Stress Testing of Financial Systems: An Overview of Issues, Methodologies, and FSAP Experiences

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Prepared by Winfrid Blaschke, Matthew T. Jones, Giovanni Majnoni, and Soledad Martinez Peria

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Abstract

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

The paper has three objectives. After a general introduction to some of the concepts and basic techniques of stress testing, the paper gives an overview of some of the conceptual issues involved in evaluating risks at the aggregated level of financial systems. Second, this study provides a basic framework and toolkit for conducting stress tests. Finally, the paper reviews some of the stress-testing analyses conducted in the context of the Financial Sector Assessment Program (FSAP) and suggests simplified approaches to deal with situations where the quantity and quality of the data is less than ideal.

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I. INTRODUCTION

1. Recent financial crises in Asia and other emerging markets have highlighted the importance of financial system stability to macroeconomic performance. Innovation and diversification in the financial services industry, coupled with dramatic increases in cross-border capital flows, have increased the complexity of supervising financial institutions and ensuring the stability of financial systems. Financial regulators have responded to this challenging environment by increasing cooperation with other regulators, as well as encouraging financial institutions to employ more sophisticated approaches to risk management. Multilateral institutions have played a facilitating role by coordinating the responses of different regulatory bodies to innovations in financial markets, and by providing analysis of country exposures and experiences. As part of this process, the IMF and World Bank have instituted the Financial Sector Assessment Program (FSAP). This program endeavors to identify vulnerabilities in the financial systems of member countries that could have significant macroeconomic consequences.²

2. A key component of the FSAP is an analysis of the resilience of financial systems to different shocks. Stress testing is one of the tools that can be employed to assess the vulnerability of portfolios to abnormal shocks and market conditions. This paper considers the experience of stress testing in the initial phase of the FSAP as a way of highlighting some of the issues involved in assessing financial systems as a whole.

3. This paper has three objectives. After a general introduction to some of the concepts and basic techniques of stress testing, the paper gives an overview of some of the conceptual issues involved in evaluating risks at the aggregated level of financial systems. Second, it provides a basic framework and toolkit for conducting stress tests. Finally, it reviews stress-testing analyses conducted in the context of the FSAP and suggests simplified and practical approaches to deal with situations where the quantity and quality of the data are less than ideal.

4. The paper is organized as follows. Section II provides an overview of some of the conceptual issues relevant to the conduct of stress tests, beginning with a discussion of stress testing at the portfolio level. The section continues by considering some of the issues involved in evaluating risks at the aggregated level of a financial system. Section III presents a basic toolkit for conducting stress tests. The section draws on finance and risk management concepts, on regulatory frameworks, and on the experience gained through the FSAPs. Section IV concludes.

² See Appendix I for a summary of the nature and purposes of the FSAP.
II. **Overview of Stress Testing Issues**

A. **Stress Testing at the Portfolio Level**

5. The term stress testing describes a range of techniques used to assess the vulnerability of a portfolio to major changes in the macroeconomic environment or to exceptional, but plausible events. The objective of a stress test is to make risks more transparent by estimating the potential losses on a portfolio in abnormal markets. Stress tests are often used to complement the internal models and management systems used by financial institutions for capital allocation decisions.

6. For most asset markets, the history of returns does not provide sufficient information about the likelihood of extreme events. Stress tests perform a useful role by complementing statistical models of returns with information about the behavior of a portfolio under exceptional circumstances. Stress tests are usually applied at the level of an individual institution or portfolio and are most often used to measure market risk. However, stress tests can be used to analyze individual components of a portfolio as well as other forms of risk. While most stress testing techniques were developed for individual portfolio applications, they can be applied to aggregate portfolios, as discussed in the following section, on aggregate stress tests.

**Specification**

7. Figure 1 shows a sequence of the different decision elements of a stress test. Stress tests begin with the specification of the type of risks to be considered and the appropriate models to use. Stress tests can focus on individual risks, such as credit risk or interest rate risk, or can encompass multiple risks. The next element of a stress test involves deciding on the range of factors to include, followed by the specification of scenarios. Stress tests can involve estimating the impact of a change in a single risk factor (a sensitivity test), or the effect of a simultaneous move in a group of risk factors (a scenario analysis). Scenarios can be designed to encompass both movements in individual market variables (such as prices) and changes in the underlying relationships between different asset markets (such as correlations and volatilities). Stress testing can be based on historical scenarios, employing shocks that occurred in the past, or can be based on hypothetical scenarios, constructed to take account of plausible changes in circumstances that have no historical precedent.

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4 Two other techniques that are often included under the rubric of stress testing are extreme value theory, which applies statistical analysis to the tails of return distributions, and the maximum loss approach, which estimates the combination factors that would cause the greatest loss to a portfolio. See Longin (1999) for a discussion of extreme value theory.
Figure 1. Decision Sequence for the Conduct of a Stress Test of Individual Portfolios

Type of risk model

- Market risk (interest-rate risk, exchange rate risk)
- Credit risk
- Other (liquidity, operational)

Type of stress test

- Sensitivity (single factor)
- Scenario (multiple factors simultaneously)
- Other (Extreme value, maximum loss)

Type of shock

- Individual market variables (e.g., prices or interest rates)
- Underlying volatilities
- Underlying correlations

Type of scenario

- Historical
- Hypothetical
- Monte Carlo simulation

Core assets to be shocked, peripheral assets to be shocked, size of shocks and time horizon

Aggregation (across business units, product lines) and re-pricing of portfolio (marked to market), comparison with present portfolio, adjustment to present portfolio and risk management techniques
8. Creating scenarios using historical data is perhaps the most intuitive approach, since the events actually happened and hence could plausibly recur. Under this approach, the pattern of changes in market risk factors observed during various historical episodes is applied to the portfolio to measure the potential losses should such a situation occur again. The disadvantage of this approach is that it is backward looking, and may lose relevance over time as markets and institutional structures change.

9. Hypothetical scenarios have the advantage that they can allow a more flexible formulation of potential events, as well as encouraging risk managers to be more forward-looking. Hypothetical scenarios can be constructed by shocking market factors, volatilities, or correlations. This approach helps to identify the sensitivity of a portfolio to different risk factors. Hypothetical scenarios can also be used to anticipate particular events to which a portfolio may be most vulnerable, such as a flight to quality. Finally, simulation techniques can be applied to a specific portfolio to search for scenarios that would cause the greatest losses. The main drawback with hypothetical scenarios is the difficulty in determining the likelihood of the event occurring, since it is beyond the range of experience. The difficulty of determining the likelihood of an event also applies to historical scenarios, but historical scenarios do have the advantage that they were observed and hence some information on the relative likelihood is available.\(^5\)

10. The specification of scenarios involves a sequence of decisions, such as which of the core assets are to be stressed, what the relevant risk factors are, how much to stress them and over what time period, and how to treat the peripheral assets. Once specified, the scenarios are applied to the portfolio to determine the potential change in the present value. This usually involves marking the portfolio to market using the new set of prices generated by the scenario. The process of creating different scenarios is the most difficult and controversial aspect of stress testing. An ideal stress test needs to be relevant to the current portfolio; it should include changes in relevant market rates, it should encompass potential regime shifts and market illiquidity; and it should consider the interaction of different risks such as market risk and credit risk. These requirements can impose significant resource costs, and involve a great deal of practical expertise and judgment by the parties involved. In practice, stress tests do not meet these ideals, because of the computational complexities involved or because of the lack of adequate data.

**Stress tests and bank regulation**

11. Stress testing techniques began to be applied widely by internationally active banks in the early 1990s, and have now spread to most large financial institutions. Banking supervisors and regulators have sanctioned the use of stress tests as an important component

\(^5\) See Berkowitz (1999) for more discussion of this point.

\(^6\) See also Basel Committee on Banking Supervision (1997) for a discussion of interest rate risk and stress testing.
of the internal-models approach to market-risk monitoring. The Basel Committee on Banking Supervision (BCBS) highlighted the need for stress testing when it published the Amendment to the Capital Accord to Incorporate Market Risks in January 1996. In that document, the BCBS states:

Banks that use the internal models approach for meeting market risk capital requirements must have in place a rigorous and comprehensive stress-testing program. Stress testing to identify events or influences that could greatly impact banks is a key component of a bank’s assessment of its capital position.

Banks’ stress scenarios need to cover a range of factors that can create extraordinary losses or gains in trading portfolios, or make the control of risk in those portfolios very difficult. These factors include low-probability events in all major types of risks, including the various components of market, credit, and operational risks. Stress scenarios need to shed light on the impact of such events on positions that display both linear and non-linear price characteristics (i.e., options and instruments that have options-like characteristics).

Banks’ stress tests should be both of a quantitative and qualitative nature, incorporating both market risk and liquidity aspects of market disturbances. Quantitative criteria should identify plausible stress scenarios to which banks could be exposed. Qualitative criteria should emphasize that two major goals of stress testing are to evaluate the capacity of the bank’s capital to absorb potential large losses and to identify steps the bank can take to reduce its risk and conserve capital. This assessment is integral to setting and evaluating the bank’s management strategy and the results of stress testing should be routinely communicated to senior management and, periodically, to the bank’s board of directors.

Banks should combine the use of supervisory stress scenarios with stress tests developed by banks themselves to reflect their specific risk characteristics.

12. National bank supervisors and regulators have issued their own guidelines on stress testing, in accordance with the principles laid down by the BCBS. The imprimatur of supervisory authorities for institutions to adopt an internal-models approach to risk management has provided great impetus to the development of stress-testing methodologies.

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7 See also Basel Committee on Banking Supervision (1997) for a discussion of interest rate risk and stress testing.

8 See also Basel Committee on Banking Supervision (1997) for a discussion of interest rate risk and stress testing.

Stress-testing methods are now regarded by regulators and market participants as an essential component of an effective risk management system.

The role of stress tests in risk management

13. Stress tests provide information on the sources of risk in a portfolio that is relevant for decision makers at all levels of management in a financial institution. At the trading level, stress tests demonstrate the potential vulnerability of a particular position or product. At the managerial level, stress tests enable a comparison of risks across different asset classes and exposures, and highlight the need for risk limits and controls. At the executive level, stress tests provide a way of comparing the risk profile of the institution with the risk appetite of the owners, helping to guide decisions on the optimal allocation of capital within the institution. For all levels of management, stress tests can help to determine if the return on a particular product or position is commensurate with the level of risk.

14. Stress tests have several shortcomings that can limit their applicability or usefulness. If the underlying model being stressed is incorrectly specified or estimated, then the conclusions drawn from a stress test may be invalid. Furthermore, stress tests that are not relevant to an institution's current portfolio or that ignore spillovers between markets and risks can provide misleading information to managers. A poorly specified stress test can provide a false sense of security to risk managers, causing them to underestimate their exposures and assume even greater risk. Stress tests also do not give any indication of the likelihood of a particular scenario, but answer the question "How much could be lost?" instead of "How much is likely to be lost?"

B. Aggregate Stress Testing of Financial Systems

15. An aggregate stress test can be defined as a measure of the risk exposure of a group of reporting firms to a specified stress scenario. Aggregate stress tests are different from portfolio stress tests because they have different objectives. The objective of an aggregate stress test is to help regulators identify structural vulnerabilities and overall risk exposures in a financial system that could lead to the disruption of financial markets. The emphasis of aggregate stress tests is on potential externalities and market failures, for example, when

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10 See Basel Committee on Banking Supervision (1999c), for details of the mixed performance of banks' stress-testing exercises during the events of late 1998. Many banks reported that their stress tests "did not anticipate the incidence or magnitude of certain shocks."


12 This section draws on Committee on the Global Financial System (2000).

there is an evaporation of liquidity or a flight to quality. In contrast, the objective of a portfolio stress test is to assist in the process of managing risks within a firm, and ensuring the optimal allocation of capital across risk-taking activities. Stress tests of individual portfolios attempt to account for possible market breakdowns, but only from the perspective of its effect on the value of the portfolio.

16. Stress tests performed by individual firms can have a tendency to underestimate the impact on market liquidity when many firms attempt to reduce their exposures simultaneously. The negative externality on market liquidity that arises from a phenomenon such as a flight to quality is often not fully incorporated into stress test scenarios of individual portfolios. Aggregation of stress-testing scenarios has the potential to expose these vulnerabilities and hence highlight the shortcomings inherent in the modeling assumptions of individual firms.

Scope

17. Stress testing of financial systems presents a different set of methodological challenges to stress tests of individual portfolios. The first issue to consider is the question of scope: while the portfolios of individual financial institutions can be clearly delineated, the aggregate portfolio of the financial system can be more problematic to define. In situations where financial institutions have diverse portfolios of complex interlocking claims, it may be difficult to aggregate across institutions. For example, aggregating across bank exposures to the interbank market may give a small figure for net exposure, but the gross figure may be quite large and a significant source of systemic risk.

18. Another aspect of scope that may present difficulties is the choice of financial institutions to include in the analysis. From the perspective of financial system stability, attention can be restricted to the major players such as banks if nonbank financial institutions do not present a systemic threat to the operation of the financial system, e.g., through a possible disruption to the payment system or the process of intermediation. However, this approach may overlook potential vulnerabilities in the financial system if nonbank institutions have major risk exposures.

19. Another issue that arises in the discussion of the scope of aggregate financial systems is the role of foreign ownership. Countries that have a significant market presence of foreign-owned banks may find the risks to their financial system increased or diminished by foreign ownership. A country with a subsidiary or branch of a large parent bank may face the potential for disruption in the event of difficulties in the parent (e.g., the collapse of BCCI). On the other hand, letters of comfort or implicit guarantees from a foreign parent bank may provide an additional buffer to disruptions arising from the domestic financial system. Foreign banks may also integrate the risk management and open positions of a local subsidiary into their global operations. Thus, foreign-owned banks have the potential to both absorb and transmit shocks, depending on the stability of the parent group. It may be difficult to determine if nonbank institutions are a source of systemic risk without including them in the analysis, which would suggest that the broadest possible approach as a first step, then restricting attention to fewer institutions once evidence is presented of a lack of systemic
risk. Thus, in conducting any systemic analysis, excluding foreign banks from the sample should only be considered once evidence is shown that the foreign banks present significantly lower risk to the financial system.

Aggregation

20. The second methodological issue that arises in the analysis of aggregate financial systems is the process of aggregation. Stress testing of an aggregate financial system can be accomplished by compiling the results of stress tests to individual portfolios, or by applying a common stress test to an aggregated portfolio. Compiling the results of stress tests done by individual banks is only viable in countries where banks employ sophisticated risk management techniques internally. In other countries, it may be necessary for the supervisor to conduct the analysis, based on data provided by individual firms.

21. Compiling the results of stress tests performed by individual banks presents difficulties of comparability, since each institution may employ different methodologies and modeling assumptions. The choice of risk measurement methodology involves a tradeoff between accuracy and computational burden. The differing degrees of sophistication among participants will imply that each institution is at a different point in that tradeoff, so aggregating across such a diverse sample is likely to introduce substantial measurement error. A lack of commonly accepted methodology for valuing certain complex financial products can further exacerbate this problem. An alternative approach may be for supervisors to provide a set of detailed scenarios and modeling assumptions and have banks perform the analysis using their own models. The weakness of this approach is that supervisors may have difficulty in persuading banks to perform a large number of common scenarios, and so will have to restrict their attention to a few key scenarios. Even the most carefully chosen scenarios may be insufficient to develop an accurate picture of the risks to a portfolio if the number of scenarios is small, and the problems associated with differences in methodologies will remain.

22. The second approach to aggregation is to gather portfolio data from the relevant institutions and for the supervisor to perform the stress tests, using common scenarios and methodology. This approach has the advantage that the aggregation of results is more meaningful, since the individual components are more comparable and the methodology is identical. However, this approach imposes a higher burden on the supervisory authorities, as they must have the resources and specialized expertise necessary to conduct such an analysis. In addition, supervisors need access to disaggregated data on individual portfolio positions to ensure an accurate analysis. Maximizing the usefulness of this approach thus requires supervisors to have detailed knowledge of the portfolio structure and strategic direction of the firms involved so that the most relevant stress-testing scenarios are considered.

23. Aggregating the results of individual stress tests performed by financial institutions is likely to provide the most informative picture of the risks and vulnerabilities of a financial

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14 See Pritsker (1997) and Gibson (1997) for a discussion of this tradeoff.
system. This is because individual institutions are likely to know their own risk exposures in greatest detail, and they have the strongest incentive to ensure an accurate and relevant series of stress tests. The supervisory authorities can promote good stress-testing methodologies by encouraging a dialogue between market participants, and collating the results of different scenarios (while maintaining confidential and proprietary information). By making common information available to all participants, regulators can give a better picture of likely scenarios and impacts on market liquidity assumed or used by participants in their tests. Such an approach will not only provide the supervisor with information on the major vulnerabilities in the financial system, but may also promote better risk management by market participants.

24. Aggregation is clearly an important and difficult issue in stress testing. Questions of how to aggregate or compare across institutions or countries and deciding how to interpret the results of such an exercise (e.g., whether one aggregation is “riskier” than another) are areas that are not well developed in the literature. While these issues are beyond the scope of this paper, they do present an interesting avenue for future research.

III. A BASIC STRESS TESTING TOOLKIT

25. The basic sequence of decisions outlined in Figure 1 provides a framework for approaching the issue of stress testing. For each type of risk (interest rate risk, market risk, exchange rate risk, credit risk, liquidity risk, etc.) it is necessary to decide on the type of stress test to perform (sensitivity, scenario, extreme value, maximum loss); the type of shock to apply (to individual market variables, to underlying volatilities, or to correlations); the type of scenario to consider (historical, hypothetical, Monte Carlo simulation); as well as which assets are to be shocked, by how much, and over what time period. For an aggregate stress test, it is also important to decide which institutions to include in the analysis, and how to aggregate, present, and interpret the results.

26. This section discusses the analytical toolkit necessary for evaluating the major risks that financial institutions assume in their operations. Each subsection begins with an overview of the models applicable to measuring that type of risk, followed by a discussion of approaches to stress testing that particular risk. Although each risk is treated separately, in practice the measurement and estimation of the different risks can be considered jointly, especially for multivariate approaches such as the Value-at-Risk framework.

27. For each individual risk, we also discuss the recent FSAP experiences to illustrate the type of limitations that may be confronted when conducting stress tests in practice. Bearing in mind some of these limitations, simplified approaches are proposed that can be used under such circumstances. Data availability and the sophistication of supervisors and financial institutions will determine the best approach to take for each risk. Individual financial institutions will normally be able to stress test their portfolios more accurately and comprehensively than supervisors. Supervisors, in turn, often have an advantage over outside assessors.
A. Interest Rate Risk$^{15}$

Definition and measurement

28. Interest rate risk is the risk incurred by a financial institution when the interest rate sensitivity of its assets and liabilities are mismatched. Interest rate changes can affect the interest income and interest expenses of a financial institution, as well as other interest sensitive components of its balance sheet. Changes in interest rates can affect the market value of assets and liabilities of the institution, since the present value of future cash flows is sensitive to changes in interest rates. Interest rate risk can be analyzed using a variety of methods. This section will survey two common approaches: the gap model and the duration model.

29. Most approaches to measuring the interest rate risk exposure of an institution begin with the compilation of a maturity/repricing schedule for all assets and liabilities. This requires a financial institution to sort its assets and liabilities according to their interest rate sensitivities (time to repricing for floating rate instruments, maturity for fixed rate instruments), and report assets and liabilities in a few time categories or “buckets.” The “gap” is the difference in the flow of earnings on the holdings of assets and liabilities in each bucket. The gap in each time band or maturity bucket shows how net interest income could be affected by a given change in interest rates. A snapshot of the repricing schedule is often provided by banks in their published accounts or annual reports.$^{16}$

Repricing or maturity gap model$^{17}$

30. The repricing-gap model of interest rate risk is based on the difference between the flow of interest earned by a financial institution on its assets and the flow of interest paid on its liabilities. For any given change in interest rates, $\Delta R$, the repricing gap can be used to calculate the change in net interest income in each bucket $i$, and for the total portfolio:

$$\Delta \text{Net interest income}_i = GAP_i \times \Delta R,$$

$$\Delta \text{Net interest income} = \text{Cumulative GAP} \times \Delta R. \tag{1}$$

31. While the repricing model provides useful information on the maturity mismatches in a portfolio, it suffers from several shortcomings. The repricing model assumes that changes in interest rates only affect the income position of an institution and not the market value of its assets, effectively valuing assets and liabilities at book value and ignoring capital gains

$^{15}$ A fuller discussion of the different models is provided in Appendix II.

$^{16}$ See Appendix II for a numerical example.

$^{17}$ This section is based on Saunders (2000).
and losses. The repricing model also ignores problems of aggregation within maturity buckets and the cash flows on assets and liabilities before their next repricing.

32. A second type of "gap" model that can be used to assess interest rate risk is the maturity-gap model, which is based on the weighted-average maturity\(^{18}\) of the assets and liabilities of the financial institution, defined as follows:

\[
M^A = \sum_{i=1}^{N} w_i^A M_i^A \quad \quad M^L = \sum_{i=1}^{N} w_i^L M_i^L,
\]

where \(M^A\) = weighted average maturity of assets \\
\(M^L\) = weighted average maturity of liabilities \\
\(M_i^A\) = maturity of asset with a maturity given by \(i\) \\
\(M_i^L\) = maturity of liability with a maturity given by \(i\) \\
\(w_i^A\) = weight of asset \(i\) in portfolio, measured by proportion of total market value \\
\(w_i^L\) = weight of liability \(i\) in portfolio, measured by proportion of total market value

A financial institution may be interested in the maturity gap, which is the difference between the weighted-average maturity of its assets and liabilities:

\[
GAP^{Maturity} = M^A - M^L.
\]

33. A severe mismatch in the maturity of its assets and liabilities exposes a financial institution to interest rate risk. A rise in interest rates will reduce the market value of assets and liabilities, with the reduction being greater for longer maturities. If interest rates rise and a financial institution has a positive maturity gap \((M^A > M^L)\), the institution will face a larger fall in the value of its assets than its liabilities, reducing its equity or net worth. Thus, the weighted-average maturity provides useful information on exposures to changes in interest rates.

34. However, the weighted-average maturity measure is not a perfect measure of interest rate risk. A bank that matches the maturity of its assets with the maturity of its liabilities may still be exposed to losses from changes in interest rates. This situation may arise if the timing of the cash flows on assets and liabilities is different. To avoid this problem, the measure of duration provides a more accurate measure of exposure to interest rate risk.

**Duration model\(^{19}\)**

35. Duration can be defined as a measure of the interest sensitivity of an asset that takes into account the maturity of the asset as well as the timing of cash flows. Duration can be calculated as the weighted-average time-to-maturity, using the present value of cash flows as

---

\(^{18}\) Where maturity is defined as the time to next repricing.

\(^{19}\) This section is based on Saunders (2000, chap. 9) and Jorion (1997, chap. 6).
weights. (See Appendix II for more details). Duration can also be defined as the interest
elasticity (sensitivity) of the price of the asset, \( P \), to small changes in the yield, \( R \).

36. Once a measure of duration is derived for a security or a group of assets in a portfolio,
the financial institution can use the duration gap to analyze its exposure to interest rate risk.
The duration gap is the difference between the duration of the assets and liabilities of the
institution:

\[
GAP_{\text{Duration}} = D^A - D^L.
\]

Portfolio managers can use the duration gap to "immunize" the portfolio, or protect it against
changes in interest rates. Immunizing a portfolio involves matching the gains and losses in
the value of assets from changes in interest rates with the gains and losses in the value of
liabilities.

37. One of the weaknesses in this approach to estimating duration is the use of a single
discount factor. The specification of a single interest rate, \( R \), to calculate the discount factor
implies that the yield curve is flat, so changes in interest rates thus imply a parallel shift in
the yield curve. Alternative measures of duration can account for the possibility of changes in
the shape of the yield curve, using specific discount factors for each maturity, as suggested
by different values of interest rates at different points of the term structure. Another
weakness of the above formulation of duration is that it is only accurate for small changes in
interest rate yields. Since stress tests typically involve large changes in interest rates, we can
include second order terms to account for convexity. Convexity shows how duration changes
in response to changes in interest rate yields, and permits a more accurate estimate of the
price sensitivity of an asset to changes in yields.\(^{20}\)

Selection of institutions

38. Since the primary objective of stress tests is to identify vulnerabilities in the financial
system, the selection of institutions to include in the analysis should be based on their
importance to the financial system. The analysis should include banks and financial
institutions that have major interest rate exposures and significant market share, in order to
capture all institutions whose failure could cause instability in the financial system or
interrupt the process of intermediation. Since almost all banks and intermediaries assume
some form of interest rate risk (because of their liquidity transformation role), the analysis
could begin by considering all reporting banks. The sample can be narrowed as appropriate,
by excluding institutions with only minor exposures and whose exclusion would not
materially influence the results.

\(^{20}\) For a discussion of more sophisticated measures of duration, see Bliss (1997).
Nature, type, and size of shocks

39. Once the exposure of the portfolio to interest rate risk is estimated, the next step in performing a stress test is to specify the nature of the shocks to be applied. For interest rate risk the simplest form of shocks are a parallel shift in the yield curve, a change in the slope of the yield curve, and a change in the spread between different interest rates with the same time horizon. These shocks are typically applied to the level of interest rates, but the underlying volatilities and correlations can also be shocked. The size of the shock can be based on historical experience, or can be a hypothetical scenario.

40. The Derivatives Policy Group (1995) recommends the use of a set of basic shocks for interest rate risk, which include parallel shifts in the yield curve of 100 basis points (up and down), steepening and flattening of the yield curve by 25 basis points; and an increase and decrease in the 3 month yield volatilities by 20 percent of prevailing levels. The commercial bank examination manual of the U.S. Federal Reserve notes that a 200 basis point parallel shift in the yield curve represents a plausible stress scenario, based on the experience of the period 1974–94, when changes in yields on constant maturity treasury securities exceeded 193 basis points only 1 percent of the time.21

41. These examples illustrate some of the possible scenarios that could be considered in assessing the sensitivity of a portfolio to interest rate risk. Larger shocks should be considered, especially in countries that have experienced or are likely to experience a greater degree of interest rate volatility than the U.S. Historical experience can guide the selection of appropriate scenarios. An assessment of the present macroeconomic environment and outlook can also form the basis of a hypothetical scenario.

42. The time horizon to be considered for an interest rate shock depends on the nature of the analysis. A longer time horizon allows for a greater degree of variation and possibility of larger shocks, but if the balance sheet of the institution concerned is likely to change frequently, then a one-year time horizon may be less relevant. Many stress tests consider a short time frame such as a 10-day or one-month horizon. Scenarios with shorter time horizons tend to be applied to institutions with large trading portfolios that are subject to more risk on a daily basis. Stress tests that consider longer horizons are more applicable to institutions with balance sheets that are more stable over time. Stress tests with longer horizons provide information that is more relevant for an analysis of the structural positions in the financial system, rather than the short-term positions taken by more active market participants.

Analyzing interest rate risks in the FSAPs and in practice

43. Table 1 summarizes the main features of the interest rate risk stress tests conducted in the twelve FSAPs recently completed. Most of the FSAPs examined interest rate risk by using maturity buckets and gap analysis. Few studies had the necessary information to

21 See Board of Governors of the Federal Reserve System (1994, sec. 4090.1, p. 9, fn. 6).
conducted duration analysis. An important limitation of the tests conducted is that the scenarios of interest rate increases considered were largely ad hoc. Two reports investigated the impact of an increase in interest rates equivalent to the largest shock observed in the past, over a given horizon.

Table 1. Interest Rate Risk Stress Tests in the Context of the FSAPs

<table>
<thead>
<tr>
<th>Data</th>
<th>Models</th>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>- In ten out of the twelve reports, individual bank data for all banks or for those banks concentrating more than 50 percent of bank deposits, assets, or loans were used.</td>
<td>- In four of twelve countries, vulnerability to changes in interest rates was analyzed using duration analysis.</td>
<td>- Impact of hypothetical interest rate increases was examined in eight cases. Changes in interest rates assumed were ad hoc. Parallel shifts in the yield curve were examined in three of five cases.</td>
</tr>
<tr>
<td>- In two cases, aggregate data for the overall financial system or for distinct groups of banks were used.</td>
<td>- Maturity buckets and gap analysis were used in 6 cases</td>
<td>- Two reports investigated the impact of an increase in interest rates equivalent to the largest shock observed over a given time horizon.</td>
</tr>
<tr>
<td></td>
<td>- A relatively sophisticated VAR scenario model was used in one case.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- No interest rate risk stress test was conducted in one country.</td>
<td></td>
</tr>
</tbody>
</table>

44. Some of the limitations observed in the analysis conducted by the FSAPs reflect the difficulties encountered when dealing with individual financial systems in practice. Inadequate data, time or resource constrains, or even lack of expertise may prevent countries from conducting sophisticated stress tests. When this is the case, it is important to be able to implement alternative methodologies that can circumvent some of the limitations in the data or in the expertise of bank supervisors.

45. For example, gap analysis can be applied with varying degrees of sophistication, depending on the information available and the complexity of the balance sheet under consideration. At a very basic level, the gap in each time band can be multiplied by various scaling factors to provide information on the sensitivity of income to changes in interest rates. For example, the average coupon on assets and liabilities in each band can be calculated and applied to the gap as a way of scaling the change in net interest income.

46. Even if no data is available on the duration of assets and liabilities, it might still be possible to obtain lower and upper bound estimates of the change in bank equity as a result of a change in interest rates. For example, taking a portfolio with a positive duration and assuming that the average duration of assets is equal to the shortest maturity observed for assets and the duration of liabilities is equal to the longest maturity observed for liabilities, it is possible to get an estimate of the lower bound fall in equity as a result of a change in interest rates. On the other hand, by assuming that assets have the longest duration possible and liabilities have the shortest duration possible, we can get an upper bound estimate of the expected fall in bank equity as a result of movements in interest rates.

47. Both for the gap and duration analysis, several benchmarks can be used to determine the relevant size of the interest rate change to consider. An obvious choice is to examine the impact of the maximum interest rate change observed in recent history, or some multiple of the standard deviation. Another possible scenario would to include interest rate changes
similar to those recently experienced by neighboring countries of similar size and level of economic development.

B. Exchange Rate Risk

Definition and measurement

48. Exchange rate risk is the risk that exchange rate changes can affect the value of an institution’s assets and liabilities, as well as its off-balance sheet items. Exchange rate risk can be direct, i.e., where a financial institution takes or holds a position in foreign currency, or indirect, i.e., where a foreign exchange position taken by one of the financial institution’s borrowers or counter-parties may affect their creditworthiness. Furthermore, exchange rate risk can arise from positions in foreign currency as well as positions in local currency that are indexed to foreign exchange rates. The latter would therefore need to be included in the calculation of foreign exchange rate risk.

49. The most commonly used measure of foreign exchange exposure is an institution’s net open foreign exchange position. The position should be calculated according to an internationally recognized methodology, such as the one recommended by the Basel Committee on Banking Supervision. Normally, both on- and off-balance sheet foreign exchange positions should be included in the stress tests, except where off-balance sheet foreign exchange positions have been confirmed as minimal. Under the Basel methodology, it is necessary to first calculate a bank’s net open position in each currency, by summing the following items.\(^22\)

- the net spot position (i.e., all asset items less all liability items, including accrued interest, denominated in the currency in question);
- the net forward position (i.e., all amounts to be received less all amounts to be paid under forward foreign exchange transactions, including currency futures and the principal on currency swaps not included in the spot position);
- guarantees (and similar instruments) that are certain to be called and are likely to be irrecoverable;
- net future income/expenses not yet accrued, but already fully hedged (at the discretion of the reporting bank);
- any other item representing a profit or loss in foreign currencies, depending on the particular accounting conventions in different countries;

\(^{22}\) For more details, see Basel Committee on Banking Supervision, *Amendment to the Capital Accord to incorporate market risks*, Basel (January 1996)
• the net delta-based equivalent of the total book of foreign currency options.²³

50. The resulting net open position in each currency can be stress-tested against variations in the exchange rate of a particular currency (sensitivity analysis). If the overwhelming majority of foreign exchange exposure is in one currency, one may use either the net open position by currency, or the aggregated net open position. The latter may be necessary if a breakdown by currency is not available. For institutions holding significant short positions in foreign currency options, it should be taken into account that for large exchange rate moves as used in stress-testing, a simple linear approximation (such as the delta) of exchange rate sensitivity may not be sufficient, and more accurate second order approximations (such as the gamma) may need to be considered.²⁴

51. For those financial institutions that use internal models, the net open positions in each currency should still be available, before aggregation into the overall position, and the model should be able to stress test by currency. Institutions using internal models should be able to provide the results of the stress tests, using their own measurement techniques. Internal models would also lend themselves to scenario analysis, taking account of correlations between currencies, although these correlations may break down during crises (see below).

52. Since foreign exchange exposures can, and often do, change rapidly and frequently, the results of stress tests are likely to be outdated very quickly, especially when they are based upon supervisory reports or published annual reports of financial institutions. For an up-to-date view of foreign exchange risk, it is therefore indispensable to have current data provided by the financial institution itself. This may then be supplemented by historical data from various sources and supervisory experience to determine the typical direction and approximate size of an institution’s foreign exchange exposure over time.

Selection of institutions

53. The first step in designing a stress test for foreign exchange risk is to define the sample of institutions that should be included. Naturally, only those institutions that are systemically important and have a significant exposure to foreign exchange risk should be included.²⁵ Considering that foreign exchange derivatives are the most widely used

²³ The “delta-based equivalent” is calculated by multiplying the market value of the underlying option by the “delta.” The “delta” is the first-order or linear approximation of changes in the value of the option with respect to the exchange rate.

²⁴ Under the Basel Committee’s methodology the gamma impact is calculated using a Taylor series expansion, by multiplying the square of 8 percent of the market value of each option position.

²⁵ For a specific yardstick, one could use the Basel Committee criteria for the imposition of capital requirements, i.e., a foreign currency business (the greater of the sum of an institution’s gross long positions and the sum of its gross short positions in all foreign (continued)
derivatives instruments worldwide (see below), one cannot automatically assume that financial institutions have low foreign exchange exposure just by looking at their balance sheet. Similarly, even if strict limits on foreign exchange exposure apply to the banking system, other financial institutions such as finance companies may take large foreign exchange positions. If this is the case, it may be necessary to include them in the stress tests.

54. Financial institutions are also subject to indirect foreign exchange risk, i.e., foreign exchange risk incurred by their borrowers that can be transmitted to the financial institution through credit risk. In order to assess the indirect foreign exchange risk, it is necessary to stress test the major borrowers of the financial institution and then estimate the impact on the loan portfolio. However, the necessary data on corporate foreign exchange exposure are often unavailable. In markets with low availability of derivatives and other hedging instruments, it may be possible to arrive at a rough approximation of foreign exchange exposure of the largest borrowers. Any stress tests for indirect foreign exchange risk should feed into the stress test for credit risk (see below).

**Nature, type, and size of shocks**

55. The basic type of shock to be used in all stress tests is a shock to the exchange rate itself. Depending on their relevance for the financial institution, one or more exchange rates will have to be shocked, either separately (sensitivity analysis) or simultaneously (scenario analysis). Where financial institutions have significant exposures to several exchange rates, scenario analysis may be more useful than sensitivity analysis, since most shocks to one exchange rate do feed through to other exchange rates.

56. For financial institutions that hold significant positions in foreign exchange options, shocks to foreign exchange volatility should also be included in stress tests. In practice, this will be most relevant where financial institutions have sold foreign currency options. Such financial institutions should be able to calculate their option vega\(^{26}\) and reprice their positions as a function of sharp volatility increases. Since shocks to volatility are often connected with shocks to the exchange rate itself, both should ideally be stress-tested simultaneously. As an example, for an institution having sold U.S. Dollar calls/domestic currency puts, a sharp depreciation of the domestic exchange rate against the U.S. Dollar may cause a loss (depending on the strike price) due to both the drop in the exchange rate and the resulting increase in volatility. For institutions using internal models such as value-at-risk (VAR), it is not sufficient to test for shocks to exchange rates, and, possibly, volatility. Since these models usually rely upon correlation assumptions, it is also necessary to stress test for breakdowns in the assumed correlations, including correlations between exchange rates, as well as volatility correlations, if applicable. Especially in crises situations, observed

\(^{26}\) The “vega” of an option is a measure of its sensitivity to changes in volatility.
correlations tend to break down, and exchange rates that were basically uncorrelated (or even negatively correlated) may suddenly move in tandem.

57. The type of scenario to be used depends in part on the recent history of the exchange rate. If the country’s exchange rate has suffered from sharp depreciations in the past, these historical scenarios could be used to assess the impact of similar market moves today on the financial institution’s current portfolio. Where the country itself has not been affected, historical currency crises in other comparable economies could be used as a yardstick. Future currency crises may, however, be quite different from past crises, especially in rapidly developing financial systems. Switches in currency regimes, capital account liberalization, increasing use of derivatives, changes in regulation and supervision, and the entry of foreign banks are just some of the factors that can make a difference in how a financial system reacts to a currency crisis.

58. It is therefore essential to combine the historical simulation with various hypothetical scenarios. These scenarios should not include only the most likely scenarios, but also include a worst-case scenario, even if it is not considered very likely. As an example, a country that has a fixed exchange rate, should nevertheless have financial institutions run stress tests to estimate the potential effect of an exchange rate revaluation. These tests should be done regularly, as a matter of principle, and introduced at time when the currency is stable, so that they will not be perceived by the market as foreshadowing an exchange rate adjustment. Even though this may be a sensitive area, the hypothetical scenarios chosen for stress tests should not be left completely up to the financial institutions, but get at least some input from supervisors.

59. The third type of simulation, Monte Carlo simulation, allows stress-tests to simulate the impact of a wide variety of different combinations of variables, and to include the effect on portfolios with non-linear characteristics, such as complex foreign exchange option portfolios. Monte Carlo simulation, however, is computationally intensive, and requires a high level of risk management expertise that can generally only be found in the most sophisticated financial institutions. Moreover, supervisors will need to have sufficient expert staff themselves to be able to verify the accuracy, and to correctly interpret the results of the simulation. At this time, therefore, Monte Carlo simulation can only be an additional tool, to be used for a limited number of institutions.

60. The size of the shocks applied to exchange rates, volatilities and correlations will depend on the economic environment that the financial institutions operate in. As discussed above, historical observations can give some indication of the likely size of exchange rate shocks that may occur, particularly during a currency crisis, but larger (hypothetical) scenarios may need to be tested as well. Although it is usually assumed that potential exchange rate shocks will be higher in an emerging market environment than in the most developed economies, sharp exchange rate adjustments have occurred in industrial countries as well, as evidenced by the 1992 European Exchange Rate Mechanism (ERM) crisis.

61. Even though there is no specific size of shock that is appropriate for stress tests in all currencies, some minimum criteria can be derived from the standards promulgated by
regulators. Concerning appreciation and depreciation of exchange rates, the Basel Committee’s capital charges for foreign exchange risk are based upon exchange rate changes of +/- 8 percent, which implies that stress tests should be based upon significantly higher fluctuations. The Derivatives Policy Group (1995) has recommended at least 6 percent for major world currencies, and at least 20 percent for other currencies. The financial instruments disclosure recommendation of the Commission of the European Communities (2000) mentions an adverse change of at least 10 percent. As pointed out above, these numbers should be regarded as absolute minimum fluctuations; in most currencies, stress tests will have to include much more dramatic exchange rate changes.

62. With regard to volatility, the Basel Committee recommends the use of volatility fluctuations of at least 25 percent to determine capital charges for options. As with exchange rates, these volatility assumptions should be increased significantly for the purpose of stress tests. For countries with managed exchange rates, in particular, historical volatility can be quite low (close to zero under fixed exchange rates), so that potential volatility changes may need to be defined in absolute terms, rather than only in percentage terms.

Analyzing exchange rate risks in the FSAPs and in practice

21. Most of the FSAPs recently conducted, analyzed foreign exchange risk by calculating the net open foreign exchange position for individual institutions or for groups of banks and by examining the banks’ capital sensitivity to shocks in the exchange rate. In the majority of the countries the shocks considered were arbitrarily determined (see Table 2).

<table>
<thead>
<tr>
<th>Data</th>
<th>Models</th>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Ten out of twelve reports, used individual bank data for all banks or for those banks concentrating more than 50 percent of bank deposits, assets, or loans. - Aggregate data for the overall financial system or for distinct groups of banks were used in two cases.</td>
<td>- In most cases, the impact on capital of a shock to the net foreign currency position was examined. - Value at risk was used in the case of one country. - No tests were conducted for one country.</td>
<td>- In eight out of the twelve cases, ad hoc devaluation scenarios were used. - In three reports, the impact of large historical shocks was evaluated.</td>
</tr>
</tbody>
</table>

63. In the context of FSAPs, data on the net foreign exchange position for each individual bank is sometimes not available or is out of date, so alternative approaches are needed. In countries where supervisors have defined hard limits on the net open foreign exchange position of financial institutions, it is often possible to use a simplified approach, as long as one can be reasonably certain that the financial institutions actually adhere to those limits at all times, and as long as foreign exchange portfolios are essentially linear, i.e., the institutions are not active in the options market.

64. If, for example, the net open position, properly defined according to the Basel rules, cannot exceed 20 percent of an institution’s capital, this implies that the institution would not
lose more than 20 percent of its capital even under the assumption of a 100 percent
devaluation. In the latter case, the 10 percent capital ratio would decline to 8 percent,
whereas a 25 percent devaluation would shave just a half percentage point off a 10 percent
capital ratio. In terms of foreign exchange losses alone, it would take a 500 percent
devaluation to completely deplete the capital of an institution holding the maximum
20 percent exposure allowed (see Table 3 below).

Table 3. Simple Framework to Measure Exchange Rate Risk
(Open position/capital 20%)

<table>
<thead>
<tr>
<th>Devaluation</th>
<th>Initial Capital Ratio</th>
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<tbody>
<tr>
<td></td>
<td>6.0%      8.0%      10.0%      12.0%      14.0%</td>
</tr>
<tr>
<td>25</td>
<td>5.7%      7.6%      9.5%      11.4%      13.3%</td>
</tr>
<tr>
<td>50</td>
<td>5.4%      7.2%      9.0%      10.8%      12.6%</td>
</tr>
<tr>
<td>100</td>
<td>4.8%      6.4%      8.0%      9.6%      11.2%</td>
</tr>
<tr>
<td>200</td>
<td>3.6%      4.8%      6.0%      7.2%      8.4%</td>
</tr>
<tr>
<td>500</td>
<td>0.0%      0.0%      0.0%      0.0%      0.0%</td>
</tr>
<tr>
<td>1000</td>
<td>-6.0%     -8.0%     -10.0%     -12.0%     -14.0%</td>
</tr>
</tbody>
</table>

65. This very rough approximation can, of course, be done on both an institution-by-
institution basis, as well as on a system-wide basis. The simplified approach can also be used
in cases where there is no hard regulatory limit, but the approximate size of the open foreign
exchange positions is known or can be estimated. In practical terms, this approach can often
obviate the need for more detailed calculations, which are not always required for stress tests,
and whose accuracy may be short-lived or misleading.

C. Credit Risk

66. Credit risk is the risk that a counter-party or obligor will default on their contractual
obligations. It refers to the risk that the cash flows of an asset may not be paid in full,
according to contractual agreements.

67. Many credit instruments are not traded or marked-to-market, so there is very little
information on the underlying value of a particular instrument. Defaults on credit instruments
are also infrequent events, and when they occur they tend to be monitored and registered less
efficiently than other events such as changes in market prices. Furthermore, the distribution
of returns associated with credit risk is inherently asymmetric, with a high probability of
small positive returns and a low probability of large negative returns. The combination of
insufficient data, infrequent observations, and asymmetric distributions makes modeling
credit risk extremely difficult, both analytically and empirically. However, a common
framework for credit risk measurement has emerged. The two main features of this
framework are the definition of credit losses, and the relationship between capital and credit
risk.
68. A distinction is often made in the literature between expected and unexpected credit losses. Expected losses are those losses that are "uncertain, but occurring on average," and refer to events that have not yet occurred.\textsuperscript{27} Financial institutions typically cover expected losses through appropriate provisioning and pricing of their credit instruments. Unexpected losses are those losses that occur with unknown frequency and that cannot be countered by provisioning policies. Unexpected losses are proportional to the dispersion of the probability density function of the portfolio,\textsuperscript{28} and are typically measured using the standard deviation of losses or a predefined percentile level. Credit risk capital is the amount of economic capital that an institution must hold in order to cover its unexpected losses. Credit risk capital is usually defined as the maximum loss within a known confidence interval (e.g., 99 percent), over an orderly liquidation period\textsuperscript{29} (see Figure 2).

Figure 2. PDF of Losses, Unexpected and Expected Losses, and Economic Capital

69. This conceptual framework has important implications for risk management. It highlights the importance of proper accounting practices, coherent regulatory structures,

\textsuperscript{27} This notion differs from that often used in the accounting literature, where expected losses are referred to as credit events that are "certain but not yet registered." Certain losses, whether registered or not, do not have a probabilistic nature and cannot therefore be included in the PDF. Accounting practices allow for general provisions against losses that are expected in a probabilistic sense, while specific provisions are required for losses that are certain but not yet registered.

\textsuperscript{28} Which shows the probability of each level of losses.

\textsuperscript{29} Wilson (1998, p. 72).
proper measurement, and structured data gathering procedures. All of these components need to be in place for accurate and proper credit risk assessments. From this perspective, it is very important to underline the dual function of stress testing exercises at the aggregate level. On the one hand, stress tests can be seen as tools for checking the validity of existing models; on the other they represent a tool for verifying the presence of the required preconditions for the effective use of credit risk models. Credit risk models can highlight institutional weaknesses that need to be addressed when they signal an excessive risk exposure, and when they cannot be built due to the lack of the necessary preconditions. From this perspective stress testing exercises should be recognized not only as a tool for detecting the balance sheet weaknesses of financial institutions, but also for identifying institutional obstacles to credit risk assessment and management.

70. Measuring the credit risk exposure of a portfolio of instruments involves the estimation of a number of different parameters. An institution must measure the likelihood of default on each instrument both on average and under extreme conditions (expected and unexpected default frequencies), the extent of losses in the event of default (the loss in the event of default, or loss given default), and the likelihood that other obligors will default at the same time (i.e., the correlation or joint distribution of losses).

Provisioning approach: expected credit losses

71. A preliminary step to the evaluation of capital adequacy is the assessment of expected losses and of commensurate loan loss provisions. Not only is capital meaningless without adequate provisioning, but also it may be useful to consider the impact of different provisioning scenarios on capital adequacy ratios. A loan reclassification scheme can help to provide more realistic capital ratios (or leverage ratios, where data on risk weighted assets are not available) by estimating the amount of extra provisioning that is required and subtracting it from both total assets and capital:

\[
\left( \frac{\text{true capital}}{\text{true assets}} \right) = \frac{\text{reported capital} - \text{extra provisions}}{\text{reported assets} - \text{extra provisions}}
\]

72. The amount of provisioning shortages can be estimated by simple reclassification methods as those based on “peer reviews,” whereby loans to the same borrower from different institutions are reclassified according to the grade assigned by the most conservative institution. The value of extra provisions associated with this reclassification can be computed according to a number of criteria, depending on data availability and the purpose of the analysis. In the first case, taking regulatory prescriptions as a benchmark, extra provisions can be computed by simply applying regulatory provisioning to the portfolios of reclassified loans. In the second case, the benchmark can be represented by the average observed historical default ratio or by the historical transition matrix (see Appendix II). Where data on individual borrowers are not available, more aggregate approaches need to be followed. Peer reviews based on countries with common geographical or economic characteristics can provide a benchmark from which to ascertain the adequacy of provisioning practices. Alternative approaches focusing on the financial strength of the non-
financial sector can also provide a method for assessing expected losses. If balance sheet information are available for non financial corporate borrowers, expected losses can also be gathered from an analysis of balance sheet ratios or from the use of scoring models, which can produce more comprehensive assessments of financial fragility.

**Non-performing loans approach: unexpected credit losses**

73. The sensitivity of unexpected credit losses to external shocks can be gathered from the empirical estimation of the determinants of observed default frequencies as captured by non-performing loans (NPL) ratios. This measure can be interpreted as a default frequency measure. The NPL/Total assets ratio can be regressed on macroeconomic factors such as real interest rates, GDP growth, and terms of trade changes. The coefficients of these regressions can be an important input to any stress testing exercise, by providing an estimate of the sensitivity of bank borrowers to the relevant macroeconomic risk factors. Generally adequate information is available about major macroeconomic variables, making it possible to relate a meaningful macroeconomic scenario to the fragility of the banking sector. The following equation represents a possible specification of such an approach:

\[
\left( \frac{NPL}{\text{Total Assets}} \right)_{it} = \alpha + \beta \cdot i_{it} + \gamma \cdot p_{it} + \delta \cdot \Delta GDP_{it} + \lambda \cdot \Delta ToT_{it} + \varepsilon_{it}
\]

(6)

where \( i_{it} \) = nominal interest rate;
\( p_{it} \) = inflation rate;
\( \Delta GDP_{it} \) = percentage change of real GDP;
\( \Delta ToT_{it} \) = percentage change of terms of trade.

The advantage of this approach is the considerable flexibility it allows the user in specifying the relevant aggregate of NPL (over homogeneous groups of banks, or borrowers grouped by geographical area or by economic sector). This specification also permits dynamic analyses that are suited to estimating both a short-run effect and a long-run (equilibrium) effect, e.g., by using an error correction model.

74. Assuming linear risk exposures, we can derive from the previous equation the following expression for the volatility of the NPL ratio:

\[
\sigma_{\left(\frac{NPL}{\text{Total Assets}}\right)} = \sqrt{\beta^2 \cdot \sigma_i^2 + \gamma^2 \cdot \sigma_p^2 + \ldots + 2 \rho_{i,p} \sigma_i \sigma_p + \ldots}
\]

(7)

where full account is taken of existing correlations. This approach offers the opportunity for an integrated treatment of both market and credit risk. Interest rates and exchange rates (which are typically among the determinants of the default frequency) are also among the relevant market risk determinants. If it is thought that macroeconomic factors affect risk exposures in a non-linear way, then the previous expression should not be used and a Monte Carlo simulation should be considered instead.
75. The main shortcomings of this approach are related to the lack of long and reliable time series on non-performing loans. This is the case not only for transition economies, but also for most developing countries. For the first group of countries, the banking system has only come into its present form in the last decade. On the other hand, most developing economies have undergone major structural reforms in the last decade, which may have vastly changed behavioral patterns of both lenders and borrowers.

76. In general, considerable caution is needed when drawing conclusions from models that may be subject to several sources of error. Model and measurement errors may be present in different combinations that are difficult to evaluate ex-ante. Conclusions obtained with one model should be cross-checked, where possible, with those derived from different approaches and/or based on different sources of information. Typically models that look at the effects of macroeconomic scenarios should be compared with scoring models used by individual banks, and vice-versa. Evidence based on some aggregate estimation of the determinants of the default probability can be cross-checked with microeconomic information of the corporate sector financial fragility.

The integrated approach

77. The calculation of expected losses, according to the provisioning approach, and of unexpected losses, following the NPL approach, makes it possible to assess whether the revised capital ratio provides an adequate buffer against external shocks. Based on the following formula we can assess whether, as a result of the measurement of unexpected losses, the capital ratio is still above some desired minimum levels:

$$\text{Revised capital ratio} = \frac{\text{reported capital} - \text{extra provisions} - \text{unexpected losses}}{\text{reported assets} - \text{extra provisions} - \text{unexpected losses}} \tag{8}$$

78. The information contained in the various capital adequacy ratios provides a numerical estimate of capital shortages. This information can indicate how much financial institutions are underestimating their exposures to unexpected losses from credit risk. The main disadvantage of this approach is some arbitrariness involved in loan provisioning, and the fact that it relies on backward-looking information that may not fully describe the underlying evolution of the creditworthiness of the borrower, especially during times of major structural changes in the financial sector.

Analyzing credit risks in the FSAPs and in practice

79. The majority of the FSAPs conducted so far, have analyzed credit risk in the banking sector by examining the impact on bank solvency of changes in the extent of provisioning. In other words, these reports estimated the capital losses that would result from increasing the level of provisioning to what are considered to be more realistic levels. In two reports, this analysis was complemented with an evaluation using regression analysis of the impact of future potential macroeconomic shocks on the behavior of non-performing loans. Most of the credit risk analysis conducted as part of the FSAPs relied on ad hoc assumptions on the
extent of under-provisioning, the future behavior of non-performing loans, and the size of macroeconomic shocks (see Table 4).

80. In those situations where no precise information exists on the extent of under-provisioning or the future behavior of non-performing loans, it might be useful to at least calculate the maximum loss that banks could bear if capital is to remain above the required level. Similarly, another possibility would be to calculate the size of the losses that would cause the largest banks to become insolvent. While it would be difficult to attach a probability to these scenarios, they should help to provide a sense of the robustness of the banking sector.

D. Liquidity Risk

Definition and measurement

There are two types of liquidity risk: asset liquidity risk and funding liquidity risk. Asset liquidity risk refers to the inability to conduct a transaction at current market prices because of the size of the transaction. This type of liquidity risk comes into play when certain assets need to be liquidated at short notice (a “fire sale”). Funding liquidity risk refers to the inability to access sufficient funds to meet payment obligations in a timely manner. In many cases of bank failure, illiquidity occurs after the bank has become insolvent. Thus, the lack of adequate funding liquidity is considered to be a key sign that a bank is in serious financial difficulties.

<table>
<thead>
<tr>
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<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Ten out of twelve reports, used individual bank data for all banks or for those banks concentrating more than 50 percent of bank deposits, assets, or loans.</td>
<td>- In five out of twelve cases, regression analysis and simulations were used to examine the response of non-performing loans to future macroeconomic shocks. The remaining FSAPs evaluated changes in bank solvency resulting from pre-determined modifications in the extent of provisioning or from an ad hoc deterioration in the quality of the loan portfolio.</td>
<td>- In one case, Monte Carlo simulations of shocks to interest rates, exchange rates, and inflation, based on the historical covariances, were used to examine the simultaneous impact on the value of bank’s capital.</td>
</tr>
<tr>
<td>- Aggregate data for the overall financial system or for distinct groups of banks were used in two cases.</td>
<td></td>
<td>- In eight cases ad hoc assumptions were made on the extent of under-provisioning, the future behavior of non-performing loans, and the size of the macro shocks that could affect the loan portfolio of banks.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- In the remaining cases, the scenarios used were chosen based on the shocks observed historically.</td>
</tr>
</tbody>
</table>
81. Banks face constant liquidity pressures because of the nature of their business. Banks fund longer-term loans with short-term liabilities, and hold an unusually large portion of their liabilities in the form of demandable debt. Therefore, banks face liquidity pressures from imbalances between the maturity dates on their assets and liabilities, which imply that incoming cash flows from assets may not match the cash outflows to cover liabilities. Interest rate changes can also lead to liquidity problems. High interest rates may cause liquidity withdrawals as depositors seek higher returns elsewhere. Also, high interest rates may bring about a decline in the market value of the assets that banks hold in order to meet withdrawals. Finally, a bank’s liquidity situation may depend on funds that can suddenly dry up (like foreign borrowings) or assets that might unexpectedly become illiquid during periods of stress. For example, banks holding government bonds as their liquid asset may find that the market may disappear during times of crisis, when they need access to liquidity the most. Even if a market for such instruments exist, it is possible that if liquidity needs are systemic, banks may have to sell them at large discounts. Similarly, banks’ deposits in other domestic banks, a type of asset that would normally be considered highly liquid, may not be redeemable in times of need, if the banks where the funds are held become insolvent.

82. In this section we review two approaches commonly used to estimate banks’ liquidity requirements: the sources and uses of fund approach and the structure of funds approach.

The sources and uses of fund approach

83. This methodology relies on the premise that bank liquidity rises as deposits or other sources of funds increase and loans or other uses of funds decrease. The liquidity gap is the difference between the sources and uses of funds. When the uses of liquidity exceed the funds available, banks face a liquidity deficit, so they must raise funds to cover the gap. There are two key steps in the sources and uses of funds approach:

1. Uses and sources of funds must be forecasted for a given liquidity planning period; and

2. The estimated change in the uses and sources of funds must be forecasted for the same planning period.

84. The estimated change in the liquidity position is equal to the difference between estimated changes in sources of funds and estimated changes in uses of funds. If an institution exhibits a deficit in its liquidity position, then alternative sources of liquidity that would allow the institution to cover the gap need to be accessed. A particular difficulty in estimating the changes in the sources and uses of funds is introduced by contingent assets and liabilities such as derivatives. Since derivatives positions can have a significant impact on liquidity, off-balance-sheet positions should not be neglected in the stress tests.
The structure of funds approach

85. This method to assess a bank’s future liquidity requirements involves a number of steps:

1. Bank deposits and other sources of funds have to be divided into categories based on their estimated probability of being withdrawn.

2. Banks need to identify sources of funds that can become illiquid or dry up under certain situations.

Having estimated (i) and (ii), it is possible to calculate the percentage of liquid liabilities that can be covered, even if certain assets generally considered liquid suddenly become illiquid. Since, in crisis situations, even government bonds (in non-“safe haven” countries) may become illiquid, these should not automatically be considered liquid assets in stress tests.

\[
\frac{\text{Liquid liabilities covered}}{\text{Liquid liabilities}} = \frac{(\text{Liquid assets} - \text{assets likely to become illiquid})}{\text{Liquid liabilities}} \times 100 \tag{9}
\]

In order to implement either of the approaches to measuring liquidity risk for the banking sector, it is crucial to determine which institutions to focus on, and what scenarios to consider.

Selection of institutions

86. Any stress test for liquidity-risk in the banking sector should take into consideration the liquidity gap for all those institutions that could disrupt the payments system, if they were to become illiquid. In other words, all those institutions that concentrate a large percentage of deposits in the financial system (especially if these are short-term) should be considered. The primary emphasis should be on those institutions that are most active in the interbank market, and that handle the largest volumes of transactions.

Nature, type, and size of shocks

87. Historical data are one important benchmark for stress tests of liquidity risk. At the minimum, it is important that banks are able to withstand shocks of a similar magnitude to those that occurred in the past. Similarly, in the case of developing countries, the authorities can use the deposit withdrawals observed in other peer group countries under crises situations as a benchmark. Also, because liquidity shocks may be the by-product of other shocks that hit the economy, it is important to account for the correlation between liquidity shocks and other shocks that may indirectly affect bank liquidity. For example, during speculative attacks on a currency, banks may face a liquidity crisis as people withdraw their funds from the banking system to exchange them for foreign currency. Since financial institutions may lose access to foreign exchange funding during a crisis, a separate stress test for foreign exchange liquidity should be considered in cases where there are large foreign currency exposures (e.g., banks with a high ratio of foreign currency deposits). In this
context, the value of foreign exchange denominated instruments issued by domestic institutions, including government Eurobonds, may need to be discounted appropriately to account for a possible reduction in liquidity during a crisis.

Analyzing liquidity risks in the FSAPs and in practice

88. Less than half of the FSAPs recently conducted, examined the banking sector’s resilience to liquidity shocks. In some cases, lack of adequate data was the reason behind such a limited evaluation of this source of risk. Most of the FSAPs made ad hoc assumptions about the extent of liabilities that would need to be re-paid without notice and the types of assets that could be used in order to do so (see Table 5).

89. Ideally, if bank level data exists on banks’ liquid assets and liabilities, it should be possible to determine the percentage of liabilities that can be covered at any moments’ notice. If no data on liquid assets exists beyond knowing what the reserve requirements ratio is, then the lower bound on the liquidity of banks would be given by the reserves held by banks with the central bank. Furthermore, if there are rules regarding the size of loans that the central bank could make to banks, then by adding this to the reserve requirement it would be possible to get a second estimate of system wide liquidity. The hardest step in examining bank liquidity is to determine which of the assets that are normally considered liquid may become illiquid in the near future. A conservative scenario would be to assume that only the cash held by banks (in domestic and foreign currency) as well as the reserve requirements are always liquid. The next step would be to add to the category of liquid assets, deposits that banks hold abroad. Deposits with local banks can become illiquid if the country is confronted with a systemic liquidity crisis. Similarly, domestic government or corporate bonds can rapidly become illiquid, when enough banks are trying to sell these assets all at once. On the other hand, to the extent that the liquidity crisis does not affect the main financial centers, then banks could dispose of their foreign bonds to meet liquidity outflows at home.

Table 5. Liquidity Risk Stress Tests in the Context of the FSAPs

<table>
<thead>
<tr>
<th>Data</th>
<th>Models Used</th>
<th>Scenarios Considered</th>
</tr>
</thead>
</table>
| - Ten out of twelve reports, used individual bank data for all banks or for those banks concentrating more than 50 percent of bank deposits, assets, or loans.  
- Aggregate data for the overall financial system or for distinct groups of banks were used in two cases. | - In four out of twelve cases, the impact on liquidity of various definitions of liquid assets was examined.  
- No analysis was conducted for the remaining countries. | - In cases where liquidity risk stress test were conducted, these involved making arbitrary re-classification of the assets that can or should be considered liquid |
E. Equity Price Risk

Definition and measurement

90. Equity price risk is the risk that stock price changes affect the value of an institution’s assets and liabilities and its off-balance-sheet items. Equity price risk consists of two components: specific equity price risk refers to the risk associated with movements in the price of an individual stock. The second component, general equity price risk, is the risk associated with movements of the stock market as a whole.

91. The starting point for measuring an institution’s equity risk exposure is its net open position. In calculating the net open position, an internationally recognized methodology should be used, such as the one recommended by the Basel Committee on Banking Supervision. Except for institutions whose off-balance sheet equity positions have been confirmed as minimal, both on- and off-balance sheet equity positions should be included in the stress tests. Under the Basel methodology, the first step is the calculation of a bank’s net open position in each equity. Equity derivative positions must be converted into notional equity positions (options are delta-weighted).³⁰

92. To stress test for specific market risk, i.e., equity risk related to the individual issuer, the stress test would have to be applied to the net open position in the equity concerned. Such a stress-test would primarily be relevant in cases where the institution is known to hold a highly concentrated trading portfolio of equities. More commonly, stress tests are conducted for general market risk, i.e., the risk related to a major change in the overall stock market, usually a market crash scenario. For this purpose, the net open positions of an institution in all equities would be aggregated, and the stress scenario applied to the institution’s aggregate position.

93. If an institution tends to hold significant short positions in stock options, a linear (or delta) approach may not be a good approximation of stock price sensitivity, and second order approximation (gamma measures) need to be considered.³¹ This is particularly relevant for the large stock market moves that are used for the purpose of stress testing.

94. Financial institutions that include equity risk factors in their internal models should conduct comprehensive stress test using their own measurement techniques, and provide the results to regulators. For those institutions, the net open positions in each equity should still be available, before aggregation into the overall position, and the model should be able to

³⁰ For more details, see Basel Committee on Banking Supervision, Amendment to the Capital Accord to incorporate market risks, Basel (January 1996)

³¹ Under the Basel Committee’s methodology the gamma impact is calculated using a Taylor series expansion, by multiplying ½ of gamma by the square of 8 percent of the market value of each option position.
stress test each equity separately. Internal models can also be used to implement scenario analysis, taking account of correlations among stock prices and/or indices, although these correlations may break down during crises (see below).

95. Equity exposures in the trading book may be subject to frequent and substantial swings, along with stock market developments. The results of stress tests can therefore be outdated fairly quickly. Whereas supervisory reports or published annual reports of financial institutions can give a reasonable “snapshot,” it is preferable to obtain more current data on the composition of an institution’s equity portfolio from the financial institution itself. Where such up-to-date data is unavailable, knowledge about the most frequently traded equities, and the stock exchange dealing and underwriting activities of the institution, can sometimes help in updating open position estimates.

**Selection of institutions**

96. When defining the sample of institutions to be included in the stress tests for equity price risk, a number of criteria should be taken into account. First, such tests should focus on institutions that have a significant exposure to equity prices and pose a systemic threat. Second, in countries where banks are not allowed to buy or sell stocks, they may nevertheless be exposed to equity prices if they are permitted to engage in equity or equity-linked derivatives transactions. Strict limits on equity exposures that apply to the banking system may not apply to other financial institutions such as securities firms, allowing the latter to take large equity positions. Securities firms that are of systemic importance may therefore need to be included in the stress tests, especially if they are subsidiaries of banks.

**Nature, type, and size of shocks**

97. In all stress tests for equity risk, the basic type of shock is a shock to the main stock market index. The prices of individual equities may be shocked as well, depending on their relevance for the financial institutions, either separately (sensitivity analysis) or simultaneously (scenario analysis). Scenario analysis may be more appropriate than sensitivity analysis in cases where the financial institution has significant exposure to several equities, given that shocks to a particular stock may be correlated with changes in other stock prices, especially in the same industry or sector.

98. Shocks to stock price volatility should also be included in stress tests for those financial institutions that hold significant positions in stock options. After calculating their option vega, such financial institutions should reprice their stock options as a function of sharp increases in stock price volatility. Because shocks to stock volatility are often correlated with shocks to stock prices themselves, it is advisable that both shocks are jointly considered in the stress tests. As an example, for an institution having sold S&P500 puts, a sharp fall in the S&P index may cause a loss (depending on the strike price) due to both the drop in the stock index and the resulting increase in volatility.

32 The “vega” of an option is a measure of its sensitivity to changes in volatility.
99. For those institutions that employ internal models such as value-at-risk (VAR), it is not sufficient to test for shocks to the stock market, and, possibly, volatility. Internal models commonly rely upon correlation assumptions. In crises situations, however, observed correlations often break down, and stocks and/or stock markets that were basically uncorrelated (or even negatively correlated) may become (positively) correlated. Therefore, stress tests should take into account the possibility of breakdowns in the assumed correlations, including correlations among stocks and/or stock indices, as well as volatility correlations.

100. The choice of an appropriate scenario for the stress test depends in part on the recent performance of the stock market. At a minimum, stress tests should examine a financial institution’s resilience to shocks comparable to previously observed stock market crashes. If a country has not been affected in the past, e.g., because of the absence of a stock market and/or capital controls, historical stock market swings in other comparable economies may serve as a suitable benchmark. However, it is important to recognize that future stock market developments may be affected by a number of institutional changes such as increases in stock market liquidity, capital account liberalization, introduction of equity derivatives, changes in the regulation and supervision of equity-related activities of financial institutions, and the entry of foreign banks and capital market intermediaries.

101. A combination of historical simulation with various hypothetical scenarios can therefore prove useful. These scenarios should include not only scenarios that have a medium to high likelihood, but also those scenarios with a low probabilities ("worst-case scenarios," such as market crashes). As with other stress tests, the equity risk tests should be done on a regular basis. Their introduction at a time when the market is unlikely to be overvalued can help to avoid the impression that regulators fear a crash in the near future. Supervisors should be able to understand and, if necessary, provide some input into, the design of the (hypothetical) scenarios used by the financial institutions.

102. As described above, stress tests using Monte Carlo methods can simulate the impact of many different combinations of variables. Monte Carlo simulation can also assess the impact on complex equity option portfolios. Due to its computational intensity and complexity, however, Monte Carlo simulation requires a high level of risk management expertise that is not available in most financial institutions. In addition, supervisory expertise will be needed to ensure the accuracy of the simulation and the correct interpretation of the results.

103. The size of the shocks to equity prices, volatilities, and correlations that are used in the stress test should be determined taking into account the economic environment that the financial institutions operate in. Whereas historical observations can give some indication of the likely size of market shocks that may occur, larger (hypothetical) shock scenarios may need to be tested as well. Potential market shocks are likely to be higher in an emerging market environment than in the most developed economies. Nevertheless, sharp stock market adjustments have occurred in industrial countries as well, as evidenced by the 1987 stock market crash. In addition, shocks applied to small- and mid-cap indices should generally be
higher than shocks to "blue-chip" stock indices, taking into account lower levels of liquidity and "flight-to-quality." 33

104. Since equity risk can vary substantially among countries, as well as among stocks and stock indices within the same country, there is no specific size of shock that is appropriate for stress tests for all stocks or stock markets. Some \textit{minimum} criteria can nevertheless be derived from regulatory standards. The Basel Committee’s capital charge for general equity price risk is based upon changes of +/-8 percent, indicating that stress tests should be based upon much higher stock price moves. Where stress tests incorporate specific market risk (in low diversification cases), the assumed price changes should be increased accordingly. The Derivatives Policy Group (1995) has recommended at least 10 percent for stock price indices. The financial instruments disclosure recommendation of the Commission of the European Communities (2000) mentions an adverse change of at least 10 percent. Stress tests will in many cases have to include much more dramatic stock price fluctuations, as pointed out above. With regard to stock price volatility, the Basel Committee recommends the use of volatility fluctuations of at least 25 percent to determine capital charges for equity options. Again, these volatility assumptions should be increased significantly for the purpose of stress tests.

\textbf{Analyzing equity price risks in the FSAPs and in practice}

105. Many financial institutions do not have large equity holdings, especially institutions in developing countries. Consequently, equity holdings seldom represent a large fraction of the banks’ assets. Thus, it is not surprising that out of twelve recently completed FSAPs, only one examined the effect of a stock market crash on bank solvency. It is also possible that lack of adequate data may have prevented the stress testing of equity risks.

106. When no bank level data is available on the net open position for each individual equity or even for the overall equity portfolio held by a bank, a simple approach can be taken in those cases when regulations exist limiting the aggregate net position that banks can take. In those situations, a crude test may be conducted by assuming that all banks have the maximum tolerable net open position. Given this assumed net open position, the impact on bank capital can be determined by shocking this position with different measures of stock market changes.

\textbf{F. Commodity Price Risk}

\textbf{Definition and measurement}

107. Commodity price risk refers to the potential losses that may result from changes in the market price of bank assets and liabilities, as well as off-balance sheet instruments, due to

\footnote{Note that liquidity can also vary significantly depending on the market segment, on the same stock exchange, that a stock is traded in.}
commodity price changes. Even if financial institutions do not take positions in commodities or commodity-linked instruments directly, they may be subject to commodity price risk indirectly via the impact on their loan portfolio. This occurs if their borrowers’ ability to repay their debts is affected by shocks to commodity prices. This indirect source of commodity risk can be particularly important for many banks in developing countries that lend to exporters and/or importers of commodities.

108. Under the methodology proposed by the Basel Committee on Banking Supervision, one can calculate the financial institution’s net position in the most relevant commodities by netting long and short positions, expressed in terms of the standard unit of measurement, in the same commodity. The net position would then be converted into the national currency at current spot rates for the commodity. Commodity derivatives should be converted into notional commodities positions, and included in the framework in the same way. Banks seldom hold significant short positions in commodity options, but if they do, stress tests will need to use second-order approximations (gamma measures), rather than just first order approximations (delta measures).

Nature, type, and size of shocks

109. In choosing stress scenarios, it is possible to look at either historical swings in commodity prices, or develop plausible new scenarios. Since many commodity prices have been volatile in the past, historical scenarios may give a good indication of the possible size of future price swings. In practice, the most relevant examples for long commodity positions may be sharp falls in the prices of energy or precious metals, and, for short positions, sharp energy price increases.

110. In addition to pure directional risk, it is important to make an allowance for liquidity risk, since many commodities markets tend to become illiquid during stress. For commodity derivatives, an allowance may also be necessary for forward curve or basis risk, i.e., the risk of changes in the relationship between the prices of a commodity for immediate versus future delivery. Forward prices can be higher (contango) or lower (backwardation) than spot prices, depending on perceptions of current and future supply. These perceptions often change in situations of stress, arguing for the inclusion of basis risk in stress tests where applicable.

111. Historical volatility varies significantly among commodities. Therefore, the Basel Committee’s broad benchmark of 15 percent (for non-stress situations) can only be taken as an indication that stress tests for commodity prices should generally consider more extreme fluctuations than stress tests for other risks. If at all possible, stress tests should incorporate fluctuations that have occurred in the same commodity, or at least in a highly correlated commodity.

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34 For a description of such a conversion, see Basel Committee on Banking Supervision, *Amendment to the Capital Accord to incorporate market risks*, Basel (January 1996).
Analyzing commodity price risks in the FSAPs and in practice

112. Two FSAPs attempted to measure the exposure of banks to commodity price risks. In general, banks are only rarely involved directly in trading commodities. On the other hand, banks could be indirectly affected by commodity price movements, if these impact the ability of bank borrowers to repay their debts.

113. To the extent that banks are only indirectly exposed to commodity price movements, then their effect on bank solvency could be analyzed by examining their relationship to the behavior of non-performing loans. This can be done for example, by incorporating them in the regressions for non-performing loans, as discussed in the credit risk section.

114. In countries where banks are allowed to trade commodities or invest heavily in them, it becomes necessary to measure their direct exposure to commodity price movements. If regulations exist limiting the maximum net open position tolerated, then a crude stress test can be conducted assuming that all banks are at the maximum level permissible and subjecting this position to various shocks. One useful benchmark is to determine the size of the shock that would cause the capital for the aggregate system or for the largest banks to fall below the amount required by regulation.

G. Market Risk and Value-at-Risk

Definition and measurement

115. The approach adopted in the paper thus far has been to consider separately the various risks to a portfolio. In practice, many financial institutions take a multivariate approach to measuring, analyzing, and managing risk, and combine the different risks under the rubric of market risk. Market risk can be defined as the risk of losses on a portfolio arising from movements in market prices.\textsuperscript{35} Broader definitions of market risk include other factors such as "the risk that a change in liquidity or in the level of one or more market prices, rates, indices, volatilities, correlations or other market factors will result in losses for a specified position or portfolio."\textsuperscript{36}

116. One of the methodologies that has been developed to deal with aggregating and measuring multiple risks is the Value-at-Risk framework. The Value-at-Risk (VaR) of a portfolio is a statistical measure that summarizes the largest expected loss that the portfolio is likely to suffer over a specified time period for a given level of confidence. For example, a portfolio may have a ten-day VaR of $100 million at a 99 percent confidence level. This implies that over the next 10 days there is a 99 percent chance that the portfolio will lose less

\textsuperscript{35} This is the definition adopted by the Basel Committee on Banking Supervision (1996, p. 1).

\textsuperscript{36} Derivatives Policy Group (1995).
than $100 million, or a less than one percent chance that the portfolio will lose more than
$100 million. For institutions using a VaR approach to risk measurement, exposures can be
calculated in terms of a numerical amount (e.g., expressed in dollar terms), or using other
similar measures such as Daily Earnings at Risk, Cash Flow at Risk, or Risk Adjusted Return
on Capital.

117. VaR measures can provide useful information to decision makers about the likely
pattern of events that will influence the value of a portfolio, but it is less useful in providing
information about unlikely events. For example, the October 1987 stock market crash was a
20 standard deviation shock; an event that is unlikely to have been captured by a model based
on a normal distribution. This anecdote illustrates the point that a VaR measure is not the
maximum amount that a portfolio could lose. Instead, VaR is a less threshold that will be
exceeded with only a small probability. Furthermore, the probability that the loss threshold is
correct is dependent upon the specification and estimation of the underlying statistical model
of portfolio returns. If the underlying distribution of returns has fatter tails than the assumed
distribution (which is typically the normal distribution), then the estimated VaR may
underestimate the extent of possible losses to the portfolio. If the underlying estimation
technique is inaccurate (e.g., because of linear approximations), then the model may also
under-predict losses on a portfolio, especially if movements in asset prices are large.

Estimation

118. There are two broad approaches to estimating a VaR. The first approach is the local
valuation method, which uses an estimate of the sensitivity of the portfolio (usually the
duration for interest sensitive assets) multiplied by the estimated price change to arrive at the
estimated change in value. This approach takes the slope and curvature of the portfolio
around its present market value to impute the new value under changed circumstances. The
delta-normal method is a widely used approximation method in this category. This method
assumes normal distributions, and uses linear derivatives (the delta) to approximate the
change in value of a portfolio. A second variant of the local valuation method is the delta-
gamma approach, which uses an additional second order term (the gamma) to improve the
accuracy of the delta-normal method.

119. The second major approach to VaR estimation is the full valuation method. Instead of
using approximations, this approach recalculates the value of the portfolio using a new vector
of prices. Two widely used full-valuation methods are the historical valuation method (where
historical prices are used to form the vector of new prices) and the Monte Carlo simulation
method (where new prices are simulated using known distributions).

37 Based on historical experience.

38 See the appendix for further details.
Nature, type, and size of shocks

120. For a VaR approach to market risk, the time horizon can be chosen according to the structure of the portfolio (e.g., the expected holding period of the assets) and the expected length of time required to liquidate a position or to take corrective action in the event of losses. The confidence interval can be chosen according to the risk appetite of the owners and managers of the firm, as well as the statistical tradeoffs involved. The data used to implement a VaR calculation can be based on historical correlations and volatilities, or on implied observations. The type of mapping applied to the different risk factors in a portfolio can be determined by the computational burden involved in specifying separate risk factors for each element of the portfolio, and the availability of data.

121. The Basel Committee on Banking Supervision (1998) recommends that banks using the internal models approach for calculating their capital on market risk exposures use a VaR based on the 99th percentile of a one-tailed confidence interval, with a holding period of ten trading days. The models should use a sample observation period of at least one year, and banks should update their data sets every three months, or more frequently during volatile markets. The Basel Committee does not provide specific recommendations on the type of model to be used by banks, but sets capital requirements based on their ex-post performance. Banks are required to hold capital based on the higher of their VaR for the previous business day or their average VaR from the previous 60 business days. The required capital is based on their estimated VaR, multiplied by a "multiplication factor." The multiplication factor is set at a minimum of 3, based on backtesting of the model.

IV. CONCLUSIONS

122. An important by-product of the recent financial crises around the world has been a renewed interest in measuring and monitoring the vulnerability of financial systems to shocks. However, there are few studies that describe the issues and appropriate methodologies in evaluating the risk exposure of financial systems. This paper has attempted to bridge some of the gaps in the literature by pursuing three main objectives. First, the paper discussed some of the conceptual issues that are relevant in evaluating the vulnerabilities in a financial system. In particular, the paper described the decision sequence to be followed in conducting a stress test for individual portfolios, and discussed some of the issues in performing aggregate stress tests. Second, the paper provided a basic toolkit of stress testing techniques concerning credit, interest rate, foreign exchange, commodity, and liquidity risks. The paper gave a description of some simple methodologies that can be used in each case, along with a discussion of the advantages and disadvantages of each approach. Finally, the paper reviewed stress tests undertaken by the IMF and World Bank to monitor financial systems around the world as part of the Financial Sector Assessment Program (FSAP). The

39 Such as the type 1 and type 2 errors.
paper highlighted some of the weaknesses and gaps in the literature, suggesting avenues for further research.\textsuperscript{40}

\textsuperscript{40} The IMF and World Bank are actively pursuing research in this and related areas.
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THE FINANCIAL SECTOR ASSESSMENT PROGRAM

123. The Financial Sector Assessment Program (FSAP) is a joint IMF-World Bank initiative designed to strengthen countries' financial systems through in-depth assessments of financial sectors' strengths, risks, and vulnerabilities. There are multiple objectives for the FSAP. "The FSAP is intended to encourage the early detection of financial system weaknesses and to develop appropriate policy responses, to support a more effective dialogue with national authorities, and to provide both institutions with a common platform for policy advice and technical assistance. The program aims also to identify key priorities for financial sector development and improve the design and delivery of support for strengthening financial systems." IMF and World Bank (1999).

124. The FSAP was launched on a pilot basis in May 1999 by the managements of the World Bank and the IMF, in response to calls by the international community for intensified cooperative efforts to strengthen the monitoring of financial systems. The assessments provide inputs to the Article IV surveillance process of the IMF, and the World Bank's financial system activities. The program is being implemented with the cooperation of a range of national central banks, supervisory agencies, and international standard-setting bodies, which adds an element of peer review to the exercise.

125. The stress testing of financial systems forms a key part of the FSAP, and is complemented by other components of the exercise, specifically the assessments of observance of financial sector standards and codes. Analyzing the implementation of these standards and codes provides a basis for understanding and evaluating the data that are used as inputs into the stress testing exercise.

126. The FSAP also tries to strengthen the link between the microeconomic analysis of institutions and the macroeconomic picture. Through its integration with the IMF's Article IV consultations, the FSAP is designed to ensure that stress tests are appropriately tailored to a country's macroeconomic situation, taking into account the main risk factors originating from current and future economic developments. Conversely, the FSAP should facilitate the analysis of feedback effects of vulnerabilities in financial institutions to the macroeconomy.

127. The International Monetary and Financial Committee (IMFC) in its April and September 2000 communiqués welcomed the work of both institutions in implementing the pilot FSAP, stating that it could serve as the primary basis for enhancing the IMF's monitoring of the financial sector in the context of Article IV surveillance. It endorsed the decision to continue the FSAP, and expand the coverage to 24 countries on a voluntary basis for the current year, encouraging member countries to participate.

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41 For more details, see http://www.imf.org/external/np/FSAP/2001/fsap.asp

TECHNICAL ADDENDUM

A. Interest Rate Risk Models

Interest rate risk arises because of repricing risk, yield curve risk, basis risk, and options risk. Repricing risk arises from mismatches in the time to maturity or time to repricing of assets and liabilities. Yield curve risk originates in changes in the slope and shape of the yield curve. Changes in the relationship between interest rates at different maturities can affect the income or capital position of a financial intermediary. For example, a long position in long-term bonds that is hedged by a short position in short-term bonds may make a bank immune to parallel movements in the yield curve, but may subject the bank to losses from a change in the shape of the yield curve. Basis risk is the risk that arises from an imperfect correlation between the adjustment of rates earned and paid on assets with the same repricing characteristics. For example, if a loan reprices annually based on a 1-year Treasury Note, and is funded by a one-year deposit that reprices annually based on the one-year LIBOR, the institution is exposed to risk if the spread between the two indices changes. Finally, options risk is the risk that an options clause embedded in some component of an institution’s portfolio will become exercised for a given change in interest rates.

Repricing-gap model

The repricing model of interest rate risk is based on the concept of the repricing gap. The repricing gap is the difference between the flow of interest earned by a financial institution on its assets and the flow of interest paid on its liabilities. Calculating the repricing gap requires a financial institution to sort its assets according to their rate sensitivities (time to repricing), and report assets and liabilities in a few time categories or “buckets,” as shown in the first column of the table below.

<table>
<thead>
<tr>
<th>Time to Repricing</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Gaps (A-L)</th>
<th>Cumulative Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 1 day</td>
<td>20</td>
<td>30</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>1 day up to 3 months</td>
<td>30</td>
<td>40</td>
<td>-10</td>
<td>-20</td>
</tr>
<tr>
<td>3 months up to 6 months</td>
<td>70</td>
<td>85</td>
<td>-15</td>
<td>-35</td>
</tr>
<tr>
<td>6 months up to 1 year</td>
<td>90</td>
<td>70</td>
<td>+20</td>
<td>-15</td>
</tr>
<tr>
<td>1 year up to 5 years</td>
<td>40</td>
<td>30</td>
<td>+10</td>
<td>-5</td>
</tr>
<tr>
<td>More than 5 years</td>
<td>10</td>
<td>5</td>
<td>+5</td>
<td>0</td>
</tr>
</tbody>
</table>

*This section is based on Saunders (2000, chaps. 8-9), and Basel Committee on Banking Supervision (1997).*

*This example is taken from Saunders (2000, p. 123).*
For any given change in interest rates, $\Delta R$, the repricing gap can be used to calculate the change in net interest income in each bucket $i$, and for the total portfolio:

\[
\Delta \text{Net interest income}_i = GAP_i \times \Delta R_i \\
\Delta \text{Net interest income} = \text{Cumulative GAP} \times \Delta R. 
\]

Using the example from the table above, the cumulative gap on rate sensitive assets and liabilities with a maturity up to one year is 15. If interest rates change by 1 percent ($\Delta R = 0.01$), the impact on net interest income at a one year horizon is 0.15. This figure can also be expressed in terms of the gap ratio by expressing the cumulative gap as a percentage of total assets. Scaling the cumulative gap using a measure such as total assets provides information on the extent of an institution's exposure to interest rate risk that is comparable across institutions. In this example, total assets are 260, so the gap ratio for one-year rate sensitive assets is $15 \div 260 = 0.058$ or 5.8%.

Table A2 provides a more sophisticated illustration of the gap approach. The weighted sum of gaps can be used as a basis for applying a given interest rate shock to the portfolio. More sophisticated analysis that allows for the possibility of changes in the slope of the yield curve can apply a different interest rate to each maturity bucket. The results can then be aggregated to give a picture of the total portfolio effect of a change in rates.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Assets</th>
<th>Liabilities</th>
<th>Gaps (A-L)</th>
<th>Weights $w_i$</th>
<th>Sums $w_i \times GAP_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 1 day</td>
<td>20</td>
<td>30</td>
<td>-10</td>
<td>0.00</td>
<td>0.0</td>
</tr>
<tr>
<td>1 day up to 3 months</td>
<td>30</td>
<td>40</td>
<td>-10</td>
<td>0.20</td>
<td>-2.0</td>
</tr>
<tr>
<td>3 months up to 6 months</td>
<td>70</td>
<td>85</td>
<td>-15</td>
<td>0.40</td>
<td>-6.0</td>
</tr>
<tr>
<td>6 months up to 1 year</td>
<td>90</td>
<td>70</td>
<td>+20</td>
<td>0.70</td>
<td>+14.0</td>
</tr>
<tr>
<td>1 year up to 5 years</td>
<td>40</td>
<td>30</td>
<td>+10</td>
<td>2.00</td>
<td>+20.0</td>
</tr>
<tr>
<td>More than 5 years</td>
<td>10</td>
<td>5</td>
<td>+5</td>
<td>4.60</td>
<td>+23.0</td>
</tr>
<tr>
<td>Totals</td>
<td>260</td>
<td>260</td>
<td>0</td>
<td></td>
<td>+49.0</td>
</tr>
</tbody>
</table>

Note: Weights are from Basel Committee on Banking Supervision (1998, Table)

The repricing-gap model has several weaknesses. The repricing model values assets at book value, ignoring capital gains and losses by assuming that changes in interest rates only affect the income position. The repricing model also ignores problems of aggregation within maturity buckets, as grouping different assets together may be misleading. For example, liabilities may tend to be repriced towards the end of the range of maturities in a bucket, while assets may tend to be repriced towards the beginning. Thus the aggregation of different assets and liabilities may mask underlying mismatches in maturities. The repricing model also suffers from the problem of runoffs, as different components of a portfolio mature at different times, and may provide some cash flow in the short-run. For example, a 30-year loan may have only one year remaining, and it may pay interest monthly. Thus including the loan in the 30-year category may tend to overstate an institution's exposure.
Maturity-gap model

134. The maturity-gap model is based on the concept of the weighted-average maturity of the assets and liabilities of the financial institution, defined as follows:

\[ M^A = \sum_{i=1}^{N} w_i^A M_i^A \]
\[ M^L = \sum_{i=1}^{N} w_i^L M_i^L, \] 

where
- \( M^A \) = weighted average maturity of assets
- \( M^L \) = weighted average maturity of liabilities
- \( M_i^A \) = maturity of asset with a maturity given by \( i \)
- \( M_i^L \) = maturity of liability with a maturity given by \( i \)
- \( w_i^A \) = weight of asset \( i \) in portfolio, measured by proportion of total market value
- \( w_i^L \) = weight of liability \( i \) in portfolio, measured by proportion of total market value

135. A financial institution may be interested in the maturity gap, which is the difference between the weighted-average maturity of its assets and liabilities:

\[ GAP^{Maturity} = M^A - M^L. \]

A severe mismatch in the maturity of its assets and liabilities exposes a financial institution to interest rate risk. A rise in interest rates will reduce the market value of assets and liabilities, with the reduction being greater for longer maturities. If interest rates rise and a financial institution has a positive maturity gap (\( M^A > M^L \)), the institution will face a larger fall in the value of its assets than its liabilities, reducing its equity or net worth. Thus, the weighted-average maturity provides useful information on exposures to changes in interest rates.

136. However, the weighted-average maturity measure is not a perfect measure of interest rate risk. A bank that matches the maturity of its assets with the maturity of its liabilities may still be exposed to losses from changes in interest rates. This situation may arise if the timing of the cashflows on assets and liabilities is different. To avoid this problem, the measure of duration provides a more accurate measure of exposure to interest rate risk.

Duration model\(^{45}\)

137. The duration model is based on the concept of duration, which is a measure of the interest sensitivity of an asset that takes into account the maturity of the asset as well as the timing of cash flows. Duration can be calculated as the weighted-average time-to-maturity, using the present value of cash flows as weights. We can define duration as follows:

\(^{45}\) This section is based on Saunders (2000, chap. 9) and Jorion (1997, chap. 6).
\[ D = \frac{\sum_{t=1}^{N} CF_t \times DF_t \times t}{\sum_{t=1}^{N} CF_t \times DF_t} = \frac{\sum_{t=1}^{N} PV_t \times t}{\sum_{t=1}^{N} PV_t}, \quad (A4) \]

where \( D \) = duration measured in years
\( CF_t \) = cash flow received on the security at the end of period \( t \)
\( N \) = last period in which the cash flow is received
\( DF_t \) = discount factor \( 1/(1+R)^t \), where \( R \) is the current yield or interest rate
\( PV_t \) = present value of the cash flow at the end of the period.

138. The duration of an asset increases with the maturity, \( M \), but at a decreasing rate:

\[ \frac{\partial D}{\partial M} > 0, \quad \frac{\partial D^2}{\partial M^2} < 0, \quad (A5) \]

and duration decreases as the yield, \( R \), or coupon (promised interest payment), \( C \), increases

\[ \frac{\partial D}{\partial R} < 0, \quad \frac{\partial D}{\partial C} < 0. \quad (A6) \]

139. Duration can also be defined as the interest elasticity (sensitivity) of the price of the asset, \( P \), to small changes in the yield, \( R \):

\[ D = -\frac{dP}{(1 + R)} \quad (P) \quad (A7) \]

140. To relate the return on an asset and changes in yields, we can use the concept of modified duration, \( D^* \):

\[ D^* = \frac{D}{(1 + R)} = \left( -\frac{1}{P} \right) \frac{dP}{dR}. \quad (A8) \]

This formulation describes the change in the price of an asset for any given present value increase in interest yield. For small changes in interest rates, we can show that the price of an asset moves inversely proportional to the modified duration, \( D^* \).

141. Duration can be used to translate volatility in yields into price volatility. We can rewrite equation (A8) to show

\[ \frac{dP}{P} = -D^* \, dR, \quad (A9) \]

implying that
\[
\frac{\sigma_{dP}}{P} = D^* \sigma_{dR} = D^* R \sigma_{dR},
\]

(A10)

where \( \sigma \) represents the standard deviation. Thus the volatility in the price of a security can be computed directly from the volatility of the yield, using the measure of modified duration.

142. However, due to the convexity of fixed income securities such as bonds, duration will tend to overestimate the fall in the price of a security for large interest rate increases. For large interest rate decreases, duration will tend to underestimate the increase in the price of the security. We can see this effect from the following diagram.

![Diagram showing price changes and yield changes](image)

Yield changes \( dR/(1+R) \)

143. The diagram above shows how the total effect of a change in interest rates on the price of a security will depend on the curvature or convexity of the price-yield curve, defined as the second derivative of the price-yield curve:

\[
\text{Convexity} = \frac{\partial^2 P}{\partial R^2} \frac{1}{P} = -\frac{dD^*}{dR}.
\]

(A11)

Thus, the effect of a change in interest rates on the price of a security can be more accurately approximated by incorporating the second order convexity term in a Taylor series expansion as follows:

\[
\frac{dP}{P} = \frac{dP}{dR} \frac{1}{P} dR + \frac{1}{2} \frac{d^2 P}{dR^2} \frac{1}{P} (dR)^2
\]

\[
= -D^* dR + \frac{1}{2} \text{Convexity} (dR)^2.
\]

(A12)

144. Once a measure of duration is derived for a security or a group of assets in a portfolio, the financial institution can use the duration gap to analyze its exposure to interest rate risk. The duration gap is the difference between the duration of the assets and liabilities of the institution:

\[
GAP_{\text{Duration}} = D^A - D^L.
\]

(A13)

Portfolio managers can use the duration gap to “immunize” the portfolio, or protect it against changes in interest rates. Immunizing a portfolio involves matching the gains and losses in the value of assets from changes in interest rates with the gains and losses in the value of liabilities.
145. Portfolio managers may be more interested in measuring the impact of interest rate changes on the net worth of the institution (or change in equity, given by $\Delta E$), which is given by

$$
\Delta E = \Delta A - \Delta L = \left[ -D^A \times A \times \frac{\Delta R}{(1 + R)} \right] - \left[ -D^L \times L \times \frac{\Delta R}{(1 + R)} \right]
$$

$$
= - \left[ D^A A - D^L L \right] \frac{\Delta R}{(1 + R)} = - \left[ D^A - k D^L \right] \times A \times \frac{\Delta R}{(1 + R)}, \quad (A14)
$$

where

- $A$ = value of assets
- $L$ = value of liabilities
- $k$ = leverage ratio, equal to $L/A$

Thus a portfolio manager will attempt to set $D^A = kD^L$ in order to immunize the balance sheet from interest rate risk.

146. One of the weaknesses in this approach to estimating duration is the use of a single discount factor. The specification of a single interest rate, $R$, to calculate the discount factor implies that the yield curve is flat, so changes in interest rates thus imply a parallel shift in the yield curve. Alternative measures of duration can account for the possibility of changes in the shape of the yield curve, using specific discount factors for each maturity, as suggested by different values of interest rates at different points of the term structure:

$$
\bar{D} = \sum_{t=1}^{N} t \times \frac{CF_t (1 + R_t)^t}{\sum_{i=1}^{N} CF_i (1 + R_i)^t}. \quad (A15)
$$

**B. Value-at-Risk (VaR)**

147. The Value-at-Risk of a portfolio is a point in the distribution of changes in the value of the portfolio. Following Pritsker (1997, p. 203), we can define $V(P_t, X_t, t)$ as the value of a portfolio at time $t$ with instruments $X_t$ and prices $P_t$, and we can denote the change in the value of the portfolio from time $t$ to $t+1$ as $\Delta V(P_{t+1}-P_t, X_t, t)$. The cumulative density function of changes in the value of the portfolio, $\Delta V(*)$ for a given set of instruments $X_t$, conditional on the information $I_t$ available at time $t$, can be written as

$$
G(k, X_t | I_t) = \text{Probability}(\Delta V(P_{t+1} - P_t, X_t) \leq k | I_t). \quad (A16)
$$

---

46 Saunders (2000, pp. 164-165) notes that maintaining a given capital-to-assets ratio (so that $\Delta(L/A) = 0$) requires the institution to set $D^A = D^L$.

47 This section draws on Jorion (2000, chap. 5) and Pritsker (1997).
The Value-at-Risk (VaR) for confidence level \( u \) is defined in terms of the inverse cumulative density function, \( G^{-1} \), of changes in the portfolio value, \( \Delta V(\cdot) \):

\[
\text{VaR}(u, X_t | I_t) = G^{-1}(u, X_t | I_t).
\]  

(A17)

Put another way, the VaR of the portfolio \( V \) is the \( u \)th quantile of the distribution of changes in the value of the portfolio.

148. In a large portfolio the VaR calculations will depend on the joint distribution of numerous different instruments. To simplify the calculation, practitioners usually make some simplifying assumptions on the risk factors, on the method of calculating the change in the value of the portfolio, and on the statistical distributions underlying the different risk factors.

149. In many empirical implementations of VaR methodology, the normal distribution is chosen as the underlying statistical model of returns. There are several reasons for this choice. First, because of the additive feature of normal distributions,\(^48\) it is simple to aggregate across portfolios. The normal distribution also permits a relatively simple analytic representation of the distribution, as the distribution is completely described by the first two moments of the distribution (i.e., the mean and the variance).

150. There are two broad approaches to estimating a VaR. The first approach is the local valuation method, which uses an estimate of the sensitivity of the portfolio multiplied by the estimated price change to arrive at the estimated change in value. This approach takes the slope and curvature of the portfolio around its present market value to impute the new value under changed circumstances. The delta-normal method is a widely used approximation method in this category. This method assumes normal distributions, and uses linear derivatives (the delta) to approximate the change in value of a portfolio. A second variant of the local valuation method is the delta-gamma approach, which uses an additional second order term (the gamma) to improve the accuracy of the delta-normal method.

151. The second major approach to VaR estimation is the full valuation method. Instead of using approximations, this approach recalculates the value of the portfolio using a new vector of prices. Two widely used full-valuation methods are the historical valuation method (where historical prices are used to form the vector of new prices) and the Monte Carlo simulation method (where new prices are simulated using known distributions).

152. The choice of estimation method involves a tradeoff between accuracy and computational burden.\(^49\) Approximation methods are easier to implement but are less accurate, especially if the portfolio has many instruments with nonlinear payoff characteristics (such as options). Full valuation methods provide greater precision, but are

\(^48\) I.e., a linear combination of two normal distributions is also normal.

\(^49\) Pritsker (1997) examines the tradeoff between efficiency and accuracy in great detail.
significantly more complex to design and implement. The high resource cost of evaluating a large portfolio of complex products using full valuation methods may preclude the regular use of the methodology, undermining its effectiveness as a risk management tool, especially on a day-to-day basis.

153. The Derivatives Policy Group (1995) recommends the use of a definition of capital at risk that equals the maximum loss expected to be exceeded with a probability of one percent over a two-week period. The changes in risk factors that they suggest are as follows:

- An increase and decrease in equity index values by 10 percent
- An increase and decrease in equity index volatilities by 20 percent;
- An increase and decrease in the exchange value (relative to the U.S. dollar) of foreign currencies by 6 percent, in the case of major currencies, and 20 percent, in the case of other currencies;
- An increase and decrease in foreign exchange rate volatilities by 20 percent; and
- An increase and decrease in swap spreads by 20 basis points.

C. Models of Credit Risk

Likelihood of default

154. Many financial institutions evaluate credit risk exposures in terms of the risk of credit losses, where losses can be defined as the difference between the present value of a portfolio and the future value at the end of some time horizon. 50 Credit losses can be measured according to two different criteria. 51 The first approach—the Default Mode (DM) approach—defines credit losses as losses that are caused by default events taking place in a predefined period. A credit instrument can either default—in which case its value is given by the discounted value of future recoveries, less workout costs—or it can remain performing and keep its book value. This approach, appropriate for “buy and hold” strategies, is typically suited for banks that have limited access to secondary markets for credit instruments. The second approach—the Mark to Market (MTM) approach—does not limit measurement of credit losses to default events, but takes into consideration variations in a borrower’s creditworthiness that may affect the value of its debt position. The value of a credit instrument granted to a solvent borrower may vary with changes in the borrower’s financial

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50 Since credit instruments are generally less liquid in comparison with many market instruments, many banks use a time horizon of a year, which corresponds to the interval required for an orderly disposal of assets.

51 See Jones and Mingo (1998).
position. In this multi-state approach the value of a credit instrument can migrate to different values within the range between its book value and the value of recovery in the event of default.

155. Given the definition of a credit event, we need to measure the likelihood that an instrument migrates from one value to another in the relevant planning horizon. In the Default-Mode approach, average historical default frequencies can provide a measure of the probability of the transition of loans from a non-default state to a default state, using a single number. In contrast, the Mark-to-Market approach employs a matrix of transition probabilities to show the probability of migrating from one credit rating to another. In this formulation, default events represent the last of a series of credit events. An example of a ratings transition matrix is given below, which shows Moody’s historical ratings transition matrix for corporate bond issuers.\(^{52}\)

156. While information about the expected frequencies over a one-year time horizon can be gathered from the historical transition matrix presented in Table A3, the volatility of these frequencies (necessary for evaluating unexpected losses) needs to be measured from the dispersion of yearly values around their average values. Since there is no unique measure of dispersion, we report, as an example, the third quartile and the maximum value of the distribution of default frequencies of bonds with different initial rating observed over a thirty years period by Moody’s.\(^{53}\)

Table A3. Moody’s Average Rating Transition Matrix For Bond Issues, 1980-99
(All corporates, in percent)

<table>
<thead>
<tr>
<th></th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
<th>Default</th>
<th>WR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>85.88</td>
<td>9.76</td>
<td>0.48</td>
<td>0.00</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>3.84</td>
</tr>
<tr>
<td>Aa</td>
<td>0.92</td>
<td>84.87</td>
<td>9.64</td>
<td>0.36</td>
<td>0.15</td>
<td>0.02</td>
<td>0.00</td>
<td>0.04</td>
<td>4.01</td>
</tr>
<tr>
<td>A</td>
<td>0.08</td>
<td>2.24</td>
<td>86.24</td>
<td>6.09</td>
<td>0.77</td>
<td>0.21</td>
<td>0.00</td>
<td>0.02</td>
<td>4.36</td>
</tr>
<tr>
<td>Baa</td>
<td>0.08</td>
<td>0.37</td>
<td>6.02</td>
<td>79.16</td>
<td>6.48</td>
<td>1.30</td>
<td>0.11</td>
<td>0.19</td>
<td>6.30</td>
</tr>
<tr>
<td>Ba</td>
<td>0.03</td>
<td>0.08</td>
<td>0.46</td>
<td>4.02</td>
<td>76.76</td>
<td>7.88</td>
<td>0.47</td>
<td>1.40</td>
<td>8.89</td>
</tr>
<tr>
<td>B</td>
<td>0.01</td>
<td>0.04</td>
<td>0.16</td>
<td>0.53</td>
<td>5.86</td>
<td>76.07</td>
<td>2.74</td>
<td>6.60</td>
<td>7.98</td>
</tr>
<tr>
<td>Caa-C</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>2.79</td>
<td>5.38</td>
<td>56.74</td>
<td>25.35</td>
<td>8.73</td>
</tr>
</tbody>
</table>

Source: Keenan et al. (2000, p. 25). WR represents the withdrawn ratings category.

\(^{52}\) The default probabilities are applicable to corporate bond issues, which may be quite different to the behavior of loan defaults.

\(^{53}\) For illustrative purposes we can consider the case of provisioning and capital required to cover the default risk of a B rated bond when the recovery ratio in the event of default is zero. General loss provisions should be equal to 6.60 per cent (average default frequency in Table A3). To cover only the first quartile of losses, capital allocation should be 1.55 percent (8.15 from Table A4 less 6.60 from Table A3). To cover the maximum losses, capital allocation should be 17.4 percent (24.00 from Table A4 less 6.60 from Table A3).
Table A4. Volatility of Default Frequencies for Different Rating Grades: 1970–99 (percent)

<table>
<thead>
<tr>
<th>Volatility Measures</th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd Quartile</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.14</td>
<td>1.84</td>
<td>8.15</td>
<td>38.33</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.00</td>
<td>0.80</td>
<td>0.35</td>
<td>1.89</td>
<td>5.32</td>
<td>24.00</td>
<td>67.50</td>
</tr>
</tbody>
</table>

Source: Keenan et al. (2000, p. 25).

**Loss in the event of default**

157. To move from the distribution of default probabilities to credit losses, it is necessary to estimate the present value of the amount that can be recovered from a defaulted loan. Recoveries are not only a function of the different types of financial assets (according to maturity, collateralization, guarantees etc.), but also a function of seniority and the different degrees of creditor protection. The recovery rate may change with the economic cycle, and it may also vary by type of instrument. The institutional environment (which includes factors such as the efficiency of the judicial system, accounting policies and procedures etc.) can also affect the timing and the present value of recoveries of defaulted loans. For simplicity, the recovery rate is often assumed to be fixed, equal to the long run average rate or set to zero.\(^{54}\)

158. Exposures to expected losses can be estimated from transition matrices, based on historical default frequencies. This information can be provided by central banks, bank supervisors, or credit registers. The use of default frequencies is consistent with the DM approach. Data on the frequency distribution for different rating categories may be available where bank supervisors provide banks with criteria for loan classification, for example based on different lags of past due payments. Evidence of this type is consistent with the MTM approach, but is often available only in a limited number of markets.

159. Exposures to unexpected credit losses can be measured using commercial products such as those provided by Oliver, Wyman and Company, JPMorgan’s Creditmetrics, Credit Suisse’s CreditRiskPlus, McKinsey’s Credit Portfolio View, and KMV’s Creditor Monitor. Most of these commercial approaches are based on the availability of a rich dataset on default frequencies and loan classifications. They are also addressed to individual institutions that can observe and keep records of their borrowers’ features. Their complexities are often ill-suited to a more general approach for detecting elements of systemic risk, but they can offer useful methodological approaches to the measurement of risk exposures. The commercial credit risk management products can also help to define the amount and nature of data that are required for promoting sound risk management practices, thus playing a very important “institution building” role. The emphasis on microeconomic determinants of credit

\(^{54}\) See Keenan et al. (2000, p. 25) for long run average recovery rates of different classes of bonds.
risk can also be a limiting factor, especially when trying to estimate the impact on the financial system of a common external shock. A proper specification of the impact of macroeconomic factors on financial institutions enables an analysis of different sources of credit risk in countries with different level of economic development, size, and financial structure. In this respect the McKinsey Credit Portfolio View is one of the more useful approaches for evaluating systemic and macroeconomic aspects of credit risk.

The prevailing approaches

160. The CreditRisk+ model developed by Credit Suisse Financial Products is an insurance-based model of credit risk. CreditRisk+ assumes that loan defaults are independent events (i.e., zero correlation between any pair of loans), and individual loan defaults are random events driven by a Poisson process.

161. CreditMetrics applies the Value-at-Risk framework to measure the risk of a portfolio of credit instruments. The Value-at-Risk on a credit portfolio is given by the product of the value of the portfolio (i.e., the market price of the different instruments) and the volatility, for a given confidence level. Since most credit instruments are not tradable, neither the value of an instrument nor its volatility are observable in most cases. CreditMetrics uses data on the credit rating of a borrower, together with a rating transition matrix, average recovery rates, and yield spreads in the bond market to derive a measure of the market price and its volatility for any group of credit instruments. Correlations between different obligors are taken into account by mapping each obligor into a country and industry index and using the correlations between the different indices.

162. The Credit Monitor Model of KMV uses an options-pricing model to calculate the expected default frequency of a given obligor. The model is based on Merton’s insight that the payoff for a bank making a loan is equivalent to a put option on the assets of a borrowing firm. The model uses information from equities markets as a proxy for the market value and volatility of assets, together with information on the face value of debt. KMV’s portfolio manager model applies portfolio theory to a loan portfolio to calculate the efficient risk-return frontier or to infer the marginal contribution of a loan to total portfolio risk. The model uses a measure of the return on the loan (all-in-spread less the expected default frequency times the loss given default), the risk of the loan (unexpected loss), and the correlation (based on the correlation of systematic equity returns) to derive an empirical measure of risk.

163. The main shortcoming of the commercial approaches relates to the availability of information:

55 For more details, see Credit Suisse First Boston (1997).

56 For more details, see Gupton et. al. (1997)
• For credit rating assessments. Rating agency assessments are used by JPMorgan’s Creditmetrics for building the transition matrix. Although rating agencies provide one of the few reliable historical data sets on default frequencies for homogeneous categories of borrowers/assets, their grades refer mostly to bond issuers and therefore to borrowers that are considerably different from bank borrowers in terms of size, market access, and financial structure. This may represent a severe limitation and provide poor guidance if used to assess the distribution of losses of bank loans.

• For stock market data. Market information is used by KMV for devising categories of borrowers with similar features of financial fragility. Stock prices, in principle provide a very important source of forward-looking information that may usefully complement backward-looking balance sheet data. Stock prices, though, are seldom available for the small and medium businesses that are the bulk of bank customers. In addition, very few stock markets are able to provide the level of liquidity necessary to reduce the amount of noise in individual stock prices. The statistical approach followed by Credit Suisse’s Credit Risk Plus is more flexible in this respect, being based on default frequencies that are more readily available and for longer time periods. Sufficiently long time series of data are important inputs in ensuring the stability of parameter estimates.