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Excessive Lending, Leverage, and Risk-Taking in the Presence of Bailout Expectations

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Abstract

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The financial crisis that began in 2007 has brought to the fore the issues of excesses in lending, leverage, and risk-taking as some of the fundamental causes of this crisis. At the same time, in dealing with the financial crisis there have been large scale interventions by governments, often referred to as bailouts of the lenders. This paper presents a framework where rational economic agents engage in ex ante excessive lending, borrowing, and risk-taking if creditors assign a positive probability to being bailed out. The paper also offers some thoughts on policy implications. It argues that it would be most productive for the long run if lending institutions were not bailed out. If the continuing existence of an institution was deemed essential, assistance should take the form of capital injections that dilute the equity of existing owners.

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I. INTRODUCTION

The financial crisis that began in 2007 has brought to the fore the issues of excesses in lending, leverage, and risk-taking as some of the fundamental causes of this crisis. At the same time, in dealing with the financial crisis there has been large scale intervention by governments—in ways that are often referred to as bailouts—to deal with the effects of the actual or expected incapacity of borrowers to pay their debts.

The broad questions that arise are as follows:
- Is there a possible connection between the actions taken that could amount to the bailout of lenders and the excesses in lending/borrowing and risk-taking that have led to the crisis?
- Can efforts to address the financial crisis by bailing out the lenders validate a moral hazard that may have been one of the causes of the recent crisis, and thus help recreate in the future some of the behaviors that led to this crisis—excessive lending and risk taking?
- Does the need to salvage the financial system necessarily imply that lenders would have to be bailed out and that the reinforcement of moral hazard is unavoidable?

This paper offers a simple analytical framework that one can use to think about the first two questions posed above. It also offers some views on the third question, as the argument about the moral hazard implications of bailouts should not be dismissed as a priori leading to unrealistic policy prescriptions.

The financial crisis that began in 2007 has a number of very complex aspects. However, it can be characterized by some simple facts:
- The crisis was preceded by an excessive buildup of debt.
- There was a parallel increase in leverage (as debt grew faster than equity).
- The buildup of excessive debt took place with lenders charging particularly low interest rates; spreads were compressed.
- Excessive risk-taking accompanied the buildup of excessive debt.

The analytical framework presented in this paper can (with some qualifications) economically reproduce the above basic facts of the recent financial crisis. It does this by analyzing the effects of the perception of lenders that they may be bailed out by a third party when borrowers cannot meet their debt servicing obligations. The framework is squarely based on the assumption that economic agents are rational and that they—both lenders and borrowers—have adequate information about the (probability distributions of the) activities that are being financed with the credit provided.

Therefore, this framework does not resort to irrationality of economic agents or opaqueness to explain the lending and risk-taking excesses that were important ingredients of the buildup to the crisis that erupted in 2007. This does not mean that there may not have been additional reasons for the excesses observed, whereby some of them may have had to do with irrational behavior. Nevertheless, it is arguable that such purportedly irrational behaviors—including

the blatant disregard for the need for information about inter alia the capacity of borrowers to repay their debts—may have been more readily permitted (say, by lending institutions' shareholders) to be committed because of the *rationally* processed expectation of a bailout.[1]

The financial crisis that erupted in 2007 had as protagonists of excessive lending, not only banks, but also non-bank financial institutions (the so-called "shadow" banking system). The analytical framework that is presented in this paper should not be thought of as modeling just a bank lending problem, but as modeling a relatively generic lending relationship. Thus, this analytical framework can be used to draw some basic conclusions about the effect of bailout expectations by lenders, irrespective of the specific modalities of the lending process, including modalities used by non-bank financial institutions.

It should be emphasized that this paper does not purport to provide the unique explanation of the conditions that led to the recent financial crisis; there were several factors that operated simultaneously to create these conditions. The paper offers for consideration that lending by rational agents in the context of bailout expectations was one of these factors. Finally, this paper does not aim to explain the precipitation, mode of propagation, or severity of the recent financial crisis.

II. THE ANALYTICAL FRAMEWORK[2]

A. The Basic Model

The arguments of this paper are presented with the help of a simple model of the loan market. The model belongs to the class of models developed by Jaffee and Modigliani (1969), Smith (1972), Jaffee and Russell (1976), Milde and Riley (1984), and Clemenz (1986). However, the model developed here highlights the peculiar features of credit market relationships when the creditors assign a positive probability to an ex post involvement by a third party in a formally two party contract. The assumptions of the model are presented below:

(i) The potential borrower/investor is confronted with an investment project whose output price is a non-negative random variable X, such that $X = q(L)Y$, where q is the quantity produced with input L, and L is the amount of funds invested in the project. The price per unit output is set equal to 1 for simplicity of presentation, and it is given exogenously. Y is a non-negative random variable, such that $Y \in [0, m]$, with distribution $F(Y)$. It is assumed that $q'(L) > 0$ and $q''(L) < 0$ for all L.

(ii) There is, for the moment, only one source of finance of the project and it is credit; this assumption will be relaxed later on.

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[1] By the same token, there would be diminished interest by lending institutions' owners in putting in place systems for penalizing lending practices by managers that turn out to be inappropriate.

[2] The analytical framework in this paper draws on the dissertation of the author: Georgiou, Andreas (1989). *Essay in Overlending and Capital Flight*, Ph.D. Dissertation, The University of Michigan, Ann Arbor.

(iii) The debtor/investor has to repay the amount borrowed times R, where R is defined as $1 +$ *interest rate*, if the return of the project is greater than $\hat{X}$, where $\hat{X} = q(L)\hat{Y} = RL$. However, if Y happens to be less than $\hat{Y}$ (i.e., $X < \hat{X}$), the borrower always remits to the lender all the returns of the project.

(iv) When Y falls below $\hat{Y}$, a third party, which was not ex ante involved in the contract, intervenes with probability π and bails out the lender by paying the rest of the contracted repayment amount, $RL - q(L)Y$.

(v) Both the lender and the debtor-investor are risk neutral in the model as originally formulated, i.e., they maximize their expected returns.

(vi) It is assumed that the lender is restricted by the zero profit condition due to the competition of alternative lenders.

Following Milde and Riley (1984) and Clemenz (1986), the borrower's problem is written:

$$\max_L B = q(L)\int_{\hat{Y}}^{m} Y f(Y)dY - RL[1 - F(\hat{Y})] \tag{1}$$

where $\hat{Y} = \dfrac{RL}{q(L)}$ and $f(Y) = F'(Y)$.

This optimization problem can be rewritten:

$$\max_L B = q(L)\{\hat{Y}[1 - F(\hat{Y})] + \int_{\hat{Y}}^{m}[1 - F(Y)]dY\} - RL[1 - F(\hat{Y})] \tag{2}$$

The first order condition is:

$$\frac{\partial B}{\partial L} = \left[R(e-1) + q'(L)H(\hat{Y})\right]\left[1 - F(\hat{Y})\right] = 0 \tag{3}$$

Where $e(L) \equiv \dfrac{q'(L)L}{q(L)}$ and $H(\hat{Y}) \equiv \dfrac{\int_{\hat{Y}}^{m}(1 - F(Y))dY}{[1 - F(\hat{Y})]}$.

The second order condition is:

$$\frac{\partial^2 B}{\partial L^2} = \left[R\frac{\partial e}{\partial L} + q''(L)H(\hat{Y}) + q'(L)H'(\hat{Y})\frac{\partial \hat{Y}}{\partial L} \right]\left[1 - F(\hat{Y})\right]$$
$$- \frac{f(\hat{Y})\frac{\partial \hat{Y}}{\partial L}\left(\frac{\partial B}{\partial L}\right)}{\left[1 - F(\hat{Y})\right]} \tag{4}$$

For the second order condition to have a negative sign, it must be that $e < 1$ (which implies $\frac{\partial \hat{Y}}{\partial L} > 0, \frac{\partial e(L)}{\partial L} \leq 0$ and $H'(\hat{Y}) < 0$. These are sufficient conditions. For $H'(\hat{Y}) < 0$, the so-called hazard rate $\frac{f(Y)}{1 - F(Y)}$ should be increasing in Y. Therefore, $e < 1$ and $H'(\hat{Y}) < 0$ imply that $\frac{\partial^2 B}{\partial L^2} < 0$, which means that $\frac{\partial B}{\partial L} = 0$ defines a maximum.

The slope of the implied credit demand curve is found by using:

$$\frac{dR}{dL} = \frac{\dfrac{\partial^2 B}{\partial L^2}}{\dfrac{\partial^2 B}{\partial L \partial R}} \tag{5}$$

where

$$\frac{\partial^2 B}{\partial L \partial R} = \left[e - 1 + q'(L)H'(\hat{Y})\frac{\partial \hat{Y}}{\partial R} \right]\left[1 - F(\hat{Y})\right] - \frac{\left[f(\hat{Y})\dfrac{\partial \hat{Y}}{\partial R}\dfrac{\partial B}{\partial L} \right]}{\left[1 - F(\hat{Y})\right]} \tag{6}$$

Since $\frac{\partial \hat{Y}}{\partial R} > 0$, $e < 1$ and $H'(\hat{Y}) < 0$, then it holds that $\frac{\partial^2 B}{\partial L \partial R} < 0$. Therefore, the following is true:

Proposition 1: *If $e(L) < 1$, $\frac{\partial e(L)}{\partial L} \leq 0$ and $\frac{f(Y)}{1 - F(Y)}$ increases with Y, then the amount of credit demanded L decreases as the interest rate R increases.*

The demand curve defined above has isoprofit curves associated with it. The properties of the isoprofit curves are spelled out in proposition 2 below.[3]

Proposition 2: $e(L) < 1, \dfrac{\partial e(L)}{\partial L} \leq 0$ and $\dfrac{f(Y)}{1 - F(Y)}$ *increases with Y, then:*

- (i) *Each isoprofit curve R(L,B) has a unique turning point at* $L^*(B)$ *such that*

$$\frac{\partial R}{\partial L} > 0 \text{ for } L < L^*(B) \text{ and } \frac{\partial R}{\partial L} < 0 \text{ for } L > L^*(B)$$

- (ii) $\dfrac{\partial L^*}{\partial B} > 0$

- (iii) $\dfrac{\partial R[L^*(B)]}{\partial B} < 0$

Turning now to the lender's problem, we take the basic formulation illustrated in Clemenz (1986) and amend it to account for the lender perceiving that a third party will intervene to bail out the lender by paying her the amount $RL - q(L)Y$ if Y realized is less than $\hat{Y}$, that is if the return to the project fails to cover the contracted repayment amount RL.

In this context, the zero profit condition of the lender is as follows:

$$P = (1 - \pi)q(L)\int_0^{\hat{Y}} Yf(Y)dY + (\pi - 1)RLF(\hat{Y}) + RL - IL = 0 \tag{7}$$

where π is the probability perceived by the lender that a third party will intervene to bail out the lender by paying her the amount $RL - q(L)Y$ if Y realized is less than $\hat{Y}$, that is if the return to the project fails to cover the contracted repayment amount RL. The per unit cost of loanable funds to the lender is I.

Proposition 3: *If e(L) < 1, then the zero profit line per borrower of a given lender has a non-negative slope for all $L \geq 0$.*

Proof: The slope of the zero-profit line is given by

$$\frac{dR}{dL} = -\frac{\dfrac{\partial P}{\partial L}}{\dfrac{\partial P}{\partial R}} \tag{8}$$

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[3] The proof of Proposition 2 can be found in Clemenz (1986), p. 134 and will not be repeated here.

We know that

$$\frac{\partial P}{\partial L} = q'(L)(1-\pi)\int_0^{\hat{Y}} Yf(Y)\,dY + (\pi-1)F(\hat{Y})R + R - I \tag{9}$$

Moreover, from the zero profit condition we have

$$I = (1-\pi)\frac{q(L)}{L}\int_0^{\hat{Y}} Yf(Y)\,dY + (\pi-1)RF(\hat{Y}) + R \tag{10}$$

Since $\dfrac{q'(L)L}{q(L)} = e < 1$, it follows that $I > (1-\pi)q'(L)\int_0^{\hat{Y}} Yf(Y)\,dY + (\pi-1)RF(\hat{Y}) + R$ [4]

This implies that $\dfrac{\partial P}{\partial L} < 0$. Furthermore, $\dfrac{\partial P}{\partial R} = L\left[1-(1-\pi)F(\hat{Y})\right] > 0$ since $0 \le \pi \le 1$.

Consequently, $\dfrac{dR}{dL} = -\dfrac{\dfrac{\partial P}{\partial L}}{\dfrac{\partial P}{\partial R}} > 0$.

The question that arises is what happens to the slope of the zero-profit line as the perceived probability of a bailout π changes.

Proposition 4: *An increase in the perceived probability of bailout of the lender by a third party π reduces the slope of the zero-profit line of the lender.*

Proof: The change in the slope for a change in the perceived probability π is

$$\frac{\partial\left(\dfrac{dR}{dL}\right)}{\partial\pi} = \frac{\int_0^{\hat{Y}}\left[q'(L)Y - I\right]f(Y)\,dY}{L\left[1-(1-\pi)F(\hat{Y})\right]^2} \tag{11}$$

Regarding the numerator in the expression, from the proof of Proposition 3, we know that

$$(1-\pi)q'(L)\int_0^{\hat{Y}} Yf(Y)\,dY + \left[(\pi-1)F(\hat{Y})+1\right]R - I < 0. \tag{12}$$

We can now write

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[4] It is noteworthy that as L increases, and the probability $F(\hat{Y})$ that the return of the project will not be enough to cover the debt service also increases, the slope of the zero-profit line increases. The rate at which the slope increases as L increases will be a factor in determining inter alia whether an increase in the cost of funds I to the creditor will lead either to an increase in the interest rate charged on the loan or to a decline in that interest rate (while the amount lent falls in both cases). This is relevant for Proposition 8 below.

$$(1-\pi)q'(L)\int_0^{\hat{Y}} Yf(Y)dY + I\left[1-(1-\pi)F(\hat{Y})\right] - I < 0, \text{ since } R \geq I. \tag{13}$$

This can be rewritten as

$$(1-\pi)\int_0^{\hat{Y}}\left[q'(L)Y - I\right]f(Y)dY < 0, \text{ since } 0 \leq \pi \leq 1. \tag{14}$$

Thus,

$$\int_0^{\hat{Y}}\left[q'(L)Y - I\right]f(Y)dY < 0. \tag{15}$$

Since the denominator of expression (11), $L\left[1-(1-\pi)F(\hat{Y})\right]^2$, is positive, and its numerator

is negative, it follows that $\dfrac{\partial\left(\dfrac{dR}{dL}\right)}{\partial\pi} < 0.$

If the expected probability of bailout is $\pi > 0$, then the interest rate at which a lender is willing to lend a given amount of funds will not fully reflect the associated credit risk, and the spread between that interest rate and the cost of funds will be compressed for a given loan amount.

Corollary: *If the perceived probability π of a bail out is equal to 1, the zero-profit line is infinitely elastic at $R = I$.*

Proof: The limit of the bank's profit function, as π approaches 1, is

$$\lim_{\pi \to 1}(1-\pi)q(L)\int_0^{\hat{Y}} Yf(Y)dY + RL\left[1-(1-\pi)F(\hat{Y})\right] - IL \tag{16}$$

In the limit, this is equal to $RL - IL$. If the profit per borrower is zero, then at $\pi = 1$, $R = I$.

In this case, the spread between the cost of funds and the interest rate the lender is willing to lend will collapse to zero because the lender perceives that he will not bear any of the credit risk associated with his lending.

Propositions 1 and 3 imply a situation as depicted in Figure 1. An equilibrium has to lie on the zero-profit line of the lender. Equilibrium occurs where the zero-profit line, $(P = 0)$, is tangent to the best feasible isoprofit curve of the borrower. Comparing any equilibrium that we get when $\pi > 0$ with the one corresponding to $\pi = 0$, we see that in the former case the

amount borrowed is larger. In Figure 1, the distance $L*L**$ is the amount overlent (overborrowed) due to the presence of expectations of bailout of the lender by a third party.[5]

More formally, the equilibrium can be found by maximizing the borrower's return, $B(R, L)$, with respect to R and L, and subject to the zero-profit condition of the lender, i.e., by maximizing

$$\mathcal{L} = B(R,L) + \lambda P(R,L) \tag{17}$$

where λ is the Lagrange multiplier. The first order conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial L} &= q'(L)\int_{\hat{Y}}^{m} Yf(Y)dY - R\left[1 - F(\hat{Y})\right] \\
&+ \lambda\left\{(1-\pi)q'(L)\int_{0}^{\hat{Y}} Yf(Y)dY - (1-\pi)RF(\hat{Y}) + R - I\right\} = 0
\end{aligned}
\tag{18}
$$

$$\frac{\partial \mathcal{L}}{\partial R} = -L\left[1 - F(\hat{Y})\right] + \lambda L\left[1 - (1-\pi)F(\hat{Y})\right] = 0 \tag{19}$$

It follows that in equilibrium it must hold that

$$\frac{q'(L)\pi\int_{\hat{Y}}^{m} Yf(Y)dY}{[1 - F(\hat{Y})]} + (1-\pi)q'(L)EY = I \tag{20}$$

$$(1-\pi)q(L)\int_{0}^{\hat{Y}} Yf(Y)dY - (1-\pi)RLF(\hat{Y}) + RL = IL \tag{21}$$

Since the Pareto-efficient allocation of capital is given by the condition $q'(L)EY = I$, which can be found by maximizing $E(X) - IL$, the following holds:

Proposition 5: *If the Pareto efficient allocation of capital is given by $q'(L)EY = I$, then the allocation of capital when $\pi > 0$ is inefficient and the amount of capital invested is larger than the Pareto-optimal amount. The deviation from the socially optimal amount is overlending/overborrowing, which increases with π and is zero when $\pi = 0$.*

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[5] The model presented above can be reformulated with the borrower being risk averse. It can be shown that if the borrower is risk averse, the quantity of credit demanded at each interest rate will be smaller than when the borrower is risk neutral. Moreover, the quantity of credit demanded at each interest rate declines as the degree of relative risk aversion increases. Furthermore, the introduction of risk averse borrowers implies (as in the case of risk neutral ones) the existence of concave isoprofit lines for each borrower. The isoprofit lines achieve a unique maximum on what may be interpreted as the downward sloping demand curve for credit of the borrower. Thus, the introduction of risk-averse borrowers does not alter the model presented above or any of its conclusions in any significant way.

Proof:

Equation (20) can be rewritten

$$q'(L)EY = I + \left[\frac{\pi}{1-(1-\pi)F(\hat{Y})}\right]\int_0^{\hat{Y}}\left[q'(L)Y - I\right]f(Y)dY \qquad (22)$$

Since $\int_0^{\hat{Y}}\left[q'(L)Y - I\right]f(Y)dY < 0$, as shown in the proof of Proposition 4, and

$\dfrac{\pi}{1-(1-\pi)F(\hat{Y})} > 0$, it follows that, when $\pi > 0$, $q'(L)EY < I$. Since $q''(L) < 0$, the amount

borrowed when $\pi > 0$ is greater than the socially optimal amount.

It should be noted that the deviation from the socially optimal amount depends on the

magnitude of the term $\phi(\pi) = \dfrac{\pi}{1-(1-\pi)F(\hat{Y})}$ in the expression (22) above. This term reduces

to zero when $\pi = 0$. Furthermore, $\phi(\pi)$ increases as π increases, since

$$\frac{\partial\phi(\pi)}{\partial\pi} = \frac{1-F(\hat{Y})}{\left[1-(1-\pi)F(\hat{Y})\right]^2} > 0 \qquad (23)$$

A question that arises is what happens to contract interest rates as the perceived probability of bailout π increases and overlending takes place. In this model, there is a possibility that interest rates charged to borrowers may not monotonically decline as π increases. However, as π tends to reach 1, the interest rates charged will unambiguously decline and the spread between these interest rates and the cost of funds will decrease towards zero.

The reason for the potentially indeterminate movement in interest rates and spreads as π increases lies in the fact that, as π rises (but remains below 1), two contradictory effects are taking place. First, as π is rising, the amount of credit risk that the lender perceives to be facing declines and the interest rate R would fall to reflect that. Second, the amount lent increases and this in turn increases the (endogenous) probability of default

$F(\hat{Y})$, since $\dfrac{\partial\hat{Y}}{\partial L} > 0$. The increase in the probability of default warrants an increase in the

interest rate charged on the loan. The net effect cannot be determined a priori, unless one is given specific values for the parameters of the problem.[6]

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[6] It should be noted that the presence of the downward slopping credit "demand curve" (shown to exist by proposition 1 above) does not automatically imply that a decrease in the slope of the zero profit line of the lender on account of a rising π will necessarily lead to a decrease in the interest rate charged R. The reason lies in the fact that equilibrium is not determined by the intersection of that demand curve of the borrower and the zero profit line of the lender; it is determined instead by the position of the tangency of the best feasible isoprofit curve of the borrower with the zero profit line of the lender.

If the probability of default does not increase too rapidly as the amount lent increases, then the intuitively appealing scenario materializes: As π becomes positive and rises, the amounts of credit provided to borrowers increase while the interest rates charged fall, compressing the observed spreads between lending rates and the cost of funds to creditors. As the cost of funds to creditors can also be expected to closely track the rate of return on benchmark government paper,[7] a positive and rising probability of bailout would also be associated with increasingly compressed spreads between effectively riskless (or low-risk) government paper and what would be quite risky credits from a social point of view. The observed spreads would therefore tend to loose their signaling content, with the degree of that loss increasing as expectations of bailout π increase.

B. Different Sources of Finance

So far in the model, it has been assumed that there is only one source of finance for the project—i.e., credit. In the case where the borrower-investor can have equity K in the project, such that $K \leq W_0$ (where W_0 is the borrower-investor's initial wealth) and the opportunity cost of equity is IK, the following holds:

Proposition 6: *If borrower-investors are risk-neutral and $\pi = 0$, then the debt-equity ratio of the project is indeterminate. If however $\pi > 0$, no equity will be used and the project will be completely debt financed.*

The proof of proposition 6 is provided in the Appendix.

The above result is important as it shows how a positive probability of bailout of the lender by a third party will result in the substitution of debt for equity in the financing of the project (i.e., an increase in leverage). In the present model, where economic agents are risk-neutral, the projects end-up being financed solely by issuing debt when π is positive.

C. Choice of Project Riskiness

In the basic model used so far, it is assumed that the riskiness of the project is not a choice variable. That assumption will now be relaxed. The risk-neutral borrower is now assumed to choose among projects of varying riskiness, Θ, where $\Theta \in [\Theta_0, \Theta_m]$. It is assumed that Θ is a continuous variable, with a larger Θ signifying a mean-preserving increase in risk.[8] The probability distribution function of returns to the project can now be written $F(Y, \Theta)$ and $f(Y, \Theta) = F'(Y, \Theta)$. Hence,

$$\frac{\partial}{\partial \Theta} \int_0^{\hat{Y}} F(Y, \Theta) dY > 0 \text{ for } \hat{Y} > 0. \tag{24}$$

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[7] This could be expected in a system where the government provides credible deposit insurance and where a large share of loanable funds available to creditors benefit from such insurance.

[8] This formulation of increasing risk is based on Rothchild and Stiglitz (1970).

and

$$\frac{\partial}{\partial\Theta}\int_{\hat{Y}}^{m}F(Y,\Theta)dY<0 \text{ for } \hat{Y}<m. \tag{25}$$

Proposition 7: *If there is choice among projects of different riskiness, in the sense of different mean-preserving spreads, and economic agents are risk-neutral, then:*

(i) if the perceived probability of bailout $\pi = 0$, the riskiness of the project undertaken in equilibrium is not determinate;

(ii) if $\pi > 0$, the riskiness of the project undertaken in equilibrium is determinate, and the riskiest project available is undertaken.

A formal proof of this result is provided in the Appendix. One can also consider the following intuitive argument that may shed some light on the result.

The crucial point is that the risk-neutral unit borrower-lender is only interested in the expected value of the returns accruing to it. When there are no expectations of bailout, changes in the riskiness of the project Θ do not change the expected value of the revenue of the unit borrower-lender. This accounts for the fact that when $\pi = 0$, the choice of project riskiness is indeterminate. On the other hand, when the perceived probability of bailout is positive, a change in Θ results in a change in the expected value of the revenue accruing to the unit borrower-lender. Specifically, the more risky the project, the greater the probability that the project will not provide a return high enough to repay the lender in full and, therefore, the greater the probability that a third party will provide the resources from without (given a $\pi > 0$). This in turn would lead to a greater expected value for the return accruing to the unit borrower-lender. As a result, when $\pi > 0$, both the borrower-investor and the lender will agree to undertake the riskiest project when considering projects characterized by different degrees of risk.

This is a particularly important result because it implies that a positive probability of bailout will not only cause overlending, but it will also increase the riskiness of the projects chosen by borrower-investors and their creditors.

D. Change in the Cost of Loanable Funds

A change in the cost of funds to lenders is represented in the model as a change in I. In the case where there is only one source of finance for the projects (i.e., debt), an increase in I will leave the demand for credit unaffected. However, an increase in I will affect the zero-profit condition of the lender. The vertical shift of the zero profit line is

$$\frac{dR}{dI}=\frac{1}{1-(1-\pi)F(\hat{Y})}\geq1 \text{ for } L=\overline{L} \tag{26}$$

It is interesting to note that the shift in the zero-profit line is not a parallel shift when $0 \leq \pi < 1$. In those cases, as I increases, the zero-profit line rotates counterclockwise at the

same time that it shifts up. However, when $\pi = 1$, the now infinitely elastic zero-profit line undergoes only a parallel shift upwards, equal in magnitude to the change in I.

The effects of the change in I on R and L can be summarized as follows.

Proposition 8: *An increase in the cost of loanable funds I will cause a decrease in the amount lent for $0 \leq \pi < 1$. However, an increase in I will cause an unambiguous increase in the loan interest rate only when $\pi = 1$.*

A heuristic proof of Proposition 8 is provided here. When $\pi = 0$, the equilibrium conditions are given by $q'(L)EY = I$ and the zero-profit condition of the lender. Taking the total differential of each condition and using Cramer's Rule, we can solve for $\dfrac{dL}{dI}$ and $\dfrac{dR}{dI}$. It is easily verified that $\dfrac{dL}{dI} < 0$ but $\dfrac{dR}{dI}$ has a ambiguous sign. In the case where $\pi = 1$, since

$R = I$, it holds that $\dfrac{dR}{dI} = 1$. Given the downward sloping demand for credit (from Proposition 1), the parallel shift upward of the (now horizontal) zero-profit line will lead to a smaller amount lent.[9] Thus, it can be argued that since an increase in I decreases the amount of the loan when $\pi = 0$ as well as when $\pi = 1$, it will also decrease the amount of the loan when $0 < \pi < 1$.

The reason it is not possible to determine what will happen to R when $\pi < 1$ is that, as I is increasing, two contradictory effects are taking place. First, when I increases, the opportunity cost of lending to the lender increases and R has to reflect that by rising too. Second, as I is rising, the amount lent decreases and this in turn decreases the (endogenous) probability of default $F(\hat{Y})$, since $\dfrac{\partial \hat{Y}}{\partial L} > 0$. The decrease in the probability of default warrants a decrease in the interest rate charged on the loan. The net effect cannot be determined a priori, unless one is given specific values for the parameters of the problem.

Proposition 9: *A larger probability of bailout π would render the amount lent less sensitive to changes in the cost of loanable funds.*

For a given change in I, the change in the amount lent would *ceteris paribus* become smaller as π increases. This is indicated in expression (26), where the shift of the zero profit line of the creditor for a given change in I is smaller for a larger π at any given amount lent $\bar{L}$. The upward shift is smallest when π reaches 1, whereby the zero profit line shifts just by the

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[9] When the perceived probability of bailout $\pi = 1$ and thus the zero profit line of the lender is infinitely elastic, the equilibrium is determined by the intersection of that zero profit line and the demand curve for credit because, in this case (and in contrast to cases where $\pi < 1$), the best feasible isoprofit curve of the borrower is tangent to the zero profit line of the lender exactly at the intersection of the latter with the borrower's demand curve.

amount of the change in the cost of loanable funds. The shift is largest when the perceived probability of bailout is zero.

As the perceived probability of bailout π tends to "desensitize" the supply of credit to changes in the cost of loanable funds, by the same token a positive π would also tend to desensitize the supply of credit to potential efforts by policy makers to put a break on excessive lending through monetary policy actions (changes in policy rates).

E. Changes in the Probability Distribution Function of the Debt-Financed Project

A change in the riskiness of the project is modeled here as a mean-preserving spread increase (see section II.C above). Its effect on L can be summarized as follows.

Proposition 10: *If the perceived probability of bailout $\pi = 0$, an increase in the riskiness of the project (in the form of a larger, mean preserving, spread) will leave the amount lent L unaffected. If, however, $\pi > 0$, an increase in project riskiness will cause L to rise.*

From expression (22) we see that when $\pi = 0$ the second term in the sum on the right-hand side of the expression becomes zero and the remaining components of the expression are not affected by changes in project riskiness Θ. Therefore, changes in Θ leave the amount lent L unaffected.

However, when $\pi > 0$, the second term in the sum on the right hand-side of expression (22) becomes a negative number, as shown in the proof of Proposition 5. The absolute value of this second term increases as the riskiness Θ of the project rises, given expressions (24) and (25). Therefore, $q'(L)EY$ becomes increasingly smaller than I, as Θ rises, which means that, since $q''(L) < 0$, the amount lent for some $\pi > 0$ becomes increasingly larger than the socially optimal amount.

These results can also be intuitively seen in the following manner. As mentioned earlier, the risk-neutral unit borrower-lender is only interested in the expected value of the return accruing to it. When the perceived probability of bailout is zero, an increase in the riskiness of the project does not alter the expected value of the return to the borrower-lender unit. For this reason, the amount borrowed and invested L does not change as riskiness exogenously increases.

If, however, the perceived probability of bailout is positive, an exogenous increase in Θ will increase the probability that a third party will provide resources from without. This in turn implies an increase in the expected value of the return accruing to the unit borrower-lender. Thus, when $\pi > 0$, both the borrower (investor) and the lender will agree to a larger loan (and to a larger investment) as the riskiness of the project rises.

III. SOME THOUGHTS ON POLICY IMPLICATIONS

To offset the effects of the expectation that a bailout will take place on lending/borrowing behavior and risk taking, policy makers have some options:

(i) Policy makers could recognize that there are expectations of bailout (occurring with some probability) and try to tax lending, calibrating the tax so that the amount lent is not allowed to exceed the socially optimal amount.

(ii) Policy makers could take the position that the owners of a lending institution will not receive more than is implied by the return to its investments, and that any assistance to maintain the institution's solvency (if that was deemed necessary) would take the form of capital injections that would dilute the equity of existing owners.

The first option mentioned above could theoretically provide the desired outcome, but the estimation of the tax needed to bring about the socially optimal result would likely be very complex and controversial as it would involve the government estimating a lender's expectation of the probability of bailout by the same government. Moreover, implementation of the tax across contracts in the economy would likely be an overwhelming task as each credit contract could in principle be characterized by a unique expected probability of bailout. Approaches that would impose a uniform tax across (broad categories of) lending contracts would likely lead to continuing distortions, with a coexistence of cases of overlending (overinvestment) and underlending (underinvestment). A positive aspect of imposing a tax would be that it would affect *current* lending behavior (in contrast to approaches which are aimed to affect future lending behavior, such as option (ii) above, where lenders have to "be taught" through the experience of an overlending event that the government will not bail them out and, thus, they then adjust their expectations and lending behavior.)

The second option mentioned above could be implemented more readily than the first one, and its relative simplicity is made more attractive by its promoting of market discipline and the rule of law (regarding the observance of economic contracts). This approach avoids reinforcing bailout expectations for the future. By aiming to have the owners of lending institutions bear the costs of the actions of their agents—the managers of the lending institutions—this option creates an environment where company shareholders have the incentive to exercise appropriate corporate governance. This approach helps relieve the government from the difficult task of devising policies such as in option (i) in an effort to generate a socially optimal outcome, and lets instead the market to work.

A potential problem with the approach is that the government could end-up as the (majority) owner of lending institutions, in case the government deems that the overlending institutions cannot be allowed to fail for systemic stability or other reasons. Such a concern could be addressed by the government putting in place (optimally beforehand) an appropriate framework for divestment that would be used in such cases.[10] Another concern about this

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[10] Such a framework would have to be carefully crafted to ensure that owners of credit institutions do not receive a transfer from the government *over time* as the government's shares are reprivatized.

approach could be that it does not affect lending behavior immediately, but only in the future, after the approach has been sternly implemented by the government and the latter has created a "track record."[11]

In the case of an investment project such as the one considered in the model above, failure of the project's outcome to cover the debt servicing required for the debt (i.e., when $Y < \hat{Y}$) implies that, other things being equal, the loss $(RL – q(L)Y)$ would be the amount by which the capital of the lending institution would decline if there is no bailout. According to option (ii) discussed above, in an environment of normally functioning markets, this amount is the upper limit of the amount of capital that would be injected into the lending institution by the government to keep it from failing, if the continuing functioning of the institution is deemed by the government as essential.[12] The injection of capital to offset the effect of losses should dilute the equity share of existing owners of the lending institution.

In addition, the capital injection should be accompanied by a prohibition of dividend payouts to existing shareholders. If such prohibition is not carried out, the capital injection by the government would effectively be diverted to make dividend payments to existing owners and thus amount to their being bailed out (at least partially). If dividends were paid out in anticipation of the bailout, this would result in a larger capital gap and thus should result in greater dilution of the shares of the existing owners. The government should also acquire voting rights in the lending institution that are fully commensurate with its capital injection, to not only ensure the appropriate payment of dividends, but also—together with existing owners—pursue as needed the allocation of appropriate penalties to the management of the lending institution as well as help create proper incentive structures for managers.

The approach suggested here indicates that there does not have to be a choice only between a bailout of the lending institution and the failure/bankruptcy of that lending institution. If the continuing existence of the institution is deemed by policy makers as essential, capital injection by the government would be consistent with the continuing functioning of the institution and with the costs of its lending practices being born by the appropriate parties, if losses of the lending institution do not exceed the capital of the existing owners. This latter qualifier is important as it implies the need for regulators to establish capital requirements that are high enough to cover potential losses.[13] Failure to do so would imply that there will be a higher probability that, even if all the capital of the owners in wiped out, there will be a transfer of resources to them via prior dividend payments (which presumably cannot be clawed back).

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[11] This points to supplementing this approach by appropriate regulations and strengthening supervision of lending practices.

[12] The effects of markets in distress (that may accompany system-wide increases in failures of lenders to collect on credits) on the solvency of institutions (via liquidity/funding constraints) would increase the amount of capital injection needed beyond the amount described here.

[13] If this approach were taken to its limit, it would resemble in this respect the so called "Chicago Plan" of banking reform, whereby lenders would be required to engage in 100 percent reserve lending (Sjaastad 1983). Credit would then be based completely on equity, rather than on debt.

It is worthwhile mentioning the implications of other approaches that could be followed in the case of failure of the project's outcome to cover the debt servicing required for the debt (i.e., when $Y < \hat{Y}$).

- A possible approach is for the government to buy from the lending institution the asset associated with the credit extended to finance the investment project. If the government does so at a price reflecting the full payment of debt service on the credit (RL), then the capital of the existing owners of the lending institution remains intact and the solution amounts to a full bailout of the lending institution's owners for any loss ($RL - q(L)Y$).

- If the government buys the asset at a discount from the price that corresponds to a full repayment of the lender, and that discount fully corresponds to the shortfall in repayment, then there will be losses accruing to the lending institution and this could lead to its insolvency. In this case there is no bailout of the owners of the lending institution. The latter may then be permitted to go bankrupt or, if the government deems its continuing existence essential, it may recapitalize it (with existing owners being "wiped out").

- In between the two approaches above involving asset purchases by the government, there is a continuum of others where, as the discount offered to the lender decreases, the degree of the bailout of existing owners increases.

The discussion in this section has not addressed the role of market valuation of the lender's asset, whether in determining the appropriate discount for the purchase of the asset by the government or in determining the capital of the lending institution as the residual of the value of assets after the value of liabilities is subtracted. It is arguable that the introduction of market valuation of assets in the discussion would bring into the calculation factors that do not necessarily have to do only with the actual repayment capacity of specific borrowers, but also with issues such as the degree to which markets function normally or the probability of intervention by the government. In that sense, in the view of this author, there may be grounds for consideration of a supervisory-guided approach to balance sheet valuation, which would be tailored to address valuation matters from a solvency perspective that would be longer-term. These matters, albeit important, go beyond the focus of this paper.

IV. Conclusion

This paper has argued that ex ante excessive lending, vis-à-vis the socially optimal amount, can occur if the creditor perceives that the debtor and creditor formally involved in the debt contract may not be the sole potential bearers of the risk of the loan (as the contract specifies). Specifically, this can arise when the creditor perceives that a positive probability exists for a third party to ex post intervene to help in the repayment of the credit not repayable in full by the formal debtor. Therefore, overlending/overborrowing is discussed in this paper not as an act of irrationality or ignorance on the part of economic agents, but as the result of profit maximizing behavior under very specific conditions. To present these ideas

and to explore further their implications, a model of loan contracting has been developed. The main conclusions are outlined below:

A perceived positive probability of bailout of the creditor by a third party will result in greater amounts lent to (and capital invested in) a project than warranted from a social optimum (Pareto efficient) point of view. The deviation from the efficient amount of lending is termed "overlending" and it increases as the probability of bailout rises.

If the expected probability of bailout is positive, the interest rate at which a lender is willing to lend a given amount of funds will not fully reflect the associated credit risk, and the spread between that interest rate and the cost of funds will be compressed *for a given loan amount*.

A perceived positive probability of bailout will affect leverage. Specifically, if a risk-neutral borrower can use her own capital, besides borrowing, to finance her project, the debt-to-equity financing mix of the project will change from indeterminate to all-debt/no-equity as the perceived probability of bailout of the *creditor* changes from zero to positive.

When there are projects of different riskiness available, the perceived positive probability of bail out will, under certain conditions, lead to the adoption of the riskiest project. Specifically, if a risk-neutral borrower is confronted with projects of varying degrees of mean-preserving riskiness, the choice among projects will become, from indeterminate, determinate when there is a perception of a positive probability of bailout of the *creditor*.

Greater project riskiness leads to greater amounts lent by risk-neutral creditors to risk-neutral borrowers when the perceived probability of bailout of the lender is positive, as opposed to no change in the amount lent when that probability of bailout is zero.

An increase in the cost of loanable funds will cause a decrease in the amount lent, irrespective of what the perceived probability of bailout of the lender is. However, the expectation of bailout tends to "desensitize" the amount of credit to changes in the cost of loanable funds, and this effect is larger the greater the probability assigned to the bailout.

This paper has also discussed options that policy makers have available to try to offset the effects of the expectation of a bailout on lending/borrowing behavior and risk taking. It has argued that, overall, the most productive approach for the long run would be for policy makers to ensure that a lending institution does not receive more than is implied by the return to its investments and that any assistance to maintain the institution's solvency take the form of capital injections by the government that dilute the equity of existing owners.

Figure 1. Equilibrium Loan Contracts
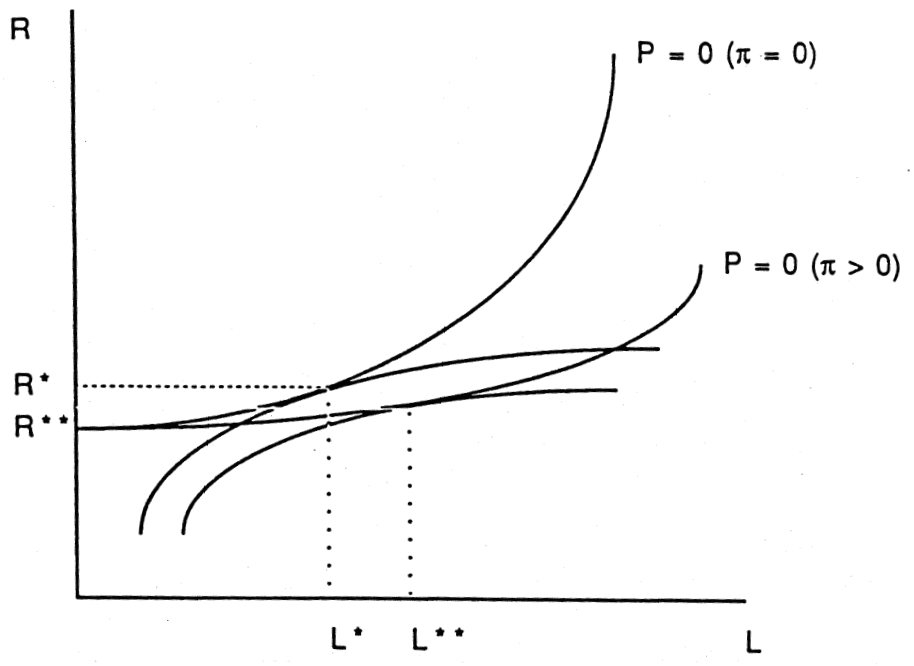
Appendix

Proof of Proposition 6: In the case when the borrower-investor can have personal equity in the project, the equilibrium can be found by maximizing Φ:

$$\max_{K,L,R}\Phi=q(L+K)\int_{\hat{Y}}^{m} Yf(Y)dY-RL[1-F(\hat{Y})]+(W-K)I$$
$$+\lambda\left\{(1-\pi)q(L+K)\int_{0}^{\hat{Y}} Yf(Y)dY-(1-\pi)RLF(\hat{Y})+RL-IL\right\}$$

(28)

The first order conditions are:

$$(i)\ \frac{\partial\Phi}{\partial L}=q'(L+K)\int_{\hat{Y}}^{m} Yf(Y)dY-R[1-F(\hat{Y})]$$
$$+\lambda\left\{(1-\pi)q'(L+K)\int_{0}^{\hat{Y}} Yf(Y)dY+R[1-(1-\pi)F(\hat{Y})]-I\right\}\le 0$$

(29)

$$(ii)\ \frac{\partial\Phi}{\partial K}=q'(L+K)\int_{\hat{Y}}^{m} Yf(Y)dY$$
$$+\lambda(1-\pi)q'(L+K)\int_{0}^{\hat{Y}} Yf(Y)dY-I\le 0$$

(30)

$$(iii)\ \frac{\partial\Phi}{\partial R}=\lambda L[1-(1-\pi)F(\hat{Y})]-L[1-F(\hat{Y})]\le 0$$

(31)

Using the Kuhn-Tucker Theorem, there are three cases when $\pi=0$.

Case 1: $R>0$, $L>0$, $K=0$ imply that $\lambda=1$, $q'(L+K)EY=I$ and $q'(L+K)EY\le I$ respectively. Therefore, in equilibrium, $q'(L+K)EY=I$ which is not a contradiction.

Case 2: $R>0$, $L>0$, $K>0$ imply that $\lambda=1$, $q'(L+K)EY=I$ and $q'(L+K)EY=I$ respectively. Therefore, in equilibrium, $q'(L+K)EY=I$ which is not a contradiction.

Case 3: $R>0$, $L=0$, $K>0$ imply that $\lambda=1$, $q'(L+K)EY\le I$ and $q'(L+K)EY=I$ respectively. Therefore, in equilibrium, $q'(L+K)=I$ which is not a contradiction.

Therefore, we conclude that in equilibrium, the borrower-investor will be indifferent with respect to the mix of debt and equity used in financing the project.

Using the Kuhn-Tucker Theorem, there are also three cases when $\pi>0$.

Case 1: $R>0$, $L>0$, $K=0$ imply that

$$\lambda=\frac{1-F(\hat{Y})}{1-(1-\pi)F(\hat{Y})},\ \pi q'(L+K)\int_{\hat{Y}}^{m} Yf(Y)dY=I[1-F(\hat{Y})]-[1-F(\hat{Y})](1-\pi)q'(L+K)EY,\ \text{and}$$

$$\pi q'(L+K)\int_{\hat{Y}}^{m} Yf(Y)dY\le I[1-(1-\pi)F(\hat{Y})]-[1-F(\hat{Y})](1-\pi)q'(L+K)EY\ \text{respectively.}$$

Combining the second and third expressions produces the expression, $I\pi F(\hat{Y})\ge 0$, which is

not a contradiction if it is interpreted as an inequality. Therefore, using the Kuhn-Tucker conditions, we cannot show that the exclusive use of debt to finance the project is contradictory.

Case 2: $R > 0$, $L > 0$, $K > 0$ imply that

$$\lambda = \frac{1 - F(\hat{Y})}{1 - (1 - \pi)F(\hat{Y})}, \quad \pi q'(L + K)\int_{\hat{Y}}^{m} Yf(Y)dY = I[1 - F(\hat{Y})] - [1 - F(\hat{Y})](1 - \pi)q'(L + K)EY, \text{ and}$$

$$\pi q'(L + K)\int_{\hat{Y}}^{m} Yf(Y)dY = I[1 - (1 - \pi)F\hat{Y})] - [1 - F(\hat{Y})](1 - \pi)q'(L + K)EY \text{ respectively.}$$

Combining the second and third expressions, we get $I\pi F(\hat{Y}) = 0$, which implies $F(\hat{Y}) = 0$. This is a contradiction since L is assumed to be positive.

Case 3: R > 0, L = 0, K > 0 imply

$$\lambda = \frac{1 - F(\hat{Y})}{1 - (1 - \pi)F(\hat{Y})}, \quad \pi q'(L + K)\int_{\hat{Y}}^{m} Yf(Y)dY \leq I[1 - F(\hat{Y})] - [1 - F(\hat{Y})](1 - \pi)q'(L + K)EY, \text{ and}$$

$$\pi q'(L + K)\int_{\hat{Y}}^{m} Yf(Y)dY = I[1 - (1 - \pi)F\hat{Y})] - [1 - F(\hat{Y})](1 - \pi)q'(L + K)EY \text{ respectively.}$$

Combining the second and third expressions, we get that $I\pi F(\hat{Y}) \leq 0$, which is not a contradiction if it is interpreted as an equality.

Therefore, using Kuhn-Tucker conditions, we cannot show that the exclusive use of equity to finance the project is contradictory. However, it can be proved that from the two extreme forms of finance, when $\pi > 0$, the borrower-investors will choose the "all debt" method instead of the "all equity" method. To show this consider the following:

It has been shown earlier that when $\pi = 0$, the borrower-investor is indifferent between "all equity" and "all debt" financing of the project. This means that

$$B(L^*, R^*) = B(K^*) \tag{32}$$

where

$$B(L^*, R^*) = q(L^*)\int_{\hat{Y}}^{m} Yf(Y)dY - R^*L^*[1 - F(\hat{Y})] + W \, I \text{ and } B(K^*) = q(K^*)EY + (W - K^*)I.$$

L^* and R^* are the equilibrium loan amount and interest factor under the "all debt" option when $\pi = 0$, while K^* is the equilibrium equity invested under the "all equity" option when $\pi = 0$. However, we already know that when $\pi > 0$, the banks would be able to offer a contract (L^{**}, R^{**}) which is more attractive to the borrower-investor than the (L^*, R^*) contract. Therefore, we can write: $B(L^{**}, R^{**}) > B(L^*, R^*) = B(K^*)$. This means that when $\pi > 0$, the "all debt" form of financing the project is superior to the "all equity" form from the borrower-investor point of view and thus, it will be the one used.

Proof of Proposition 7: For simplicity, in the text below, $F(Y, \Theta)$ will be written $F(Y)$. When $\pi = 0$, and the riskiness of the project is a choice variable, equilibrium is found by solving the following maximization problem:

$$\max_{R,L,\Theta} \mathcal{L} = q(L)\int_{\hat{Y}}^{m} [1-F(\hat{Y})]dY$$

$$+ \lambda \left\{ EYq(L) - IL - q(L)\int_{\hat{Y}}^{m} [1-F(Y)]dY \right\} \tag{33}$$

$$+ \mu(\Theta - \Theta_m)$$

The first order conditions are:

$$(i)\frac{\partial \mathcal{L}}{\partial L} = q'(L)\int_{\hat{Y}}^{m} Yf(Y)dY - R[1-F(\hat{Y})]$$

$$+ \lambda \left\{ q'(L)\int_{0}^{\hat{Y}} Yf(Y)dY + R[1-F(\hat{Y})] - I \right\} \leq 0 \tag{34}$$

$$(ii)\frac{\partial \mathcal{L}}{\partial R} = L[1-F(\hat{Y})] - \lambda L[1-F(\hat{Y})] \leq 0 \tag{35}$$

$$(iii)\frac{\partial \mathcal{L}}{\partial \Theta} = -q(L)\int_{\hat{Y}}^{m} \frac{\partial F(Y)}{\partial \Theta} dY$$

$$+ \lambda q(L)\int_{\hat{Y}}^{m} \frac{\partial F(Y)}{\partial \Theta} dY + \mu \leq 0 \tag{36}$$

From first order condition (ii), we get that $\lambda = 1$. Substituting this into first order condition (iii), we get:

$$-q(L)\int_{\hat{Y}}^{m} \frac{F(Y)}{\partial \Theta} dY + q(L)\int_{\hat{Y}}^{m} \frac{\partial F(Y)}{\partial \Theta} dY + \mu \leq 0 \tag{37}$$

which implies $\mu \leq 0$. This means that Θ can take any value, including the extreme values Θ_0 and Θ_m. Therefore, in the case where $\pi = 0$, the riskiness of the agreed upon project (by the borrower and lender) is not determinate.

Now consider a situation in which $\pi > 0$. All the rest of the assumptions are left unchanged. The equilibrium is found by solving the following:

$$\max_{R,L,\Theta} \mathcal{L}' = q(L)\int_{\hat{Y}}^{m} [1-F(Y)]dY$$

$$+ \lambda \left\{ (1-\pi)q(L)EY - (1-\pi)q(L)\hat{Y}[1-F(\hat{Y})] \right\}$$

$$- \lambda \left\{ (1-\pi)q(L)\int_{\hat{Y}}^{m} [1-F(Y)]dY - (\pi-1)RLF(\hat{Y}) - RL + IL \right\} \tag{38}$$

$$+ \mu\{\Theta - \Theta_m\}$$

The first order equations are:

$$(i') \quad \frac{\partial \mathcal{L}'}{\partial L} = q'(L)\int_{\hat{Y}}^{m} Yf(Y)\,dY - R[1-F(\hat{Y})]$$

$$+ \lambda \left\{ (1-\pi)q'(L)\int_{0}^{\hat{Y}} Yf(Y)\,dY \right. \tag{39}$$

$$\left. - (1-\pi)RF(\hat{Y}) + R - I \right\} \le 0$$

$$(ii') \quad \frac{\partial \mathcal{L}'}{\partial R.} = L[1-F(\hat{Y})] + \lambda L[1-(1-\pi)F(\hat{Y})] \le 0 \tag{40}$$

$$(iii') \quad \frac{\partial \mathcal{L}'}{\partial \Theta} = -\pi q(L)\int_{\hat{Y}}^{m} \frac{\partial F(Y)}{\partial \Theta}\,dY + \mu \le 0 \tag{41}$$

Equation (iii') is presented after substituting in the fact that $\lambda = \dfrac{1-F(\hat{Y})}{1-(1-\pi)F(\hat{Y})}$ from equation (ii').

Now consider expression (iii'). We know that for $\Theta < \Theta_m$, $\mu = 0$. This means that for an internal Θ, it must hold that:

$$-\pi q(L)\frac{\partial}{\partial \Theta}\int_{\hat{Y}}^{m} F(Y)\,dY = 0 \tag{42}$$

However, this expression is a contradiction because we know that $\dfrac{\partial}{\partial \Theta}\int_{\hat{Y}}^{m} F(Y)\,dY < 0$. So Θ cannot be smaller than the maximum, Θ_m. We also know that when $\Theta = \Theta_m$, $\mu < 0$. So (iii') can be written:

$$-\pi q(L)\frac{\partial}{\partial \Theta}\int_{\hat{Y}}^{m} F(Y)\,dY = -\mu \tag{43}$$

which is not a contradiction. Therefore, when $\pi > 0$, the choice of Θ is determinate; borrowers and lenders are best off with the riskiest project available. This result hinges crucially on the assumption of risk neutrality.

References

Clemenz, Gerhard, (1986), *Credit Markets With Asymmetric Information*, (Berlin: Springer-Verlag).

Jaffee, Dwight M. and Franco Modigliani, (1969), ''A Theory and Test of Credit Rationing,'' *American Economic Review*, No. 59, pp. 850-72.

Jaffee, Dwight M. and Thomas Russell, (1976), "Imperfect Information, Uncertainty, and Credit Rationing," *The Quarterly Journal of Economics*, MIT press, No. 90 Vol. 4, pp. 651-66.

Georgiou, Andreas, (1989), *Essays in Overlending and Capital Flight*, Ph. D. Dissertation, The University of Michigan, (Michigan: Ann Arbor).

Milde, H. and J. G. Riley, (1984), "Signaling in Credit Markets," *Diskussionsbeitrage*, Series No. 185, (Germany: University of Konstanz).

Rothchild, M and J. E. Stiglitz (1970), "Increasing Risk: A Definition," *Journal of Economic Theory*, 2, 225-43.

Sjaastad, Larry A. (1983), The international debt quagmire: to whom do we owe it?, *Graduate papers in international economics*, No. 8302, (Genève: Institut universitarie de hautes études internationals).