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Fiscal Rules and Countercyclical Policy:  
Frank Ramsey Meets  
Gramm-Rudman-Hollings

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**Fiscal Rules and Countercyclical Policy:  
Frank Ramsey Meets Gramm-Rudman-Hollings<sup>1</sup>**

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Authorized for distribution by Enzo Croce

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**Abstract**

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Fiscal rules—legal restrictions on government borrowing, spending, or debt accumulation (like the Gramm-Rudman-Hollings Act in the United States)—have recently been adopted or considered in several countries, both industrial and developing. Previous literature stresses that such laws restrict countercyclical government borrowing, thus preventing intertemporal equalization of marginal deadweight losses of taxation—an idea associated with Frank Ramsey. However, such literature typically abstracts from persistent current deficits that are financed by future tax increases. Eliminating such deficits may substantially reduce tax rate variability—the very goal of countercyclical borrowing—even over a *finite horizon*. Thus, Gramm-Rudman-Hollings and Frank Ramsey are not necessarily enemies and they may even be good friends!

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## I. INTRODUCTION

Fiscal rules—legal restrictions on government borrowing, spending, or debt accumulation—have recently been adopted in several countries and are being discussed in several others (both industrialized and developing). While the details of fiscal rules may differ across countries, debates regarding their adoption involve similar issues.

Opponents of fiscal rules emphasize that they prevent the government from smoothing tax rates and expenditures over the business cycle, and may even prohibit discretionary countercyclical policy.<sup>2</sup> By contrast, their proponents argue that fiscal rules supplement weak institutions to promote fiscal responsibility and credibility.<sup>3</sup> This issue may be especially important in those Latin American countries that have suffered from chronic fiscal indiscipline.<sup>4, 5</sup>

Economic theory should be able to help policy makers evaluate alternative fiscal policies, including legal restrictions on fiscal policy (balanced budget restrictions or debt ceilings). A sizeable literature already examines fiscal policy from a welfare theoretic perspective. Much of this work builds on Frank Ramsey's (1927) insight that governments should equate the marginal deadweight losses from different tax sources. Robert Barro (1979) applied this insight to determine the optimal path of tax rates over time. He concluded that deadweight losses would be minimized under a policy of *tax smoothing*. Moreover, under such a policy, fiscal policy is optimally countercyclical in the sense that the government is permitted to borrow during economic recessions (but must save during upturns).<sup>6</sup>

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<sup>2</sup> Fiscal rules may also encourage governments to use questionable accounting procedures (Milesi-Ferretti (2000)).

<sup>3</sup> Drazen (2000, 2002) argues that fiscal rules compensate for inherent pro-deficit biases. Aizenman, Gavin, and Hausmann (2000) and Stockman (2001a,b) formally model the credibility gains from fiscal rules.

<sup>4</sup> For example, among Latin American countries, fiscal rules have been recently enacted in Argentina, Brazil, Colombia, Peru and Chile, and they have been proposed in elsewhere.

<sup>5</sup> Empirical evidence on fiscal rules (largely for industrialized countries) supports both claims. For example, Bayoumi and Eichengreen (1995) suggest that jurisdictions (U.S. states) that have more restrictive fiscal rules do also run smaller deficits. At the same time, fiscal policy in such jurisdictions tends to be more procyclical in nature than jurisdictions without fiscal rules.

<sup>6</sup> In work after Barro (1979)—for example Lucas and Stokey (1983), Chari and Kehoe (1999), and Aiyagari, Marcet, Sargent and Seppälä (2002)—a “Ramsey approach” came to be understood as one in which the authority *directly* maximizes a the utility of a

(continued...)

In this vein, several recent papers have compared a balanced-budget fiscal rule like the U.S. Gramm-Rudman-Hollings (GRH) Act with the optimal (Ramsey) policy discussed above.<sup>7</sup> Unsurprisingly, such research has generally confirmed that welfare under the Ramsey policy is higher than under the more restrictive fiscal rule.<sup>8</sup>

However, it may be relevant to compare such a balanced-budget restriction against a broader range of policies. As one alternative, a government might run primary deficits today but is nonetheless expected to finance its debt service with future surpluses. Since tax rates are not smoothed over time, such a policy is not optimal. Nonetheless, such a policy may more closely resemble those of actual governments than the optimal (Ramsey) policy. How would a country's tax rates—level and variability—change if the government replaced such a policy with a GRH-like fiscal rule? Which regime would the country's residents prefer?

This paper compares a restrictive fiscal law—one that prohibits government from borrowing more than the minimum required to keep debt-GDP constant—against several policy alternatives.<sup>9</sup> Under a benchmark policy, tax rates are completely smooth. Also, a general fiscal reaction function that may resemble more closely policies of actual governments is considered. This policy links tax rates to debt (thus ensuring long-run solvency) but also allows for a constant (potentially deficit) component.

Uncertainty from two sources is assumed: cyclical output and access to credit. However *permanent* output is known and the ratio of government expenditures to *permanent* output is constant. Under these assumptions, countercyclical fiscal policy is synonymous with smooth tax rates. Welfare is assumed to fall when either the mean or the variance of tax rates rises.

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representative consumer over an infinite horizon. However, the position taken in this paper is that a “Ramsey approach” may also be one where the authority *indirectly* maximizes utility, including by minimizing a deadweight loss function (as did Barro (1979)). Note also that the “countercyclical policy” in this paper refers specifically to the smoothing of taxes or expenditures. Issues of endogenous countercyclical spending (automatic stabilizers) or discretionary policy are left for another paper.

<sup>7</sup> Reference to Gramm-Rudman-Hollings (GRH), a legislative act that has been modified and weakened, is for rhetorical purposes only. Fiscal rules outside the United States may differ considerably either from GRH or the fiscal rule proposed in this paper.

<sup>8</sup> For example see Schmitt-Grohe and Uribe (1997).

<sup>9</sup> *Ex-ante*, the rule permits just enough borrowing to keep the debt ratio constant. However, since tax rates are set before output is known, *ex post* borrowing also reflects a forecasting error. Such net borrowing might be thought of as variations in public sector bank deposits.

Previous literature has stressed that the restrictive nature of a fiscal rule like Gramm-Rudman-Hollings hinders countercyclical public borrowing and thus also hinders tax smoothing. By contrast, this paper notes that the removal of persistent current primary deficits—through either a fiscal rule or a once-and-for-all fiscal reform—permits smoother tax rates than otherwise.<sup>10</sup>

Over an infinite horizon, the welfare gains implied by moving closer to a Ramsey regime—lower and less variable tax rates—should be immediately apparent. However, over shorter horizons, the issue is not as clear-cut. If policy makers choose to finance some level of government expenditures today by accumulating debt, taxes *today* may be lower, but they will also be more variable. Simulations presented in this paper suggest the magnitude of this tradeoff.<sup>11</sup>

Thus, under certain conditions, a fiscal rule may reduce tax rate variability—precisely the goal of countercyclical borrowing—even over a finite horizon. Put differently, Frank Ramsey and Gramm-Rudman-Hollings are not necessarily enemies. In fact, they may be good friends!<sup>12</sup>

The paper is organized as follows. Section II presents basic identities and discusses optimal fiscal policy over an infinite horizon under certainty. A general fiscal rule is introduced. Section III presents an alternative model in which the authority may favor current taxpayers at the expense of future ones. In Section IV, uncertainty in output and borrowing restrictions are introduced. Section V presents the central analysis of the paper: alternative fiscal regimes are defined and simulated. Section VI extends the framework to include variable government expenditures. Section VII presents some evidence regarding public sector *size* and expenditure variability. Section VIII summarizes and concludes.

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<sup>10</sup> In the model, the deficit is eliminated by raising taxes. Potential examples of such a policy might include a *one-time* tax rate increase or improvement in tax collection. More broadly, *one-time, permanent expenditure reductions* may also help reduce the deficit. In this sense, there may be a distinction between a *fiscal rule* and a *fiscal reform*. A fiscal rule, according to Kopits and Symansky (1998), is a permanent restriction on fiscal policy, while a reform occurs at one point in time. Of course, the two measures may complement one another. And, as a legal matter, the two may be combined. For example, Brazil's Fiscal Responsibility Law (FRL) not only limits borrowing but also mandated a one-time, permanent reduction in public sector employment. For further details, see Guardia and Messenberg (2002).

<sup>11</sup> Whether or not the costs of increased tax rate variability exceeds the benefits of lower tax rate *levels* depends on the precise form of the loss function (the degree of risk aversion).

<sup>12</sup> Note that, in Latin America fiscal policy in the region has generally been procyclical even *without* fiscal rules (see, for example, Gavin and Perotti (1997), Talvi and Végh (2000b)).

## II. THE OPTIMAL FISCAL RULE: CERTAINTY, INFINITE HORIZON

In any period, the government's budget constraint is:

$$b_{t-1}\theta - ps_t = b_t \quad (1)$$

where  $b$  is the ratio of government debt to GDP,  $\theta$  is the growth adjusted discount factor  $(1+r)/(1+\lambda)$ ,  $r$  = real interest rate (constant),  $\lambda$  = permanent real GDP growth,  $\lambda < r$ ,  $ps_t$  is the primary (non-interest) surplus (ratio to GDP). The intertemporal budget constraint is obtained by successive substitution of (1) over an infinite horizon:

$$b_{-1} - \sum_{t=0}^{\infty} ps_t / \theta^{t-1} = \lim_{t \rightarrow \infty} b_t / \theta^{t-1} \quad (2)$$

The transversality (or “no-Ponzi game”) condition is:

$$\lim_{t \rightarrow \infty} b_t / (1+r)^{t-1} = 0 \quad (3)$$

The primary surplus is the difference between tax ratio  $\tau$  and noninterest expenditures  $\gamma$ . For convenience, we assume a constant and exogenous expenditure ratio.<sup>13</sup>

A general expression for fiscal regimes—including legally stipulated fiscal rules—is now introduced. Tax rates, primary expenditures and debt are linked according to:

$$\tau_t = \gamma - \kappa + \beta b_{t-1} \quad (4)$$

Fiscal policy is therefore summarized by the government's choice of  $\kappa$  and  $\beta$  (for given values of initial  $b_{-1}$ , and constants  $\gamma$ ,  $\lambda$  and  $r$ ). The term  $\kappa$  may be thought of as a “tax gap” (Blanchard et al. (1990)) reflecting either a decision to keep taxes low or inefficient tax collection.

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<sup>13</sup> Other authors have used such an expression; see, for example, Leeper (1991). In this section, since growth is constant, actual and permanent output are identical. Note also that the assumption of constant  $\gamma$  is made for simplicity. An extension to the case of variable expenditure ratios is presented later in the paper.

In any period, losses are assumed to increase in both first and second derivatives:  $\phi' > 0$ ,  $\phi'' > 0$ .<sup>14</sup> Thus, the government choose a path of tax rates over time  $\tau_t$  that minimizes the *discounted* loss function subject to (3):<sup>15</sup>

$$\sum_{t=0}^{\infty} \phi(\tau_t) \theta^{-t} \quad (5)$$

The intertemporal first order condition is:  $\phi'(\tau_t) = \phi'(\tau_{t+1})$  for all  $t$ . Thus, a result similar to Barro (1979) obtains. For an optimum,  $\kappa = 0$  and  $\beta = (r - \lambda)/(1 + \lambda)$ . Doing so ensures both *tax-smoothing* ( $\tau_t = \tau_{t+1}$ , all  $t$ ) and that the debt ratio remains constant at  $b_{-1}$ .

Note that this result resembles the widely-used policy framework associated with Blanchard et. al. (1990) and Talvi and Végh (2000a).<sup>16</sup> Both papers note that sustainable fiscal policy (satisfaction of (3) without inflation or default) requires a permanent primary surplus of  $[r - \lambda]/[1 + \lambda] * b^P$  ( $\kappa = 0$  and  $\beta = [r - \lambda]/[1 + \lambda]$ ). However, strict satisfaction of (3) does *not* require  $\kappa = 0$  and  $\beta = [r - \lambda]/[1 + \lambda]$ . Rather, sustainability only requires  $\beta > 0$ . But, the analysis shows conditions under which  $\kappa = 0$  and  $\beta = [r - \lambda]/[1 + \lambda]$  is an *optimal* policy.<sup>17</sup>

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<sup>14</sup> Previous literature in this area typically specifies household preferences. For a closed economy doing so is important, since intertemporal fluctuations in the tax rate may affect intertemporal allocations of consumption and leisure. By contrast, for an open economy such a specification would be inessential, since the government smooths its expenditure stream over time through international capital markets (the current account). In this context, the loss function should be interpreted as one of *collection costs* on a commodity tax – for example tax evasion. While consumers might wish to smooth such costs over time, incomplete markets prohibit them from doing so.

<sup>15</sup> Default, through inflation or otherwise, is not considered in this paper.

<sup>16</sup> This idea is widely used in policy literature regarding fiscal sustainability. See, for example, Chalk and Hemming (2000), Chalk (2002) and Croce and Juan-Ramón (2002).

<sup>17</sup> However, as discussed above, a sustainable policy  $\kappa > 0$  and  $\beta < [r - \lambda]/[1 + \lambda]$  has an important drawback: debt and tax rates increase over time. In some cases, the discounted debt may also grow, but only initially. Note also that formally debt *per se* is not “bad” if the borrower sticks to its chosen values of  $\kappa$  and  $\beta$  and lenders continue to lend—*no matter what*. Such a case is formally equivalent to one discussed by McCallum (1984).



### III. FAVORING THE PRESENT OVER THE FUTURE?

Much evidence suggests that governments do not pursue policies that resemble  $\kappa = 0$  and  $\beta = [r-\lambda]/[1+\lambda]$ . For example, Table 1 presents selected recent fiscal variables—the primary surplus and debt (in percent of GDP) and real GDP growth—for several Latin American countries. In several key cases—Argentina, Brazil, Colombia, Peru, Uruguay—government debt grows substantially, while primary balances are persistently below values consistent with  $\kappa = 0$  and  $\beta = [r-\lambda]/[1+\lambda]$ , and in some years negative.

Governments may instead favor present over future taxpayers.<sup>18</sup> If they do so, their loss functions would instead contain two distinct components:  $\phi_P(\tau)$  for the present (P,  $t = 0$  to J) and  $\phi_F(\tau)$  for the future (F,  $t = J+1$  to  $\infty$ ). As before,  $\phi_i' > 0$ ,  $\phi_i'' < 0$ ,  $i = P, F$ . As above, governments choose a path of tax rates  $\tau_t$  to minimize

$$\sum_{t=0}^J \phi_P(\tau_t)\theta^{-t} + \sum_{t=J+1}^{\infty} \phi_F(\tau_t)\theta^{-t} \quad (5')$$

subject to (3). And, as before, within periods P and F, the authority sets  $\phi_i'(\tau_t) = \phi_i'(\tau_{t+1})$ ,  $i = P, F$ . *Across* periods P and F, the rate at which the authority prefers P over F is summarized by an interperiod marginal of substitution, P and F,  $\mu_{PF}$ . This number is assumed to be constant. Thus, for an optimum, the authority must equate the ratio of average tax rates between periods P and F  $\tau_P/\tau_F$  to the marginal rate of substitution  $\mu_{PF}$ .

To see that problem (5') is compatible with a fiscal rule like (4), suppose that the government gives a tax cut  $\kappa_P$  to period P but requires period F to pay a surcharge  $-\kappa_F$ . Sustainability thus requires:

$$\kappa_P = -\kappa_F / \{\theta^{J+1} - 1\} \quad (6)$$

For simplicity, assume  $\beta$  is constant. If  $\tau_P/\tau_F = \mu_{PF}$ , the optimal “tax break” for current taxpayers  $\kappa_P^*$  is:

$$\kappa_P^* = \{\gamma(1-\mu_{PF}) + \beta(b_P - b_F)\} / \{1 + \mu_{PF}(\theta^{J+1} - 1)\} \quad (7)$$

That is,  $\kappa_P^*$  is a function of  $\mu_{PF}$ ,  $\gamma$ ,  $r, \lambda$ ,  $\beta$ , and debt in both periods,  $b_P$  and  $b_F$  respectively.

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<sup>18</sup> One kind of default is unanticipated inflation. For simplicity, we do not consider such issues in this paper.

Table 1. Primary Surplus, Debt and GDP Growth  
Selected Latin American Countries

	1996	1997	1998	1999	2000		
<b>Argentina</b>							
Primary surplus (ps <sub>t</sub> )		0.3	0.5	-0.8	0.5	Period avg.	0.1
Debt (b <sub>t</sub> )		38.1	41.3	47.4	50.8	Period avg. chg	4.2
GDP Growth ( $\lambda_t$ )		8.1	3.8	-3.4	-0.5	Period avg.	2.0
<b>Brazil</b>							
Primary surplus (ps <sub>t</sub> )	-0.1	-1	0	3.1	3.5	Period avg.	1.1
Debt (b <sub>t</sub> )	33.3	34.6	42.4	47	49.2	Period avg. chg	4.0
GDP Growth ( $\lambda$ )	2.7	3.3	0.2	0.8	0.8	Period avg.	1.5
<b>Chile</b>							
Primary surplus (ps <sub>t</sub> )	2.7	0.4	-1.6	-3.2	-2.1	Period avg.	-0.8
Debt (b <sub>t</sub> )	6.7	6.5	8.1	9.2	8.4	Period avg. chg	0.4
GDP Growth	7.4	7.4	3.9	-1.1	5.4	Period avg.	4.6
<b>Colombia</b>							
Primary surplus (ps <sub>t</sub> )	-0.1	-1.4	-0.7	-2.1	0.8	Period avg.	-0.7
Debt (b <sub>t</sub> )	24.5	26.7	29.9	38.2	35.5	Period avg. chg	2.8
GDP Growth	2.1	3.4	0.5	-4.2	2.8	Period avg.	0.9
<b>Costa Rica</b>							
Primary surplus (ps <sub>t</sub> )			2	1.4	1	Period avg.	1.5
Debt (b <sub>t</sub> )			33.6	27.7	30.3	Period avg. chg	-1.7
GDP Growth			8.4	8.4	1.7	Period avg.	6.2
<b>Dominican Republic</b>							
Primary surplus (ps <sub>t</sub> )			-1.2	-1.2	-1.1	Period avg.	-1.2
Debt (b <sub>t</sub> )			28.4	26.8	26	Period avg. chg	-1.2
GDP Growth			7.3	8	7.8	Period avg.	7.7
<b>El Salvador</b>							
Primary surplus (ps <sub>t</sub> )			-0.1	-0.2	-0.2	Period avg.	-0.2
Debt (b <sub>t</sub> )			24.8	25.6	27.5	Period avg. chg	1.4
GDP Growth			3.2	3.4	2	Period avg.	2.9
<b>Guatemala</b>							
Primary surplus (ps <sub>t</sub> )			-0.3	-1.4	-0.7	Period avg.	-0.8
Debt (b <sub>t</sub> )			15.5	18.4	18	Period avg. chg	1.3
GDP Growth			5.1	3.8	3.6	Period avg.	4.2
<b>Honduras</b>							
Primary surplus (ps <sub>t</sub> )			5.6	1.8	0.2	Period avg.	2.5
Debt (b <sub>t</sub> )			76.1	78.6	72.4	Period avg. chg	-1.8
GDP Growth			2.9	-1.9	5.0	Period avg.	2.0
<b>Mexico</b>							
Primary surplus (ps <sub>t</sub> )	3.5	2.2	0.4	1	1.5	Period avg.	1.3
Debt (b <sub>t</sub> )	50	46.6	50	46.7	41.7	Period avg. chg	-2.1
GDP Growth	5.2	6.8	5.0	3.6	6.6	Period avg.	5.5
<b>Peru</b>							
Primary surplus (ps <sub>t</sub> )			1.2	-0.8	-0.9	Period avg.	-0.2
Debt (b <sub>t</sub> )			42.7	42.8	45.9	Period avg. chg	1.6
GDP Growth			-0.5	0.9	3.1	Period avg.	1.2
<b>Uruguay</b>							
Primary surplus (ps <sub>t</sub> )	0.5	0.5	0.9	-2.1	-1.2	Period avg.	-0.8
Debt (b <sub>t</sub> )	31.3	31.8	34.2	40.1	45.8	Period avg. chg	4.7
GDP Growth	5.6	4.9	4.7	-3.2	-1.0	Period avg.	0.2

Sources: IMF Staff Reports and Recent Economic Developments (various); for Central American and Caribbean Countries, Offerdahl (2002).

#### IV. OPTIMIZATION UNDER UNCERTAINTY: OUTPUT AND ACCESS TO CREDIT

Uncertainty is an essential aspect of most policy environments. This paper assumes uncertainty in both output and access to credit markets. For simplicity, both forms of uncertainty are assumed to be exogenous. However, even if the uncertainty were instead to be endogenous, the qualitative implications of the analysis would be similar.

First, output  $Y$  is the sum of its (certain) permanent and (uncertain) temporary components:

$$Y_t = Y_t^P + v_t \quad (8)$$

where  $Y_t^P = Y_t^P (1+\lambda)$  is permanent (trend) output, and  $v_t$  is mean-zero temporary income (deviation from trend) whose known variance is constant relative to  $Y_t^P$ . Note that government spending as a fraction of permanent income  $\gamma$  remains constant.

Second, a random element to credit markets is introduced. The country is assumed to face a cutoff from access to borrowing with probability  $\pi^c$  which is uniformly distributed between 0 and 1. If  $\pi^c = 0$ , the country will have access to borrowing with certainty. If  $\pi^c$  is 0.5, there is a 50 percent chance that the country will not be able to borrow.

If a government is denied access to credit in a period that it would have otherwise have borrowed, it must raise taxes in that period.<sup>19</sup> At the same time, a cutoff from borrowing has an asymmetric effect: it limits a country's deficit but not its surplus. Thus, suppose the country follows the tax smoothing policy discussed in the previous section, but sets an (ad hoc) target level of debt  $b^P$ . With unfettered access to credit, taxes in each period are simply  $\tau(U) = \gamma + (r-\lambda)/(1+\lambda)b^P$ . By contrast, if borrowing is constrained, taxes are linked to the level of debt in the previous period according to  $\tau(C)_t = \gamma + (r-\lambda_t)/(1+\lambda_t)b_{t-1}$ , where  $\lambda_t$  is prospective total growth in period  $t$ . Thus, taxes in any period are the minimum of  $\tau(U)$  and  $\tau(C)_t$ .

However, under such assumptions,  $\kappa=0$  may no longer be optimal. Rather, if  $\pi^c > 0$ , fiscal policy has a *precautionary* element: governments may want to self-insure against prospective

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<sup>19</sup> Introducing a random exclusion from borrowing also indirectly brings in the issue of default. If borrowers know with certainty that they will be cut off from all borrowing in the present and future, and there is no other default penalty, the borrower will default. By contrast, if the government knows with certainty that they will never be cut off from credit markets, default is less likely than otherwise. In the scheme presented here, governments find themselves somewhere between these two extremes. More realistically, the probability of a borrowing cut-off should be modeled as a function of debt itself. Such a task is left for another paper.

borrowing cutoffs. As a simple two period example, suppose that a currently unconstrained government wishes to equate current (period 1) marginal deadweight loss with expected future (period 2) deadweight loss, by choosing the optimal surplus  $(r-\lambda)/(1+\lambda)b_0 - \kappa_1^*$  ( $\kappa_1 < 0$ ) in period 1. Therefore,  $\kappa_1^*$  will be chosen to satisfy:

$$\phi'(\tau(U) - \kappa_1) = (1 - \pi^C) E\{\phi'(\tau(U))\} + \pi^C E\{\phi'(\tau(C)_2)\} \quad (9)$$

Current marginal deadweight loss	Future marginal deadweight loss, unfettered access to borrowing	Future expected marginal deadweight loss, no access to borrowing
-------------------------------------	--	--

In (9), note that if  $\pi^C=0$ ,  $\kappa_1^*=0$ . More generally,  $\partial \kappa_1^* / \partial \pi^C < 0$ : as borrowing becomes more restricted (probabilistically), the optimal primary *surplus* rises.<sup>20</sup>

To extend a framework like (9) to many periods also requires that an *ad hoc* debt target must be specified. Otherwise, the continually run surpluses and become a net creditor. Such a result is formally derived in Aiyagari, Marcet, Sargent and Seppälä (2002). Specifically, they show that under “natural” debt limits—“the maximum debt that could be repaid almost surely under an optimal tax policy”—the optimal tax rate will be zero, with expenditures financed entirely from interest receipts. By contrast, under an *ad hoc* debt limit, their model predicts that taxes will be positive and will follow a random walk, similar to Barro (1979).

## V. A COMPARISON OF FISCAL REGIMES

As a practical matter, formulating a policy explicitly based on an optimizing framework may pose difficulties.<sup>21</sup> For this reason, some have emphasized the importance of clear and simple policy rules—even if they are *ad hoc*. As discussed in further detail below, authors like Drazen (2000) have suggested that such simple rules—that is, balanced budget rules—may improve policy making since they provide a substitute for otherwise weak institutions.

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<sup>20</sup> In expression (9), the current period marginal loss from taxation is  $\phi'(\tau(U) - \kappa_1) = \phi'[\gamma + (r-\lambda) / (1+\lambda)b_0 - \kappa_1]$ , the future marginal loss, unconstrained case is  $\phi'(\tau(U))_2 = (1-\pi^C) \phi'[\gamma + (r-\lambda) / (1+\lambda)b_0]$ , and the future marginal loss, constrained case is  $\phi'(\tau(C))_2 = \phi'[\gamma + (r-\lambda_2) / (1+\lambda_2)(b_1 + \kappa_1)]$ , where  $b_0 = b^P$ .

<sup>21</sup> The optimal zero-tax policy derived by Aiyagari, Marcet, Sargent and Seppälä (2002), discussed above, is such an example.

In this section, three such rules are formulated and evaluated. First, under a benchmark policy regime (R0), tax rates are completely smooth (similar to optimizing under certainty) and the government is free to borrow over the business cycle while at the same time maintaining solvency. Second, under a restrictive fiscal law (R1), borrowing beyond the minimum required to keep debt-GDP constant is prohibited. Third, a general fiscal reaction function (R2) that may more closely resemble actual policies is considered. This policy links tax rates to debt (thus ensuring long-run solvency) but also allows for a constant (potentially deficit) component.

Successful implementation of (R0) may require strong institutions. However, if institutions are not sufficiently strong, and the authorities are not constrained by a fiscal law like (R1), they may instead choose a less restrictive fiscal regime like (R2).

As mentioned above, the loss function for taxation is assumed (in standard fashion) to increase in both first and second derivatives. Thus, losses increase with both the level and the *variability* of tax rates, and the authority will willingly trade off one against the other at some rate. In turn, tax rates and variability depend both on exogenous factors (that is,  $\text{var}(v_t)$  and  $\pi^c$ ) and the choice of regime.

One way to compare regimes would be to assume a loss function with an explicit functional form. However, doing so may be unnecessarily restrictive. Instead, an alternative strategy is used: simulated values for the average and standard deviation of tax rates from different regimes are presented. Doing so thus yields the implicit marginal rate of substitution between tax rate levels and standard deviations at which the government would be *indifferent* between regimes.

### A. The Regimes Defined

The fiscal regime that gives the government the most freedom to borrow over the business cycle while at the same time maintaining solvency is similar to the tax smoothing regime under certainty. The constant tax rate is:

$$\tau(0) = \gamma + (r-\lambda)/(1+\lambda)b^P \quad (R0)$$

where  $b^P$  is assumed to be equal to *initial* debt  $b_{-1}$ . Access to borrowing is unfettered. To see that the debt remains in the long run close to  $b^P$  and that solvency condition (3) is satisfied, note first that the primary surplus / GDP ratio in any period is:

$$ps(0)_t = \gamma[1 - w_t] + (r-\lambda)/(1+\lambda)b^P \quad (10)$$

where  $w_t = Y_t^P / Y_t$  is the ratio of permanent to total output in any period.<sup>22</sup> Thus, the borrowing requirement *beyond* the minimum required to keep debt / GDP constant  $br_t$  is:

$$br(0)_t = \gamma[w_t - 1] + (r - \lambda)/(1 + \lambda)[b_{t-1} - b^P] \quad (11)$$

Consider next a fiscal rule that explicitly ties today's tax rates to the previous period's debt. The tax rate is chosen when  $b_{t-1}$  is known but *before*  $Y_t$  is known. According to (5)  $Y_t^P$  is also the expected value of  $Y_t$ . Thus, taxes are set according to:

$$\tau(1)_t = \gamma + (r - \lambda_t^*)/(1 + \lambda_t^*)b_{t-1}. \quad (R1)$$

where  $\lambda_t^* = [Y_t^P / Y_{t-1} - 1]$  is *expected* output growth in any period. Thus, this rule aims *ex-ante* to maintain a constant debt / GDP ratio. Debt inherited from the previous period limits new borrowing more under (R1) than (R0). *Ex post*, governments may borrow or save.<sup>23</sup> The primary surplus ratio in any period is:

$$ps(1)_t = \gamma[1 - w_t] + [(1+r)/(1+\lambda^*)]b_{t-1} \quad (12)$$

while the *ex post* borrowing requirement is:

$$br(1)_t = \gamma[w_t - 1] + [(1+r)/(1+\lambda_t^*)\varepsilon_t]b_{t-1} \quad (13)$$

where  $\varepsilon_t = (\lambda_t - \lambda_t^*)/(1 + \lambda_t)$  reflects the forecasting error in period  $t$ .

Regimes (R0) and (R1) appear similar: in both cases  $\kappa = 0$  and  $\beta = r - \lambda/(1 + \lambda)$ . Therefore, average tax rates should be roughly equal across the two regimes. Tax rate variance is zero under (R0) but is positive under (R1). It is difficult to compare rule (R1) and (R0) without reference to institutional context.<sup>24</sup> We therefore assume that rule (R1) is *legally stipulated* while (R0) is not. As Drazen (2000) argues, explicit rules can bolster otherwise weak credibility and institutions.

<sup>22</sup> As a probability limit,  $E(1 - w_t)$  converges to zero. Thus, solvency is ensured, since discounted debt converges over an infinite horizon to:

$$\lim_{t \rightarrow \infty} b_t / \theta^t = \gamma \sum_{t=0}^{\infty} [w_t - 1] / \theta^t$$

<sup>23</sup> We might assume that such *ex post* variations in the debt ratio are reflected in changes in government bank deposits or other liquid assets.

<sup>24</sup> Schmitt-Grohe and Uribe (1997) make a similar comparison in the context of a growth model.

If institutions are weak and there is no fiscal rule, choice of  $b^P$  may be problematic; there may be incentives to revise  $b^P$  on a period-by-period basis. Without a rule like (R1), such revisions would likely be asymmetric: instead of adjusting, a government might simply raise its debt ceiling ( $b^P$ ).

However, (R0) may not be the relevant alternative to (R1). Instead, we may want to compare (R1) against a broader range of alternative fiscal regimes. Left without an explicit rule, a country's institutional structure may not abide by optimality conditions  $\kappa = 0$  and  $\beta = r - \lambda / (1 + \lambda)$ . Rather, the tax rule might take a more general form:

$$\tau(2)_t = \gamma - \kappa + \beta b_{t-1}, \quad \kappa > 0, \beta > 0. \quad (R2)$$

While (R1) is a legal requirement, (R2) is not. As mentioned above  $\kappa > 0$  reflects a tax gap. The primary surplus as a fraction of GDP is:

$$ps(2)_t = -\kappa + \beta b_{t-1}; \quad (14)$$

Incremental new borrowing is

$$br(2)_t = \kappa + [(r - \lambda) / (1 + \lambda) - \beta] b_{t-1}. \quad (15)$$

## B. Interaction Between Borrowing Constraints and Tax Regimes

As mentioned above, even under the constrained regime, countries may borrow in order to cover their forecasting error.<sup>25</sup> Thus, under (R1), neither taxes nor borrowing will be affected. For regimes (R0) and (R2), if a borrowing constraint holds in a given period, borrowing is the maximum of what obtains under (R1) (namely,  $br(1)_t = [w_t - 1] + [(1+r)/(1+\lambda^*)\varepsilon_t]b_{t-1}$ ) and the amount that would have been borrowed otherwise; the tax rate is the minimum of  $\tau(1)_t = \gamma + (r - \lambda) / (1 + \lambda_t^*) b_{t-1}$  and the unconstrained rate.<sup>26</sup> In this sense, governments benefit from past discipline: all else equal, lower debt levels imply smaller tax increases.

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<sup>25</sup> As mentioned above, such borrowing might reflect reductions in government bank deposits.

<sup>26</sup> More precisely, borrowing and taxes under borrowing constraints are:  $br(0^C)_t = \max\{br(1)_t, br(0)_t\}$ ,  $\tau(0^C)_t = \min\{\tau(1)_t, \tau(0)_t\}$ ,  $br(2^C)_t = \max\{br(1)_t, br(2)_t\}$ , and  $\tau(2^C)_t = \min\{\tau(1)_t, \tau(2)_t\}$ .

### C. Simulation Results

It should be immediately apparent—without simulations—that over an infinite horizon, the level of tax rates under (R0) and (R1) should be close to one another, but taxes are more variable under (R1).<sup>27</sup> By contrast, under a regime like (R2), with values of  $\kappa$  and  $\beta$  that differ from their infinite horizon optima (0,  $(r-\lambda)/(1+\lambda)$ , respectively) tax rates will be higher and more variable than under either (R0) or (R1).

However, such a distinction is not as clear-cut over shorter horizons (finite J). Policy makers choose (R2) if they want to provide a given level of government expenditures today but delay tax financing until some future date.<sup>28</sup> Doing so may noticeably increase both tax rate variability and debt accumulation over this shorter horizon. That is, as  $\kappa$  and  $\beta$  move further away from their infinite horizon optima, tax rate variability rises, thus frustrating the very goal of countercyclical borrowing. Whether or not the costs of increased tax rate variability exceed the benefits of lower tax rate *levels* depends on the marginal rate of substitution implied by loss function  $\phi(\tau)$ . Also, as  $\kappa$  and  $\beta$  move further away from their optimal values, more debt is accumulated, an important factor if credibility is imperfect.

Tables 2 through 4 present simulations of regimes (R0), (R1) and (R2). These simulations are intended to convey a flavor of how such regimes differ for shorter horizons—5, 10, and 20 years. The tables show the mean, variance, minima and maxima for three key variables: the tax rate ( $\tau_t$ ), the primary surplus (ratio to GDP) ( $ps_t$ ), and the end of period debt ( $b_J$ , J = 5, 10, 20). In all cases, 500 random draws are taken.

As mentioned above, regimes may be compared by tradeoffs. Such tradeoffs are summarized in Table 5. Moving from (R2) to (R1), by how much will taxes rise ( $\Delta$  Average)? By how much will the standard deviation fall ( $\Delta$  Standard Deviation)? What is the tax increase required to ‘buy’ a one-percent reduction in the standard deviation? (Tradeoff Ratio =  $\Delta$  Average /  $\Delta$  Standard Deviation)?

In all cases, the initial debt ratio is assumed to be 50% ( $b^P = .5$ ); permanent growth  $\lambda$  is assumed to be 4 percent; the variance of temporary income is assumed to be 5% of permanent income; the constant interest rate is 7% ( $r=.07$ ), and the permanent spending ratio is  $\gamma = 20$  percent. Assumption that borrowing restrictions will be imposed with probabilities  $\pi^c = 0, 0.3, \text{ and } 0.5$  are presented in Tables 2, 3, and 4, respectively. For regime (R2), all tables present alternative values for  $\kappa$  and  $\beta$ , namely  $\kappa = .03, \beta=.8$  and  $\kappa = .05, \beta=.8$ .

Consider first the case of no borrowing constraints  $\pi^c=0$  in Table 2. Note that for (R0) the (benchmark) constant tax rate is  $\tau = .2144$  (21.44%) over all horizons. Moving to the near-

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<sup>27</sup> Taxes under (R0) *can* vary but only if there are borrowing constraints.

<sup>28</sup> This policy may be thought of as a naïve application of problem (5') that ignores tax rate variability.



Table 2. Alternative Fiscal Regimes: Simulation Results  
Probability of borrowing constraint  $\pi^C = 0.0$

	5-year horizon (J=5)				10-year horizon (J=10)				20-year horizon (J=20)			
	$\tau$ Tax Rate	ps Primary Surplus	$b_j$ Debt (End Per.)	$\tau$ Tax Rate	ps Primary Surplus	$b_j$ Debt (End Per.)	$\tau$ Tax Rate	ps Primary Surplus	$b_j$ Debt (End Per.)	$\tau$ Tax Rate	ps Primary Surplus	$b_j$ Debt (End Per.)
Smoothing (R0)												
Average	0.2144	0.0137	0.5033	0.2144	0.0137	0.5027	0.2144	0.0138	0.5065	0.2144	0.0138	0.5065
Standard Deviation	0.0000	0.0093	0.0417	0.0000	0.0098	0.0500	0.0000	0.0099	0.0705	0.0000	0.0099	0.0705
Minimum	0.2144	0.0029	0.3931	0.2144	-0.0022	0.3818	0.2144	-0.0062	0.3360	0.2144	-0.0062	0.3360
Maximum	0.2144	0.0238	0.6476	0.2144	0.0279	0.6773	0.2144	0.0312	0.7539	0.2144	0.0312	0.7539
"Balanced Budget" Rule (R1)												
Average	0.2127	0.0119	0.5105	0.2125	0.0118	0.5220	0.2131	0.0125	0.5418	0.2131	0.0125	0.5418
Standard Deviation	0.0203	0.0197	0.0752	0.0238	0.0098	0.1114	0.0257	0.0272	0.1653	0.0257	0.0272	0.1653
Minimum	0.1889	-0.0104	0.3305	0.1734	-0.0277	0.2897	0.1616	-0.0404	0.1915	0.1616	-0.0404	0.1915
Maximum	0.2347	0.0334	0.7395	0.2485	0.0494	1.0340	0.2611	0.0636	1.0905	0.2611	0.0636	1.0905
Gen'l Fiscal Reaction (R2), $\kappa = .03, \beta = .8$												
Average	0.1812	-0.0196	0.6425	0.1827	-0.0180	0.8241	0.1869	-0.0137	1.2099	0.1869	-0.0137	1.2099
Standard Deviation	0.0183	0.0179	0.0720	0.0247	0.0256	0.1161	0.0340	0.0350	0.2097	0.0340	0.0350	0.2097
Minimum	0.1598	-0.0395	0.4614	0.1427	-0.0584	0.5624	0.1198	-0.0815	0.7195	0.1198	-0.0815	0.7195
Maximum	0.2012	0.0000	0.8703	0.2205	0.0211	1.3335	0.2540	0.0546	1.9449	0.2540	0.0546	1.9449
Gen'l Fiscal Reaction (R2), $\kappa = .05, \beta = .8$												
Average	0.1618	-0.0389	0.7237	0.1644	-0.0363	1.0099	0.1709	-0.0297	1.6197	0.1709	-0.0297	1.6197
Standard Deviation	0.0197	0.0190	0.0748	0.0284	0.0290	0.1271	0.0427	0.0434	0.2491	0.0427	0.0434	0.2491
Minimum	0.1390	-0.0601	0.5325	0.1189	-0.0819	0.7177	0.0873	-0.1136	1.0277	0.0873	-0.1136	1.0277
Maximum	0.1834	-0.0180	0.9624	0.2084	0.0086	1.5650	0.2581	0.0580	2.5098	0.2581	0.0580	2.5098

Notes: *Unconstrained* regimes are: (R0) smoothing:  $\tau(0) = \gamma + (\tau - \lambda)b^p$ ,  $ps(0)_t = \gamma[1 - w_t] + (\tau - \lambda)b^p$ ; (R1) near-balanced budget:

$\tau(1)_t = \gamma + (\tau - \lambda^*)b_{t-1}$ ,  $ps(1)_t = \gamma[1 - w_t] + [(1 + \lambda^*)b_{t-1}]$ ; (R2) general reaction:  $\tau(2)_t = \gamma - \kappa + \beta b_{t-1}$ ,  $\kappa > 0$ ,  $\beta > 0$ ,  $ps(2)_t = -\kappa + \beta b_{t-1}$ ;

*Constrained* regimes are: (R0):  $\tau(0^C)_t = \max\{\tau(1)_t, ps(0)_t\}$ ; (R2):  $\tau(2^C)_t = \max\{\tau(1)_t, ps(2)_t\}$ ,  $ps(2^C)_t = \max\{br(1)_t, br(2)_t\}$ ;

$\pi^C$  = probability of constraint in any period,  $\tau$  = tax rate,  $\gamma$  = ratio of government expenditures to permanent output,  $r$  = interest rate,  $\lambda$  = growth of permanent output,

$\lambda^*$  = *expected* output growth,  $b$  = ratio of debt to output,  $w_t$  = ratio, permanent/total output in period  $t$ ;

For all simulations,  $\lambda = 4\%$ ,  $r = 7\%$ ,  $b(\text{initial}) = b^p = 0.5$ ,  $\gamma = 0.2$ , variance of temporary output = 0.5 \* permanent output. Number of draws = 500.

balanced budget regime (R1) reduces tax rates (but only slightly) while variability becomes positive.<sup>29</sup> For example, during the first five years ( $J = 5$ ) tax rate variance rises from zero under (R0) to 0.05 percent of GDP under (R1).

Thus, in this period, tax rates range from a minimum of 18.9 percent to a maximum of about 23.5 percent under (R1). For longer horizons ( $J = 10, 20$ ) the variance of  $\tau$  rises, as does the gap between the between minima and maxima. Note also that, under both regimes, the primary surpluses range from 1.1 to 1.3 percent of GDP. And, end-period debt  $b_J$  remains on average close to its initial value of 0.5 under these regimes.

Unsurprisingly, for horizons presented here, taxes are lower under regime (R2)—in both cases—than either (R0) or (R1); tax rate variability is about the equal under (R1) and (R2) for  $J = 5$ ; However, as  $J$  rises to 10 and 20, tax rate variability does as well. For example, for  $\kappa = 0.03$ —a regime under which primary *deficits* average about 2 percent of GDP—tax rates in the first five years ( $J = 5$ ) range from just under 16% to just over 20%. However, over a ten-year horizon ( $J = 10$ ), tax rates range from about 14% to 22%, and for a twenty-year horizon ( $T = 20$ ) tax rates range from about 12% to about 25%. In all cases, tax rates become more variable  $\kappa$  is increased to 0.05 (primary *deficits* of 3 to 4 percent of GDP).

Unsurprisingly, debt accumulation is also substantially greater under (R2), and debt builds up ever more as the period grows. For example, in the case of  $J = 5$ , debt accumulation averages about 60% of GDP for  $\kappa = 0.03$  and 65% for  $\kappa = 0.05$ . For  $J = 10$ , the end-period debt ratio  $b_J$  rises to about 72% and 85%; for  $J = 20$ , the debt ratio rises to just under 100% and 127%, respectively.

Thus, as Table 5 shows, for  $J=10$  the tax rate increase required to obtain a one percent decrease in the standard deviation appears to be prohibitive: 35% for  $\kappa = 0.03$  and 10% for  $\kappa = 0.05$ . However, as the horizon grows to  $J = 20$ , this tradeoff drops to dramatically: 3.1% for  $\kappa = 0.03$  and 2.5% for  $\kappa = 0.05$ .

As Tables 3 and 4 show, if uncertain borrowing constraints are assumed ( $\pi^c=0.3$  and  $\pi^c=0.5$ ), regime (R0) becomes slightly less attractive relative to (R1): both the level and variability of tax rates under (R0) rise for the cases shown.<sup>30</sup> Unsurprisingly, (R0) debt accumulation is greater under  $\pi^c = 0$  than  $\pi^c > 0$  (since borrowing is not always available). Tables 3 and 4 also show, that the (R2) regimes become less attractive relative to both (R0) and (R1). As  $\pi^c$  rises from 0 to 0.3 and 0.5, so do both the level and variability of tax rates under (R2).

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<sup>29</sup> Note that results for (R1) are invariant to  $\pi^c$ .

<sup>30</sup> Note that, at some point, an increase in  $\pi^c$  should *decrease* both tax levels and variability under (R0) and (R2), since  $\pi^c = 1$  is the same as an (R1) regime.

Table 3. Alternative Fiscal Regimes: Simulation Results

	Probability of borrowing constraint $\pi^C = 0.3$								
	5-year horizon (J=5)		10-year horizon (J=10)		20-year horizon (J=20)				
	$\tau$	ps	b <sub>j</sub>	$\tau$	ps	b <sub>j</sub>	$\tau$	ps	b <sub>j</sub>
	Tax Rate	Primary Surplus	Debt (End Per.)	Tax Rate	Primary Surplus	Debt (End Per.)	Tax Rate	Primary Surplus	Debt (End Per.)
Smoothing (R0)									
Average	0.2164	0.0157	0.4949	0.2168	0.0161	0.4789	0.2168	0.0162	0.4480
Standard Deviation	0.0037	0.0108	0.0477	0.0056	0.0118	0.0608	0.0067	0.0123	0.0861
Minimum	0.2144	0.0038	0.3361	0.2144	-0.0012	0.3173	0.2144	-0.0055	0.2208
Maximum	0.2220	0.0281	0.6476	0.2305	0.0362	0.6657	0.2403	0.0442	0.7332
"Balanced Budget" Rule (R1)									
Average	0.2127	0.0119	0.5105	0.2125	0.0118	0.5220	0.2131	0.0125	0.5418
Standard Deviation	0.0203	0.0197	0.0752	0.0238	0.0248	0.1114	0.0257	0.0272	0.1653
Minimum	0.1889	-0.0104	0.3305	0.1734	-0.0277	0.2897	0.1616	-0.0404	0.1915
Maximum	0.2347	0.0334	0.7395	0.2485	0.0494	1.0340	0.2611	0.0636	1.0905
Gen'l Fiscal Reaction (R2), k = .03, b = .8									
Average	0.1908	-0.0099	0.6007	0.1919	-0.0088	0.7267	0.1951	-0.0055	0.9857
Standard Deviation	0.0240	0.0240	0.0787	0.0292	0.0300	0.1222	0.0355	0.0366	0.2001
Minimum	0.1645	-0.0356	0.3848	0.1485	-0.0529	0.4148	0.1300	-0.0714	0.4244
Maximum	0.2183	0.0177	0.8091	0.2394	0.0398	1.1669	0.2685	0.0697	1.6753
Gen'l Fiscal Reaction (R2), k = .05, b = .8									
Average	0.1774	-0.0233	0.6576	0.1792	-0.0215	0.8573	0.1842	-0.0164	1.2697
Standard Deviation	0.0308	0.0308	0.0900	0.0370	0.0376	0.1440	0.0460	0.0467	0.2439
Minimum	0.1454	-0.0549	0.4171	0.1277	-0.0735	0.4871	0.1043	-0.0968	0.5569
Maximum	0.2136	0.0132	0.8715	0.2410	0.0412	1.3886	0.2813	0.0820	2.1524

Notes: Unconstrained regimes are: (R0) smoothing:  $\tau(0) = \gamma + (1 - \lambda)b^p$ ,  $ps(0) = \gamma[1 - w_d + (1 - \lambda)b^p]$ ,  $ps(1) = \gamma[1 - w_d + (1 + \lambda^*)b_{t-1}]$ ,  $ps(2) = \gamma - \kappa + \beta b_{t-1}$ ,  $\kappa > 0$ ,  $\beta > 0$ ,  $ps(2)_t = -\kappa + \beta b_{t-1}$ ;

Constrained regimes are: (R0):  $\tau(0) = \max\{\tau(1), \tau(0)\}$ ,  $ps(0) = \max\{ps(1), ps(0)\}$ ; (R2):  $\tau(2) = \max\{\tau(1), \tau(2)\}$ ,  $ps(2) = \max\{br(1), br(2)\}$ ;

$\pi^C$  = probability of constraint in any period,  $\tau$  = tax rate,  $\gamma$  = ratio of government expenditures to permanent output,  $r$  = interest rate,  $\lambda$  = growth of permanent output,  $\lambda_t^*$  = expected output growth,  $b$  = ratio of debt to output,  $w_t$  = ratio, permanent/total output in period  $t$ ;

For all simulations,  $\lambda = 4\%$ ,  $r = 7\%$ ,  $b(\text{initial}) = b^p = 0.5$ ,  $\gamma = 0.2$ , variance of temporary output = 0.5 \* permanent output. Number of draws = 500.

Table 4. Alternative Fiscal Regimes: Simulation Results

	5-year horizon (J=5)				10-year horizon (J=10)				20-year horizon (J=20)			
	$\tau$	ps	b <sub>j</sub>	b <sub>j</sub>	$\tau$	ps	b <sub>j</sub>	b <sub>j</sub>	$\tau$	ps	b <sub>j</sub>	b <sub>j</sub>
	Tax Rate	Primary Surplus	Debt (End Per.)	Debt (End Per.)	Tax Rate	Primary Surplus	Debt (End Per.)	Debt (End Per.)	Tax Rate	Primary Surplus	Debt (End Per.)	Debt (End Per.)
Probability of borrowing constraint $\pi^C = 0.5$												
Smoothing (R0)												
Average	0.2178	0.0171	0.4894	0.4894	0.2183	0.0176	0.4639	0.4639	0.2181	0.0175	0.4135	0.4135
Standard Deviation	0.0057	0.0113	0.0502	0.0502	0.0081	0.0128	0.0652	0.0652	0.0088	0.0133	0.0889	0.0889
Minimum	0.2144	0.0048	0.3311	0.3311	0.2144	-0.0004	0.2979	0.2979	0.2144	-0.0050	0.2022	0.2022
Maximum	0.2261	0.0302	0.6476	0.6476	0.2373	0.0400	0.6657	0.6657	0.2463	0.0486	0.7286	0.7286
"Balanced Budget" Rule (R1)												
Average	0.2127	0.0119	0.5105	0.5105	0.2125	0.0118	0.5220	0.5220	0.2131	0.0125	0.5418	0.5418
Standard Deviation	0.0203	0.0197	0.0752	0.0752	0.0238	0.0248	0.1114	0.1114	0.0257	0.0272	0.1653	0.1653
Minimum	0.1889	-0.0104	0.3305	0.3305	0.1734	-0.0277	0.2897	0.2897	0.1616	-0.0404	0.1915	0.1915
Maximum	0.2347	0.0334	0.7395	0.7395	0.2485	0.0494	1.0340	1.0340	0.2611	0.0636	1.0905	1.0905
Gen'l Fiscal Reaction (R2), $\kappa = .03, \beta = .8$												
Average	0.1973	-0.0034	0.5729	0.5729	0.1980	-0.0027	0.6631	0.6631	0.2004	-0.0002	0.8448	0.8448
Standard Deviation	0.0252	0.0251	0.0797	0.0797	0.0299	0.0307	0.1207	0.1207	0.0346	0.0357	0.1884	0.1884
Minimum	0.1689	-0.0312	0.3311	0.3311	0.1530	-0.0480	0.3585	0.3585	0.1367	-0.0648	0.3827	0.3827
Maximum	0.2254	0.0248	0.7835	0.7835	0.2456	0.0460	1.0272	1.0272	0.2696	0.0711	1.5422	1.5422
Gen'l Fiscal Reaction (R2), $\kappa = .05, \beta = .8$												
Average	0.1879	-0.0128	0.6134	0.6134	0.1891	-0.0116	0.7566	0.7566	0.1928	-0.0078	1.0482	1.0482
Standard Deviation	0.0324	0.0324	0.0928	0.0928	0.0381	0.0386	0.1433	0.1433	0.0444	0.0452	0.2296	0.2296
Minimum	0.1517	-0.0487	0.3311	0.3311	0.1343	-0.0669	0.3987	0.3987	0.1149	-0.0864	0.4878	0.4878
Maximum	0.2239	0.0235	0.8715	0.8715	0.2489	0.0492	1.2082	1.2082	0.2811	0.0823	1.9347	1.9347

Notes: Unconstrained regimes are: (R0) smoothing:  $\tau(0) = \gamma + (r - \lambda) / (1 + \lambda) b^P$ ,  $ps(0)_t = \gamma [1 - w_t] + (r - \lambda) / (1 + \lambda) b^P$ ; (R1) near-balanced budget:

$\tau(1)_t = \gamma + (r - \lambda) / (1 + \lambda) b_{t-1}^*$ ,  $ps(1)_t = \gamma [1 - w_t] + (1 + \lambda) b_{t-1}^*$ ; (R2) general reaction:  $\tau(2)_t = \gamma - \kappa + \beta b_{t-1}$ ,  $\kappa > 0$ ,  $\beta > 0$ ,  $ps(2)_t = -\kappa + \beta b_{t-1}$ ;

Constrained regimes are: (R0):  $\tau(0)_t = \max\{\tau(1)_t, ps(0)_t\}$ ; (R1):  $\tau(1)_t = \max\{\tau(1)_t, ps(1)_t\}$ ; (R2):  $\tau(2)_t = \max\{\tau(1)_t, ps(2)_t\}$ ;

$\pi^C$  = probability of constraint in any period,  $\tau$  = tax rate,  $\gamma$  = ratio of government expenditures to permanent output,  $r$  = interest rate,  $\lambda$  = growth of permanent output,  $\lambda_t^* = expected$  output growth,  $b_t$  = ratio of debt to output,  $w_t$  = ratio, permanent/total output in period  $t$ ;

For all simulations,  $\lambda = 4\%$ ,  $r = 7\%$ ,  $b(\text{initial}) = b^P = 0.5$ ,  $\gamma = 0.2$ , variance of temporary output = 0.5 \* permanent output. Number of draws = 500.

Table 5. Summary, Simulation Results, Tradeoffs  
Tax Rate Level versus Standard Deviation, R2 versus R1

	$\pi^c=0$			$\pi^c=0.3$			$\pi^c=0.5$		
	J=5	J=10	J=20	J=5	J=10	J=20	J=5	J=10	J=20
Gen'l Fiscal Reaction (R2), $\kappa = .03$ , $\beta = .8$									
Δ Average	-0.032	-0.030	-0.026	-0.022	-0.021	-0.018	-0.015	-0.014	-0.013
Δ Standard Deviation	-0.002	0.001	0.008	0.004	0.005	0.010	0.005	0.006	0.009
Tradeoff Ratio	15.950	-34.149	-3.159	-5.857	-3.843	-1.823	-3.158	-2.382	-1.420
Gen'l Fiscal Reaction (R2), $\kappa = .05$ , $\beta = .8$									
Δ Average	-0.0509	-0.0481	-0.0422	-0.0352	-0.0333	-0.0289	-0.0248	-0.0234	-0.0203
Δ Standard Deviation	-0.0006	0.0045	0.0170	0.0105	0.0131	0.0203	0.0121	0.0142	0.0187
Tradeoff Ratio	78.562	-10.619	-2.476	-3.367	-2.533	-1.423	-2.043	-1.644	-1.084

Note: Δ Average = Difference in average tax rate ( $\tau$ ), (R2) minus (R1); Δ Standard Deviation = Difference in standard deviation of tax rate ( $\tau$ ), (R2) minus (R1); Tradeoff ratio = Δ Average/ Δ Standard Deviation. Negative number indicates tax hike required to obtain a one-percent decrease in standard deviation.

Furthermore tax rates are always more variable under (R2) than (R1) if  $\pi^c > 0$ . For example, for  $\kappa = 0.03$ —a regime under which primary *deficits* average between 0.5 and 1 percent of GDP—tax rates in the first five years ( $J = 5$ ) range from about 16.5% to about 21%. However, over a ten-year horizon ( $J = 10$ ), tax rates range from about 15% to just under 24%, and over a twenty-year horizon ( $J = 20$ ) tax rates range from about 13% to about 27%. As before, tax rates become more variable when  $\kappa$  is increased to 0.05 (primary deficits of 1.6 to 2 percent of GDP). Note also that debt buildup under (R2) falls when  $\pi^c$  rises, but is nonetheless substantially higher than under (R0) or (R1).

Table 5 reveals that, for  $\pi^c = 0.3$  and 0.5, the tax rate increases required to obtain a one percent decrease in the standard deviation are substantially lower than for the case of  $\pi^c = 0$ . For example, in the case of  $\pi^c = 0.5$ ,  $J = 10$ , these tradeoffs are: 2.4% for  $\kappa = 0.03$  and 1.6% for  $\kappa = 0.05$ . For a longer horizon of  $J = 20$ , these tradeoffs are 3.4% and 1.1%, respectively.

## VI. EXTENSION: VARIABLE GOVERNMENT EXPENDITURES

In the preceding discussion, expenditures have been assumed to be constant (exogenously set) fraction of permanent GDP. While such an assumption is standard in the literature, it is made primarily for convenience rather than realism.<sup>31</sup> Typically, expenditures also suffer cuts during adverse periods. Thus, consider a more general framework. Expenditures and taxes, without borrowing constraints, are determined by:<sup>32</sup>

$$\tau_t = \gamma^P - \kappa + \omega\beta b_{t-1} \quad (16a)$$

$$\gamma_t = \tau_t + \kappa - (1-\omega)\beta b_{t-1} \quad (16b)$$

where  $\gamma^P$  and  $\gamma_t$  are permanent and total government expenditures respectively, and  $0 \leq \omega \leq 1$ . In expressions (16a) and (16b) the exogenous (long-run), fiscal adjustment is distributed between taxes and expenditures according to  $\omega$ : if  $\omega = 1$ , the entirety of the adjustment falls

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<sup>31</sup> It is easier to make welfare statements about tax rates than about expenditures. For example, government expenditures in the form of lump-sum transfers have no welfare implications.

<sup>32</sup> Note that, under (R0), both taxes and expenditures are exogenous.

on taxes (equivalent to the previous section's model). By contrast, if  $\omega = 0$  all adjustment falls on expenditures. In this case,  $\tau$  is constant and  $\gamma^P \equiv \tau + \kappa$ .<sup>33</sup>

For  $\pi^c > 0$ , the corresponding expressions are:

$$\tau_t^C = \max [\gamma^P - \kappa + \omega\beta b_{t-1}, \gamma^P - \omega(r-\lambda_t^*)/(1+\lambda_t^*)b_{t-1}] \quad (17b)$$

$$\gamma_t^C = \min [\tau_t + \kappa - (1-\omega)\beta b_{t-1}, \tau_t - (1-\omega)(r-\lambda_t^*)/(1+\lambda_t^*)b_{t-1}] \quad (17b)$$

In (17b), if  $\omega = 0$ , if a borrowing government is denied access to credit, it cuts expenditures in that period.

## VII. EXTENSION: PUBLIC SECTOR SIZE AND VOLATILITY

Fiscal reforms generally envisage *permanent* cuts of less productive expenditures.<sup>34</sup> Doing so helps transfer resources to either higher priority public expenditures, the private sector (through tax cuts), or both. This is perhaps the most widely recognized benefit of such an adjustment. However, doing so may also permit essential public goods and services to be provided more *smoothly*—with fewer cuts or interruptions.

Moreover, the previous discussion suggests that level of permanent government expenditures  $\gamma^P$  and their volatility should be related. For example, under the endogenous expenditure regime ( $\omega=0$ ) the average level of government expenditures ( $\gamma^P = \tau + \kappa$ ) and the variability of expenditures  $\text{var}(\gamma)$  should be positively related.<sup>35</sup> For more general cases, ( $0 \leq \omega \leq 1$ ) higher  $\gamma^P$  should raise the variability of both expenditures and revenues.

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<sup>33</sup> Presumably, political considerations would determine the value of  $\omega$ . However, this topic is left for another paper.

<sup>34</sup> That is, alignment between taxes and primary expenditures—removal of the “tax gap”  $\kappa$ —may be achieved by a once-and-for-all reduction in  $\gamma$ . Moreover, as Alesina and Ardanga (1998) suggest, fiscal adjustments that emphasize expenditure reduction rather than tax increases are both more durable and more likely to increase economic growth.

<sup>35</sup> For discussions of related issues in Latin America see Gavin and Perotti (1996) and Talvi and Végh (2000b).

To investigate this issue in Latin America, Figure 1 presents a plot of the level of *real consumption* expenditures (relative to GDP) against its coefficient of variation (variance / mean). (The average ratio of government consumption / GDP thus proxies for the permanent expenditure ratio  $\gamma$ ). According to this chart, casual observation may favor such a positive relationship among Latin American countries.

What is the relationship between the level of government expenditures and real GDP volatility?<sup>36</sup> In the traditional public finance literature, stabilization was a key role of the public sector. Moreover, according to recent evidence presented by Fatas and Mihov (2001), amongst industrialized countries, a larger public sector is associated with *lower* output variability. However, such a relationship is not evident for Latin America. Figure 2 presents a plot of the level of *real consumption* expenditures (relative to GDP) against the variance of real GDP. According to this chart, casual observation may also favor such a positive relationship among Latin American countries between these two variables.

### VIII. Summary and Conclusions

This paper attempted to clarify several widely-held but informal notions regarding restrictive fiscal rules and the conduct of fiscal policy over the business cycle. Fiscal rules (like Gramm-Rudman-Hollings) are often cast as an “enemy” of the first-best (Ramsey) optimum of tax smoothing. Of course, in any welfare comparison, it is essential to be clear about exactly *what* are the alternatives under consideration. Fiscal policy in many emerging markets—and particularly in Latin America—is plagued by budgetary rigidities, weak tax administration, and volatile tax rates, expenditures, and debt / GDP ratios.

As a theoretical construct, the benefits of a Ramsey-style tax smoothing regime are clear: over an infinite horizon, consumers benefit from lower and less variable tax rates. A more difficult question involves shorter horizons. If a persistent tax gap is eliminated (through once-and-for-all measure tax and expenditure measures) will tax rates or expenditures become appreciably smoother? Simulations presented in this paper suggested that the answer to this question is “yes.” Moreover, while such once-and-for-all measures may be distinct from a balanced-budget law or other fiscal restriction, the two may nonetheless complement one another. In these ways, Gramm-Rudman-Hollings and Frank Ramsey may be “friends” rather than “enemies.”

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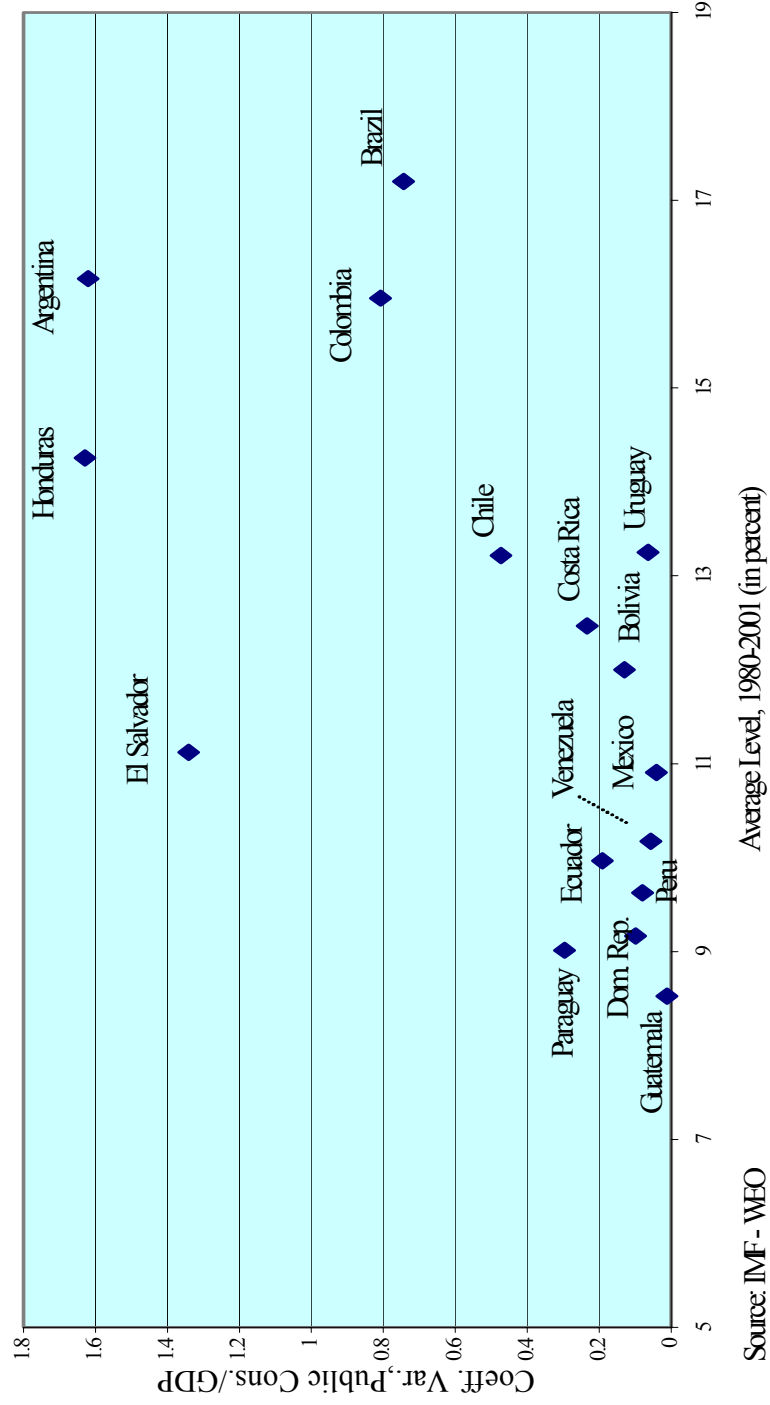
<sup>36</sup> This idea is not undisputed. For example, in the traditional public finance literature stabilization was one of the public sector’s key roles. More recently, Fatas and Mihov (2001) provide evidence—for *industrialized* countries—that a larger public sector is associated with lower output variability.



While the assumptions in this paper were simple, more realistic ones might be used in future work. For example, both expenditures and taxes might share some of the burden in further simulations. Also, future work might specify consumer preferences and the production technology more fully.

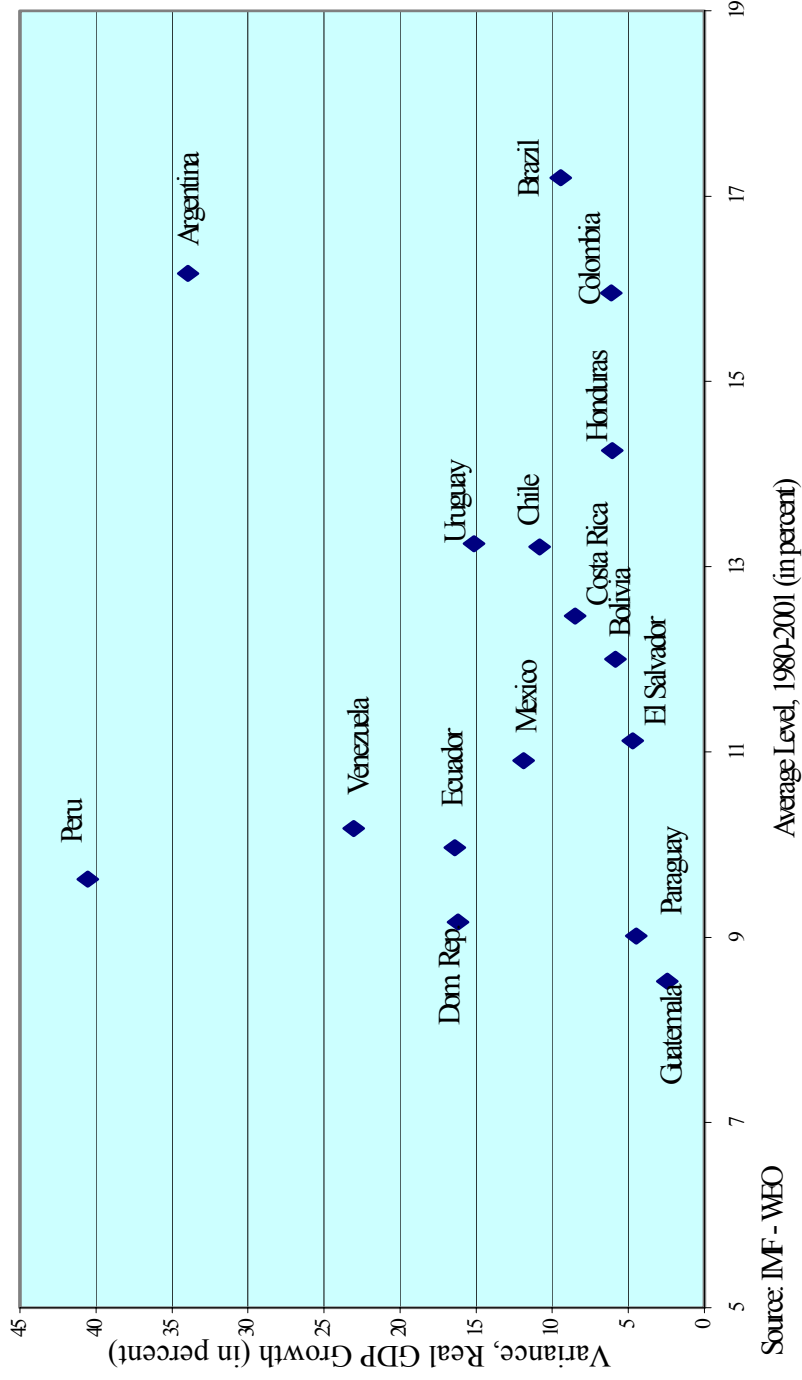
There were also some key topics that, while omitted, would be fruitful extensions in future work. For example, an extension of this work might include a motivation for default and endogenous borrowing constraints, as discussed in previous sections. Also, the model might be extended to include changes in the price level, interest rate changes, or both (according to, for example, the recently developed “fiscal theory of the price level”). Ultimately, economic theory ought to be able to compare a GRH-like rule against policies that countries currently pursue. The agenda for future research on this topic thus remains sizeable.

Figure 1. Latin America: Public Consumption/GDP Level and Coefficient of Variation



Source: IMF- WEO

Figure 2. Latin America: Public Consumption/GDP and Output Volatility



Source: IMF - WEO

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