

On the production side, *firms* produce the final goods, an array of differentiated intermediate goods, and provide intermediation services.

In each country there are two final goods—a consumption good (A) and an investment good (E)—produced by perfectly competitive firms. The consumption good is consumed either by domestic households or by the government (G_C). Similarly, demand for the investment good is split between private agents (I) and the public sector (G_I). Final goods are produced by using all available intermediate goods as inputs.

There are many varieties of *intermediate* goods, each produced by a single firm under conditions of monopolistic competition. Each intermediate good is produced by using domestic labor inputs and domestic capital. Intermediate goods are either nontraded (N) or traded internationally (T). The nontraded intermediate goods can be purchased by the government (G_N) or used in the production of the final goods (N_N). Domestic tradables used by domestic firms are denoted Q, imports from all other country blocs are denoted M. Imports are subject to short-term adjustment costs that temporarily lower the response of demand to changes in relative prices. Prices of intermediate goods are subject to adjustment costs (nominal price rigidities).

Finally, the *government* purchases the two national final goods, as well as nontradable services. As treasury, the government finances its expenditures with net taxes on the domestic private sector. As central bank, the government manages the national short-term nominal interest rate. Monetary policy is specified in terms of a credible commitment to guarantee price stability by managing the domestic nominal short-term interest rate.

II. General Considerations

Country Size

The world economy consists of a set $\mathcal{N}$ of regional blocs ("countries"). The size of the world economy is normalized to one. The *size* of each country H is denoted s^H, with $0 < s^H < 1$ and $\sum_H s^H = 1$ for $H \in \mathcal{N}$. The country size measures the world share of private agents who are resident in the country: both households and firms in country H are defined over a continuum of mass s^H.

Discussion

For applications focused on short- and medium-term analysis, the country size s^H can be treated as a constant parameter. This specification is problematic in simulation exercises in which one or more countries grow for a prolonged period of time below (or above) the common trend (the latter is defined below). In these cases, maintaining the country size constant over time leads to an upward (downward) bias of the long-term economic relevance of these countries in a global context. Also, because the number of product varieties and labor inputs in each country is normalized to s^H, constant country sizes imply that the sectoral extensive margins (the number of firms and varieties in any given sector) remain constant over time as well, and there is no labor immigration. For the time being, the current version of GEM does not encompass endogenous firms' entry and exit across sectors and countries. A possible way to deal with these issues is to let s^H be time varying and equal (for instance) to the size of GDP (or alternative measures of a country's economy) in country H relative to the world GDP. The model-based notion of GDP is discussed later, in Section IV.

Growth Trend

There is a common stochastic trend for the world economy (the variable $TREND$), whose gross rate of growth between time t and time τ is denoted $g_{t,\tau} \equiv TREND_\tau / TREND_t$. All quantity variables in each country are expressed in detrended terms, that is, as ratios of $TREND$. The exception is labor effort ℓ, bounded by endowment. In the long run, $g_{t,t+1}$ converges to g_{SS} and $g_{t,\tau}$ converges to $g_{SS}^{\tau-t}$, where g_{SS} is a constant.

Discussion

Each (detrended) real variable is stationary, that is, it converges to a well-defined steady-state level. This applies to all relative prices, including terms of trade and real exchange rates. In the long term there is balanced growth (at the rate g) across countries and sectors. The assumption is less restrictive than it appears, as it is always possible to engineer persistent (albeit not permanent) deviations from balanced growth for an arbitrarily long period of time. Variants of the model could be considered to account for unit roots in relative prices, but they are not discussed here.

Prices and Inflation Rates

As a convention throughout the model, nominal prices expressed in domestic currency are denoted with uppercase variables, but relative prices are denoted with lowercase variables. Without loss of generality, in each country the consumption good A is the *numeraire* of the economy and all national relative prices are expressed in terms of domestic consumption units, that is relative to the consumer price index (CPI). For instance, if P_A denotes the nominal CPI and P_E the nominal price of one unit of the E good, $p_E \equiv P_E / P_A$ denotes the price of one unit of E in terms of A. Of course, by definition we have $p_A = 1$.

Also we denote the (gross) CPI inflation rate between time t and time τ with $\pi_{t,\tau} \equiv P_{A,\tau}/P_{A,t}$. The inflation rate in sector E is therefore equal to

$$\pi_{Et,\tau} \equiv \frac{p_{E,\tau}}{p_{E,t}} \pi_{t,\tau}. \tag{1}$$

In steady state the inflation rate $\pi_{t,t+1}$ converges to π_{SS} and $\pi_{t,\tau}$ converges to $\pi_{SS}^{\tau-t}$, where π_{SS} is a constant equal to the inflation target of the government.

Discussion

GEM is coded up after transforming all prices in relative terms, so that nominal prices do not appear in the model. Precisely for the same reason why all quantities are defined in detrended terms, by normalizing all prices relative to the domestic nominal trend P_A we avoid dealing with unit roots, either nominal or real, in quantitative simulations of the model over very long time horizons. Because the inflation rate $\pi_{t,\tau}$ is part of the model solution, one can always reconstruct the nominal path for the CPI level by arbitrarily setting the value of P_A at some initial time $t=0$ and computing $P_{A,t} = P_{A,0}\pi_{0,t}$. In the main text we typically adopt the notation with relative (lowercase) prices. We only switch to the notation with nominal (uppercase) variables when appropriate in order to simplify the exposition.

As observed above, all relative prices p converge to well-defined steady-state levels. If two countries have different steady-state inflation rates, reflecting different policy preferences for the domestic nominal anchor, in steady state the nominal exchange rate between the two countries depreciates at a rate equal to the difference between the two inflation rates but the CPI-based real exchange rate remains constant. We return to this point in Section III.

Usually inflation variables carry a double time index. In most applications time t is measured in quarters, so that $\pi_{t-1,t}$ measures the quarterly inflation rate at time t, $(\pi_{t-1,t})^4$ measures the annualized quarterly inflation rate at time t, and $\pi_{t-4,t}$ measures the year-over-year inflation rate at time t. When there is no risk of confusion, we adopt the notation π_t as shorthand for quarterly inflation $\pi_{t-1,t}$.

Notational Conventions and Other Formal Aspects

The convention throughout the model is that variables that are not explicitly indexed (to firms or households) are expressed in domestic per-capita terms. For instance, $A_t \equiv (1/s)\int_0^s A_t(x)\,dx$ and $\ell_t \equiv (1/s)(\int_0^s \ell_t(n)\,dn + \int_0^s \ell_t(h)dh)$. Variables without time indices as well as variables with subscript SS are used interchangeably to denote steady-state levels. For instance, $g_{SS} = g$.

GEM allows for a rich menu of stochastic processes. As a general convention throughout the model, when we state that variable X follows an autoregressive process, we mean that the process for X is coded as

$$X_t = (1 - \lambda_X)X_{SS} + \lambda_X X_{t-1} + e_{X,t}, \tag{2}$$

where $0<\lambda_X<1$, X_{SS} is the steady-state value of X_t, and $e_{X,t}$ is an i.i.d. shock. If variable X is strictly positive, a logarithmic transformation is considered:

$$\ln X_t = (1 - \lambda_X)\ln X_{SS} + \lambda_X \ln X_{t-1} + e_{X,t}. \tag{3}$$

Needless to say, the assumptions about the dynamic structure of the random variables and the variance-covariance matrix play a crucial role in volatility exercises. In general, alternative specifications of the stochastic processes can be introduced, depending on the specific nature of the simulation project.

It is worth emphasizing that GEM has been developed and coded up in nonlinear terms. This choice not only enhances the transparency of the model code relative to the theoretical apparatus but also allows for seamless higher-order extensions of the analysis beyond the traditional first-order approximations around the nonstochastic steady state. This flexibility is particularly relevant for welfare analyses involving at least second-order expansions around the steady-state equilibrium. In one case below, however, we find it useful to focus the presentation on the linear approximations of the GEM equations, in order to facilitate the comparison between our framework and similar analytical models.

III. The Domestic Macroeconomy in Partial Equilibrium

This section is devoted to the country-specific elements of the model that do not involve international interactions. Thus, in what follows we consider a representative country under the working assumption that trade-related variables are exogenous and determined outside the equilibrium. For this reason, country-specific indices play no role in this section. General equilibrium considerations and a fuller notation involving country indices are introduced in Section IV.

Final Goods

In each country there is a continuum of symmetric firms producing two final goods, A (the consumption good) and E (the investment good). Both goods are produced under perfect competition.

Consider first the consumption sector. Each firm is indexed by $x \in [0, s]$. Firm x's output at time t is denoted $A_t(x)$. The consumption good is produced with the following nested constant elasticity of substitution

(CES) technology:

$$A_t(x) = \left\{ (1-\gamma_A)^{\frac{1}{\varepsilon_A}} N_{A,t}(x)^{1-\frac{1}{\varepsilon_A}} + \gamma_A^{\frac{1}{\varepsilon_A}} \left[\nu_A^{\frac{1}{\mu_A}} Q_{A,t}(x)^{1-\frac{1}{\mu_A}} + (1-\nu_A)^{\frac{1}{\mu_A}} M_{A,t}(x)^{1-\frac{1}{\mu_A}} \right]^{\frac{\mu_A}{\mu_A-1}\left(1-\frac{1}{\varepsilon_A}\right)} \right\}^{\frac{\varepsilon_A}{\varepsilon_A-1}} \tag{4}$$

Three intermediate inputs are used in the production of the consumption good A: a basket N_A of nontradable goods, a basket Q_A of domestic tradable (import-competing) goods, and a basket M_A of imported goods. The elasticity of substitution between tradables and nontradables is $\varepsilon_A > 0$, and the elasticity of substitution between domestic and imported tradables is $\mu_A > 0$. The weights of the three inputs are, respectively, $1-\gamma_A$, $\gamma_A \nu_A$ and $\gamma_A(1-\nu_A)$ with $0 < \gamma_A, \nu_A < 1$.

Firm x takes as given the prices of the three inputs and minimizes its costs $p_N N_A(x) + p_Q Q_A(x) + p_{MA} M_{A,t}(x)$subject to the technological constraint (4). Cost minimization implies that firm x's demands for intermediate inputs are

$$N_{A,t}(x) = (1-\gamma_A) p_{N,t}^{-\varepsilon_A} A_t(x), \tag{5}$$

$$Q_{A,t}(x) = \gamma_A \nu_A p_{Q,t}^{-\mu_A} p_{XA,t}^{\mu_A-\varepsilon_A} A_t(x), \tag{6}$$

$$M_{A,t}(x) = \gamma_A (1-\nu_A) p_{MA,t}^{-\mu_A} p_{XA,t}^{\mu_A-\varepsilon_A} A_t(x), \tag{7}$$

where p_N, p_Q, and p_{MA} are the relative prices of the inputs in terms of final consumption baskets, and p_{XA} is the cost-minimizing price of the composite basket of domestic and foreign tradables, or:

$$p_{XA,t} \equiv [\nu_A p_{Q,t}^{1-\mu_A} + (1-\nu_A) p_{MA,t}^{1-\mu_A}]^{\frac{1}{1-\mu_A}}. \tag{8}$$

The production technologies in the consumption and investment sectors can be quantitatively different but their formal characterization is similar, with self-explanatory changes in notation. For instance, a firm $e \in [0, s]$, that produces the investment good, demands nontradable goods according to

$$N_{E,t}(e) = (1-\gamma_E)(p_{N,t}/p_{E,t})^{-\varepsilon_E} E_t. \tag{9}$$

Discussion

Note that p_{MA} and p_{ME} are sector-specific as they reflect the different composition of imports in the two sectors, but p_N and p_Q are identical across sectors. In Section IV we discuss the role of import adjustment costs and their effects on relative prices.

The weights γ_A, ν_A, γ_E, and ν_E can be modeled as constant parameters or as autoregressive processes. In the latter case, they can be interpreted as preference shifters, reflecting shifts in households' consumption demand from tradables to nontradables, or from import-competing goods to foreign imports.

The CES specification is notationally cumbersome but widely adopted in DSGE models to allow for a flexible parametrization of elasticities. Of course, when the elasticities are equal to one the equations collapse to the traditional Cobb-Douglas specification. In most applications the elasticities of substitution between import-competing goods and imports, μ_A and μ_E, are likely to be larger than the elasticities of substitution between nontradables and tradables, ε_A and ε_E. Note however that there are no theoretical restrictions on the size of these elasticities.

Demand for Intermediate Goods

Intermediate inputs come in different varieties (brands) and are produced under conditions of monopolistic competition. In each country there are two kinds of intermediate goods, tradables and nontradables. Each kind is defined over a continuum of mass s. Without loss of generality, we assume that each nontradable good is produced by a single domestic firm indexed by $n \in [0, s]$, and each tradable good is produced by a firm $h \in [0, s]$.

Focusing first on the basket N_A, this is a CES index of all domestic varieties of nontradables. Denoting as $N_A(n, x)$ the demand by firm x of an intermediate good produced by firm n, the basket $N_A(x)$ is

$$N_{A,t}(x) = \left[\left(\frac{1}{s} \right)^{\frac{1}{\theta_N}} \int_0^s N_{A,t}(n, x)^{1-\frac{1}{\theta_N}} dn \right]^{\frac{\theta_N}{\theta_N - 1}}, \tag{10}$$

where $\theta_N > 1$ denotes the elasticity of substitution among intermediate nontradables.

Firm x takes as given the prices of the nontradable goods $p(n)$ and minimizes its costs $\int_0^s p(n) N_{A,t}(n, x)\, dn$ subject to (10), obtaining

$$N_{A,t}(n, x) = \frac{1}{s} \left(\frac{p_t(n)}{p_{N,t}} \right)^{-\theta_N} N_{A,t}(x), \tag{11}$$

where p_N (the Lagrange multiplier) is the cost-minimizing price of one unit of the nontradable basket, or

$$p_{N,t} = \left[\left(\frac{1}{s} \right) \int_0^s p_t(n)^{1-\theta_N} dn \right]^{\frac{1}{1-\theta_N}}. \tag{12}$$

The basket N_E is similarly characterized. Aggregating across firms, and accounting for public demand of nontradables—here assumed to have the same composition as private demand—we obtain the total demand for good n as

$$\int_0^s N_{A,t}(n,x)dx + \int_0^s N_{E,t}(n,e)de + G_{N,t}(n)$$
$$= \left(\frac{p_t(n)}{p_{N,t}}\right)^{-\theta_N} (N_{A,t} + N_{E,t} + G_{N,t}) = \left(\frac{p_t(n)}{p_{N,t}}\right)^{-\theta_N} N_t, \quad (13)$$

where $N_{A,t} = (1/s)\int_0^s N_{A,t}(x)\,dx$ (and similar).

Following similar steps we can derive the domestic demand schedules for the intermediate goods h:

$$\int_0^s Q_{A,t}(h,x)dx + \int_0^s Q_{E,t}(h,e)de = \left(\frac{p_t(h)}{p_{Q,t}}\right)^{-\theta_T} Q_t. \quad (14)$$

Demand for imported intermediate goods will be characterized below.

Discussion

The elasticity of substitution θ, either in the nontradables or the tradables sector, can be modeled as a constant parameter or as a time-varying stationary process.

In Section IV we discuss how (13) changes when distribution services are considered.

Supply of Intermediate Goods

Each nontradable good n is produced with the following CES technology:

$$N_t(n) = Z_{N,t}\left[(1-\alpha_N)^{\frac{1}{\xi_N}} \ell_t(n)^{1-\frac{1}{\xi_N}} + \alpha_N^{\frac{1}{\xi_N}} K_t(n)^{1-\frac{1}{\xi_N}}\right]^{\frac{\xi_N}{\xi_N - 1}}. \quad (15)$$

Firm n uses labor $\ell(n)$ and capital $K(n)$ to produce $N(n)$ units of its variety. $\xi_N > 0$ is the elasticity of input substitution, and Z_N is a stochastic process for productivity, common to all producers of nontradables.

Following the notational convention regarding prices, we let mc_t, w_t, and r_t denote marginal costs, wages, and rental rates in consumption units. Firm n minimizes its costs $w_t\ell_t(n) + r_t K_t(n)$ subject to (15). Cost minimization yields the marginal cost in nontradables production as

$$mc_t(n) = \frac{1}{Z_{N,t}}\{(1-\alpha_N)w_t^{1-\xi_N} + \alpha_N r_t^{1-\xi_N}\}^{\frac{1}{1-\xi_N}}, \quad (16)$$

and the capital-labor ratio is

$$\frac{K_t(n)}{\ell_t(n)} = \frac{\alpha_N}{1-\alpha_N}\left(\frac{r_t}{w_t}\right)^{-\xi_N}. \quad (17)$$

In each country, labor inputs are differentiated and come in different varieties (skills). Each input is associated to one household, defined over a continuum of mass equal to the country size and indexed by $j \in [0, s]$. Each firm n uses a CES combination of all available labor inputs:

$$\ell_t(n) = \left[\left(\frac{1}{s}\right)^{\frac{1}{\psi_L}} \int_0^s \ell(n,j)^{1-\frac{1}{\psi_L}} dj \right]^{\frac{\psi_L}{\psi_L - 1}}, \tag{18}$$

where $\ell(n,j)$ is the demand of labor input of type j by the producer of good n and $\psi_L > 1$ is the elasticity of substitution among varieties of labor inputs. Cost minimization implies that $\ell(n,j)$ is a function of the relative wage:

$$\ell_t(n,j) = \frac{1}{s}\left(\frac{w_t(j)}{w_t}\right)^{-\psi_L} \ell_t(n), \tag{19}$$

where $w(j)$ is the wage paid to labor input j and the wage index w is defined as

$$w_t = \left[\left(\frac{1}{s}\right) \int_0^s w_t(j)^{1-\psi_L} dj \right]^{\frac{1}{1-\psi_L}}. \tag{20}$$

Similar considerations hold for the production of tradables. We denote by $T(h)$ the supply of each intermediate tradable h. Using self-explanatory notation, we have:

$$T_t(h) = Z_{T,t}\left[(1-\alpha_T)^{\frac{1}{\xi_T}} \ell_t(h)^{1-\frac{1}{\xi_T}} + \alpha_T^{\frac{1}{\xi_T}} K_t(h)^{1-\frac{1}{\xi_T}} \right]^{\frac{\xi_T}{\xi_T - 1}}, \tag{21}$$

where Z_T is total factor productivity. Aggregating across firms, we obtain the total demand for labor input j as:

$$\begin{aligned} \int_0^s \ell_t(n,j)dn + \int_0^s \ell_t(h,j)dh \quad &= \left(\frac{w_t(j)}{w_t}\right)^{-\psi_L} \frac{1}{s}\left(\int_0^s \ell_t(n)dn + \int_0^s \ell_t(h)dh \right) \\ &= \left(\frac{w_t(j)}{w_t}\right)^{-\psi_L} \ell_t, \end{aligned} \tag{22}$$

where ℓ is per capita total labor in the economy.

Discussion

Recall that all variables are defined in detrended terms. The implicit assumption is that in each country the effectiveness of labor effort ℓ grows at the same rate as *TREND*, so that a shock Z (either in the tradables or in the nontradables sector) is defined as a total factor productivity deviation from the common world trend. The model allows for country-specific changes in Z that do not affect the long-run balanced-growth properties of the model. Therefore, one can consider a scenario in which the level of Z changes permanently, or one in which Z grows or falls for some time, but not a scenario in which Z grows or falls permanently in steady state. Variants of the model allow for the possibility of transitory shocks to the effectiveness of labor or capital in addition to total factor productivity.

A variant of the model considered in Juillard and others (2006) introduces adjustment frictions in the labor market. The adjustment terms reflect the fact that it takes time for labor inputs to be fully productive in production, so that from the viewpoint of national producers their effective costs are higher in the short term than in steady state. Rewrite Equation (15) as

$$N_t(n) = Z_{N,t}\left[(1-\alpha_N)^{\frac{1}{\xi_N}}\ell_t^*(n)^{1-\frac{1}{\xi_N}} + \alpha_N^{\frac{1}{\xi_N}}K_t(n)^{1-\frac{1}{\xi_N}}\right]^{\frac{\xi_N}{\xi_N-1}}, \tag{23}$$

where $\ell^*(n)$ is "effective" labor, defined as the product of two components:

$$\ell_t^*(n) = \ell_t(n)(1-\Gamma_N[\ell_t(n)]). \tag{24}$$

In the expression above, $\ell(n)$ is the same CES basket of differentiated labor inputs as defined in (18). However, now we assume that changes in labor are subject to firm-specific adjustment costs. These costs are specified relative to the past observed level of labor effort in the sector and are zero in steady state. Specifically, $\Gamma_N[\ell(n)]$ can be modeled as a quadratic term:

$$\Gamma_N[\ell_t(n)] = \frac{\phi_L}{2}\left(\frac{\ell_t(n)}{\ell_{N,t-1}} - 1\right)^2, \tag{25}$$

where as usual $\ell_{N,t-1} = (1/s)\int_0^s \ell_{t-1}(n)dn$. In this case, expression (17) is replaced by

$$\frac{K_t(n)}{\ell_t^*(n)} = \frac{\alpha_N}{1-\alpha_N}\left(\frac{r_t}{w_t/(1-\Gamma_{N,t}(n)-\ell_t(n)\Gamma'_{N,t}(n))}\right)^{-\xi_N},$$

and the marginal cost $mc(n)$ is given by

$$mc_t(n) = \frac{1}{Z_{N,t}}\left((1-\alpha_N)\left(\frac{w_t}{1-\Gamma_{N,t}(n)-\ell_t(n)\Gamma'_{N,t}(n)}\right)^{1-\xi_N} + \alpha_N r_t^{1-\xi_N}\right)^{\frac{1}{1-\xi_N}}. \tag{26}$$

The adjustment terms in Equation (26) reflect the fact that, if $\ell_t(n)$ differs from $\ell_{N,t-1}$, producers' costs are temporarily higher to account for the losses of efficiency associated with the change in labor inputs.

Another variant of GEM allows for heterogeneity in labor skills, to model situations in which differences between high- and low-skill workers may be relevant. As we discuss below, there are two types of households, *LC*-type households and *FL*-type households (these indices will be explained later). *FL*-type households represent a share $(1-s_{LC})$ of domestic households and are indexed by $j\in[0, s(1-s_{LC})]$. *LC*-type households represent a share s_{LC} of domestic households and are indexed by

$j \in (s(1-s_{LC}), s]$. Suppose each type of household supplies a different type of labor input, and each type comes in many differentiated varieties. In this case, (18) is replaced with

$$\ell_t(n) = \left[s_{LC}^{\frac{1}{\psi_L}} \ell_{LC,t}(n)^{1-\frac{1}{\psi_L}} + (1-s_{LC})^{\frac{1}{\psi_L}} \ell_{FL,t}(n)^{1-\frac{1}{\psi_L}} \right]^{\frac{\psi_L}{\psi_L-1}}, \tag{27}$$

where $\ell_{LC}(n)$ is a basket of LC-type labor inputs, $\ell_{FL}(n)$ a basket of FL-type inputs, and ψ_L the elasticity of substitution between the two types. The two baskets are defined as

$$\ell_{FL,t}(n) = \left[\left(\frac{1}{s(1-s_{LC})} \right)^{\frac{1}{\psi_{FL}}} \int_0^{s(1-s_{LC})} \ell(n,j)^{1-\frac{1}{\psi_{FL}}} dj \right]^{\frac{\psi_{FL}}{\psi_{FL}-1}}, \tag{28}$$

$$\ell_{LC,t}(n) = \left[\left(\frac{1}{s \times s_{LC}} \right)^{\frac{1}{\psi_{LC}}} \int_{s(1-s_{LC})}^{s} \ell(n,j)^{1-\frac{1}{\psi_{LC}}} dj \right]^{\frac{\psi_{LC}}{\psi_{LC}-1}}, \tag{29}$$

where $\ell(n,j)$ is the demand of labor input j by the producer of good n and ψ_{FL}, $\psi_{LC} > 1$ are the elasticities of substitution among skills. Cost minimization implies that $\ell(n,j)$ is a function of the relative wages

$$\ell_t(n,j) = \begin{cases} \frac{1}{s} \left(\frac{w_t(j)}{w_{FL,t}} \right)^{-\psi_{FL}} \left(\frac{w_{FL,t}}{w_t} \right)^{-\psi_L} \ell_t(n) & \text{for } FL \text{ inputs} \\ \frac{1}{s} \left(\frac{w_t(j)}{w_{LC,t}} \right)^{-\psi_{LC}} \left(\frac{w_{LC,t}}{w_t} \right)^{-\psi_L} \ell_t(n) & \text{for } LC \text{ inputs} \end{cases}, \tag{30}$$

where the wage indices w_{LC}, w_{FL}, and w are defined as

$$w_{FL,t} = \left[\left(\frac{1}{s(1-s_{LC})} \right) \int_0^{s(1-s_{LC})} w_t(j)^{1-\psi_{FL}} dj \right]^{\frac{1}{1-\psi_{FL}}}, \tag{31}$$

$$w_{LC,t} = \left[\left(\frac{1}{s \times s_{LC}} \right) \int_{s(1-s_{LC})}^{s} w_t(j)^{1-\psi_{LC}} dj \right]^{\frac{1}{1-\psi_{LC}}}, \tag{32}$$

$$w_t = [s_{LC} w_{LC,t}^{1-\psi_L} + (1-s_{LC}) w_{FL,t}^{1-\psi_L}]^{\frac{1}{1-\psi_L}}. \tag{33}$$

Similar considerations hold for the tradables sector. Aggregating across firms, we obtain the total demand for FL-type labor input j as

$$\int_0^{s(1-s_{LC})} \ell_t(n,j)dn + \int_0^{s(1-s_{LC})} \ell_t(h,j)dh = \left(\frac{w_t(j)}{w_{FL,t}}\right)^{-\psi_{FL}} \left(\frac{w_{FL,t}}{w_t}\right)^{-\psi_L} (1-s_{LC})\ell_t, \quad (34)$$

where once again ℓ is per-capita total labor in the economy. Similarly, total demand for LC-type labor input j is

$$\int_{s(1-s_{LC})}^{s} \ell_t(n,j)dn + \int_{s(1-s_{LC})}^{s} \ell_t(h,j)dh = \left(\frac{w_t(j)}{w_{LC,t}}\right)^{-\psi_{LC}} \left(\frac{w_{LC,t}}{w_t}\right)^{-\psi_L} s_{LC}\ell_t. \quad (35)$$

The elasticities ψ_L, ψ_{FL}, and ψ_{LC} can be modeled as constants or as time-varying autoregressive processes.

Price Setting in the Nontradables Sector

Consider now profit maximization in the intermediate nontradables sector. The key element here is the presence of nominal rigidities. They are modeled as costs to nominal price adjustment measured in terms of total profits foregone, building on Rotemberg (1982) and Ireland (2001). To illustrate as clearly as possible the role of nominal inertias, we find it useful to cast the analysis first in terms of nominal prices, and later move back to our usual notation involving relative prices.

Each firm n takes into account the demand (13) for its product and sets its nominal price $P_t(n)$ to maximize the present discounted value of profits. The adjustment cost is denoted $G_{PN,t}[P_t(n), P_{t-1}(n)]$ and is a function of both current and lagged prices. The benchmark parameterization we adopt allows the model to reproduce realistic nominal dynamics:

$$G_{PN,t}(n) \equiv \frac{\phi_{PN}}{2}\left(\frac{P_t(n)/P_{t-1}(n)}{\pi_{N,t-1}} - 1\right)^2. \quad (36)$$

The adjustment cost is related to changes of the nominal price of nontradable n relative to the lagged inflation rate in the nontradables sector $\pi_{N,t-1}$. Underlying this specification is the notion that firms should not be penalized when their price hikes are indexed to some (publicly observable) measures of aggregate or sectoral inflation.

The price-setting problem is then characterized as

$$\max_{P_t(n)} \frac{TREND_t}{P_{A,t}} E_t \sum_{\tau=t}^{\infty} D_{t,\tau} g_{t,\tau} [P_\tau(n) - MC_\tau(n)]$$

$$\left(\frac{P_\tau(n)}{P_{N,\tau}}\right)^{-\theta_N} N_\tau (1 - G_{PN,\tau}(n)), \tag{37}$$

where $D_{t,\tau}$ (with $D_{t,t} = 1$) is the appropriate nominal discount rate, to be defined below in Equation (66). As real variables are detrended, Equation (37) includes the rate of growth of the global trend between t and τ.

The first-order condition is

$$0 = \left[1 - \theta_N \left(\frac{P_t(n) - MC_t(n)}{P_t(n)}\right)\right] \left(\frac{P_t(n)}{P_{N,t}}\right)^{-\theta_N} (1 - G_{PN,t}(n))$$

$$- [P_t(n) - MC_t(n)] \left(\frac{P_t(n)}{P_{N,t}}\right)^{-\theta_N} \frac{\partial G_{PN,t}}{\partial P_t(n)}$$

$$- E_t D_{t,t+1} g_{t,t+1} [P_{t+1}(n) - MC_{t+1}(n)]$$

$$\times \left(\frac{P_{t+1}(n)}{P_{N,t+1}}\right)^{-\theta_N} \frac{N_{t+1}}{N_t} \frac{\partial G_{PN,t+1}}{\partial P_t(n)}, \tag{38}$$

where

$$\frac{\partial G_{PN,t}}{\partial P_t(n)} = \phi_{PN} \left(\frac{P_t(n)/P_{t-1}(n)}{\pi_{N,t-1}} - 1\right) \frac{1}{\pi_{N,t-1}} \frac{1}{P_{t-1}(n)}, \tag{39}$$

$$\frac{\partial G_{PN,t+1}}{\partial P_t(n)} = -\phi_{PN} \left(\frac{P_{t+1}(n)/P_t(n)}{\pi_{N,t}} - 1\right) \frac{P_{t+1}(n)/P_t(n)}{\pi_{N,t}} \frac{1}{P_t(n)}. \tag{40}$$

Because marginal costs are symmetric across nontradables producers, say $MC_t(n) = MC_{Nt}$, firms n charge the same equilibrium price $P(n) = P_N$. The first-order condition can therefore be simplified as

$$0 = \{[P_{Nt}(1 - \theta_N) + \theta_N MC_{Nt}](1 - G_{PN,t}(n))\}$$

$$- \left\{[P_{Nt} - MC_{Nt}] \frac{\partial G_{PN,t}}{\partial P_t(n)} P_{Nt}\right\}$$

$$- \left\{E_t D_{t,t+1} g_{t,t+1} [P_{Nt+1} - MC_{Nt+1}] \frac{N_{t+1}}{N_t} \frac{\partial G_{PN,t+1}}{\partial P_t(n)} P_{Nt}\right\}. \tag{41}$$

The right-hand side of the previous equation consists of three expressions in curly brackets. When prices are fully flexible ($G_{PN} = 0$), only the first expression matters and the optimization problem collapses to the standard markup rule:

$$P_{Nt} = \frac{\theta_N}{\theta_N - 1} MC_{Nt}, \tag{42}$$

where the gross markup is a negative function of the elasticity of input substitution. Deviations from markup pricing occur if firms are penalized for modifying their prices in the short term. The speed of adjustment in response to shocks depends on the trade-off between current costs (second expression in curly brackets) and future expected costs (third expression), making the price-setting process forward looking.

We can now return to the standard notation in terms of relative prices: the optimization problem can be written as

$$\max_{p_t(n)} TREND_t E_t \sum_{\tau=t}^{\infty} D_{t,\tau} g_{t,\tau} \pi_{t,\tau} [p_\tau(n) - mc_\tau(n)]$$

$$\left(\frac{p_\tau(n)}{p_{N,\tau}}\right)^{-\theta_N} N_\tau(1 - \Gamma_{PN,\tau}(n)), \tag{43}$$

where the adjustment costs Γ are now expressed as a function of relative prices:

$$\Gamma_{PN,t}(n) \equiv \frac{\phi_{PN}}{2}\left(\pi_t \frac{p_t(n)/p_{t-1}(n)}{\pi_{N,t-1}} - 1\right)^2. \tag{44}$$

Note that $\Gamma_{PN,t} = G_{PN,t}$, $p_t(n)\partial\Gamma_{PN,t}/\partial p_t(n) = P_t(n)\partial G_{PN,t}/\partial P_t(n)$ and $p_t(n)\partial\Gamma_{PN,t+1}/\partial p_t(n) = P_t(n)\partial G_{PN,t+1}/\partial P_t(n)$. The first-order condition is then

$$\begin{aligned}
0 =& (1 - \Gamma_{PN,t}(n))[p_t(n)(1 - \theta_N) + \theta_N mc_t(n)] \\
&- [p_t(n) - mc_t(n)] \frac{\partial \Gamma_{PN,t}}{\partial p_t(n)} p_t(n) \\
&- E_t D_{t,t+1} \pi_{t,t+1} g_{t,t+1} [p_{t+1}(n) - mc_{t+1}(n)] \frac{N_{t+1}}{N_t} \frac{\partial \Gamma_{PN,t+1}}{\partial p_t(n)} p_t(n).
\end{aligned} \tag{45}$$

Discussion

Note that when θ is very large, the first-order condition is approximately solved by $p_t(n) \approx mc_t(n)$ regardless of how sizable ϕ_{PN} is. This implies that in a competitive economy (large θ_N) prices must move in tandem with the shocks affecting marginal costs, even though such flexibility entails large adjustment costs. Instead, if price setters have strong monopoly power (θ_N is close to one, its minimum value), they can charge a high average markup over marginal costs. In this case, when marginal costs increase owing to cyclical conditions, firms find it optimal to maintain relatively stable prices and absorb the change in production costs through a markup squeeze. In other words, when θ_N is small, firms are able to keep their prices well above marginal costs and accommodate changes in demand through supply adjustments, without corresponding changes in prices. Other things being equal, an increase in θ_N reduces firms' ability to use markup fluctuations as a shock absorber.

It can be useful to express the pricing equation above as a first-order approximation around the steady state where $\Gamma_{PN,t} = \partial\Gamma_{PN,t}/\partial p_t(n) = \partial\Gamma_{PN,t+1}/\partial p_t(n) = 0$, $\pi_{Nt} = \pi_{Nt-1} = \pi_{Nt+1} = \pi$, $D_{t,t+1}\pi_{Nt+1}g_{t,t+1} \equiv 1/(1+r)$ and $p(n) = \theta_N/(\theta_N - 1)mc(n)$. Also for simplicity consider the case in which there is zero growth in steady state or $g = 1$, so that $1/(1+r) = \beta$. Defining $y_{Nt} \equiv mc_t(n)/p_t(n)$, we can rewrite (45) as

$$\begin{aligned} 0 = &\{(1 - \theta_N + \theta_N y_{Nt})(1 - \Gamma_{PN,t}(n))\} \\ &- \left\{(1 - y_{Nt})\frac{\partial\Gamma_{PN,t}}{\partial p_t(n)}p_t(n)\right\} \\ &- \left\{E_t D_{t,t+1}g_{t,t+1}\pi_{t,t+1}(1 - y_{Nt+1})\frac{N_{t+1}}{N_t}\frac{\partial\Gamma_{PN,t+1}}{\partial p_t(n)}p_t(n)\right\}. \end{aligned} \tag{46}$$

Now linearize in the neighborhood of the steady state:

$$\begin{aligned} 0 = &\theta_N dy_{Nt} - \frac{1}{\theta_N}\phi_{PN}\left(\frac{d\pi_{Nt} - d\pi_{Nt-1}}{\pi}\right) \\ &+ E_t\beta\frac{1}{\theta_N}\phi_{PN}\left(\frac{d\pi_{Nt+1} - d\pi_{Nt}}{\pi}\right). \end{aligned} \tag{47}$$

Define $\widehat{\pi_N}_t = d\pi_{Nt}/\pi$ and $\widehat{y_N t} = dy_{Nt}/y_N$. Obtain

$$\begin{aligned} (\theta_N - 1)\hat{y}_t = &\frac{1}{\theta_N}\phi_{PN}(\widehat{\pi_N}_t - \widehat{\pi_N}_{t-1}) \\ &- \frac{1}{\theta_N}\phi_{PN}E_t\beta(\widehat{\pi_N}_{t+1} - \widehat{\pi_N}_t), \end{aligned} \tag{48}$$

which can be rewritten as a log-linear Phillips curve with full indexation, an expression that relates changes in inflation to expected changes in inflation and real marginal costs:

$$\Delta\widehat{\pi_N}_t = \gamma\widehat{y_N}_t + \beta E_t\Delta\widehat{\pi_N}_{t+1}, \quad \gamma \equiv \frac{(\theta_N - 1)\theta_N}{\phi_{PN}}. \tag{49}$$

Similar considerations apply to the tradables sector.

Notice that markup/monopoly power in our setup (θ) directly affects the slope of the Phillips curve γ. The important implication is that a reduction of monopoly power in GEM (higher θ) makes the Phillips curve steeper and reduces the sacrifice ratio faced by the economy. Similar considerations apply to an increase in price flexibility (lower ϕ_{PN}).

Variants of the model can exploit alternative assumptions about the degree of indexation and inflation inertia in the model, simply by modifying the denominator of the term in brackets in (36) or (44). For instance, a model in which

$$\Gamma_{PN,t}(n) \equiv \frac{\phi_{PN}}{2}\left(\pi_t \frac{p_t(n)/p_{t-1}(n)}{\pi^{\alpha}_{N,t-1}\pi^{1-\alpha}_t} - 1\right)^2, \tag{50}$$

implies weighted indexation with respect to past sectoral inflation and current economy-wide inflation, and the resulting Phillips curve is characterized by asymmetries between forward- and backward-looking components of the inflation process:

$$\begin{aligned}\widehat{\pi_{N}}_t &= \alpha\widehat{\pi_{N}}_{t-1} + (1-\alpha)\hat{\pi}_t + \gamma\widehat{y_N}_t \\ &\quad + \beta E_t(\widehat{\pi_N}_{t+1} - \alpha\widehat{\pi_N}_t - (1-\alpha)\hat{\pi}_{t+1}), \quad \gamma \equiv \frac{(\theta_N - 1)\theta_N}{\phi_{PN}}.\end{aligned} \tag{51}$$

Price Setting in the Tradables Sector

In this section we only consider optimal price setting for domestic firms selling in the domestic market, and we abstract from the role of the distribution sector. Later we consider the price-setting problem faced by exporters and consider a variant of the model encompassing distribution services.

The analysis is similar to the nontradables sector above. Adopting a self-explanatory notation, the price-setting problem of firm h at time t can be characterized as follows:

$$\begin{aligned}&\max_{p_t(h)} TREND_t E_t \sum_{\tau=t}^{\infty} D_{t,\tau}\pi_{t,\tau}g_{t,\tau}[p_\tau(h) - mc_\tau(h)] \\ &\left(\frac{p_\tau(h)}{p_{Q,\tau}}\right)^{-\theta_T} Q_\tau(1 - \Gamma_{PQ,\tau}(h)),\end{aligned} \tag{52}$$

and the first-order condition in a symmetric equilibrium with $p(h) = p_Q$ is

$$\begin{aligned}0 =& (1 - \Gamma_{PQ,t}(h))[p_t(h)(1-\theta_T) + \theta_T mc_t(h)] \\ & - [p_t(h) - mc_t(h)]\frac{\partial\Gamma_{PQ,t}}{\partial p_t(h)}p_t(h) \\ & - E_t D_{t,t+1}\pi_{t,t+1}g_{t,t+1}[p_{t+1}(h) - mc_{t+1}(h)]\frac{Q_{t+1}}{Q_t}\frac{\partial\Gamma_{PQ,t+1}}{\partial p_t(h)}p_t(h).\end{aligned} \tag{53}$$

Consumer Preferences

In each country there is a continuum of households indexed by $j \in [0, s]$, the same index of labor inputs. Some households have access to capital markets, some do not. The latter finance their consumption by relying exclusively on their labor incomes. We refer to the first type as "Ricardian" or "forward-looking" (*FL*). We refer to the second type as "non-Ricardian" or "liquidity-constrained" (*LC*). The two types of households can also be heterogeneous in the labor market, as discussed above.

For each household *j*, we denote with $\mathcal{W}_t(j)$ the lifetime expected utility and specify its preferences as

$$\mathcal{W}_t(j) \equiv TREND_t E_t \sum_{\tau=t}^{\infty} \beta_{t,\tau} g_{t,\tau}^{1-\sigma} u_\tau(C_\tau(j), \ell_\tau(j)), \tag{54}$$

where the instantaneous felicity is a function of (detrended) consumption C and labor effort ℓ:

$$u_t(C_t(j), \ell_t(j)) = \frac{Z_U}{1-\sigma} \left[C_t(j) - b_c \frac{C_{j,t-1}}{g_{t-1,t}} - \frac{Z_V}{1+\zeta} \frac{(\ell_t(j) - b_\ell \ell_{j,t-1})^{1+\zeta}}{(1-b_\ell)^\zeta} \right]^{1-\sigma}. \tag{55}$$

In the expressions above, $\beta_{t,\tau}$ is the discount rate between time t and time τ, possibly time-varying and different across countries. In steady state $\beta_{t,\tau}$ converges to $\beta_{SS}^{\tau-t}$ where β_{SS} is a constant.

The term $g_{t,\tau}^{1-\sigma}$ in (54) implies that the disutility of labor effort increases with the common trend, an assumption required to guarantee balanced growth. The implicit assumption is that technological progress associated with home production activities follows the same trend as the effectiveness of labor in manufacturing production. The restriction $lim_{\tau\to\infty} \beta_{t,\tau} g_{t,\tau}^{1-\sigma} < 1$ is imposed to ensure that utility is bounded.

The parameter σ in (55), which affects the curvature of consumption utility, is the reciprocal of the elasticity of intertemporal substitution. The parameter ζ, which affects the curvature of labor disutility, is the reciprocal of the Frisch elasticity of labor supply.

There is habit persistence in consumption with coefficient $0 < b_c < 1$. The term $C_{j,t-1}$ in (55) is past per capita consumption of household *j*'s peers (that is, either forward-looking or liquidity-constrained agents). Similarly, there is habit persistence in leisure with coefficient $0 < b_\ell < 1$. Households' preferences are therefore symmetric within their respective categories but, because of different reference groups in habit formation, they are not symmetric across categories.

The marginal utilities of consumption and leisure are

$$\frac{\partial u_t(j)}{\partial C_t(j)} = Z_U\left[C_t(j) - b_c\frac{C_{j,t-1}}{g_{t-1,t}} - \frac{Z_V}{1+\zeta}\frac{(\ell_t(j) - b_\ell \ell_{j,t-1})^{1+\zeta}}{(1-b_\ell)^\zeta}\right]^{-\sigma}, \tag{56}$$

$$-\frac{\partial u_t(j)}{\partial \ell_t(j)} = Z_U\left[C_t(j) - b_c\frac{C_{j,t-1}}{g_{t-1,t}} - \frac{Z_V}{1+\zeta}\frac{(\ell_t(j) - b_\ell \ell_{j,t-1})^{1+\zeta}}{(1-b_\ell)^\zeta}\right]^{-\sigma} \times Z_V\left(\frac{\ell_t(j) - b_\ell \ell_{j,t-1}}{1-b_\ell}\right)^\zeta \tag{57}$$

and the marginal rate of substitution is

$$MRS_t(j) = -\frac{\partial u_t(j)/\partial \ell_t(j)}{\partial u_t(j)/\partial C_t(j)} = Z_V\left(\frac{\ell_t(j) - b_\ell \ell_{j,t-1}}{1-b_\ell}\right)^\zeta \tag{58}$$

Discussion

The specification of the utility function builds on Greenwood, Herkowitz, and Huffman (1988). The main reason underlying the choice of this parameterization relatively to alternative options (such as additive separability or Cobb-Douglas aggregators) is that it generates relatively high volatility in consumption and countercyclical trade balances, consistent with empirical stylized facts. This is because hours worked are determined exclusively by the real wage (see (71) below after accounting for (58)), leading to a direct link between fluctuations in labor effort and consumption growth.

The terms Z_U and Z_V can be modeled as positive parameters or autoregressive processes. Note that Z_V is normalized so that in a steady state with $\ell_t(j) = \ell_{j,t-1}$, the marginal rate of substitution is independent of habit persistence.

Budget Constraint (Forward-Looking Households)

The individual flow budget constraint for Ricardian agent $j \in [0, (1-s_{LC})s]$ is

$$\begin{aligned} B_t(j) + \varepsilon_t B_t^*(j) \leqslant {} & (1+i_{t-1})\frac{B_{t-1}(j)}{\pi_{t-1,t}g_{t-1,t}} \\ & + (1+i_{t-1}^*)[1-\Gamma_{B,t-1}]\frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^* g_{t-1,t}} \\ & + (1-\tau_K)r_t K_t(j) + (1-\tau_L)w_t(j)\ell_t(j)(1-\Gamma_{W,t}(j)) \\ & - C_t(j) - p_{E,t}I_t(j) + \phi_t(j) - TT_t(j). \end{aligned} \tag{59}$$

Households hold two nominal bonds, one denominated in domestic currency and one denominated in an international currency. We will refer to the country issuing the international currency as the "center." In terms of our notation, $B_t*(j)$ is holdings of the domestic bond by household j, expressed in terms of domestic consumption units, $B_t(j)$ is holdings of the international bond, expressed in terms of center consumption units, and ε_t is the CPI-based real exchange rate, expressed as the price of one center consumption basket in terms of domestic consumption. If the domestic currency is also the international currency, ε is equal to 1. Below, when we introduce explicit country indices, $\varepsilon^{H,J}$ is the price of one consumption basket in country J in terms of country H's consumption baskets, and similarly $\varepsilon^{H,*}$ is the bilateral real exchange rate of country H relative to the center.

The short-term nominal rates i_t and i_t^* are paid at the beginning of period $t+1$ and are known at time t. The two rates are directly controlled by their respective national governments, so that i is the onshore rate in the center country. The spread between the onshore rate paid in the center country and the offshore rate received by domestic investors is denoted Γ_B. Only the center-currency bond is traded internationally and is in zero net supply worldwide. The domestic bond is in zero net supply at the domestic level (although later we consider a model variant encompassing public debt).

Agents who take a position in the international bond market must deal with financial intermediaries who charge a transaction fee Γ_B on sales/purchases of the international bond. The presence of the financial friction Γ_B guarantees that international net asset positions follow a stationary process and the economies converge asymptotically to a well-defined steady state. This transaction cost is a function of the average net asset position of the whole economy. Specifically, we adopt the following functional form:

$$1 - \Gamma_{B,t} = \left(1 - \phi_{B1} \frac{\exp(\phi_{B2}[\varepsilon_t B_t^*/GDP_t - b^*_{FDES}]) - 1}{\exp(\phi_{B2}[\varepsilon_t B_t^*/GDP_t - b^*_{FDES}]) + 1} - Z_{B,t}\right) \frac{\beta^*_{t-1,t}}{\beta_{t-1,t}}, \quad (60)$$

where $0 \le \phi_{B1} \le 1$, $\phi_{B2} > 0$, and $\varepsilon_t B^* \equiv (1/s)\varepsilon_t \int_0^{s(1-s_{LC})} B^*(j)dj$ represents the per capita net asset position of the country in consumption units. The term b^*_{FDES} is the "desired" net asset position of the country expressed as a ratio of GDP. This variable measures the degree of international exposure that financial intermediaries consider appropriate for the economy, based on their assessment of the global economic outlook.

To understand the role played by Γ_B, suppose first that $b^*_{FDES} = Z_B = 0$ and $\beta^* = \beta$. In this case, when the net asset position of the country is equal to its "desired" level of zero, it must be the case that $\Gamma_B = 0$ and the return on the international bond is equal to $1 + i*$. If the country is a net creditor, worldwide Γ_B rises above zero, implying that the country's households lose an increasing fraction of their international bond returns to financial intermediaries. When holdings of the international bond go to infinity, the return on the international bond approaches $(1+i^{*})(1-\phi_{B1})$. By the same token, if the country is a net debtor worldwide, Γ_B falls from zero to $-\phi_{B1}$, implying that households pay an increasing intermediation premium on their international debt. When net borrowing goes to infinity, the cost of borrowing approaches $(1+i^{*})(1+\phi_{B1})$. In nonlinear applications of GEM the parameter ϕ_{B2} controls the flatness of the Γ_B function: if $\phi_{B2}=0$ then $\Gamma_B=0$ regardless of the net asset position; if ϕ_{B2} tends to infinity then $1-\Gamma_B=(1-\phi_{B1})$ for any arbitrarily small net lending position, and $1-\Gamma_B=(1+\phi_{B1})$ for any arbitrarily small net borrowing position. An appropriate parameterization allows the model to generate realistic dynamics for net asset positions and current account.

Consider now the other components of (60). The term b^{*}_{FDES} can be positive or negative. The above considerations are still valid after reinterpreting the concepts of "net creditor" or "net borrower" in terms of deviations from the desired levels.

The variable $Z_{B,t}$ can be modeled as a stochastic process with zero mean in steady state, provided that fluctuations in Z_B are not large enough to push Γ_B above 1. In our framework, uncertainty in international financial intermediation plays the same role that "uncovered interest parity shocks" or risk-premium fluctuations play in similar open-economy models.

Finally, when rates of time preference diverge across countries and $\beta*\neq\beta$, the transaction cost is appropriately modified to account for asymmetries in real interest rates across countries, as in Faruqee and others (2007).

Let us consider now the remaining components in the budget constraint. Households accumulate physical capital which they rent to domestic firms at the after-tax rate $r(1-\tau_K)$. Gross investment before depreciation is denoted I. The law of motion of capital is

$$K_{t+1}(j)g_{t,t+1} = (1-\delta)K_t(j) + \Gamma_{I,t}(j)K_t(j), \quad 0<\delta\leq 1, \tag{61}$$

where δ is the country-specific depreciation rate of capital. Capital accumulation is subject to adjustment costs: $\Gamma_I(.)$ is an increasing, concave, and twice-continuously differentiable function of the investment/capital ratio $I_t(j)/K_t(j)$ with two properties entailing no adjustment costs in steady state: $\Gamma_I(\delta+g-1)=\delta+g-1$ and $\Gamma'_I(\delta+g-1)=1$. The specific functional form we adopt is quadratic and encompasses inertia in investment:

$$\Gamma_{I,t}(j) \equiv \frac{I_t(j)}{K_t(j)}(1+Z_{I,t}) - \frac{\phi_{I1}}{2}\left(\frac{I_t(j)}{K_t(j)} - (\delta+g-1)\right)^2 - \frac{\phi_{I2}}{2}\left(\frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}}\right)^2, \tag{62}$$

where ϕ_{I1}, $\phi_{I2} \geq 0$, $Z_{I,t}$ is a transitory shock (modeled as a negative adjustment cost) and g is the steady-state growth rate.

Labor incomes $w\ell$ are taxed at the rate τ_L. Each FL-type household is the monopolistic supplier of a specific labor input and sets the nominal wage for its labor input j accounting for its demand $\ell(j) = (w(j)/w)^{-\psi_L}\ell$. There is sluggish wage adjustment due to resource costs that are measured in terms of the total wage bill. The adjustment cost is denoted Γ_{WFL} (for Wage Forward-Looking) and its specification is the analog of (44) above:

$$\Gamma_{WFL,t}(j) \equiv \frac{\phi_{WFL}}{2}\left(\pi_t \frac{w_t(j)/w_{t-1}(j)}{\pi_{W,t-1}} - 1\right)^2, \tag{63}$$

where π_W is the wage inflation rate.

Ricardian households own all domestic firms and there is no international trade in claims on firms' profits. The variable Φ includes all dividends accruing to shareholders, plus all revenue from nominal and real adjustment rebated in a lump-sum way to all Ricardian households, plus revenue from financial intermediation which is assumed to be provided by domestic firms exclusively. A formal definition of Φ is given below in Equation (119).

Finally, agents pay lump-sum (nondistortionary) net taxes TT denominated in consumption units.

Discussion

In GEM it is assumed that all intermediation firms are owned by the country's residents, and that their revenue is rebated to domestic households in a lump-sum fashion. A simple variant of the model in which intermediation firms are owned by foreign residents leaves the basic results virtually unchanged. There are no intermediation costs for center residents entering the international bond market, that is, there is no difference between onshore and offshore center interest rates. Note that the choice of currency denomination of the international bond is arbitrary, and any available country currency is viable.

Both desired (b^*_{FDES}) and actual ($\varepsilon B^*/GDP$) net asset positions converge over the long term to their steady-state value $b^*_{F,SS}$.

Consumer Optimization (Forward-Looking Households)

The representative Ricardian household chooses bond holdings, capital and consumption paths, and sets wages to maximize its expected lifetime utility (54) subject to (59) and (61), taking into account (22) (or (34) if there are different types of labor).

For expositional convenience, it is worthwhile to write explicitly the maximization problem of agent $j \in [0, (1-s_{LC})s]$ in terms of the following Lagrangian:

$$\max_{C_t(j), I_t(j), B_t(j), B_t^*(j), K_{t+1}(j), w_t(j)}$$

$$TREND_t E_t \sum_{\tau=t}^{\infty} \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \{u(C_\tau(j), w_\tau(j)^{-\psi_L} w_\tau^{\psi_L} \ell_\tau)$$

$$+ \mu_\tau(j)(-B_\tau(j) - \varepsilon_\tau B_\tau^*(j) + \frac{(1+i_{\tau-1})B_{\tau-1}(j)}{\pi_{\tau-1,\tau} g_{\tau-1,\tau}}$$

$$+ \frac{(1+i_{\tau-1}^*)(1-\Gamma_{B,\tau-1})\varepsilon_\tau B_{\tau-1}^*(j)}{\pi_{\tau-1,\tau}^* g_{\tau-1,\tau}}$$

$$+ (1-\tau_L) w_\tau(j)^{1-\psi_L} w_\tau^{\psi_L} \ell_\tau (1 - \Gamma_{W,\tau}[w_\tau(j), w_{\tau-1}(j)])$$

$$+ (1-\tau_K) r_\tau K_\tau(j) - C_\tau(j) - p_{E,\tau} I_\tau(j) + \Phi_\tau(j) - TT_\tau(j))$$

$$+ \lambda_\tau(j)(-K_{\tau+1}(j) g_{\tau,\tau+1} + (1-\delta) K_\tau(j)$$

$$+ \Gamma_{I,\tau}[I_\tau(j)/K_\tau(j)] K_\tau(j))\}, \quad (64)$$

where μ and λ are the multipliers associated with, respectively, the budget constraint and the capital accumulation process.

The first-order conditions with respect to $C_t(j)$ and $I_t(j)$ yield

$$\mu_t(j) = \partial u_t(j)/\partial C_t(j) = \lambda_t(j) \Gamma'_{\mathrm{I},t}(j)/p_{E,t}. \quad (65)$$

In a symmetric setup, $\partial u_t(j)/\partial C_t(j)$ is the same across Ricardian agents j. Their stochastic discount rate and pricing kernel is therefore the variable $D_{t,\tau}$, which is defined as

$$D_{t,\tau} \equiv \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \frac{\mu_\tau}{\mu_t} \frac{1}{\pi_{t,\tau}} \frac{1}{g_{t,\tau}}. \quad (66)$$

Accounting for the above expressions, the first-order conditions with respect to $B_t(j)$ and $B_t(j)$ are, respectively,

$$1 = (1+i_t) E_t D_{t,t+1}, \quad (67)$$

$$1 = (1+i_t^*)(1-\Gamma_{B,t}) E_t (D_{t,t+1} \Delta_{t,t+1}), \quad (68)$$

where Δ denotes the rate of nominal exchange rate depreciation against the center country, or

$$\Delta_{t,\tau} = \frac{\varepsilon_\tau}{\varepsilon_t} \frac{\pi_{t,\tau}}{\pi^*_{t,t}}. \tag{69}$$

The first-order condition with respect to $K_{t+1}(j)$ is

$$\begin{aligned} \frac{p_{E,t}}{\Gamma'_{I,t}(j)} E_t g_{t,t+1} = & E_t \Big\{ D_{t,t+1} \pi_{t,t+1} g_{t,t+1} \big((1-\tau_K) r_{t+1} \\ & + \frac{p_{E,t+1}}{\Gamma'_{I,t+1}(j)} \Big[1 - \delta + \Gamma_{I,t+1}(j) \\ & - \Gamma'_{I,t+1}(j) \frac{I_{t+1}(j)}{K_{t+1}(j)} \Big] \big) \Big\}. \end{aligned} \tag{70}$$

Expression (70) links capital accumulation to the behavior of the after-tax price of capital $(1-\tau_K)r$. In a nonstochastic steady state $1+(1-\tau_K)r/p_E$ is equal to the sum of the natural real rate g^σ/β and the rate of capital depreciation δ.

Finally, taking the first-order condition with respect to $w(j)$, the Ricardian household's wage rate is set according to

$$\begin{aligned} \psi_L MRS_t(j) \frac{1}{w_t(j)} = & (\psi_L - 1)[1 - \Gamma_{WFL,t}(j)](1-\tau_L) \\ & + \frac{\partial \Gamma_{WFL,t}(j)}{\partial w_t(j)} w_t(j)(1-\tau_L) \\ & + E_t D_{t,t+1} \pi_{t,t+1} g_{t,t+1} \frac{w_{t+1}(j)}{w_t(j)} \\ & \times \frac{(w_{t+1}(j)/w_{t+1})^{-\psi_L}}{(w_t(j)/w_t)^{-\psi_L}} \frac{\ell_{t+1}}{\ell_t} \\ & \times \frac{\partial \Gamma_{WFL,t+1}(j)}{\partial w_t(j)} w_t(j)(1-\tau_L), \end{aligned} \tag{71}$$

where MRS has been defined in (58) above. The interpretation of (71) is similar to (112) above. In a nonstochastic steady state the real wage $w(j)$ is equal to the marginal rate of substitution between consumption and leisure, $MRS = -u_\ell/u_c$, augmented by the markup $\psi_L/(\psi_L-1)$, which reflects monopoly power in the labor market. For an analysis of wage rigidities in open-economy general equilibrium models, see Corsetti and Pesenti (2001).

Discussion

In a nonstochastic steady state (67) implies $(1+i_{SS})/\pi_{SS}=g_{SS}^{\sigma}/\beta_{SS}$: recall that π_{SS} is the (gross steady-state quarterly) inflation rate, $(1+i_{SS})/\pi_{SS}$ is the equilibrium real interest rate, g_{SS} is the (gross steady-state quarterly) rate of growth of the world economy, $1/\beta_{SS}$ is the rate of time preference, and $g_{SS}^{\sigma}/\beta_{SS}$ is the steady-state "natural" real interest rate of the economy. International differences in natural rates can arise from asymmetric rates of time preference. The financial friction Γ_B in (60) is appropriately adjusted to take into account these asymmetries.

Expressions (67) and (68) yield the risk-adjusted uncovered interest parity, recalling that the return on international bond holdings is modified to account for the costs of intermediation Γ_B. In steady state the interest differential $(1+i_{SS})/[(1+i_{SS}*)(1-\Gamma_{B,SS})]$ is equal to the steady-state nominal depreciation rate of the currency vis-à-vis the United States, and relative purchasing power parity holds.

Note that the expectation operator on the left-hand side of (70) is needed as shocks to the trend $g_{t,t+1}$ are not part of the information set at time t. This is because variables are expressed as deviations from the current trend. An alternative specification which expresses variables as deviations from the lagged trend would make little difference.

When the two types of households also represent different types of labor inputs with different elasticities, the first-order condition (71) is replaced by

$$\begin{aligned}\psi_{FL}MRS_t(j)\frac{1}{w_t(j)} =&(\psi_{FL}-1)[1-\Gamma_{WFL,t}(j)](1-\tau_L)\\ &+\frac{\partial\Gamma_{WFL,t}(j)}{\partial w_t(j)}w_t(j)(1-\tau_L)\\ &+E_tD_{t,t+1}\pi_{t,t+1}g_{t,t+1}\frac{w_{t+1}(j)}{w_t(j)}\\ &\times\frac{(w_{FL,t+1}/w_{t+1})^{-\psi_L}}{(w_{FL,t}/w_t)^{-\psi_L}}\frac{\ell_{FL,t+1}}{\ell_{FL,t}}\\ &\times\frac{\partial\Gamma_{WFL,t+1}(j)}{\partial w_t(j)}w_t(j)(1-\tau_L).\end{aligned}\tag{72}$$

A variant of the model considers the role of money. Define as $\mathcal{M}$ the stock of real money balances held by household j. The budget constraint (59) becomes

$$\mathscr{M}_t(j) + B_t(j) + \varepsilon_t B_t^*(j) \leq \mathscr{M}_{t-1}(j) + (1 + i_{t-1}) \frac{B_{t-1}(j)}{\pi_{t-1,t} g_{t-1,t}}$$
$$+ (1 + i_{t-1}^*)[1 - \Gamma_{B,t-1}] \frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^* g_{t-1,t}} + (1 - \tau_K) r_t K_t(j)$$
$$+ (1 - \tau_L) w_t(j) \ell_t(j) (1 - \Gamma_{W,t}(j))$$
$$- C_t(j)[1 + \Gamma_{S,t}(j)] - p_{E,t} I_t(j) + \Phi_t(j) - TT_t(j). \tag{73}$$

Consumption spending is subject to a proportional transaction cost Γ_S that depends on the household's money velocity v, where

$$v_t(j) \equiv \frac{C_t(j)}{\mathscr{M}_t(j)}. \tag{74}$$

A suggested functional form for the transaction cost (implying a satiation point for the demand of real balances) is

$$\Gamma_S(v_t) = \phi_{S1} v_t + \frac{\phi_{S2}}{v_t} - 2(\phi_{S1}\phi_{S2})^{1/2}. \tag{75}$$

Agents optimally choose their stock of real money holdings $\mathscr{M}$ so that at the margin shopping costs measured in terms of foregone consumption are equal to the benefits from investing in yield-bearing assets. The first-order condition with respect to $\mathscr{M}_t(j)$ is

$$1 - \Gamma'_{S,t}(j) v_t^2(j) = E_t D_{t,t+1}, \tag{76}$$

which defines real money balances $\mathscr{M}$ as a positive function of consumption and a negative function of the nominal interest rate. Other equations of the model need to be modified appropriately to account for the presence of money. For instance, the asset pricing kernel is now equal to

$$D_{t,\tau} \equiv \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \frac{\mu_\tau}{\mu_t} \frac{1}{\pi_{t,\tau}} \frac{1}{g_{t,\tau}} \frac{1 + \Gamma_{S,t}(j) + \Gamma'_{S,t}(j) v_t(j)}{1 + \Gamma_{S,\tau}(j) + \Gamma'_{S,t}(j) v_\tau(j)}, \tag{77}$$

and the government budget constraint is modified so that seigniorage revenue is rebated in a lump-sum fashion through net transfers.

Consumer Optimization (Liquidity-Constrained Households)

As liquidity-constrained households have no access to capital markets, their optimal choices are confined to labor supply. Similar to the Ricardian households, they can optimally set their wages to exploit their market power. Also similar to the Ricardian households, they face adjustment costs for their wages. These costs are denoted $\Gamma_{WLC,t}$ (for wage liquidity constrained) and are similar to (63). Different from the Ricardian households, however, their optimal choices are purely static and do not entail forward-looking components.

The maximization problem of agent $j \in ((1-s_{LC})s, s]$ can be written in terms of the following static Lagrangian:

$$\max_{C_t(j), w_t(j)} u(C_t(j), w_t^{-\psi_L}(j) w_t^{\psi_L} \ell_t) + \mu_t(j)[-C_t(j) - TT_t(j)$$
$$+ (1-\tau_L) w_t(j)^{1-\psi_L} w_t^{\psi_L} \ell_t (1 - \Gamma_{WLC,t}(j)]. \tag{78}$$

It is assumed that redistributive policies TT rebate to LC-type households the income losses associated with wage adjustment, so that their consumption level is:

$$C_t(j) = (1-\tau_L) w_t(j) \ell_t(j). \tag{79}$$

The first-order conditions with respect to $C(j)$ and $w(j)$ determines partial adjustment of wages:

$$\psi_L MRS_t(j) \frac{1}{w_t(j)} = (1-\tau_L)\Big[(\psi_L - 1)(1 - \Gamma_{WLC,t}(j)) + \frac{\partial \Gamma_{WLC,t}(j)}{\partial w_t(j)} w_t(j)\Big]. \tag{80}$$

Denoting w_{FL} the wage rate $w(j)$ that solves (71), and w_{LC} the wage rate $w(j)$ that solves (80), Equation (20) determines the wage rate for the whole economy as

$$w_t^{1-\psi_L} = s_{LC} w_{LC,t}^{1-\psi_L} + (1 - s_{LC}) w_{FL,t}^{1-\psi_L}. \tag{81}$$

Discussion

When two types of labor inputs are considered, Equation (80) is replaced by

$$\psi_{LC} MRS_t(j) \frac{1}{w_t(j)} = (1-\tau_L)\Big[(\psi_{LC} - 1)(1 - \Gamma_{WLC,t}(j)) + \frac{\partial \Gamma_{WLC,t}(j)}{\partial w_t(j)} w_t(j)\Big]. \tag{82}$$

Fiscal Policy

Public spending falls on nontradable goods, both final and intermediate. In per-capita terms, G_C is government consumption, G_I is government investment, and G_N denotes public purchases of intermediate nontradables. There are four sources of (net) tax revenue: taxes on capital income τ_K, taxes on labor income τ_L, import tariffs *tar*, and lump-sum taxes net of transfers to households *TT*. In the benchmark version of GEM the government follows a balanced budget rule:

$$0 = G_t - G_{REV,t}, \tag{83}$$

where

$$G_t \equiv G_{C,t} + p_{E,t} G_{I,t} + p_{N,t} G_{N,t}, \tag{84}$$

and G_{REV} is aggregate government revenue, to be defined below.

Discussion

Although GEM has not been designed to analyze fiscal policy issues in detail, variants of the model can be designed to provide a satisfactory quantitative assessment of budgetary dynamics. In what follows we consider a possible extension in this direction, following Faruqee and others (2007, forthcoming).

The government finances the excess of public expenditure over net taxes by issuing debt denominated in nominal currency, denoted *B* in per-capita terms. All national debt is held exclusively by domestic (Ricardian) agents. The budget constraint of the government is

$$B_t \geq (1 + i_{t-1}) \frac{B_{t-1}}{\pi_{t-1,t} g_{t-1,t}} + G_t - G_{REV,t}. \tag{85}$$

Define now the average tax rate for the economy τ as

$$\tau_t \equiv G_{REV,t} / GDP_t. \tag{86}$$

Similarly, define the deficit-to-GDP ratio as

$$\frac{DEF_t}{GDP_t} = \left(B_t - \frac{B_{t-1}}{\pi_{t-1,t} g_{t-1,t}} \right) \Big/ GDP_t. \tag{87}$$

From (85), in steady state we have

$$\begin{aligned} \frac{B_{SS}}{GDP_{SS}} &= \frac{\pi_{SS} g_{SS}}{\pi_{SS} g_{SS} - (1 + i_{SS})} \left(\frac{G_{SS}}{GDP_{SS}} - \tau_{SS} \right) \\ &= \frac{\pi_{SS} g_{SS}}{\pi_{SS} g_{SS} - 1} \frac{DEF_{SS}}{GDP_{SS}}. \end{aligned} \tag{88}$$

The previous equations define the relationships between the debt-to-GDP, average tax rate, and deficit-to-GDP ratio that are sustainable in the long term. In what follows we treat the long-run debt-to-GDP ratio as a policy parameter set by the government, and let τ_{SS} and DEF_{SS}/GDP_{SS} be determined by (88).

The government is assumed to control lump-sum taxes, trade policy parameters, τ and τ_K directly, while τ_L is endogenously determined. A possible specification for the fiscal rule for τ is

$$\tau_t = (\tau_{t-1} + \tau_t + E_t\tau_{t+1})/3 + \phi_{TAX1}\left(\frac{B_t}{GDP_t} - \phi_{TAX2}b_{TAR,t} - (1-\phi_{TAX2})\frac{B_{t-1}}{GDP_{t-1}}\right) + \phi_{TAX3}\left(\frac{DEF_t}{GDP_t} - \frac{DEF_{SS}}{GDP_{SS}}\right) + \phi_{TAX4}\left(\frac{G_t}{GDP_t} - \frac{G_{SS}}{GDP_{SS}}\right), \tag{89}$$

where b_{TAR} is the targeted debt-to-GDP ratio, a variable that converges to B_{SS}/GDP_{SS} in steady state. The tax rate is a smoothed function of past and expected future rates, adjusted upward when the current debt-to-GDP ratio is above the average of its current target and its past observed level, when the current deficit-to-GDP ratio is above its sustainable steady-state level, and when current government spending as a share of GDP is above its long-run level.

By construction, public debt is exclusively held by domestic agents, and the net asset position of the country is independent of the extent of public borrowing. This feature of the model is of course highly unrealistic. A way to enhance the realism of the simulations is by introducing a link between the desired net asset position of country H and the debt-to-GDP ratios in the world economy as follows:

$$b^{*H}_{FDES,t} = b^{*H}_{FNEUT} - \phi^{H}_{F1}\frac{B^H_t}{GDP^H_t} + \sum_{J\neq H}\phi^{J,H}_{F2}\frac{B^J_t}{GDP^J_t}. \tag{90}$$

According to the previous expression, b^{*H}_{FDES} is equal to a country-specific constant, b^{*H}_{FNEUT}, adjusted to account for changes in the debt-to-GDP ratios in either the domestic economy (B^H/GDP^H) or in the other countries in the world (B^J/GDP^J).

This specification provides a plausible (albeit judgmental) link between debt imbalances and net asset positions. When the national debt-to-GDP ratio increases, domestic agents reduce the share of foreign securities in their portfolios by selling the international bond to foreigners. By the same token, if the debt-to-GDP ratio increases in the center country, international investors would require a higher return on center securities, leading to a higher share of center assets in their portfolios or a reduction of net borrowing from the center.

Of course, our approach should be viewed only as a crude approximation to the actual determinants of cross-country spreads and interest rate premiums in response to macroeconomic imbalances, whose endogenization should be eventually incorporated in a self-contained model. It remains unclear, however, whether the final benefits of such a framework significantly outstrip the costs of incorporating the large amount of complications from which we abstract here.

Quantitatively, one could take b_{FDES^*} as a free variable and estimate the ϕ_{F1} and ϕ_{F2} parameters on the basis of empirical evidence on the link between net asset positions and debt levels. Alternatively, one could rely on cross-fertilization with respect to alternative theoretical models able to shed light on the structural determinants of these parameters. In Faruqee and others (2007), for instance, the calibration of (90) has relied on results based on the Global Fiscal Model, an overlapping-generation multicountry model developed at the International Monetary Fund.

Monetary Policy

The government controls the short-term rate i_t. Monetary policy is specified in terms of annualized interest rate rules. The specification of the interest rule is likely to change according to the nature of the simulation exercise. A benchmark specification is

$$(1 + i_t)^4 = \omega_i(1 + i_{t-1})^4 + (1 - \omega_i)(1 + i_t^{neut})^4 + \omega_1 E_t(\pi_{t-1,t+3} - \Pi_{t-1,t+3}). \quad (91)$$

The current interest rate i_t is an average of the lagged rate i_{t-1} and the current "neutral" rate i_t^{neut}, defined as

$$1 + i_t^{neut} \equiv \frac{\Pi_{t-4,t}^{0.25}(g_{t-1,t})^{\sigma}}{\beta_{t-1,t}}, \quad (92)$$

where $\Pi_{t-\tau,\,t-\tau+4}$ is the year-over-year gross CPI inflation target (either explicit or implicit) prevailing at time t for the four-quarter period between $t-\tau$ and $t-\tau+4$. This average is adjusted to account for the expected inflation gap three quarters in the future. In a steady state when all constant targets are reached it must be the case that the nominal interest rate is equal to the neutral level, equal to the product of the equilibrium "natural" real interest rate g^{σ}/β times the inflation target:

$$1 + i_{SS} = 1 + i_{SS}^{neut} = \frac{\Pi_{SS}^{0.25} g_{SS}^{\sigma}}{\beta_{SS}} = \frac{\pi_{SS} g_{SS}^{\sigma}}{\beta_{SS}}. \quad (93)$$

Discussion

The rule (91) could be modified to include policy responses to a set of other variables (such as measures of the output gap level or growth, the exchange rate, the current account, etc.) expressed as deviations from their targets. For an extension to price-level path targeting, the reader is referred to Laxton, N'Diaye, and Pesenti (2006). For an introduction to optimal monetary policy in open economies, see Corsetti and Pesenti (2005).

Market Clearing in the Domestic Economy

Maintaining international variables (including government revenue G_{REV}, which depends on import tariffs) exogenous for the time being, the model is closed by imposing the following resource constraints and market clearing conditions.

In each country, the domestic resource constraints for capital and labor are, respectively

$$\int_0^{s(1-s_{LC})} K_t(j)dj \geq \int_0^s K_t(n)dn + \int_0^s K_t(h)dh, \tag{94}$$

and

$$\ell_t(j) \geq \int_0^s \ell_t(n,j)dn + \int_0^s \ell_t(h,j)dh. \tag{95}$$

The resource constraint for the nontradable good n^H is

$$N_t(n) \geq \int_0^s N_{A,t}(n,x)dx + \int_0^s N_{E,t}(n,e)de + G_{N,t}(n), \tag{96}$$

while the tradable h can be used by domestic firms or imported by foreign firms (see below).

The final good A can be used for private or public consumption:

$$\int_0^s A_t(x)dx \geq \int_0^s C_t(j)dj + sG_{C,t} \tag{97}$$

and similarly for the investment good E:

$$\int_0^s E_t(e)de \geq \int_0^{(1-s_{LC})s} I_t(j)dj + sG_{I,t}. \tag{98}$$

Market clearing in the domestic bond market requires

$$\int_0^{s(1-s_{LC})} B_t(j)dj = sB_t, \tag{99}$$

where $B_t = 0$ in the benchmark model (see discussion above for the treatment of public debt $B_t > 0$).

IV. World Interdependencies in General Equilibrium

So far all trade-related variables have been taken as exogenous. Now we close the model by considering a multicountry general-equilibrium setting. The notation becomes slightly more complicated, as explicit country indices must be introduced. We will refer to H as the "home" country and to $J \neq H$ as one of the remaining "foreign" countries. When a double country index is considered in the case of bilateral trade variables, the first index refers to the importing (destination) country and the second index to the exporting (source) country. Multicountry applications of GEM can be found in Faruqee and others (2007, forthcoming).

Demand for Imports

The derivation of the foreign demand schedule for good h is analytically more complex but, as we show in (108) at the end of this section, it shares the same functional form as (13) and (14) above, and thus can be written as a function of the relative price of good h (with elasticity θ_T) and total foreign demand for imports.

We focus first on import demand in the consumption good sector of country H. Denote the representative firm in the consumption sector as $x^H \in [0, s^H]$. Its imports $M_A^H(x^H)$ are a CES function of baskets of goods imported from the other countries, or

$$M_{A,t}^{H}(x^{H})^{1-\frac{1}{\rho_A^H}} = \sum_{J \neq H} (b_A^{H,J})^{\frac{1}{\rho_A^H}} (M_{A,t}^{H,J}(x^H)(1 - \Gamma_{MA,t}^{H,J}(x^H)))^{1-\frac{1}{\rho_A^H}}, \qquad (100)$$

where

$$0 \leq b^{H,J} \leq 1, \quad \sum_{J \neq H} b^{H,J} = 1. \qquad (101)$$

In (100) above, ρ_A^H is country H's elasticity of substitution across exporters: the higher is ρ_A^H, the easier it is for firm x^H to replace imports from one country with imports from another. The parameters $b_A^{H,J}$ determine the composition of the import basket across countries. $M_A^{H,J}(x^H)$ denotes imports from country J by firm x^H located in country H.

The response of import volumes to changes in demand as well as their price elasticities is typically estimated to be smaller in the short term than in the long run. To model realistic import dynamics, such as the delayed and sluggish adjustment to changes in relative prices typically referred to as the "J curve," we assume that bilateral imports are subject to bilateral adjustment costs $\Gamma_{MA}^{H,J}$. These costs are specified in terms of import shares relative to firm x^H's output. They are zero in steady state. Specifically, GEM adopts the parameterization

$$\Gamma_{MA,t}^{H,J}\left[\frac{M_{A,t}^{H,J}(x^H)}{A_t^H(x^H)} \Big/ \frac{M_{A,t-1}^{H,J}}{A_{t-1}^H}\right]$$
$$= \frac{\phi_{MA}^{H,J}}{2} \frac{[(M_{A,t}^{H,J}(x^H)/A_t^H(x^H))/(M_{A,t-1}^{H,J}/A_{t-1}^H) - 1]^2}{(1 + [(M_{A,t}^{H,J}(x^H)/A_t^H(x^H))/(M_{A,t-1}^{H,J}/A_{t-1}^H) - 1]^2)}, \qquad (102)$$

with $\phi_{MA}^{H,J} \geq 0$. The specification is such that $\Gamma_{MA}^{H,J}[1] = 0$, $\Gamma_{MA}^{H,J}[\infty] = \phi_{MA}^{H,J}/2$, and $\Gamma_{MA}^{H,J}[0] = \Gamma_{MA}^{H,J}[2] = \phi_{MA}^{H,J}/4$. Alternative parameterizations (for instance, quadratic) could be considered, although the suggested one has proven to be useful in nonlinear simulation exercises with relatively large shocks.

Denoting $p_M^{H,J}$ the price in country H of a basket of intermediate inputs imported from J, firm x^H minimizes its costs $\sum_{J \neq H} p_M^{H,J} M_A^{H,J}(x^H)$ subject to (100). Cost minimization implies

$$\frac{M_{A,t}^{H,J}(x^H)(1-\Gamma_{MA,t}^{H,J}(x^H))}{(1-\Gamma_{MA,t}^{H,J}(x^H) - M_{A,t}^{H,J}(x^H)\Gamma'_{MA,t}H,J(x^H))^{\rho_A^H}}$$

$$= b_A^{H,J}\left(\frac{p_{M,t}^{H,J}}{p_{MA,t}^H(x^H)}\right)^{-\rho_A^H} M_{A,t}^H(x^H), \tag{103}$$

where $\Gamma'^{H,J}_{MA}(x^H)$ is the first derivative of $\Gamma^{H,J}_{MA}(x^H)$ with respect to $M_A^{H,J}(x^H)$ and the cost-minimizing import price index $p_{MA}^H(x^H)$ is the Lagrangian multiplier:

$$p_{MA,t}^H(x^H) = \left[\sum_{J \neq H} b^{H,J}\left(\frac{p_{M,t}^{H,J}}{1-\Gamma_{MA,t}^{H,J}(x^H) - M_{A,t}^{H,J}(x^H)\Gamma'_{MA,t}H,J(x^H)}\right)^{1-\rho_A^H}\right]^{\frac{1}{1-\rho_A^H}}. \tag{104}$$

In principle, the import price $p_{MA}^H(x^H)$ is firm-specific, as it depends on firm x^H's import shares. To the extent that all firms x^H are symmetric within the consumption sector, however, there will be a unique import price p_{MA}^H. It follows that $p_{MA}^H M_A^H = \sum_{J \neq H} p_M^{H,J} M_A^{H,J}(1-\Gamma_{MA}^{H,J})/(1-\Gamma_{MA}^{H,J} - M_A^{H,J}\Gamma_{MA}{'}^{H,J})$.

Consider now the basket $M_A^{H,J}(x^H)$ in some detail. In analogy with (10) above, it is a CES index of all varieties of tradable intermediate goods produced by firms h^J operating in country J and exported to country H. Denoting as $M_A^{H,J}(h^J, x^H)$ the demand by firm x^H of an intermediate good produced by firm h^J, the basket $M_A^{H,J}(x^H)$ is

$$M_{A,t}^{H,J}(x^H) = \left[\left(\frac{1}{s^J}\right)^{\frac{1}{\theta_T^J}} \int_0^{s^J} M_{A,t}^{H,J}(h^J, x^H)^{1-\frac{1}{\theta_T^J}} dh^J\right]^{\frac{\theta_T^J}{\theta_T^J - 1}}, \tag{105}$$

where $\theta_T^J > 1$ is the elasticity of substitution among intermediate tradables, the same elasticity entering (14) in country J.

The cost-minimizing firm x^H takes as given the prices of the imported goods $p^H(h^J)$ and determines its demand of good h^J according to

$$M_{A,t}^{H,J}(h^J, x^H) = \frac{1}{s^J}\left(\frac{p_t^H(h^J)}{p_{M,t}^{H,J}}\right)^{-\theta_T^J} M_{A,t}^{H,J}(x^H), \tag{106}$$

where $M_{A,t}^{H,J}(x^H)$ has been defined in (103) and $p_M^{H,J}$ is

$$p_{M,t}^{H,J} = \left[\left(\frac{1}{s^J}\right)\int_0^{s^J} p_t^H(h^J)^{1-\theta_T^J} dh^J\right]^{\frac{1}{1-\theta_T^J}}. \tag{107}$$

The import demand schedules in the investment good sector can be derived in perfect analogy with the analysis above. As a last step, we can derive country J's demand schedule for country H's intermediate good h^H, that is, the analog of (14). Aggregating across firms (and paying attention to the order of the country indices) we obtain

$$\int_0^{s^J} M_{A,t}^{J,H}(h^H, x^J)dx^J + \int_0^{s^J} M_{E,t}^{J,H}(h^H, e^J)de^J$$

$$= \frac{s^J}{s^H}\left(\frac{p_t^J(h^H)}{p_{M,t}^{J,H}}\right)^{-\theta_T^H}(M_{A,t}^{J,H} + M_{E,t}^{J,H}). \tag{108}$$

Discussion

Import adjustment costs $\Gamma_{MA}^{H,J}$ and $\Gamma_{ME}^{H,J}$ are treated in GEM as expenditures associated with intermediation activities (transportation, distribution, training, etc.) Thus, they show up somewhere else in the economy as revenue for the firms that provide these services, and as dividend incomes for the households who own these firms. Below, we include these components in the definition of Φ.

Variants of the model can include trade in commodities, parts, raw materials, and other ‘‘upstream’’ intermediate goods. The reader is referred to Laxton and Pesenti (2003) for a detailed algebraic treatment.

Price Setting in the Tradables Sector and Exchange Rate Pass-Through

In Section III we characterized the optimal price set by a firm producing tradable intermediate goods for the local market. We now reconsider the price-setting problem in the tradables sector from the vantage point of the firm h^H located in country H and exporting to all other countries $J \neq H$. We also introduce the distinction between import prices at the national level and at the border level. National import prices $p^J(h^H)$ are paid by firms located in country J to purchase one unit of the variety h^H. These are the prices that enter equations such as (103) above. Border import prices are indexed with a bar (for example $\bar{p}^J(h^H)$). These are the prices set by the exporting firm h^H. The difference between the two prices stems from trade barriers such as tariffs. Below we consider a more general specification of the model in which the gap between the two prices reflects distribution costs and retail margins.

In terms of our notation, we have

$$p_t^J(h^H) = (1 + tar^{J,H})\bar{p}_t^J(h^H), \tag{109}$$

where $tar^{J,H}$ is a proportional tariff duty imposed by country J over its imports from country H.

To the extent that different country blocs represent segmented markets in the global economy, each firm h^H in country H has to set different prices for the domestic market and all other export markets. Because the firm faces the same marginal costs regardless of the scale of production in each market, the different price-setting problems are independent of each other.

Exports are invoiced (and prices are set) in the currency of the destination market. Accounting for (108), the price-setting problems of firm h in country H at time t can then be characterized as follows:

$$\begin{aligned}
&\max_{\bar{p}_t^J(h^H)} TREND_t E_t \sum_{\tau=t}^{\infty} D_{t,\tau}^H \pi_{t,\tau}^H g_{t,\tau} \\
&\times[\varepsilon_\tau^{H,J}\bar{p}_\tau^J(h^H) - mc_\tau^H(h^H)]\frac{s^J}{s^H}\left(\frac{(1+tar^{J,H})\bar{p}_\tau^J(h^H)}{p_{M,\tau}^{J,H}}\right)^{-\theta_T^H} \\
&\times(M_{A,\tau}^{J,H} + M_{E,\tau}^{J,H})(1-\Gamma_{PM,\tau}^{J,H}(h)),
\end{aligned} \tag{110}$$

where $p_M^{J,H}$ is the price of the basket of country J's imports from country H, and $M_A^{J,H} + M_E^{J,H}$ is country J's aggregate imports from country H. The term $\varepsilon^{H,J}$ is the bilateral real exchange rate between country H and country J (an increase in $\varepsilon^{H,J}$ represents a real depreciation of country H's currency against country J). The term $\Gamma_{PM}^{H,J}(h^H)$ denotes adjustment costs related to changes of the price of good h^H in country J. These costs are the analogs of (44) above:

$$\Gamma_{PM,t}^{J,H}(h^H) \equiv \frac{\phi_{PM}^{J,H}}{2}\left(\pi_t^J \frac{\bar{p}_t^J(h^H)/\bar{p}_{t-1}^J(h^H)}{\bar{\pi}_{M,t-1}^{J,H}} - 1\right)^2, \tag{111}$$

where $\bar{\pi}_M$ is the inflation rate for bilateral imports prices. Despite its fastidiousness, the notation above is straightforward and the equations are self-explanatory.

Accounting for firms' symmetry ($\bar{p}^J(h^H) = {}_M^{J,H}$ and $(1 + tar_t^{J,H})\bar{p}^J(h^H) = p_M^{J,H}$ in equilibrium), profit maximization yields

$$\begin{aligned} 0 =& (1 - \Gamma_{PM,t}^{J,H}(h^H))[\varepsilon_t^{H,J}\bar{p}_t^J(h^H)(1 - \theta_T^H) + \theta_T^H mc_t^H(h^H)] \\ & - [\varepsilon_t^{H,J}\bar{p}_t^J(h^H) - mc_t^H(h^H)]\frac{\partial \Gamma_{PM,t}^{J,H}}{\partial \bar{p}_t^J(h^H)}\bar{p}_t^J(h^H) \\ & - E_t\Big\{D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1}[\varepsilon_{t+1}^{H,J}\bar{p}_{t+1}^J(h^H) - mc_{t+1}^H(h^H)] \\ & \times \left(\frac{M_{A,t+1}^{J,H} + M_{E,t+1}^{J,H}}{M_{A,t}^{J,H} + M_{E,t}^{J,H}}\right)\frac{\partial \Gamma_{PM,t+1}^{J,H}}{\partial \bar{p}_t^J(h^H)}\bar{p}_t^J(h^H)\Big\}. \end{aligned} \tag{112}$$

If adjustment costs in the export market are relatively large, the prices of country H's goods in the foreign markets are characterized by significant stickiness in local currency. In this case, the degree to which short-term exchange rate movements (and other shocks to marginal costs in country H) affect import prices in country J is rather small—as in Chari, Kehoe, and McGrattan (2002). If instead the $\phi_{PM}^{J,H}$ coefficients are small, expression (112) collapses to a markup rule, and exchange rate pass-through is full:

$$\varepsilon_t^{H,J}\bar{p}_t^J(h^H) = \varepsilon_t^{H,J}\bar{p}_{M,t}^{J,h} = \frac{\theta_T^H}{\theta_T^H - 1}mc_t^H. \tag{113}$$

If firm h^H faces small adjustment costs in all sales markets, both domestic and foreign, the law of one price holds at the border level:

$$p_{Q,t}^H = \varepsilon_t^{H,J}\bar{p}_{M,t}^{J,H} = \frac{\theta_T^H}{\theta_T^H - 1}mc_t^H. \tag{114}$$

Discussion

In the previous paragraph, low exchange rate pass-through is the result of nominal price stickiness in export prices. There is no need, however, to limit our analysis of the determinants of pass-through to the role of nominal rigidities. As an example of an alternative approach, the variant of GEM studied in Laxton and Pesenti (2003) and based on work by Corsetti and Dedola (2005) considers the role of the distribution sector. In this section we briefly summarize this extension and its implications for export prices.

Suppose that firms producing the final goods A and E in country J do not import intermediate tradables directly from the foreign producers. Instead, firms in the distribution sector purchase tradables abroad and distribute them to the firms producing the final good. The distribution technology is Leontief: to make one unit of an intermediate good available to downstream producers, firms in the distribution sector require $\eta \geq 0$ units of the nontradables basket N. Thus, total demand for nontradables (13) in country J is appropriately modified as $(p_t^J(n^J)/p_{N,t}^J)^{-\theta_N^J}(N_{A,t}^J + N_{E,t}^J + G_{N,t}^J + \eta^J \sum_{H \neq J} (M_{A,t}^{J,H} + M_{E,t}^{J,H})$.

Firms in the distribution sector are perfectly competitive. Because of distribution costs, there is a wedge between producer (border) and consumer (retail) prices even in the absence of tariffs and trade barriers. It follows that

$$p_t^J(h^H) = (1 + tar^{J,H})\bar{p}_t^J(h^H) + \eta^J p_{N,t}^J. \tag{115}$$

From the vantage point of firm h^H exporting to country J, the price-setting equation (112) then becomes

$$\begin{aligned}
0 =& (1 - \Gamma_{PM,t}^{J,H}(h^H))\Big[\varepsilon_t^{H,J}\bar{p}_\tau^J(h^H)(1 - \theta_T^H) \\
&+ \frac{\varepsilon_t^{H,J}\eta^J p_{N,t}^J}{1 + tar^{J,H}} + \theta_T^H mc_t^H(h^H)\Big] \\
&- [\varepsilon_t^{H,J}\bar{p}_t^J(h^H) - mc_t^H(h^H)]\frac{\partial\Gamma_{PM,t}^{J,H}}{\partial\bar{p}_t^J(h^H)}\Big(\bar{p}_\tau^J(h^H) \\
&+ \frac{\eta^J p_{N,t}^J}{1 + tar^{J,H}}\Big) - E_t\Big\{D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1} \\
&\times[\varepsilon_{t+1}^{H,J}\bar{p}_{t+1}^J(h^H) - mc_{t+1}^H(h^H)]\left(\frac{M_{A,t+1}^{J,H} + M_{E,t+1}^{J,H}}{M_{A,t}^{J,H} + M_{E,t}^{J,H}}\right) \\
&\times\frac{\partial\Gamma_{PM,t+1}^{J,H}}{\partial\bar{p}_t^J(h^H)}\left(\bar{p}_t^J(h^H) + \frac{\eta^J p_{N,t}^J}{1 + tar^{J,H}}\right)\Big\}.
\end{aligned} \tag{116}$$

The key implication of the presence of a distribution sector is that, even in the absence of adjustment costs, pass-through is no longer full. In fact, when the $\phi_{PM}^{J,H}$ coefficients are small, the above expression collapses to a double-markup rule:

$$\varepsilon_t^{H,J}\bar{p}_t^J(h^H) = \varepsilon_t^{H,J}\bar{p}_{M,t}^{J,H} = \frac{\theta_T^H}{\theta_T^H - 1}mc_t^H + \frac{\eta^J}{\theta_T^H - 1}\frac{\varepsilon_t^{H,J}p_{N,t}^J}{1 + tar^{J,H}}, \tag{117}$$

$$\begin{aligned}
\varepsilon_t^{H,J}p_t^J(h^H) = \varepsilon_t^{H,J}p_{M,t}^{J,H} =& \frac{\theta_T^H}{\theta_T^H - 1}mc_t^H(1 + tar^{J,H}) \\
&+ \frac{\eta^J}{\theta_T^H - 1}\varepsilon_t^{H,J}p_{N,t}^J.
\end{aligned} \tag{118}$$

Market Clearing in the World Economy

All profits and intermediation revenue accrue to Ricardian households:

$$
\begin{aligned}
&\int_0^{s^H(1-s^H_{LC})} \Phi^H_t(j^H)dj^H \\
&= \int_0^{s^H(1-s^H_{LC})} (1+i^*_{t-1})\Gamma^H_{B,t-1}\frac{\varepsilon^{H,*}_t B^{*H}_{t-1}(j^H)}{\pi^*_{t-1,t}g_{t-1,t}}dj^H \\
&+ \int_0^{s^H(1-s^H_{LC})} \Gamma^H_{WFL,t}(j^H)(1-\tau^H_{L,t})w^H_t(j^H)dj^H \\
&+ \int_{s^H(1-s^H_{LC})}^{s^H} \Gamma^H_{WLC,t}(j^H)(1-\tau^H_{L,t})w^H_t(j^H)dj^H \\
&+ \int_0^{s^H} [p^H_t(n^H) - mc^H_t(n^H)] \\
&\times\left(\int_0^{s^H} N^H_{A,t}(n^H,x^H)dx^H + \int_0^{s^H} N^H_{E,t}(n^H,e^H)de^H + G^H_{N,t}(n^H)\right)dn^H \\
&+ \int_0^{s^H} [p^H_t(h^H) - mc^H_t(h^H)] \\
&\times\left(\int_0^{s^H} Q^H_{A,t}(h^H,x^H)dx^H + \int_0^{s^H} Q^H_{E,t}(h^H,e^H)de^H\right)dh^H \\
&+ \sum_{J\neq H}\int_0^{s^H} [\varepsilon^{H,J}_t \overline{p}^J_t(h^H) - mc^H_t(h^H)] \\
&\times\left(\int_0^{s^J} M^{J,H}_{A,t}(h^H,x^J)dx^J + \int_0^{s^J} M^{J,H}_{E,t}(h^H,e^J)de^J\right)dh^H \\
&+ \sum_{J\neq H}\int_0^{s^H}\left(\frac{M^{H,J}_A(x^H)\Gamma'^{H,J}_{MA,t}(x^H)}{1-\Gamma^{H,J}_{MA,t}(x^H) - M^{H,J}_A\Gamma'^{H,J}_{MA,t}(x^H)}\right)p^{H,J}_M M^{H,J}_A(x^H)dx^H \\
&+ \sum_{J\neq H}\int_0^{s^H}\left(\frac{M^{H,J}_E\Gamma'^{H,J}_{ME,t}(e^H)}{1-\Gamma^{H,J}_{ME,t}(e^H) - M^{H,J}_E\Gamma'^{H,J}_{ME,t}(e^H)}\right)p^{H,J}_M M^{H,J}_E(e^H)de^H, \qquad (119)
\end{aligned}
$$

where $\varepsilon^{H,*}$ is the bilateral real exchange rate between country H and the center country.

It may be helpful to go through the single elements on the right-hand side. The first expression (integral) is revenue associated with financial intermediation in the bond market. The second and third expressions are revenue associated with wage adjustment by either forward-looking or liquidity-constrained agents. Note that revenue associated with price adjustment is not included here, as it is a cost for some firms and a revenue for others. The fourth expression is monopoly profits in the nontradables sector (if "entry costs" were considered, they would appear here as negative items offsetting these profits). The fifth expression is domestic monopoly profits in the tradables sector. The sixth expression is export profits.

The last two expressions in (119) are revenue associated with import adjustment, both in the consumption sector and in the investment sector. The sum of these last components is referred to as *IMPADJ* (for import adjustment) in what follows.

The tradable good h^H can be used by domestic firms or imported by foreign firms:

$$T_t(h^H) \geq \int_0^{s^H} Q_{A,t}(h^H, x^H)dx^H + \int_0^{s^H} Q_{E,t}(h^H, e^H)de^H$$
$$+ \sum_{J \neq H} \left(\int_0^{s^J} M_{A,t}^{J,H}(h^H, x^J)dx^J + \int_0^{s^J} M_{E,t}^{J,H}(h^H, e^J)de^J \right). \quad (120)$$

Market clearing in the international bond market requires

$$\sum_J \int_0^{s^J(1-s^J_{LC})} B_t^{*J}(j^J)dj^J = 0. \quad (121)$$

Finally, government revenue is given by

$$G^H_{REV,t} \equiv \frac{1}{s^H}\left(\int_0^{s^H} TT_t^H(j)dj + \tau_K^H r_t^H \int_0^{s^H(1-s^H_{LC})} K_t^H(j)dj \right.$$
$$\left. + \tau_L^H \int_0^{s^H} w_t^H(j)\ell_t^H(j)dj \right)$$
$$+ \frac{1}{s^H} \sum_{J \neq H} tar_t^{H,J} \int_0^{s^J} \bar{p}_t^H(h^J)(M_{A,t}^{H,J}(h^J) + M_{E,t}^{H,J}(h^J))dh^J. \quad (122)$$

Together with the appropriate transversality conditions, this concludes the description of the equilibrium.

Measuring Output and Trade Balance

GEM codes all quantity variables in per-capita terms. For the vast majority of the equations above the aggregation is straightforward. In this section we focus on the two most important macrovariables in the model, gross domestic product and the current account.

Define first per capita net financial wealth in country H as

$$F_t^H \equiv \frac{1}{s^H}(1+i_{t-1}^*)[1-\Gamma_{B,t-1}]\int_0^{s^{H(1-s_{LC}^H)}} \frac{\varepsilon_t^{H,*}B_{t-1}^{*H}(j^H)}{\pi_{t-1,t}^* g_{t-1,t}} dj^H. \tag{123}$$

Aggregating the budget constraints across private and public agents after imposing the appropriate transversality conditions, the law of motion for financial wealth is

$$\begin{aligned} E_t D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1} F_{t+1}^H = {} & F_t^H + \Gamma_{B,t-1}^H \frac{(1+i_{t-1}^*)\varepsilon_t^{H,*}B_{t-1}^{*H}}{\pi_{t-1,t}^* g_{t-1,t}} \\ & + p_{N,t}^H N_t^H + p_{T,t}^H T_t^H \\ & + \sum_{J \neq H} (p_{M,t}^H - \bar{p}_{M,t}^{H,J})(M_{A,t}^{H,J} + M_{E,t}^{H,J}) \\ & + IMPADJ_t^H - C_t^H - p_{E,t}^H I_t^H - G_t^H, \end{aligned} \tag{124}$$

where the total value of tradables is defined as

$$p_{T,t}^H T_t^H \equiv p_{Q,t}^H (Q_{A,t}^H + Q_{E,t}^H) + \sum_{J \neq H} \frac{s^J}{s^H} \varepsilon_t^{H,J} \bar{p}_{M,t}^{J,H} (M_{A,t}^{J,H} + M_{E,t}^{J,H}). \tag{125}$$

Recall that the variable $IMPADJ$ is the sum of the last two terms in (119). Expression (125) can be written as

$$CURBAL_t^H = \varepsilon_t^{H,*}\left(B_t^{*H} - \frac{B_{t-1}^{*H}}{\pi_{t-1,t}^* g_{t-1,t}}\right) = NFP_t^H + TBAL_t^H. \tag{126}$$

The left-hand side of (126) is country H's current account in domestic consumption units. The first term on the right-hand side is net factor payments from the rest of the world to country H:

$$NFP_t^H = \frac{i_{t-1}^* \varepsilon_t^{H,*} B_{t-1}^{*H}}{\pi_{t-1,t}^* g_{t-1,t}}. \tag{127}$$

$TBAL$ is the trade balance or net exports:

$$TBAL_t^H = EX_t^H - IM_t^H, \tag{128}$$

where total exports EX are evaluated at border prices:

$$EX_t^H = p_{T,t}^H T_t^H - p_{Q,t}^H (Q_{A,t}^H + Q_{E,t}^H), \tag{129}$$

and, similarly, total imports IM are evaluated at border prices:

$$IM_t^H = \sum_{J \neq H} \bar{p}_{M,t}^{H,J} (M_{A,t}^{H,J} + M_{E,t}^{H,J}). \tag{130}$$

Using the definition above, the model-based gross domestic product (in consumption units) is

$$\begin{aligned} GDP_t^H &= A_t^H + p_{E,t}^H E_t^H + p_{N,t}^H G_{N,t}^H + TBAL_t^H \\ &= p_{N,t}^H N_t^H + p_{T,t}^H T_t^H + \sum_{J \neq H} (p_{M,t}^H - \bar{p}_{M,t}^{H,J})(M_{A,t}^{H,J} + M_{E,t}^{H,J}) \\ &\quad + IMPADJ_t^H. \end{aligned} \tag{131}$$

Note that there is a discrepancy between GDP measured according to national accounting standards (goods output), and GDP measured as manufacturing output. This discrepancy reflects the portion of the revenue from sales of goods associated with making imports available to downstream users, such as costs incurred with imports adjustment and wedges between border- and market-prices of imported goods.

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