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The Art of Making Everybody Happy: How to Prevent a Secession

Michel Le Breton and Shlomo Weber

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The Art of Making Everybody Happy: How to Prevent a Secession

Prepared by Michel Le Breton and Shlomo Weber

Authorized for distribution by Teresa Ter-Minassian

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Abstract

In this paper we consider a model of the country with heterogeneous population and examine compensation schemes that may prevent a threat of secession by dissatisfied regions. We show that horizontal imbalances are combatable with *secession-proof* compensation schemes that entail a degree of *partial equalization*: the disadvantageous regions should be subsidized but the burden on advantageous regions should not be too excessive. In the case of uniform distribution, we establish the 50-percent compensation rule for disadvantageous regions. Thus, we argue for a limited *gap reduction* between advantageous and disadvantageous regions and show that neither *laissez faire* nor *Rawlsian allocation* is secession-proof.

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Author's E-Mail Address: Sweber@mail.smu.edu

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I. Introduction

The world political map has undergone dramatic changes since World War II. The number of independent countries in the world almost tripled over the second half of the last century, rising from a mere 74 in 1946 to 193 today; 45 percent of countries which exist today have a population under 5 million people. The abolition of colonial rule in Africa in the 1960s created twenty-five new countries. The last decade brought the next major wave of border changes, highlighted by the break-ups of the former Soviet Union, Yugoslavia, Czechoslovakia and the reunification of Germany. Russia, Yugoslavia, Indonesia, and Nigeria currently face serious secession threats by several regions. Quebec's recent secession bid was defeated by a majority of less than one percent. One can also point out Belgium's process of "federalization" and Scotland and Wales' path to devolution, as well as many separatist movements and conflicts over fiscal redistribution, regional power and autonomy that mushroom all over the globe.

Such spectacular changes have stimulated a body of research concerning the reasons behind the phenomena and the issue of efficiency with respect to the world's organization into nations. Most of the contributions focus on the trade-off between the benefits of large countries and the costs of heterogeneity in big populations. The conflict between small and large points to the fundamental trade-off between economies of scale and costs of heterogeneity. Barro (1991) puts it quite succinctly: "We can think of a country's optimal size as emerging from a trade-off: A large country can spread the cost of public goods over many taxpayers, but a large country is also likely to have a diverse population that is difficult for the central government to satisfy." Larger political jurisdictions bring about several benefits: the per capita cost of producing public goods declines with the population size of the country[1]; larger countries rely more heavily on more efficient taxes[2] and enjoy economies of scale in the utilization of computer hardware and software systems in their tax collection;[3] the size of a country's potential market is affected by the size of the jurisdiction in a world with barriers to trade;[4] larger countries are better equipped to absorb uninsurable shocks in different regions;[5] influence and security considerations may also matter.[6] In many countries,

[1] A large population of taxpayers can share the cost of public goods such as roads, a telephone network, defense, civil servants, and education. Alesina, Spolaore, and Wacziarg (1997) show that small countries tend to have bigger governments, and bigger government consumption, as a share of GDP. Smaller countries also face substantial costs of maintaining their distinctive language and culture. For example, the economic cost of Iceland's language is about 3 percent of the country GNP (*The Economist* (1998)).

[2] See Easterly and Rebello (1993).

[3] See Ter-Minassian (1997).

[4] See Friedman (1977), Casella (1992), Casella and Feinstein (1990).

[5] See Persson and Tabellini (1996a,b)

a majority of citizens do not particularly value their country's political and military might, but in some large countries, particularly China, France, Russia, India, and Pakistan, the citizens do care about their country's standing and influence in the world. As evidence of this phenomenon, Easterly and Rebello (1993) confirm that large countries spend relatively more on their defense. On the other hand, being small has its advantages: due to their relative ethnic, religious, and cultural homogeneity, decisions of its elected representatives are, on the average, more on line with the citizens' choices[7] for an analysis of countries' language and ethnic diversity; small countries are usually more open to trade,[8] and better adjust to dealing with technological changes in the world markets; and interest groups and unproductive activities play a lesser role in smaller countries.[9]

Alesina and Spolaore (1997) (henceforth AS) provide an analytical framework for the normative and positive analysis of this trade-off between large and small. They formulate the following fundamental question: What is the "optimal" number and size of countries and how does this configuration compare to the one resulting from a democratic process? AS examine a spatial model with a heterogeneous world population, where each individual consumes a public good and incurs a transportation cost proportional to the distance between her location and the location of the government in her country. Each government chooses its location by majority rule and uses a proportional tax scheme to produce a public good. AS consider various constitutional rules to describe the democratic process governing border redrawing. In particular, they examine rules B and C: under rule B, a new nation can be created, or an existing nation can be eliminated, if the approval of the majority in each of the existing countries affected by the border redrawing is obtained; rule C deals with unilateral secessions and allows for a set of citizens belonging to an existing country to create a new country by unanimously voting in favor of secession.[10] A configuration of countries is *B*-stable (*C*-stable) if it is immune to border redrawing under rule B (rule C, respectively). The main conclusion of AS is that, without lump-sum transfers within countries, the first welfare theorem would fail under rule B but not under rule C as *the efficient number of countries is not B-stable but is C-stable*. AS then turn to the issue of whether allowing lump-sum transfers would reconcile the fundamental conflict between B-stability and efficiency.[11]

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[6] See Alesina and Spolaore (1996).

[7] See Mauro (1995).

[8] See Alesina, Spolaore, and Wacziarg (1997).

[9] See Barro (1998).

[10] For additional discussion on constitutional provisions on country formation see Jéhiel and Scotchmer (1997), Bordignon and Brusco (1999) and, in the context of government formation, Diermeier and Merlo (1999).

[11] Since public goods are financed on a per capita basis, the quasi-linear utilities imply that Pareto efficiency can be viewed as an outcome of utilitarian optimization. If lump-sum equalization transfers between citizens are ruled out, the consistency of the whole analysis would require that efficiency and stability are examined under the same constraints on the set of available tax instruments. AS conclude that in this case there is no gap

(continued...)

"With an appropriate scheme of lump sum distribution, a social planner could move the equilibrium to the efficient one without making anybody worse off. This scheme would reward individuals who are located far from borders... Can individuals who are located far from the government be compensated so that they would not vote in favor of creating a number of countries greater than the efficient one? In other words, can we support an equilibrium with fewer countries than in the stable configuration using compensation schemes?"

AS point out certain difficulties with implementation of compensation schemes, such as waste in the redistributive process and the issue of commitment. These are valid arguments but the widespread use of compensation schemes[12] makes it worthwhile to examine the impact on stability caused by introducing them. For purposes of consistency in this investigation, one should assume that if lump sum transfers are available to the social planner, they are also available to a group of citizens contemplating the possibility of forming a new nation.

Under this modified notion of stability, lump sum transfers allow the social planner a wider range of options. However, lump sum transfers also extend the set of secession opportunities for new countries and, a priori, the aggregated effect of gains and losses on both sides is far from certain. Exploring this issue is the first goal of this paper. To proceed, we focus on rule C, since the appropriate redefinition of rule B would require a more elaborated analysis left for future research. Indeed, under the original rule C, the efficient number of countries is C stable, and thus there is no need for a compensation scheme to prevent a threat of secession.[13] However, when compensation schemes are available, rule C must be modified to take into account the possibility that the group of citizens contemplating a secession can also use a compensation scheme within the group to persuade the less eager to leave the country. The perspective, in a way, is now reversed: *to enforce the optimal configuration of countries, the social planner may need to use a compensation scheme in order to counterbalance a larger set of secession threats.*

Our first result shows, somewhat surprisingly, that under quite general assumptions, the social planner can design compensation schemes to accommodate these more stringent secession constraints. To better explain this result, let us examine the issue of efficiency and stability from a somewhat different angle, namely, in terms of benefits of cooperation

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between stability and *constrained* efficiency: "In fact, the stable number of countries solves the problem of a Rawlsian social planner, who maximizes the utility of the least well off individual but cannot use lump-sum redistribution."

[12] See, e.g., Ter-Minassian (1997) and the country-specific chapters in the same book.

[13] Note that AS's conclusion that the optimal configuration of countries is C-stable is derived in the case of uniform distribution and it is unclear whether this result extends to a larger class of distributions considered in this paper.

between different regions of a country. We consider two notions, *desirability* and *sustainability*. Cooperation is *desirable* if there are gains to be had from it or, in our context, all regions are better off under a single national government. The desirability of cooperation does not necessarily imply that the gains from cooperation can be allocated in such a way that no region can ensure all its citizens a higher payoff than that guaranteed by a smaller central government. If such a region exists, it may pose a threat of secession, and we call the region *secession-prone*. If it is possible to design an equalization scheme which does not generate secession-prone regions, the scheme is called *secession-proof* and cooperation is called *sustainable*. Thus, if cooperation is both desirable and sustainable, the efficient country configuration is not threatened by a secession and efficiency is reconciled with stability. Indeed, our first result suggests that, under quite general assumptions, a very costly public good eliminates the gap between sustainability and desirability and, thus, between efficiency and stability.

Our second contribution relates to the degree of equalization in a transfer scheme that would guarantee no region of the country is prone to secession. As we already pointed out, "disadvantageous" regions of a country may be vulnerable to secessionist threats in the absence of equalization transfers. Separatist movements represent a serious risk to a country's political stability and territorial integrity;[14] thus, one of the central government's objectives is to design an equalization scheme which would eliminate or, at least reduce, the *horizontal imbalances* between the regions and deter their threat of secession.

There are many instances when the transfers are (explicitly or implicitly) targeted to deter imbalances. As Ahmad and Craig (1997) note "... national governments may wish to ensure that citizens in different regions and localities have access to a certain modicum of publicly provided services." The horizontal imbalances in fiscal capacity should be addressed by equalization transfers from the center or between regions. In fact, Australia, Canada, Denmark, and Germany use horizontal imbalances as the basis of equalization policy between the regions.

In our model, we allow for a general class of distribution functions, which includes triangular, exponential, and various types of bimodal distributions. We examine equalization transfer schemes that address the issue of horizontal imbalances and, at the same time, eliminate a secession threat by any of its regions. We establish the principle of *partial equalization*, which asserts that:

- in order to prevent a threat of secession by disadvantageous regions, they must be subsidized by advantageous regions; and

- in order to deter a threat of secession by advantageous regions, their required contributions should not be excessive.

[14] Especially in Indonesia and Nigeria.

The principle of partial equalization suggests that, though the gap between advantageous and disadvantageous regions must be reduced, it should not be completely eliminated. Some equalization is necessary, but it cannot be complete.[15]

We also show, that in absence of any redistribution, the laissez-faire approach may leave disadvantageous areas of the country prone to secession. On the other hand, the Rawlsian transfer scheme that completely equalizes the fiscal capacities of all regions would cause advantageous regions to threaten to secede.

A. Some Related Literature

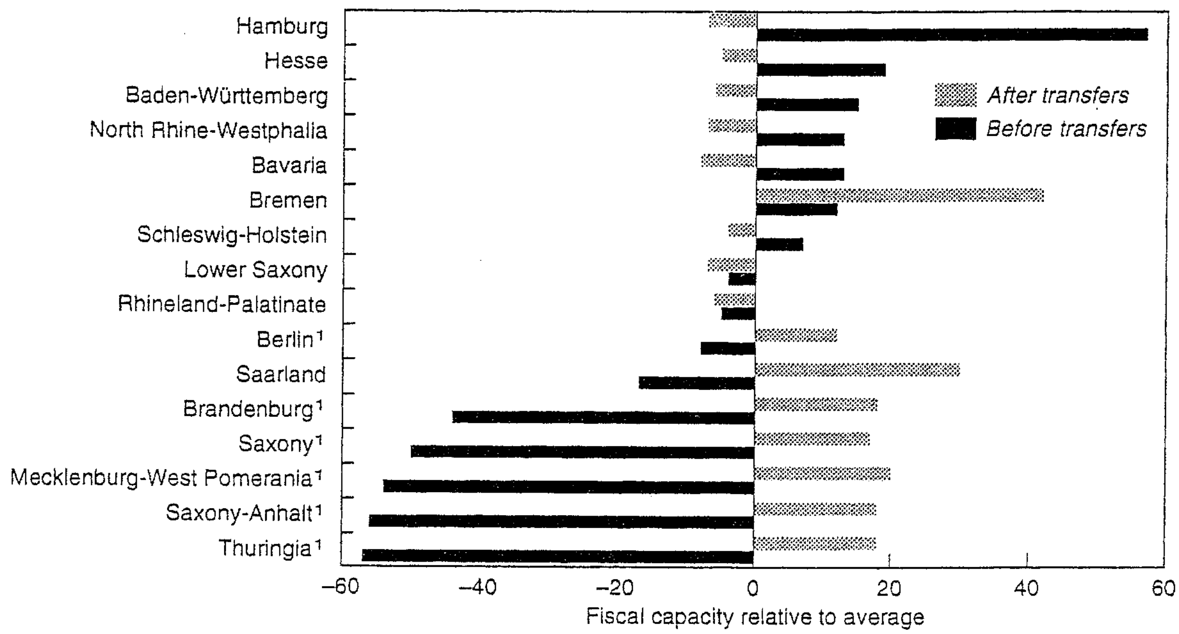
Before proceeding with the body of our analysis, let us briefly discuss the relationship of this work to the most closely related literature.[16] The framework developed by AS is also employed in papers by Casella (1992), Casella and Feinstein (1990), Feinstein (1992), Wei (1991a,b), which focuses on heterogeneity of citizens' preferences and utilizes (implicitly or explicitly) Hotelling's location model to represent the heterogeneity of preferences among voters over the provision of public goods. Cremer, De Kerchove, and Thisse (1985) develop a model that examines the number and location of public facilities. There is also a literature including Guesnerie and Oddou (1981), (1986), Greenberg and Weber (1986), Weber and Zamir (1986), Jehiel and Scotchmer (1997) rooted in the Tiebout tradition, where the heterogeneity of preferences among individuals and the impossibility of lump sum financing of public group provision lead to the formation of small jurisdictions. The main focus of these papers is the existence and the characterization of stable partitions of the individuals into jurisdictions, where equilibrium and stability notions capture various scenario concerning the mobility of individuals and groups of individuals across jurisdictions and the decision making process about the level of the public good provision.

Another related group of papers focuses primarily on the heterogeneity in income rather than individuals' preferences. The first contribution to this line of research has been made by Buchanan and Faith (1987) who explore the limits that the threat of secession puts on the tax burden imposed by the majority (which can be rich or poor). This question is the subject of Bolton and Roland (1997), who develop a model of a two-region nation with different gross income distributions. Their main focus is to understand how the threat of secession determines the choice of a purely distributive taxation rate under the assumption that if secession takes place, all gross incomes are deflated by a common factor. They show that fiscal accommodation in the union reduces the likelihood of secession, but by no means

[15] It is interesting to point out that, due to the heavy economic burden of the unification of West and East Germany, the transfer scheme used there exhibited a degree of *over-equalization*. As Figure 1 demonstrates, the fiscal capacity of poorer former East German provinces increased after the transfer, but the contributions paid by rich former West German states reduced their fiscal capacity below the average (Spahn and Föttinger (1997)). We argue in Section 3 that secession-proofness and over-equalization are incompatible in our model, so that the elimination of a threat of secession would rule out any over-equalizing transfer scheme.

[16] For survey of some of this literature see Bolton, Roland, and Spolaore (1996) and Young (1998).

Figure 1. Germany: Per Capita Fiscal Capacity of States Before and After Equalization
(In percent of average)



Source: P.B. Spahn and W. Fottinger (1997)

prevents the break-up of the nation under all circumstances. In addition, fiscal accommodation may surprisingly lead to higher taxes. As explained in Persson and Tabellini (1999), the identification of the equilibrium secession-proof tax rate is not straightforward due to the fact that individual preferences may fail to be single-peaked. The Bolton and Roland model has been extended to allow for mobility across borders (Olofsgard (1999)) and the introduction of region-specific shocks (Fidrmuc (1999)).

Federations may also be considered from a contractual perspective. What sort of arrangements or constitution, including secession clauses, should be considered to promote efficiency? Bolton and Roland (1997) have interesting insights on the determinants of the most preferred arrangement. Persson and Tabellini (1996a,b) examine a risk sharing argument under moral hazard considerations. More recently, Bordignon and Brusco (1999) offer an interesting analysis of secessions rules, arguing that the absence of explicit secession rules can be seen as a commitment device to increase the stability of the federation.

Finally, our paper also relates to the huge empirical public finance literature on transfers across regions targeted at reducing their horizontal imbalances across them. Implicit transfers across regions are often generated by taxation systems and the design of public spending programs. Many countries, including Canada, Belgium, Germany, and Switzerland, have also adopted explicit interregional transfer rules. These rules are motivated mostly by equity and solidarity considerations. The literature has focused on whether these rules lead to under-equalization or over-equalization. Although equalization is not driven by equity considerations in our paper, it turns out that secession-proof transfer schemes will entail some form of partial equalization.

The paper is organized as follows. In the next section, we present the model and formally introduce the concepts of desirability and sustainability of cooperation. In Section 3, we introduce the conditions that guarantee that desirable cooperation is also sustainable and yield the existence of a secession-proof allocation. In Section 4, we sketch the proof of our main result. In Section 5, we consider the case of a uniform distribution and demonstrate that a *laissez-faire* allocation that does attempt to correct horizontal imbalances between the regions is not, in general, secession-proof. We also show that neither is the Rawlsian allocation that imposes complete equalization by eliminating the gap between "advantageous" and "disadvantageous" citizens. Finally, we demonstrate that the 50 percent compensation rule is the unique linear allocation that remains secession-proof for all values of government costs. The proofs of all lemmas and propositions are relegated to the Appendix.

II. The Model

We consider a country whose citizens have preferences over the unidimensional policy space I given by the interval [0,1] with a mass of 1. Each citizen has preferences over the set I, which may represent tax policy, level of defense spending, and attitude towards minorities or any other issue of a national interest. Each citizen's preferences are single-peaked and each has an ideal point in I. For simplicity, we identify each citizen with her ideal point. The distribution of all ideal points (and, thus, of all citizens' preferences) is given by a cumulative distribution function F, defined over the space I. We assume that F has a density function f which is positive and continuous everywhere on the interval [0,1].

In the event where the country breaks up into several smaller countries $S_1, S_2, ..., S_k$, they will form a partition of the interval [0,1]. Every citizen belongs to only one country and nobody is left without a country. That is, a citizenship is guaranteed to every individual but no "double citizenship" is allowed. We do not impose restrictions on country formation, and, in principle, every group of individuals S can form its own country. Only for simplicity we require that each potential new country within the interval [0,1] consists of the union of a finite number of connected intervals; we will use the term *region* for such a subset of citizens when it is a part of an existing country.

Every country chooses a policy in the issue space I. In this paper, we adopt a spatial interpretation of our model and identify a policy with a location of the government. As in AS, we do not distinguish between geographical and preference dimensions. Moreover, we assume that if an individual t is a citizen of country S, whose government chooses a location

$p \in I$, then the disutility or "transportation" cost incurred by the individual t, $d(t, p)$, is determined by the distance between t and the location of the government:

$$d(t,p) = \alpha|t-p|,$$

where α is a positive cost coefficient. Denote by

$$D(S) = \min_{p \in I} \alpha \int_S d(t,p) f(t)\, dt$$

the transportation cost of the citizens of S.[17]

Every country S, whether it is big or small, has to cover the cost of public goods g which we simply call *government cost* and assume that this cost is independent of the size and the population composition of the country. Let us introduce the notion of a *S-cost allocation* which determines the monetary contribution of each individual t towards the cost of government g.

Definition 2.1: A measurable function x defined on the set S is called an S-cost allocation if it satisfies the budget constraint:

$$\int_S x(t) f(t) dt = g.$$

Since in our set-up advantages and disadvantages of a single country or its break-up are common knowledge, we allow for lump sum transfers and do not restrict the mechanism for reallocation of gains from cooperation within each country. Thus each country S will minimize its total cost given by the sum of government and transportation costs:

$$g + D(S).$$

Since the minimization of transportation cost for country S implies the selection of its median as the government location, the cost allocation x would imply that the total contribution of a citizen $t \in S$ would be:

$$\alpha|t - m(S)| + x(t)$$

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[17] Since each country consists of a finite number of connected regions, there always exists an optimal location of the government and, therefore, the cost function is well defined. It is useful to note that for any country S the total transportation cost is minimized when the government location chooses its location at a median $m(S)$ of S. If S is an interval, then its median is uniquely defined. However, if country S consists of a several intervals separated from each other, the median of S is not necessarily unique. To avoid ambiguity we then denote by $m(S)$ the leftmost median of S.

For notational simplicity, we assume hereafter that α, the marginal rate of substitution between money and the distance to the location of the government is equal to 1. With an obvious change of the variables, the analysis remains unchanged with $g\alpha$ instead of g.

Since the transportation cost incurred by a citizen is represented by the distance between her location and the policy chosen by the country to which she belongs, it again points out the aforementioned conflict between heterogeneity and increasing returns to size. Indeed, on one hand, a larger country would require a smaller per capita contribution towards government costs g given by an S-cost allocation x. On the other hand, the bigger the country the larger the chance that the government's location could be far away from citizens living on the margin. One would expect that higher government costs would strengthen the cooperation so that increasing returns to size would outweigh secession tendencies created by heterogeneity of citizens' preferences.

To examine this issue formally, we introduce the notions of *desirability* and *sustainability* of cooperation. Cooperation between the different regions of the country would be desirable if no break-up of the country into smaller parts can provide the total benefit exceeding that generated by the united country.[18]

Consider all possible partitions of the interval I into several connected or disconnected intervals. A typical partition P of I would consist of a number of smaller countries $\{S_1, S_2, ..., S_K\}$ where each individual $t \in I$ belongs to one and only one country in P. The following definition in the game-theoretic terminology amounts to *super-additivity*:

Definition 2.2: The cooperation is *desirable* if for every partition $P = (S_1, ..., S_K)$ we have

$$D(I)+g \leq \sum_{k=1}^{k}[D(S_k)+g].$$

It is useful to point out that if the country is broken up into two parts, S and T, the desirability condition implies that

$$g \leq D(I)-D(S)-D(T) \qquad (1)$$

Let us now turn to sustainability of cooperation. Sustainability requires not only positive gains from cooperation, but also a mechanism that will allocate those gains in such a way that no separate region S can generate a higher payoff to all its members than that guaranteed to them by the central government. Given a cost allocation and location of the central government, regions of a country may contemplate the possibility of secession. If a region S

[18] As Wittman (1991) puts it: "...two nations would join together (separate) if the economies of scale and scope and the synergy produced by their union created greater (smaller) benefits than the cost."

can make all its members better off than under the central government, then S would be *prone to secession*:

Definition 2.3: Consider a pair (p, x), where p is a location of the national government and x is an I-cost allocation. We say that the region S is prone to secession (given (p, x)) if

$$\int_S (d(t, p)+(t))\, f(t)dt > D(S)+g\,.$$

If no region is prone to secession, then the pair (p, x) is called *secession-proof.* The cooperation is called *sustainable* if there exists a secession-proof allocation.

Since throughout the rest of the paper we deal only with cost allocations defined for the entire interval I, we shall call an I-cost allocation simply a cost allocation.

We now state an important property of secession-proof allocations. It implies that under secession-proof allocation each region is required to make a nonnegative contribution towards the government costs. That is, *secession-proofness rules out over-equalization.* The reason is obvious: if region S receives a net transfer via cost allocation, the burden of the government costs will fall on the rest of the country $T=I\setminus S$. We show that this burden would make region T prone to secession.

Lemma 2.4: Let x be a cost allocation. Suppose that there exists a region S such that $\int_S x(t)f(t)dt < 0$. Then for any location of the government p the region $T = I \setminus S$ is prone to secession, and therefore the pair (p, x) is not secession-proof.

To complete this section we would like to point out that cooperation is sustainable or desirable only if the government cost is sufficiently high. Indeed, if the government cost g is low, there is little incentive for different regions to stay together in one country. In the extreme case where g is zero, every cost allocation would be secession-prone and no cooperation would emerge. That is:

Proposition 2.5: There is a cut-off value of government costs g_{des} such that cooperation is desirable if and only if $g \geq g_{des}$.

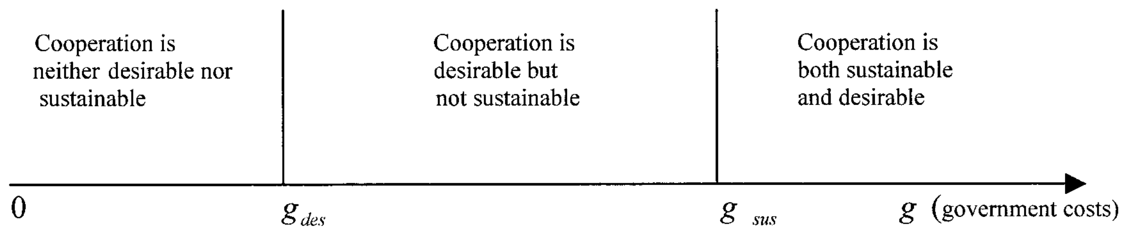
Proposition 2.6: There is a cut-off value of government costs g_{sus} such that cooperation is sustainable (and there exists a secession-proof allocation) if and only if $g \geq g_{sus}$.

As we mentioned above, sustainability of cooperation requires not only positive gains from being together but also the ability to distribute these gains without creating secession-prone regions. That is, the sustainability requirement is stronger than the desirability one:

Proposition 2.7: If cooperation is *sustainable*, it is also desirable, i.e., $g_{sus} \geq g_{des}$.

The relationship between the value of government costs g and type of possible cooperation (or lack thereof) is exhibited by Figure 2.

Figure 2. Relationship Between Government Cost and Type of Coordination



In the next section we derive the conditions under which sustainability and desirability yield the same cut-off value, $g_{sus} = g_{des}$, which would represent the lower bound on government costs that yield a secession-proof allocation.

III. The Main Result

It is important to identify the conditions under which the mere existence of gains from cooperation yields the possibility of reallocating these gains without creating regions that are prone to secession. It is also much easier to verify whether cooperation is desirable by simply observing the economies of scale rather than examining threats of secessions by every region.

We use two conditions to obtain our equivalence result. The first is

Symmetry: $f(.)$ is symmetric with respect to the center, i.e., $f(t) = f(l - t)$ for all $t \in I$

This assumption is quite standard. It implies that the point $m(I) = \frac{1}{2}$ is not only the geographical center of the country is also the median of the distribution of citizens' location. In our analysis of secession-proof allocations, we therefore restrict our attention to the situations where the government is located in the middle of the country. Thus, instead of considering a pair (p, x) in Definition 2.3, we shall focus only on cost allocation, assuming that the national government is always located at the point $\frac{1}{2}$.

To state our second assumption, we need some additional notation. For each $t \in I$ let L_t and R_t be the sets of citizens to the left and right of the point t, respectively, i.e., $L_t = [0, t]$ and $R_t = [t, 1]$. For the sets, L_t and R_t denote by $l(t)$ and $r(t)$ their respective medians, i.e., $l(t) = m(Lt)$ and $r(t) = m(R_t)$. It is easy to verify that both functions l and r are differentiable and increasing in t, with $l(0) = 0$, $l(1) = \frac{1}{2}$, $r(0) = \frac{1}{2}$, and $\mathrm{r}(1) = 1$ $l(0) = 0$,

$l(1) = ½$, $r(0) = ½$, and $r(1)=1$. Moreover, the symmetry of the distribution implies that for every $t \in I$

$$r(t) + l(1-t) = 1 \qquad (2)$$

Our second assumption is:

Gradually Escalating Median: $l'(t) < 1$ on the $[0,1]$

This assumption implies that if we increase the length of the interval $L_t = [0, t]$ by a small positive number δ, then the median of the interval $L_{t+\delta} = [0, t+\delta]$ increases by the amount less than δ. Obviously, the symmetry of the distribution represented by (2) immediately implies that if $l'(t) < 1$ then $r'(t) < 1$.

The class of distribution functions satisfying the condition of gradual escalation is quite large. In particular, it includes all *log-concave functions*,[19] i.e. those for which the logarithm of the cumulative distribution function F is concave on the interval [0, 1]:

Remark 3.1: If the distribution function is log-concave it satisfies the assumption of gradually escalating median.

The log-concavity assumption is satisfied for a wide range of symmetric distribution functions. For example, all symmetric distribution functions which are concave and have an increasing density on the interval $\left[0, \frac{1}{2}\right]$ are log-concave. This class includes a family of symmetric triangular distributions, whose densities are given by $f(t) = at + b$ for all $t \leq \frac{1}{2}$ where $0 \leq a, b$ and $\frac{a}{4} + b = 1$. This family, in turn, contains the uniform distribution, whose density is given by $f(t) = 1$ for all $t \in [0,1]$ and the triangle distribution whose density is given by $f(t) = 4t$ for all $t \leq \frac{1}{2}$ and $f(t) = 4 - 4t$ for all $t \geq \frac{1}{2}$.

Obviously, concavity is not a necessary condition for log-concavity. For example, a convex exponential function given by $f(t) = ce^t$ for all $t \leq \frac{1}{2}$, is also log-concave, where the

[19] Log-concavity is a special case of a more general concept of ρ-concavity studied in Hardy, Littlewood, and Polya (1934). The applications of log-concavity are relatively novel to economic and political science theory (see Caplin and Nalebuff (1991) and Weber (1992)). The difference between our set-up and the models discussed in Caplin and Nalebuff (1991) is that they impose log-concavity on density functions whereas we consider log-concavity of the distribution function.

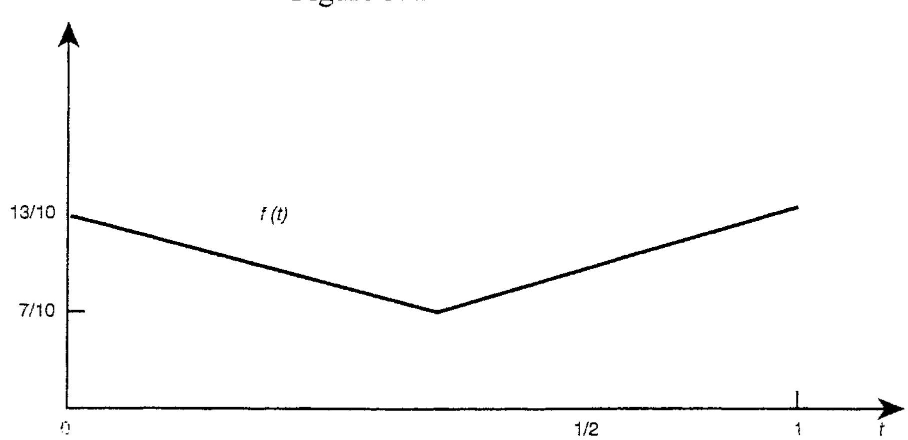
constant c is chosen to ensure that the mass of the population on the interval $\left[0,\frac{1}{2}\right]$ is equal to $\frac{1}{2}$.

It is important to stress that log-concavity does not imply single-peakedness of the density function and a higher population density in the middle region of the country. In particular, the class of log-concave distribution functions includes the following class of bimodal distributions:

$$F_{a,b}(t)=\begin{cases}2(a-b)t+b & \text{if } t\le\frac{1}{2}\\ f(1-t) & \text{if } t\ge\frac{1}{2}\end{cases}$$

where the parameters a and b satisfy $0<a<b$ and $a+b=2$. Simple algebra shows that if $\sqrt{3}-1<a<1$, then the corresponding distribution is log-concave on $[0,1]$. This class of distribution functions also serves to illustrate that the assumption of gradual escalation of the median is weaker than log-concavity of the distribution function. Indeed, it is easy to show that a distribution in the above bimodal class satisfies the assumption of gradual escalation of the median if and only if $\frac{2}{3}<a<1$. For example, the distribution, given by the density function $f(t)=\frac{10}{13}-\frac{12}{10}t$ for all $t\le\frac{1}{2}$ satisfies the gradually escalating median condition whereas its cumulative distribution function is *not* log-concave (Figure 3.)

Figure 3. Biomodal Distribution



It is interesting to note that the assumptions of gradual escalation of the median and log-concavity allow for populations exhibiting higher density on the borders than in the center as long as the variance of density is bounded from above.

In order to formally state our main result, let us turn to the examination of a structure of secession-proof allocations. Lemma 2.4 implies that every citizen makes a nonnegative contribution towards government costs. Since we assume the symmetry of the citizen's distribution with respect to the median, it is crucial to examine how the contribution of each citizen is correlated with her distance to the location of the government. We have to take into account *horizontal imbalances* between regions and to design an equalization mechanism between advantageous citizens and regions (those close to the center) and disadvantageous ones (those close to the borders). To what extent, if at all, should the more disadvantageous regions be compensated via the resulting cost allocation?

To this end, consider the cost allocation $x_g(t)$ which is defined as follows:

$$x_g(t) = \begin{cases} r(t) + \lambda & \text{if } t \leq \frac{1}{2} \\ x_g(1-t) & \text{if } t \geq \frac{1}{2} \end{cases}$$

where λ is chosen in such a way in order to satisfy the budget constraint of the country:

$$\lambda = g - 2\int_0^{\frac{1}{2}} r(t) f(t) dt.$$

It is important to note that the assumption of gradually escalating median guarantees that the allocation $x_g(t)$ satisfies the principle of partial equalization. Indeed, the fact that the cost allocation $x_g(t) = r(t) + \lambda$ is increasing whereas the total cost $d(t,m) + x_g(t) = \frac{1}{2} - t + r + \lambda$ is decreasing on the interval $\left[0, \frac{1}{2}\right]$ guarantees that the closer a citizen is to the center, the larger her contribution towards government costs is, while the total cost is still higher for those close to the borders. Thus, while some equalization takes place, it is not complete. It is interesting to note that in the case of uniform distribution the equalization rate is 50 percent (see section 5).

The Main Result: The Symmetry and Gradually Escalating Median assumptions imply $g_{des} = g_{sus}$. Moreover, if the level of government costs g satisfies $g \geq g_{des}$, the allocation x_g (t) is secession-proof.

To prove this result we consider a level of government costs g which guarantees that cooperation is desirable, i.e., $g \geq g_{des}$. Then we consider the cost allocation x_g and show that it is secession-proof. Since by Proposition 2.7, $g_{des} \geq g_{sus}$, it would imply that the cooperation is indeed sustainable.

Note that Remark 3.1 guarantees the following:

Corollary 3.2: Under symmetry and log-concavity we have $g_{\text{des}} \geq g_{sus.}$

In the next section we discuss the intuition and the way we prove this result.

IV. Sketch of the Proof of the Main Result

Although the complete proof of the main result is relegated to the Appendix, we would like to describe the method of the proof, which has an independent interest.

Since the central issue of the main result is the examination of the secession-proofness of a given cost allocation, we would like to indicate the major difficulty with verifying the secession-proofness. The difficulty stems from the fact that one cannot rule out a possibility of secession-prone regions that consist of *disconnected* intervals. If we were able to restrict our analysis to connected regions only, we could have used the Greenberg and Weber (1986) result, which yields a stable outcome when only connected or "consecutive" coalitions are considered. Unfortunately, it is not even true that if there is a secession-prone disconnected region, there exists a connected region prone to secession. The assumption of gradually escalating median plays a major role to remove this obstacle and to allow us to consider connected regions as the only ones potentially prone to secession.

We proceed in two steps. First we show that if the set of cost allocations is restricted in an appropriate way, then only a specific class of connected regions may be prone to secession. Then we show that the issue of secession-proofness translates into a variational problem.

Precisely, we consider a set X of continuous, nonnegative and symmetric cost allocations that satisfy the principle of partial equalization. In other words, let X be a set of cost allocations x which satisfy:

(α) x is a continuous and nonnegative function on the interval [0, 1];
(β) x is symmetric: $x(t) = x(1- t)$ for all $t \in [0, 1]$;
(γ) x is increasing on the interval $0, \frac{1}{2}$;
(δ) $x(t) + d\left(t \frac{1}{2}\right)$ is decreasing in t on the interval $\left[0, \frac{1}{2}\right]$.

The following lemma, the proof of which heavily relies upon the assumption of gradually escalating median, plays the central role in our proof:

Lemma 4.3: Let $x \in X$ be a cost allocation which is not secession-proof. Then, there exists $t \in [0, 1]$ such that either $L_t = [0, t]$ or $R_t = [t, 1]$ is prone to secession.

The intuition is as follows. If a cost allocation entailing some degree of partial equalization is prone to secession by a disconnected region, it is also prone to secession by a connected region that contains at least one of the end points of the interval [0,1]. The technique consists in filling the gaps in a convenient way.

Lemma 4.3 implies that for any secession-proof allocation $x \in X$ neither L_t or R_t should be prone for secession for any $t \in I$, or, equivalently, the following two conditions should be satisfied for all $t \in [0,1]$:

$$\int_{L_t} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt \leq g + D(L_t)$$

$$\int_{R_t} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt \leq g + D(R_t)$$

where, to recall, $D(S)$ denotes the minimum of the aggregated transportation cost of members of S. For every t denote by $H(t)$ the aggregated transportation cost of citizens of L_t to the location of the government at 1/2, i.e.,

$$H(t) = \int_0^t d\left(t, \frac{1}{2}\right) f(t)dt.$$

Using the symmetry of the citizens' distribution and rearranging the above inequalities, we obtain the necessary and sufficient conditions for a cost allocation $x\in X$ to be secession-proof, namely, that the inequalities

$$D(I) - D(R_t) - H(t) \leq \int_0^t x(t) f(t)dt \leq g + D(L_t) - H(t). \tag{3}$$

hold for all t ≤ ½.

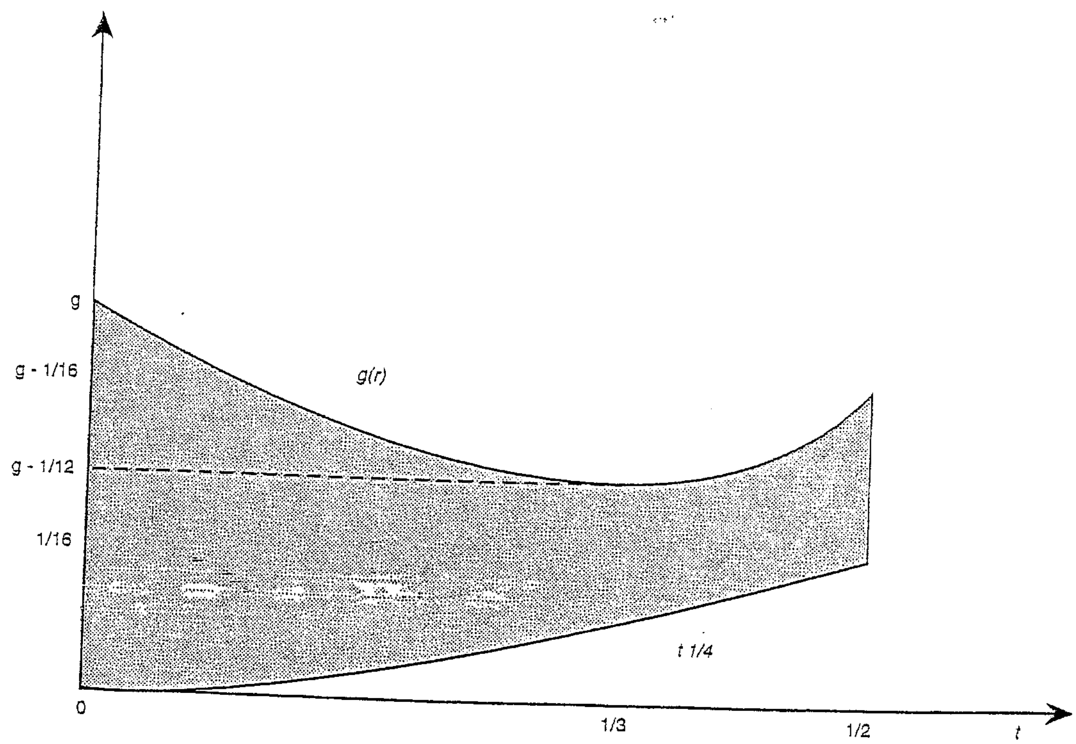
The problem is now reduced to a variational problem: find a cost allocation in X to satisfy the integral conditions in (3). To illustrate this problem consider the AS framework of a uniform distribution. (This case will be studied in the next section). It is easy to verify that in this case the necessary and sufficient conditions for secession-proofness in (3) turn into:

$$\frac{t^2}{4} \leq \int_0^t x(s)\, ds \leq \frac{3t^2 - 2t}{4} + g \tag{4}$$

This means that in order to obtain a secession-proof cost allocation, which entails a partial equalization, for a given value of g one has to find a function y sandwiched between

$\frac{t^2}{4}$ and $\varphi(t) \equiv \frac{3t^2 - 2t}{4} + g$, such that $y(0) = 0$, $y\left(\frac{1}{2}\right) = \frac{g}{2}$, $y' > 0$, and $0 < y'' < 1$. Note that the function φ is convex and obtains its minimum of $g - \frac{1}{16}$ at $t = \frac{1}{3}$. Moreover, $\varphi(0) = g$ and $\varphi\left(\frac{1}{2}\right) = g - \frac{1}{12}$. Therefore the search for a secession-proof cost allocation amounts to finding an increasing and convex (but not "too convex") function satisfying two boundary conditions and with a graph within the area depicted on Figure 4.

Figure 4. The Secession-Proof Area



The complete details of the proof are presented in the Appendix. In the next section we examine the structure of secession-proof allocations in the case of uniform distribution. Since every secession-proof allocation requires each individual to make a nonnegative contribution, the question is how the cost of the government g is shared. To show the need for a balanced equalization, we demonstrate in the next section that neither the allocation without any equalization nor the allocation with a complete equalization is secession-proof.

V. Uniform Distribution

In this section we consider the case where the density function is uniform on the interval I, i.e. $f(t) = 1$ for all $t \in [0,1]$. For $t \leq \frac{1}{2}$, we have

$$D(L_t)=\frac{t^2}{4},\ D(R_t)=\frac{(1,t)^2}{4},\ D(I)=\frac{1}{4},\ H(t)=\frac{t-t^2}{2}$$

First note that the cooperation is sustainable if the inequality (1) holds for all L_t and R_t, i.e.,

$$g \leq D(I)-D(L_t)-D(R_t)$$

or

$$g \leq \frac{t}{2}-\frac{t^2}{4}.$$

Since the maximum of the right-hand side is $\frac{1}{8}$, it follows that $g_{sus} = \frac{1}{8}$, i.e., the cooperation is sustainable if $g \geq \frac{1}{8}$.[20]

We consider two allocations, *Laissez-faire allocation*, under which each citizen contributes an equal amount towards government costs (no equalization), and *Rawlsian Egalitarian allocation* that assigns equal total contributions (including the transportation cost) to every citizen (complete equalization).[21]

Laissez-faire allocation: $LF(t) = g$ for every $t \in I$. We have the following proposition:

Proposition 5.1: There exists a level government costs $g^* > \frac{1}{8}$ such that for all $g, \frac{1}{8} \geq g < g^*$, the laissez-faire allocation LF is not secession-proof.

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[20] Indeed, proceeding similarly, we obtain that splitting the interval [0, t] in two smaller intervals is beneficial if and only if $g \geq \frac{t^2}{8}$. Since $t^2 \leq 1$, it suffices to check super-additivity for partitions with two sets only.

[21] In our analysis, the population is distributed on the bounded interval [0, 1]. If instead, the citizens were distributed (uniformly) over the entire real line, the complete equalization is the unique secession-proof compensation scheme. We thank Jacques Drèze for proving this assertion and bringing it to our attention. Although simple, the argument is too tedious to be reproduced here.

The inequality (3) can be rewritten in this case as

$$\frac{t^2}{4} \le gt \le \frac{3t^2 - 2t}{4} + g.$$

The left side of this inequality is always satisfied for $g \ge \frac{1}{8}$ as $t \le \frac{1}{2}$. This means that for $t \le \frac{1}{2}$ no region R_t is prone to secession. The reason is that the citizens in the middle of the country spared from equalization transfers to support distant regions would not wish to secede. The secession-prone regions are, therefore, those of the form L_t. It is interesting to mention that only relatively large regions, containing more than 33 percent of the population, could be prone to secession. Indeed, small regions on the margin are not prone to secession because of a heavy burden of per capita government costs if they wish to go alone.

Rawlsian Egalitarian allocation: $RE(t) + d(t, \frac{1}{2}) = g + \frac{1}{4}$, yielding $RE(t) = g + t - \frac{1}{4}$. It follows that for the range of government costs $\frac{1}{8} \le g < \frac{1}{4}$, the values of $RE(t)$ are negative for all t satisfying $0 \le t < \frac{1}{4} - g$. Then Lemma 3.4 implies that there will be regions R_t that are prone to secession. This immediately yields the following:

Proposition 5.2: For the level of government costs g satisfying $\frac{1}{8} \le g < \frac{1}{4}$, the Rawlsian Egalitarian allocation is not secession-proof.

The reason for the intervals R_t to be prone to secession is that since complete equalization puts a heavy burden on those in the center, a threat of secession comes from central regions. Moreover, by (4), we have

$$\frac{t^2}{4} \le gt + \frac{t^2}{2} - \frac{t}{4} \le \frac{3t^2 - 2t}{4} + g$$

for all $t \le \frac{1}{2}$.

The right-hand side of this inequality is

$$gt + \frac{t}{4} \le \frac{t^2}{4} + g$$

or $g \geq \frac{t}{4}$, which holds for all $t \leq \frac{1}{2}$ whenever the cooperation is sustainable. Thus, the regions L_t that do not contain the center of the country are not prone to secession as they are adequately compensated by those close to the center.

Note that $r(t) = \frac{t+1}{2}$, $\lambda = \frac{1}{8}$ and for all $t \leq \frac{1}{2}$ and $g \geq \frac{1}{8}$, the secession-proof allocation x_g given by:

$$x_g(t) = \frac{t}{2} + g - \frac{1}{8}.$$

We complete this section by a characterization of the set of linear secession-proof allocations.[22] In the case of linear allocations, the rate of equalization is uniquely determined by a slope of the allocation function. We know that the linear allocation x_g is secession-proof for all values of government costs $g \geq g_{des}$. To examine the range of equalization rates of secession-proof schemes, we consider, for a given value of government costs $g \geq \frac{1}{8}$, a symmetric linear cost allocation x. It is defined by $x(t) = \alpha t + \beta$ for all $t \in \left[0, \frac{1}{2}\right]$, where $0 \leq \alpha \leq 1$ and βos chosen to balance the government budget:

$$\int_0^{\frac{1}{2}} x(s)\,ds = \frac{g}{2}$$

It follows that $\beta = g - \frac{\alpha}{4}$ and

$$x^{\alpha}(t) = \alpha t + \frac{4g - \alpha}{4}$$

for all $t \in \left[0, \frac{1}{2}\right]$. The following proposition determines the range of secession-proof rates of equalization.

Proposition 5.3: The allocation x^{α} is secession-proof if and only if:

$$\psi(g) \leq \alpha \leq 4g \ \textit{for}\ g \in \left[\frac{1}{8}, 1 - \frac{\sqrt{3}}{2}\right],\ \textit{where}\ \psi(g) = 2 - 12g - \sqrt{128g^2 - 16g},$$

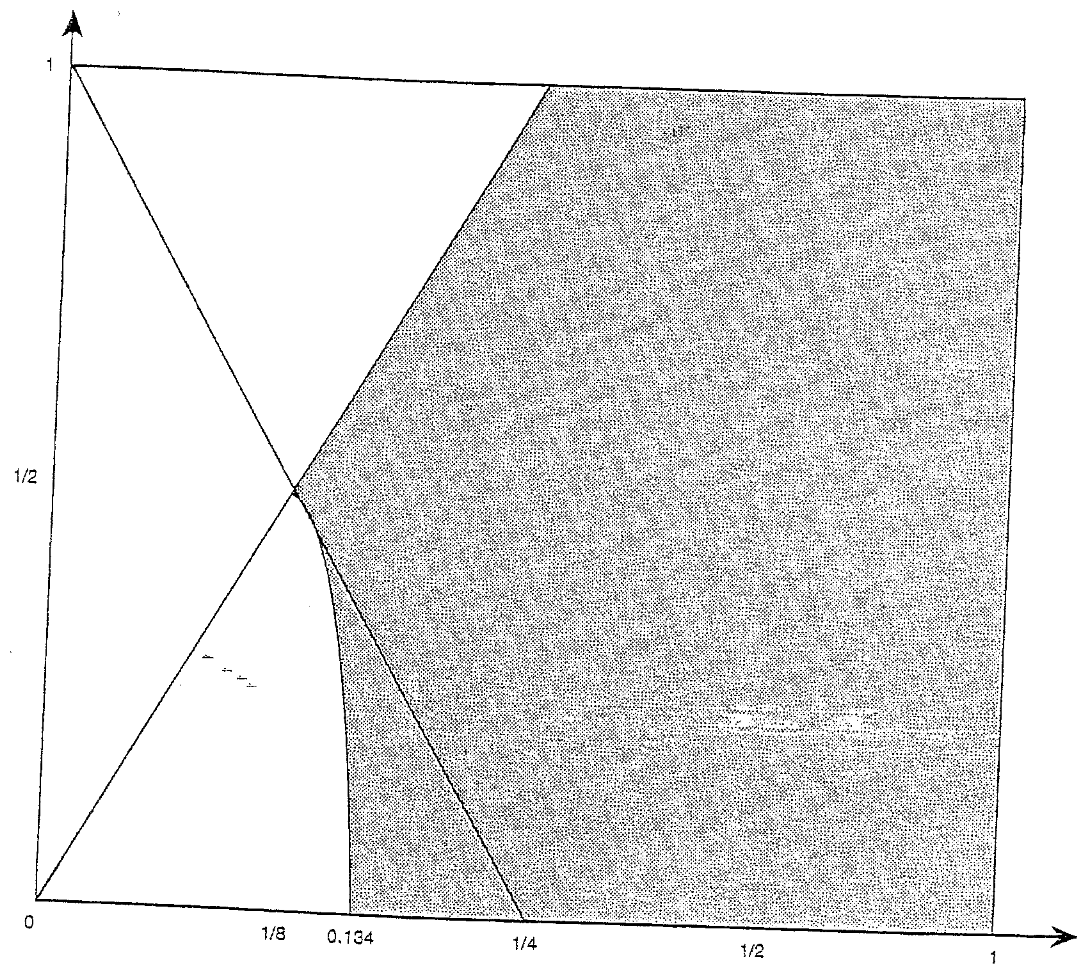
[22] This is the set of allocations examined in AS.

$$0 \le \alpha \le 4g \quad for \quad g \in \left[1-\frac{\sqrt{3}}{2}, \frac{1}{4}\right],$$

$$0 \le \alpha \le 1 \quad for \quad g \ge \frac{1}{4}.$$

(The hatched area in Figure 5 represents the secession-proof equalization rates for all possible values of g).

Figure 5. Secession-Proof Rates as g Varies



Proposition 5.3 yields two important observations. First, the equality $\psi(g) = 4g = \frac{1}{2}$ when the value of government costs g are equal to $\frac{1}{8}$, implies that $\alpha \frac{1}{2}$ is the only secession-proof rate for $g = \frac{1}{8}$. Moreover, since $\frac{1}{2}$ is always a secession-proof rate, it follows that 50 percent equalization rule provides the unique rate which is the secession-proof rate for all possible values of g exceeding $g_{des.}$ Second, if the value of government costs is at least double of g_{des}, then secession-proofness has no implication whatsoever on the range of equalization. Indeed, if the government costs rise, a desire to secede would diminish and at certain point, $g = \frac{1}{4}$ it ceases to affect the equalization rates.

Proof of Lemma 2.4: Let a cost allocation x and a region S be such that $\int_S x(t)f(t)dt < 0$. Consider the region $T = I \setminus S$ which represents all the individuals outside of S. It will have to contribute more than g toward government costs, i.e., $\int_T x(t)f(t)\,dt > g$. Suppose that the government is located at point p. The total cost of region T is:

$$\int_T (x(t)) + d(t,p)f(t)dt.$$

However, since $\int_T x(t)f(t)dt > g$ *and* $\int_T d(t,p)f(t)dt \geq D(T)$, it follows that

$$\int_T (x(t)) + d(t,p)f(t)dt > g + D(T),$$

and T is prone to secession.

Proof of Proposition 2.5: The cooperation is desirable if for any partition (S_1, …, S_k) the following inequality is satisfied:

$$g + D(I) \leq \sum_{k=1}^{k} [g + D(S_k)].$$

Since it is trivially satisfied for $K = 1$, we may consider only partitions in to $K > 1$ countries. Thus, the last inequality can be rewritten as

$$g \geq \frac{1}{K-1}\left[D(I) - \sum_{k=1}^{k} D(S_k)\right]$$

Denote by

$$g_{des} + \sup_p \frac{1}{|P|-1}\left[D(I) - \sum_{s \in p} D(S)\right],$$

where supremum is taken over the set of all partitions P of the nation I into more than one country and $|P|$ stands for the number of countries in P. Note that $g_{des} \leq \frac{D(I)}{2}$ is bounded. Thus, the cooperation is desirable if and only if $g \leq g_{des}$.

Proof of Proposition 2.6: First, suppose that the value of government costs g is such that there exists a secession-proof allocation (p, a). We shall demonstrate that for every value g', exceeding g, there is also a secession-proof allocation. Indeed, let $g' > g$ and consider a new allocation (p, a'), where $a'(t) = \frac{g'}{g} a(t)$ for every (t). Then $\int_I a'(t)f(t)dt = g'$ and since

$$\int_S (d(t,p) + a(t))f(t)dt \leq g + D(S)$$

for every region $S \subset N$, we have

$$0 \leq \int_S d(t, p) f(t) dt - D(S) \leq g - \int_S a(t) f(t) dt \leq g' - \int_S a'(t) f(t) dt$$

or

$$\int_S (d(t, p) + a'(t) f(t) dt \leq g' + D(S).$$

Thus, (p, a') is a secession-proof allocation, and there exists a level of government costs g_{sus} such that the cooperation is sustainable if and only if $g \geq g_{sus}$. To complete the proof of the Lemma, it remains to show that g_{sus} is bounded. For this end, consider an allocation $(\frac{1}{2}, b)$ with $b(t) = g$ for all $t \in I$. It is secession-proof if and only if the inequality

$$\int_S d\left(t, \frac{1}{2}\right) f(t) dt + \int_S b(t) f(t) dt \leq g + D(S)$$

is satisfied for every region S. Consider an arbitrary region S. We have

$$\int_S d\left(t, \frac{1}{2}\right) f(t) dt - D(S) \leq g(1 - F_S),$$

where $F_S = \int_S f(t) dt$. Moreover,

$$\int_S d\left(t, \frac{1}{2}\right) f(t) dt - D(S) = \int_S \left(\left|t - \frac{1}{2}\right| - |t - m(S)|\right) f(t) dt \leq \int_S \left(\left|\frac{1}{2} - m(S)\right|\right) f(t) dt.$$

Since $\bar{f} = \min_{t \in I} f(t) > 0$,

$$\left|\frac{1}{2} - m(S)\right| \leq \frac{1 - F_S}{2f}.$$

Thus,

$$\int_S d\left(t, \frac{1}{2}\right) f(t) dt - D(S) \leq \int_S \left(\left|\frac{1}{2} - m(S)\right|\right) f(t) dt \leq \frac{1 - F_S}{2f}$$

Then, for every value $g \geq \frac{1}{2f}$, the allocation $\left(\frac{1}{2}, b\right)$ is secession-proof and g_{sus} is, indeed, a finite number.

Proof of Proposition 2.7: Suppose that the value of government costs g is such that cooperation is desirable. Then, there exists a secession-proof allocation (p, a). Consider an arbitrary partition $(S_1, \ldots, S_k)$. The inequality

$$\int_{Sk} (d(t, p) + a(t)) f(t) dt \leq D(S_k) + g$$

holds for every $k = 1, \ldots, K$. We have

$$D(I) + g = \int_I d\left(t, \frac{1}{2}\right) f(t)\, dt + g \le \int_I d(t,p) f(t) dt + g.$$

However,

$$\int_I d(t,p) f(t)\, dt + g = \sum_{k=1}^{K} \int_{Sk} (d(t,p) + a(t))\; f(t) dt \le \sum_{k=1}^{K} (D(S_k) + g),$$

which proves the desirability of cooperation.

Proof of Remark 3.1: Let F be a log-concave distribution function. By the Implicit Functions Theorem,

$$l'(t) = \frac{f(t)}{2f(l(t))},$$

Since F is log-concave, we have

$$\frac{f(l(t))}{F(l(t))} > \frac{f(t)}{F(t)},$$

and, therefore,

$$l'(t) < \frac{F(t)}{2F(l(t))} = 1.$$

Proof of Lemma 4.3: Let $x \in X$ be a cost allocation such that a region S is prone to secession. Assume, without loss of generality, that $m(S) \le \frac{1}{2}$. We shall carry out the proof of the lemma in four steps.

(*i*) $S \cup [0, m(S)]$ is prone to secession as well: It suffices to show that if there are p and q with $0 \le p < q \le m(S)$ and $S \cap [p,q] = \varnothing$, then $S^1 = S \cup [p,q]$ is prone to secession. Indeed, by condition (α), we have

$$\int_{S^1} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t) dt \ge \int_p^q d\left(t, \frac{1}{2}\right) f(t) dt + \int_S \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t) dt > \int_p^q d\left(t, \frac{1}{2}\right) f(t) dt + g + D(S).$$

However,

$$\int_{[p,q]} d\left(t,\frac{1}{2}\right) f(t)dt + D(S) \le \int_{S^1} |t - m(S)| f(t)dt \ge D(S^1)$$

and S^1 is prone to secession.

(*ii*) Suppose that $S \cap \left[m(S), \frac{1}{2}\right] \neq \varnothing$ and there are p and q with $m(S) \le p < q < \frac{1}{2}$ and $S \cap [p,q] = \varnothing$. Let $\hat{t} \in \left[m(S), \frac{1}{2}\right]$ be such that

$$F(t) - F((S)) = \int_{S \cap \left[m(S), \frac{1}{2}\right]} f(t)dt.$$

Then $S^2 \setminus \left[m(S), \frac{1}{2}\right] \cap [m(S), t]$ is prone to secession: Since the shift from S to S^2 is a measure-preserving transformation, it follows that m(S)= $m(S^2)$ and, therefore, $D(S^2) \le D(S)$. Moreover, condition (δ) implies that the difference

$$\int_{S^2} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt - \int_{S} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt =$$

$$\int_{S^2 \setminus S} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt - \int_{S \setminus S^2} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt > 0.$$

However, since

$$\int_{S} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt > D(S) + g,$$

it follows that

$$\int_{S^2} \left(x(t) + d\left(t, \frac{1}{2}\right)\right) f(t)dt \, D(S^2) + g.$$

Thus, S^2 is prone to secession.

(*iii*) Suppose that $S \cap \left[\frac{1}{2}, 1\right] \neq \varnothing$ and there are p and q with $\frac{1}{2} \le p < q < 1$ and $S \cap [p,q] = \varnothing$. Let $\bar{t} \in \left[\frac{1}{2}, 1\right]$ be such that

$$F(t) - \frac{1}{2} = \int_{S \cap \left[\frac{1}{2}, 1\right]} f(t)dt.$$

Then, $S^3 = S \cap \left[0, \frac{1}{2}\right] \cup \left[\frac{1}{2}, t\right]$ is prone to secession. As in the previous case, the shift from S to S^3 is a measure-preserving transformation. Thus, $m(S^3) = m(S)$. Moreover,

$$D(S)-D(S^3)=\int_{S\setminus S^3}(t-m(S))f(t)dt-\int_{S^3\setminus S}t-m(S)f(t)dt=\int_{S\setminus S^3}tf(t)dt-\int_{S^3\setminus S}tf(t)dt.$$

Consider now the difference

$$\int_{S\setminus S^3}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt-\int_{S^3\setminus S}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt.$$

It can be presented as

$$\int_{S\setminus S^3}tf(t)dt-\int_{S^3\setminus S}tf(t)dt+\int_{S\setminus S^3}x(t)f(t)dt-\int_{S^3\setminus S}x(t)f(t)dt.$$

The property (γ) yields

$$D(S)-D(S^3)<\int_S\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt-\int_{S^3}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt.$$

Finally, the inequality

$$\int_S\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt>D(S)+g.$$

implies that

$$\int_S\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt>D(S^3)+g.$$

and S^3 is prone to secession.

(*iv*) Suppose that there is $q>\frac{1}{2}$ such that $\left[\frac{1}{2},q\right]\subset S$ and $S\cap\left[1-q,\frac{1}{2}\right]=\varnothing$. Then $S^4=S\setminus\left[\frac{1}{2},q\right]\cup\left[1-q,\frac{1}{2}\right]$ is prone to secession: Since $D(S^4)\le D(S)$, the symmetry property (β) implies that the region S^4 is prone to secession.

It is easy to verify that the proof of the Lemma follows from (*i*) – (*iv*).

Proof of the Main Result: It suffices to demonstrate that if $g \geq g_{des}$, the allocation x_g is secession-proof. It is useful to recall that $g \geq g_{des}$, implies that (1) holds.

As we indicated in Section 4, Lemma 4.3 yields that for any secession-proof allocation $x \in X$, the following two conditions should be satisfied for all $t \in [0, 1]$:

$$\int_{L_t}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt\le g+D(L_t),$$

$$\int_{R_t}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt \le g\ldots + D(R_t).$$

Note that

$$\int_{Rt}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt = \int_I\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt -$$

$$\int_{Lt}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)\,dt = g + D(I) - \int_{Lt}\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt.$$

Thus, the necessary and sufficient condition for a cost allocation $x \in X$ to be secession-proof is that the inequality

$$D(I)-D(R_t) \le \int_{Lt} x\left(x(t)+d\left(t,\frac{1}{2}\right)\right)f(t)dt \le g + D(L_t)$$

holds for all $t \le \frac{1}{2}$. This inequality is equivalent to (3), given in Section 4:

$$D(I)-(R_t)-H(t) \le \int_0^t x(t)f(t)dt \le g + D(L_t) - H(T).$$

To proceed, we need the following Lemma:

Lemma A.1: For every $t \le \frac{1}{2}$, we have the following two equations:

$$-\left(D(R_t)+H(t)\right)' = \left(r(t)-\frac{1}{2}\right)f(t) \qquad (5)$$

$$\left(D(L_t)-H(t)\right)' = \left(2t-\frac{1}{2}-l(t)f(t)\right) \qquad (6)$$

Proof of Lemma A.1: Note that

$$-\left(D(R_t)+H(t)\right) = \int_0^t sf(s)\,dt - \int_{r(t)}^1 sf(s)\,ds - \int_0^t d\left(s,\frac{1}{2}\right)f(s)ds,$$

and

$$-\left(D(R_t)-H(t)\right)' = -tf(t)+2r'(t)r(t)f(r(t)) - \left(\frac{1}{2}-t\right)f(t).$$

But since

$$r'(t) = \frac{f(t)}{2f(r(t))},$$

it follows that, indeed,

$$-\left(D(R_t) + H(t)\right)' = \left(r(t) - \frac{1}{2}\right)f(t).$$

Similarly,

$$D(L_t) - H(t) = -\int_0^{l(t)} sf(s)dt + \int_{l(t)} sf(s)ds - \int_0 d\left(s, \frac{1}{2}\right)f(s)ds,$$

and

$$\left(D(L_t) - H(t)\right)' = -2l'(t)l(t)f(l(t))(l(t)) + tf(t) - (\frac{1}{2} - t)f(t).$$

Denote $a(t) = \frac{1}{2}$ for all $t \leq \frac{1}{2}$ and let us show that a (.) satisfies (3). Equation (5) implies that

$$\int_0 a(t)f(t)dt = D(R_t) - D(R_t) - H(t),$$

and (1) yields

$$\int_0 a(t)f(t)dt \leq g + D(L_t) - H(t).$$

Note that the assumption of gradually escalating median implies that the function a would be a solution of our problem if it were to satisfy the budget constraint. However, the value of $2\int_0^{\frac{1}{2}} a(t)f(t)dt$ is not necessarily sufficient to cover the government costs g. Let us, therefore, modify the function a by adding to each individual a fixed payment λ such that $\lambda = g - 2\int_0^{\frac{1}{2}}(r(t) - \frac{1}{2})f(t)dt.$

We shall show that inequality (3) would not be violated by the function x_g(t) = $a(t) + \lambda$, i.e.,

Lemma A.2:

$$D(I) - D(R_t) - H(t) \leq \int_0 x_g(t)f(t)dt \leq g + D(L_t) - H(t).$$

Proof of Lemma A.2: Recall that the left side of (3) was actually an equality for a. To show that it would still hold for x_g (t), one has to demonstrate that $\lambda \geq 0$. By (4),

$$\lambda = g - 2\int_0^{\frac{1}{2}}\left(r(t) - \frac{1}{2}\right)f(t)\,dt = g - 2D(I) + D2D\left(L_{\frac{1}{2}}\right) + 2H\left(\frac{1}{2}\right).$$

Since $2H\left(\frac{1}{2}\right) = D(I)$ and $D\left(L_{\frac{1}{2}}\right) = D\left(R_{\frac{1}{2}}\right)$, we have, by (1),

$$\lambda = g - D\ (I) + \left(L_{\frac{1}{2}}\right) + D\left(R_{\frac{1}{2}}\right) \geq 0.$$

Before turning to the right side of (3), consider the expression

$$g - H(t) + D\ (L_2) - \int_0^t x^g(T)f(t)dt.$$

Its derivative is $2t\ r(t) - l(t)$, which, by the assumption of gradually escalating median, is increasing in t. But, $1 - r\left(\frac{1}{2}\right) - l\left(\frac{1}{2}\right) = 0$, yielding $2t - r(t) - l(t) < 0$ for $t < \frac{1}{2}$. That is, the expression $c - H(t) + D(L_t) - \int_0^t x(t)f(t)dt$ is decreasing on the interval $\left[0, \frac{1}{2}\right]$. Thus, to complete the proof of the Lemma, it remains to verify that the right side of (3) holds for $t = \frac{1}{2}$ or

$$\frac{g}{2} \geq 2H\left(\frac{1}{2}\right) - 2D(L_{\frac{1}{2}}) = D(I) - D\left(L_{\frac{1}{2}}\right) - D\left(R_{\frac{1}{2}}\right),$$

which is guaranteed by (1).

This completes the proof of the main result.

Proof of Proposition 5.1: Consider the right side of (3) for the laissez-faire allocation LF, which can be rewritten as

$$tg \leq \frac{3t^2 - 2t}{4} + g.$$

Then, for all $t \leq \frac{1}{2}$

$$g \geq \phi(t) \equiv \frac{2t - 3t^2}{4(1-t)}.$$

It is easy to see that $\varnothing(.)$ is concave and its maximum given by a solution of the equation $3t^2 - 6t + 2 = 0$ whose root is $t^* = 1 - \frac{1}{\sqrt{3}} = .423$. Thus, $g \geq \varnothing(t^*) = 1 - \frac{\sqrt{3}}{2} = .134 > g_{sus}$.
That is, for the range of government costs g, satisfying $.125 < g < .134$, the inequality (4) is violated. Thus, the allocation *LF* is not secession-proof as for this range of values of g, there are regions L_t, in particular for $t = .423$, that are prone to secession.

Proof of Proposition 5.3: Consider a linear allocation $x^{\alpha} = \alpha t + g = \frac{\alpha}{4}$. The secession-proof conditions (4) for this allocation are:

$$\frac{t^2}{4} \le \frac{\alpha t^2}{2} + gt - \frac{\alpha t}{4} \le \frac{3t^2 - 2t + 4g}{4}$$

or, equivalently,

$$\left(\frac{\alpha}{2} - \frac{3}{4}\right)t^2 + \left(g - \frac{\alpha}{4} + \frac{1}{2}\right)t - g \le 0$$

for all $t \in \left[0, \frac{1}{2}\right]$. Since $\alpha \le 1$, the left-hand side of the above inequality is a concave function whose maximal value is

$$\frac{(4g - \alpha + 2)^2}{16(3 - 2\alpha)} - g$$

obtained at

$$\tilde{t} = \frac{4g - \alpha + 2}{2(3 - 2\alpha)}.$$

Two cases should be considered:

Case 1: $\tilde{t} \le \frac{1}{2}$. This occurs when $4g + \alpha \le 1$. Note that since $g \le \frac{1}{8}$, it also implies that $\alpha \le 4g$. Simple algebra shows that

$$\frac{(4g - \varepsilon + 2)^2}{16(3 - 2\alpha)} - g \le 0$$

or

$$\alpha^2 + \alpha(24g - 4) + (4 + 16g^2 - 32g) \le 0.$$

The last inequality holds if and only if

$$\psi(g) \le \alpha \le \psi(g).$$

where

$$\psi(g) = 2 - 2 - 12g - \sqrt{128g^2 - 16g},$$
$$\psi(g) = 2 - 12g + \sqrt{128g^2}\ 16g.$$

Since $g \le \frac{1}{8}$, it is simple to verify that

$$\psi(g) \leq 1 - 4g \leq \psi(g).$$

Therefore, in Case (1), the range of secession-proof values of α is the interval $[\max(0, \psi(g)), \max(0, 1-4g)]$.

Case 2: $\tilde{t} \geq \frac{1}{2}$. This occurs when $4g + \alpha \geq 1$. Note that the function $\left(\frac{\varepsilon}{2} - \frac{3}{4}\right)t^2 + \left(g - \frac{\alpha}{4} + \frac{1}{2}\right)t - g$ is increasing on the interval $\left(0, \frac{1}{2}\right)$ and its value at $t = \frac{1}{2}$ is $-\frac{1}{16} - \frac{5_g}{8}$, which is always negative. Therefore, the only upper bounds for α are 1 and $4g$. Thus, in Case (2), the range of secession-proof values of α is the interval $[\max(0, 1-g), \min(4g, 1)]$.

It is easy to see that the function ψ decreases and $\psi(g) = 0$ at $g = 1 - \frac{\sqrt{3}}{2}$.

If $\frac{1}{8} \leq g \leq 1 \frac{\sqrt{3}}{2}$, the range of secession-proof values of α is the union of the two intervals, $[\psi(g), 1-4g]$ (generated by Case 1) and $[1 - 4g, 4g]$ (generated by Case 2). Thus, we obtain the interval $[\Psi(g), 4g]$.

If $\left[1 \frac{\sqrt{3}}{2} \leq g \leq \frac{1}{4}\right]$, the range of secession-proof values of α is the union of the two intervals, $[0, 1 - 4g]$ (generated by Case 1) and $[1 - 4g, 4g]$ (generated by Case 2). Thus, we obtain the interval $[0, 4g]$.

If $g \leq \frac{1}{4}$, only Case 2 can occur and the range of secession-proof values of α is the interval $[0, 1]$.

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