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Precautionary Monetary and Fiscal Policies

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Abstract

This paper explains why the debt reduction motive for countries that are subject to borrowing constraints and a volatile environment is greater than for those with a more stable environment and relatively better access to the financial markets. In particular, it shows that the possibility of losing the ability to borrow in the future induces precautionary debt reduction. When a government loses its ability to borrow, shocks are more costly to the economy, since they cannot be spread over time. The precautionary motive arises from the need to make these adjustments less painful when the borrowing constraints bind. To avoid large losses in the constrained period, the government prefers to raise taxes and inflation in earlier periods more than would be implied otherwise, leaving less debt to the future periods, and thereby shifting some of the required adjustment to the earlier periods. In other words, the coexistence of large shocks and borrowing constraints forces the government to be more prudent in its management of debt.

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I. INTRODUCTION

The optimal monetary and fiscal policy choices of an economy subject to a borrowing constraint are different from those of a country that is not restricted, even in periods when the constraint is not binding. This paper analyzes the policy implications of such constraints by using a simple three period model, and concludes that the existence of such constraints not only limits current policy actions, but also has implications for policy choices in previous periods.

The notion of high public sector debt is different for emerging and industrial countries. Usually the threshold above which debt stops being sustainable is lower for emerging countries than for industrial countries, which is the so-called debt intolerance phenomenon (Reinhart, et al, 2003). No matter how high this threshold is or how the economy ends up there, once the debt level reaches this point, it becomes harder to borrow additional amounts. This is more so for emerging market economies.

When financing options are limited or nonexistent, the country has to bear the cost of adjustment. This adjustment is usually painful, and involves welfare losses during the period when the adjustment takes place. Leaving out the default equilibrium, this paper shows that even if the existing public debt is below the threshold, the possibility of being in such a situation in the future, where costly adjustments are necessary, will have an effect on current policies. Given that the threshold is lower for developing countries, and that they usually face larger swings compared with industrial countries, this mechanism is more likely to be in effect for developing countries. The incentives to reduce debt in these countries exist not only because this is the only way to gain credibility, and reduce inflation, but also because low debt in the future will help ease the pain of adjustment, since it necessitates less inflation and less stringent fiscal contraction during bad times.

The policy conclusions of this paper are similar to Blanchard (2004). He shows that under a certain set of conditions – such as high level of debt and a large size of foreign currency denominated debt – aggressive interest rate responses, by causing default risk to increase, may lead to depreciation rather than appreciation, which in turn fuels inflation even further. Therefore, he concludes that fiscal policy becomes more important in fighting inflation in these situations. Blanchard’s argument provides an explanation of why the standard monetary policy advice is not appropriate for high-debt (and low-credibility) economies. This paper offers an additional mechanism that produces a similar policy mix namely higher taxation (and for a given level of government expenditure, tighter fiscal policy) and relatively less tight monetary policy based on the existence of borrowing constraints and uncertainty.

To show this, the paper formulates the problem as follows. Assume that there is a threshold on public debt (as a share of GDP) above which the incentives for default are higher than those for rolling over the debt. Observing these incentives, rational private agents will not lend to the government once this threshold is reached.[2]

In addition, the economy is subject to a shock that may take the public debt to the threshold. Any unexpected reduction in tax revenues, increase in government expenditures, or increase in the world interest rate may potentially increase the debt burden of an economy. Although a reduction in tax revenues originated by a decline in world commodity prices is different from an increase in government expenditures – due to war, some national catastrophe or a social need – the impact on debt accumulation is similar.

When the shock hits, the unconstrained optimal response is to adjust partially, and transfer some of the adjustment to future periods by borrowing. The necessary adjustments involve some combination of inflation, to the extent that a fraction of the debt is nominal, and a raise in taxes.[3] These adjustments are costly, and involve welfare losses for the periods when the adjustment takes place. Therefore, the ability to borrow allows the smoothing of this pain over time.

However, when the government finds itself in a situation where it cannot borrow anymore, it is not possible to smooth over the effects of the shock. Rather, the government has to adjust fully, which involves higher inflation and tax rates compared with the unconstrained situation. This will raise the welfare costs for that period significantly.

Assume that a country is in a period in which it is not constrained. But it also knows that its economy is vulnerable to shocks, and that there is a possibility in the future that an adverse shock may carry the economy to a point where nobody lends in a dire situation, raising the costs associated with adjustments for that period drastically. Then the question is whether the existence of such a possibility affects the policy choices today even if the economy is not constrained at all.

This question is similar to the one posed for the optimal consumption/saving decision with liquidity constraints. The answer to the question in that literature is that the existence of liquidity constraints induces precautionary saving behavior (see for example, Carroll and Kimball, 2001). The basic argument is that because constrained agents cannot spread the effects of the shocks over time, uncertainty has a bigger negative effect on the expected value function for constrained agents compared with unconstrained ones, leading to precautionary saving in an effort to make the constraints less likely to bind.

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[2] A more realistic approach is to introduce soft borrowing constraints, which means that the cost of borrowing increases with the debt level. Even though the precautionary motive still exists, it is not possible to get closed-form solutions with the soft constraints (see, for example, Fernandez-Corugedo, 2002).

[3] The exact policy mix depends on factors such as the availability of insurance, credibility, and the existence of nominal stickiness.

In addition, Carroll and Kimball (2001) prove that even if the utility function is quadratic – so the marginal utility is linear – the introduction of the liquidity constraints produces a kink in the marginal value function. This in turn induces prudence – defined as the convexity of the marginal value function, $-V'''(w)/V''(w)$ (Kimball, 1990) – around the level of wealth where the constraint is binding. Furthermore, once this prudence is created, it transmits back to the previous periods.

A similar logic applies to the optimal monetary and fiscal policy choices of a government that is subject to the borrowing constraint. Because the required adjustment – i.e., some combination of higher inflation and tax rate – is usually very costly to the economy, the possibility of an adverse shock raises the expected marginal loss. Consequently, there is a strong incentive for the government to be more prudent in previous periods by keeping the debt level low. The term prudence here refers to the motive to leave less debt behind. However, to the extent that the debt is nominal and monetary policy has an impact on the real activity due to sticky prices, this will also imply higher inflation – and implicitly lower interest rates – than otherwise. In other words, in an effort to reduce the pain associated with the adjustment, the government raises taxes and inflation more than otherwise to leave less debt to the future – that is, it increases its saving. This, in turn, will compensate for the inability to borrow in the future periods, and help spread the pain of a shock through time by an earlier adjustment of the economy, thereby reducing the severity of adjustment in the period where the borrowing constraint is binding.

Aiyagari et al. (2002) take a similar approach in examining optimal taxation by imposing lower and upper bounds on government debt accumulation. They find that the fiscal policy implications of consumption smoothing differ depending on market completeness, the persistence of shocks, and debt and asset limits. In particular, they show that in an incomplete market set up with natural asset limits,[4] the government accumulates assets. If, however, there is a more stringent limit on the government's asset holdings, they establish that debt and the tax rate contain unit roots within the band created by the debt and asset limits.

This paper differs from Aiyagari et al. (2002) in a number of ways. First, the sticky-price setup allows us to analyze both monetary and fiscal policies. Second, this paper has only a limit on debt, not on assets. Last, while they assume credible commitment, this paper focuses on discretionary policies.

II. The Model

In order to capture the idea in a transparent way, and keep the analytical solutions tractable, I employ a three-period version of the model used by Angeletos (2003). Although the model is simple and ad hoc, it captures the elements standard to the literature. For example, Benigno

[4] Natural debt limit is defined as "the maximum debt that could be paid almost surely under optimal tax policy" (Aiyagari et al., 2002, p. 1225).

and Woodford (2003) drive a similar model from first principles. The log-linearized version of the model is essentially of the form standard to the monetary policy literature; however, all the parameters in reduced-form equations (including those in the loss function) are driven from structural parameters. This in a way justifies (and provides the theoretical foundation for) the use of the linear-quadratic approach that has been traditionally employed by the monetary policy literature.

This is a three-period economy, where the initial and final periods are deterministic, and the second period is stochastic. In this economy, the government maximizes the social welfare, which is equivalent to minimizing the following loss function under a certain set of assumptions (for details of the equivalence of utility maximization and loss minimization, see Woodford, 2003, Chapter 6; and Benigno and Woodford, 2003).

$$\Lambda = \frac{1}{2}\sum_{t=1}^{3}\beta^{t} E_t \left(y_t - y_t^{*} \right)^2 + \omega \pi_t^2 \ . \tag{1}$$

The first term represents the utility loss stemming from the deviation of output from the efficient level. The second term represents the utility loss originating from sticky prices; ω is the weight given to the utility loss associated with inflation; β is the discount factor.

In this economy prices are sticky; in particular, a fraction of the prices is fixed in advance for a period, and the remaining fraction is flexible. This allows one to use the following type of neoclassical Phillips curve.

$$y_t = -\psi \tau_t + \chi \left(\pi_t - \beta E_{t-1} \pi_t \right), \tag{2}$$

where $\psi > 0$ and $\chi \geq 0$. As χ approaches zero, the economy gets closer to flexible price setting. Conversely, as χ approaches infinity, the degree of price stickiness increases. Because prices are sticky, monetary policy has real effects. In addition, sales revenue taxes, represented by τ, create distortion in the economy, which is captured by ψ.

The final equation that summarizes the economy is the government's budget constraint, which is also standard. Assume that the government issues one-period nominal and real bonds, say N_t and B_t, respectively (both as share of GDP). Then the government budget is as follows

$$\left(B_{t-1} + \frac{N_{t-1}}{P_t} \right) + g_t = \tau_t + \left(\frac{1}{1+r_t} B_t + \frac{1}{1+R_t} \frac{N_t}{P_t} \right). \tag{3}$$

While the left-hand side represents the government's outstanding real liabilities at the beginning of the period, plus its expenditures during the period, the right-hand side stands for the government's financing options: distortionary taxes and the new issue of bonds. From consumption Euler equations of the underlying model; we can approximate the real and nominal interest rates by, $(1+r_t) \approx \beta^{-1}$ and $(1+R_t) \approx \beta^{-1}(1-E_t\pi_{t+1})^{-1}$, and also let $1/P_t \approx (1-\pi_t)/P_{t-1}$. So far all the assumptions and approximations are standard. However the next assumption is specific to Angeletos (2003), and simplifies the algebraic solutions significantly. The government is assumed to keep the nominal bonds as a constant share of GDP, but to adjust the level of real bonds without restraint. One can, then, treat the nominal bonds as a parameter, say $\bar{d} = N_t/P_t = N_{t-1}/P_{t-1}$, allowing inflation to enter in the budget constraint in a nonmultiplicative way. Finally, the seignorage revenues are implicitly assumed to be distributed back to the public through lump-sum transfers, so they drop out of the budget. Then, the total level of debt as a share of GDP evolves according to

$$b_t = \beta^{-1}b_{t-1} - \beta^{-1}\left[\tau_t + \bar{d}\left(\pi_t - \beta E_{t-1}\pi_t\right) - g_t\right], \tag{4}$$

where $b_t = B_t + \dfrac{N_t}{P_t}$. All terms are as a share of GDP. The second term in the parentheses captures the impact of unexpected inflation on nominal debt, and g is the level of government expenditures, which is the only stochastic element in the model.[5] The government starts the first period with an initial level of debt, say b_0.

The final step to complete the structure of the economy is to designate the threshold above which the incentives for default dominate the incentives for rolling the debt over and, therefore, beyond which the rational lenders refuse to lend to the government.[6] As mentioned earlier, this threshold is different for developing and industrial countries – much lower for the former-- and varies from country to country. This paper does not model the threshold explicitly, since the question asked is not how this point is determined, but rather given that such a point exists, what the implications on optimal monetary and fiscal policies are. For that purpose, what matters for this paper is that the government loses its ability to borrow after that point, say $\bar{b}$, and therefore has less flexibility in responding to shocks. Consequently, as long as debt is lower than this threshold, $b_t < \bar{b}$, the level of debt is determined by equation (4). When the debt level reaches the threshold, it is not a control variable for the government anymore, but instead is simply given by $\bar{b}$.

[5] This is a common way of introducing a fiscal shock to the model in the literature.

[6] A similar situation would be a sudden reduction in the risk tolerance of investors, in which case the country finds itself in a situation where it cannot borrow anymore. This requires a different modeling approach, but in both cases the mechanism would be similar: the country, being unable to borrow, has to roll over the existing debt with distortionary sources, which is costly to the economy.

Given the structure of the economy, the next section analyzes optimal monetary and fiscal policies. First, I focus on the standard case where the government is never subject to the borrowing constraint. Then I introduce the borrowing constraint and focus on the precautionary motive.

III. The Policy Problem

This paper focuses on discretionary policy rather than on precommitment; therefore, the government reoptimizes each period instead of optimizing once at the initial period. In a discretionary environment, the government takes expectations as given, $E_{t-1}\pi_t = \pi_t^e$, which are formed rationally. This policy formulation is more suitable to developing countries where the credibility of monetary and fiscal policies is hard to achieve. The recent literature on optimal monetary and fiscal policies, on the other hand, focuses mainly on credible precommitment (Benigno and Woodford, 2003; Schmitt-Grohé and Uribe, 2004; Siu, 2004).

A. The Standard Case

In order to understand the effect of the introduction of a borrowing constraint better, this section solves the unconstrained problem, where the government can borrow as much as it wants to smooth over the effects of the shock. For simplicity, I will assume that the model is deterministic for the first and third periods, during which the level of government expenditure is constant at $\bar{g}$. Although the economy is hit by a shock during the second period, it is not subject to the borrowing constraint. Therefore, the results obtained in this section are standard to the literature in the sense that the effects of the shocks are smoothed over through time.[7] The problem will be solved backward, so the following subsection focuses on the last period.

The last period

During the last period, the government has to pay back everything borrowed in the previous periods. Therefore $b_3 = 0$. Because this period is deterministic, government expenditure is constant and is normalized to zero to save space. The budget constraint then reduces to

$$b_2 = \tau_3 + \bar{d}\left(\pi_3 - \beta\pi_3^e\right). \tag{5}$$

One can represent taxes in terms of inflation and output by using aggregate supply (AS). Therefore, when the government decides on the optimal choice of inflation and output, the following relation determines taxes endogenously.

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[7] The optimal policy mix can change depending on assumptions regarding market structure and nominal rigidities.

$$\tau_3 = \frac{\chi}{\psi}\left(\pi_3 - \beta\pi_3^e\right) - \frac{1}{\psi}y_3 . \tag{6}$$

Using AS to substitute out taxes from the budget constraint and solving for inflation obtains

$$\pi_3 = \beta\pi_3^e + \frac{1}{\left(\frac{\chi}{\psi}+\bar{d}\right)}b_2 + \frac{1}{\psi\left(\frac{\chi}{\psi}+\bar{d}\right)}y_3 . \tag{7}$$

The relevant period's loss function is given simply by,

$$\Lambda_3 = \frac{1}{2}\left[\left(y_3 - y^*\right)^2 + \omega\pi_3^2\right]. \tag{8}$$

The policy problem is then minimizing the period loss function, equation (8), with respect to equation (7) by choosing inflation and output. Given the optimal combination of these two, the optimal tax level is determined endogenously through AS relation, equation (6). The first-order condition to this problem is

$$y_3 - y^* = -\frac{\omega}{\psi\left(\frac{\chi}{\psi}+\bar{d}\right)}\pi_3 . \qquad (9)[8]$$

The implications of this first-order condition are standard for the literature. As the importance given to inflation stabilization increases, the implied inflation declines. Similarly, as the nominal component of the debt stock increases, the benefits from unexpected inflation also increase. Finally, as the degree of price stickiness increases (higher χ, implying flatter AS), the real effects of demand policies are also magnified.

Using the first-order condition (9) in budget constraint (7) and imposing rational expectations give the optimal level of inflation;

$$\pi_3 = \frac{\psi^2\left(\frac{\chi}{\psi}+\bar{d}\right)}{\left(1-\beta\right)\psi^2\left(\frac{\chi}{\psi}+\bar{d}\right)^2 + \omega}\left[b_2 + \frac{1}{\psi}y^*\right]. \tag{10}$$

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[8] A more standard representation for the monetary policy is $\pi_3 = \frac{\psi\left(\frac{\chi}{\psi}+\bar{d}\right)}{\omega}y^* - y_3$

Accordingly, the higher the debt inherited from the second period is, the higher is the inflation in this period. In order to save space the term $\psi\left(\frac{\chi}{\psi}+\bar{d}\right)$ will be represented by 'a' hereafter. Given first-order condition, equation (9), and implied optimal level of inflation, equation (10), the optimal tax rate is determined through AS relation:

$$\tau_3 = \frac{(1-\beta)\chi a+\omega}{(1-\beta)a^2+\omega}b_2 - \left(\frac{(1-\beta)\bar{d}\psi a}{(1-\beta)a^2+\omega}\right)y^*. \tag{11}$$

The intuition is again simple: a higher debt stock necessitates a higher tax rate to pay everything back. On the other hand, as the distortion created by taxes increases, the optimal tax rate declines.[9]

The second period

The second period is more complicated than the third period. Not only does the government need to decide on the optimal level of borrowing, but this period is also stochastic. In this period, the government takes into account both the period loss function and the expected third-period loss:

$$\Lambda_2 = \frac{1}{2}\sum_{t=2}^{3}\beta^t E_t\left[\left(y_t - y^*\right)^2 + \omega\pi_t^2\right]. \tag{12}$$

When one uses AS to substitute out the tax rate from the budget and solve for inflation, one obtains the budget constraint for the second period:

$$\pi_2 = \beta\pi_2^e + \frac{\psi}{a}b_1 - \frac{\psi\beta}{a}b_2 + \frac{1}{a}y_2 + \frac{\psi}{a}g_2. \tag{13}$$

Therefore, the budget in the second period is different from the budget in the last period in two ways. First, the government has the option to borrow in this period, so b_2 does not drop out of the budget constraint automatically. Notice that it enters negatively, implying that borrowing allows the government to deliver less inflation in a sense that now it does not have to finance the entire inherited debt and plus the effect of the shock through inflation and taxation.

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[9] The distortionary effect of taxes, ψ, affects the term, a, positively. As it gets larger, so is the term, a, raising the denominator more than the numerator.

Second, government expenditure is stochastic and assumed to take the form of a two-point mean-zero risk, say [$-\mu,\mu$]. This section focuses on relatively uninteresting situation that adverse realization of μ is not large enough to take the level of debt to the upper bound – that is, the support of the shock lies in the unconstrained part of the debt stock. Because the government does not lose its ability to borrow in the case of an adverse shock, the budget constraint is given by equation (13) for both realizations of the world. Therefore, b_2 is always a control variable in the second period.

The government minimizes the loss function (12) subject to the budget constraint (13) given inflation expectations by choosing inflation and output. The first-order condition is

$$y_2 - y^* = -\frac{\omega}{a}\pi_2, \tag{14}$$

which provides the optimal allocation of inflation and output.

The first-order condition obtained from the optimal choice of debt, b_2, is

$$\pi_2 = E\pi_3, \tag{15}$$

which implies that the government chooses a debt level such that the expected marginal loss from inflation – and implicitly from taxation – is equalized across periods.

Focusing on the "good" realization of the shock, [10]by using first-order condition (14) in the budget constraint to substitute out output, and using optimal inflation for the third period to substitute out b_2, and finally using the first-order condition (15) to substitute out expected inflation for the third period, one obtains the inflation for the second period (for details, see Appendix I)

$$\begin{aligned}\pi_2^G = &\frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2+2\omega}b_1 + \frac{\beta a^2}{\left[1+\beta(1-\beta)\right]a^2+2\omega}\pi_2^e \\ &+ \frac{(1+\beta)a}{\left[1+\beta(1-\beta)\right]a^2+2\omega}y^* - \frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2+2\omega}\mu,\end{aligned} \tag{16}$$

where π_2^G refers to the inflation rate in the good realization of the world. Accordingly, when the government is not constrained, inflation will be a function of the inherited debt, inflation expectations, and the state of government expenditures.

[10] Good realization of the shock refers to a decline in government expenditures and bad realization refers to an increase.

The problem is symmetric for the "bad" realization of the world; therefore, the implied optimal inflation rate is given by

$$\pi_2^B = \frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2+2\omega} b_1 + \frac{\beta a^2}{\left[1+\beta(1-\beta)\right]a^2+2\omega} \pi_2^e + \frac{(1+\beta)a}{\left[1+\beta(1-\beta)\right]a^2+2\omega} y^* + \frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2+2\omega} \mu. \tag{17}$$

Similarly, π_2^B is used to represent the inflation rate in the case of an adverse shock.

Agents form their expectations rationally, so they also know the optimal responses for the possible realizations and shape their expectations accordingly. Assuming that there is an equal probability that one of the cases will occur, the expected inflation is merely the average of the two possible outcomes.

$$\pi_2^e = \frac{1}{2}\pi_2^G + \frac{1}{2}\pi_2^B . \tag{18}$$

By simply substituting the respective outcomes, and solving for the expected inflation, one obtains

$$\pi_2^e = \frac{\psi a}{\left(1-\beta^2\right)a^2+2\omega} b_1 + \frac{\left(1+\beta\right)a}{\left(1-\beta^2\right)a^2+2\omega} y^* . \tag{19}$$

As expected, both the upper bound on debt and the size of the shock are irrelevant for the expectations of the private agents in the unconstrained economy. Expectations are a function of the inherited debt stock.

Then, given inflation expectations, one can solve for the inflation level for both good and bad realizations.

$$\pi_2^G = \vartheta_b b_1 + \vartheta_{y^*} y^* - \vartheta_\eta \mu , \tag{20}$$

and

$$\pi_2^B = \vartheta_b b_1 + \vartheta_{y^*} y^* + \vartheta_\eta \mu . \tag{21}$$

The coefficients are defined in Appendix II. Given optimal inflation and output, the optimal tax rate can be derived from the AS relation endogenously. Accordingly, the level of inflation and the tax rate in this period are functions of the beginning-of-period debt level and the shock. Once again, in an unconstrained economy the upper bound on debt is irrelevant for inflation. In response to a bad shock, inflation -- and the tax rate-- increases, but part of the shock is transferred to the next period through borrowing.

The third period

During the first period, which is deterministic, the government minimizes the social loss function in a similar fashion to the second and the third periods,

$$\Lambda_1 = \frac{1}{2}\sum_{t=1}^{3}\beta^t E_t\left[\left(y_t - y^*\right)^2 + \omega\pi_t^2\right], \tag{22}$$

subject to its budget constraint and AS of the economy,

$$\pi_1 = \beta\pi_1^e + \frac{\psi}{a}b_0 - \frac{\psi\beta}{a}b_1 + \frac{1}{a}y_1, \tag{23}$$

where the government inherits a certain amount of debt stock, b_0. The necessary first-order conditions are again

$$y_1 - y^* = -\frac{\omega}{a}\pi_1 \tag{24}$$

and

$$\pi_1 = E_1\pi_2, \tag{25}$$

where the former determines optimal allocation of inflation and output – and through AS the tax rate – the latter is the product of the optimal borrowing decision. Note that equation (25) also implies $\tau_1 = E\tau_2$. When the government borrows less, the resulting marginal loss today – due to higher inflation and taxation to roll over the inherited debt – is the left-hand side of equation (25). On the other hand, the marginal benefit of leaving less debt for tomorrow is the right-hand side of equation (25) – less taxation and inflation to roll over the debt in the later period. As shown earlier, inflation expectations for period 2, given by equation (19), are not affected by the size of the shock. In such a case, under our assumptions, the expected marginal benefit of leaving less debt to the second period is not affected by the shock, because even though the second period is stochastic, the government is not subject to a borrowing constraint.

Because the first period is deterministic, rational expectations imply that $\pi_1 = E_0\pi_1$. Given rational expectations, the expected inflation for the second period, equation (19), and the first-order condition (25) one obtains the first-period inflation (the intermediate steps are presented in Appendix III).

$$\pi_1 = \frac{\psi a}{\left(1-\beta^3\right)a^2 + \left(1+2\beta\right)\omega} b_0 + \frac{a\left(1+\beta\left(1+\beta\right)\right)}{\left(1-\beta^3\right)a^2 + \left(1+2\beta\right)\omega} y^* . \tag{26}$$

Inflation in the first period is only a function of initial debt and the efficient level of output. The upper bound on debt and the level shock are irrelevant in this period, because they do not affect the expected inflation (and tax rate) in the second period. As a result, the expected marginal loss for the second period is also unaffected by these variables.

This section has presented the optimal monetary and fiscal policy responses of a stochastic economy that has full access to financial markets. The next section analyzes the precautionary motive by introducing a limit on borrowing.

B. Precautionary Motive

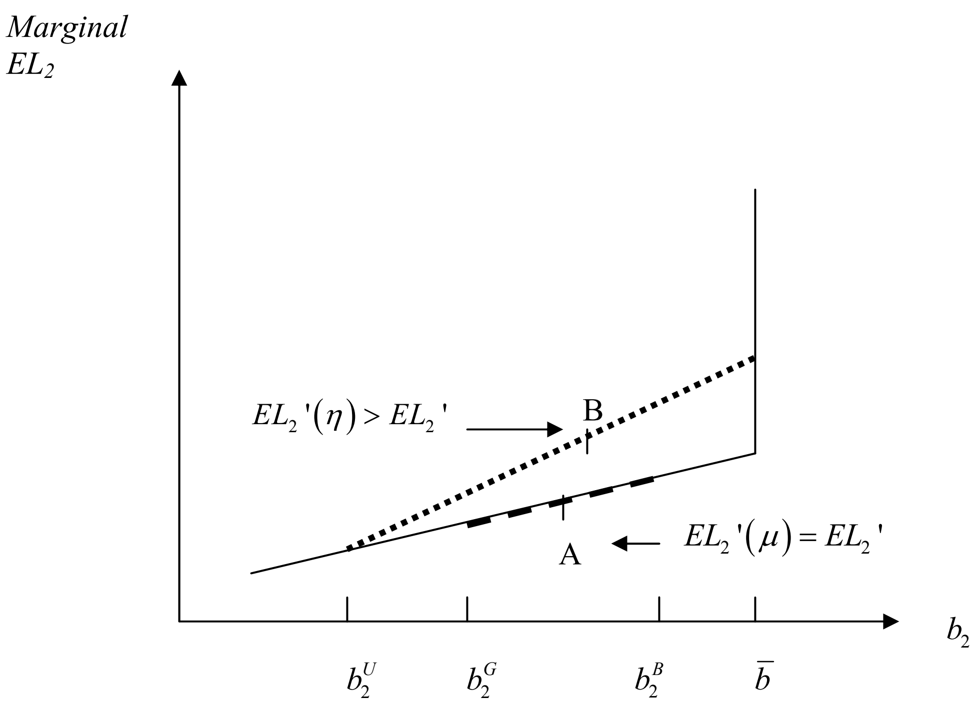
In this section, the government expenditures are not only stochastic during the second period, but also can push the level of debt beyond the margin. In such a case, the level of debt is given by the limit, $\bar{b}$, and cannot be chosen optimally. Whereas in the absence of the constraint, the government would like to smooth over the effects of a shock through time, in the presence of such an upper bound on borrowing, it does not have this option. The only choice of a constrained government is then to adjust inflation, output, and the tax level optimally to alleviate the effects of the shock.

The main argument of this paper is that the possibility of a shock pushing the economy to the limit in the second period will alter the behavior of the government in the first period, even if the economy is unconstrained in the first period. Such a possibility in future periods results in higher expected welfare loss for a constrained government than for an unconstrained one. Consequently, just as the existence of liquidity constraints induces precautionary saving, the existence of borrowing constraints will force the government to be more prudent in its debt management, implying, in this setup, relatively loose monetary policy –higher inflation – together with tight fiscal policy – higher taxation. As was mentioned above, precautionary motive arises from the need to make adjustments less painful when they bind.

In an analogy to the precautionary saving decision (Caroll and Kimball, 2001), the main idea can be summarized as in Figure 1. Because our loss function is quadratic, the marginal loss function takes the linear form. Debt cannot increase further after the upper bound, $\bar{b}$. The marginal loss depends on how much inflation and taxation are necessary to meet the budget constraint. When the size of the shock is μ, even with the adverse realization of the shock, we are still below the threshold. In such a case, the expected marginal loss for the second period is unaffected by the existence of the shock. In effect, the expected marginal loss for the second period is at point A regardless of the existence of the shock. This follows from the fact that expected inflation in the second period is unaltered by the shock.

When the shock gets larger, say η, the adverse realization of the shock takes the debt to the threshold. Any additional effects of the shock have to be matched by increased inflation and taxation. In such a case, the expected marginal loss for the second period is at point B, which is larger than the unconstrained situation. Since the expected marginal loss is larger, the expected marginal benefit of reducing the debt for this period is greater than in the unconstrained case. For a given initial debt, the inflation and tax rate implied by the unconstrained problem no longer satisfy the first-order condition for the constrained problem. As a consequence, the government increases tax and inflation rates in the first period more than those implied by the unconstrained problem. In other words, the government engages in precautionary debt reduction.

Figure 1. Expected Marginal Loss for the Second Period



The last period

The problem will be solved backward as in the first section. The third period's policy problem is the same. The government has to pay back everything that it borrowed in earlier periods. Therefore, given the debt level inherited from the second period, the inflation and tax level will be given by equations (10) and (11), respectively.

The second period

During the second period, it still is assumed that government expenditure is stochastic and takes the form of a two-point mean-zero risk $[-\eta, \eta]$. This time, however, η is large enough to carry the debt stock to the threshold, so the support of the shock interacts with the region of debt where the borrowing constraint is binding. Then, the policy response in the second period will depend on the realization of the shock. What is more important is that the marginal benefit of borrowing less in the first period will be higher as well. After solving the policy problem for the second period, we will come back to this point when we discuss the policy implications of the borrowing constraint and shock in the first period.

This section focuses on the case where η is large enough to make the borrowing constraint binding. The adverse realization of the shock, which prevents the government from borrowing, will be referred to as the constrained case hereafter. Conversely, the case where the government can freely borrow – favorable realization of the shock – will be referred to as the unconstrained case.

Let us concentrate first on the constrained case. This case is more like the policy problem in the third period, since the government does not have the freedom to decide the level of debt optimally; it is simply determined by $\bar{b}$. Indeed, under the special case of $\bar{b}=0$, the solutions to both periods are exactly the same. Then, total debt is the modified version of equation (7), with an additional term for the end-of-period debt stock, $b_2=\bar{b}$.

$$\pi_2 = \beta\pi_2^e + \frac{\psi}{a}\left(b_1 - \beta\bar{b}\right) + \frac{1}{a}y_2 + \frac{\psi}{a}\eta \, . \tag{27}$$

The minimization of the loss function (12) subject to the budget constraint (27) produces the following first-order condition.

$$y_2 - y^* = -\frac{\omega}{a}\pi_2 \, . \tag{28}$$

For a given level of debt, the optimal allocation of inflation and output (and taxes) is still the same.

Using this first-order condition in equation (27), one can obtain inflation in the second period as a function of inflation expectations, inherited debt from the first period, the debt level today, and the shock:

$$\pi_2^C = \frac{\psi a}{a^2+\omega}\left(b_1 - \beta\bar{b}\right) + \frac{\beta a^2}{a^2+\omega}\pi_2^e + \frac{a}{a^2+\omega}y^* + \frac{\psi a}{a^2+\omega}\eta \, , \tag{29}$$

where π_2^C is used to denote constrained outcome for the second period.

When, on the other hand, the realization of government expenditure is $-\eta$, the government is not constrained, and it can decide on how much to borrow optimally. Therefore, the unconstrained budget constraint takes the form,

$$\pi_2 = \beta\pi_2^e + \frac{\psi}{a}\left(b_1 - \beta b_2\right) + \frac{1}{a}y_2 - \frac{\psi}{a}\eta \,. \tag{30}$$

The rest of the problem is the same as the standard case for the second period. Thus, optimal inflation with the good realization of the shock is given by

$$\begin{aligned}\pi_2^U =& \frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2 + 2\omega} b_1 + \frac{\beta a^2}{\left[1+\beta(1-\beta)\right]a^2 + 2\omega}\pi_2^e \\ &+ \frac{(1+\beta)a}{\left[1+\beta(1-\beta)\right]a^2 + 2\omega} y^* - \frac{\psi a}{\left[1+\beta(1-\beta)\right]a^2 + 2\omega}\eta,\end{aligned} \tag{31}$$

where π_2^U represents the unconstrained outcome in the second period. Accordingly, when the government is not constrained, inflation will be a function of the inherited debt, inflation expectations, and the state of government expenditures. Note that for a given level of expected inflation, inflation with the constrained case is higher than inflation with the unconstrained case. The difference is higher for large realizations of the shock, and lower in the case where the constraint becomes binding at a relatively high debt-to-GDP ratio.

To solve for inflation under both cases, one needs to determine expected inflation for the second period. Given that there is an equal probability that favorable and adverse shocks hit, expected inflation is simply the weighted average of the two outcomes:

$$\pi_2^e = \frac{1}{2}\pi_2^U + \frac{1}{2}\pi_2^C \,. \tag{32}$$

Plugging in the corresponding values and solving for expected inflation obtains:

$$\pi_2^e = \varphi_b b_1 - \varphi_{\bar{b}}\bar{b} + \varphi_{y^*}y^* + \varphi_\eta \eta \,. \tag{33}$$

All coefficients are smaller than 1 and are defined at Appendix IV.

Accordingly, inflation expectations for the second period will be higher, the higher are the inherited debt stock and the size of the shock. Conversely, expected inflation will be lower, the higher is the upper bound on debt.

Given inflation expectations, the actual inflation rates delivered in constrained and unconstrained cases are as follows.

$$\begin{aligned}\pi_2^C =& \frac{a\psi}{a^2+\omega}\left(1+\beta a\varphi_b\right)b_1 - \frac{\beta a\psi}{a^2+\omega}\left(1+a\varphi_{\bar{b}}\right)\bar{b} \\ &+ \frac{a}{a^2+\omega}\left(1+\beta a\varphi_{y^*}\right)y^* + \frac{a\psi}{a^2+\omega}\left(1+\beta a\varphi_\eta\right)\eta\end{aligned} \tag{34}$$

and

$$
\begin{aligned}
\pi_2^U = & \frac{a\psi}{\left(1+\beta(1-\beta)\right)a^2+\omega}\left(1+\beta a\varphi_b\right)b_1 - \frac{\beta a\psi}{\left(1+\beta(1-\beta)\right)a^2+\omega}\left(a\varphi_{\bar{b}}\right)\bar{b} \\
& + \frac{a}{\left(1+\beta(1-\beta)\right)a^2+\omega}\left(1+\beta+\beta a\varphi_{y^*}\right)y^* - \frac{a\psi}{\left(1+\beta(1-\beta)\right)a^2+\omega}\left(1-\beta a\varphi_\eta\right)\eta
\end{aligned} \tag{35}
$$

Once again, given optimal inflation and output, the implied optimal tax rate can be derived from the AS relation endogenously.

In short, when the bad shock hits, the government cannot spread the effects of the shock through time due to its borrowing constraint. Consequently, the portion of the increase in expenditures that cannot be financed through borrowing is financed through higher tax and inflation rates. Seeing this in advance, private agents form their expectations by taking into account the probability of the bad shock. In this simple setup, then, expectations are a weighted average of good and bad outcomes. Therefore, as long as the probability of an adverse shock is positive, inflation expectations are higher than in the standard case.

Now, we will move to the first period in which the possibility that the constraint may be binding in the future has an effect on the current policy choices of the government even if the constraint is not binding at the moment.

The third period

In this period, the government minimizes the loss function given by equation (22), subject to its budget constraint (23). The first-order conditions are the same as for the standard problem. The government decides on the optimal allocation between inflation and output (and also taxes) through the first-order condition (24). Similarly, the optimal level of borrowing is determined through the first-order condition (25). What is different in this case is expected inflation for the second period, which is higher than in the previous section. Therefore, the marginal benefit of less debt in the future is higher for a constrained government than for an unconstrained one. As it was shown in equation (32), expected inflation for the second period is higher than the unconstrained inflation expectations. Thus, marginal benefit from less inflation (and less taxes) tomorrow is bigger. In other words, even if there is no shock and the government is not constrained in the first period, the possibility of hitting the borrowing limit in future periods has an effect on the current inflation and tax rates. In particular, in order to avoid large losses in the second period, where the government cannot smooth over, there is an incentive for the government to raise taxes and inflation today more than what would be implied otherwise, leaving less debt to the second period and thereby shifting some of the adjustment necessary in such a case to the first period. In sum, the presence of borrowing constraints, even when they are not binding, causes the government to be more prudent, by borrowing less and raising the taxes and inflation more in the first period.

By following the same steps as in the standard problem, we obtain the inflation rate for the first period as (the intermediate steps are provided at Appendix V):

$$
\begin{aligned}
\pi_1 = {} & \frac{\varphi_b}{\beta\left((1-\beta)a^2+\omega\right)} b_0 - \frac{\left(2+\beta(1-\beta)\right)a^2+3\omega}{(1-\beta)a^2+\omega}\varphi_{\bar{b}}\bar{b} \\
& + \frac{\left(2+\beta(1-\beta)\right)a^2+3\omega}{(1-\beta)a^2+\omega}\varphi_\eta \eta + \frac{\psi\varphi_{y^*}+\varphi_b}{(1-\beta)a^2+\omega} y^*.
\end{aligned}
\tag{36}
$$

As expected, inflation in the first period is higher, the higher is the initial level of debt. In addition, as the debt limit increases, there is more room for the government to mitigate the effects of the shock by borrowing in the second period, and therefore the implied inflation in the first period to smooth over the effect of the shock gets smaller. Naturally, as the shock itself gets harsher, for a given debt limit, its effect on inflation and tax rates in both periods is amplified by the necessity to cover the portion that cannot be borrowed.

Because the first period is deterministic, implying that inflation in this period is equal to the expected inflation, most of the effort in reducing the debt burden for the next period comes from the fiscal policy side – namely, from higher taxation. The government will deliver a higher tax rate in the first period as a precaution against the shock in the second period. To see this, let us use equation (36) and first-order equation (28) in the AS equation. The optimal tax rate in the first period is then given by

$$
\begin{aligned}
\tau_1 = {} & \left(\frac{(1-\beta)\chi a+\omega}{\psi}\right)\Bigg[\frac{\varphi_b}{\beta\left((1-\beta)a^2+\omega\right)} b_0 - \frac{\left(1+\beta(1-\beta)\right)a^2+3\omega}{(1-\beta)a^2+\omega}\varphi_{\bar{b}}\bar{b} \\
& + \frac{\left(1+\beta(1-\beta)\right)a^2+3\omega}{(1-\beta)a^2+\omega}\varphi_\eta + \frac{\psi\varphi_{y^*}+\varphi_b}{(1-\beta)a^2+\omega} y^*\Bigg] - \frac{1}{\psi} y^*,
\end{aligned}
\tag{37}
$$

where the coefficients are the multiple of the ones in the inflation equation. Accordingly, higher initial debt and a more severe shock also imply higher taxation for the first period.

To put it briefly, the existence of a possibility of a future adverse shock that may constrain the economy's ability to borrow will cause precautionary government saving in earlier periods. While for fiscal policy this means higher taxation, it implies relatively loose monetary policy because it allows higher inflation than otherwise.

IV. Conclusions

This paper shows that the possibility of losing the ability to borrow in the future induces precautionary debt reduction. When the government loses its ability to borrow, shocks are more costly to the economy, since they cannot be spread over time. In this setup, the effects of the shocks have to be matched by increased inflation and taxation, which in turn raise the welfare costs for the relevant period. The precautionary motive arises from the need to make these adjustments less painful when the borrowing constraints bind. In order to avoid large losses in the constrained period, the government prefers to raise taxes and inflation in earlier periods more than those implied otherwise, leaving less debt to the future periods, thereby shifting some of the adjustment necessary to the earlier periods. In other words, the coexistence of large shocks and borrowing constraints forces the government to be more prudent by reducing or limiting the increase in debt.

The extent of the precautionary motive depends on numerous factors. First, if the size of shocks that the economy faces is relatively small, then the precautionary motive disappears. Second, if the borrowing constraint is relatively relaxed – i.e. it starts to bind at higher debt to GDP ratios – there is less need for precautionary debt reduction. Finally, for highly indebted economies, there is less room for precautionary policies, since their initial debt-to-GDP ratio is already close to or at the point where the constraint is binding.

This model also implies that bad shocks tend to be more inflationary and produce tighter fiscal policy responses in countries that are subject to borrowing constraints. These implications are in line with the findings that inflation and its volatility are higher in emerging market economies (Calvo and Guidotti, 1993; and Fraga et al, 2003) and that, among inflation targeters, the average fiscal balance of emerging market economies is higher than that of advanced economies during the three-year period prior to the switch to the inflation targeting regime (Schaechter et al, 2000).

Appendix I. Derivation of Inflation for the Second Period in the Standard Case

Using first-order condition (14) in budget constraint, one obtains

$$\pi_2^G = \frac{\psi a}{a^2+\omega}\left(b_1 - \beta b_2\right) + \frac{\beta a^2}{a^2+\omega}\pi_2^e + \frac{a}{a^2+\omega}y^* - \frac{\psi a}{a^2+\omega}\mu.$$

By the help of optimal inflation in the third period, one can get rid of b_2:

$$\pi_2^G = \frac{\psi a}{a^2+\omega}b_1 - \frac{\beta(1-\beta)a^2+\omega}{a^2+\omega}E_2\pi_3 + \frac{\beta a^2}{\psi a+\omega}\pi_2^e + \frac{(1+\beta)a}{\psi a+\omega}y^* - \frac{\psi a}{\psi a+\omega}\mu\ .$$

Finally, using first-order condition (15) to substitute out expected inflation for the third period obtains equation (16) in the main text.

Appendix II. The Coefficients of Inflation for the Second Period in the Standard Case

$$\vartheta_b = \psi a \left(\frac{\left(1+\beta\left(1-\beta\right)\right)a^2 + 2\omega}{\left[\left(1+\beta\left(1-\beta\right)\right)a^2 + 2\omega\right]\left[\left(1-\beta^2\right)a^2 + 2\omega\right]} \right)$$

$$\vartheta_{y^*} = \left(1+\beta\right) a \left(\frac{\left(1+\beta\left(1-\beta\right)\right)a^2 + 2\omega}{\left[\left(1+\beta\left(1-\beta\right)\right)a^2 + 2\omega\right]\left[\left(1-\beta^2\right)a^2 + 2\omega\right]} \right)$$

$$\vartheta_\eta = \frac{\psi a}{\left[\left(1+\beta\left(1-\beta\right)\right)a^2 + 2\omega\right]}.$$

Appendix III. Derivation of Inflation for the First Period in the Standard Case

Using rational expectations, the first-period inflation can be written as

$$\pi_1 = \frac{\psi a}{(1-\beta)a^2+\omega}(b_0-\beta b_1)+\frac{a}{(1-\beta)a^2+\omega}y^*.$$

Using the solution for expected inflation for the second period, equation (19), one can solve for the end-of-period debt level:

$$b_1 = \frac{(1-\beta^2)a^2+2\omega}{\psi a}E_1\pi_2-\frac{1+\beta}{\psi}y^*.$$

When one substitutes this in first-period inflation equation, and uses the first-order condition (25) to substitute out the expected second-period inflation, and finally solves for the first-period inflation, one obtains equation (26) in the main text.

Appendix IV. The Coefficients of Expected Inflation for the Second Period in Section B

$$\varphi_b = \frac{0.5\psi a\left[\left(2+\beta(1-\beta)\right)a^2+3\omega\right]}{\left[\left(1+\beta(1-\beta)\right)a^2+2\omega\right]\left[a^2+\omega\right]-0.5\beta a^2\left[\left(2+\beta(1-\beta)\right)a^2+3\omega\right]}$$

$$\varphi_{\bar{b}} = \frac{0.5\beta\psi a\left[\left(1+\beta(1-\beta)\right)a^2+2\omega\right]}{\left[\left(1+\beta(1-\beta)\right)a^2+2\omega\right]\left[a^2+\omega\right]-0.5\beta a^2\left[\left(2+\beta(1-\beta)\right)a^2+3\omega\right]}$$

$$\varphi_{y^*} = \frac{0.5a\left[\left(2+\beta(2-\beta)\right)a^2+\left(3+\beta\right)\omega\right]}{\left[\left(1+\beta(1-\beta)\right)a^2+2\omega\right]\left[a^2+\omega\right]-0.5\beta a^2\left[\left(2+\beta(1-\beta)\right)a^2+3\omega\right]}$$

$$\varphi_\eta = \frac{0.5\psi a\left[\beta(1-\beta)a^2+\omega\right]}{\left[\left(1+\beta(1-\beta)\right)a^2+2\omega\right]\left[a^2+\omega\right]-0.5\beta a^2\left[\left(2+\beta(1-\beta)\right)a^2+3\omega\right]}.$$

Appendix V. Derivation of Inflation for the First Period in the Precautionary Case

Using first-order condition (24) in budget constraint, equation (23), one obtains

$$\pi_1 = \frac{\psi a}{a^2 + \omega}\left(b_0 - \beta b_1\right) + \frac{\beta a^2}{a^2 + \omega}\pi_1^e + \frac{a}{a^2 + \omega}y^* .$$

Because the first period is deterministic, rational expectations imply that $\pi_1 = E_0\pi_1$. Then, inflation can be written as

$$\pi_1 = \frac{\psi a}{\left(1-\beta\right)a^2 + \omega}\left(b_0 - \beta b_1\right) + \frac{a}{\left(1-\beta\right)a^2 + \omega}y^* .$$

One still needs to substitute out the debt level in order to solve for inflation. Given the solution for the expected inflation in the second period equation (33), the implied b_1 is

$$b_1 = \frac{1}{\varphi_b}\pi_2^e + \frac{\varphi_{\bar{b}}}{\varphi_b} - \frac{\varphi_{y^*}}{\varphi_b}y^* - \frac{\varphi_\eta}{\varphi_b}\eta .$$

Using this to substitute out b_1 and first-order condition (25) to replace expected second-period inflation, one finally obtains inflation in the first period as shown in equation (36) in the main text.

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