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Government Debt and Long-Term Interest Rates

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Abstract

This paper examines the relationship between government debt and long-term interest rates. A dynamic general equilibrium model that incorporates debt nonneutrality is specified and solved, and numerical simulations using the model are undertaken. In addition, empirical evidence using panel data for 19 industrial countries is examined. The estimation provides some evidence supporting the theoretical predictions: the paper finds that the simulated and estimated interest rate effects of government debt tend to be small. However, an increase in government consumption and debt leads to a considerably larger effect. The paper also argues that, although the interest rate effects of pure crowding out may be limited, the economic impact of accumulating government debt cannot be ignored.

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I. INTRODUCTION

The relationship between government debt and long-term interest rates has been one of the most keenly debated subjects in macroeconomics. At the theoretical level, the Ricardian equivalence theorem—which asserts that the level of government debt (or timing of nondistortionary taxation) should not affect the real equilibrium of an economy—has been challenged persistently, and questions have been raised regarding its underlying assumptions. At the empirical level, a number of studies have examined the relationship using a wide variety of statistical and econometric techniques. In spite of the extensive theoretical and empirical literature, the debate remains open.

Developments in recent years in the major industrial countries have created a renewed interest in this subject. In the United States, the long-term budget outlook has deteriorated markedly in the last few years, and questions have been raised as to whether and to what extent the projected rise in the federal government debt will affect long-term interest rates. Government finances have been even more stressed, and for a longer period, in Japan, although the negative impact on Japanese government bond yields has so far been limited. In the euro area, there has been a debate as to whether the differences in fiscal position across the member countries have been priced in the yield differentials on government securities. A few other countries, such as the United Kingdom and Canada, that undertook fiscal consolidation in the latter half of the 1990s—resulting in a significant reduction in the ratio of government debt to GDP—saw their long-term interest rates fall at the same time. These observations from different country experiences underline the need for further analysis.

This paper examines the relationship between government debt and long-term interest rates from both a theoretical and an empirical perspective. A dynamic general equilibrium model which incorporates debt nonneutrality is specified and solved, and numerical simulations using the model are conducted. The paper then considers empirical evidence using data for 19 industrial countries. It finds that the simulated and estimated interest rate effects of government debt tend to be small. However, if an increase in government debt is combined with an increase in government consumption, the effects would be considerably larger. It also argues that, although the interest rate effects of pure crowding out may be limited, the economic impact of accumulating government debt on long-run consumption level cannot be ignored.

The rest of the paper is organized as follows. The next section provides a brief overview of the theoretical and empirical literature. The third section considers the effects of government debt on interest rates based on numerical simulations using a dynamic general equilibrium model. The fourth section reports results of estimation using panel data from 19 industrial countries. The fifth section concludes. Two appendicies provide further detail on the model and data, respectively.

II. An Overview of the Literature

The theoretical predictions regarding the effect of government debt on interest rates depend on the assumed structure of the economy.[2] At one extreme, when Ricardian equivalence holds, government debt is neutral: changing the path of government debt does not affect the equilibrium of the economy.[3] Therefore, it has no impact on interest rates.

One of the ways to break debt neutrality is to incorporate new generations entering the economy every period. Such an overlapping-generations model in a continuous time framework was introduced by Blanchard (1985), and Buiter (1988) and Weil (1989) further contributed to its generalization. The theoretical analysis of the present paper employs their framework to explore the interest rate effects of government debt.

Recent studies by Laubach (2003) and Engen and Hubbard (2004) consider theoretical predictions for the interest rate effect of government debt in a partial equilibrium analysis, taking an aggregate production function and arbitrarily assuming crowding out effects of government debt on physical capital. Laubach (2004) discusses the interest rate effects of temporary tax cuts qualitatively using an overlapping-generations model without production. Unlike these studies, the present paper considers the determination of interest rates in a full-fledged general equilibrium setting, which could be calibrated and used to conduct numerical simulations.

There is a large body of empirical literature on the relationship between interest rates and the fiscal deficit or government debt.[4] Different specifications as well as estimation methods have been applied to data from a number of countries and from different periods. While it is difficult to argue that there is a consensus view, the main conclusions and some methodological issues arising from the most recent studies are highlighted below.

Recent studies for the United States present evidence on the interest rate effects of expected deficits and debt. Canzoneri, Cumby, and Diba (2002), Laubach (2003), and Engen and Hubbard (2004) all use official fiscal projections (from the Congressional Budget Office or the Office of Management and Budget). The latter two studies examine the evidence on the effects of fiscal forecasts on expected future interest rates. There is a clear advantage of using projected government debt and expected future interest rates instead of current values in the regression analysis: this way, it is possible to isolate the long-run influences of fiscal policy on interest rates from other factors related to the state of the business cycle.

Not surprisingly, as these studies rely on an overlapping dataset from the United States, they find broadly similar interest rate effects of fiscal deficits or government debt: a one

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[2] For a survey of macroeconomic effects of government debt, see Elmendorf and Mankiw (1999).

[3] For a survey of Ricardian equivalence theorem, see Bernheim (1987) and Seater (1993).

[4] See Brook (2003) and European Commission (2004) for recent surveys.

percentage point increase in the deficit-to-GDP ratio would increase the expected real interest rate by 19-45 basis points, while a one percentage increase in the debt-to-GDP ratio would increase the expected real interest rate by 3-5 basis points. An important insight highlighted by Laubach (2003) and Engen and Hubbard (2004) is that the estimated impact of fiscal policy on real interest rates depends on whether the deficit-to-GDP ratio or rather the debt-to-GDP ratio is used as an explanatory variable. For example, Laubach (2003) reports that the estimated effects of a one percentage point increase in the expected deficit-to-GDP ratio is about 5-8 times larger than the effects of a one percentage point increase in the expected debt-to-GDP ratio.

For euro area countries, Faini (2004) finds significant evidence that a deterioration in the cyclically adjusted primary fiscal balance in one country would boost both the euro area real long-term interest rate and the spread between domestic and euro area interest rates, with a much smaller magnitude for the latter effect. In the estimation, the government debt-to-GDP ratio of the euro area and the ratios of individual countries are included, and the debt ratio of the euro area is found to affect significantly the real long-term interest rate: a 5-7 basis point increase in the real interest rate for a one percentage point increase in the public debt-to-GDP ratio of the euro area. It is difficult to interpret the results because both deficit and debt measures are used in the same estimation. Nevertheless, the author concludes that loosening fiscal discipline not only affects domestic interest rates but also has significant spill-over effects in other euro area countries.

Evidence for a wider range of industrial countries is also available. Ford and Laxton (1995) examine the data for nine OECD countries, and conclude that the increase in OECD-wide government debt since the early 1970s was a major factor in the rise in real interest rates in the 1980s and early 1990s. Orr, Edey, and Kennedy (1995) report estimation results from pooled time-series regressions for 17 OECD countries, finding a significant long-run effect of an increase in the deficit-to-GDP ratio on long-term real rates. More recently, Ardagna, Caselli, and Lane (2004) analyze panel data of 16 OECD countries and find that an increase in government debt affects long-term interest rates only for countries with above-average debt levels.

III. Theoretical Analysis

In this section, we develop a dynamic general equilibrium model of an economy that features debt nonneutrality. Government debt is nonneutral when changing the path of the debt (while maintaining government solvency through adjusting the path of lump-sum taxes) affects the equilibrium path of the economy. In the economy considered here, new generations enter the economy at a constant rate. When the government postpones taxation (by issuing debt today and committing to increase taxes in the future), the generations born in the future will bear part of the future tax increase. The generations currently alive do not fully internalize the future tax burden associated with today’s tax cuts, which implies that the tax cut today is worth more than the present discounted value of all future taxes. In this way, the current generations perceive part of the debt as net wealth and increase consumption by running down physical capital. Therefore, by increasing the current level of debt and shifting wealth across generations, the government depresses capital accumulation and causes the interest rate to rise.

A. The Model

We employ a continuous time, overlapping-generations model to explore the interest rate effects of government debt. The model extends Buiter (1988) by incorporating a neoclassical production function and a utility function with constant elasticity of marginal utility. After discussing the features of the model in a nontechnical manner, we use a numerical model to simulate the steady-state effects of changes in government debt and expenditure.

Consider an economy in which a new generation is born at a constant rate, β, of the existing population and all individuals face a constant probability of death, λ, regardless of their age. The population grows at a constant rate, $n = \beta - \lambda$. Each individual supplies a fixed amount of labor, receives wages, and pays a lump-sum tax; both wage and tax rates are assumed to be identical irrespective of the individual's age, work hours, or efforts. The individual chooses the path of consumption and saving to maximize expected lifetime utility: the instantaneous utility is characterized by a constant elasticity of marginal utility and the pure rate of time preference, ρ, is constant.

The production side of the economy is characterized by a constant-returns-to-scale production function in which physical capital and labor are inputs. Physical capital depreciates at a constant rate, δ.

The government raises revenue from lump-sum taxes, τ; purchases the public good, g; which is consumed by the government and does not affect individual's utility; and issues government bonds, b, to finance the balance. Savings by individuals accumulate as additional nonhuman wealth, a, either in the form of physical capital, k, or financial assets; the only financial asset we consider is government bonds.

We assume that physical capital and government bonds are perfect substitutes so that yields on government bonds, or interest rates, are always equal to the marginal product of physical capital. This assumption is key when we relate one-to-one the level of government debt and physical capital on the one hand, and interest rates on the other.

The formal derivation of the equilibrium path is presented in Appendix I. Given the level of government debt ($\overline{b}$) and government consumption ($\overline{g}$), the steady-state solution is derived by solving the following system of equations for consumption (c) and physical capital (k):

(1) $$\theta^{-1}\left[f'(k)-\rho\right]c = \beta\left[\left(1-\theta^{-1}\right)r + \lambda + \rho\theta^{-1}\right](k+\overline{b}),$$

(2) $$f(k) = c - \overline{g} - \delta k.$$

The key to understanding debt nonneutrality under a positive birth rate is to note that the solution depends on $\bar{b}$ when the birth rate, β, is positive. When the birth rate is zero, equation (1) reduces to $r = f'(k) = \rho$, and the steady state of the economy is independent of government debt, i.e., government debt is neutral.

Given all the structural parameters of the model, the steady-state interest rate is determined by government debt and government consumption:

(3) $$r = R(\beta, \lambda, \theta, \rho, \alpha, \delta; \bar{b}, \bar{g})$$

The specification for the estimation in the next section is motivated by this equation. In Appendix I, we also derive an equation (A30), relating a change in interest rates to changes in government debt and public consumption, which dictate how interest rates are affected by incremental moves in government debt and public consumption.

In this economy, when the birth rate is positive, government debt will affect equilibrium interest rates by crowding out physical capital. This could easily be understood by comparing two steady states: one with low government debt and the other with high debt.[5] Assume the following simple rule for the government budget: when the level of debt changes, the government will adjust the lump-sum tax so as to maintain a balanced budget. When more government bonds are issued, the generations currently alive do not assume all of the tax increases associated with the higher debt today because the generations born at future dates will bear part of the tax increases. The generations currently alive therefore perceive part of the debt as net wealth and increase consumption by running down physical capital.

The key condition that leads to debt nonneutrality is the entry of new generations, i.e., the positive birth rate. In other words, when the birth rate is positive, government bonds crowd out physical capital through a wealth effect. A higher level of government debt is associated with a lower level of physical capital, which translates into a higher marginal return on capital. This is in a nutshell the mechanism through which government debt affects interest rates.

B. Simulation Results

We use a numerical representation of the economy to simulate the steady-state effects of government debt on interest rates. The calibration of the baseline model follows closely Barro and Sala-i-Martin (1995). The share of capital income in the total output is $\alpha = 0.3$; the intertemporal elasticity of substitution is $\theta = 3$; and the rate of depreciation of physical

[5] We do not consider the dynamic adjustment of the economy as the path of lump-sum tax changes. In general, the interest rate effects of temporary deficits are ambiguous, depending on whether the government solvency is ensured by future tax increases or expenditure cuts.

capital is $\delta = 0.05$.[6] As for birth and death rates, we initially set $\beta = 0.03$ and $\lambda = 0.02$, which imply that the rate of change in the population, $n \equiv \beta - \lambda = 0.01$.

Once these parameters are fixed, the only free parameter is the pure rate of time preference, ρ. We find that setting $\rho = 0.02$, as in Barro and Sala-i-Martin (1995), would result in too high a capital-output ratio: close to 4, compared with 2½ reported by Laubach (2003) for the United States. Accordingly, we start with the higher value of the time preference parameter ($\rho = 0.06$), which gives rise to the capital-output ratio that is close to the observed value, and check the sensitivity of the solutions for different values of ρ. We also conduct simulations by allowing birth and death rates to deviate from the baseline. For each value of death rate, we take three different values of birth rate; we only consider the cases of positive or no population growth, i.e., $\beta \geq \lambda$.

In our simulations, the steady-state interest rate effects of government debt are computed by changing the steady-state value of government debt, b, with a view to examining how the government debt-to-output ratio and real interest rates are related. We also conduct the same computation with respect to government consumption, g. Note that we assume that lump-sum taxes, τ, would adjust to ensure an instantaneous balanced budget; therefore, we allow b and g to move independently at the steady state.

Table 1 summarizes the simulation results. Panel (A) exhibits the results for different values of the rate of pure time preference while keeping birth and death rates constant. Column 1 shows the steady-state capital-to-output ratio; column 2 shows the crowding-out effect on the stock of capital with respect to a unit change in government debt; columns 3 and 4 show the interest rate effects of government debt and government consumption, respectively; and column 5 shows the long-run effect on private consumption (in percent) of a one percentage point increase in the government debt-to-output ratio. The results for different values of birth and death rates are shown in panel (B).

The simulation results show that the steady-state interest rate effects of government debt depend on the rate of pure time preference (ρ) and birth rate (β). When ρ is higher, the steady-state capital-output ratio is lower and the interest rate effect of government debt is larger. This is consistent with the formula derived by Laubach (2003), who relates the interest rate effect of a one percentage point increase in the government debt-to-output ratio to the capital-output ratio, the share of capital income in total output, and the crowding-out effect.

More significantly, the interest rate effect of government debt depends on birth rate (β). When β is lower, the interest rate effect of government debt is smaller. This is due to both a higher steady-state capital-output ratio and a smaller crowding-out effect. The crowding-out

[6] These parameter values are the same as those used in Barro and Sala-i-Martin (1995).

effect is smaller because, as we discussed in the previous subsection, a lower birth rate implies that the current generations internalize a larger part of the future tax burden associated with an increase in government debt. In other words, β could be seen as a parameter determining the burden sharing of all future taxes between the generations currently alive and the generations born in the future. As we see in the bottom of the table, when the birth rate is zero, there is no crowding-out effect of capital by government debt: the government debt is neutral in this case.

Table 1. Simulated Effects of Government Debt and Consumption on Interest Rates

(A) For different values of the time preference

Time preference	Birth rate	Death rate	Steady state		Interest rate effects 2/		Long-run effects on consumption 3/
			Capital-output ratio	Crowding-out effect 1/	Government debt	Government consumption	
			(1)	(2)	(3)	(4)	(5)
0.06	0.03	0.02	2.57	-0.53	1.6	8.3	-0.06
0.05	0.03	0.02	2.80	-0.57	1.5	8.1	-0.06
0.04	0.03	0.02	3.06	-0.62	1.3	8.0	-0.05
0.03	0.03	0.02	3.38	-0.68	1.2	7.8	-0.05
0.02	0.03	0.02	3.76	-0.74	1.1	8.3	-0.04

(B) For different values of the birth and death rate

Time preference	Birth rate	Death rate	Steady-state		Interest rate effects 2/		Long-run effects on consumption 3/
			Capital-output ratio	Crowding-out effect 1/	Government debt	Government consumption	
			(1)	(2)	(3)	(4)	(5)
0.04	0.04	0.02	2.66	-0.69	2.1	10.7	-0.08
0.04	0.03	0.02	3.06	-0.62	1.3	8.0	-0.05
0.04	0.02	0.02	3.64	-0.54	0.9	6.2	-0.03
0.04	0.03	0.01	3.22	-0.62	1.3	8.0	-0.05
0.04	0.02	0.01	3.82	-0.53	0.8	6.0	-0.03
0.04	0.01	0.01	4.80	-0.39	0.4	3.9	-0.01
0.04	0.02	0.00	4.03	-0.52	0.7	5.6	-0.03
0.04	0.01	0.00	5.03	-0.37	0.3	3.7	-0.01
0.04	0.00	0.00	7.50	0.00	0.0	0.0	0.00

1/ Change in the stock of capital with respect to a unit increase in government debt.
2/ Change in interest rates, in basis points, with respect to a one percentage point increase in the ratio of government debt or consumption to GDP.
3/ Change in the level of consumption, in percent, with respect to a one percentage point increase in the ratio of government debt to GDP.

In sum, the simulation results show that a one percentage point increase in the government debt-to-output ratio would raise the steady-state real interest rate by up to 2 basis points, depending on the underlying parameter values. The results also indicate that the interest rate effects of an increase in government consumption would be several times larger than the effects of government debt. In this model, if higher government debt is caused by higher government spending, the combined effect of government debt and government consumption would be considerably larger than if higher government debt is caused by lower revenue.

It should be emphasized that the simulated interest rate effect of government debt is likely to be the lower bound as it captures purely the long-run (steady-state) physical crowding-out effect. As there is no uncertainty, risk does not play any role in the model. Furthermore, if the government were to increase income taxes (as opposed to the lump-sum tax) to balance the budget in response to a higher debt level, there would be a larger impact on real interest rates.

Overall, the results may be taken to indicate that the pure effects of government debt on interest rates are limited. However, it should be noted that even if the interest rate effects are small, the crowding out of physical capital could be significant. As a result, the level of production and consumption would be permanently lower when government debt is higher. The simulated impact shows that consumption would be permanently lowered by up to 0.08 percent when the government debt-to-GDP ratio increases by one percentage point. This suggests that government debt could entail substantial welfare losses even if a lump-sum tax is available to the government.

IV. Empirical Evidence

The theoretical model in the previous section shows, in the case of debt nonneutrality, a positive effect of government debt on interest rates through crowding out of physical capital. In this section, we empirically test this hypothesis and estimate the size of the effect using panel data from 19 OECD countries spanning 1971–2004.[7]

A. Specification and Data

Consistent with the theoretical analysis, we focus on the steady-state relations between government debt, government expenditure, and real interest rates. We maintain the closed economy assumption of the theoretical model and abstract from issues such as international capital flows. The model also abstracts from short-run dynamics of interest rates over a business cycle. We therefore do not consider effects of output growth, inflation, and current account balance—and their variations—on long-term interest rates. We eliminate short-run fluctuations of the sample data by taking the averages over five years; each country provides seven observations at a maximum. The limited time number of data points would preclude us from running an individual country regression.[8] On the other hand, if we were to run a cross-

[7] The following countries are included in the dataset: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[8] The theoretical model suggests that the interest rate effects of government debt would be different across countries depending on the underlying structural parameters of the economy. However, variables that are suited to estimate a long-run relationship between fiscal policy and interest rates—such as expected forward rates and official fiscal projections as employed by Laubach (2003) and Engen and Hubbard (2004) for the United States—are not readily available across OECD countries. We therefore relied on a simple averaging to eliminate

(continued…)

country regression using the data from individual countries averaged over the entire sample period, we would altogether ignore heterogeneity across countries. In the following analysis, we use pooled least squares estimation with or without fixed country effects and assume that any country-specific factors, if present, would be subsumed in the fixed country effects.

Based on the theoretical assumption that government debt and physical capital are perfect substitutes and that their rates of return are equalized in equilibrium, we use real long-term interest rates on government bonds as the dependent variable. Real long-term interest rates are measured by subtracting expected inflation from long-term government (or public sector) bond yields. The maturity class for which the data are readily available is generally 10 years. An alternative option would be to use yield spreads between short- and long-term bonds or implied forward rates. However, since we are concerned with the direct influence of government debt on the level of the real interest rate, we only use current nominal bond yields. To proxy inflation expectations, we first follow Orr, Edey, and Kennedy (1995) and take the rate of change in the trend consumer price index, with the latter obtained by applying the Hodrick-Prescott filter. As an alternative specification, we also take the actual inflation rate to calculate real long-term interest rates.

As for government debt, we use the ratio of current financial liabilities of the general government to nominal GDP. As discussed above, recent studies for the United States emphasize the relevance of a projected measure of government debt, but such data are not widely available for the other OECD countries. One issue is how to treat the central bank holdings of government securities. It is possible that the crowding-out effect of government debt is mitigated to the extent that a portion of the debt is held by the central bank.[9] As a first approximation, we do not consider such a possibility. In the estimation, we consider both gross and net financial liabilities.

In the theoretical model of the previous section, an exogenous increase in government consumption would raise the equilibrium interest rate by directly crowding out physical capital. Such effects are considered by Engen and Hubbard (2004) by including a dummy variable denoting a period of significant military buildups. In our model, we include the ratio of government final consumption expenditure to nominal GDP in the estimation.

According to the specification discussed above, we estimate the following equation:

$$R_{it} = \alpha_i + \beta B_{it} + \gamma G_{it} + \varepsilon_{it}$$

where R is the real long-term interest rate, B is the government debt-to-GDP ratio, G is the government consumption-to-GDP ratio, the subscript i denotes the country, and t denotes

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short-run influences on interest rates. For this reason, the methods to estimate long-run relationships from heterogeneous panels proposed by Pesaran and Smith (1995) are not an option in this paper.

[9] Engen and Hubbard (2004) observe that purchases of U.S. Treasury securities by the Federal Reserve have a significant negative impact on real long-term interest rates.

the time period. All the data are taken from the OECD Analytical Database; they are described in Appendix II.

B. Estimation Results

The upper panel of Table 2 reports estimation results assuming that all countries have common intercepts. The first two columns show coefficient estimates for regressions of the real long-term interest rate (using trend inflation as a proxy for inflation expectation) on government debt and government consumption. The first regression uses net government debt and the second one uses gross debt; both regressions provide similar estimates, which are statistically significantly different from zero. The results indicate that a one percentage point increase in the government debt-to-GDP ratio raises the real long-term interest rate by about 2 basis points. This is close to the upper bound of the steady-state effect obtained in the numerical simulations in the previous section.

Table 2. Estimated Interest Rate Effects of Government Debt and Consumption 1/

No country effect

Dependent variable	Real interest rates (using trend inflation)		Real interest rates (using actual inflation)	
Government debt (net)	0.018		0.018	
	(0.007)**		(0.006)**	
Government debt (gross)		0.023		0.024
		(0.009)**		(0.008)**
Government consumption	0.25	0.19	0.23	0.18
	(0.06)**	(0.06)**	(0.05)**	(0.05)**
Number of observations	111	111	111	111
Adjusted R-squared	0.15	0.15	0.17	0.17
Standard error of estimated equation	2.45	2.45	2.22	2.21

Fixed country effects

Dependent variable	Real interest rates (using trend inflation)		Real interest rates (using actual inflation)	
Government debt (net)	0.053		0.052 **	
	(0.013)**		(0.012)	
Government debt (gross)		0.040		0.042 **
		(0.012)**		(0.011)
Government consumption	0.89	0.86	0.73 **	0.68 **
	(0.17)**	(0.19)**	(0.16)	(0.17)
Number of observations	111	111	111	111
Adjusted R-squared	0.42	0.38	0.41	0.38
Standard error of estimated equation	2.03	2.09	1.87	1.91

1/ ** indicates that the estimated coefficients are significant at the 5 percent level.

In both regressions, the estimated coefficients on government consumption are also significantly different from zero. These regressions show that the interest rate effects from a one percentage point increase in the government consumption-to-GDP ratio are around 20-25 basis points, considerably larger than those from a one percentage point increase in the government debt-to-GDP ratio. The results are broadly consistent with the numerical simulations in the previous section, where we find that the interest rate effects of government consumption are several times larger than those of government debt. The third and fourth columns indicate that the coefficient estimates are little affected by replacing the proxy for inflation expectations with actual inflation.

The lower panel of Table 2 reports the regression results when fixed country effects are allowed. All the estimated coefficients are statistically significantly different from zero. The results imply that a one percentage point increase in the government debt-to-GDP ratio raises real long-term interest rates by about 4-5 basis points, which is somewhat larger than in the specification not allowing for any country heterogeneity. These esimates are close to those obtained by Laubach (2003) and Engen and Hubbard (2004), who report about a 3-5 basis points increase in the real interest rate stemming from a one percentage point increase in the government debt-to-GDP ratio using very different frameworks for the United States. The estimated effects from a one percentage point increase in the government consumption-to-GDP ratio, estimated at around 90 basis points, are substantially larger than in the specification without country effects.

We note that the estimated interest rate effects of government debt and government consumption under the fixed effects specification are larger than the steady-state effect obtained in the numerical simulations. Two interpretations are possible. First, the estimated interest rate effects could be larger because of a risk premium on government debt. The theoretical model does not incorporate uncertainty and such effect is therefore excluded. Second, the estimated interest rate effects could be larger because the lump-sum tax assumed in the theoretical model is usually not available to the government. Instead, the government may need to raise distortionary income taxes in response to higher government debt or expenditure. Higher income taxes would affect the pre-tax return on government bonds, i.e., the observed long-term interest rate. In order to bring the theoretical prediction and the empirical evidence closer in line, one option would be to incorporate uncertainty and income taxation in the theoretical model. Alternatively, one could estimate crowding-out effects more directly rather than try to capture them through the impact of government debt on interest rates. These are possibilities that future research could usefully explore.

V. Conclusion

This paper has examined the relationship between government debt and long-term interest rates using a theoretical model and by empirical analysis. Its main conclusions are as follows:

- A theoretical model incorporating debt nonneutrality clarifies the channels through which government debt affects interest rates. It shows that the interest rate effects of government debt depend on the structural parameters of the economy—most notably birth rate and time preference. Numerical simulations using the model offer insight into the quantitative impact of higher government debt and government consumption on interest rates. The results suggest that, although the interest rate effects of government debt alone tend to be small, an increase in government consumption and debt leads to a considerably larger effect.

- Panel estimation results using OECD country data support the view that an increase in the government debt-to-GDP ratio has a small impact on real long-term interest rates. However, consistent with the simulation results, if an increase in government debt results from an increase in government consumption, the effect is considerably larger.

- Even though the simulated and estimated interest rate effects of government debt tend to be small, the overall economic impact can be significant. Indeed, accumulation of government debt can be expected to entail real crowding out of productive capital and a welfare loss in the long run. Further theoretical and empirical investigations are desirable with regard to this important subject.

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DERIVATION OF DYNAMIC EQUILIBRIUM CONDITIONS

The model employed for the numerical analysis is based on Buiter (1988). We extend it by incorporating a neoclassical production function and a utility function that has constant elasticity of marginal utility.

1. Production

Consider an economy where perfectly competitive firms operate a production technology for producing the single good. The production requires physical capital which is owned by individual consumers and rented to the firms, and labor which is supplied inelastically by the consumer. The production is characterized by constant-returns-to-scale technology. The production function is described as:

(A1) $y = f(k)$,

where y is gross output per unit of labor and k is the stock of physical capital per unit of labor. $f(k)$ is assumed to satisfy both $f'(k) \equiv f_k > 0$ and $f''(k) < 0$. Denoting the rental rate of physical capital (or real interest rate) by r, profit maximization by the perfectly competitive firms implies that the rental rate is equalized with the marginal product of physical capital:

(A2) $r = f'(k)$.

2. Individual consumer's behavior

Suppose the economy is populated with individual consumers who maximize their expected lifetime utility. During their lifetime, they face a common and constant probability of death, λ. At each instant t, each consumer born at time $s \le t$ solves the following maximization problem:

(A3) $$\max_{\{\overline{c}(s,z)\}} W(s,t) = \max_{\{\overline{c}(s,z)\}} \int_t^{\infty} e^{-(\rho+\lambda)(z-t)} u\left[\overline{c}\left(s,z\right)\right] dz$$

where $\overline{c}\left(s,z\right)$ is consumption of the good at time z by the consumer who is born at time s; ρ is the pure rate of time preference which is common to all the consumers. The instantaneous utility function is described as:

(A4a) $u\left[\overline{c}\left(s,z\right)\right] = \dfrac{\overline{c}\left(s,z\right)^{1-\theta} - 1}{1-\theta}$, if $\theta \neq 1$,

and

(A4b) $u\left[\overline{c}\left(s,z\right)\right] = \ln \overline{c}\left(s,z\right)$, if $\theta = 1$,

where θ is elasticity of intertemporal substitution.

The consumer's instantaneous flow budget identity is given by:

(A5) $$\frac{d}{dt}\bar{a}(s,t) = (r(t)+\lambda)\bar{a}(s,t) + \bar{w}(s,t) - \bar{\tau}(s,t) - \bar{c}(s,t),$$

where $\bar{a}(s,t)$ is the consumer's nonhuman wealth which consists of holdings of physical asset, k, and government debt, b; $\bar{w}(s,t)$ is the real wage from exogenously fixed supply of labor; $\bar{\tau}(s,t)$ is a lump-sum tax. In what follows, we assume that real wage and lump-sum-tax are age independent. The meaning of the term $\lambda\bar{a}$ in the budget identity is lucidly explained by Buiter (1988).

In addition, every consumer has to obey the following solvency constraint:

(A6) $$\lim_{z\to\infty} \bar{a}(s,z)\exp\left(-\int_t^z (r(\zeta)+\lambda)\,d\zeta\right) = 0.$$

By integrating forward the instantaneous budget identity and imposing the solvency constraint, the individual consumer's intertemporal budget constraint is obtained:

(A7) $$\bar{a}(s,t) = \int_t^\infty \exp\left(-\int_t^z (r(\zeta)+\lambda)\,d\zeta\right)\left[\bar{c}(s,z)+\bar{\tau}(s,z)-\bar{w}(s,z)\right]dz.$$

By solving the maximization problem subject to the intertemporal budget constraint, the following first-order necessary condition is obtained describing the optimal intertemporal allocation of consumption:

(A8) $$\frac{d}{dz}\bar{c}(s,t) = \theta^{-1}\left[r(t)-\rho\right]\bar{c}(s,t).$$

This equation together with the intertemporal budget constraint yields the following consumption function:

(A9) $$\bar{c}(s,z) = m(t)\left[\bar{a}(s,t)+\bar{h}(s,t)\right],$$

where

(A10) $$m(t) = \left(\int_t^\infty \exp\left\{-\left[\left(\frac{\theta-1}{\theta}\right)\int_t^z r(\zeta)d\zeta + \left(\lambda+\frac{\rho}{\theta}\right)(z-t)\right]\right\}dz\right)^{-1}$$

and

(A11) $$\overline{h}(s,t)=\int_t^{\infty}\exp\left(-\int_t^{z}\left(r(\zeta)+\lambda\right)d\zeta\right)\left[\overline{w}\left(s,z\right)-\overline{\tau}\left(s,z\right)\right]dz\ .$$

Equation (A9) shows that each consumer spends a fraction, $m(t)$, of human and nonhuman wealth for consumption. In equation (A10), $m(t)$, the marginal propensity to consume from total wealth depends on the entire path of the real interest rate. Equation (A11) defines the human wealth, $\overline{h}(s,t)$, by the present discounted value of the future real wages net of lump-sum taxes.

3. Aggregation

Let us define the population aggregate $X(t)$ as corresponding to any individual's stock or flow variable $\overline{x}(s,t)$. Buiter (1988) shows that, when the population's size at time zero is normalized as $N(0)=1$ and the birth rate is constant at β, $X(t)$ is given by:

(A12a) $$X(t)=\beta e^{-\lambda t}\int_{-\infty}^{t}\overline{x}(s,t)e^{\beta s}ds \quad \text{if } \beta>0$$

and

(A12b) $$X(t)=\overline{x}(s,t)e^{-\lambda t} \quad \text{if } \beta=0\ .$$

Since we have assumed that all individuals receive the same wage rate and pay the same lump-sum tax irrespective of their ages: $\overline{w}(s,t)=\overline{w}(t)$, $\overline{\tau}(s,t)=\overline{\tau}(t)$, and $\overline{h}(s,t)=\overline{h}(t)$ for all s. The aggregate human wealth $H(t)$ is then defined by:

(A13) $$H(t)=\overline{h}(t)N(t)\ ,$$

where $N(t)=e^{nt}$ is the size of the population at time t and $n=\beta-\lambda$ is the rate of growth of population.

A straightforward calculation yields the following relationships defining the aggregate consumption function as well as the motion of nonhuman and human wealth:

(A14) $$C(t)=m(t)\left[A(t)+H(t)\right],$$

(A15) $$\dot{A}(t)=r(t)A(t)+W(t)-T(t)-C(t)\ ,$$

(A16) $$\dot{H}(t)=\left[r(t)+\beta\right]H(t)+T(t)-W(t)\ .$$

Let us define per capita variable $x(t) = X(t)e^{-nt}$ as corresponding to any stock or flow aggregate variable $X(t)$. In per capita terms, the above relationships can be written as follows:

(A17) $c(t) = m(t)\left[a(t) + h(t)\right],$

(A18) $\dot{a}(t) = \left[r(t) - n\right]a(t) + w(t) - \tau(t) - c(t)$,

(A19) $\dot{h}(t) = \left[r(t) + \lambda\right]h(t) - w(t) + \tau(t)$.

The above three equations imply the following differential equation defining the motion of per capita consumption:

(A20) $\dot{c}(t) = \theta^{-1}\left[r(t) - \rho\right]c(t) - \beta m(t)a(t)$.

In this equation, the marginal propensity to consume, $m(t)$, involves an integral of real interest rate, $r(t)$. It is possible to show that the differential equation for $m(t)$ is expressed as:

(A21) $\dot{m}(t) = m(t)\left\{m(t) - \left[\left(1 - \theta^{-1}\right)r(t) + \lambda + \rho\theta^{-1}\right]\right\}$.

4. Economy-wide equilibrium path

The government raises revenue from lump-sum taxes and purchases the good for public consumption denoted by $g(t)$. The evolution of government debt in per capita terms is described by the following differential equation:

(A22) $\dot{b}(t) = \left(r(t) - \left(\beta - \lambda\right)\right)b(t) + g(t) - \tau(t)$.

We will assume a balanced budget rule. In other words, the government instantaneously adjusts lump-sum taxes so as to maintain a balanced budget continuously, i.e., $\dot{b}(t) = 0$ for every t. This implies the following equation for lump-sum tax:

(A23) $\tau(t) = \left(r(t) - \left(\beta - \lambda\right)\right)b(t) + g(t)$

The economy-wide resource constraint determines the accumulation of physical capital. We assume that physical capital depreciates at the rate δ. The differential equation for physical capital is therefore written as:

(A24) $\dot{k}(t) = f(k(t)) - c(t) - g(t) - \delta k(t)$

The three differential equations for $c(t)$, $m(t)$, and $k(t)$ and the balanced budget equation provide a complete description of the equilibrium path of this economy.

5. Steady state

The steady state of the economy is determined by putting $\dot{c}(t) = \dot{m}(t) = \dot{k}(t) = 0$ in the equations (A20), (A21), and (A24) and solving the system of equations for c, k, and m. By substituting the solution for m, we obtain the following two equations:

(A25) $\theta^{-1}(r-\rho)c = \beta\left[\left(1-\theta^{-1}\right)r + \lambda + \rho\theta^{-1}\right](k+b)$,

(A26) $f(k) = c - g - \delta k$,

where $r = f'(k)$.

We find that when the birth rate is positive, the steady state of the economy depends on government debt, b. When the birth rate is zero, equation (A25) yields $r = f'(k) = \rho$, and the steady state of the economy is independent of government debt.

Substituting (A26) for c into (A25) yields the following equation determining the steady-state value of k:

(A27) $\theta^{-1}\left[f'(k)-\rho\right]\left[f(k)-g-\delta k\right] - \beta\left[\left(1-\theta^{-1}\right)f'(k)+\lambda+\rho\theta^{-1}\right](k+b) = 0$.

6. Comparative static

The impact from a change in government debt on physical capital can be derived by totally differentiating the equation (A27) with respect to k and b, and solving the equation for $\partial k / \partial b$ as follows:

(A28) $\partial k / \partial b = \dfrac{\beta\Omega}{\theta^{-1}\left\{f''(k)\left[f(k)-g-\delta k\right]+\Psi\right\} - \beta\left\{\left(1-\theta^{-1}\right)f''(k)k+\Omega\right\}}$,

where

$\Psi \equiv \left(f'(k)-\rho\right)\left(f'(k)-\delta\right)$,
and
$\Omega \equiv \left(1-\theta^{-1}\right)f'(k)+\lambda+\rho\theta^{-1}$.

Similarly, the impact from a change in government debt on physical capital can be derived as follows:

(A29) $$\partial k / \partial g = \frac{\theta^{-1}\left[f'(k) - \rho \right]}{\theta^{-1}\left\{ f''(k)\left[f(k) - g - \delta k \right] + \Psi \right\} - \beta\left\{ \left(1 - \theta^{-1}\right) f''(k) k + \Omega \right\}}.$$

Accordingly, a change in interest rate due to changes in government debt and spending is expressed by the following equation:

(A30) $$dr = \frac{\partial r}{\partial k} dk = f''(k) \frac{\partial k}{\partial b} db + f''(k) \frac{\partial k}{\partial g} dg,$$

where $\partial k / \partial b$ is given in equation (A28) and $\partial k / \partial g$ is in equation (A29).

DATA SOURCES AND DEFINITIONS

All data are from the OECD Analytical Database, except for some series that are supplemented by data from national sources. The definition of the variables is as follows.

Real long-term interest rates: the nominal long-term interest rate minus expected inflation. Inflation expectations are determined by taking year-over-year percentage change in trend and actual CPI. Trend CPI is calculated by applying a Hodrick-Prescott filter to quarterly CPI with a λ value of 1600.

Nominal long-term interest rates: yields on 10-year government bonds for the United States, Japan, France, the United Kingdom, Canada, Australia, Denmark, Finland, Ireland, Netherlands, New Zealand, Sweden. For Germany, yields on 9- to 10-year public sector bonds. For Austria, yields on public sector bonds with maturity of more than 1 year up to 1989, and 10-year government bonds from 1990 onwards. For Belgium, yields on central government bonds with maturity of more than 5 years. For Italy, yields on government bonds with maturity of more than 2 years up to 1991, and 9- to10-year government bonds from 1992 onwards. For Norway, yields on 6- to 10-year government bonds up to 1992, and 10-year government bonds from 1993 onwards. For Spain, yields on government bonds with maturity more than 2 years. For Switzerland, yields on confederation government bonds with maturity more than 5 years.

Government debt: gross and net financial liabilities of the general government.

Government consumption: final consumption of the general government.

The table below indicates the sample availability.

	Real interest rates	Government debt	Government consumption
Australia	1971-2004	1988-2003	1971-2004
Austria	1971-2004	1980-2003	1971-2004
Belgium	1971-2004	1971-2003	1971-2004
Canada	1971-2004	1971-2004	1971-2004
Denmark	1971-2004	1988-2002	1988-2004
Finland	1971-2004	1975-2002	1971-2004
France	1971-2004	1977-2003	1971-2004
Germany	1971-2004	1971-2002	1971-2004
Ireland	1985-2004	1974-2003	1971-2004
Italy	1971-2004	1971-2002	1971-2004
Japan	1971-2004	1971-2002	1971-2004
Netherlands	1971-2004	1971-2003	1971-2004
New Zealand	1971-2004	1992-2002	1971-2004
Norway	1971-2004	1979-2002	1971-2004
Spain	1971-2004	1990-2002	1971-2004
Sweden	1971-2004	1971-2002	1971-2004
Switzerland	1971-2004	1971-2002	1971-2004
United Kingdom	1971-2004	1971-2003	1971-2004
United States	1971-2004	1971-2004	1971-2004