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Corruption, Competition, and Contracts:
A Model of Vote Buying

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Abstract

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In the presence of competing interest groups, this paper examines how the form of vote-buying contracts affects policy outcomes. We study contracts contingent upon individual votes, policy outcomes, and/or vote shares. Voters either care about their individual votes, or about the policy outcome. We find that vote buying is cheaper when what can be contracted upon coincides with what voters care about. Vote buying becomes extremely costly, or even impossible, when there is no such coincidence. Finally, vote buying is extremely cheap, or even free, when contracts can be contingent upon both individual votes and vote shares.

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I. Introduction

In 1757, George Washington ran for a seat on the Virginia House of Burgesses, the main legislative body in the state at that time. To aid his chances, Washington offered his constituents an informal contract: they received an average of one and a half quarts of various alcoholic beverages in exchange for their votes.[2] Not just in the America of the 18th century, but also in today's world is vote buying a rather common phenomenon. Pasuk *et al.* (2000) present survey evidence that in the 1996 Thai general elections, fully 30 percent of heads of households were offered money in exchange for their votes.

While Washington offered a simple "contract" specifying a payment in exchange for a vote, vote-buying contracts are often more complex. For example, the reverse of Washington's contract was a common strategy in 19th-century Maryland.[3] This practice, often referred to as negative vote buying, offers a payment in exchange for *not* voting in an election. Other contracts offer contingencies based on aggregate outcomes rather than individual votes. For example, a common vote buying practice in England after the introduction of the secret ballot in 1872 was to offer villages or neighborhoods material incentives contingent on the aggregate turnout in the election.[4] Indirect means may also be used to achieve a similar end. For instance, in the 2000 Taiwan Presidential election, the ruling National Party subsidized betting parlors to offer extremely favorable betting odds on the event that the party's candidate was elected.[5]

In some vote buying schemes, the amount of compensation promised also depends on the vote share. For instance, in boroughs in England in the 1830s, it was common for the price of a vote to be low early on election day and to increase as the day progressed, especially when the election was thought to be close. Once a party judged that a sufficient margin of victory was achieved, the price of votes fell precipitously.[6]

Competition among various interest groups, such as that between the rival parties in the English elections of the 1830s, plays an important role in determining the magnitude and contractual structure of vote-buying schemes. A more modern illustration of this is the scandal involving the International Olympic Committee (IOC). It was reported that certain key members of the IOC were paid an up-front amount in exchange for their individual vote as well as a "bonus" conditional on the outcome of the vote—the success of a competing city's Olympic bid.[7]

In this paper, we study how variation in the type of contracts available for vote buying affects the "buyability" of voters in a model where interest groups compete. As we will show, it is the combination of what is contractible and what voters care about that determines the success of vote buying. What voters care about may differ depending on the context in which the vote is taking place. For example, when voters are legislators, they will be primarily concerned with how their voting record is perceived by their

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[2] See Thayer (1973, p. 25).

[3] See Argersinger (1987).

[4] See, Seymour (1915).

[5] See Shaffer (2002, p. 5).

[6] See Lehoucq (2002).

[7] See "Olympic 'vote buying' scandal" BBC News, December 12, 1998. Available via the Internet: http://news.bbc.co.uk/1/hi/world/europe/233742.stm

constituents. On the other hand, when voters are citizens voting in an election, they will be primarily concerned with the outcome—who ultimately gets elected. Hence, we study both of these cases.

The main results of the paper are as follows:

1. Vote buying is cheaper when what is contractible coincides with what voters care about. For instance, contracts contingent on an individual's vote are more effective when voters mainly care about their own vote, as would typically be the case for the votes of legislators who are accountable to their constituents. Contracts contingent on policy outcomes are more effective when voters mainly care about policy outcomes, as would typically be the case in large general elections.

2. Vote buying can become extremely costly, or even impossible, when what is contractible does not coincide with what voters care about. When voters care about their individual votes but only outcomes are contractible, there is no "price" at which an interest group can guarantee its preferred outcome through vote buying.

3. Vote buying becomes extremely cheap, or even free, when both individual votes and vote shares are contractible. Furthermore, competition among interest groups does little to counteract this effect. We construct a contract that guarantees a supermajority of votes to an interest group at a cost equal to or only slightly above the level that would obtain in the absence of competition.

The remainder of the paper proceeds as follows: In Section II we present the basic model. Section III examines equilibrium outcomes of vote buying under three contractual variations: contracts contingent only on votes, contracts contingent only on outcomes, and contracts contingent on votes and vote shares. Section IV places these results in the context of the extant literature. Finally, Section V concludes.

II.   The Model

Suppose that there are an odd number, n, of voters choosing between two policies. The policies, which one could also think of as candidates or party platforms, are labeled a (the new policy) and b (the status quo). The policy receiving the majority of votes is adopted.

Two interest groups, labeled A and B, are trying to affect the policy choice. In a setting where the voters are legislators, the interest groups can be thought of as lobbyists or political action committees. In a setting where voters are citizens voting in an election, the interest groups may be thought of as political parties. In this interpretation, the policy options refer to which party gets to form the government. Group A prefers policy a, the new policy, while group B prefers policy b, the status quo.

Excluding the cost of buying votes, group A enjoys a payoff $W_A > 0$ when a is adopted and zero when b is adopted. Group B, on the other hand, enjoys a payoff $W_B > 0$ when b is adopted and zero when a is adopted. Thus, groups A and B have diametrically opposed

policy preferences. To induce voters to vote for its preferred policy, each group can offer enforceable vote buying contracts.[8] We will vary, however, the contingencies on which the contract can be based. The net payoff to a group is its payoff associated with the adopted policy less any vote buying costs.

Voters, indexed by $i = 1, 2, ..., n$, care about their actual votes, the policy outcome, and any transfers from the interest groups. Specifically, voter i's payoff is

$$U_i\left(c_i, p, t_i\right) = u_i\left(c_i, p\right) + t_i$$

where c_i indicates voter i 's vote, p indicates the adopted policy, and t_i denotes monetary transfers received from the interest groups. For the specification of u_i, we will consider two polar cases: (1) where voters only care about their own vote, i.e., u_i is independent of p; (2) where voters care only about which policy is adopted, i.e., u_i is independent of c_i. So that we may directly compare the effect of preferences over votes to preferences over policies, we assume that the difference in payoffs to voter i from changing his vote (respectively, the policy outcome) from b to a, is the same and equal to v_i. Formally, suppose that voter i cares only about his own vote, then

$$v_i = u_i\left(c_i = a\right) - u_i\left(c_i = b\right)$$

Now suppose that voter i cares only about the policy, then

$$v_i = u_i\left(p = a\right) - u_i\left(p = b\right)$$

These v_i's are common knowledge. Suppose that all voters have strict preferences over a and b; that is, $v_i \neq 0$, for all i, and that voters are ordered such that v_i is a strictly decreasing function of index i. Furthermore, we assume that the median voter, $M \equiv \frac{n+1}{2}$, prefers policy b; that is, $v_M < 0$. Hence, in the absence of interest group A, policy b would always be adopted, while policy a is only adopted when interest group A has managed to buy the vote.

When u_i is independent of p, we shall say that voters have "vote-based motives." In the context where voters are legislators, vote-based motives arise naturally from maximizing reelection probabilities, which depend on the legislator's voting record. This is the interpretation offered by Mayhew (1974) and Groseclose and Snyder (1996), among others. Alternatively, these voters might represent ideologues who measure the costs and benefits of casting votes on issues in terms of their own ideology—regardless of whether the individual vote has any effect on policy.

When u_i is independent of c_i, we shall say that voters have "outcome-based motives." The outcome-based model seems appropriate for describing voters in general election, where the main concern is typically who gets elected.

The extensive form of the game is as follows: First A proposes a contract. Next, after observing A's proposal, B proposes a contract. Finally, votes are cast and payoffs are realized. If B is indifferent between proposing a contract and staying out, we assume that he stays out.

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[8] We will sometimes refer to vote buying schemes as "bribes" to voters. This is simply for succinctness and not an expression of the legality (or lack thereof) of a particular scheme.

Notice that the vote-based version of our model (and indeed much of the notation) is identical to that of Groseclose and Snyder (hereafter GS). Readers desiring additional details or justification for this case should refer to their paper.

Multiplicity of equilibria for a given contract does not arise in the setting proposed by GS, because the payoffs to each voter are independent of the actions of the other voters. In general, however, this will not be the case. In contractual variations in which contingencies based on vote shares or outcomes are permitted, the contract itself creates interactive incentives. In variations where preferences are based on outcomes rather than individual votes, incentives are likewise interactive regardless of the contractual form. To address this issue, we shall take a conservative view about the cost to A of successful vote buying by adopting the following definition.

Definition 1 *A vote buying contract is **successful** if and only if it **guarantees** adoption of policy a.*

Formally, a vote buying contract is successful if and only if all subgame perfect equilibria following the contract lead to the adoption of policy a.

III. Variation in Contracts

As a benchmark, we first analyze the case where individual votes are contractible and where voters care only about their individual votes. This case was first analyzed in GS. Starting from this benchmark, we show how variation in the motives of voters affects the cost of vote buying. In Sections III.B and III.C, we then explore how variation over what is contractible interacts with variation in voters' motives to affect the costs of vote buying.

A. Contracting on Votes

Vote-Based Motives

Before proceeding, it is useful to examine what would happen absent any competition. In that case, group A can do no better than to offer a transfer $t_i = -v_i$ to all voters $i \leq M$ whose intrinsic preferences are for policy b; that is, voters for whom $v_i < 0$. No transfers are needed for voters whose preferences already favor policy a. Thus, the vote buying cost to A absent competition is

$$C_A^0 = \sum_{i=1}^{M} \max\{0, -v_i\}$$

where the 0 superscript in the expression above denotes the fact that A is operating as a monopoly (i.e., group A faces zero competitors).

Now consider the case where there is a competing interest group. First, fix some coalition size m, such that $n \geq m \geq M$. Next define K to be the minimum expected payoff earned by any voter $i = \{1, 2, ..., m\}$, where payoffs include transfers from group A. We will choose

K such that group B will (just) not wish to "invade" A's coalition of voters to implement policy b. That is, for a fixed m, K is the least-cost successful vote buying scheme.

Figure 1. Voters' Payoffs (linear case)
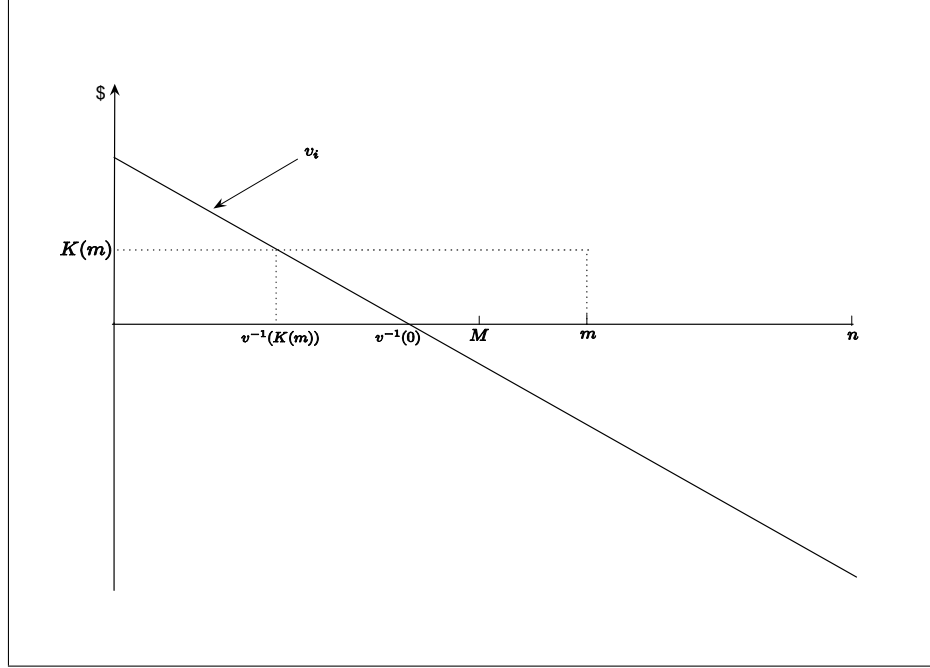
Specifically, for B to obtain its desired policy it must re-bribe at least $m - M + 1$ voters. Thus, B needs to offer transfers that exceed the voters' expected payoff under the vote buying scheme proposed by A. By definition, this amount is at least K. Hence, successfully re-bribing will cost B at least

$$C_B(K, m) = (m - M + 1) K$$

For A to deter B, it must be that these costs exceed the benefits to B of switching the policy form a to b. That is,

$$C_B(K, m) \geq W_B$$

Solving this expression under equality yields:

$$K(m) = \frac{W_B}{m - M + 1}$$

Thus, conditional on m, $K(m)$ implicitly describes the least-cost successful vote buying scheme available to A. We now turn to the details of this scheme: Define $v^{-1}(K(m))$ to be the lowest index number i such that $v_i < K(m)$. Notice that all voters with indices less than $v^{-1}(K(m))$ earn more than $K(m)$ by voting for a.

We are now in a position to characterize the contract proposed by A as a function of the coalition size m. Specifically, suppose that A offers:

For $v^{-1}\left(K\left(m\right)\right) \leq i \leq m$

$$t_i = \begin{cases} K\left(m\right) - v_i & if \quad c_i = a \\ 0 & if \quad c_i = b \end{cases}$$

For $i < v^{-1}\left(K\left(m\right)\right)$ or $i > m$, no contract is offered.

For future reference, we refer to a contract of this form as a $K\left(m\right)$ contract. Of course, the particular contingency on which t_i is based will differ depending on the contractual environment.

The cost of this scheme is

$$C_A\left(m\right) = \sum_{i=v^{-1}\left(K\left(m\right)\right)}^{m} t_i$$

Clearly, A wishes to minimize $C_A\left(m\right)$ by choosing m optimally. Since there are a finite number of feasible indices $m \in \{M, ..., n\}$, there exists at least one value $m^* \in \arg\min C_A\left(m\right)$ that minimizes these costs. In the event that the arg min set is non-singleton, let m^* denote the largest integer i in the arg min set.

Finally, it remains to check that it is optimal for group A to engage in such a vote buying scheme in the first place. Recall that the benefit to A of switching the policy from b (which will occur if A does not intervene since $v_M < 0$) to policy a is W_A. Hence, we require that

$$C_A\left(m^*\right) < W_A. \tag{1}$$

For the remainder of the analysis, we will assume that the benefits to A of changing policy are sufficient to justify its vote buying costs. We will examine how these costs differ depending on what is contractible and what voters care about.

We summarize the above arguments as follows:

Proposition 1 *A $K\left(m^*\right)$ contract is a least-cost successful contract under vote-based motives where only individual votes are contractible.*

How expensive is this scheme for A? Notice that A's cost may be rewritten as

$$C_A\left(m^*\right) = C_A^M + W_B + \left\{ \frac{M - v^{-1}\left(K\left(m^*\right)\right)}{m^* - M + 1} W_B - \sum_{i=v^{-1}\left(K\left(m^*\right)\right)}^{v^{-1}\left(0\right)-1} v_i \right\} + \sum_{i=M+1}^{m^*} \left(-v_i\right)$$

The expression in the curly brackets may be readily shown to be positive; hence, the cost to A of vote buying is strictly greater than its monopoly cost plus the entire value to group B. Thus, A must have a very high valuation, W_A, associated with policy a for it to be willing to undertake any vote buying scheme if A can only condition on votes.

It is interesting to note that, if A could contract with B, a cheaper alternative would be available. Group A could pay group B an amount W_B in exchange for its absence from the vote buying competition. This would allow A to obtain policy a at a vote buying cost

of C_A^M. In that case, A's total cost of obtaining policy A would be $W_B + C_A^M < C_A(m^*)$. Following GS, we exclude this possibility from the analysis, but do so with misgivings: There seems little *a priori* reason to rule out such a strategy.

Outcome-Based Motives

How do the costs of successful vote buying change when voters care about outcomes rather than their individual votes? On the one hand, it might seem as though voters with outcome based motives would be easier to influence since, apart from the circumstance in which the voter is pivotal, he is content to merely sell his vote to the highest bidder regardless of his intrinsic preferences. On the other hand, since interest groups can contract only on votes rather than what voters care about, outcomes, one might think that successful contracting by A is more difficult.

The crucial difference between outcome-based motives and vote-based motives entails the considerations affecting a voter's decision when presented with competing offers from A and B. If a voter believes that his vote is pivotal, then the margins of choice are identical to the case of vote-based motives. Therefore, by offering a $K(m)$ contract, group A can successfully deter B from trying to recruit a coalition comprised of a bare majority of voters.

If a voter believes that his vote is not pivotal (i.e., B attempts to recruit a supermajority), then, in evaluating the competing offers of A and B, it is as though the v_i associated with voter i were zero. Since B's cost is increasing in the size of the supermajority, group A must also defend against a coalition consisting of $M+1$ voters. To determine the costs associated with this, consider the following "alternate" problem for group A :

Suppose that $v_i = 0$ for all i. Then, to deter B from recruiting a coalition consisting of $M+1$, group A must offer $m \geq M$ voters transfers of at least

$$\tau(m) = \frac{W_B}{m - M + 2} \equiv K(m+1)$$

In this case, B obtains $n - m$ voters for free and must pay an additional $m - M + 2$ voters at least $\tau(m)$ each. Hence

$$
\begin{aligned}
C_B &\geq (m - M + 2)\tau(m) \\
&= W_B
\end{aligned}
$$

and thus B is deterred from attempting to alter the adopted policy from a to b.

Since A must deter B from both possibilities, a candidate for the least-cost successful contract that recruits m voters is the following:

For $i \leq m$

$$t_i = \begin{cases} \max\{K(m+1), K(m) - v_i\} & if \quad c_i = a \\ 0 & if \quad c_i = b \end{cases}$$

For $i > m$, no contract is offered.

We first determine the optimal coalition size under such a scheme. Define $z(m)$ to be the largest integer such that

$$K(m+1) \geq K(m) - v(z(m))$$

Then, the cost to A of this contract is

$$C_A(m) = z(m) K(m+1) + \sum_{i=z(m)+1}^{m} (K(m) - v_i)$$

Let m^{**} be an integer $m \leq n$ that minimizes A's cost. Note that such an m^{**} always exists and, as above, if there are multiple values of m that minimize the above expression, we choose m^{**} to be the largest of these.

We are now in a position to characterize the least-cost successful contract when voters have outcome-based motives:

Proposition 2 *The following is a least-cost successful contract under outcome-based motives where only individual votes are contractible.*

For $i \leq m^{**}$, $t_i = \max\{K(m^{**}+1), K(m^{**}) - v_i\}$

For $i > m^{**}$, no contract is offered.

Furthermore, this contract is more costly than the least cost successful contract under ***vote****-based motives where only individual votes are contractible.*

Proof. We have already shown that no successful contract could have costs lower than that given in Proposition 2. It remains to show that the contract above is indeed successful. Suppose that A proposes the above scheme, group B does nothing, and voters $i = 1, ..., m^{**}$ vote for a.

First consider the incentives of voters $i = 1, ..., m^{**}$: Since group A has successfully recruited a supermajority, these voters correctly ascribe infinitesimal probability to being pivotal. Therefore, voting comes down to whichever side offers the higher bribe amount and this is Group A. For unbribed voters, voting turns on the small probability of being pivotal, therefore these voters vote for policy a if and only if $v_i > 0$. Hence, voters $i = \{1, 2, ..., m^{**}\}$ optimally vote for policy a while the remainder vote for policy b.

Next, we need to show that Group B can do no better than to offer no contract when faced with A's contract. B can either offer bribes such that every voter perceives himself to be pivotal or B can offer bribes such that no voter considers himself to be pivotal. In either case, voters must be paid the amount of their foregone rents if they switch from a to b. By construction, when all voters are pivotal, the cost to B of recruiting a bare majority is at least W_B. For no voters to be pivotal and b to be adopted, B must construct a supermajority. By the definition of $K(m^{**}+1)$, the cost of constructing such a coalition is at least W_B. Therefore, B can do no better than to do nothing. Hence, the above is a successful contract. ∎

As Propositions 1 and 2 make clear, A's optimal coalition size depends on the motives of the voters. In general, there is no unambiguous ranking of m^*, the optimal coalition size

under vote-based motives, and m^{**}, the optimal coalition size under outcome-based motives. To see this, suppose, as in GS, that v_i is a linear function of the index i; that is $v_i = \alpha - \beta (i - M)$. Further, suppose that $\beta = 1$, $\alpha = (n+1)\left(\frac{1-\sqrt{2}}{4\sqrt{2}}\right)$, and $W_B = \frac{1}{2}(M)^2$. Then, when $n = 9$, the optimal coalition size under vote-based preferences is $m^* = 8$ while the optimal coalition size under outcome-based preference falls to $m^{**} = 7$. However, when $n = 99$, the optimal coalition size under vote-based preferences is $m^* = 84$ while the optimal coalition size under outcome-based preferences increases to $m^{**} = 86$.

To see the competing economic forces pulling in opposite directions, it is helpful to temporarily ignore the integer constraint on m and use calculus to compare the marginal cost of a change in the coalition size under outcome-based motives when $m = m^*$. That is,

$$\frac{\partial C_A(m)}{\partial m}\Big|_{m=m^*} = K'(m^*+1)\, z(K(m^*)) + \left(v^{-1}(K(m^*)) - z(K(m^*))\right) K'(m^*)$$

This expression consists of two parts. The first part, $K'(m^*+1)\, z(K(m^*))$, represents the savings on defending against a supermajority when recruiting a larger coalition than m^*. The other expression, $\left(v^{-1}(K(m^*)) - z(K(m^*))\right) K'(m^*)$, represents the loss from defending against a bare majority when recruiting a larger coalition than m^*. As the above example shows, either effect can dominate. When intrinsic preferences of voters can be approximated by linear function of the index we can show the following:

Remark 1 *In large elections where intrinsic preferences are approximately linear, larger fractions of the electorate will be bribed under outcome-based motives than under vote-based motives.*

Formally, if $v_i = \alpha - \beta (i - M)$, then for n sufficiently large, $m^{**} > m^*$.

The least-cost successful contract has several features in common with the vote buying strategies of major parties in a certain South-East Asian country, as described by Quimbo (2002):

> The amounts [paid to voters] may vary among supporters, the undecided, and those on the other side. Undecided voters sometimes get three times as much as supporters. Key supporters from the other side receive even more if they switch sides.

Discussion

In comparing the least-cost successful contracts in Propositions 1 and 2, several features are worth noting. First, when voters care about outcomes, successful contracts require that even the staunchest supporters of policy a are paid for their votes. The ubiquitousness of vote buying in this case appears broadly consistent with vote buying patterns in recent general elections as described by Pasuk *et al.* (2000) and Shaffer (2002).

B. Contracting on Outcomes

To curb the incentives for vote buying, the form of voting in many elections was changed to secret ballots. The response to this change in the U.K. and the U.S. is well documented.[9] After the introduction of the secret ballot in the U.K. in 1872, parties often resorted to payments conditional on outcomes or turnout in villages or neighborhoods (Seymour, 1915).

How are the incentives for vote buying affected when contracts can only be made contingent on outcomes rather than individual votes? Is vote buying more costly under outcome-based motives in this context as well? To examine these questions, suppose that the two interest groups are limited to offering contracts contingent only on the policy outcome, b or a. Thus, a "bribe" by group A to voter i consists of a transfer conditional on the a policy being adopted and a transfer in the event that the b policy is adopted. Importantly, groups cannot offer a transfer contingent on i's vote. It is straightforward to see that it is never optimal for a group to offer positive transfers in the event that the policy it does not favor is passed. We now proceed to analyze least-cost successful contracts under vote and outcome-based motives.

Vote-Based motives

One might imagine that vote buying would be relatively easy under vote-based motives, since the only contractual contingency—what policy is adopted—is not directly payoff relevant to the voters. As we show in the following propositions, however, this intuition is incorrect. It ignores the fact that the contingency about which the voters care intensely—their own voting record—is not contractible, which makes the contracting problem immensely more difficult for the interest group. Indeed, the combination of opacity (contracting over outcomes alone) and "career concerns" (vote-based motives) is a prescription for reduced vote buying. Specifically, when it is the case that, absent vote buying, policy b enjoys a supermajority (formally, $v_{M-1} < 0$) then we obtain the following impossibility result:

Proposition 3 *If $v_{M-1} < 0$, then successful vote buying contracts do not exist under vote-based motives where only outcomes are contractible.*

Proof. We will show that for contract offered by A, there exists a subgame perfect equilibrium where policy b is adopted. To see this, suppose group A offers some arbitrary contract, group B does nothing and voters vote according to their intrinsic preferences (i.e., voter i votes for a if and only if $v_i > 0$). Since A's contract is only contingent on outcomes, it is payoff relevant to voter i only to the extent that i is in a position to alter the outcome by his vote. Furthermore, since b commands a supermajority of intrinsic support, then, under the putative equilibrium, each voter has only an infinitesimal probability of affecting the policy by changing his vote. At the same time, changing one's vote from b to a leads to a first order payoff effect in the amount v_i. Therefore, voters can

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do no better than to vote according to their intrinsic preferences. Further, since B obtains its preferred outcome at no cost, it can do not better than to do nothing. ∎

The inability of the group A to influence the voters stems from the fact that the incentives from contracts contingent only on outcomes are only relevant to a voter when his vote is pivotal. In situations where, absent any interest groups, intrinsic support for the status quo policy is sufficiently great, the chances of being pivotal if all other voters vote "honestly" are vanishingly small and, as a consequence, the incentive effects of any contract proposed by A are also vanishingly small.

In circumstances where intrinsic support for b commands only a bare majority, i.e., $v(M-1) > 0$, successful vote buying is once again possible; however, buying a supermajority is impossible. Thus the cost to A of successful vote buying is raised when it can only contract over outcomes. Formally:

Proposition 4 *If $v(M-1) > 0$, then the $K(M)$ contract is a least-cost successful contract under vote-based motives where only outcomes are contractible.*

Proof. We first establish some properties of voter behavior and (potentially) successful contracts by A.

Suppose that, following contract proposals by A and B, a voter ascribes infinitesimal probability of being pivotal, then she votes for policy a if and only if $v_i > 0$. If instead, a voter ascribes probability close to one of being pivotal, then she votes for policy a if and only if $v_i + t_i > b$ where b is the bribe offered by group B contingent on outcome b.

From this, we may now deduce that any potentially successful contract offered by A must induce a bare majority and not a supermajority. To see this, suppose to the contrary that there exists a successful contract offered by A where supermajority of voters vote for A. In that case, none of the voters is pivotal. However, this cannot be optimal for voters since, in the event that a voter is not pivotal, he should vote according to his intrinsic preferences regardless of the contracts offered by A and B. This leads to a majority of votes voting for policy b, which is a contradiction. Therefore, any successful contract by A must command a bare majority of support.

Notice that a successful $K(M)$ contract leads to a bare majority of voters to vote for a. When a bare majority vote for a, then payoffs to all voters under the $K(M)$ scheme are equivalent to the case where contracts are contingent on individual votes as in Proposition 1. Therefore, a $K(m = M)$ contract is the least-cost way to achieve a bare majority. Moreover, if B does nothing, then this contract is successful.

We finally show that B can do no better than to offer no contract. By the same argument as above, B cannot offer a contract which commands a supermajority since, if it did, voters would vote according to their intrinsic preferences, which would lead to a bare majority for B. Thus, any contract B offers must lead to a bare majority; however, as we showed above, in that circumstance, the incentives of voters are identical to those in Proposition 1. Furthermore, we showed that, under these circumstances, B could do no better than to offer no contract.

Hence, a $K(M)$ contract is a least-cost successful contract. ∎

If one takes a less conservative view of group A's chances of vote buying, then, using arguments identical to Proposition 4, one can show:

Remark 2 *A $K(M)$ contract is a least-cost contract such that there exists a subgame perfect equilibrium in which policy a is adopted.*

As Remark 2 highlights, even under the best of circumstances, when voters care about their individual votes, it is more costly to contract on outcomes than to contract on votes. Indeed, the bluntness of the contractual instrument limits A to a situation where it buys a bare majority rather than a supermajority of the voters. The reason is that, were A to try to buy a supermajority, then the incentive effects of its contract, which depend on a voter being pivotal, are completely undermined and, ironically, A's preferred proposal would go down to defeat. Buying a bare majority rescues the incentive effects but generally requires a greater cost to implement in order to deter group B from re-buying than were A to buy a supermajority. The upshot is that the incentives for legislative capture are reduced.

Propositions 3 and 4 suggests that the introduction of the secret ballot offers a powerful remedy against vote buying. The cost of introducing the secret ballot is that it reduces accountability. Such a remedy may be especially undesirable in legislative settings, where the accountability of the legislator's voting record is of paramount importance to his constituents.

Outcome-Based motives

How is the situation different when voters have outcome-based motives? Here we shall see that, in contrast to the case of vote-based motives, successful vote buying schemes are relatively cheap. Formally,

Proposition 5 *The $K(m^*)$ contract is a least-cost successful contract under outcome-based motives where only outcomes are contractible.*

*Furthermore, the costs of this scheme are equal to the costs of a least-cost successful contract under **vote**-based motives where only **votes** are contractible.*

Proof. Notice that the difference in a voter's payoff from voting for policy a versus policy b hinges entirely on the circumstance where his vote is pivotal. Furthermore, conditional on being pivotal, the voters' payoffs are identical to the case of vote-based motives where contracts are conditional only on individual votes. Therefore, by an argument identical to that in Proposition 1, the result follows. ∎

Discussion

Taken together, the previous results show that vote buying is most effective when the contractual contingencies coincide with the motives of voters. Voters with vote-based

motives are bought most cheaply through vote-contingent contracts, while outcome-based voters are bought most cheaply through outcome contingent contracts. The misalignment of the contractual contingency with the voters' preferences raises the cost of successful vote buying or, in some cases, makes successful vote buying impossible.

C. Contracting on Votes and Vote Shares

Suppose that contracting on votes is once again feasible for the interest groups. It seems sensible to think that these groups might also wish to take into account other contingencies in determining their contracts, such as the successful passage of preferred legislation or the margin of victory. Hence, we study contracts contingent on both votes and vote shares.

To deal with these new contractual contingencies, some additional notation is required. Let $\#a$ denote the number of votes cast for policy a. The number of votes for b is then $n - \#a$.

Vote-Based motives

Recall that, in the absence of competition, it cost group A and amount C_A^0 to obtain outcome a. When contracting solely on the basis of votes, the presence of competition drove group A's costs of vote-buying to amounts in excess of $C_A^0 + W_B$. In contrast, our next Proposition shows that, when contracts are also contingent on vote shares, competition has virtually no effect: The least-cost successful contract costs A an amount $C_A^0 - v\,(M+1)$.

Proposition 6 *The following is a least-cost successful contract under vote-based motives where individual votes and votes shares are contractible.*

For $v^{-1}\,(K\,(M+1)) \leq i \leq M+1$

$$
t_i = \begin{cases}
\max\,(-v_i, 0) & if \quad c_i = a \ and \ \#a \geq M+1 \\
K\,(M+1) - v_i & if \quad c_i = a \ and \ \#a < M+1 \\
0 & if \quad\quad\quad c_i = b
\end{cases}
$$

For $i < v^{-1}\,(K\,(M+1))$ *or* $i > M+1$, *no contract is offered.*

Proof. First, we show that the contract in the proposition is least cost. Recall that the minimum cost of obtaining $\#a = m$ votes is

$$
C_A\,(m) = \sum_{i=1}^{m} \max\,(-v_i, 0)
$$

Therefore, the only contract with potentially lower costs is where $\#a = M$. Under such a contract, group B needs only to capture a single voter to shift the outcome. To induce a voter to switch, B must pay that voter more than his outside option from changing the policy and receiving compensation from A. Since the value of this outside option is zero

for voter $i = M$, B can obtain this vote at arbitrarily small cost. Therefore, such a contract does not lead to the adoption of policy a.

It remains to show that the contract in the proposition is a successful contract. We claim that if A offers this contract, B can do no better than to do nothing, and voters $i \leq M + 1$ can do no better than voting for a.

First, suppose B does nothing and consider a deviation by any voter i currently voting for a. By deviating from a to b, the voter earns

$$\Delta U_i = -v_i - t_i$$

For $i < v^{-1}\left(K\left(M+1\right)\right)$, $v_i > 0$ and $t_i = 0$ hence this is strictly unprofitable. For $v^{-1}\left(K\left(M+1\right)\right) \leq i \leq M+1, t_i \geq -v_i$; therefore $\Delta U_i \leq 0$; hence this is unprofitable.

Next, we show that B has no profitable deviation. Clearly, if B offers a contract that does not alter the policy, it does not benefit. Suppose B alters the policy by recruiting $k \geq 2$ voters from A's coalition. To induce these voters to switch, each must be paid the value of his outside option conditional on policy b being adopted or policy a being adopted with a bare majority (since these are the two possible contingencies associated with deviating from b to a). That is, for all k, each voter must be paid an amount at least $K\left(M+1\right)$ and, by construction

$$kK\left(M+1\right) \geq W_B$$

for $k \geq 2$. Therefore, B can has no profitable deviation. This completes the proof. ■

Group A is able to deflect the effects of competition by compensating "loyal" voters in the event of a defeat or a narrow victory for policy a. That is, voters receive the most compensation from group A when they are most needed—when the success of adopting policy a is in doubt. In contrast, when success is assured, i.e. when a enjoys a supermajority of support, the contract calls for minimal compensation for loyalty. The upshot is that competition from B is deflected by the high promised payments when a is at risk while keeping equilibrium payments low owing to the successful "entry deterrence" strategy.

Outcome-Based motives

Vote buying can be made arbitrarily cheap when voters have outcome-based motives as the following proposition shows:

Proposition 7 *The following is a least-cost successful contract under outcome-based motives where individual votes and votes shares are contractible.*

For voters $v^{-1}\left(K\left(M+1\right)\right) \leq i \leq M+1$

$$t_i = \begin{cases} 0 & if \quad c_i = a \ and \ \#a \geq M+1 \\ K\left(M+1\right) - v_i & if \quad c_i = a \ and \ \#a < M+1 \\ 0 & if \qquad \quad c_i = b \end{cases}$$

For $i < v^{-1}\left(K\left(M+1\right)\right)$ or $i > M+1$, no contract is offered.

Proof. Since the cost of the contract is zero in equilibrium, it is clearly a least-cost contract. It remains to show that it is a successful contract. We claim that if A offers this contract, B can do no better than to do nothing, and voters $i \leq M + 1$ can do no better than voting for a.

First, suppose B does nothing and consider a deviation by any voter $v^{-1}(K(M+1)) \leq i \leq M + 1$ currently voting for a. Let π_1 denote the probability that strictly less than $M - 1$ other voters vote for a. Let π_2 denote the probability that exactly $M - 1$ other voters vote for a. Finally, let π_3 denote the probability that exactly M other voters vote for a. By deviating from a to b, the voter earns

$$\Delta U_i = -(\pi_1 + \pi_3)(K(M+1) - v_i) - \pi_2(K(M+1))$$

Note that $\Delta U_i < 0$ for all positive probabilities π_1, π_2 and π_3. Hence this is strictly unprofitable. For voters $i < v^{-1}(K(M+1))$, since $v_i > 0$, voting for b is strictly less profitable than voting for a.

Next, we show that B has no profitable deviation. Clearly, if B offers a contract that does not alter the policy, it does not benefit. Suppose B alters the policy by recruiting $k \geq 2$ voters from A's coalition. To induce these voters to switch, each must be paid the value of his outside option conditional on policy b being adopted or policy a being adopted with a bare majority (since these are the two possible contingencies associated with deviating from b to a). That is, for all k, each voter must be paid an amount at least $K(M+1)$ and, by construction

$$kK(M+1) \geq W_B$$

for $k \geq 2$. Therefore, B can has no profitable deviation. This completes the proof. ∎

Discussion

Thus, the key for A to successfully defend against an invasion by B is to create good incentives both in the event that B recruits a bare majority and in the event that B recruits a supermajority. The constructions given in Propositions 6 and 7 achieve this by having A offer to pay quite generously in the event that its proposal *loses* or just barely wins and a particular voter is loyal to A. Since, to successfully invade, B must pay each voter an amount equal to their opportunity cost of foregoing A's proposal, this strategy dramatically raises B's cost of a successful invasion while not affecting A's costs in the event achieves a supermajority.

By conditioning on votes and vote shares, A is in a position to offer these deterring incentives without actually having to pay for them in equilibrium. Propositions 6 and 7 highlight the extreme susceptibility of voters to vote buying schemes in rich contractual environments. The policy prescription here is clear. Contingent contracts along the line specified above must be made extremely costly, perhaps by penalties such as forfeiture of office or heavy fines.

IV.   Related Literature

Having presented the results of the paper, it is useful to place our analysis in the context of the larger theoretical literature on vote buying. Our paper is closely related to the seminal work of Groseclose and Snyder (1996). Using a model similar to ours, Groseclose and Snyder focused on the case of pure vote-based motives and where two competing interest groups offer contracts contingent only on an individual's vote. Their main result is to show that, generally, contracting for a supermajority is optimal.

Dal Bo (2004) considers the case where motives are purely outcome-based and where the contract space is arbitrary, but where competition is absent. He shows that a monopoly interest group operating in this setting can use a contract contingent on an individual's vote as well as whether that vote turns out to be pivotal to obtain its outcome at arbitrarily small cost.

Dekel, Jackson, and Wolinsky (2005) consider a model of vote buying where budget constraints play a key role. They allow competing interest groups to contract based on votes and outcomes. Unlike our paper and the previous works above, the extensive form of the competition between the interest groups is via alternating offers. Their work also differs from ours and from the above papers in that voters are not modeled as strategic players. Instead, voters are assumed to follow simple heuristic rules. Dekel, *et al.* find conditions in which a budget constrained interest group offering a contract whose payment is contingent on both votes and outcomes can achieve its preferred policy at arbitrarily small cost.

Our paper builds and extends the Groseclose and Snyder framework by allowing for outcome-based motives on the part of voters as well as by enriching the contract space. Our paper complements that of Dal Bo by illustrating that in contractual settings where conditioning on votes and votes shares is possible, an interest group can still obtain its preferred outcome at arbitrarily small cost even in the presence of competition. While our paper shares similar concerns with that of Dekel, *et al.*, a key dimension in which it differs is in the modeling of the voters. We model voters as strategic players. Indeed, this difference is important in explaining some of our results. Specifically, the potential immunity of the voters to pressure from interest groups highlighted in our Proposition 3 follows from the fact that voters are strategic.

There is a larger literature on the buying and selling of influence, that differs significantly in both its concerns and modeling approach from our work and the papers above. Specifically, voting plays little role in this branch of the literature—the policy is typically determined by a single player and competition among interest groups is typically modeled as some type of auction. Some of the earliest work in this area (see Tullock, 1972, 1980) models the policy maker as non-strategic and supposes that competition among interest groups takes the form of an imperfectly discriminating all-pay auction. One of the primary concerns of this literature is on how variation in the structure of the auction affects rent-seeking expenditures by lobbying groups. The interested reader should see Nitzan (1994) for an excellent survey. Another important approach is to use menu auctions (Bernheim and Whinston, 1986) to model competition for influence. Unlike the rent-seeking literature, here the policy-maker is modeled as a strategic player. The seminal

work along these lines is Grossman and Helpman (1994) who apply this analysis to trade policy. Other notable work in this vein includes Grossman and Helpman (1996, 1999).

V.   Conclusions

We have shown that the cost and frequency with which vote buying occurs depends crucially on two factors: the available contingencies in (implicit or explicit) vote buying contracts, as well as the motives of the voters. Indeed, an important finding is that, when what is contractible coincides with what voters care about, the cost of vote buying is substantially reduced. That is, outcome-contingent contracts are more cost effective when voters mainly care about policy outcomes, while vote-contingent contracts are more cost effective when voters mainly care about their individual vote. Indeed, when what is contractible does not coincide with the motives of voters, we have shown that vote buying can be extremely expensive or altogether impossible.

Enriching the space of contractible contingencies to allow for conditioning on an individual's vote as well as an aggregate measure such as vote share substantially increases the power of interest groups to buy votes cheaply. Indeed, we have shown that the following stark case is possible: Suppose that a supermajority of a large electorate is against a given policy and that there is a competing interest group with the same policy preference as the majority of voters. Then, in spite of these formidable obstacles, an interest group with opposite policy preferences can construct a vote buying scheme that delivers a supermajority of votes to its preferred policy at arbitrarily small cost. In short, competition offers little in the way of an antidote to the influence of interest groups in contractually rich settings. And mere size of the voting body does not act as a deterrent to vote buying either, as the interest group's total expenditure does not increase much with the number of voters needed to win passage of the unpopular policy.

The introduction of secret ballots is widely believed to undermine the feasibility of vote buying schemes. Our results suggest that the efficacy of this measure depends on the motivations of voters. When voters care mostly about their own votes and less about policy outcomes, secret ballots are indeed a very effective antidote to vote buying. In contrast, when voters care mostly about policy outcomes, secret ballots do much less to raise the price of successful vote buying.

Another interpretation of our results is to suggest that increased transparency, combined with the much lower communication and contracting costs associated with advances in information technology, can quickly lead to a situation where even modest amounts spent on vote buying can dramatically affect policy outcomes.

To summarize, our results suggest that there is no single measure that prevents vote buying in all or even most circumstances. Instead, a successful policy requires a combination of secret ballots, enfranchisement, and strategic enforcement.

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