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Chile's Structural Fiscal Surplus Rule: A Model-Based Evaluation

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Abstract

The paper analyzes Chile's structural balance fiscal rule in the face of copper price shocks originating in foreign copper demand. It uses a version of the IMF's Global Integrated Monetary and Fiscal Model (GIMF) that includes a copper sector. Two results are obtained. First, Chile's current fiscal rule performs well if the policymaker puts a small weight on output volatility (relative to inflation volatility) in his/her objective function. A more aggressive countercyclical fiscal rule can attain lower output volatility, but there is a trade-off with (somewhat) higher inflation volatility and (much) higher volatility of fiscal variables. Second, given its current stock of government assets, Chile's adoption of a 0.5% surplus target starting in 2008 is desirable from a business cycle perspective. This is because the earlier 1% target would have required significant further asset accumulation that could only have been accomplished at the expense of greater volatility in fiscal instruments and therefore in GDP.

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I. Introduction

Chile's fiscal policy since 2000 has been conducted in accordance with a structural surplus rule.[1] The introduction of that rule confirmed and intensified Chile's commitment to fiscal responsibility since the mid-1980s by introducing a more explicit medium-term orientation. The rule was initially not enshrined in law, but this changed with the 2006 Fiscal Responsibility Law, which also introduced new rules on the investment of accumulating assets. The structural surplus rule only covers the central government. The main elements of the public sector left outside the rule are the central bank, public non-financial enterprises, the military sector, and municipalities.

The structural surplus rule implies a counter-cyclical behavior of ex-ante government surpluses. It states that the central government's overall structural balance should in every year equal a surplus of 1% (0.5% effective 2008) of actual GDP. The structural balance equals structural revenues plus interest on net government assets (which are positive in Chile) minus actual expenditures on goods and services. Structural revenue is determined by two independent panels of experts and reflects what tax revenue would have been if the economy had operated at potential rather than actual output, and what copper revenue would have been at a long-term reference world copper price rather than the actual price. The rule therefore specifies permissible annual expenditures on goods and services as a residual, given the values of the target, structural revenues, the level of government assets, interest rates, and GDP. The resulting counter-cyclicality of government deficits isolates government expenditures on goods and services from the cycle and keeps them growing with trend output. No distinction is made between government consumption and investment expenditures, because this is difficult to do in practice.

A positive surplus target implies significant asset accumulation by the government. It was adopted to provide for future social commitments and to address contingent liabilities. The 2006 Fiscal Responsibility Law formalized this by establishing rules for the investment of surpluses. These rules envision investment in a government pension fund, gradual central bank recapitalization, and a Fund for Economic and Social Stabilization. In May 2007 a reduction in the surplus target from 1% to 0.5% of GDP was announced, effective in 2008. The additional resources that thereby become available for current spending will be devoted primarily to education.

In this paper we analyze the effects of Chile's structural surplus rule on business cycle volatility. We inquire into two questions. The first is whether the performance of the rule could be improved through a more explicitly countercyclical stance, specifically by letting deficits respond more strongly to excess fiscal revenue than what is allowed under the current rule. While this gives up one clear advantage of the existing rule, namely the fact that it implies only small and gradual changes in fiscal instruments in response to shocks, it may have the offsetting benefit of smaller volatility of GDP and inflation. For example, in response to an increase in world copper prices the existing rule implies only small changes in tax rates in the short run (assuming tax rates are the fiscal tool of choice), while a more aggressive rule might respond to the post-shock increase in demand by raising tax rates and thereby dampening the boom. The second question we ask is whether there are advantages to aligning the level of the surplus target more closely with

[1] The rule is described in detail in Marcel, Tokman, Valdes and Benavides (2001).

the existing stock of net government debt. There is a proportional long-run relationship between the surplus to GDP ratio and the government assets to GDP ratio, so that if the targeted surplus to GDP ratio is different from the current assets to GDP ratio, actual short-run surpluses will have to vary over time until assets reach their long-run value. This leads to a fiscal policy driven business cycle even in the complete absence of shocks.

The analytical framework employed is a 2-country version (Chile and rest of the world) of the IMFs Global Integrated Monetary and Fiscal Model (GIMF). This is a state-of-the-art dynamic general equilibrium model of the kind that is increasingly being deployed at central banks around the world, but with a far wider range of fiscal features. Like a conventional business cycle model, GIMF incorporates a range of nominal and real rigidities that are useful for short-run business cycle analysis, and an interest rate reaction function that is common in Inflation-Targeting countries such as Chile. In addition GIMF incorporates multiple and powerful non-Ricardian features that give an important role to fiscal policy, because in a non-Ricardian model the timing of taxes and transfers affects economic activity. These features include: overlapping generations of agents; life-cycle income profiles; liquidity-constrained consumers; and multiple distortionary taxes. This framework makes it meaningful to also incorporate a fiscal policy reaction function, specifically Chile's structural surplus rule.

We use a two-country version of GIMF that is carefully calibrated to reproduce structural features of the Chilean economy. These include the breakdown of GDP into its expenditure and income components, the breakdown of trade into its raw materials, intermediates and finished goods components, debt-to-GDP ratios of the public and private sectors, trend real and nominal growth rates, the composition of tax revenue between labor, consumption, capital income, and other taxes, and the composition of government outlays between expenditures on goods and services, transfers and interest expenses.

The key addition to the standard version of GIMF for the purpose of analyzing Chile's fiscal rule is a world copper market. This is critical due to the importance, especially most recently, of cyclical copper revenue for Chile's fiscal balance. Global copper output is modeled as an endowment, 38% of which accrues to Chile, as in the data. The copper price fluctuates with shocks to foreign industrial demand for copper, and the world copper market exhibits perfect price arbitrage. Total copper revenue is divided between domestic capital and labor, the domestic government, and foreigners, in the proportion observed in the data.

The rest of this paper is organized as follows. Section II presents the model. Section III discusses calibration. Section IV analyzes the effects of different parameterizations of the structural surplus rule on business cycle volatility. Section V analyzes the consequences of choosing a government surplus target that is not aligned with the existing debt stock. Section VI concludes.

II. The Model

The world consists of 2 countries, Chile and Foreign, where Foreign represents the rest of the world. In our exposition we will ignore country indices except when interactions between the two countries are concerned. It is understood that all parameters except population and technology growth can differ across countries. Figure 1 illustrates the flow of goods and factors.

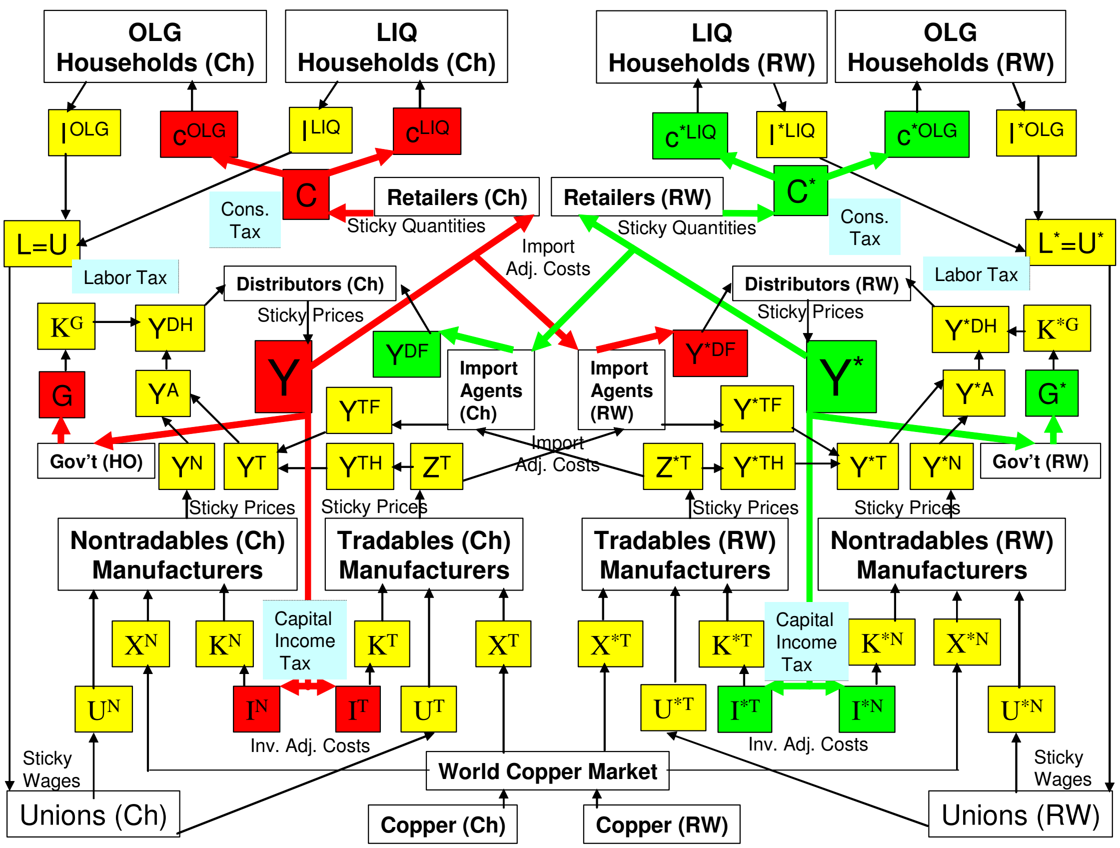
Countries are populated by two types of households, both of which consume final retailed output and supply labor to unions. First, there are overlapping generations households with finite planning horizons as in Blanchard (1985), and exhibiting external habit persistence. Each of these agents faces a constant probability of death $(1-\theta)$ in each period, which implies an average planning horizon of $1/(1-\theta)$.[2] In each period, $N(j)n^t(1-\psi(j))\left(1-\frac{\theta}{n}\right)$ of such individuals are born, where $N(j)$, $j \in \{Chile, Foreign\}$, indexes absolute population sizes in period 0, n is the world population growth rate, and $\psi(j)$ is the share of liquidity constrained agents in j. Second, there are liquidity constrained households who do not have access to financial markets, and who consequently are forced to consume their after tax income in every period. The number of such agents born in each period is $N(j)n^t\psi(j)\left(1-\frac{\theta}{n}\right)$. Aggregation over different cohorts of agents implies that the total numbers of agents in country j is $N(j)n^t$. For computational reasons we do not normalize world population to one. Instead we assume $N(Chile) = 1$, and set $N(Foreign)$ such that $N(Chile)/(N(Chile) + N(Foreign))$ equals the share of Chilean agents in the world population. In addition to the probability of death households also experience labor productivity that declines at a constant rate over their lifetimes. This simplified treatment of lifecycle income profiles is justified by the absence of explicit demographics in our model, and adds another powerful channel through which fiscal policies can have non-Ricardian effects. Households of both types are subject to a uniform labor income tax and a uniform consumption tax. We will denote variables pertaining to these two groups of households by OLG and LIQ.

Firms are managed in accordance with the preferences of their owners, myopic OLG households, and they therefore also have finite planning horizons. Each country's primary production is carried out by manufacturers producing tradable and nontradable goods. Manufacturers buy investment goods from distributors, and they buy labor from monopolistically competitive unions that are subject to nominal wage rigidities, and who in turn buy that labor from households. Manufacturers are subject to nominal rigidities in price setting as well as real rigidities in investment. Manufacturers' domestic sales go to domestic distributors. Their foreign sales go to import agents that are domestically owned but located in each export destination country. Import agents in turn sell their output to foreign distributors. When the pricing-to-market assumption is made these import agents are subject to nominal rigidities in foreign currency. Distributors first assemble nontradable goods and domestic and foreign tradable goods, where changes in the volume of imported inputs are subject to an adjustment cost. This private sector output is then combined with a publicly provided capital stock (infrastructure) as an essential further input. This capital stock is maintained through government investment expenditure that is financed by tax revenue. The combined domestic private and public sector output is

[2] In general we allow for the possibility that agents may be more myopic than what would be suggested by a planning horizon based on a biological probability of death.

then combined with foreign final output to produce domestic final output. Foreign final output is purchased through a second set of import agents that can price to the domestic market, and again changes in the volume of imported goods are subject to an adjustment cost. This second layer of trade at the level of final output is critical for allowing the model to produce the high trade to GDP ratios typically observed in small, highly open economies. Domestic final output is sold to domestic consumption goods retailers, domestic manufacturing firms (in their role as investors), the domestic government, and to final goods import agents located in foreign economies. Distributors are subject to another layer of nominal rigidities in price setting. This cascading of nominal rigidities from upstream to downstream sectors has important consequences for the behavior of aggregate inflation. Retailers, who are also monopolistically competitive, face real instead of nominal rigidities. While their output prices are flexible they find it costly to rapidly adjust their sales volume. This feature contributes to generating inertial consumption dynamics.

Figure 1. Goods and Factor Flows in GIMF



The world economy experiences constant positive trend growth $g_t = T_t/T_{t-1}$, where T_t is the level of labor augmenting world technology, and constant positive population growth n. When the model's real variables, say x_t, are rescaled, we divide by the level of technology T_t and by population, but for the latter we divide by n^t only, meaning real figures are not in per capita terms but rather in absolute terms adjusted for population growth. We use the notation $\check{x}_t = x_t/(T_t n^t)$, with the steady state of $\check{x}_t$ denoted by $\bar{x}$. An exception to this is quantities of labor, which are only rescaled by n^t.

Asset markets are incomplete. There is complete home bias in government debt, which takes the form of nominally non-contingent one-period bonds denominated in domestic currency. The only assets traded internationally are nominally non-contingent one-period bonds denominated in the currency of Foreign. There is also complete home bias in ownership of domestic firms. In addition equity is not traded in domestic financial markets, instead households receive lump-sum dividend payments. This assumption is required to support our assumption that firm and not just household preferences feature myopia.

A. Overlapping Generations Households

We first describe the optimization problem of OLG households. A representative member of this group and of age a derives utility at time t from consumption $c_{a,t}^{OLG}$ relative to the consumption habit $h_{a,t}^{OLG}$, leisure $(1-\ell_{a,t}^{OLG})$ (where 1 is the time endowment), and real balances $(M_{a,t}/P_t^R)$ (where P_t^R is the retail price index). The lifetime expected utility of a representative household of age a at time t has the form

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left[\frac{1}{1-\gamma} \left(\left(\frac{c_{a+s,t+s}^{OLG}}{h_{a+s,t+s}^{OLG}} \right)^{\eta^{OLG}} \left(1-\ell_{a+s,t+s}^{OLG}\right)^{1-\eta^{OLG}} \right)^{1-\gamma} + \frac{u^m}{1-\gamma} \left(\frac{M_{a+s,t+s}}{P_{t+s}^R} \right)^{1-\gamma} \right], \tag{1}$$

where E_t is the expectations operator, β is the discount factor, $\theta < 1$ is the degree of myopia, $\gamma > 0$ is the coefficient of relative risk aversion, $0 < \eta^{OLG} < 1$[3], and $u^m > 0$.

As for money demand, in the following analysis we will only consider the case of the cashless limit advocated by Woodford (2003), where $u^m \longrightarrow 0$. As a result the optimality conditions for money will be ignored throughout our analysis. Note that this does not involve a great loss of generality in our case, and in fact it has one major advantage. The reason is that the combination of separable money in the utility function and monetary policy specified as an interest rate rule implies that the money demand equation becomes redundant and that inflation is not directly distortionary for the consumption-leisure decision. But money also has a fiscal role through the government budget constraint, and any reduction in inflation tax revenue must be accompanied by an offsetting increase in other forms of distortionary taxation.[4] Because of this indirect distortionary effect, an increase in inflation in this model would actually reduce overall distortions unless we consider the case of the cashless limit, in which case inflation causes no distortions in either direction.

[3]For flexible model calibration we allow for the possibility that OLG households attach a different weight η^{OLG} to consumption than liquidity constrained households. This allows us to model both groups as working during an equal share of their time endowment in steady state, while OLG households have much higher consumption due to their accumulated wealth.

[4]Except for the special case of lump-sum taxation.

The consumption habit is given by lagged per capita consumption of OLG households[5], which in turn equals lagged aggregate consumption divided by the size of this population group,

$$h^{OLG}_{a,t} = \left(\frac{c^{OLG}_{t-1} g_t}{Nn^{t-1}(1-\psi)} \right)^{v} , \tag{2}$$

and where v parameterizes the degree of habit persistence. This is the external, catching up with the Joneses variety of habit persistence. Consumption $c^{OLG}_{a,t}$ is given by a CES aggregate over retailed consumption goods varieties $c^{OLG}_{a,t}(i)$, with elasticity of substitution σ_R:

$$c^{OLG}_{a,t} = \left(\int_0^1 \left(c^{OLG}_{a,t}(i) \right)^{\frac{\sigma_R - 1}{\sigma_R}} di \right)^{\frac{\sigma_R}{\sigma_R - 1}} . \tag{3}$$

This gives rise to a demand for individual varieties

$$c^{OLG}_{a,t}(i) = \left(\frac{P^R_t(i)}{P^R_t} \right)^{-\sigma_R} c^{OLG}_{a,t} , \tag{4}$$

where $P^R_t(i)$ is the retail price of variety i, and the aggregate retail price level P^R_t is given by

$$P^R_t = \left(\int_0^1 \left(P^R_t(i) \right)^{1-\sigma_R} di \right)^{\frac{1}{1-\sigma_R}} . \tag{5}$$

A household can hold two types of bonds, domestic government bonds $B_{a,t}$ denominated in domestic currency, and foreign bonds denominated in the currency of Foreign, $F_{a,t}$. The nominal exchange rate vis-a-vis Foreign is denoted by $\mathcal{E}_t$, and $\mathcal{E}_t F_{a,t}$ are nominal net foreign asset holdings in terms of domestic currency. In each case the time subscript t denotes financial claims held from period t to period $t+1$. For Chile, gross nominal interest rates on domestic and foreign currency denominated assets held from t to $t+1$ are i_t and $i_t(Foreign)(1+\xi^f_t)$, where $i_t(Foreign)$ is the nominal interest rate determined in Foreign, and ξ^f_t is a foreign exchange risk premium that is external to the household's asset accumulation decision and payable to Foreign households. Its functional form is given by

$$\xi^f_t = y_1 + \frac{y_2}{\left(y_4 - \frac{ca_t}{gdp_t} \right)^{y_3}} , \tag{6}$$

where $y_1 - y_4$ are parameters, ca_t/gdp_t is the current account to GDP ratio, and y_1 is constrained by the condition $y_1 = -y_2/y_4^{y_3}$. We have found this functional form to be more suitable for applied work than conventional quadratic specifications because it is asymmetric, allowing for a steeply increasing risk premium at large current account deficits.

Participation by households in financial markets requires that they enter into an insurance contract with companies that pay a premium of $\frac{(1-\theta)}{\theta}$ on a household's financial wealth for each period in which that household is alive, and that encash the household's entire financial wealth in the event of his death.[6]

[5] Multiplied by the aggregate technology growth rate.

[6] The turnover in the population is assumed to be large enough that the income receipts of the insurance companies exactly equal their payouts.

Apart from returns on financial assets, households also receive labor and dividend income. Households sell their labor to "unions" that are competitive in their input market and monopolistically competitive in their output market, vis-à-vis manufacturing firms. The productivity of a household's labor declines throughout his lifetime, with productivity $\Phi_{a,t} = \Phi_a$ of age group a given by

$$\Phi_a = \kappa \chi^a \ , \tag{7}$$

where $\chi < 1$. The overall population's average productivity is assumed without loss of generality to be equal to one. Household pre-tax nominal labor income is therefore $W_t \Phi_{a,t} \ell_{a,t}^{OLG}$. Dividends are received in a lump-sum fashion from all firms in the nontradables (N) and tradables (T) manufacturing sectors, the distribution (D), retail (R) and import agent (M) sectors, from the domestic (X) and foreign (F) copper sectors, and from all unions (U) in the labor market, with after-tax nominal dividends received from firm/union i denoted by $D_{a,t}^j(i)$, $j = N, T, D, R, U, M, X, F$. OLG households are liable to pay lump-sum transfers $\tau_{T_{a,t}}^{OLG}$ to the government, which in turn redistributes them to the relatively less well off LIQ agents. Household labor income is taxed at the rate $\tau_{L,t}$, consumption is taxed at the rate $\tau_{c,t}$, and in addition there is a lump-sum tax $\tau_{ls,t}^{OLG}$. It is assumed that retailers face costs of rapidly adjusting their sales volume. To limit these costs they therefore offer incentives (or disincentives) that are incorporated into the effective retail purchase price P_t^R. The consumption tax $\tau_{c,t}$ is however assumed to be payable on the pre-incentive price P_t, which equals the price at which retailers purchase consumption goods from distributors.[7] We choose the aggregate final goods price level P_t (determined by distributors) as our numeraire. We denote the real wage by $w_t = W_t/P_t$, the relative price of any good x by $p_t^x = P_t^x/P_t$, gross inflation for any good x by $\pi_t^x = P_t^x/P_{t-1}^x$, and gross nominal exchange rate depreciation by $\varepsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$.[8]

The household's budget constraint in nominal terms is

$$P_t^R c_{a,t}^{OLG} + P_t c_{a,t}^{OLG} \tau_{c,t} + B_{a,t} + \mathcal{E}_t F_{a,t} = \frac{1}{\theta} \left[i_{t-1} B_{a-1,t-1} + i_{t-1}(Foreign) \mathcal{E}_t F_{a-1,t-1} (1 + \xi_{t-1}^f) \right] \tag{8}$$

$$+ W_t \Phi_{a,t} \ell_{a,t}^{OLG} (1 - \tau_{L,t}) + \sum_{j=N,T,D,R,U,M,X,F} \int_0^1 D_{a,t}^j(i) di - P_t \tau_{T_{a,t}}^{OLG} - P_t \tau_{ls,t}^{OLG} \ .$$

The OLG household maximizes (1) subject to (2), (3), (7) and (8). The derivation of the first-order conditions for each generation, and aggregation across generations, is discussed in detail in the Appendices 1-3. Aggregation takes account of the size of each age cohort at the time of birth, and of the remaining size of each generation. Using the example of overlapping generations households' consumption, we have

$$c_t^{OLG} = N n^t (1 - \psi) \left(1 - \frac{\theta}{n} \right) \Sigma_{a=0}^{\infty} \left(\frac{\theta}{n} \right)^a c_{a,t}^{OLG} \ . \tag{9}$$

This also has implications for the intercept parameter κ of the age-specific productivity distribution. Under the assumption of an average productivity of one, and for given parameters χ and θ, we obtain $\kappa = (n - \theta\chi)/(n - \theta)$. Several of the optimality conditions

[7] Without this assumption consumption tax revenue could become too volatile in the short run.

[8] We adopt the convention throughout the paper that all nominal price level variables are written in upper case letters, and that all relative price variables are written in lower case letters.

that need to be aggregated are nonlinear Euler equations. In such conditions, aggregation requires nonlinear transformations that are only valid under certainty equivalence. Tractable aggregate consumption optimality conditions therefore only exist for the cases of perfect foresight and of first-order approximations. For our purposes this is not problematic as our application of GIMF will use only log-linear approximations. However, for the purpose of exposition we find it preferable to present optimality conditions in nonlinear form. We therefore adopt the notation $\tilde{E}_t$ to denote an expectations operator that is understood in this fashion.

The first-order conditions for the goods varieties and for the consumption/leisure choice are given by

$$\check{c}_t^{OLG}(i) = \left(\frac{P_t^R(i)}{P_t^R}\right)^{-\sigma_R} \check{c}_t^{OLG} \;, \tag{10}$$

$$\frac{\check{c}_t^{OLG}}{N(1-\psi) - \check{\ell}_t^{OLG}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} \check{w}_t \frac{(1-\tau_{L,t})}{(p_t^R + \tau_{c,t})} \;. \tag{11}$$

The arbitrage condition for foreign currency bonds (the uncovered interest parity relation) is given by

$$i_t = i_t(Foreign) \tilde{E}_t \varepsilon_{t+1} (1 + \xi_t^f) \;. \tag{12}$$

The consumption Euler equation on the other hand cannot be directly aggregated across generations. For each generation we have

$$E_t c_{a+1,t+1} = E_t j_t c_{a,t} \;, \tag{13}$$

$$\begin{aligned} j_t \;=\; & \left(\beta \frac{i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}} \left(\chi g_{t+1} \frac{\check{w}_{t+1}(1-\tau_{L,t+1})(p_t^R + \tau_{c,t})}{\check{w}_t(1-\tau_{L,t})(p_{t+1}^R + \tau_{c,t+1})}\right)^{(1-\eta^{OLG})(1-\frac{1}{\gamma})} \\ & \left(\frac{p_t^R + \tau_{c,t}}{p_{t+1}^R + \tau_{c,t+1}}\right)^{\frac{1}{\gamma}} \left(\frac{\check{c}_t^{OLG} g_{t+1}}{\check{c}_{t-1}^{OLG}}\right)^{\upsilon \eta^{OLG}(1-\frac{1}{\gamma})} \;. \end{aligned} \tag{14}$$

We introduce some additional notation. The production based real exchange rate vis-a-vis Foreign is $e_t = (\mathcal{E}_t P_t(Foreign))/P_t$, where $P_t(Foreign)$ is the price of final output in Foreign. We adopt the convention that each nominal asset is deflated by the final output price index of the currency of its denomination, so that real domestic bonds are $b_t = B_t/P_t$ and real foreign bonds are $f_t = F_t/P_t(Foreign)$. The real interest rate in terms of final output is $r_t = i_t/\pi_{t+1}$. The subjective and market nominal discount factors are given by

$$\tilde{R}_{t,s} = \Pi_{l=1}^s \frac{\theta}{i_{t+l-1}} \text{ for } s > 0 \; (=1 \text{ for } s = 0) \;, \tag{15}$$

$$R_{t,s} = \Pi_{l=1}^s \frac{1}{i_{t+l-1}} \text{ for } s > 0 \; (=1 \text{ for } s = 0) \;, \tag{16}$$

and the subjective and market real discount factors by

$$\tilde{r}_{t,s} = \Pi_{l=1}^s \frac{\theta}{r_{t+l-1}} \text{ for } s > 0 \; (=1 \text{ for } s = 0) \;, \tag{17}$$

$$r_{t,s} = \Pi_{l=1}^s \frac{1}{r_{t+l-1}} \text{ for } s > 0 \; (=1 \text{ for } s = 0) \;. \tag{18}$$

In each case the subjective discount factor incorporates an agent's probability of economic death, which ceteris paribus makes him value near term receipts more highly than receipts in the distant future.

We now discuss a key condition of GIMF, the optimal aggregate consumption rule of OLG households. The derivation of this condition is algebraically complex and is therefore presented in Appendix 3. The final result expresses current aggregate consumption of OLG households as a function of their real aggregate financial wealth fw_t and human wealth hw_t, with the marginal propensity to consume of out of wealth given by $1/\Theta_t$. Human wealth is in turn composed of hw_t^L, the expected present discounted value of households' time endowments evaluated at the after-tax real wage, and hw_t^K, the expected present discounted value of capital or dividend income net of lump-sum transfer payments to the government. After rescaling by technology we have

$$\check{c}_t^{OLG}\Theta_t = \check{f}w_t + \check{h}w_t^L + \check{h}w_t^K \ , \tag{19}$$

where

$$\check{f}w_t = \frac{1}{\pi_t g_t n}\left[i_{t-1}\check{b}_{t-1} + i_{t-1}(Foreign)\varepsilon_t(1+\xi_{t-1}^f)\check{f}_{t-1}e_{t-1}\right] \ , \tag{20}$$

$$\check{h}w_t^L = (N(1-\psi)(\check{w}_t(1-\tau_{L,t}))) + \tilde{E}_t\frac{\theta\chi g_{t+1}}{r_t}\check{h}w_{t+1}^L \ , \tag{21}$$

$$\check{h}w_t^K = \left(\check{d}_t^N + \check{d}_t^T + \check{d}_t^D + \check{d}_t^R + \check{d}_t^U + \check{d}_t^M + \overline{d^X} + d_t^F - \check{\tau}_{T,t} - \check{\tau}_{ls,t}^{OLG}\right) + \tilde{E}_t\frac{\theta g_{t+1}}{r_t}\check{h}w_{t+1}^K \ , \tag{22}$$

$$\Theta_t = \frac{p_t^R + \tau_{c,t}}{\eta^{OLG}} + \tilde{E}_t\frac{\theta j_t}{r_t}\Theta_{t+1} \ . \tag{23}$$

The intuition of (19) is key to GIMF. Financial wealth (20) is equal to the domestic government's and foreign households' *current* financial liabilities. For the government debt portion, the government services these liabilities through different forms of taxation, and these *future* taxes are reflected in the different components of human wealth (21) and (22) as well as in the marginal propensity to consume (23). But unlike the government, which is infinitely lived, an individual household factors in that he might not be around by the time higher future tax payments fall due. Hence *a household discounts future tax liabilities by a rate of at least r_t/θ, which is higher than the market rate r_t*, as reflected in the discount factors in (21), (22) and (23). The discount rate for the labor income component of human wealth is even higher at $r_t/\theta\chi$, due to the decline of labor incomes over individuals' lifetimes.

A fiscal consolidation through higher taxes represents a tilting of the tax payment profile from the more distant future to the near future, so as to effect a reduction in the debt stock. The government has to respect its intertemporal budget constraint in effecting this tilting, and this means that the expected present discounted value of its future primary surpluses has to remain equal to the current debt $i_{t-1}b_{t-1}/\pi_t$ *when future surpluses are discounted at the market interest rate r_t.* But when individual households discount future taxes at a higher rate than the government, the same tilting of the tax profile represents a decrease in human wealth because it increases the expected value of future taxes for which the household expects to be responsible. This is true both for the direct effect of labor income taxes on labor income receipts, and for the indirect effect of corporate taxes on dividend receipts. For a given marginal propensity to consume, these reductions in human wealth lead to a reduction in consumption.

The marginal propensity to consume $1/\Theta_t$ is, in the simplest case of logarithmic utility and exogenous labor supply, equal to $(1-\beta\theta)$. For the case of endogenous labor supply, household wealth can be used to either enjoy leisure or to generate purchasing power to buy goods. The main determinant of the split between consumption and leisure is the consumption share parameter η^{OLG}, which explains its presence in the marginal propensity to consume (23). While other forms of taxation affect the different components of wealth, the time profile of consumption taxes affects the marginal propensity to consume, reducing it with a balanced-budget shift of such taxes from the future to the present. The intertemporal elasticity of substitution $1/\gamma$ is another key parameter for the marginal propensity to consume. For the conventional assumption of $\gamma > 1$ the income effect of an increase in the real interest rate r is stronger than the substitution effect and tends to increase the marginal propensity to consume, thereby partly offsetting the contractionary effects of a higher r on human wealth $\check{hw}_t$. Expression (14) also reflects the effects of habit persistence on current consumption.

B. Liquidity Constrained Households

The objective function of liquidity constrained (LIQ) households is assumed to be nearly identical to that of OLG households:

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left[\frac{1}{1-\gamma} \left(\left(\frac{c^{LIQ}_{a+s,t+s}}{h^{LIQ}_{a+s,t+s}} \right)^{\eta^{LIQ}} \left(1 - \ell^{LIQ}_{a+s,t+s} \right)^{1-\eta^{LIQ}} \right)^{1-\gamma} \right] , \tag{24}$$

$$h^{LIQ}_{a,t} = \left(\frac{c^{LIQ}_{t-1} g_t}{N n^{t-1} \psi} \right)^{v} , \tag{25}$$

$$c^{LIQ}_{a,t} = \left(\int_0^1 \left(c^{LIQ}_{a,t}(i) \right)^{\frac{\sigma_R - 1}{\sigma_R}} di \right)^{\frac{\sigma_R}{\sigma_R - 1}} . \tag{26}$$

These agents can consume at most their current income, which consists of their after tax wage income plus government transfers $\tau^{LIQ}_{T_{a,t}}$. Their budget constraint is

$$P^R_t c^{LIQ}_{a,t} + P_t c^{LIQ}_{a,t} \tau_{c,t} \leqq W_t \Phi_{a,t} \ell^{LIQ}_{a,t} (1 - \tau_{L,t}) + P_t \tau^{LIQ}_{T_{a,t}} - P_t \tau^{LIQ}_{ls,t} . \tag{27}$$

The aggregated first-order conditions for this problem, after rescaling by technology, are

$$\check{c}^{LIQ}_t(i) = \left(\frac{P^R_t(i)}{P^R_t} \right)^{-\sigma_R} \check{c}^{LIQ}_t , \tag{28}$$

$$\check{c}^{LIQ}_t (p^R_t + \tau_{c,t}) = \check{w}_t \ell^{LIQ}_t (1 - \tau_{L,t}) + \check{\tau}_{T,t} - \check{\tau}^{LIQ}_{ls,t} , \tag{29}$$

$$\frac{\check{c}^{LIQ}_t}{N\psi - \check{\ell}^{LIQ}_t} = \frac{\eta^{LIQ}}{1 - \eta^{LIQ}} \check{w}_t \frac{(1 - \tau_{L,t})}{(p^R_t + \tau_{c,t})} . \tag{30}$$

To obtain aggregate consumption demand and labor supply we simply add the respective optimality conditions for OLG and LIQ households:

$$\check{C}_t = \check{c}_t^{OLG} + \check{c}_t^{LIQ} \ , \tag{31}$$

$$\check{L}_t = \check{\ell}_t^{OLG} + \check{\ell}_t^{LIQ} \ . \tag{32}$$

C. Manufacturers

There is a continuum of manufacturing firms indexed by $i \in [0,1]$ in two separate manufacturing sectors indexed by $J \in [N,T]$, where N represents nontradables and T tradables. For prices in these two sectors we introduce a slightly different index $\tilde{J} \in [N,TH]$, because the index T for prices is reserved for a different goods aggregate produced by distributors (see below). Manufacturers buy labor inputs from unions, copper inputs from copper producers, and capital inputs from distributors. Sector N and T manufacturers sell to domestic distributors, and sector T manufacturers also sell to import agents in foreign countries, who in turn sell to distributors in those countries.[9] Manufacturers are perfectly competitive in their input markets and monopolistically competitive in the market for their output. Their price setting is subject to nominal rigidities. We first analyze the demands for their output, then turn to their technology, and finally describe their optimization problem.

Demands for manufacturers' output varieties are given by

$$Y_t^J(z) = \left(\int_0^1 Y_t^J(z,i)^{\frac{\sigma_J - 1}{\sigma_J}} di \right)^{\frac{\sigma_J - 1}{\sigma_J}} \quad , \quad Y_t^{TX}(z) = \left(\int_0^1 Y_t^{TX}(z,i)^{\frac{\sigma_J - 1}{\sigma_J}} di \right)^{\frac{\sigma_J - 1}{\sigma_J}} \quad , \tag{33}$$

where $Y_t^J(z,i)$ and $Y_t^J(z)$ are variety i and total demands from domestic distributor z in sector J, and $Y_t^{TX}(z,i)$ and $Y_t^{TX}(z)$ are variety i and total demands for exports from Chile to import agent z in Foreign. Cost minimization by distributors and import agents generates demands for varieties

$$Y_t^J(z,i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Y_t^J(z) \quad , \quad Y_t^{TX}(z,i) = \left(\frac{P_t^{TH}(i)}{P_t^{TH}} \right)^{-\sigma_J} Y_t^{TX}(z) \ , \tag{34}$$

with price indices defined as

$$P_t^{\tilde{J}} = \left(\int_0^1 P_t^{\tilde{J}}(i)^{1-\sigma_J} di \right)^{\frac{1}{1-\sigma_J}} \ . \tag{35}$$

The aggregate demand for variety i produced by sector J can be derived by simply integrating over all distributors, import agents and all other sources of manufacturing output demand. We obtain

$$Z_t^J(i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J \ , \tag{36}$$

[9] There are also some small sales of aggregate manufacturing output back to manufacturing firms, related to manufacturers' need for resources to pay for adjustment costs.

where $Z_t^J(i)$ and Z_t^J remain to be specified by way of market clearing conditions for manufacturing goods.

The **technology** of each manufacturing firm is given by a nested CES production function. This first combines copper inputs $X_t^J(i)$ with a capital-labor composite $M_t^J(i)$, with elasticity of substitution ξ_{XJ}. A sub–production function then defines $M_t^J(i)$ as a CES aggregate in capital $K_t^J(i)$ and union labor $U_t^J(i)$, with elasticity of substitution ξ_{ZJ} and labor augmenting productivity T_t:[10,11]

$$\begin{aligned} Z_t^J(i) &= F(M_t^J(i), X_t^J(i)) && (37)\\ &= \mathcal{T}\left(\left(1-\alpha_t^X\right)^{\frac{1}{\xi_{XJ}}}\left(M_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} + \left(\alpha_t^X\right)^{\frac{1}{\xi_{XJ}}}\left(X_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}}\right)^{\frac{\xi_{XJ}}{\xi_{XJ}-1}} , \end{aligned}$$

$$\begin{aligned} M_t^J(i) &= F(K_t^J(i), U_t^J(i)) && (38)\\ &= \left(\left(1-\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}}\left(K_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}}\left(T_t U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}}\right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} . \end{aligned}$$

The only shock we will consider in this model, due to its great importance for Chile's fiscal policy, is to copper demand and therefore, by implication, to copper prices. Specifically, we assume that the Foreign copper share parameter $\alpha_t^X(Foreign)$, which is equal across sectors, follows the following stochastic process:

$$\alpha_t^X(Foreign) = \left(1-\rho^X\right)\overline{\alpha^X(Foreign)} + \rho^X \alpha_{t-1}^X(Foreign) + \mathbf{e}^X . \tag{39}$$

Manufacturing firms are subject to two types of adjustment costs. First, quadratic inflation adjustment costs $G_{P,t}^J(i)$ are real resource costs that represent a demand for the output of sector J. Following Ireland (2001) and Laxton and Pesenti (2003), they are quadratic in changes in the rate of inflation rather than in price levels, which is essential in order to generate realistic inflation dynamics. Compared to versions of the Calvo (1983) price setting assumption such adjustment costs have the advantage of greater analytical tractability. We have:

$$G_{P,t}^J(i) = \frac{\phi_{PJ}}{2} Z_t^J \left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1\right)^2 . \tag{40}$$

Second, investment adjustment is subject to quadratic adjustment costs $G_{I,t}(i)$:

$$G_{I,t}^J(i) = \frac{\phi_I}{2} K_t^J(i) \left(\frac{I_t^J(i)}{K_t^J(i)} - \frac{I_{t-1}^J}{K_{t-1}^J}\right)^2 . \tag{41}$$

[10] Note that, for the sake of clarity, we make a notational distinction between two types of elasticities of substitution. Elasticities between continua of goods varieties, which give rise to market and pricing power, are denoted by a σ subscripted by the respective sectorial indicator. Elasticities between factors of production, both in manufacturing and in final goods distribution, are denoted by a ξ subscripted by the respective sectorial indicator.

[11] The factor $\mathcal{T}$ is a constant that can be set different from one to obtain different levels of GDP per capita across countries.

The law of motion of capital is described by

$$K_{t+1}^{J}(i) = (1-\delta)\, K_t^J(i) + I_t^J(i)\ , \tag{42}$$

where δ represents the depreciation rate of capital.

It is assumed that each firm pays out each period's after tax nominal net cash flow as dividends $D_t^J(i)$. It maximizes the expected present discounted value of dividends $D_t^J(i)$. The discount rate it applies in this maximization includes the parameter θ so as to equate the discount factor of firms θ/r_t with the pricing kernel for nonfinancial income streams of their owners, myopic households, which equals $\beta\theta E_t\left(\lambda_{a+1,t+1}/\lambda_{a,t}\right)$. This equality follows directly from OLG households' first order condition for government debt holdings $\lambda_{a,t} = \beta E_t\left(\lambda_{a+1,t+1}\frac{i_t}{\pi_{t+1}}\right)$. Pre-tax net cash flow equals nominal revenue $P_t^{\tilde{J}}(i)Z_t^J(i)$ minus nominal cash outflows. The latter include the wage bill $V_t U_t^J(i)$, where V_t is the aggregate wage rate charged by unions, spending on copper $P_t^X X_t^J(i)$, where P_t^X is the nominal domestic currency price of copper, investment $P_t I_t^J(i)$ and investment adjustment costs $P_t G_{I,t}(i)$ that represent a demand for final output Y_t, and a fixed cost $P_t^{\tilde{J}} T_t \omega^J$ and price adjustment costs $P_t^{\tilde{J}} G_{P,t}^J(i)$ that represent a demand for sectorial manufacturing output Z_t^J. The fixed resource cost arises as long as the firm chooses to produce positive output. Net output in sector J is therefore equal to $\max(0, Z_t^J(i) - T_t\omega^J)$. The fixed cost is calibrated to make the steady state shares of economic profits, labor and capital in GDP consistent with the data. This becomes necessary because the model counterpart of the aggregate income share of capital equals not only the return to capital but also the profits of monopolistically competitive firms. With several layers of such firms the profits share becomes very large, and the capital share in the production function has to be reduced accordingly, unless fixed costs are assumed.

Notice also that net cash flow does not equal economic profit because investment expenditure represents a cash outflow but not an expenditure. The capital related expenditure relevant for profits is the nominal return to capital $R_{k,t}^J K_t^J(i)$. The total after tax net cash flow or dividend of the firm is[12]

$$\begin{aligned} D_t^J(i) \quad = \quad & \left[P_t^{\tilde{J}}(i)Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - P_t I_t^J(i) - P_t G_{I,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - P_t^{\tilde{J}} T_t \omega^J\right] \\ & -\tau_{k,t}\left[R_{k,t}^J - \delta P_t q_t^J\right] K_t^J(i)\ . \end{aligned} \tag{43}$$

The **optimization problem** of each manufacturing firm is given by

$$\underset{\left\{P_{t+s}^{\tilde{J}}(i), U_{t+s}^J(i), I_{t+s}^J(i), K_{t+s+1}^J(i)\right\}_{s=0}^{\infty}}{Max} \quad E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i)\ , \tag{44}$$

subject to the definition of dividends (43), demands (36), production functions (37) and (38), and adjustment costs (40), (41). The first-order conditions for this problem are derived in some detail in Appendix 4. A key step is to recognize that all firms behave identically in equilibrium, so that $P_t^{\tilde{J}}(i) = P_t^{\tilde{J}}$ and $Z_t^J(i) = Z_t^J$. Let λ_t^J denote the real

[12] Note that the last term assumes that the depreciation allowance for capital income taxation purposes is evaluated at current market prices of installed capital $P_t q_t^J K_t^J$, as opposed to the book value of installed capital. While this may not correspond exactly to most real world tax systems, it does correspond exactly to the nominal economic loss to the firm due to capital depreciation.

marginal cost of producing an additional unit of manufacturing output. Also, rescale the optimality conditions by technology and population as discussed above. Then the condition for $P_t^{\tilde{J}}(i)$ is

$$\left[\frac{\sigma_J}{\sigma_J - 1}\frac{\lambda_t^J}{p_t^{\tilde{J}}} - 1\right] = \frac{\phi_{P^J}}{\sigma_J - 1}\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}\right)\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}} - 1\right) \tag{45}$$

$$-E_t\frac{\theta g_{t+1} n}{r_t}\frac{\phi_{P^J}}{\sigma_J - 1}\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}\right)\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}} - 1\right) .$$

The first order condition for labor demand $U_t^J(i)$ and for copper demand $X_t^J(i)$ are

$$\check{v}_t = \lambda_t^J \check{F}_{U,t}^J , \tag{46}$$

$$p_t^X = \lambda_t^J \check{F}_{X,t}^J , \tag{47}$$

where $\check{F}_{U,t}^J$ and $\check{F}_{X,t}^J$ are the marginal products of labor and copper, for which closed form solutions are shown in Appendix 4. There is no equivalent condition determining the real return to capital $r_{k,t}^J$, because capital is owned by the firm and not rented through a market. However, in order to determine the profits and capital income taxes payable to them, the fiscal authorities must impute $r_{k,t}^J$. We assume that it is imputed to be equivalent to what would be obtained if capital was rented through a market, namely

$$r_{k,t}^J = \lambda_t^J \check{F}_{K,t}^J . \tag{48}$$

The first order condition for investment demand $I_t^J(i)$ is

$$q_t^J = 1 + \phi_I\left(\frac{\check{I}_t^J}{\check{K}_t^J} - \frac{\check{I}_{t-1}^J}{\check{K}_{t-1}^J}\right) , \tag{49}$$

while the Euler equation for capital, i.e. the first order condition with respect to $K_{t+1}^J(i)$, is

$$q_t^J = \frac{\theta}{r_t}E_t\left[q_{t+1}^J(1-\delta) + r_{k,t+1}^J - \tau_{k,t+1}\left(r_{k,t+1}^J - \delta q_{t+1}^J\right)\right] \tag{50}$$

$$+\phi_I\frac{\theta}{r_t}E_t\left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J}\right)\left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J} - \frac{\check{I}_t^J}{\check{K}_t^J}\right) - \frac{\phi_I}{2}\frac{\theta}{r_t}E_t\left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J} - \frac{\check{I}_t^J}{\check{K}_t^J}\right)^2 .$$

Finally, the rescaled aggregate dividends of firms in each sector are

$$\begin{aligned} \check{d}_t^J \quad &= \quad \left[p_t^{\tilde{J}}\check{Z}_t^J - \check{v}_t U_t^J - p_t^X \check{X}_t^J - \check{I}_t^J - \check{G}_{I,t}^J - p_t^{\tilde{J}}\check{G}_{P,t}^J - p_t^{\tilde{J}}\omega^J\right] \\ &\quad -\tau_{k,t}\left[r_{k,t}^J - \delta q_t^J\right]\check{K}_t^J(i) . \end{aligned} \tag{51}$$

D. Copper Producers

Copper supply or output in each country is specified, for simplicity, as an exogenous endowment X_t^{sup}. In this paper we will only consider the cases where copper price shocks originate in demand rather than supply shocks, that is we will treat the endowment as a constant. The world copper market is subject to perfect worldwide price arbitrage, with the domestic copper prices denoted by p_t^X. Total copper price revenues are paid out to three recipients. We denote the payments to foreigners as f_t^X, the payments to the domestic government as g_t^X, and the payments to domestic factors of production as d_t^X. Furthermore, we assume that the payments going to domestic factors do not change with the business cycle, $d_t^X = \overline{d^X}$, so that all cyclical excess revenue goes to either foreigners or the domestic government. To summarize, we have

$$p_t^X X_t^{sup} = g_t^X + \overline{d^X} + f_t^X \ . \tag{52}$$

The net exports of the domestic copper sector are given by

$$X_t^x = p_t^X \left(X_t^{sup} - X_t^{dem} \right) \ . \tag{53}$$

E. Unions

There is a continuum of unions indexed by $i \in [0,1]$. Unions buy labor from households and sell labor to manufacturers. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their wage setting is subject to nominal rigidities. We first analyze the demands for union output and then describe their optimization problem.

Demand for unions' labor output varieties comes from manufacturing firms $z \in [0,1]$ in sectors $J \in [N,T]$. The demand for union labor by firm z in sector J is given by

$$U_t^J(z) = \left(\int_0^1 \left(U_t^J(z,i) \right)^{\frac{\sigma_U - 1}{\sigma_U}} di \right)^{\frac{\sigma_U}{\sigma_U - 1}} , \tag{54}$$

where $U_t^J(z,i)$ is the demand by firm z for the labor variety supplied by union i. Given imperfect substitutability between the labor supplied by different unions, they have market power vis-à-vis manufacturing firms. Their demand functions are given by

$$U_t^J(z,i) = \left(\frac{V_t(i)}{V_t} \right)^{-\sigma_U} U_t^J(z) \ , \tag{55}$$

where $V_t(i)$ is the wage charged to employers by union i and V_t is the aggregate wage paid by employers, given by

$$V_t = \left(\int_0^1 V_t(i)^{1-\sigma_U} di \right)^{\frac{1}{1-\sigma_U}} \ . \tag{56}$$

The demand (55) can be aggregated over firms z and sectors J to obtain

$$U_t(i) = \left(\frac{V_t(i)}{V_t} \right)^{-\sigma_U} U_t \ , \tag{57}$$

where U_t is aggregate labor demand by all manufacturing firms. Nominal wage rigidities in this sector take the form familiar from (40):

$$G^U_{P,t}(i) = \frac{\phi_{P^U}}{2} U_t T_t \left(\frac{\frac{V_t(i)}{V_{t-1}(i)}}{\frac{V_{t-1}}{V_{t-2}}} - 1 \right)^2 . \tag{58}$$

Note that these adjustment costs are zero in steady state even though real wages grow at the rate of world technological progress. Also, the level of world technology enters as a scaling factor in (58), as otherwise these costs would become insignificant over time.

The **optimization** problem of a union consists of maximizing the expected present discounted value of nominal wages paid by firms $V_t(i)U_t(i)$ minus nominal wages paid out to workers $W_t U_t(i)$, minus nominal wage inflation adjustment costs $P_t G^U_{P,t}(i)$. Unlike manufacturers, this sector does not face fixed costs of operation. It is assumed that each union pays out each period's nominal net cash flow as dividends $D^U_t(i)$. The objective function of unions is

$$\underset{\{V_{t+s}(i)\}_{s=0}^{\infty}}{Max} \; E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(V_{t+s}(i) - W_{t+s} \right) U_{t+s}(i) - P_{t+s} G^U_{P,t+s}(i) \right] , \tag{59}$$

subject to labor demands (57) and adjustment costs (58). We obtain the first order condition for this problem. As all unions face an identical problem, their solutions are identical and the index i can be dropped in all first-order conditions of the problem, with $V_t(i) = V_t$ and $U_t(i) = U_t$. Letting $\pi^V_t = V_t / V_{t-1}$, the gross rate of wage inflation, and rescaling by technology, we obtain the condition

$$\begin{aligned} \left[\frac{\sigma_U}{\sigma_U - 1} \check{w}_t - \check{v}_t \right] &= \frac{\phi_{P^U}}{\sigma_U - 1} \left(\frac{\pi^V_t}{\pi^V_{t-1}} \right) \left(\frac{\pi^V_t}{\pi^V_{t-1}} - 1 \right) \\ &- E_t \frac{\theta g_{t+1} n}{r_t} \frac{\phi_{P^U}}{\sigma_U - 1} \frac{\check{U}_{t+1}}{\check{U}_t} \left(\frac{\pi^V_{t+1}}{\pi^V_t} \right) \left(\frac{\pi^V_{t+1}}{\pi^V_t} - 1 \right) . \end{aligned} \tag{60}$$

Real "dividends" from union organization, denominated in terms of final output, are distributed lump-sum to households in proportion to their share in aggregate labor supply. After rescaling they take the form

$$\check{d}^U_t = (\check{v}_t - \check{w}_t) \check{U}_t - \check{G}^U_{P,t} . \tag{61}$$

We also have $\check{v}_t / \check{v}_{t-1} = (V_t / P_t T_t) / (V_{t-1} / P_{t-1} T_{t-1})$, so that

$$\frac{\check{v}_t}{\check{v}_{t-1}} = \frac{\pi^V_t}{\pi_t g_t} . \tag{62}$$

F. Import Agents

Each country owns two continua of import agents in its export destination market, one for manufactured intermediate tradable goods and another for final goods, each indexed by $i \in [0,1]$ and by $J \in [T, D]$, where T stands for manufactured intermediate tradable goods and D for final goods. Import agents buy tradable goods (or final goods) from manufacturers (or distributors) in their owners' country and sell these goods to distributors in the destination country. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. We first analyze the demands for their output and then describe their optimization problem.

Demand for the output varieties supplied by import agents comes from distributors $z \in [0,1]$. This is true for goods in both sectors T and D, but those goods enter at different stages of production, see Figure 1. Chile distributors z require a CES imports aggregate $Y_t^{JM}(z)$ from the import agents of Foreign. That aggregate consists of varieties supplied by different import agents i, $Y_t^{JM}(z,i)$, with respective prices $P_t^{JM}(i)$, and is given by

$$Y_t^{JM}(z) = \left(\int_0^1 \left(Y_t^{JM}(z,i) \right)^{\frac{\sigma_{JM}-1}{\sigma_{JM}}} di \right)^{\frac{\sigma_{JM}}{\sigma_{JM}-1}} . \tag{63}$$

This gives rise to demands for varieties of

$$Y_t^{JM}(z,i) = \left(\frac{P_t^{JM}(i)}{P_t^{JM}} \right)^{-\sigma_{JM}} Y_t^{JM}(z) , \tag{64}$$

$$P_t^{JM} = \left(\int_0^1 P_t^{JM}(i)^{1-\sigma_{JM}} di \right)^{\frac{1}{1-\sigma_{JM}}} , \tag{65}$$

and these demands can be aggregated over distributors z to yield

$$Y_t^{JM}(i) = \left(\frac{P_t^{JM}(i)}{P_t^{JM}} \right)^{-\sigma_{JM}} Y_t^{JM} . \tag{66}$$

Nominal rigidities in this sector take the form familiar from (40):

$$G_{P,t}^{JM}(i) = \frac{\phi_{P^{JM}}}{2} Y_t^{JM} \left(\frac{\frac{P_t^{JM}(i)}{P_{t-1}^{JM}(i)}}{\frac{P_{t-1}^{JM}}{P_{t-2}^{JM}}} - 1 \right)^2 . \tag{67}$$

Import agents' cost minimizing solution for inputs of manufactured intermediate tradable goods (or final goods) varieties follows equations (33) - (35) (or similar conditions for final goods demand derived in the section on distributors) above when we recognize that $Y_t^{JM}(Chile,i) = Y_t^{JX}(Foreign,i)$ and $Y_t^{JM}(Chile) = Y_t^{JX}(Foreign)$. We denote the price of inputs imported from Foreign at the border of Chile by $P_t^{JM,cif}(Chile)$, the cif (cost, insurance, freight) import price. By purchasing power parity this satisfies $P_t^{JM,cif}(Chile) = P_t^{JH}(Foreign)\ \mathcal{E}_t(Chile)$, or

$$p_t^{JM,cif}(Chile) = p_t^{JH}(Foreign) e_t , \tag{68}$$

where we note that $p_t^{DH}(Foreign) = 1$ because final output is the numeraire in each economy.

The **optimization** problem of import agents consists of maximizing the expected present discounted value of nominal revenue $P_t^{JM}(i)Y_t^{JM}(i)$ minus nominal costs of inputs $P_t^{JM,cif}Y_t^{JM}(i)$, minus nominal inflation adjustment costs $P_t G_{P,t}^{JM}(i)$. The latter represent a demand for final output. Like unions, this sector does not face fixed costs of operation. It is assumed that each import agent pays out each period's nominal net cash flow as dividends $D_t^{JM}(i)$. The objective function of import agents is

$$\underset{\left\{P_{t+s}^{JM}(i)\right\}_{s=0}^{\infty}}{Max} E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[P_{t+s}^{JM}(i) Y_{t+s}^{JM}(i) - P_{t+s}^{JM,cif} Y_{t+s}^{JM}(i) - P_{t+s} G_{P,t+s}^{JM}(i) \right] , \tag{69}$$

subject to demands (66) and adjustment costs (67). The first order condition for this problem, after dropping firm specific subscripts and rescaling by technology, has the form:

$$\left[\frac{\sigma_{JM}}{\sigma_{JM}-1} p_t^{JM,cif} - p_t^{JM} \right] = \frac{\phi_{PJM}}{\sigma_{JM}-1} \left(\frac{\pi_t^{JM}}{\pi_{t-1}^{JM}} \right) \left(\frac{\pi_t^{JM}}{\pi_{t-1}^{JM}} - 1 \right) \tag{70}$$

$$-E_t \frac{\theta g_{t+1} n}{r_t} \frac{\phi_{PJM}}{\sigma_{JM}-1} \frac{\check{Y}_{t+1}^{JM}}{\check{Y}_t^{JM}} \left(\frac{\pi_{t+1}^{JM}}{\pi_t^{JM}} \right) \left(\frac{\pi_{t+1}^{JM}}{\pi_t^{JM}} - 1 \right) ,$$

where

$$\frac{p_t^{JM}}{p_{t-1}^{JM}} = \frac{\pi_t^{JM}}{\pi_t} . \tag{71}$$

The total dividends received by OLG households in Chile, expressed in terms of Chile output, are

$$\check{d}_t^{JM}(Chile) = \left[(p_t^{JM}(Foreign) - p_t^{JM,cif}(Foreign)) \check{Y}_t^{JM}(Foreign) - \check{G}_{P,t}^{JM}(Foreign) \right] e_t . \tag{72}$$

G. Distributors

This sector produces final output. There is a continuum of distributors indexed by $i \in [0,1]$. Distributors buy goods from manufacturers and import agents. They also use the stock of public infrastructure free of a user charge. Distributors sell final output to consumption goods retailers, manufacturing firms (in their role as investors), the government, final goods import agents located in foreign countries, and to various other sectors for fixed costs and adjustment costs. They are perfectly competitive in their input markets and monopolistically competitive in their output market. Their price setting is subject to nominal rigidities. We first analyze the demand for final output, then we turn to distributors' technology, starting upstream and finishing with final output, and finally we describe their profit maximization problem.

Demand for the final output varieties supplied by distributors comes from multiple sources. Let z be an individual purchaser of final output. Then his demand $\mathcal{D}_t(z)$ is for a CES composite of final output varieties i, with elasticity of substitution σ_D:

$$\mathcal{D}_t(z) = \left(\int_0^1 \left(\mathcal{D}_t(z,i) \right)^{\frac{\sigma_D - 1}{\sigma_D}} di \right)^{\frac{\sigma_D}{\sigma_D - 1}} , \tag{73}$$

with associated demands

$$\mathcal{D}_t(z,i) = \left(\frac{P_t(i)}{P_t}\right)^{-\sigma_D} \mathcal{D}_t(z) \ , \tag{74}$$

where $P_t(i)$ is the price of variety i of final output, and P_t is the aggregate price level for final output given by

$$P_t = \left(\int_0^1 (P_t(i))^{1-\sigma_D} di\right)^{\frac{1}{1-\sigma_D}} \ . \tag{75}$$

Furthermore, the total demand facing a distributor of final goods variety i can be obtained by aggregating over all sources of demand z. We obtain

$$\mathcal{D}_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\sigma_D} \mathcal{D}_t \ , \tag{76}$$

where $\mathcal{D}_t(i)$ and $\mathcal{D}_t$ remain to be specified by way of a market clearing condition for final output.

We divide our description of the **technology** of distributors into a number of stages. In the first stage a tradables composite is produced by combining foreign tradables with domestic tradables, subject to an adjustment cost that makes rapid changes in the share of foreign tradables costly. In the second stage a tradables-nontradables composite is produced. In the third stage the tradables-nontradables composite is combined with a publicly provided stock of infrastructure. And in the fourth stage the private-public composite is combined with final output originating in the foreign economy and sold to distributors by import agents, and again subject to an import adjustment cost. We discuss each of these stages in turn.

The **tradables composite** $Y_t^T(i)$ is produced by combining foreign produced tradables $Y_t^{TF}(i)$ with domestically produced tradables $Y_t^{TH}(i)$, in a CES technology with elasticity of substitution ξ_T. A key concern in open economy DSGE models is the potential for an excessive short-term responsiveness of international trade to real exchange rate movements. This model avoids that problem by introducing adjustment costs $G_{F,t}^T(i)$ that make it costly to vary the share of Foreign produced tradables in total tradables production $Y_t^{TF}(i)/Y_t^T(i)$ relative to the value of that share in the aggregate distribution sector in the previous period Y_{t-1}^{TF}/Y_{t-1}^T. The sub-production function for tradables therefore has the following form:[13]

$$Y_t^T(i) = \left((\alpha_{TH})^{\frac{1}{\xi_T}} \left(Y_t^{TH}(i)\right)^{\frac{\xi_T-1}{\xi_T}} + (1-\alpha_{TH})^{\frac{1}{\xi_T}} \left(Y_t^{TF}(i)(1-G_{F,t}^T(i))\right)^{\frac{\xi_T-1}{\xi_T}} \right)^{\frac{\xi_T}{\xi_T-1}} \ , \tag{77}$$

$$G_{F,t}^T(i) = \frac{\phi_{FT}}{2} \frac{\left(aux_t^T - 1\right)^2}{1+\left(aux_t^T - 1\right)^2} \ , \tag{78}$$

$$aux_t^T = \frac{\frac{Y_t^{TF}(i)}{Y_t^T(i)}}{\frac{Y_{t-1}^{TF}}{Y_{t-1}^T}} \ . \tag{79}$$

[13] Home bias in tradables use depends on the parameter α_{TH} and on a similar parameter α_{DH} at the level of final goods imports.

After dropping the firm specific indices and expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the aggregate tradables sub-production function from (77) - (79). We also obtain the following first-order conditions for optimal input choice:

$$\check{Y}_t^{TH} = \alpha_{TH} \check{Y}_t^T \left(\frac{p_t^{TH}}{p_t^T} \right)^{-\xi_T} , \tag{80}$$

$$\check{Y}_t^{TF} \left[1 - G_{F,t}^T \right] = (1 - \alpha_{TH}) \check{Y}_t^T \left(\frac{p_t^{TF}}{p_t^T} \right)^{-\xi_T} \left(X_t^T \right)^{\xi_T} , \tag{81}$$

$$X_t^T = 1 - G_{F,t}^T - \phi_{FT} \frac{aux_t^T \left(aux_t^T - 1 \right)}{\left[1 + \left(aux_t^T - 1 \right)^2 \right]^2} . \tag{82}$$

The **tradables-nontradables composite** $Y_t^A(i)$ is produced with another CES production function with elasticity of substitution ξ_A:

$$Y_t^A(i) = \left((1 - \alpha_N)^{\frac{1}{\xi_A}} \left(Y_t^T(i) \right)^{\frac{\xi_A - 1}{\xi_A}} + (\alpha_N)^{\frac{1}{\xi_A}} \left(Y_t^N(i) \right)^{\frac{\xi_A - 1}{\xi_A}} \right)^{\frac{\xi_A}{\xi_A - 1}} . \tag{83}$$

The real marginal cost of producing $Y_t^A(i)$ is, with obvious notation for sectorial price levels,

$$p_t^A = \left[(1 - \alpha_N) \left(p_t^T \right)^{1 - \xi_A} + \alpha_N \left(p_t^N \right)^{1 - \xi_A} \right]^{\frac{1}{1 - \xi_A}} . \tag{84}$$

After dropping the firm specific indices and expressing prices in terms of the numeraire, and after rescaling by technology, we obtain the aggregate tradables-nontradables sub-production function from (87), and the following first-order conditions for optimal input choice:

$$\check{Y}_t^N = \alpha_N \check{Y}_t^A \left(\frac{p_t^N}{p_t^A} \right)^{-\xi_A} , \tag{85}$$

$$\check{Y}_t^T = \alpha_T \check{Y}_t^A \left(\frac{p_t^T}{p_t^A} \right)^{-\xi_A} . \tag{86}$$

The **private-public composite** $Y_t^{DH}(i)$ is produced with the following production function:

$$Y_t^{DH}(i) = Y_t^A(i) \left(K_t^G \right)^{\alpha_G} \mathcal{S} . \tag{87}$$

The inputs are the tradables-nontradables composite $Y_t^A(i)$ and the stock of public infrastructure K_t^G, which is identical for all firms and provided free of charge to the end user (but not of course to the taxpayer). Note that this production function exhibits constant returns to scale in private inputs while the public capital stock enters externally, in an analogous manner to exogenous technology. The term $\mathcal{S}$ is a technology scale factor that can be used to normalize steady state technology to one, $\left(\bar{K}^G \right)^{\alpha_G} \mathcal{S} = 1$.

The real marginal cost of $Y_t^{DH}(i)$ is given by p_t^{DH}, while the real marginal cost of $Y_t^A(i)$ is p_t^A. After dropping the firm specific indices and expressing prices in terms of the numeraire, and after rescaling by technology and population, we obtain the aggregate Y_t^{DH} from (87), and the following first-order condition:

$$p_t^{DH} \left(\check{K}_t^G\right)^{\alpha_G} \mathcal{S} = p_t^A \ . \tag{88}$$

Final output $Y_t(i)$ is produced by combining foreign produced final output $Y_t^{DF}(i)$ with the domestically produced private-public composite $Y_t^{DH}(i)$, in a CES technology with elasticity of substitution ξ_D. Adjustment costs $G_{F,t}^D(i)$ make it costly to vary the share of foreign produced final output in domestic final output $Y_t^{DF}(i)/Y_t(i)$ relative to the value of that share in the aggregate distribution sector in the previous period Y_{t-1}^{DF}/Y_{t-1}. The sub-production function for final output therefore has the following form:

$$Y_t(i) = \left((\alpha_{DH})^{\frac{1}{\xi_D}} \left(Y_t^{DH}(i)\right)^{\frac{\xi_D - 1}{\xi_D}} + (1-\alpha_{DH})^{\frac{1}{\xi_D}} \left(Y_t^{DF}(i)(1 - G_{F,t}^D(i))\right)^{\frac{\xi_D - 1}{\xi_D}} \right)^{\frac{\xi_D}{\xi_D - 1}} , \tag{89}$$

$$G_{F,t}^D(i) = \frac{\phi_{FD}}{2} \frac{\left(aux_t^D - 1\right)^2}{1 + \left(aux_t^D - 1\right)^2} , \tag{90}$$

$$aux_t^D = \frac{\frac{Y_t^{DF}(i)}{Y_t(i)}}{\frac{Y_{t-1}^{DF}}{Y_{t-1}}} . \tag{91}$$

After dropping the firm specific indices and expressing prices in terms of the numeraire, and after rescaling by technology, we obtain the aggregate tradables sub-production function from (89) - (91). We also obtain the following first-order conditions for optimal input choice:

$$\check{Y}_t^{DH} = \alpha_{DH} \check{Y}_t \left(\frac{p_t^{DH}}{p_t^D}\right)^{-\xi_D} , \tag{92}$$

$$\check{Y}_t^{DF} \left[1 - G_{F,t}^D\right] = (1-\alpha_{DH}) \check{Y}_t \left(\frac{p_t^{DF}}{p_t^D}\right)^{-\xi_D} \left(X_t^D\right)^{\xi_D} , \tag{93}$$

$$X_t^D = 1 - G_{F,t}^D - \phi_{FD} \frac{aux_t^D \left(aux_t^D - 1\right)}{\left[1 + \left(aux_t^D - 1\right)^2\right]^2} . \tag{94}$$

We finally turn to the **profit maximization** problem. It consists of maximizing the expected present discounted value of nominal revenue $P_t(i)Y_t(i)$ minus nominal costs of production $P_t^D Y_t(i)$, a fixed cost $P_t T_t \omega^D$, and inflation adjustment costs $P_t G_{P,t}^D(i)$. The latter are real resource costs that have to be paid out of final output Y_t. Their functional form is by now familiar:

$$G_{P,t}^D(i) = \frac{\phi_{PD}}{2} Y_t \left(\frac{\frac{P_t(i)}{P_{t-1}(i)}}{\frac{P_{t-1}}{P_{t-2}}} - 1 \right)^2 . \tag{95}$$

It is assumed that the distributor pays out each period's nominal net cash flow as dividends $D_t^D(i)$. The objective function is

$$\underset{\{P_{t+s}(i)\}_{s=0}^{\infty}}{Max} \ E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[\left(P_{t+s}(i) - P_{t+s}^D\right) Y_{t+s}(i) - P_{t+s} G_{P,t+s}^D(i) - P_{t+s} T_{t+s} \omega^D \right] , \tag{96}$$

subject to product demands (76) and given marginal cost P_t^D. We obtain the first order condition for this problem, again using the fact that all firms behave identically in equilibrium. We obtain

$$\left[\frac{\sigma_D}{\sigma_D - 1} p_t^D - 1\right] = \frac{\phi_{P^D}}{\sigma_D - 1} \left(\frac{\pi_t}{\pi_{t-1}}\right) \left(\frac{\pi_t}{\pi_{t-1}} - 1\right) \tag{97}$$
$$-E_t \frac{\theta g_{t+1} n}{r_t} \frac{\phi_{P^D}}{\sigma_D - 1} \frac{\check{Y}_{t+1}}{\check{Y}_t} \left(\frac{\pi_{t+1}}{\pi_t}\right) \left(\frac{\pi_{t+1}}{\pi_t} - 1\right) .$$

Finally, the rescaled aggregate dividends of distributors are

$$\check{d}_t^D = \check{Y}_t - p_t^{DF} \check{Y}_t^{DF} - p_t^N \check{Y}_t^N - p_t^{TH} \check{Y}_t^{TH} - p_t^{TF} \check{Y}_t^{TF} - \check{G}_{P,t}^D - \omega^D . \tag{98}$$

H. Retailers

There is a continuum of retailers indexed by $i \in [0, 1]$. Retailers buy final output from distributors and sell to households. They are perfectly competitive in their input market and monopolistically competitive in their output market. Their price setting is subject to real rigidities in that they find it costly to rapidly adjust their sales volume to changing demand conditions. We first analyze the demands for retailers' output and then describe their optimization problem.

Demand for the output varieties $C_t(i)$ supplied by retailers comes from households, and follows directly from (10) and (28) as

$$C_t(i) = \left(\frac{P_t^R(i)}{P_t^R}\right)^{-\sigma_R} C_t . \tag{99}$$

The **optimization** problem of retailers consists of maximizing the expected present discounted value of nominal revenue $P_t^R(i)C_t(i)$ minus nominal costs of inputs $P_t C_t(i)$, minus nominal quantity adjustment costs $P_t G_{Y,t}^R(i)$, where the latter represent a demand for final output. Like unions, this sector does not face fixed costs of operation. The quantity adjustment costs take the form[14]

$$G_{Y,t}^R(i) = \frac{\phi_C}{2} C_t \left(\frac{(C_t(i)/(g_t n)) - C_{t-1}(i)}{C_{t-1}(i)}\right)^2 . \tag{100}$$

It is assumed that each retailer pays out each period's nominal net cash flow as dividends $D_t^R(i)$. The objective function of retailers is

$$\underset{\{P_{t+s}^R(i)\}_{s=0}^{\infty}}{Max} \; E_t \Sigma_{s=0}^{\infty} \tilde{R}_{t,s} \left[P_{t+s}^R(i) C_{t+s}(i) - P_{t+s} C_{t+s}(i) - P_{t+s} G_{Y,t+s}^R(i)\right] , \tag{101}$$

subject to demands (99) and adjustment costs (100). The first order condition for this problem, after dropping firm specific subscripts and rescaling by technology and

[14] The presence of the growth terms in (100) ensure that adjustment costs are zero along the balanced growth path.

population, has the form:

$$\left[\frac{\sigma_R-1}{\sigma_R}p_t^R-1\right]=\phi_C\left(\frac{\check{C}_t-\check{C}_{t-1}}{\check{C}_{t-1}}\right)\frac{\check{C}_t}{\check{C}_{t-1}} \tag{102}$$
$$-E_t\frac{\theta g_{t+1}n}{r_t}\phi_C\left(\frac{\check{C}_{t+1}-\check{C}_t}{\check{C}_t}\right)\left(\frac{\check{C}_{t+1}}{\check{C}_t}\right)^2 .$$

The real **dividends** and rescaled adjustment costs of this sector are given by

$$\check{d}_t^R=(p_t^R-1)\check{C}_t-\check{G}_{Y,t}^R , \tag{103}$$

$$\check{G}_{Y,t}^R=\frac{\phi_C}{2}\check{C}_t\left(\frac{\check{C}_t-\check{C}_{t-1}}{\check{C}_{t-1}}\right)^2 . \tag{104}$$

I. Government

Fiscal policy consists of a specification of public investment spending G_t^{inv}, public consumption spending G_t^{cons}, transfers $\tau_{T,t}$, and of four different taxes $\tau_{L,t}$, $\tau_{c,t}$, $\tau_{k,t}$ and $\tau_{ls,t}$. Nominal government investment and consumption spending $P_t(G_t^{inv}+G_t^{cons})$ represents a demand for final output. Government investment spending has a critical function in this economy. It augments the stock of publicly provided infrastructure capital K_t^G, the evolution of which is, after rescaling by technology, given by

$$\check{K}_{t+1}^G g_{t+1}=(1-\delta)\,\check{K}_t^G+\check{G}_t^{inv} . \tag{105}$$

Government consumption spending on the other hand is unproductive. Both types of government spending are taken as exogenous in the model unless the fiscal rule is specified in such a way that government spending rather than taxes becomes the main fiscal tool. The government's policy rule for transfers partly compensates for the lack of asset ownership of LIQ agents by redistributing a small fraction of OLG agents's dividend income receipts to LIQ agents. Specifically, dividends of the retail and union sector are redistributed in proportion to LIQ agents' share in consumption and labor supply, while the redistributed share of dividends in the four remaining sectors is ι, which we will typically calibrate as being smaller than the share ψ of LIQ agents in the population. After rescaling by technology we therefore have the following rule:

$$\check{\tau}_{T,t}=\iota\left(\check{d}_t^N+\check{d}_t^T+\check{d}_t^D+\check{d}_t^M+\overline{d^X}+d_t^F\right)+\frac{\check{c}_t^{LIQ}}{\check{C}_t}\check{d}_t^R+\frac{\check{\ell}_t^{LIQ}}{\check{L}_t}\check{d}_t^U . \tag{106}$$

The sources of nominal tax revenue are labor income taxes $\tau_{L,t}W_tL_t$, consumption taxes $\tau_{c,t}P_tC_t$, taxes on the return to capital $\tau_{k,t}\Sigma_{j=N,T}\left[R_{k,t}^J-\delta P_t q_t^J\right]K_t^J$, and lump-sum taxes $\tau_{ls,t}$. We assume that the latter is apportioned between OLG and LIQ agents in proportion to their consumption shares. We define the rescaled aggregate real tax variable as

$$\check{\tau}_t=\tau_{L,t}\check{w}_tL_t+\tau_{c,t}\check{C}_t+\tau_{k,t}\Sigma_{j=N,T}\left[r_{k,t}^J-\delta q_t^J\right]\check{K}_t^J+\check{\tau}_{ls,t} .$$

Furthermore, the government issues nominally non-contingent one-period nominal debt B_t at the gross nominal interest rate i_t. The rescaled real government budget constraint therefore takes the form

$$\check{b}_t=\frac{i_{t-1}}{\pi_t g_t n}\check{b}_{t-1}+\check{G}_t^{inv}+\check{G}_t^{cons}-\check{\tau}_t-\check{g}_t^X . \tag{107}$$

A key assumption of the model is that fiscal policy is conducted in accordance with a structural fiscal surplus rule of the following form:

$$gs_t^{rat} = gs_t^{rat^*} + d^{tax}\left(\frac{\check{\tau}_t - \check{\tau}_t^{pot}}{g\check{d}p_t}\right) + d^{cop}\left(\frac{\check{g}_t^X - \check{g}_t^{pot}}{g\check{d}p_t}\right) , \tag{108}$$

where gs_t^{rat} is the overall, interest inclusive government surplus to GDP ratio, given by

$$gs_t^{rat} = -\frac{B_t - B_{t-1}}{GDP_t} = -\frac{\check{b}_t - \frac{\check{b}_{t-1}}{\pi_t g_t n}}{g\check{d}p_t} = \frac{\check{\tau}_t + \check{g}_t^X - \check{G}_t^{inv} - \check{G}_t^{cons} - \frac{i_{t-1}-1}{\pi_t g_t n}\check{b}_{t-1}}{g\check{d}p_t} , \tag{109}$$

$\check{\tau}_t^{pot}$ is tax revenue at potential, that is at current tax rates multiplied by the tax base in steady state,

$$\check{\tau}_t^{pot} = \tau_{L,t}\bar{w}\bar{L} + \tau_{c,t}\bar{C} + \tau_{k,t}\Sigma_{j=N,T}\left[\bar{r}_k^J - \delta\right]\bar{K}^J + \bar{\tau}_{ls} , \tag{110}$$

and $\check{g}_{X_t}^{pot}$ is government copper revenue evaluated at a reference or long-run value for world copper prices $\bar{p}^X(Foreign)$:

$$g_{X_t}^{pot} = \left(e_t\bar{p}^X(Foreign)\bar{X}^s - \bar{d}^X\right)(1 - s_f^x) \ . \tag{111}$$

In the last expression we first deduct constant payments to domestic factors $\bar{d}^X$ from copper revenue at the reference price $e_t\bar{p}^X(Foreign)\bar{X}^s$. The remainder is split between the domestic government and foreign owners, with the share of foreign owners given by s_f^x.

Chile's 2006 Fiscal Responsibility Law specifies that savings are to be accumulated in an Economic and Social Stabilization Fund. As of the end of 2008 this fund had accumulated over 20 billion U.S. dollars (about 14 percent of GDP), principally from excess copper revenues. Chile's January 2009 fiscal stimulus plan, which amounts to 4 billion U.S. dollars, is being funded from this source. This plan includes a significant component of increased transfers, the baseline policy instrument of our paper, as well as tax reductions. In terms of the rule it is being formalized as a temporary reduction of the structural surplus target $\overline{gs}^{rat}$ from 0.5 to 0.

The rule (108) makes two key assumptions about fiscal policy. The first concerns dynamic stability, and the second stabilization of the business cycle.

With respect to *dynamic stability*, fiscal policy ensures a non-explosive government assets to GDP ratio by adjusting one of the tax rates to generate sufficient revenue, or by reducing one of the expenditure items. This rules out partial default on government debt, and it also rules out fiscal dominance over monetary policy, implying that inflation will not be used as a tool of discretionary fiscal revenue generation. The rule accomplishes this by stabilizing gs_t^{rat} at a long-run level $gs_t^{rat^*}$, given that on average it must be true that $\check{\tau}_t = \check{\tau}_t^{pot}$ and $\check{g}_t^X = \check{g}_t^{pot}$. Denoting the long-run government assets to GDP ratio by $gassets^{rat^*}$, we obtain the following relationship between government surplus and government assets to GDP ratios:

$$gs^{rat^*} = \frac{\bar{\pi}\bar{g}n - 1}{\bar{\pi}\bar{g}n} gassets^{rat^*} \ . \tag{112}$$

In other words, choosing a deficit target gs^{rat^*} implies an assets target $gassets^{rat^*}$ and therefore keeps assets from exploding.

With respect to *business cycle stabilization*, fiscal policy ensures that the government surplus to GDP ratio, while satisfying its long-run target of gs^{rat^*}, can also flexibly respond to the business cycle. A structural fiscal balance rule chooses $d^{tax} = 1$. Under this rule the realized fiscal surplus is allowed to rise with cyclical excess tax revenue and cyclical excess copper revenue, where potential is evaluated at current tax rates and real exchange rates, but holding the tax base, world copper prices and copper output at their steady-state values. The implication is that during a boom, when tax revenue exceeds its long run value, the government uses the extra funds to pay off government debt (or to accumulate government assets) by increasing the surplus above its long run value. The main effect of this rule is to minimize the variability of fiscal instruments, but of course it also reduces the variability of output and inflation relative to a balanced budget rule, which would set $d^{tax} = 0$. On the other hand, a more explicitly counter-cyclical rule would set $d^{tax} > 1$. As we will show, this would imply more volatile fiscal instruments but less volatile output and inflation.[15]

The rule (108) is not an instrument rule but rather a targeting rule. Any of the available tax and spending instruments can be used to make sure the rule holds. Our default setting, applied in Section IV, is that this instrument is the labor tax rate $\tau_{L,t}$, because this is the most plausible choice. However, other instruments or combinations of multiple instruments are possible. To illustrate this, we will use lump-sum taxes as the instrument in Section V.

Monetary policy uses an interest rate rule to stabilize inflation, the output gap and output growth. We posit a rule that features interest rate smoothing and which responds to deviations of one year ahead year-on-year inflation $\pi_{4,t+4}$ from the inflation target $\bar{\pi}$, and to the output gap (with steady state GDP given by $\overline{gdp}$).

$$i_t = (i_{t-1})^{\mu_i} \left(\bar{r}\pi_{4,t+4}\right)^{1-\mu_i} \left(\frac{\pi_{4,t+4}}{\bar{\pi}}\right)^{(1-\mu_i)\mu_\pi} \left(\frac{gdp_t}{\overline{gdp}}\right)^{(1-\mu_i)\mu_y} , \tag{113}$$

$$\pi_{4,t} = \left(\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}\right)^{\frac{1}{4}} . \tag{114}$$

We define a *government policy* to be a sequence of policy instruments $\left\{G_s^{inv}, G_s^{cons}, \tau_{L,s},\right.$ $\left.\tau_{c,s}, \tau_{k,s}, \tau_{ls,s}, i_s\right\}_{s=t}^{\infty}$ such that (106), (107), (108) and (113) hold at all times.

J. Equilibrium and Balance of Payments

An equilibrium is an allocation, a price system, and a government policy such that:

1. *OLG* households maximize lifetime utility (1) subject to (2), (3), (7) and the sequence of budget constraints (8).

2. *LIQ* households maximize lifetime utility (24) subject to (25), (26) and the sequence of budget constraints (27).

[15] In this more general form of the rule, Chile's recent stimulus package could be reinterpreted in terms of the rule as a more aggressive countercyclical behavior $d^{tax} > 1$, rather than as a temporary reduction in the structural surplus target gs^{rat^*}.

3. Manufacturers maximize the expected present discounted value of cash flows (44) subject to the definition of after-tax dividends (43), a process for capital accumulation (42), and subject to demands (36), production functions (37), (38) and adjustment costs (40) and (41).

4. The copper sector divides its revenues in accordance with (52).

5. Unions maximize the expected present discounted value of cash flows (59) subject to demands (57) and adjustment costs (58).

6. Import agents maximize the expected present discounted value of cash flows (69) subject to demands (66) and adjustment costs (67).

7. Distributors maximize the expected present discounted value of cash flows (96) subject to demands (76), production functions (77), (78), (79), (83), (87), (89), (90), (91), and adjustment costs (95).

8. Retailers maximize the expected present discounted value of cash flows (101) subject to demands (99) and adjustment costs (100).

9. The labor market clears:

$$\begin{aligned} U_t &= U_t^N + U_t^H \ , \\ L_t &= \ell_t^{OLG} + \ell_t^{LIQ} \ , \\ U_t &= L_t \ . \end{aligned} \tag{115}$$

10. The nontradables market clears:

$$\check{Z}_t^N = \check{Y}_t^N + \omega^N + \check{G}_{P,t}^N \ . \tag{116}$$

11. The tradables market clears:

$$\check{Z}_t^T = \check{Y}_t^{TH} + \omega^T + \check{G}_{P,t}^T + \check{Y}_t^{TX} \ . \tag{117}$$

12. The markets for exports and imports clear:

$$\begin{aligned} \check{Y}_t^{TM}(Chile) &= \check{Y}_t^{TX}(Foreign) \ , \\ \check{Y}_t^{DM}(Chile) &= \check{Y}_t^{DX}(Foreign) \ . \end{aligned} \tag{118}$$

13. The market for final output clears:

$$\check{Y}_t = \check{C}_t + \check{I}_t^N + \check{I}_t^T + \check{G}_t^{inv} + \check{G}_t^{cons} + \check{Y}_t^{DX} + \omega^D + \check{G}_{I,t}^N + \check{G}_{I,t}^T + \check{G}_{P,t}^D + \check{G}_{P,t}^U + \check{G}_{P,t}^M + \check{G}_{Y,t}^R \ . \tag{119}$$

14. The world copper market clears:

$$X_t^N(Chile) + X_t^T(Chile) + X_t^N(Foreign) + X_t^T(Foreign) = X_t^{sup}(Chile) + X_t^{sup}(Foreign) \tag{120}$$

Combining these market clearing conditions with the budget constraints of households and the government and with the expressions for firm dividends we obtain an expression for the current account:

$$
\begin{aligned}
e_t \check{f}_t \quad = \quad & \frac{i_{t-1}(Foreign)\varepsilon_t\left(1+\xi^f_{t-1}\right)}{\pi_t g_t n} e_{t-1}\check{f}_{t-1} \\
& + p_t^{TH}\check{Y}_t^{TX} + \check{d}_t^{TM} - p_t^{TF}\check{Y}_t^{TF} \\
& + \check{Y}_t^{DX} + \check{d}_t^{DM} - p_t^{DF}\check{Y}_t^{DF} \\
& + X_t^x - f_t^X \ .
\end{aligned}
\tag{121}
$$

When we repeat the same exercise for Foreign we obtain the market clearing condition for international bonds:

$$
\check{f}_t(Chile) + \check{f}_t(Foreign) = 0 \ . \tag{122}
$$

The level of GDP is given by the following expression:

$$
\begin{aligned}
g\check{d}p_t \quad = \quad & \check{C}_t + \check{I}_t^N + \check{I}_t^T + \check{G}_t^{inv} + \check{G}_t^{cons} \\
& + p_t^{TH}\check{Y}_t^{TX} + \check{d}_t^{TM} - p_t^{TF}\check{Y}_t^{TF} \\
& + \check{Y}_t^{DX} + \check{d}_t^{DM} - p_t^{DF}\check{Y}_t^{DF} + X_t^x \ .
\end{aligned}
\tag{123}
$$

Finally, we obtain expressions for international relative prices. In this model there is no exact counterpart of either the consumption based price index (because both P_t and P_t^R would be justifiable) or of the GDP deflator (where P_t comes closest). However, it is straightforward (and implemented in GIMF) to derive a GDP deflator by dividing nominal GDP by an appropriately constructed Fisher index of real GDP. The main definition of the real exchange rate required by the model is the one based on the relative price of the final output numeraire goods in Chile and Foreign, that is $e_t = E_t P_t(Foreign)/P_t$. Together with the uncovered interest parity condition (12) this implies

$$
\frac{e_{t+1}}{e_t} = \frac{\varepsilon_{t+1}\pi_{t+1}(Foreign)}{\pi_{t+1}} = \frac{r_t}{r_t(Foreign)} \ . \tag{124}
$$

III. Calibration

We calibrate the steady state of the economy to reflect key features of the Chilean economy. Clearly a more exhaustive exercise would have to include a more complete set of shocks, and a careful calibration of the parameters that drive the model's dynamics by way of estimation. However that is beyond the scope of this exercise, which simply aims to illustrate the consequences of alternative fiscal rules. It will however be considered in future work.

The model is quarterly, and the denomination of international bonds is in the currency of Foreign. Chile represents one third of one percent of the world economy, both in terms of GDP and in terms of population. It faces a long-run world real interest rate of 3% per annum, which is calibrated by the appropriate choice of foreign households' discount factor. The risk premium function is calibrated to produce a 50 to 60 basis points premium over international interest rates at the steady state net foreign liabilities to GDP ratio of 20 percent, in line with recent values for Chile. The specific calibrated values are $y_1 = 0.00125$, $y_2 = 0.005$, $y_3 = 2$, $y_4 = -6$. The government debt to GDP ratio is zero in Chile and 50 percent in Foreign. The real world growth rate is assumed to equal 2% per annum, and the population growth rate 1% per annum. The long-run inflation rate in Chile, equal to the central bank's inflation target, is assumed to equal 3% per annum, and 2% per annum in the rest of the world. The critical parameters θ and χ, which determine the degree of non-Ricardian behavior in the model, are set to equal 0.98125, which corresponds to a 15 year planning horizon or average life expectancy $1/(1-\theta)$ in the case of θ, and to a 15 year remaining working life for χ. These values were chosen based on our experience with U.S. calibrations of the model, where this parameter choice produces an elasticity of the real interest rate with respect to a 1 percentage point increase in the government debt to GDP ratio of 4 basis points. This value is towards the lower end of the estimates of Engen and Hubbard (2004) and Laubach (2003).

Household preferences are further characterized by an intertemporal elasticity of substitution of 0.2, or $\gamma = 5$, and by habit persistence $v = 0.7$. The wage elasticity of labor supply depends on the steady state value of labor supply among both OLG and LIQ households, which is in turn determined by the leisure share parameters η^{OLG} and η^{LIQ}. We adjust these parameters to obtain a wage elasticity of 0.5. Pencavel (1986) reports that most microeconomic estimates of the Frisch elasticity are between 0 and 0.45, and our calibration is at the upper end of that range, in line with much of the business cycle literature.[16] The assumed share ψ of liquidity constrained agents in the population is 50 percent for Chile and 40 percent in Foreign. The share of these agents in dividend income is assumed to be half of their share in the population.

We now turn to the calibration of technologies. The elasticities of substitution between capital, labor and copper in both tradables and nontradables are assumed to be equal to one. The elasticities of substitution between domestic and foreign traded intermediates and final goods, which correspond to the long-run price elasticities of demand for imports, are assumed to be equal to 1.5 as in Erceg, Guerrieri and Gust (2005). Finally, the elasticity of substitution between tradables and nontradables is assumed to equal 0.8, based on the evidence cited in Mendoza (2005). The real adjustment cost parameters are chosen to yield reasonable aggregate dynamics.

As for price setting in different sectors, the degree of market power is reflected in the markup of price over marginal cost. We assume that this markup is equal to 10 percent in the two manufacturing sectors and in the labor market. This is a typical assumption in the monetary business cycle literature. For the distribution and retail sectors we assume smaller markups of 5 percent, and for import agents of 2.5 percent. The key parameter for nominal rigidities is the inflation adjustment cost. Here we choose values that yield plausible dynamics over the first two to three years following a shock.

A number of share and other parameters is calibrated by reference to long-run values for the shares of different expenditure and income categories in GDP. The manufacturing labor share parameters are set to ensure a labor income share of 55 percent in Chile and 64 percent in the rest of the world, while the nontradables labor shares are assumed to

[16] As discussed by Chang and Kim (2005), a very low Frisch elasticity makes it difficult to explain cyclical fluctuations in hours worked, and they present a heterogenous agent model in which aggregate labor supply is considerably more elastic than individual labor supply.

equal 64 percent in both Chile and Foreign. This reflects the low labor share in the Chilean tradable goods sector. The nontradables share parameter is adjusted to ensure a nontradables share in GDP of 50 percent. The steady state shares of investment spending in GDP are calibrated based on historical averages to equal 20 percent in both countries. The steady state shares of government spending in GDP are 12 percent in Chile and 18 percent in Foreign, with government investment shares of 2 percent and 3 percent. Ratios of transfers to GDP are set to 10 percent in both countries. On the revenue side we have calibrated the shares of different tax revenues in overall tax revenue based on Chilean data as 50 percent for consumption tax revenue, 14 percent for capital income tax revenue, 15 percent for labor income tax revenue, and 21 percent for all other tax revenue, classified as lump-sum taxes. The corresponding shares for Foreign are 30, 10, 30 and 30.

Calibrating the depreciation rate of private capital would ordinarily present a problem given that we have already fixed the two capital income shares and the investment to GDP ratio. The only three free parameters available for to fix these four values would typically be α_N^U, α_T^U and δ. But in our model the income of capital consists not only of the return to capital in manufacturing, but also of economic profits due to market power in multiple sectors. We have introduced fixed costs in manufacturing and distribution that partly or wholly eliminate these profits. The percentage of steady state economic profits that is eliminated by fixed costs can therefore be specified as a fourth free parameter. This allows us to calibrate the annual depreciation rate of private capital at the conventional 10 percent while maintaining the investment to GDP ratio and capital income shares stated above.

The copper endowments and demand share parameters are calibrated in order to approximately reproduce Chile's historic ratios of copper output and copper exports to GDP, which we set to 12.38 percent and 12.3 percent, and to normalize the steady state world copper price to one. The copper demand shock is assumed to be highly persistent, as in the data, with $\rho^X = 0.95$.

We calibrate the trade share parameters to produce Chilean ratios to GDP of intermediate and final goods exports of 8.7 percent and 14 percent, and of final goods imports of 5 percent, and to normalize the initial steady state final output based real exchange rate to 1. Taken together with net copper exports the current account equation then determines the intermediate goods imports ratio as a residual given the net foreign liabilities to GDP ratio.

As for the division of copper revenue between the different parties, we assume that in steady state domestic factors of production receive 65 percent, with the remainder split evenly between the domestic government and the foreign private sector. The net excess revenue following a shock is shared evenly between the domestic government and the foreign private sector.

For the monetary policy rules in each country we assume $\mu_i = 0$ and $\mu_\pi = 1.0$. This produces reasonable monetary policy responses to the shocks we investigate. The focus of course is less on the monetary policy behavior and is rather on the fiscal rules, where we will investigate the consequences of a number of rule calibrations.

IV. Choice of Countercyclical Coefficients

The parameters of the fiscal rule (108) are critical for its effect on the business cycle. In this section we focus on the coefficients d^{tax} and d^{cop}, which determine the countercyclicality of fiscal policy in response to shocks, and which can therefore be used to represent a variety of different policy rules. Setting both equal to zero corresponds to a balanced-budget rule, which requires lower taxes (or higher spending) in response to a boom in demand. This is highly procyclical and therefore undesirable. Setting both equal to one corresponds to Chile's structural surplus rule. It implies minimal short-run changes in fiscal instruments in response to shocks, and it implies a countercyclical overall deficit. This is somewhat countercyclical and far superior to a balanced-budget rule. Finally, setting both coefficients at values greater than one is even more countercyclical, because it implies not only a countercyclical overall deficit but also countercyclical fiscal instruments, such as higher tax rates (or lower spending) in response to a boom in demand. Investigating this possibility is a key part of our analysis.

To quantify the performance of different choices of the coefficients d^{tax} and d^{cop} we adopt the following conventional loss function

$$Loss = sd(\pi) + \lambda * sd(gdp) \ , \tag{125}$$

where sd stands for standard deviation. This function penalizes a weighted sum of the standard deviations of inflation and output. To trace out an inflation-output efficiency frontier we vary λ. For each λ we choose the weights d^{tax} and d^{cop} in the fiscal policy rule that minimize the loss function. The resulting efficiency frontier represents the best available combinations of output and inflation volatility, given the model and the shock distributions. We keep it simple by focusing only on shocks to copper prices, calibrated to reproduce the unconditional variance and autocorrelation of international copper prices. We choose the labor income tax rate $\tau_{L,t}$ as the fiscal instrument that adjusts endogenously to satisfy the structural surplus rule.

Figure 2 and Table 1 show the results. First, different fiscal policies affect, as expected, mostly the volatility of output rather than inflation. Second, Chile's rule, with both coefficients equal to one, performs far better than a balanced-budget rule with both coefficients equal to zero. Third, Chile's structural surplus rule is very close to the efficiency frontier and implies a relative weight on output volatility in the policymaker's objective function of approximately 0.13. Fourth, a rule aimed at more aggressively stabilizing output, by increasing the weight on excess copper revenue to around two, results in significantly less output volatility, but at the cost of an increase, albeit smaller, in inflation volatility, and more importantly at the cost of much higher volatility in tax rates, fiscal deficits, and government debt. For the rule represented by the far left portion of the efficiency frontier the relative weight of output in the loss function is greater than one. Fifth, the volatility of fiscal instruments is minimized by the structural surplus rule with coefficients equal to one. Fiscal instruments become more volatile for deviations from this rule in either direction, but with opposite signs. Specifically, fiscal instruments are very volatile and procyclical for a balanced budget rule, and very volatile but countercyclical for the case of fiscal rule coefficients significantly greater than one.

Figure 2. Policy Efficiency Frontiers

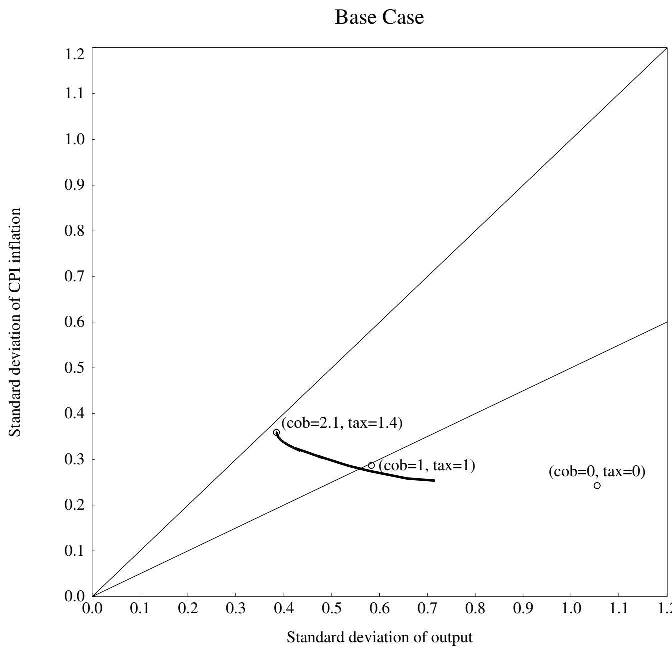


Table 1. Fiscal Policy Rules and Macroeconomic Volatility

Weight of Copper in Fiscal Rule $\Longrightarrow$	*Balanced Budget*	0.4	*SFS (Chile)*	$\simeq 1$	2.1
Real GDP std.	1.05	0.72	0.58	0.58	0.38
Inflation std.	0.24	0.25	0.29	0.27	0.36
Deficit std.	0	0.89	1.07	1.41	1.96
Debt std.	0	9.9	10.5	16.2	19.2
Δ Tax std.	0.67	0.32	0.04	0.06	0.68
Output Weight in Loss Fct.	-	0.05	-	0.13	>1

Figures 3 and 4 show the performance of the structural surplus rule by way of impulse responses (40 quarters) to a one standard deviation copper price shock. We observe an expansion of GDP accompanied by a reduction in inflation. This causes an accommodative response of monetary policy, which boosts investment. The real exchange rate appreciates and net exports of non-copper goods and services decline. On the fiscal side, the surge in copper related revenue accruing to the government is allowed to reduce deficits and debt. Tax rates change very little in the short run, but as debt and interest charges on debt decline tax rates start to fall, thereby providing a stimulus to consumption for an extended period of time. The boom in GDP is much more short-lived than that in consumption, as the monetary stimulus disappears quickly and the effect of the real appreciation dominates over the medium term.

Figures 5 and 6 show impulse responses for a balanced-budget rule. The main difference to the previous case is that the surge in copper revenue is not allowed to affect deficits and instead leads to an immediate reduction in taxes. The result is an amplification of the boom in GDP.

Figures 7 and 8 show impulse responses for a strongly countercyclical rule. The main difference to the structural surplus rule is that the surge in copper prices is now accompanied by an immediate increase in taxes that causes much higher fiscal surpluses. The short-run effect on GDP is now contractionary rather than expansionary.

V. Choice of Surplus Target

Chile's government surplus to GDP target before May 2007 was 1%, and its government assets to GDP ratio was around 8%-9%. The simple manipulation of the government budget constraint shown in equation (112) shows that the targeted or long-run government surplus to GDP ratio equals the long-run government assets to GDP ratio multiplied by the sum of the long-run growth rates of technology, population, and prices, which we assume to equal 2%, 1% and 3% per annum for Chile. By this calculation a 1% surplus target implies a long-run government assets to GDP ratio of 17.4%, which represents a substantial asset accumulation beyond the current level. The 0.5% surplus target adopted in May 2007 (and effective 2008) is however consistent with the current assets to GDP ratio and implies no significant changes in assets in the long run. This has advantages for business cycle stabilization, because further asset accumulation would require higher taxes and/or lower spending today relative to the future, which would induce intertemporal effects in consumption and investment.

Figures 9 and 10 illustrate the consequences of choosing a surplus target that is inconsistent with current asset stocks. The thought experiment is as follows. Assume a steady state where the government has been pursuing a 0.5% government surplus target, which is consistent with Chile's current assets to GDP ratio. Next assume that the target is permanently raised to 1% at time 0. In this case we choose lump-sum transfers (negative lump-sum taxes) as our fiscal instrument.

The rule itself induces a business cycle even in the absence of shocks. We observe that transfers have to be temporarily reduced to allow the government to accumulate the desired assets. Due to the non-Ricardian nature of agents' behavior, this temporarily crowds out consumption and crowds in investment and net exports. This consideration is of course only one of many in evaluating the merits of different levels of the surplus target.

Figure 3. SFS Rule - Survey

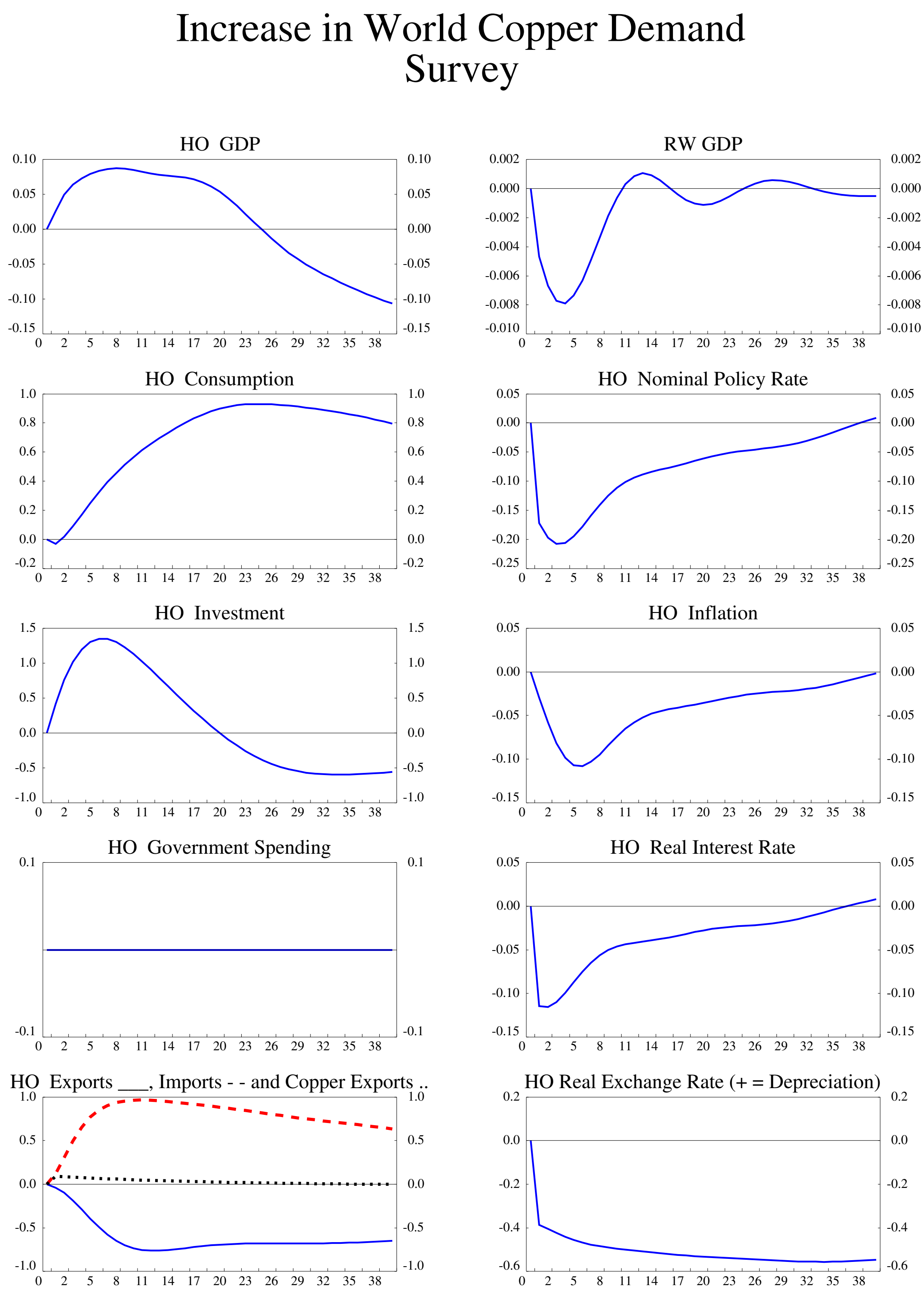


Figure 4. SFS Rule - Fiscal Accounts

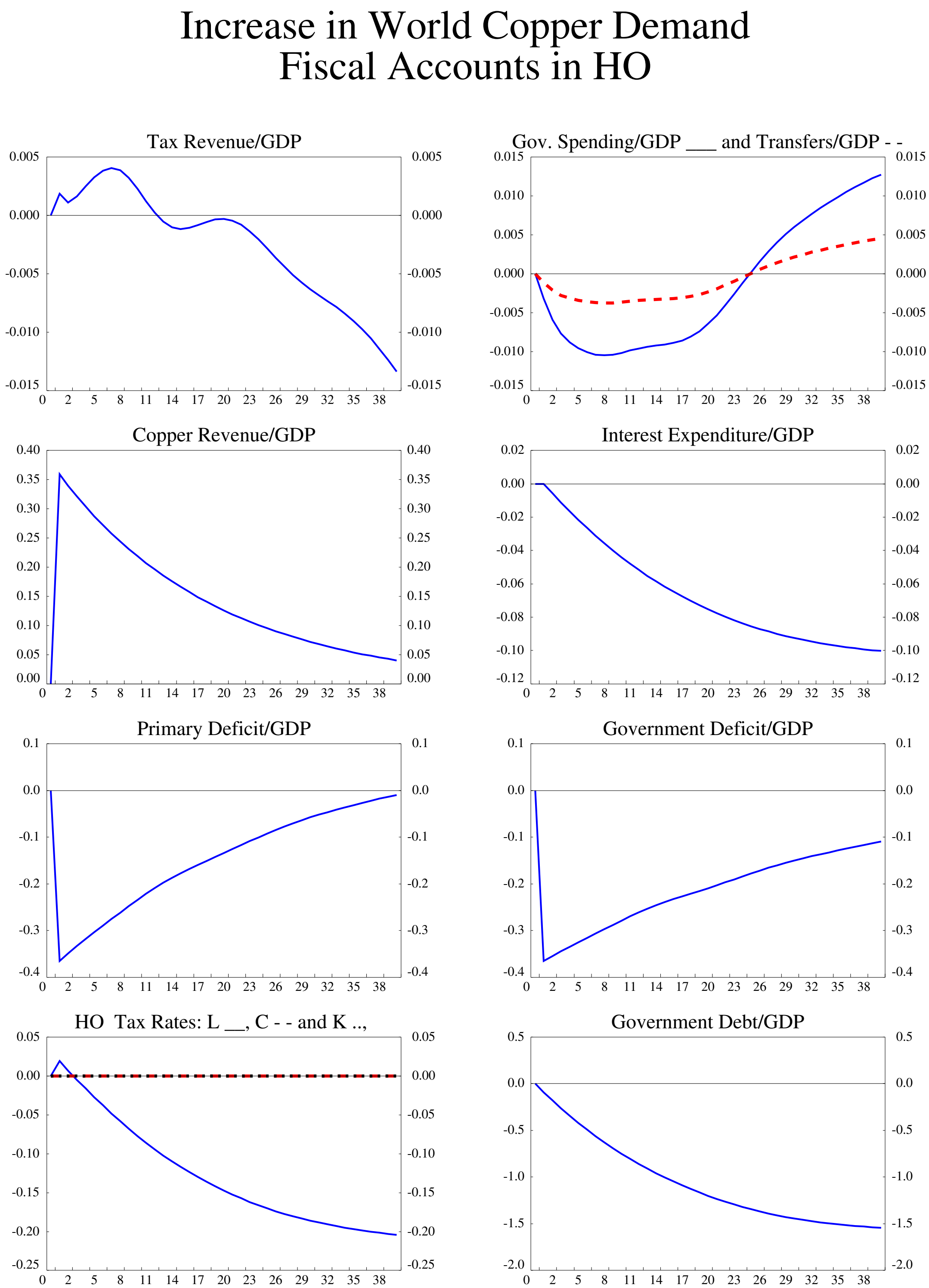


Figure 5. Balanced Budget Rule - Survey

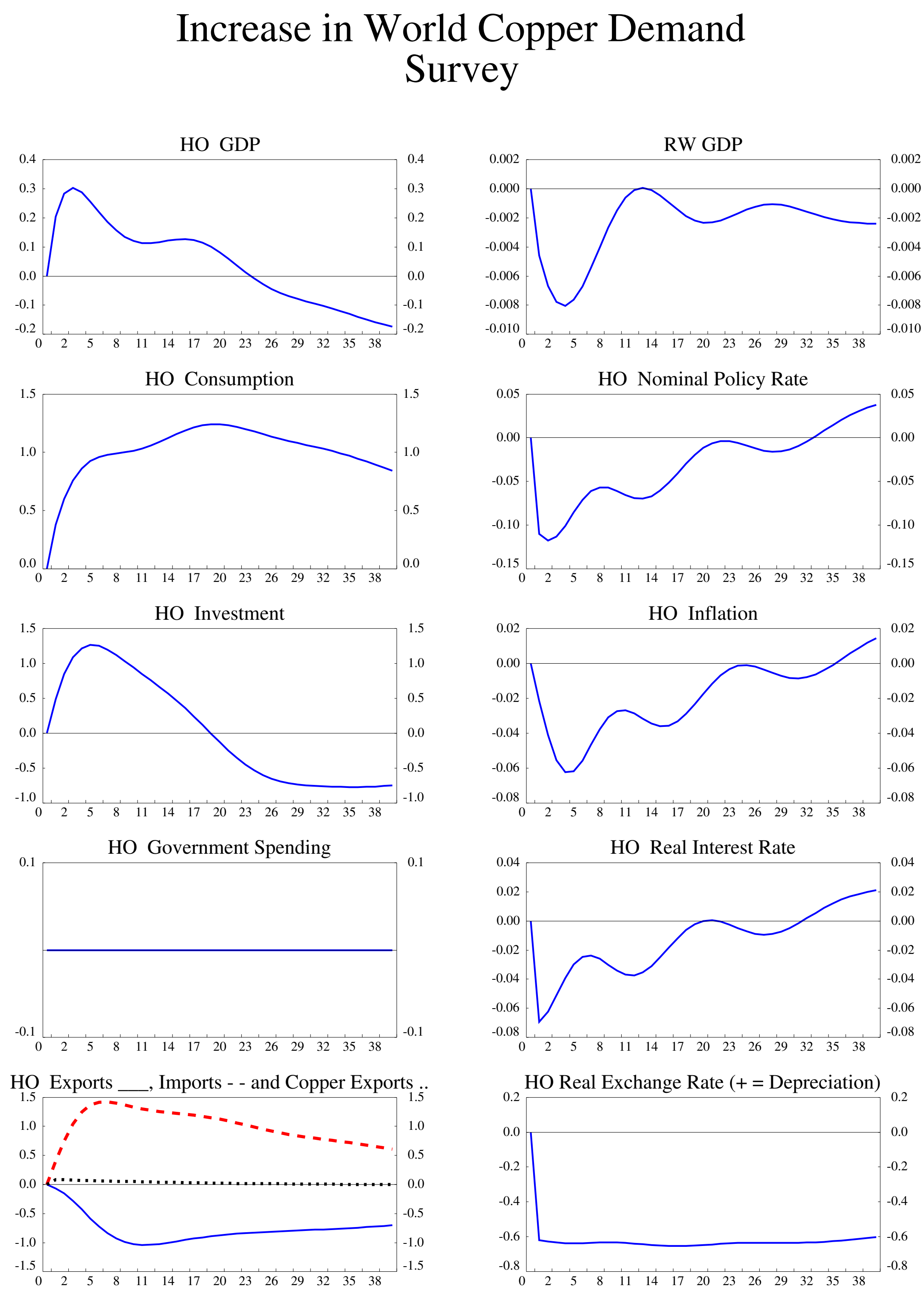


Figure 6. Balanced Budget Rule - Fiscal Accounts

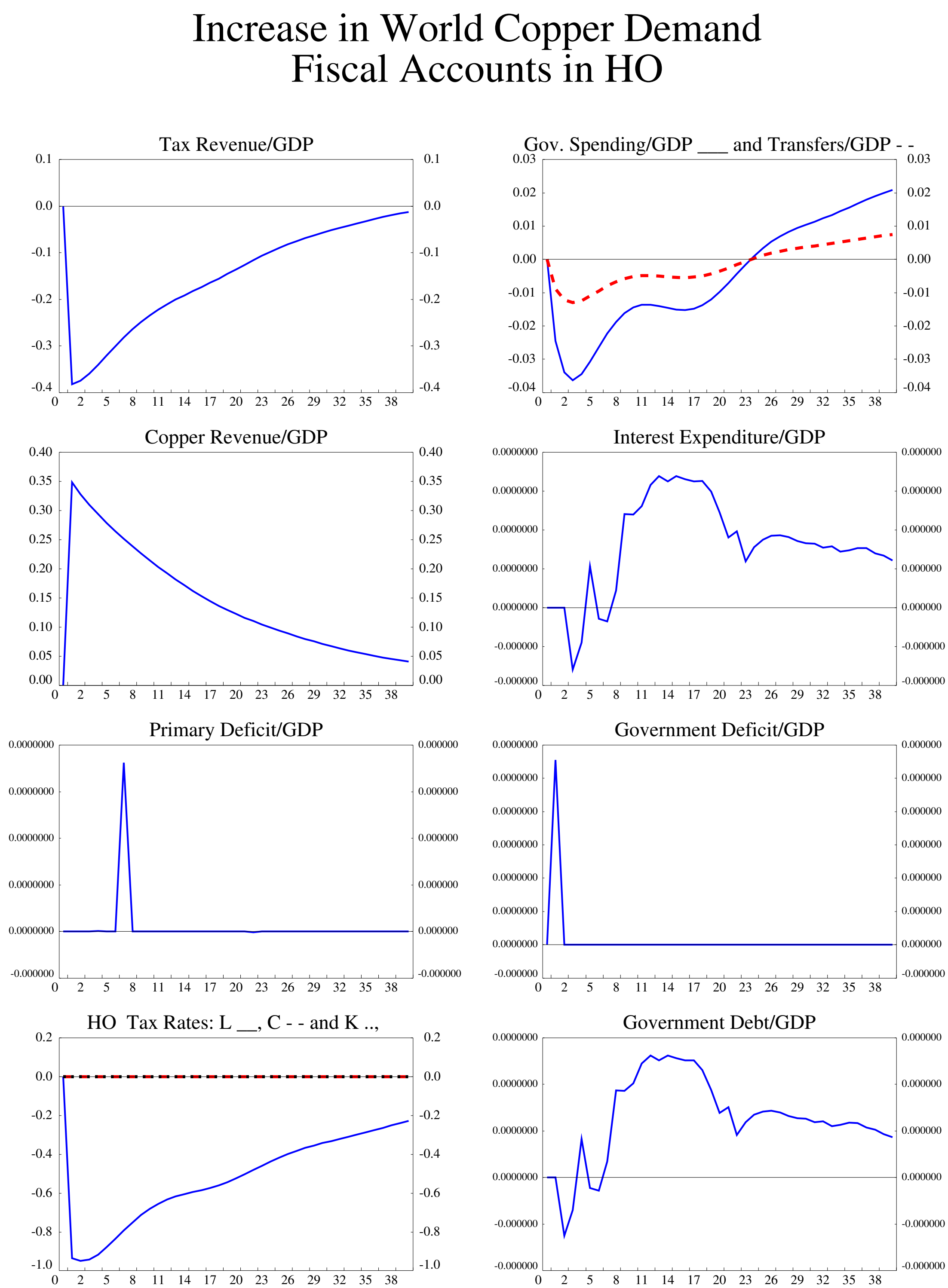


Figure 7. Aggressive Countercyclical Rule - Survey

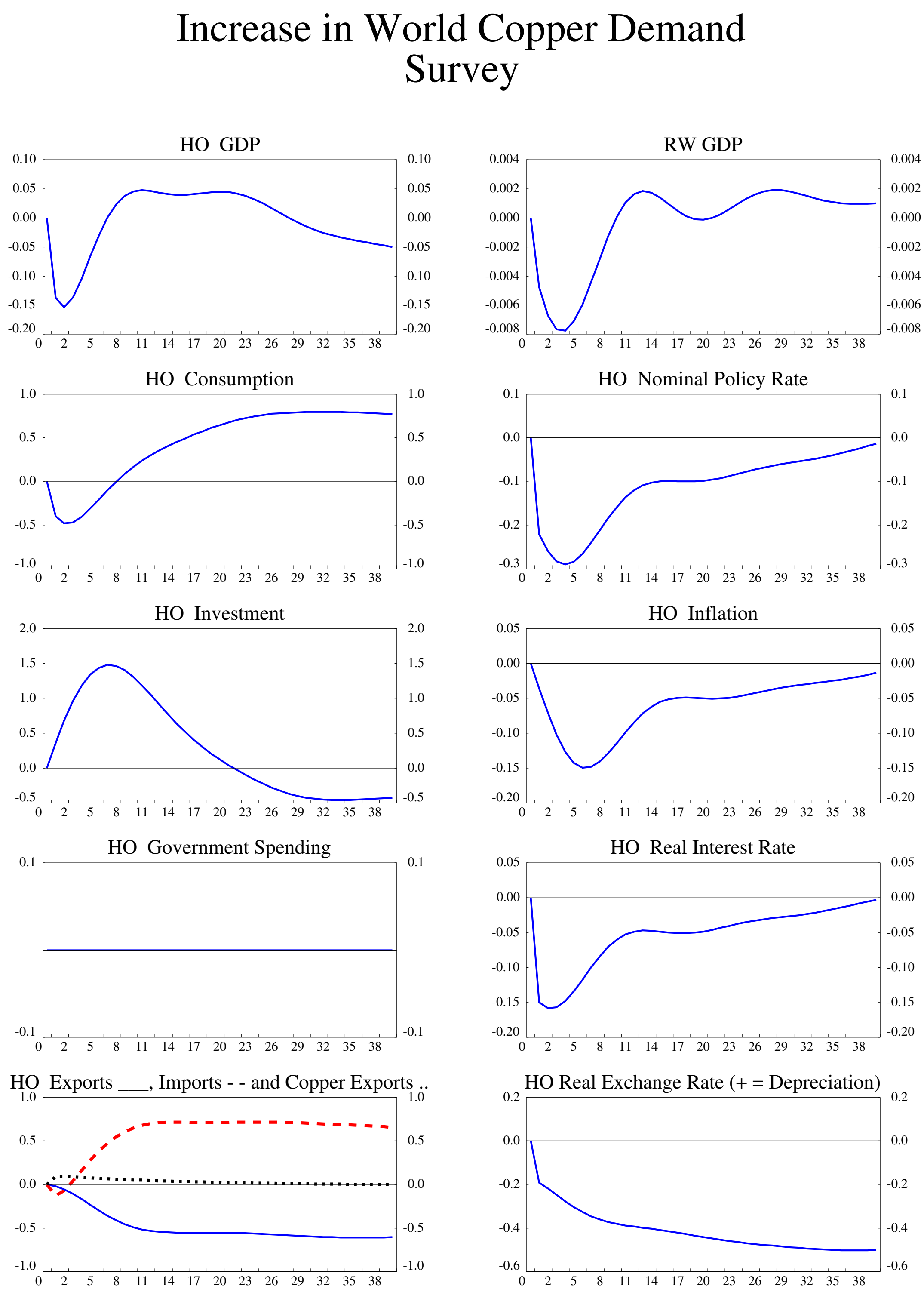


Figure 8. Aggressive Countercyclical Rule - Fiscal Accounts

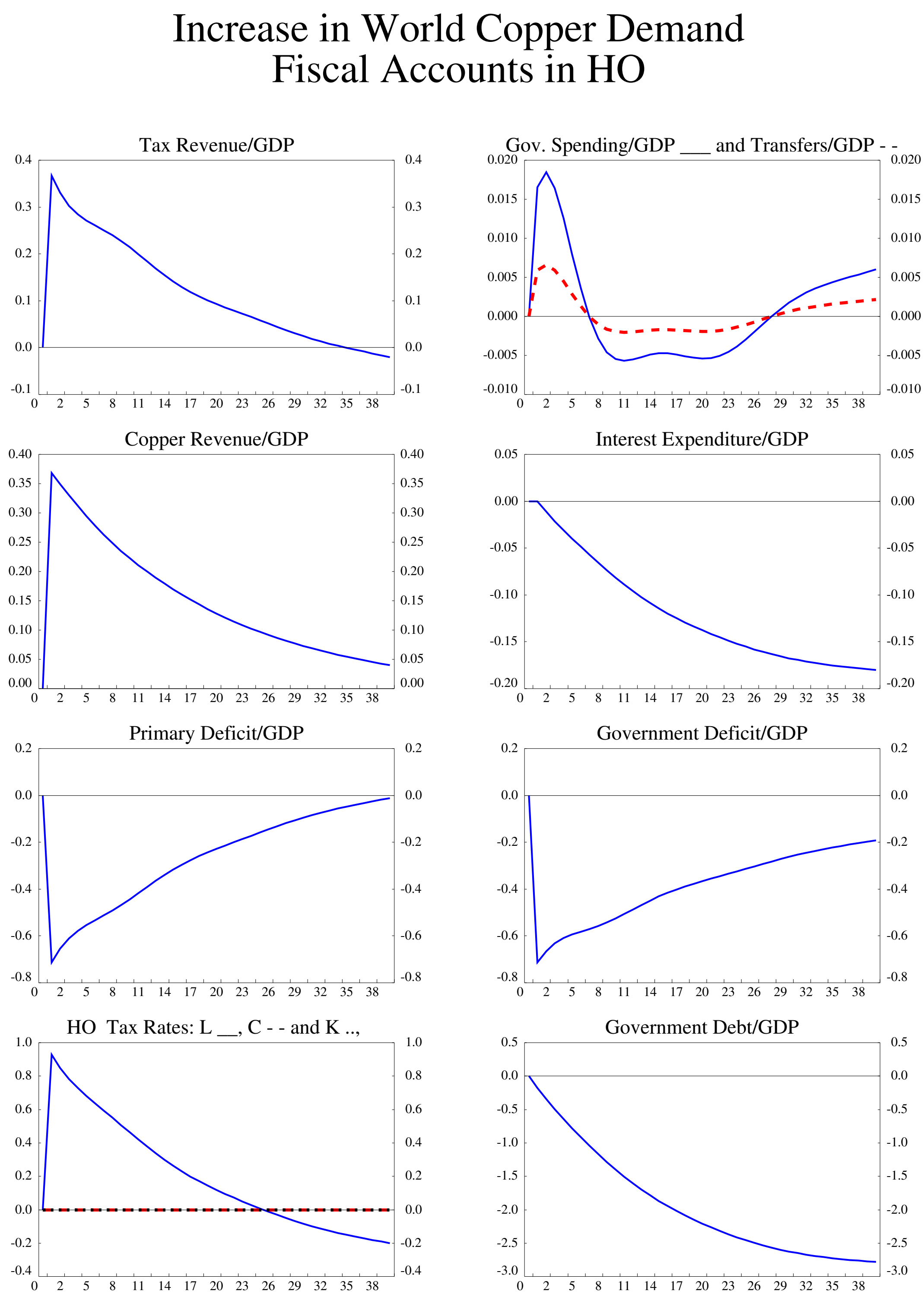


Figure 9. Surplus Target Shock - Survey

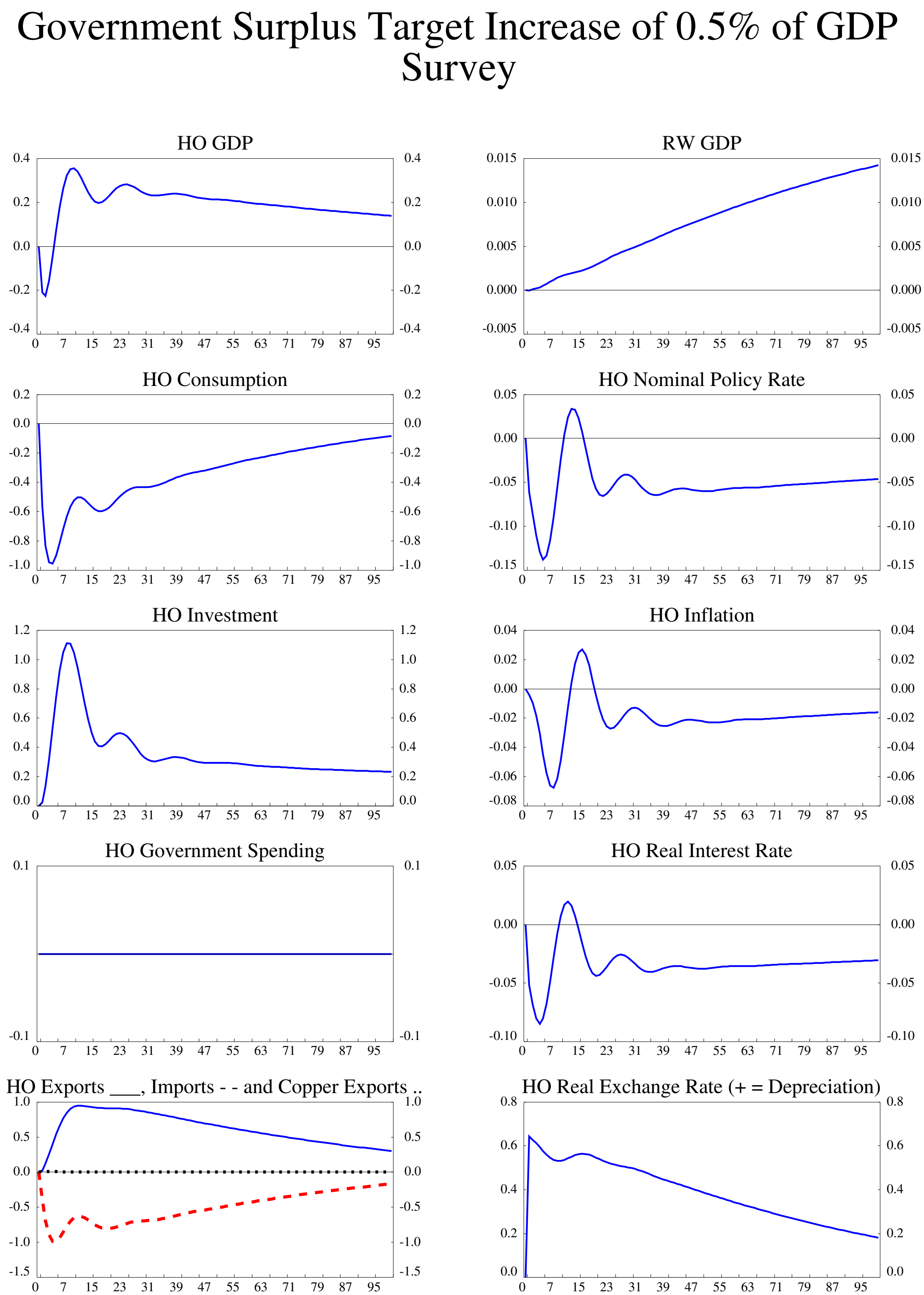
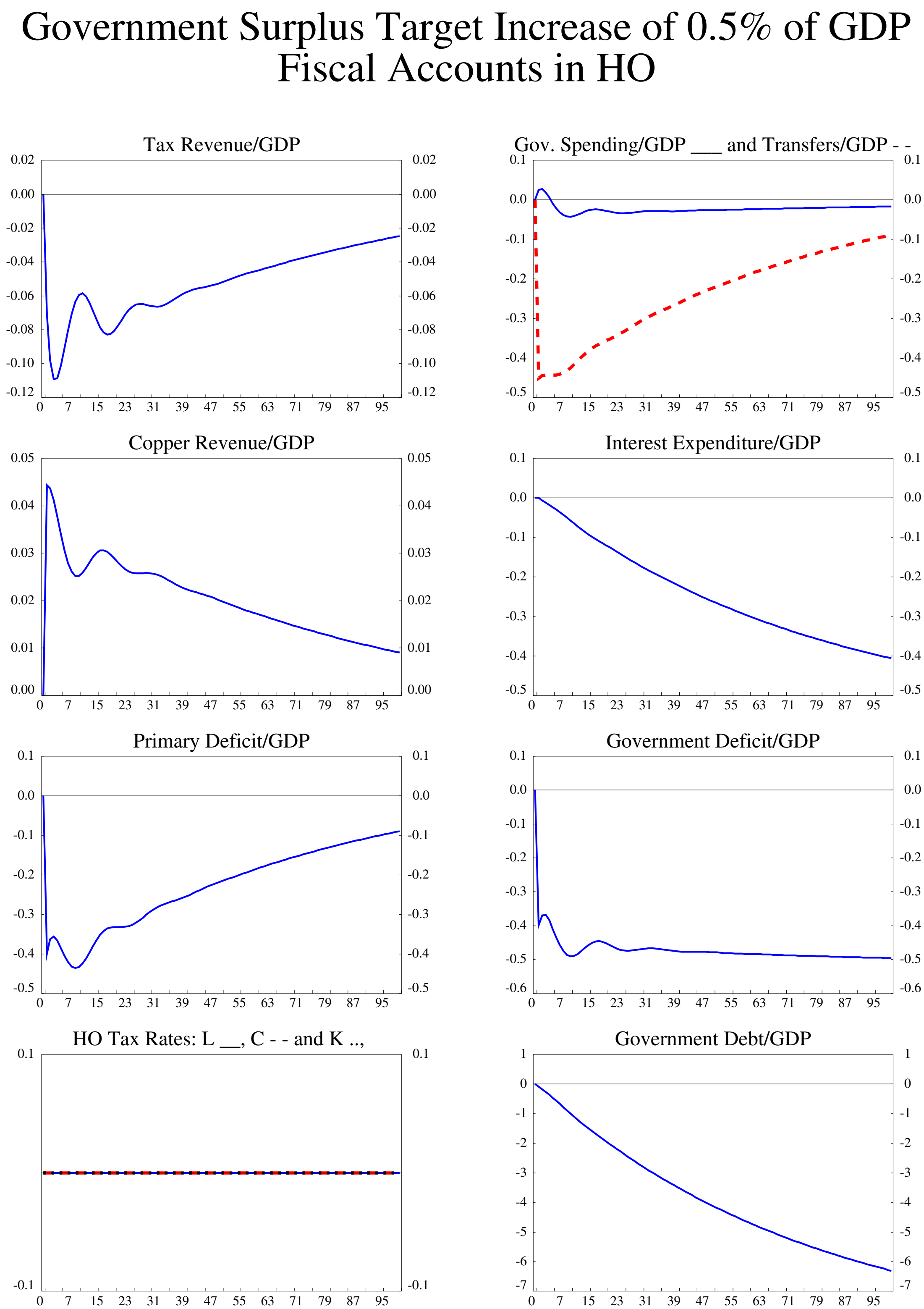


Figure 10. Surplus Target Shock - Fiscal



VI. Summary

This paper has analyzed the performance of Chile's structural fiscal surplus rule in the face of copper price shocks originating in foreign copper demand. The objective was to explore whether the performance of this rule could be improved by making it more countercyclical and/or by making its target government surplus to GDP ratio more consistent with pre-existing government debt stocks. We have obtained two results.

First, Chile's current structural surplus rule performs well if the policymaker puts a small weight on output volatility (relative to inflation volatility) in his/her objective function. A more aggressive countercyclical fiscal rule can attain lower output volatility, but there is a trade-off with (somewhat) higher inflation volatility and (much) higher volatility of fiscal variables.

Second, given its current stock of government assets, Chile's adoption of a 0.5% surplus target starting in 2008 is desirable from a business cycle perspective. This is because the earlier 1% target would have required significant further asset accumulation that could only have been accomplished at the expense of greater volatility in fiscal instruments and therefore in GDP.

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Appendices

Population Growth

The population size at time 0 is assumed to equal N, with $N(1-\psi)$ OLG households and $N\psi$ LIQ households. The size of a new cohort born at time t is given by $Nn^t\left(1-\frac{\theta}{n}\right)$, so that by time $t+k$ this cohort will be of size $Nn^t\left(1-\frac{\theta}{n}\right)\theta^k$. When we sum over all cohorts at time t we obtain

$$
\begin{aligned}
& Nn^t\left(1-\frac{\theta}{n}\right)+Nn^{t-1}\left(1-\frac{\theta}{n}\right)\theta+Nn^{t-2}\left(1-\frac{\theta}{n}\right)\theta^2+\ldots \\
= \quad & Nn^t\left(1-\frac{\theta}{n}\right)\left(1+\frac{\theta}{n}+\left(\frac{\theta}{n}\right)^2+\ldots\right) \\
= \quad & Nn^t \, .
\end{aligned}
$$

This means that the overall population grows at the rate n. When we normalize real quantities, we divide by the level of technology T_t and by population, but for the latter we divide by n^t only, meaning real figures are not in per capita terms but rather in absolute terms adjusted for population growth.

Optimality Conditions for OLG Households

We have the following Lagrangian representation of the optimization problem of OLG households:[17]

$$\mathcal{L}_{a,t} = E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left\{ \left[\frac{1}{1-\gamma} \left(\left(\frac{c^{OLG}_{a+s,t+s}}{h_{t+s}} \right)^{\eta^{OLG}} \left(1 - \ell^{OLG}_{a+s,t+s}\right)^{1-\eta^{OLG}} \right)^{1-\gamma} \right] \right\} \tag{126}$$

$$+\Lambda_{a+s,t+s} \left[\frac{1}{\theta} \left[i_{t-1+s} B_{a-1+s,t-1+s} + i^*_{t-1+s} \mathcal{E}_{t+s} F_{a-1+s,t-1+s} (1 + \xi^f_{t-1}) \right] \right.$$

$$+W_{t+s}\Phi_{a+s,t+s}\ell^{OLG}_{a+s,t+s}(1-\tau_{L,t+s}) + \sum_{j=N,T,D,R,U,M,X,F} \int_0^1 D^j_{a+s,t+s}(i)di - P_t\tau^{OLG}_{T_{a,t}} - P_t\tau^{OLG}_{ls,t}$$

$$\left. - \left[P_{t+s} c^{OLG}_{a+s,t+s} (p^R_{t+s} + \tau_{c,t+s}) + B_{a+s,t+s} + \mathcal{E}_{t+s} F_{a+s,t+s} \right] \right\} ,$$

where $\Lambda_{a,t}$ is the marginal utility to the generation of age a at time t of an extra unit of domestic currency. Define the marginal utility of an extra unit of final output Y_t as

$$\lambda_{a,t} = \Lambda_{a,t} P_t \ , \tag{127}$$

and let

$$u^{OLG}_{a,t} = \left(\frac{c^{OLG}_{a,t}}{h_t} \right)^{\eta^{OLG}} \left(1 - \ell^{OLG}_{a,t}\right)^{1-\eta^{OLG}} \ , \tag{128}$$

$$h_t = \left(\frac{c^{OLG}_{t-1} g_t}{N n^{t-1}(1-\psi)} \right)^{v} \ . \tag{129}$$

Then we have the following first-order conditions for consumption and labor supply

$$\frac{\eta^{OLG} \left(u^{OLG}_{a,t}\right)^{1-\gamma}}{c^{OLG}_{a,t}} = \lambda_{a,t}(p^R_t + \tau_{c,t}) \ , \tag{130}$$

$$\frac{(1-\eta^{OLG}) \left(u^{OLG}_{a,t}\right)^{1-\gamma}}{1 - \ell^{OLG}_{a,t}} = \lambda_{a,t} w_t \Phi_{a,t} (1 - \tau_{L,t}) \ , \tag{131}$$

which can be combined to yield

$$\frac{c^{OLG}_{a,t}}{1 - \ell^{OLG}_{a,t}} = \frac{\eta^{OLG}}{1-\eta^{OLG}} w_t \Phi_{a,t} \frac{(1-\tau_{L,t})}{(p^R_t + \tau_{c,t})} \ . \tag{132}$$

We can aggregate this as

$$\frac{c^{OLG}_t}{N n^t (1-\psi) - \ell^{OLG}_t} = \frac{\eta^{OLG}}{1-\eta^{OLG}} w_t \frac{(1-\tau_{L,t})}{(p^R_t + \tau_{c,t})} \ , \tag{133}$$

and normalize it as

$$\frac{\check{c}^{OLG}_t}{N(1-\psi) - \check{\ell}^{OLG}_t} = \frac{\eta^{OLG}}{1-\eta^{OLG}} \check{w}_t \frac{(1-\tau_{L,t})}{(p^R_t + \tau_{c,t})} \ . \tag{134}$$

[17] For simplicity we ignore money given the cashless limit assumption.

In this aggregation we have made use of the following assumptions about labor productivity:

$$\Phi_{a,t} = \kappa \chi^a \ , \tag{135}$$

$$Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a \Phi_{a,t} = Nn^t(1-\psi) \ , \tag{136}$$

$$\kappa = \frac{(n-\theta\chi)}{(n-\theta)} \ , \tag{137}$$

$$Nn^t(1-\psi)\left(1-\frac{\theta}{n}\right)\Sigma_{a=0}^{\infty}\left(\frac{\theta}{n}\right)^a \left(\ell_{a,t}^{OLG}\Phi_{a,t}\right) \equiv \ell_t^{OLG} \ . \tag{138}$$

Equation (135) is our specification of the profile of labor productivity over the lifetime. Equation (136) is the assumption that average labor productivity equals one. Equations (135) and (136), for a given productivity decline parameter χ, imply the initial productivity level κ in (137). Equation (138) is the definition of effective aggregate labor supply.

Next we have the first-order conditions for domestic and foreign bonds $B_{a,t}$ and $F_{a,t}$:

$$\lambda_{a,t} = \beta E_t \lambda_{a+1,t+1}\frac{i_t}{\pi_{t+1}} \ , \tag{139}$$

$$\lambda_{a,t} = \beta E_t \lambda_{a,t+1}\frac{i_t^* \varepsilon_{t+1}(1+\xi_t^f)}{\pi_{t+1}} \ . \tag{140}$$

Together these yield the uncovered interest parity condition

$$i_t = i_t^* \tilde{E}_t \varepsilon_{t+1}(1+\xi_t^f) \ . \tag{141}$$

To write the marginal utility of consumption $\lambda_{a,t}$ in terms of quantities that can be aggregated, specifically in terms of consumption, we use (128) and (132) in (130) to get

$$\lambda_{a,t} = \eta^{OLG}\left(\frac{c_{a,t}^{OLG}}{h_t}\right)^{-\gamma} h_t^{-1}(p_t^R+\tau_{c,t})^{-1}\left(h_t\frac{(1-\eta^{OLG})(p_t^R+\tau_{c,t})}{\eta^{OLG}w_t\Phi_{a,t}(1-\tau_{L,t})}\right)^{(1-\eta^{OLG})(1-\gamma)} \ . \tag{142}$$

We use (142) in (139) to obtain the generation specific consumption Euler equations

$$\begin{aligned} \tilde{E}_t c_{a+1,t+1}^{OLG} &= \tilde{E}_t j_t c_{a,t}^{OLG} \quad \text{, where} \qquad (143)\\ j_t &= \left(\frac{\beta i_t}{\pi_{t+1}}\right)^{\frac{1}{\gamma}}\left(\frac{p_t^R+\tau_{c,t}}{p_{t+1}^R+\tau_{c,t+1}}\right)^{\frac{1}{\gamma}}\left(\chi g_{t+1}\frac{\check{w}_{t+1}(1-\tau_{L,t+1})(p_t^R+\tau_{c,t})}{\check{w}_t(1-\tau_{L,t})(p_{t+1}^R+\tau_{c,t+1})}\right)^{(1-\eta^{OLG})(1-\frac{1}{\gamma})} \\ &\quad \left(\frac{\check{c}_t^{OLG} g_{t+1}}{\check{c}_{t-1}^{OLG}}\right)^{v\eta^{OLG}(1-\frac{1}{\gamma})} \ . \end{aligned}$$

Consumption and Wealth

The key equation for OLG households is the one relating current consumption to current wealth. We start deriving this by reproducing the budget constraint:

$$P_t c^{OLG}_{a,t}(p^R_t + \tau_{c,t}) + B_{a,t} + \mathcal{E}_t F_{a,t} = \frac{1}{\theta}\left[i_{t-1}B_{a-1,t-1} + i^*_{t-1}\mathcal{E}_t F_{a-1,t-1}(1+\xi^f_{t-1})\right] \tag{144}$$

$$+W_t\Phi_{a,t}\ell^{OLG}_{a,t}(1-\tau_{L,t}) + \sum_{j=N,T,D,R,U,M,X,F}\int_0^1 D^j_{a,t}(i)di - P_t\tau^{OLG}_{T_{a,t}} - P_t\tau^{OLG}_{ls,t} .$$

We now derive an expression that decomposes human wealth into labor and dividend income. First, we note that after-tax wage income can be decomposed as follows:

$$W_t\Phi_{a,t}\ell^{OLG}_{a,t}(1-\tau_{L,t}) = W_t\Phi_{a,t}(1-\tau_{L,t}) - W_t\Phi_{a,t}(1-\tau_{L,t})(1-\ell^{OLG}_{a,t}) . \tag{145}$$

The first expression on the right-hand side of (145) is the labor component of income, which equals the marginal value of the household's entire endowment (one unit) of time. The second expression in (145), by (132), can be rewritten as

$$W_t\Phi_{a,t}(1-\tau_{L,t})(1-\ell^{OLG}_{a,t}) = \frac{1-\eta^{OLG}}{\eta^{OLG}}P_t c^{OLG}_{a,t}(p^R_t + \tau_{c,t}) , \tag{146}$$

which can be combined with the consumption expression in (144) to obtain, on the left-hand side of (144), $P_t c^{OLG}_{a,t}(p^R_t+\tau_{c,t})/\eta^{OLG}$. The second component of income is dividend income net of redistribution to LIQ agents, the expression for which can be simplified by noting that in equilibrium all firms in a given sector pay equal dividends, so that we can drop the firm specific index and write $\int_0^1 D^j_{a,t}(i)di = D^j_{a,t}$. We also assume that per capita dividends received by each OLG agent are identical. Finally, we incorporate the assumptions about shares of dividend income that are redistributed to LIQ agents:

$$P_t\tau^{OLG}_{T_{a,t}} = \iota\sum_{j=N,T,D,M,X,F} D^j_{a,t} + \frac{c^{LIQ}_t}{C_t}D^R_{a,t} + \frac{\ell^{LIQ}_t}{L_t}D^U_{a,t} . \tag{147}$$

These assumptions imply

$$\sum_{j=N,T,D,R,U,M,X,F}\int_0^1 D^j_{a,t}(i)di - P_t\tau^{OLG}_{T_{a,t}} = \sum_{j=N,T,D,M,X,F}\frac{D^j_t(1-\iota)}{Nn^t(1-\psi)} + \frac{c^{OLG}_t}{C_t}D^R_t + \frac{\ell^{OLG}_t}{L_t}D^U_t . \tag{148}$$

The preceding arguments, taken together with our assumption about the share of total lump-sum taxes $\tau_{ls,t}$ levied on OLG agents, imply that total nominal wage and dividend income of households of age a in period t is given by

$$Inc_{a,t} = W_t\Phi_{a,t}(1-\tau_{L,t}) + \sum_{j=N,T,D,M,X,F}\frac{D^j_t(1-\iota)}{Nn^t(1-\psi)} + \frac{c^{OLG}_t}{C_t}\left(D^R_{a,t} - P_t\tau_{ls,t}\right) + \frac{\ell^{OLG}_t}{L_t}D^U_{a,t} . \tag{149}$$

We now rewrite the household budget constraint as follows:

$$P_t c^{OLG}_{a,t} \frac{(p^R_t + \tau_{c,t})}{\eta^{OLG}} + B_{a,t} + \mathcal{E}_t F_{a,t} \tag{150}$$

$$= Inc_{a,t} + \frac{1}{\theta}\left[i_{t-1} B_{a-1,t-1} + i^*_{t-1} \mathcal{E}_t F_{a-1,t-1} (1 + \xi^f_{t-1}) \right] .$$

We proceed to derive a condition relating current consumption to lifetime wealth through successive forward substitutions of (150). In doing so we use the arbitrage condition (140) to cancel terms relating to foreign bonds. After the first substitution we obtain

$$\frac{\theta}{i_t} \tilde{E}_t \left\{ B_{a+1,t+1} + \mathcal{E}_{t+1} F_{a+1,t+1} \right\} \tag{151}$$

$$+ P_t c^{OLG}_{a,t} \frac{(p^R_t + \tau_{c,t})}{\eta^{OLG}} + \frac{\theta}{i_t} \tilde{E}_t \left\{ P_{t+1} c^{OLG}_{a+1,t+1} \frac{(p^R_{t+1} + \tau_{c,t+1})}{\eta^{OLG}} \right\} =$$

$$Inc_{a,t} + \frac{\theta}{i_t} \tilde{E}_t \left\{ Inc_{a+1,t+1} \right\} + \frac{1}{\theta}\left[i_{t-1} B_{a-1,t-1} + i^*_{t-1} \mathcal{E}_t F_{a-1,t-1} (1 + \xi^f_{t-1}) \right] ,$$

and successively substitute forward in the same fashion. We impose the following no-Ponzi condition on the household's optimization problem:

$$\lim_{s \longrightarrow \infty} \tilde{E}_t \tilde{R}_{t,s} \left[B_{a+s,t+s} + \mathcal{E}_{t+s} F_{a+s,t+s} \right] = 0 . \tag{152}$$

Furthermore, we let

$$FW_{a-1,t-1} = \frac{1}{\theta}\left[i_{t-1} B_{a-1,t-1} + i^*_{t-1} \mathcal{E}_t F_{a-1,t-1} (1 + \xi^f_{t-1}) \right] . \tag{153}$$

This expression denotes nominal financial wealth inherited from period $t-1$. Next we define a variable $HW_{a,t}$ denoting lifetime human wealth, which equals the present discounted value of future income variables Inc_t. We have

$$HW_{a,t} = \tilde{E}_t \Sigma^{\infty}_{s=0} \tilde{R}_{t,s} Inc_{a+s,t+s} . \tag{154}$$

Further forward substitutions on (151), and application of the transversality condition (152), then yields the following:

$$\tilde{E}_t \Sigma^{\infty}_{s=0} \tilde{R}_{t,s} \left[P_{t+s} c^{OLG}_{a+s,t+s} \frac{(p^R_{t+s} + \tau_{c,t+s})}{\eta^{OLG}} \right] = HW_{a,t} + FW_{a-1,t-1} . \tag{155}$$

The left-hand side of this expression can be further evaluated by using (143) for all future consumption terms. We let

$$\begin{aligned} j_{t,s} &= 1 \quad \text{for } s = 0, \\ &= \Pi^s_{l=1} j_{t+l-1} \ \text{ for } s \geq 1 . \end{aligned} \tag{156}$$

Then we can write

$$P_t c^{OLG}_{a,t} \tilde{E}_t \left(\Sigma^{\infty}_{s=0} \tilde{r}_{t,s} j_{t,s} \frac{(p^R_{t+s} + \tau_{c,t+s})}{\eta^{OLG}} \right) = HW_{a,t} + FW_{a-1,t-1} . \tag{157}$$

The infinite summation on the left-hand side is recursive and can be written as

$$\Theta_t = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} j_{t,s} \frac{(p_{t+s}^R + \tau_{c,t+s})}{\eta^{OLG}} = \frac{(p_t^R + \tau_{c,t})}{\eta^{OLG}} + \tilde{E}_t \frac{\theta j_t}{r_t} \Theta_{t+1} \ , \tag{158}$$

so we finally obtain

$$P_t c_{a,t}^{OLG} \Theta_t = HW_{a,t} + FW_{a-1,t-1} \ . \tag{159}$$

We want to express this equation in real aggregate terms. We begin with real aggregate human wealth, denoted by hw_t:

$$hw_t = Nn^t (1-\psi) \left(1 - \frac{\theta}{n}\right) \Sigma_{a=0}^{\infty} \left(\frac{\theta}{n}\right)^a \frac{HW_{a,t}}{P_t} \ . \tag{160}$$

We break this down into its labor income and dividend income components hw_t^L and hw_t^K. For hw_t^L we have

$$hw_t^L = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} \chi^s \left(Nn^t (1-\psi) w_{t+s} (1 - \tau_{L,t+s}) \right) \ ,$$

where we have used (135) and (137). In recursive form, and scaling by technology, the last equation equals

$$\check{h}w_t^L = (N(1-\psi)\check{w}_t(1-\tau_{L,t})) + \tilde{E}_t \frac{\theta \chi g_{t+1}}{r_t} \check{h}w_{t+1}^L \ . \tag{161}$$

For hw_t^K we have, using (148) and letting $d_t^j = D_t^j / P_t$,

$$hw_t^K = \tilde{E}_t \Sigma_{s=0}^{\infty} \tilde{r}_{t,s} \left(\Sigma_{j=N,T,D,M,X,F} d_t^j (1-\iota) + \frac{c_t^{OLG}}{C_t} \left(d_t^R - \tau_{ls,t} \right) + \frac{\ell_t^{OLG}}{L_t} d_t^U \right) \ ,$$

which has the recursive representation, again after scaling by technology, of

$$\check{h}w_t^K = \left(\Sigma_{j=N,T,D,M,X,F} \check{d}_t^j (1-\iota) + \frac{c_t^{OLG}}{C_t} \left(d_t^R - \tau_{ls,t} \right) + \frac{\ell_t^{OLG}}{L_t} d_t^U \right) + \tilde{E}_t \frac{\theta g_{t+1}}{r_t} \check{h}w_{t+1}^K \ . \tag{162}$$

Finally, we have

$$\check{h}w_t = \check{h}w_t^L + \check{h}w_t^K \ . \tag{163}$$

Next we aggregate over the financial wealth of different age groups. We note here that aggregation cancels the $1/\theta$ term in front of the bracket in (153). This is because the period by period budget constraint (144) from which (153) was derived is the budget constraint of the agents that have in fact survived from period $t-1$ to t. Aggregation has to take account of the fact that $(1-\theta)$ agents did not survive and their wealth passed, through the insurance company, to surviving agents. Noting that $B_{-1,t-1} = 0$, we therefore have[18]

$$B_{t-1} = Nn^t (1-\psi) \left(1 - \frac{\theta}{n}\right) \Sigma_{a=0}^{\infty} \left(\frac{\theta}{n}\right)^{a-1} B_{a-1,t-1} \ .$$

For total nominal financial wealth, we therefore have

$$FW_{t-1} = \left[i_{t-1} B_{t-1} + i_{t-1}^* \mathcal{E}_t F_{t-1} (1 + \xi_{t-1}^f) \right] \ .$$

[18] Take the example of bonds held by those of age 0 at time $t-1$. Only θ of those agents survive into period t, but those that do survive obtain $1/\theta$ units of currency for every unit they held in $t-1$. Their weight in period t bonds aggregation is therefore $\theta \frac{1}{\theta} = 1$.

To express this in real terms, we define the real domestic currency asset stock as $b_t = B_t/P_t$. We adopt the convention that each nominal asset is deflated by the consumption based price index of the currency of its denomination, so that $f_t = F_t/P_t^*$. With the real exchange rate in terms of final output denoted by $e_t = \mathcal{E}_t P_t^*/P_t$, and after scaling by technology and population, we can then write

$$\check{f}w_t = \frac{FW_{t-1}}{P_t T_t n^t} = \frac{1}{\pi_t g_t n}\left[i_{t-1}\check{b}_{t-1} + i_{t-1}^* \varepsilon_t \check{f}_{t-1} e_{t-1}(1+\xi_{t-1}^f)\right] . \tag{164}$$

Finally, using (159)-(164) we arrive at our final expression for current period consumption:

$$\check{c}_t^{OLG}\Theta_t = \check{h}w_t + \check{f}w_t . \tag{165}$$

The linearized form of the aggregate equation (165) can instead be derived by linearizing an individual age group's budget constraint, using its linearized optimality conditions, and then aggregating over all generations. As mentioned above, it is therefore appropriate to use the expectations operator $\tilde{E}_t$ in nonlinear equations as long as it is understood that this is valid only up to first-order approximations of the system.

Optimality for Manufacturing Firms

The objective function facing each manufacturing firm in sectors $J \in [N, T]$ is

$$\underset{P_s^J(i),U_s^J(i),I_s^J(i),K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) \; .$$

The price (and inflation) terms in the two sectors will be indexed with $\tilde{J} \in [N, TH]$. Then dividend terms are given by

$$\begin{aligned} D_t^J(i) \;\; = \;\; & \left[P_t^{\tilde{J}}(i) Z_t^J(i) - V_t U_t^J(i) - P_t^X X_t^J(i) - P_t I_t^J(i) - P_t G_{I,t}^J(i) - P_t^{\tilde{J}} G_{P,t}^J(i) - P_t^{\tilde{J}} T_t \omega^J\right] \\ & -\tau_{k,t} \left[R_{k,t}^J - \delta P_t q_t^J\right] K_t^J(i) \; . \end{aligned}$$

Optimization is subject to the equality of output with demand

$$F(K_t^J(i), U_t^J(i), X_t^J(i)) = Z_t^J(i) \; , \text{ where}$$

$$F(K_t^J(i), U_t^J(i), X_t^J(i)) = \mathcal{T} \left(\left(1-\alpha_t^X\right)^{\frac{1}{\xi_{XJ}}} \left(M_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} + \left(\alpha_t^X\right)^{\frac{1}{\xi_{XJ}}} \left(X_t^J(i)\right)^{\frac{\xi_{XJ}-1}{\xi_{XJ}}} \right)^{\frac{\xi_{XJ}}{\xi_{XJ}-1}} \; ,$$

$$M_t^J(i) = \left(\left(1-\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(K_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} + \left(\alpha_J^U\right)^{\frac{1}{\xi_{ZJ}}} \left(T_t U_t^J(i)\right)^{\frac{\xi_{ZJ}-1}{\xi_{ZJ}}} \right)^{\frac{\xi_{ZJ}}{\xi_{ZJ}-1}} \; ,$$

$$Z_t^J(i) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J \; .$$

We also have the following capital accumulation equation and adjustment costs:

$$K_{t+1}^J(i) = (1-\delta) K_t^J(i) + I_t^J(i) \; ,$$

$$G_{I,t}^J(i) = \frac{\phi_I}{2} K_t^J(i) \left(\frac{I_t^J(i)}{K_t^J(i)} - \frac{I_{t-1}^J}{K_{t-1}^J} \right)^2 \; ,$$

$$G_{P,t}^J(i) = \frac{\phi_{PJ}}{2} Z_t^J \left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1 \right)^2 \; .$$

We write out the profit maximization problem of a representative manufacturing firm in Lagrangian form. Terms pertaining to period t and $t+1$ are sufficient. We introduce a multiplier Λ_t^J for the market clearing condition $F(K_t^J(i), U_t^J(i), X_t^J(i)) = \left(\frac{P_t^{\tilde{J}}(i)}{P_t^{\tilde{J}}} \right)^{-\sigma_J} Z_t^J$. The variable Λ_t^J equals the nominal marginal cost of producing one more unit of good i in sector J. We also introduce a multiplier q_t^J for the capital accumulation equation, which represents the shadow value of an additional unit of installed capital (Tobin's q) in terms of current investment goods. We have

$$\underset{P_s^{\tilde{J}}(i),U_s^J(i),I_s^J(i),K_{s+1}^J(i)}{Max} E_t \Sigma_{s=t}^{\infty} \tilde{R}_{t,s} D_{t+s}^J(i) =$$

$$\Bigg[\left(P_t^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_t^{\tilde{J}}\right)^{\sigma_J}Z_t^J - V_tU_t^J(i) - P_t^XX_t^J(i) - P_tI_t^J(i) - \tau_{k,t}\left(R_{k,t}^J - \delta P_tq_t^J\right)K_t^J(i)$$

$$-P_t\frac{\phi_I}{2}K_t^J(i)\left(\frac{I_t^J(i)}{K_t^J(i)} - \frac{I_{t-1}^J}{K_{t-1}^J}\right)^2 - P_t^{\tilde{J}}Z_t^J\frac{\phi_{P^J}}{2}\left(\frac{\frac{P_t^{\tilde{J}}(i)}{P_{t-1}^{\tilde{J}}(i)}}{\frac{P_{t-1}^{\tilde{J}}}{P_{t-2}^{\tilde{J}}}} - 1\right)^2 - P_t^{\tilde{J}}T_t\omega^J\Bigg]$$

$$+\Lambda_t^J\left[F(K_t^J(i), U_t^J(i), X_t^J(i)) - P_t^{\tilde{J}}(i)^{-\sigma_J}P_t^{\tilde{J}\sigma_J}Z_t^J\right]$$

$$-q_t^JP_t\left[K_{t+1}^J(i) - (1-\delta)K_t^J(i) - I_t^J(i)\right]$$

$$+E_t\Bigg\{\frac{\theta}{i_t}\Bigg[\left(P_{t+1}^{\tilde{J}}(i)\right)^{1-\sigma_J}\left(P_{t+1}^{\tilde{J}}\right)^{\sigma_J}Z_{t+1}^J - V_{t+1}U_{t+1}^J(i) - P_{t+1}^XX_{t+1}^J(i) - P_{t+1}I_{t+1}^J(i)$$

$$-\tau_{k,t+1}\left(R_{k,t+1}^J - \delta P_{t+1}q_{t+1}^J\right)K_{t+1}^J(i)$$

$$-P_{t+1}\frac{\phi_I}{2}K_{t+1}^J(i)\left(\frac{I_{t+1}^J(i)}{K_{t+1}^J(i)} - \frac{I_t^J}{K_t^J}\right)^2 - P_{t+1}^{\tilde{J}}Z_{t+1}^J\frac{\phi_{P^J}}{2}\left(\frac{\frac{P_{t+1}^{\tilde{J}}(i)}{P_t^{\tilde{J}}(i)}}{\frac{P_t^{\tilde{J}}}{P_{t-1}^{\tilde{J}}}} - 1\right)^2 - P_{t+1}^{\tilde{J}}T_{t+1}\omega^J\Bigg]$$

$$+\frac{\Lambda_{t+1}^J\theta}{i_t}\left[F(K_{t+1}^J(i), U_{t+1}^J(i), X_{t+1}^J(i)) - P_{t+1}^{\tilde{J}}(i)^{-\sigma_J}P_{t+1}^{\tilde{J}}{}^{\sigma_J}Z_{t+1}^J\right]$$

$$-\frac{q_{t+1}^JP_{t+1}\theta}{i_t}\left[K_{t+2}^J(i) - (1-\delta)K_{t+1}^J(i) - I_{t+1}^J(i)\right]\Bigg\}$$

+terms pertaining to periods $t+2, t+3, ...$

We take the first-order condition with respect to $P_t^{\tilde{J}}(i)$ and then impose symmetry by setting $P_t^{\tilde{J}}(i) = P_t^{\tilde{J}}$ and $Z_t^J(i) = Z_t^J$ because all firms face an identical problem. We let $\lambda_t^J = \Lambda_t^J/P_t$ and rescale by technology. Then we obtain

$$\left[\frac{\sigma_J}{\sigma_J - 1}\frac{\lambda_t^J}{p_t^{\tilde{J}}} - 1\right] = \frac{\phi_{P^J}}{\sigma_J - 1}\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}}\right)\left(\frac{\pi_t^{\tilde{J}}}{\pi_{t-1}^{\tilde{J}}} - 1\right)$$
$$-E_t\frac{\theta g_{t+1}n}{r_t}\frac{\phi_{P^J}}{\sigma_J - 1}\left\{\frac{p_{t+1}^{\tilde{J}}}{p_t^{\tilde{J}}}\frac{\check{Z}_{t+1}^J}{\check{Z}_t^J}\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}}\right)\left(\frac{\pi_{t+1}^{\tilde{J}}}{\pi_t^{\tilde{J}}} - 1\right)\right\} . \tag{166}$$

For $U_t^J(i)$, $X_t^J(i)$, $I_t^J(i)$, and $K_{t+1}^J(i)$ we have

$$\check{v}_t = \check{\lambda}_t^J\check{F}_{U,t}^J , \tag{167}$$

$$p_t^X = \check{\lambda}_t^J\check{F}_{X,t}^J , \tag{168}$$

$$q_t^J = 1 + \phi_I\left(\frac{\check{I}_t^J}{\check{K}_t^J} - \frac{\check{I}_{t-1}^J}{\check{K}_{t-1}^J}\right) , \tag{169}$$

$$q_t^J = \frac{\theta}{r_t} E_t \left[q_{t+1}^J (1-\delta) + r_{k,t+1}^J - \tau_{k,t+1} \left(r_{k,t+1}^J - \delta q_{t+1}^J \right) \right] \tag{170}$$

$$+ \frac{\theta}{r_t} E_t \left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J} \right) \phi_I \left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J} - \frac{\check{I}_t^J}{\check{K}_t^J} \right) - \frac{\theta}{r_t} E_t \frac{\phi_I}{2} \left(\frac{\check{I}_{t+1}^J}{\check{K}_{t+1}^J} - \frac{\check{I}_t^J}{\check{K}_t^J} \right)^2 ,$$

where we have used

$$r_{k,t}^J = \check{\lambda}_t^J \check{F}_{K,t}^J , \tag{171}$$

$$\check{F}_{U,t}^J = \mathfrak{T} \left(\frac{\left(1-\alpha_{J_t}^X\right) \check{Z}_t^J}{\mathfrak{T} \check{M}_t^J} \right)^{\frac{1}{\xi_{XJ}}} \left(\frac{\alpha_J^U \check{M}_t^J}{\check{U}_t^J} \right)^{\frac{1}{\xi_{ZJ}}} , \tag{172}$$

$$\check{F}_{K,t}^J = \mathfrak{T} \left(\frac{\left(1-\alpha_{J_t}^X\right) \check{Z}_t^J}{\mathfrak{T} \check{M}_t^J} \right)^{\frac{1}{\xi_{XJ}}} \left(\frac{\left(1-\alpha_J^U\right) \check{M}_t^J}{\check{K}_t^J} \right)^{\frac{1}{\xi_{ZJ}}} , \tag{173}$$

$$\check{F}_{X,t}^J = \mathfrak{T} \left(\frac{\alpha_J^X \check{Z}_t^J}{\mathfrak{T} \check{X}_t^J} \right)^{\frac{1}{\xi_{XJ}}} . \tag{174}$$