

IMF Working Paper

Constructing Forecast Confidence Bands During the Financial Crisis

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Research

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Abstract

This Working Paper should not be reported as representing the views of the IMF.

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

We derive forecast confidence bands using a Global Projection Model covering the United States, the euro area, and Japan. In the model, the price of oil is a stochastic process, interest rates have a zero floor, and bank lending tightening affects the United States. To calculate confidence intervals that respect the zero interest rate floor, we employ Latin hypercube sampling. Derived confidence bands suggest non-negligible risks that U.S. interest rates might stay near zero for an extended period, and that severe credit conditions might persist.

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I. INTRODUCTION

This paper describes the construction of confidence intervals for an illustrative economic forecast deriving from the Global Projection Model (GPM) project. This family of models has a standard macroeconomic basis, focusing on inflation, output and/or the unemployment rate, a short-term interest rate, and the exchange rate. This facilitates their practical use for forecasting and monetary policy simulations. Various central banks, as well as the IMF, employ them.

The paper focuses on results for the United States, for which the model includes an empirically important bank lending tightening effect. The model also covers the euro area, and Japan.² Within it, monetary policy provides a nominal anchor, by targeting a stable, low rate of inflation, to which end it uses a short-term interest rate instrument. The model incorporates an endogenous world oil price (in inflation-adjusted US dollars), which allows us to trace out the effects on inflation and output of shocks to the level and growth rate of oil prices.

The present model includes several key nonlinearities:

- zero bound to interest rates
- amplified bank lending tightening effect beyond a specified threshold
- asymmetric output gap effect on inflation—excess demand raises inflation by more than excess supply reduces it
- adjustment mechanism to avoid implosive disinflation in the event of very large negative output gaps

These nonlinearities result in asymmetric model-derived confidence bands, especially in situations that are far from long-run equilibrium.

We use Bayesian estimation methods, which give weight both to theoretical priors and to the data. Efficient blending of priors and data produces estimated models with comparable predictive accuracy to other models, but with more plausible dynamics. A further advantage of the technique is that it allows estimation of flexible stochastic processes, and thereby timely, model-consistent measures of unobservable variables such as potential output.

² Carabenciov and others (2008, 2008b, 2008c) describe the model used here in some detail.

To calculate empirically-based confidence intervals, we simulate the model with multiple drawings from estimated distributions of the disturbance terms. Since the sample space is large, we employ computationally efficient Latin hypercube sampling.

Section 2 describes the model structure, and the stochastic inputs on which the confidence intervals are built. Section 3 discusses the estimated intervals and their potential implications. Section 4 contains some brief conclusions.

II. MODEL STRUCTURE

2.1 Overview

GPM stems from the small quarterly macroeconomic models, originally associated with inflation targeting, which over the past decade have become the workhorse for quantitative monetary policy analysis.³ Typically, these have 4 or 5 behavioral equations determining:

- the output gap and/or the unemployment rate;
- the inflation rate (an inflation expectations augmented Phillips curve);
- the exchange rate, and
- the interest rate (a monetary policy rule).

The simplicity of the structure brings 2 practical advantages. The first is the ease with which forecasts can be updated and alternative assumptions examined. The second is the mainstream economic logic, with a direct focus on the main goal and instrument of monetary policy. These features facilitate timely analysis, and they provide policymakers with a coherent framework for outlining their views on key issues, and for explaining their decisions to a wide audience.

For the United States, this version of GPM includes an endogenous credit-conditions effect. We construct an index of bank lending tightening (BLT) in the United States from the Federal Reserve Board's Senior Loan Officer Survey.⁴ The index subtracts the percentage of "eased" responses against the percentage of "tightened." A BLT in excess of 50 percent means an exceptional degree of tightening; in 2008 the index rose above 80 percent, a previously unprecedented level.

³ We refer to the models used, e.g., in the vast literature on monetary policy rules. For applications of GPM itself, see Clinton and others (2008, 2009) and Decressin and Laxton (2009).

⁴ This specification is based on Carabenciov and others (2008b), which contains an extensive discussion of credit-channel effects and their modeling. It is also in the spirit of earlier work by Bayoumi and Swiston (2007). Bayoumi and Melander (2008), Lown, Morgan and Rohatgi (2000), Lown and Morgan (2002), Lown (2006), and Swiston (2008).

It is convenient to express all variables as deviations from equilibrium (from the monetary policy target in the case of inflation)—i.e. in gap format. Underlying real equilibrium values, which determine the long-run paths of real variables, are not directly observable, but within the context of the model, the Bayesian technique allows us to estimate these values, given a specified stochastic process.⁵

2.2 Model components

2.2.1 Variable definitions

$\overline{RPOIL}_{US,t}$: logarithm of the real equilibrium price of oil in real U.S. dollar terms

$RPOIL_{US,t}$: logarithm of the real price of oil in U.S. dollar terms

$P_{US,t}$: CPI of US.

$\bar{Y}_{i,t}$: potential output of country i

$\bar{U}_{i,t}$: equilibrium rate of unemployment or NAIRU of country i

$I_{i,t}$: nominal interest rate of country i

$R_{i,t}$: real interest rate of country i

$\bar{R}_{i,t}$: equilibrium real interest rate of country i

$\pi_{i,t}$: annualized quarterly inflation of country i

$Z_{i,t}$: real exchange rate of country i

$\bar{Z}_{i,t}$: equilibrium real exchange rate of country i

$Z_{i,t}^e$: expected exchange rate of next period of country i

$BLT_{US,t}$: bank lending tightening variable for the United States

$\overline{BLT}_{US,t}$: equilibrium credit tightening variable for the United States

2.2.2 Underlying equilibrium values and stochastic processes

The model has 5 observable variables for each economy. The inflation rate is the annualized quarterly change in the CPI, measured as 400 times the first difference of the log of the CPI. Output is represented by GDP. Unemployment refers to the standard labor force measure. The real interest rate is the difference between the nominal interest rate and the expected rate of inflation next quarter. The U.S. exchange rate is euro or yen per dollar (an increase corresponding to U.S. dollar appreciation); the real exchange rate adjusts for differential changes in CPIs (an increase implying an increase in the relative price of U.S. goods).

⁵ See, e.g., Collyns, Laxton and Tamirisa, 2008.

Oil price

The oil price block of equations determine the level and rate of change of real oil prices, denominated in the currencies of each of the countries in the model. The change in the rate of inflation in oil prices has important implications for the level of potential output and the rate of inflation.

In equation (1), the log of the real equilibrium price of oil (in inflation-adjusted U.S. dollars), $\overline{RPOIL}_{US,t}$, is defined as a function of its own lagged value plus the growth in the real price of oil in U.S. dollar terms, $\overline{g}_{US,t}^{RPOIL}$, and a disturbance term $\varepsilon_{US,t}^{RPOIL}$ that captures shocks to the level of real oil prices in U.S. dollar terms

$$\overline{RPOIL}_{US,t} = \overline{RPOIL}_{US,t-1} + \overline{g}_{US,t}^{RPOIL} + \varepsilon_{US,t}^{RPOIL}. \quad (1)$$

The growth of oil in real U.S. dollar terms is also subject to shocks, $\varepsilon_{US,t}^{g^{RPOIL}}$. In the absence of shocks, the one-period growth rate $\overline{g}_{US,t}^{RPOIL}$ converges to zero, at a rate defined by the adjustment coefficient $1 - \rho_{g,US}$:

$$\overline{g}_{US,t}^{RPOIL} = (1 - \rho_{g,US}) \overline{g}_{US,t-1}^{RPOIL} + \varepsilon_{US,t}^{g^{RPOIL}}. \quad (2)$$

In equation (3), the log of the level of the real oil price, $RPOIL_{US,t}$, is defined as the sum of the log of the equilibrium real oil price and the gap between the level and its equilibrium value, $rpoil_{us}$.

$$RPOIL_{US,t} = \overline{RPOIL}_{US,t} + rpoil_{US,t}. \quad (3)$$

The gap between the current price and the equilibrium price, $rpoil_{us,t}$, follows a damped process:

$$rpoil_{US,t} = \rho_{rpoil,us} rpoil_{US,t-1} + \varepsilon_{US,t}^{rpoil}. \quad (4)$$

In model simulations, the nominal price of oil in the United States is also affected by the U.S. inflation rate, and in other countries by the rate of exchange.

The rate of inflation of real oil prices in US dollar is defined by

$$\pi_{US,t}^{RPOIL} = 4(RPOIL_{US,t} - RPOIL_{US,t-1}), \quad (5)$$

and the rate of inflation of real oil prices denominated in the currency country i by

$$\pi_{i,t}^{RPOIL} = \pi_{US,t}^{RPOIL} + 4(Z_{i,t} - Z_{i,t-1}). \quad (6)$$

The log of the nominal US dollar price of oil is defined as

$$POIL_{US,t} = RPOIL_{US,t} + \log P_{US,t}. \quad (7)$$

Potential output and NAIRU

$$\bar{Y}_{i,t} = \bar{Y}_{i,t-1} + g_{i,t}^{\bar{Y}} / 4 - \sigma_i \left(\sum_{j=0}^3 \pi_{i,t-j}^{RPOIL} \right) + \varepsilon_{i,t}^{\bar{Y}}. \quad (8)$$

In equation (8), potential output follows a stochastic trend with disturbances which may affect its level permanently, and its growth rate $g_{i,t}^{\bar{Y}}$ over a finite period. In addition, increases in the international price of oil have a negative effect on the level of potential output.

The steady-state growth rate is $g_i^{\bar{Y}ss}$. However, disturbances may affect the short-run rate, which returns to equilibrium at a pace determined by the adjustment coefficient τ_i :

$$g_{i,t}^{\bar{Y}} = \tau_i g_i^{\bar{Y}ss} + (1 - \tau_i) g_{i,t-1}^{\bar{Y}} + \varepsilon_{i,t}^{g^{\bar{Y}}}. \quad (9)$$

Equations (10) and (11), using similar notation, embody level and growth-rate shocks for the non-accelerating rate of unemployment (NAIRU):

$$\bar{U}_{i,t} = \bar{U}_{i,t-1} + g_{i,t}^{\bar{U}} + \varepsilon_{i,t}^{\bar{U}}. \quad (10)$$

$$g_{i,t}^{\bar{U}} = (1 - \alpha_{i,3}) g_{i,t-1}^{\bar{U}} + \varepsilon_{i,t}^{g^{\bar{U}}}. \quad (11)$$

Real equilibrium interest rate and exchange rate

The equilibrium real interest rate may diverge from the steady-state value in the short run because of transitory shocks. Permanent shocks can cause a long-run change in the equilibrium rate. Equation (12) defines the equilibrium real interest rate, $\bar{R}_{i,t}$, as a function of the steady-state rate, \bar{R}_i^{ss} , plus a stochastic shock:

$$\bar{R}_{i,t} = \rho_i \bar{R}_i^{ss} + (1 - \rho_i) \bar{R}_{i,t-1} + \varepsilon_{i,t}^{\bar{R}} \quad (12)$$

The equilibrium real exchange rate, $\bar{Z}_{i,t}$, follows a random walk:

$$\bar{Z}_{i,t} = \bar{Z}_{i,t-1} + \varepsilon_{i,t}^{\bar{Z}} \quad (13)$$

2.2.3 Bank lending tightening

The effects of normal cyclical variations in the availability of credit on the level of activity are accounted for implicitly by the interest rate and the lagged dependent variable. However, shocks to lending practices will have an independent impact, which is not so captured. We estimate these shocks empirically, as the residual, $\varepsilon_{US,t}^{BLT}$, from equation (14).

$$BLT_{US,t} = \overline{BLT}_{US,t} - \kappa_{US} y_{US,t+4} - \varepsilon_{US,t}^{BLT}. \quad (14)$$

In this equation, $BLT_{US,t}$ is a function of the expected output gap 4-quarters ahead and equilibrium credit tightening, $\overline{BLT}_{US,t}$, which we define to be a random walk:

$$\overline{BLT}_{US,t} = \overline{BLT}_{US,t-1} + \varepsilon_{US,t}^{BLT}. \quad (15)$$

The BLT shock effect on the output gap is represented by the term $\eta_{US,t}$, a distributed lag of the non-systematic component, $\varepsilon_{US,t}^{BLT}$, which builds up for 5 quarters and then tapers off:

$$\begin{aligned} \eta_{US,t} = & 0.04\varepsilon_{US,t-1}^{BLT} + 0.08\varepsilon_{US,t-2}^{BLT} + 0.12\varepsilon_{US,t-3}^{BLT} + 0.16\varepsilon_{US,t-4}^{BLT} + 0.20\varepsilon_{US,t-5}^{BLT} + 0.16\varepsilon_{US,t-6}^{BLT} \\ & + 0.12\varepsilon_{US,t-7}^{BLT} + 0.08\varepsilon_{US,t-8}^{BLT} + 0.04\varepsilon_{US,t-9}^{BLT} \end{aligned} \quad (16)$$

In model simulations we impose a nonlinear effect, under which the marginal impact on the output gap doubles when the index is above a 50 percent threshold. This modification amplifies the effect of BLT levels at and beyond the threshold of previous experience—i.e. situations in which lenders cut off credit, and borrowers have no alternative sources. It is a pragmatic device to recognize the notion of a financial accelerator, as described, e.g., by Bernanke, Gertler and Gilchrist, (1996).

2.2.4 Output gap

$$\begin{aligned} y_{US,t} = & \beta_{US,1} y_{us,t-1} + \beta_{US,2} y_{us,t+1} - \beta_{US,3} r_{us,t-1} + \beta_{US,4} \sum_j \omega_{us,4,j} (-z_{j,US,t-1}) + \beta_{US,5} \sum_j \omega_{us,5,j} y_{j,t-1} - \theta_{us} \eta_{us,t} \\ & + 0.05 [\varepsilon_{us,t}^{yneg} \cdot y_{us,t}^{yneg}] + \varepsilon_{US,t}^y \end{aligned} \quad (17)$$

This determines the output gap as a function of:

- its own lead and lagged values,
- the real interest rate gap
- bank lending tightening (U.S. equation only)

- output gaps in its trading partners
- the real exchange rate gap, and
- a disturbance term

The lag from the output gap itself captures both intrinsic delays due to adjustment costs, and the adaptive component of expectations or habit persistence. The lead corresponds to forward-looking investment and consumption behavior, in anticipation of expected future output and income.⁶

The product of the terms $\varepsilon_{US,t}^{yneg}$ (negative shocks to output) and $y_{US,t}^{yneg}$ (negative output gaps) amplifies the impact of negative demand shocks in a recession. This state-dependent effect recognizes 2 strands of empirical research. First, studies of the Great Depression suggest that negative demand shocks have larger effects when there is significant excess supply in the economy (e.g., Temin, 1976, Romer, 1990, 1993, and Greasley, Madsen, and Oxley, 2001). Second, Desroches and Gosselin (2004) find threshold effects on consumption from large swings in consumer confidence. Given the emerging severity of the current recession, these findings have obvious relevance.

2.2.5 Unemployment

Unemployment is related to the output gap via a dynamic version of Okun's law:

$$u_{i,t} = \alpha_{i,1}u_{i,t-1} + \alpha_{i,2}y_{i,t} + \varepsilon_{i,t}^u \quad (18)$$

This equation does not have an impact on the simulations discussed in the next section, but the implied correlation does provide useful information for empirical estimation of the output gap.

2.2.6 Inflation

$$\pi_{i,t} = \lambda_{i,1}\pi_{i,t+4} + (1 - \lambda_{i,1})\pi_{i,t-1} + \lambda_{i,2}y_{i,t-1} + \lambda_{i,3}\sum_j \omega_{i,j,3}\Delta Z_{i,j,t} + v_{i,1}\pi_{i,t}^{RPOIL} + v_{i,2}\pi_{i,t-1}^{RPOIL} - \varepsilon_{i,t}^\pi \quad (19)$$

This augmented Phillips curve writes inflation as a function of

- inflation expectations—a weighted average of past and model-consistent future rates
- the lagged output gap
- the change in the real exchange rate

⁶ For example, Laxton and Pesenti (2003) derive Euler equations for consumption that contain lagged and expected consumption, real interest rates and a habit-persistence parameter.

- the current and lagged increase in the real price of oil, and
- a disturbance term

The model-consistent element relates to price setting based on predictions of future inflation. When monetary policy maintains a stable policy rule, expectations eventually converge on the rate of inflation targeted by the central bank. The greater the weight on the forward-looking component ($\lambda_{i,1}$), the more rapid is the convergence. The backward-looking component in expectations reflects adaptive behavior.

The lagged output gap embodies the familiar short-run trade-off, a crucial link between the real sector and the price level.⁷ The equation includes a mechanism to limit the negative effect on inflation of large negative output gaps: we set the marginal effect at zero for gaps wider than -3 percent of potential output. Without such a mechanism, the model could implode in response to large contractionary shocks. An empirical justification for this constraint is that while the price level has declined, it has not dropped precipitously, even during the deepest recessions.

The coefficients on the changes in the real exchange rate and in the real price of oil contain the pass-through to the CPI of changes to import and oil prices. Oil price shocks are particularly unfavorable, in that they directly raise headline inflation and, at the same time, reduce output.

2.2.7 Policy rule for the interest rate

$$I_{i,t} = (1 - \gamma_{i,1}) \left[\bar{R}_{i,t} + \pi 4_{i,t+3} + \gamma_{i,2} (\pi 4_{i,t+3} - \pi_i^{tar}) + \gamma_{i,4} y_{i,t} \right] + \gamma_{i,1} I_{i,t-1} + \varepsilon_{i,t}^I \quad (20)$$

This Taylor-type, forward-looking, policy rule determines a key short-term interest rate (the Federal Funds rate for the United States). Because of the zero interest floor, the equation holds only for non-negative $I_{i,t}$. The own lag provides a smoothing device, in line with the incremental adjustments typical of central bank interest rate decisions.⁸

Monetary policy responds to the deviation of the expected inflation rate from its target, in such a way that the target is ultimately achieved. While the U.S. announces no official target, published Fed forecasts suggest a fairly precise implicit target. For example, the range of the

⁷ The equation also contains a limiting upper value to the output gap (y_{max}), which implies that the short-run Phillips curve becomes vertical—at some point, because of very low excess capacity, increases in demand would fail to induce increases in output, and result only in accelerating inflation. This mechanism does not, however, come into play in the recessionary environment of the current forecasts.

⁸ Woodford (2003) provides a theoretical rationale for the smoothed interest rate response. In essence, smoothing increases the impact of changes in short-term rates on long-term rates, because the changes are likely to have some persistence.

long-run FOMC forecasts for the consumption deflator (PCE) in February 2009 was 1.7-2.0 percent.⁹ Since monetary policy determines the inflation rate in the long run, this forecast must reflect the effective policy target. In terms of the CPI, the FOMC range would imply a long-run target of about 2 ¼ to 2 ½ percent, given the historical difference between the PCE and CPI.¹⁰ On these grounds, we assume for current purposes that the Fed target is 2.5 percent.

In a steady state, with inflation on target at 2.5 percent, the central bank sets the actual rate at the long-run equilibrium level. Otherwise, it opens a gap between the two. From the equation, the long-run impact on the interest rate of a change in the inflation rate is equal to $(1 + \rho_{i,2})$. This is greater than unity, so that the real interest rises (or declines) when inflation increases (or decreases) from the target rate. It also responds to non-zero output gaps, which have potential effects on inflation in future quarters.

The equation also includes a disturbance term to allow for interest rate actions not indicated by the equation.

The model respects the zero lower bound to the interest rate.

2.2.8 Exchange rate

$$4(Z_{i,t+1}^e - Z_{i,t}) = (R_{i,t} - R_{u,st}) - (\bar{R}_{i,t} - \bar{R}_{u,st}) + \varepsilon_{i,t}^{Z-Z^e} \quad (21)$$

This embodies a modified uncovered interest parity condition. Uncovered interest parity would imply equality of all exchange-rate adjusted short-term interest rates. In contrast, the equation allows for exchange-rate adjusted interest rates to differ even in the long-run equilibrium. That is, each currency has a risk premium, which may be positive or negative, or zero. The expected real exchange rate for the next period is a weighted average of the lagged real exchange rate and the one-period ahead model-consistent solution:

$$Z_{i,t+1}^e = \phi_i Z_{i,t+1} + (1 - \phi_i) Z_{i,t-1} \quad (22)$$

2.2.9 Variance and covariance of disturbances

⁹ *Monetary Policy Report to Congress*, February 24, 2009 (Table 1).

¹⁰ See McCully, Moyer, and Stewart (2007).

(continued...)

Shocks to the variables are not independent. The present version of GPM contains 3 cross-equation correlations, between:

- potential output and inflation disturbances:- negative covariance representing supply shocks.- e.g. a positive shock to potential output reduces the current inflation rate;
- potential output growth and output gap disturbances: - positive covariance representing the expected income effect of a change in growth, which has an immediate effect on spending and output, implying excess demand in the short run;
- and potential output growth and BLT disturbances in the United States: - a negative covariance representing asset market/output market interactions.- e.g. higher growth of potential eases bank credit (because higher growth implies higher asset values and returns); in the short run an anticipatory increase in spending would produce a positive output gap.

III. GPM-GENERATED CONFIDENCE BANDS

3.1 Construction

Given the number of interacting variables in GPM, it would require an unfeasibly large number of samples if we were to use routine Monte-Carlo methods to draw the samples to calculate the confidence bands. To avoid this problem, we employ Latin hypercube sampling, which ensures – with a high probability – that the samples are representative of the real variability of the random terms.

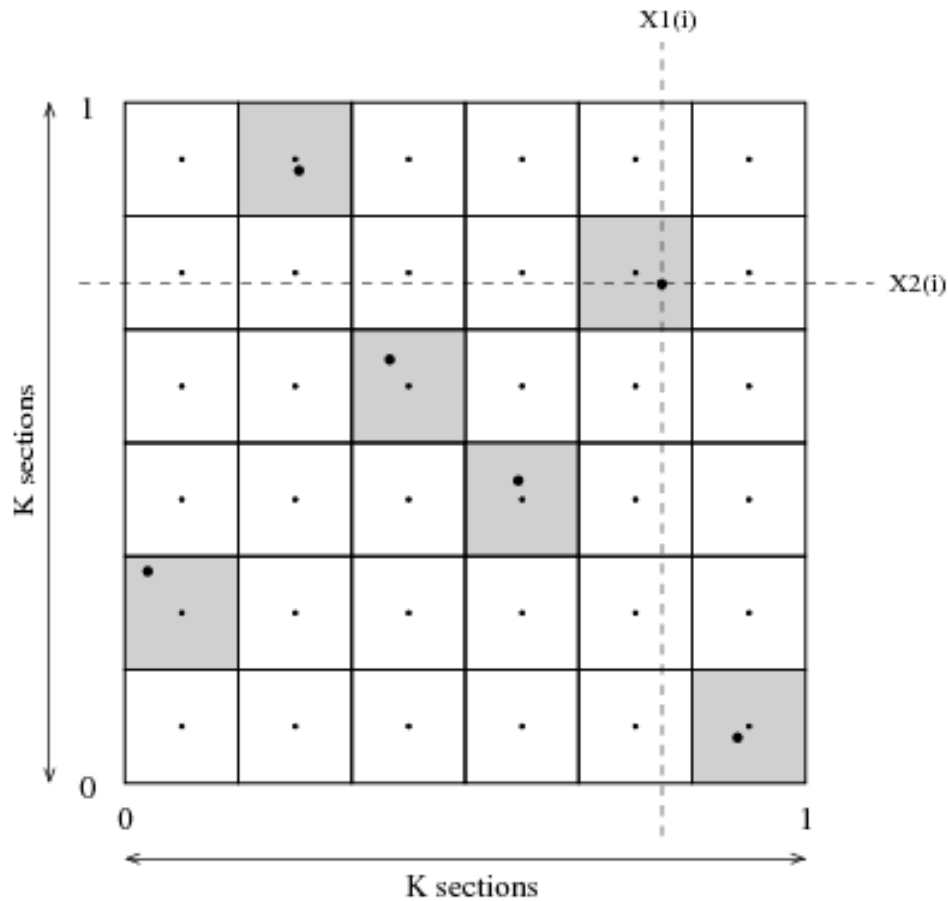
The choice of numerical method to estimate confidence bands depends mainly on the dimensionality of the underlying joint distribution. This is equal to $N \cdot M$, where N is the number of periods, and M is the minimum of state variables and exogenous shocks. In our exercise $N \cdot M$ is equal to $20 \times 36 = 720$. Given the time needed to simulate 20 periods of unanticipated shocks, an application of standard Monte-Carlo methods, as in Davis and Rabinowitz (2007), based on random drawings is infeasible.

Therefore, we apply the Latin Hypercube sampling technique, as in McKay, Beckman and Conover (1979), which samples the series of shocks more evenly, and reduces the variance of the simulation outcomes. This also helps to reduce the number of simulations needed to obtain convergence of the confidence band estimates.

In order to generate Latin Hypercube samples, we use the LHSA algorithm (Fang, Li and Sudjianto, 2006). The algorithm aims to evenly spread a given number of points in a higher

dimensional hypercube. Let $D = N \cdot M$ be the dimension, and K be a number of samples. For the experiment, we set $K = 4800$. The hypercube is split to K^D D -dimensional hypercubes by splitting each coordinate to K sections. Latin hypercube sampling technique selects K cubes from the K^D cubes, such that in each coordinate hyperplane, there is one and only one selection. The LHSA algorithm achieves this by generating D random independent permutations π_1, \dots, π_D of integers $1, \dots, K$. Each selection is then defined as $[\pi_1(k), \dots, \pi_D(k)]$ for $k = 1, \dots, K$. The Latin Hypercube samples are then obtained as centers of the selected cubes perturbed by uniform D -dimensional distributions not exceeding the boundaries of the selected cubes. When the samples are scaled by length of edges of the hypercube, we obtain K draws from D -dimensional uniform distribution.

The following picture depicts the LHSA algorithm for $D = 2$ and $K = 6$. First, the two dimensional cube (square) is divided into 6×6 squares. Second, two random permutations of integers $1 \dots 6$ select 6 squares, such that in each column and row, there is one and only one selection. Third, each center of the selected squares is perturbed by a uniform distribution. In this way we obtain 6 draws $[X1(i), X2(i)]$ for $i = 1 \dots 6$.



For our experiment, the LHS algorithm produces K points which try to evenly spread the $D = N \cdot M$ dimensional unit hypercube. However, we need to obtain K series of N independent draws from $N(0, V)$, where V is a variance-covariance matrix of the M exogenous shocks. Therefore, for each of K points having $N \cdot M$ coordinates, we split the coordinates to N sections containing M numbers. In this way we obtain K series of N independent draws from M dimensional uniform distribution. Now it suffices to convert each of these draws to represent a draw from M dimensional Gaussian distribution $N(0, I_M)$, and then multiply each draw by a matrix A , where $A \cdot A^* = V$ to obtain draws of the Gaussian shocks with the given variance-covariance matrix V .

One may decompose the calculated confidence intervals to identify the contributions of the main sources of variance. For this purpose, step by step, we eliminate the respective random components from the model, and recalculate the confidence intervals. Each of these counterfactual steps would generally (but not always, because of negative covariances) result in narrower intervals; eliminating an important factor for a given variable would in practice result in a substantial narrowing.

The sequence we followed was to remove, cumulatively:

- first, foreign shocks other than the oil price
- second, domestic supply shocks, and
- third, oil-price shocks—leaving U.S. demand shocks as the only contributor to the confidence interval

3.2 U.S. inflation

Figure 1 shows an illustrative 20-quarter forecast, with an associated fan of 95 percent confidence intervals for CPI inflation in the United States. The solid line, at the centre of the fan, labeled *No Shocks*, is the base case.¹¹ The pair of plain lines towards the outside of the fan, labeled *All Shocks*, delineates the model-generated, empirically-based, confidence interval around the forecast path. This is calculated from multiple simulations, incorporating all sources of variance specified in the model.

The illustrative baseline forecast, which was developed in March 2009, assumes the price of oil rises steadily from about \$40 per barrel to almost \$60 over the 20 quarters (Figure 5). The

¹¹ We use the *No Shocks* solution to represent the central tendency of all simulation-derived distributions. In every scenario, the median path of the solutions underlying the confidence intervals differed negligibly from the *No Shocks* path.

calculated confidence interval is about \$30 to \$70 at the 4-quarter horizon, \$30 to \$90 at 8-quarters, and \$25 to \$115 at the 12-quarter horizon.¹²

Baseline inflation falls from an initial rate near zero to a trough of -3 percent in quarter 3. The decline in the price level largely reflects a lagged response to the drop in energy prices already experienced. As these stabilize, the rate of increase in the CPI turns up, and becomes positive again at about quarter 8. Thereafter, however, inflation increases at a very moderate rate, and after 20 quarters it remains short of 2.5 percent, the assumed implicit target of the Federal Reserve.

At 4 quarters, the *All Shocks* confidence interval for the annual inflation rate is about -2.0 to +1.0 percent.¹³ At 8 quarters, the interval is just inside the ± 2 percent range. As the lower *All Shocks* boundary line stays well below zero throughout the forecast period, the calculations imply a considerable risk that deflation might become a lasting problem.

The first step in decomposing the interval—eliminating foreign shocks—does not materially change the estimated confidence band around the U.S. inflation forecast (in Figure 1, the fan lines with circle markers). This reflects that the relevant foreign variables (e.g. output, interest rates, and real exchange rates) have their main impact on inflation through their effect on spending; and that this effect is relatively small for the United States. Moreover, the Federal Reserve policy rule works to keep spending and inflation stable in the face of such shocks.

The second counterfactual step—eliminating domestic supply shocks—does narrow the band (the fan lines with plus-sign markers). These shocks impinge directly on the augmented Phillips curve. They involve changes to the level and growth rate of potential output, to the equilibrium unemployment rate, and to the exogenous residual term in the inflation equation itself. Moreover, since supply shocks involve negative covariance between output and inflation, monetary policy has no easy remedy—interest rate responses that reduce the short-run variance of inflation would increase that of the output gap. At horizons of 4 quarters and beyond, their removal slices about 1 percent from each side of the band in the *All Shocks* calculation.

The third step—eliminating shocks to the world oil price—further narrows the band (the fan lines with triangle markers). Indeed, the band now stays quite close to the central line. This means that, in the model, the latter 2 shocks account for almost all of the risk to the inflation forecast; the remaining source of variance—U.S. demand shocks—therefore makes only a

¹² The lack of symmetry in the confidence band for the level of the oil price results mainly from conversion from the logarithmic equation.

¹³ This is in line with the width of the range that inflation targeters often announce (Roger and Stone, 2005), Table 6.

small contribution to the confidence interval. This result is consistent with the intent of the monetary policy rule, which is to dampen disturbances to the inflation rate arising from domestic demand shocks.

3.3 U.S. interest rate

The baseline policy-determined rate of interest stays near zero for 8 quarters (Figure 2). Since the expectations process implies that expected inflation is also near zero, the real interest is well below the long-run equilibrium rate. The baseline interest rate rises slowly, and is still below the equilibrium level at the 20-quarter horizon. This prolonged stimulative monetary stance represents a response to below-target inflation, and to the negative output gap.

Confidence intervals for the interest rate are distinctly non-symmetric because of the zero lower bound; moreover, they widen steadily. At 20 quarters, the *All Shocks* confidence interval goes from near zero to almost 7 percent (plain lines, on the outside of the fan). This indicates a clear risk that the Fed might have to contend with the zero interest floor over an extended period. At the same time, it is clear that the range of uncertainty surrounding the interest rate is large. The steep increase in the upper bound of the confidence interval reflects that an empirically plausible set of inflationary shocks could require the Fed to withdraw stimulus at a smart pace.

Removal of foreign non-oil shocks significantly narrows the confidence interval for the interest rate—by at least $\frac{1}{2}$ percentage point after 4 quarters. The reason is that monetary policy has one less set of shocks to respond to.

Eliminating US supply shocks has little effect on the interval for the interest rate. With output and inflation moving in opposite directions, these shocks have weak implications for the interest rate, and their removal does not cause much decline in rate variability. Removal of oil price shocks, in contrast, produces a major narrowing.

The remaining source of risk to the forecast, domestic demand shocks, accounts for a substantial part of the confidence interval for the interest rate, at all forecast horizons. This is as one would expect, as the interest rate rule responds directly to these shocks.

3.4 U.S. output gap

In Figure 3, the *No Shocks* baseline forecast for the U.S. output gap shows a 5-quarter decline, with a trough near -6 percent. Stimulative monetary conditions, and the restoration of normal bank lending, encourage a recovery. But baseline output is not back to potential until the very end of the 20-quarter horizon.

The fan lines show a clear non-symmetry at, and for several quarters after, the trough. Thus, at 8 quarters, the 95 percent confidence interval extends from about 4 percent below, to 2

percent above, the 5 percent midpoint. The greater downside risk is attributable to the nonlinearities in the financial sector—i.e. the zero bound on the interest rate, and the threshold effect of BLT—both of which amplify large negative output shocks.

Eliminating foreign shocks, for reasons previously discussed, has no material effect on uncertainty surrounding the U.S. output gap. More surprisingly, perhaps, removal of *U.S.* supply shocks also has virtually no effect. Endogenous mechanisms, e.g. expectations effects on current spending, apparently help keep demand roughly in balance with the shocks to supply.

Removal of oil price shocks, however, results in a halving of the interval. The upshot is that oil price and domestic demand shocks, together, account for almost the entire confidence interval around the forecast output gap.

3.5 Bank lending tightening

In Figure 4, the *No Shocks* forecast path for the BLT index shows a reversion, from an extremely high starting point, to near zero within 5 quarters. While zero would be a normal value for the tightening index in normal times, under current circumstances it suggests that an unusually high degree of bank lending tightness will persist. To get back to a normal *level* of tightness would require an extended period of *easing*—i.e. *negative* BLT values.¹⁴

The uncertainties around the forecast path, moreover, are considerable. This is reflected in 2 results from the model. First, the calculated confidence interval is very wide. For example, at the 8-quarter horizon the width of the *All Shocks* band is about 80 percentage points; and it does not diminish much as the horizon moves ahead. Second, the interval is asymmetric, showing much more risk of unexpected tightening than easing. This would reflect feedbacks from the real sector, which intensify the impact of negative shocks when credit tightening is at extreme levels—i.e. the financial accelerator.

The decomposition exercise reveals that oil price shocks dominate the confidence interval for BLT. Domestic demand shocks (which include own BLT shocks) have a surprisingly small independent influence.

IV. CONCLUSIONS

The technique for constructing confidence intervals, around forecasts from the GPM model, contains several innovations. The model includes nonlinearities of material import, notably the zero bound to interest rates, which allow asymmetric confidence bands. The asymmetries

¹⁴ One might conjecture that respondents to the survey confuse tightening (the derivative) with tight (the level).

are especially noticeable in situations, such as the 2008-09 recession, that are far from long-run equilibrium.

We use Bayesian estimation methods, which efficiently blend prior knowledge with information in the data. These ensure that the system has economically plausible properties, while allowing for flexible stochastic processes. They also provide model-consistent estimates of unobservable variables such as potential output.

To calculate confidence intervals, we take multiple drawings from estimated distributions of the disturbance terms, employing Latin hypercube sampling. This is an efficient technique for a multi-variable model using lengthy time series.

Results for the United States suggest that these innovations are useful. The nonlinearities give rise to visibly asymmetric forecast confidence intervals for key financial variables—the short-term interest rate and bank lending tightening. The lower bound for the 95 percent confidence interval for the policy interest rate touches the zero axis almost throughout the 20-quarter forecast period, while the interval for BLT suggests more risk of unusual tightening than easing. Together, these results warn that there is a non-negligible risk that the Fed might continue to be constrained, over the medium term, by the zero bound to the interest rate, in the face of severe credit market conditions.

Consistent with this, the confidence interval around the forecast path for the output gap shows more downside than upside risk at the trough of the recession. The interval for inflation, however, is roughly symmetric; this would be due, in part, to the forward-looking component of the inflation expectations process in the model, which is anchored by the policy target.

A decomposition of the random disturbances suggests that oil price shocks and U.S. domestic demand shocks tend to have a stronger influence on the width of the confidence intervals than non-oil foreign shocks or U.S. supply shocks.

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Figure 1: U.S. CPI Inflation (year-over-year)

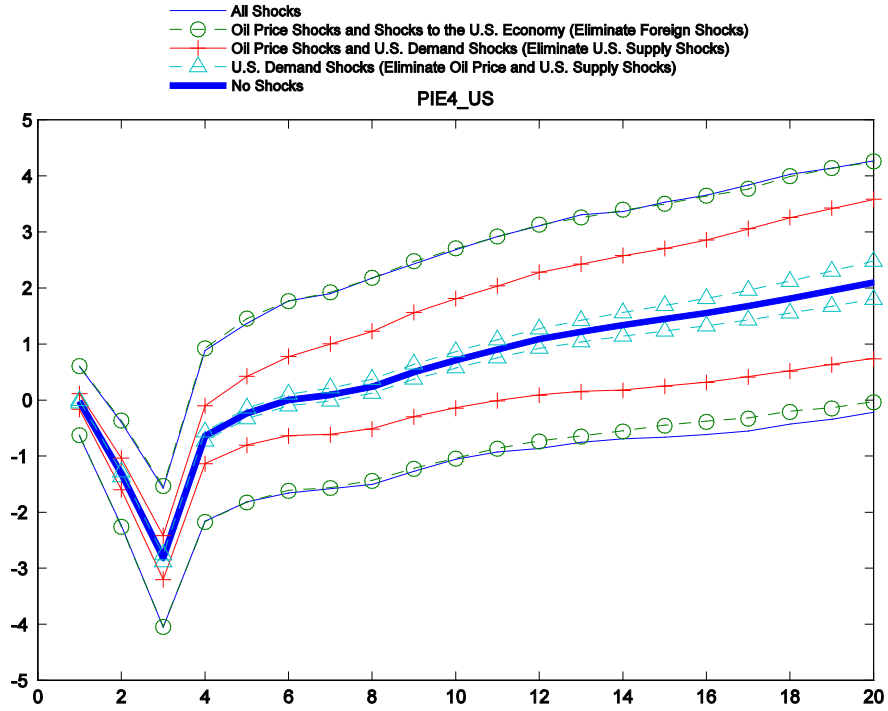


Figure 2: U.S. Interest Rate

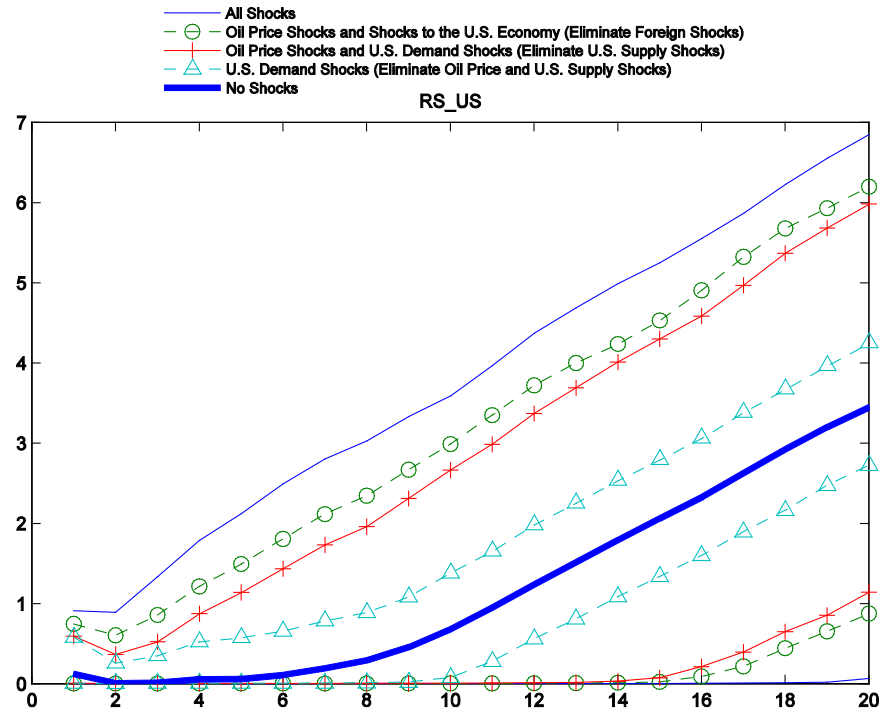


Figure 3: U.S. Output Gap

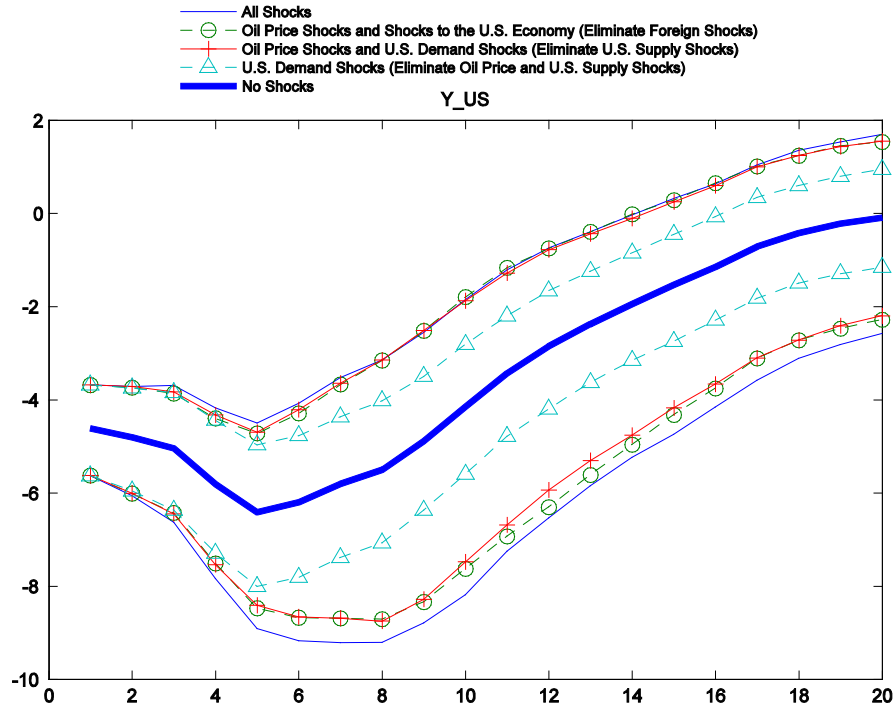


Figure 4: Bank Lending Tightening

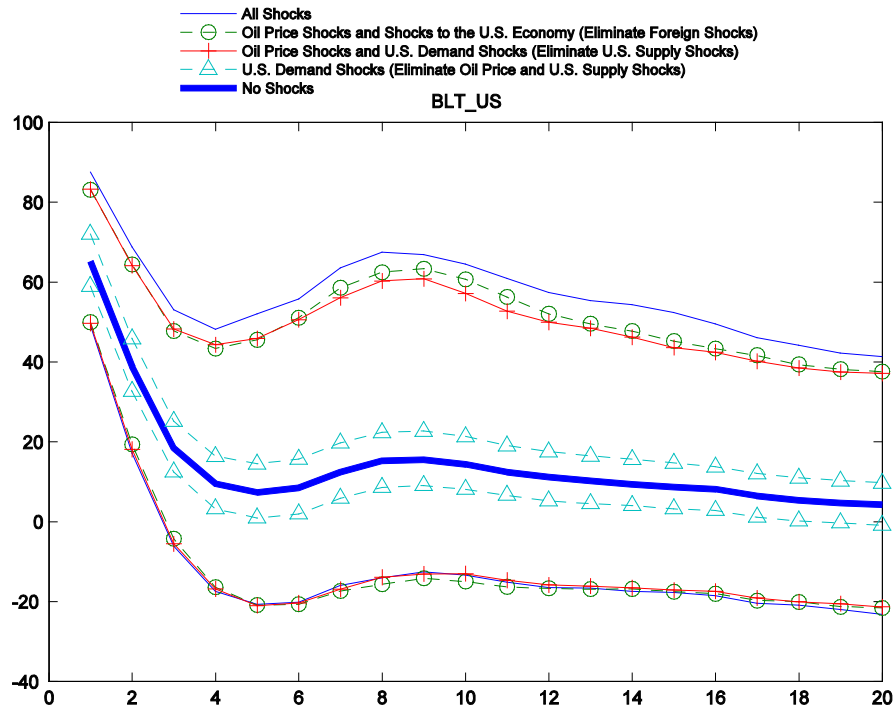


Figure 5: Oil Price

