



# IMF Working Paper

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## Systemic Risk and Optimal Regulatory Architecture

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**IMF Working Paper**

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**Systemic Risk and Optimal Regulatory Architecture<sup>1</sup>**

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**Abstract**

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Until the recent financial crisis, the safety and soundness of financial institutions was assessed from the perspective of the individual institution. The financial crisis highlighted the need to take systemic externalities seriously when rethinking prudential oversight and the regulatory architecture. Current financial reform legislation worldwide reflects this intent. However, these reforms have overlooked the need to also consider regulatory agencies' forbearance and information sharing incentives. In a political economy model that explicitly accounts for systemic connectedness, and regulators' incentives, we show that under an expanded mandate to explicitly oversee systemic risk, regulators would be more forbearing towards systemically important institutions. We also show that when some regulators have access to information regarding an institutions' degree of systemic importance, these regulators may have little incentive to gather and share it with other regulators. These findings suggest that (and we show conditions under which) a unified regulatory arrangement can reduce the degree of systemic risk vis-à-vis a multiple regulatory arrangement.

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## I. Introduction

Recent financial regulatory reforms include changes to the allocation of regulatory functions aimed at lessening systemic risk. For example, referring to the financial reform bill recently passed by the U.S. Congress, Senator Dodd stated: “with this bill, we have protected taxpayers from being forced to bail out companies that threaten to bring down the economy. . . regulators will no longer be able to ignore emerging threats to the economy.”

This bill and other relevant legislation around the globe have been advanced under the assumption that the incentives of regulators, charged with different oversight functions, are perfectly aligned with each other. As pointed out by Repullo (2000), if the agencies charged with the different regulatory functions were social welfare maximizers, their joint action would also lead to the optimal implementation of regulatory oversight. However, this is not the case when agencies’ objective functions differ across regulatory agencies. In this case, the optimal regulatory response is a second best that depends on the comparison of alternative allocation of regulatory functions. This comparison is potentially more difficult when regulators are asked to explicitly take into account the systemic importance of the institutions they oversee (as in recent financial regulatory reforms).

In our view, an analytical framework that is useful for evaluating the alternative allocation of regulatory functions should explicitly model financial institutions facing two potential types of shock: a shock to their liquidity (represented by unexpected withdrawals by depositors) and a shock to their solvency (represented by a decrease in the value of their assets). And key regulatory functions to be analyzed should include lender of last resort, early intervention powers and the provision of short-term bank-liabilities insurance. Furthermore, because the regulatory reforms are intent on dealing with intermediaries’ systemic connectedness risks, the model should explicitly account for systemic externalities. The model should also explicitly include political economy considerations as in, for example, Repullo (2000) and Kahn and Santos (2005), to explicitly account for regulators’ strategic actions regarding data sharing and forbearance incentives.<sup>1</sup>

Regulatory forbearance arises primarily because the failure of a financial intermediary is “politically costly” for a regulator. Regulators often have the incentive to keep an institution afloat, even when insolvent, because regulators strongly dislike closing institutions under their watch, especially because in some cases, given enough time, an institution may be back on its feet. Similarly, how or whether regulators share information will reflect the strategic considerations of the different agencies’ objectives. For example, agencies possessing information regarding firms’ degree of systemic importance may have an incentive to misrepresent this information to other agencies.

The main findings of the paper are that: i) under an expanded mandate to explicitly oversee systemic interconnectedness, regulators would be more forbearing towards systemically important institutions, because the systemically important institutions will have a more damaging effect on other institutions under the regulators’ purview and stricter with non-systematic institutions; and ii) in the presence of efficient resolution mechanisms and high political costs for shutting down a financial institution, a unified regulatory arrangement could reduce systemic risk *vis-a-vis* a multiple regulatory arrangement because: a) forbearance would be reduced, b) when only a subset of regulators have access to private information regarding an institution’s degree of systemic importance, these regulators may have no incentive to share it with other regulators, and c) a multiregulator

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<sup>1</sup>Important contributions in the banking regulation literature [spanning the analysis of: lender of last resort and the role of policy precommitment (e.g., Goodfriend and Lacker (1999) and Freixas (1999)); deposit insurance and moral hazard (e.g., Kareken and Wallace (1978) and Merton (1977)); deposit insurance pricing and closure policies (e.g., Pennacchi (1987), Mailath and Mester (1994), Acharya and Dreyfus (1989), Allen and Saunders (1993), and Fries et al. (1997)); lender of last resort and deposit insurance (e.g., Kanatas (1986), Sleet and Smith (2000)); and supervision and bank monitoring (e.g., Campbell et al (1992))] abstracted from strategic interactions among regulators.

setting decreases the amount of information gathered even if information can be collected at no cost.

The remainder of the paper is organized as follows. Section 2 describes the model without private information. In Section 3, we solve the model with private information and examine its main results. Section 4 concludes the paper. All proofs are in the Appendix.

## II. The Model without Private Information

In order to analyze the optimal regulatory implications of explicitly accounting for systemically important institutions, our political economy model features two “banks” indexed by  $i \in \{A, B\}$ . Their only source of financing for their long term projects is demand deposits. Each bank faces the possibility of two types of shocks: liquidity shocks (represented by a sudden drop in deposits) and solvency shocks (represented by low probability of success of the bank’s investment project). Lastly, banks  $A$  and  $B$  are identical except in two respects: (i) bank  $A$  is the systemic institution in the sense that its failure lowers the probability that bank  $B$ ’s projects succeed, and (ii) the timing of each bank’s actions differ slightly in order for bank  $A$ ’s performance to affect bank  $B$ .

Figure 1 summarizes the timing. There are four periods and no time discounting. At time  $t = 0$ , banks invest in long-term projects  $Y_i$ , that pay a return  $\tilde{R}_i$  at a later date. These projects are financed by capital,  $K_i$ , and short-term deposits,  $D_i$ . Banks are subject to liquidity shocks, which are publicly observable at date  $t = 1$ : if the new level of deposits  $\tilde{D}_i$  at  $t = 1$  is such that  $D_i > \tilde{D}_i$ , banks are forced to seek emergency liquidity from a lender-of-last-resort to bridge the liquidity gap, defined as  $\omega_i \equiv D_i - \tilde{D}_i$ .<sup>2</sup> At  $t = 1$ , the probability of success of bank  $A$ ’s project,  $u_A$ , is publicly revealed and the return of its investment project is realized at  $t = 2$ . Similarly, bank  $B$ ’s probability of success,  $u_B$ , is publicly known at  $t = 2$ , and the return on this investment is realized at  $t = 3$ . Furthermore, as of date 0, the signal  $u_i$  containing information on  $\tilde{R}_i$  (i.e. supervisory information) has the following properties:

**Assumption 1** For each bank  $i = A, B$ , the level of deposits available at date 1,  $\tilde{D}_i$ , are independent random variables with distribution  $G(D)$ , and  $G'(0) > 0$ .

**Assumption 2** The financial condition signal of bank  $A$ ’s portfolio of loans is given by the random variable  $u_A$ , with support  $[\underline{u}_A, \bar{u}_A] \subset [0, 1]$  and distribution  $F_A(u_A)$ , and it is publicly observable at date 1 but not verifiable. If bank  $A$  invests  $Y_A$  in loans at period 0, it will receive  $Y_A \tilde{R}_A$  in period

$t = 2$  where

$$\tilde{R}_A = \begin{cases} R & \text{with probability } u_A \\ 0 & \text{with probability } 1 - u_A \end{cases}$$

**Assumption 3** The financial condition signal of bank  $B$ ’s portfolio of loans is given by the random variable  $\tilde{u}_B - \gamma\chi$  ( $\chi$  is an indicator variable which equals 1 if bank  $A$  failed at  $t = 2$  and 0 otherwise.), where  $\tilde{u}_B$  has distribution  $F_B(u_B)$  with support  $[\underline{u}_B, \bar{u}_B] \subset [0, 1]$  and  $\gamma \in [0, \underline{u}_B]$ . The financial condition signal is publicly observable at date  $t = 2$  but not verifiable. If bank  $B$  invests  $Y_B$  in loans at period 0, it will receive  $Y_B \tilde{R}_B$  in period  $t = 3$ , where

$$\tilde{R}_B = \begin{cases} R & \text{with probability } u_B - \gamma\chi \\ 0 & \text{with probability } 1 - u_B + \gamma\chi \end{cases}$$

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<sup>2</sup>The rollover risk serves the same role as does liquidity demand for early withdrawals in previous models such as Diamond and Dybvig (1983), but allows for a simpler stochastic structure.

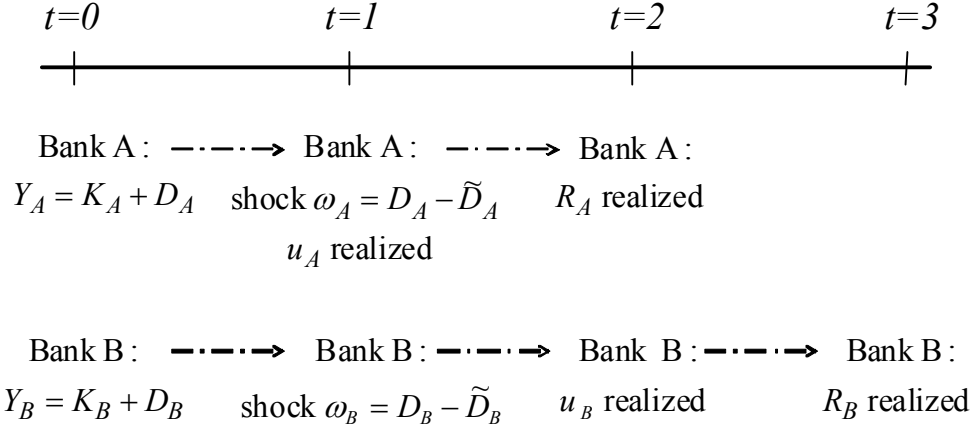


Figure 1: Timing of information.

Each bank can be liquidated before the realization of the investment outcome (that is, before  $t = 2$  for bank  $A$  and before  $t = 3$  for bank  $B$ ). If this occurs, the liquidation value of the bank's projects is  $L_i Y_i$ , where  $L_i \in (0, 1)$  determines the relative loss that banks suffer for canceling their investments early. In addition, there is a bankruptcy cost of  $c_i > 0$ , such that, as of time 0 the expected return from bank lending (net of second period bankruptcy costs) exceeds the zero return from holding liquid assets, for  $i = A, B$ ,

$$E(\tilde{u}_i)[R + c_i] > 1 + c_i. \quad (1)$$

### A. Regulatory Architectures: Unified *vs* Multiple Regulators

We are interested in considering different regulatory architectures. In particular, we want to compare a unified regulator ( $UR$ ) against an alternative set-up where regulatory functions are divided between two agencies: a lender of last resort ( $LoLR$ ) charged with the provision of emergency liquidity to banks, and a deposit insurer ( $DI$ ) responsible for guaranteeing bank deposits and that has early intervention powers (the  $DI$  can intervene even if there is no liquidity shortfall).<sup>3</sup>

All regulators have private objective functions; that is, regulators do not maximize social welfare. Instead, regulators care about their income and face a trade-off between the political cost of closing a bank in distress and the expected financial cost of not closing—that is, forbearing—the bank. In other words, on the one hand regulators are interested in avoiding the reputational cost associated with a bank closure, while on the other hand, they want to minimize their financial costs in case the bank actually fails. For the  $LoLR$ , these costs are the funds lent to a distressed bank (at a penalty rate  $P$ ) that eventually defaults. For the  $DI$ , the costs are the deposit insurance

<sup>3</sup>Note that this is akin to the prompt corrective actions framework of the FDIC in the United States.

payouts (to depositors) that are made if a bank eventually fails. For the *UR*, the costs are the sum of the *LoLR* and *DI* costs.

The regulatory decisions of the *DI* and the *LoLR* regarding bank  $i$  are respectively given by  $d_i, l_i \in \{0, 1\}$ , where  $d_i = 1$  if the *DI* decides not to close the bank and  $d_i = 0$  if otherwise, and  $l_i = 1$  if the *LoLR* provides emergency funds and  $l_i = 0$  if otherwise. In terms of the model's timing this can be expressed as follows: at  $t = 1$ , signal  $u_A$  is obtained and the *DI* may choose to close bank  $A$  or the *LoLR* may refuse to provide liquidity if  $\omega_A > 0$ . In period  $t = 2$ , the signal  $u_B$  is observed and the regulators make similar decisions regarding bank  $B$ .

Let  $s_i \equiv (d_i, l_i)$ . The utility function of regulators are given by,

$$E \left[ I \left( s_A, \tilde{R}_A \right) - \alpha c_{AX} \left( s_A, \tilde{R}_A \right) \theta_A + E \left[ I \left( s_B, \tilde{R}_B \right) - \alpha c_{BX} \left( s_B, \tilde{R}_B \right) \theta_B \mid \chi \left( s_A, \tilde{R}_A \right), \tilde{u}_B \right] \mid \tilde{u}_A \right] \quad (2)$$

where  $I(\cdot)$  is the net income accruing to the regulator,  $\alpha c_i$  measures the regulator's political cost of bankruptcy, and  $\theta$  is an indicator variable which is 0 if and only if the regulator's action is to not close the bank while the other regulator chooses to close the bank.<sup>4</sup>

## B. Equilibrium in a Multiple-Regulator Architecture

To find equilibrium outcomes, we solve the regulators' problem by backwards induction. The first step consists of solving the regulators' problem for bank  $B$  once it is known whether bank  $A$  is closed. Proposition 1, states "the closing rules" for bank  $B$ .

**Proposition 1** *Consider a multi-regulator architecture with separate LoLR and DI agencies. At  $t=2$ , once it is known whether bank  $A$  has been closed ( $\chi = 0$ ) or not ( $\chi = 1$ ), bank  $B$  is closed if the realized value of  $u_B$  falls below the threshold  $u_B^m(\gamma)$  given by:*

$$u_B^m(\chi) \equiv \max \{ u_B^{DI}(\chi), u_B^{LoLR}(\chi) \},$$

where

$$u_B^{LoLR}(\chi) = \frac{\omega_B}{(P+1)\omega_B + \alpha c_B} + \gamma\chi,$$

$$u_B^{DI}(\chi) = \frac{L_B Y_B - \max\{0, \omega_B\}}{D_B - \max\{0, \omega_B\} + \alpha c_B} + \gamma\chi.$$

The intuition for the proposition is as follows. From the *LoLR* perspective, taking *DI*'s actions as given, the size of the liquidity shock determines the degree of forbearance: larger injections of liquidity,  $\omega_B$ , require greater likelihood of success (i.e., higher values of  $u_B^{LoLR}$ ). If the *LoLR* does not provide the loan, the bank is forced to close and the *LoLR* would bear the bankruptcy cost  $\alpha c$ . If, on the other hand, the *LoLR* makes the loan but the bank fails, then in addition to the bankruptcy cost, the *LoLR* forgoes the liquidity injection,  $\omega_B$ , which the failed bank is unable to repay. Therefore, the higher the need for liquidity injections, the greater is the probability of success  $u$  required by the *LoLR* to extend the liquidity support.

From the *DI*'s perspective, taking the *LoLR*'s actions as given, the *DI* is more forbearing the greater the liquidity assistance supplied by the *LoLR*. This is because (unpaid) debts to the *LoLR* are outside the responsibility of the *DI* regulator if the bank fails *and* reduce the potential need for deposit insurance outlays.<sup>5</sup> This increases the temptation for an independent *DI* agency to engage in forbearance as liquidity shortfalls increase.

<sup>4</sup>For the unified regulator  $\theta = 1$  whenever the regulator chooses to close a bank.

<sup>5</sup>Notice that the liquidity injection transforms the bank's debt to depositors (which are insured by the *DI*) into debt to the *LoLR* (which is not insured by the *DI*).

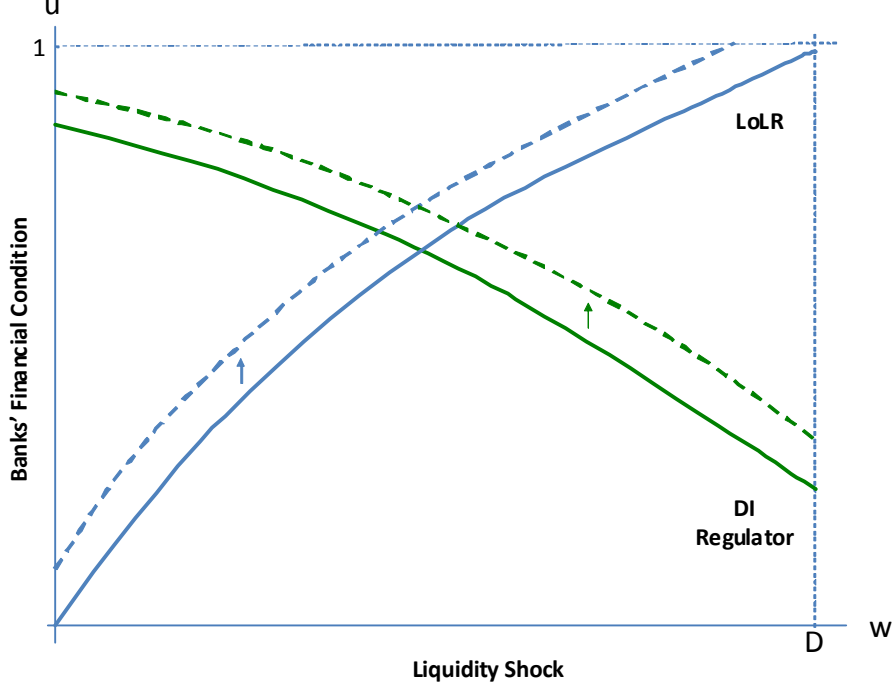


Figure 2: Higher probability of closing non-systemic institutions in crisis under multiregulator arrangements. The solid lines represent the liquidation cutoffs of the *LoLR* and the *DI* for the non-systemic bank *B* when bank *A* is not closed. The dashed lines represent the liquidation cutoffs of the *LoLR* and the *DI* for the non-systemic bank *B* when bank *A* is closed.

Taking as given the regulatory response towards bank *B*, the second step consists of solving the regulators' problem for bank *A*. Proposition 2, states “the closing rules” for bank *A*.

**Proposition 2** *Consider a multi-regulator architecture with separated LoLR and DI agencies. Bank A is closed for  $u_A$  values below*

$$u_A^m(\gamma) \equiv \max \{u_A^{DI}(\gamma), u_A^{LoLR}(\gamma)\},$$

where

$$u_A^{LoLR}(\gamma) = \frac{\omega_A}{(P+1)\omega_A + \alpha c_A + \gamma[(P+1)\max\{0, \omega_B\} + \alpha c_B] \Pr[u_B \geq u_B^m(\chi=0)]},$$

$$u_A^{DI}(\gamma) = \frac{L_A Y_A - \max\{0, \omega_A\}}{D_A - \max\{0, \omega_A\} + \alpha c_A + \gamma[D_B - \max\{0, \omega_B\} + \alpha c_B] \Pr[u_B \geq u_B^m(\chi=0)]}.$$

The insights of propositions 2 and 3 are illustrated in Figures 2 and 3, where the solid lines represent the case in which  $\chi = 0$  and the dashed lines represent the case in which  $\chi = 1$ . The horizontal axis depicts the liquidity shock  $\omega$ , whereas the vertical axis represents the bank's financial condition,  $u$ . The higher the required *LoLR*'s liquidity injection, the less forbearing the *LoLR* is (i.e., higher  $u$ )—this explains why the *LoLR* line is upward sloping. On the other hand, higher liquidity injections reduce the size of potential outlays from the deposit insurance, thus inducing the *DI* agency to become more forbearing—this is represented in the downward sloping *DI* line in Figure 2.



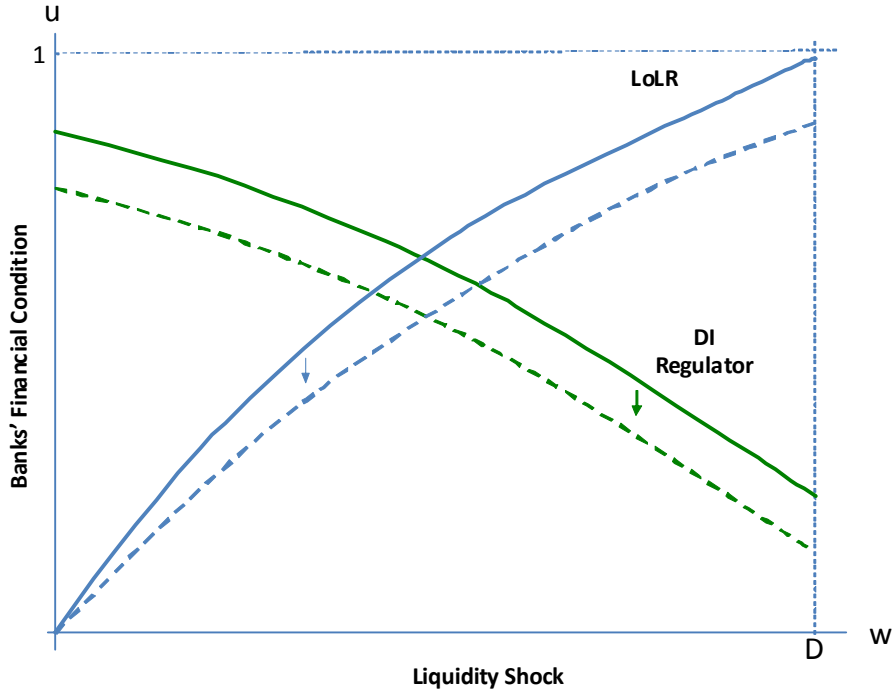


Figure 3: More forbearance toward systemic institutions under single regulator and multiregulator arrangements. The solid lines represent the liquidation cutoffs of the *LoLR* and the *DI* for the systemic bank *A*, in the absence of systemic risk. The dashed lines represent the liquidation cutoffs of the *LoLR* and the *DI* for the systemic bank *A* when bank *A*'s degree of systemic risk is positive.

Figure 2 also illustrates two results arising under a mandate, to monitor the systemic importance of financial institutions. Under this mandate, both, the *LoLR* and the *DI* supervisors will be: (i) less forbearing towards non-systemic institutions (Figure 2); and (ii) more forbearing towards systemically important institutions (Figure 3). This follows from interpreting the liquidation cutoffs for which  $\chi = 0$  as representing the decision to close bank *B* in the absence of systemic risk ( $\gamma = 0$ ). In this case, regulation outcomes regarding bank *B* are the same regardless of what happens to bank *A*.

Figure 3 illustrates the results of Proposition 2. The solid lines represent liquidation cutoffs in the absence of systemic risk ( $\gamma = 0$ ) and the dashed lines represent the environment in the presence of systemic risk ( $\gamma > 0$ ). The presence of systemic risk makes regulators more forbearing towards the systemic institution. The decision to close bank *A* increases the probability that bank *B* is liquidated — due to more stringent solvency standards, increasing the expected costs associated with its failure. As a result, regulators become less willing to liquidate bank *A* in order to avoid the expected downside consequences of systemic risk.

## C. Equilibrium in a Unified Regulator Architecture

### 1. Non-systemic bank

Because under a *UR*, the *LoLR* and a *DI* problems are solved simultaneously, intuition would suggest that a *UR* would internalize the excessive forbearance incentives faced by each individual regulator leading to the lowest degree of excessive regulatory forbearance. However, it is important

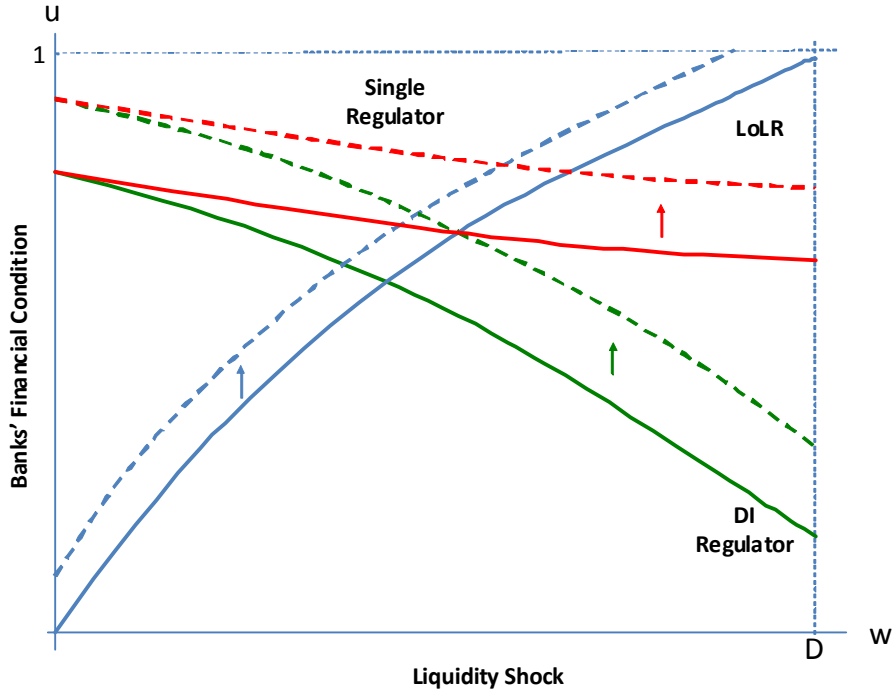


Figure 4: Higher probability of closing non-systemic institutions in crisis under single and multi-regulator arrangements. The solid lines represent the liquidation cutoffs of the *LoLR*, the *DI*, and the single regulator for the non-systemic bank *B* when bank *A* is not closed. The dashed lines represent the liquidation cutoffs of the *LoLR*, the *DI*, and the single regulator for the non-systemic bank *B* when bank *A* is closed.

to recall that what is at stake is the maximization of regulators' utility and not necessarily social welfare. Propositions 3 and 4 formally state the conditions under which *UR* leads to the lowest oversight forbearance of non systemic and the systemic banks, respectively.

**Proposition 3** *Consider the case of a unified regulator with *DI* and *LoLR* functions under its purview. At  $t=2$ , once it is known whether bank *A* remains open ( $\chi = 0$ ) or not ( $\chi = 1$ ), bank *B* is closed if the realized value of  $u_B$  falls below*

$$u_B^u(\chi) \equiv \frac{L_B Y_B}{P \max\{0, \omega_B\} + \alpha c_B + D_B} + \gamma \chi.$$

The unified regulator has the same payoff as that of the *DI* when the decision is to close bank *B*. However, their payoffs are different when bank *B* is allowed to continue operating. If bank *B* fails, the unified regulator has to back all the deposits while the *DI* only responds for those not yet covered by the *LoLR*. This makes the unified regulator less forbearing. If bank *B* succeeds, the unified regulator receives the liquidity injection times the penalty rate  $P$  while the *DI* has a payoff of zero. For moderate liquidity shocks, the unified regulator's payoff is more weighted towards that of when bank *B* fails, implying lower forbearance. Since the *DI* is less forbearing than the *LoLR* for moderate shocks, this implies that the unified architecture results in less forbearance than the multiregulator setting.

The unified regulator has a higher payoff than that of the *LoLR* when the decision is to close bank *B* due to the salvage value. This makes the unified regulator less forbearing for a given size of

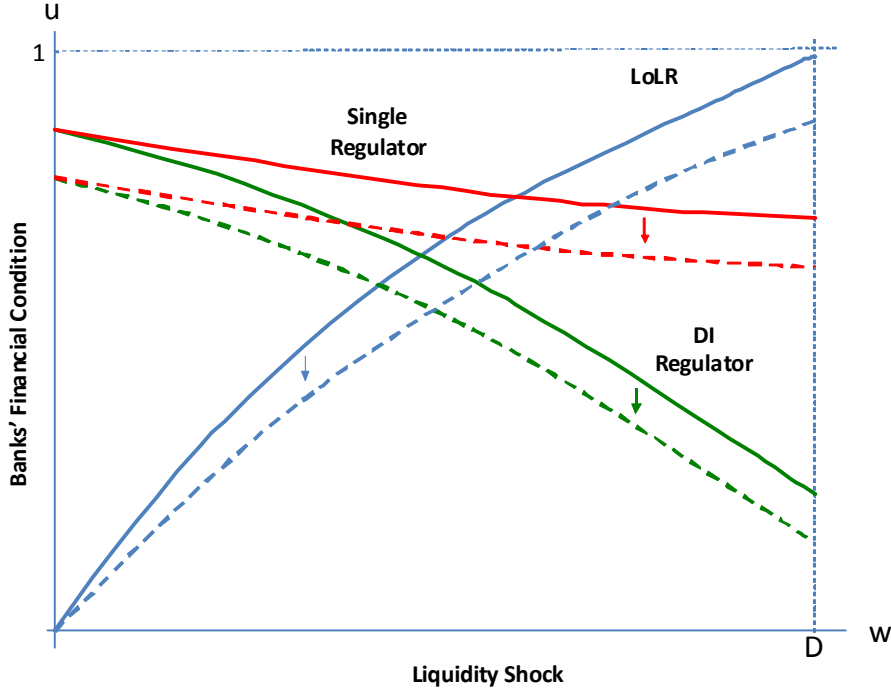


Figure 5: More forbearance toward systemic institutions under single regulator and multiregulator arrangements. The solid lines represent the *LoLR*, the *DI*, and the single regulator liquidation cutoffs for the systemic bank *A* in the absence of systemic risk. The dashed lines represent the *LoLR*, the *DI*, and the single regulator liquidation cutoffs for the systemic bank *A* when bank *A*'s degree of systemic risk is positive.

liquidity shock. However, their payoff differ when bank *B* is not closed. If bank *B* succeeds, both get the liquidity injection times the penalty rate  $P$ . However, the unified regulator's payoff when bank *B* fails is independent of the liquidity injection — all deposits need to be backed — while the *LoLR* payoff is decreasing in the liquidity shock. It follows that the unified regulator's payoff when he does not close bank *B* is increasing in the liquidity shock, implying greater forbearance. The opposite is true in the multiregulator arrangement. Therefore, when the liquidity shock is sufficiently high, the unified regulator setting is more forbearing than the multiregulator one. This is illustrated in Figure 4.

## 2. Systemic bank

**Proposition 4** *Consider the case where a unified regulator has *DI* and *LoLR* functions. Then bank *A* is closed for  $u_A$  values below*

$$u_A^u(\gamma) \equiv \frac{L_A Y_A}{P \max\{0, \omega_A\} + D_A + \alpha c_A + \gamma (P \max\{0, \omega_B\} + \alpha c_B + D_B) \Pr[u_B \geq u_B^u(\chi = 0)]}.$$

The regulator will be softer on potentially systemic institutions. Consolidating standard regulatory functions without additional tools to preclude institutions from becoming systemic in the first place will not eliminate regulators' incentives to be lenient with systemically important institutions. In fact, these incentives could be exacerbated.

The failure of a systemically important institution increases the likelihood of failures among non-systemic institutions. These increased costs mean that any regulator will be more lenient with a systemically important institution, as illustrated by the downward shift in lines in Figure 5. On the other hand, since distress in the systemic institution negatively affects the chances of survival of the non-systemic institution, the regulator is less likely to save the non-systemic institution as illustrated by the upward shift of lines in Figure 4. The presence of systemic risk does not relax a regulator’s standards for rescue of non-systemic institutions.

## D. Comparing Multiple- vs Unified-Regulator Architectures

In this subsection we compare the multiregulator and unified regulator arrangements in terms of forbearance. We derive several comparative statics associating the levels of forbearance in each case with the model’s primitive parameters.

### 1. Non-systemic bank

Given the liquidity shock  $\omega_B^*$ , the liquidation cutoffs of the unified regulator and the multiregulator cross each other, i.e., both regulators are equally forbearing. Liquidity shocks below (above) this point imply that the multiregulator arrangement is more (less) forbearing than the unified one. Therefore, increases in  $\omega_B^*$  reduces the “dominance region”, i.e., the set of liquidity shocks for which the unified regulator arrangement is less forbearing than multiregulator arrangement. The following proposition states that the dominance region is increasing in the liquidation value, the bankruptcy cost, the penalty rate, and decreasing in the level of deposits.

**Proposition 5** *Consider a multiregulator arrangement, which has agencies LoLR and DI, and a unified regulator, which has DI and LOLR functions. Let  $u_B^m(\cdot)$  and  $u_B^u(\cdot)$  be their respective liquidation thresholds for bank B. Then*

1. *If  $\omega_B = 0$ , then  $u_B^u(\cdot) = u_B^m(\cdot)$ , i.e., the unified regulator and the multiregulator arrangements are equally forbearing.*
2. *Suppose  $\omega_B > 0$ .*
  - (i) *If  $u_B^u(\cdot) > u_B^{LoLR}(\cdot)$  for all  $\omega_B \in (0, D_B]$ , then the multiregulator arrangement is always more forbearing than the unified regulator arrangement.*
  - (ii) *If  $\exists \omega_B^* \in (0, D_B]$  such that  $u_B^u(\cdot) = u_B^{LoLR}(\cdot)$ , then the multiregulator arrangement is more forbearing than the unified regulator arrangement for  $\omega_B \in (0, \omega_B^*)$  and less forbearing for  $\omega_B \in [\omega_B^*, D_B]$ . In addition, if  $\alpha$  is sufficiently large, then  $\omega_B^*$  is increasing in  $L_B, c_B, P$ , and decreasing in  $D_B$ .*

The intuition behind the results is as follows. Increases in the liquidation value  $L_B$  make the unified regulator less forbearing — increase the payoff when bank  $B$  is closed — and do not change the behavior of the *LoLR*. Therefore, the *LoLR* and the unified regulator are equally forbearing only at higher levels of liquidity shock, which make the *LoLR* less forbearing and the unified regulator more forbearing. It follows that increases in the liquidation value improves the unified regulator arrangement over the multiregulator setting. Higher values of political cost  $c_B$  make both the unified regulator and the multiregulator more forbearing. The reason is that closing bank  $B$  now results in higher costs with certainty while the realization of these costs when bank  $B$  is not closed is uncertain — only if bank  $B$  fails. However, because the unified regulator has to back all the deposits (while the *LoLR* only loses the liquidity injection) when bank  $B$  fails, the decrease in the liquidation cutoff of the unified regulator is relatively smaller. Therefore, the impact of  $c_B$  on forbearance is lower for the unified regulator. It follows that higher political costs reduce the region for which

the multiregulator setting is less forbearing than the unified regulator arrangement. A similar logic applies to changes in  $P$ . Increases in  $D_B$  make the unified regulator more forbearing — increase the costs of closing the bank — and do not change the incentives of the *LoLR*. As a result, the higher the amount of deposits the larger the relative performance of the multiregulator arrangement.

## 2. Systemic bank

**Proposition 6** *Consider a multiregulator arrangement, which has agencies *LoLR* and *DI*, and a unified regulator, which has *DI* and *LoLR* functions. Suppose  $\omega_B \leq 0$  and let  $u_A^m(\gamma)$  and  $u_A^u(\gamma)$  be their respective liquidation thresholds for bank  $A$ . Then*

1. *If  $\omega_A = 0$ , then  $u_A^u(\gamma) = u_A^m(\gamma)$ , i.e., the unified regulator and the multiregulator arrangements are equally forbearing.*

2. *Suppose  $\omega_A > 0$ .*

(i) *If  $u_A^u(\gamma) > u_A^{LoLR}(\gamma)$  for all  $\omega_A \in (0, D_A]$ , then the multiregulator arrangement is more forbearing than the unified regulator arrangement.*

(ii) *If  $\exists \omega_A^* \in (0, D_A]$  such that  $u_A^u(\gamma) = u_A^{LoLR}(\gamma)$ , then the multiregulator arrangement is more forbearing than the unified regulator arrangement for  $\omega_A \in (0, \omega_A^*)$  and less forbearing for  $\omega_A \in [\omega_A^*, D_A]$ . In addition, if  $\alpha$  is sufficiently large, then  $\omega_A^*$  is increasing in  $L_A, L_B, c_A, c_B, D_B, P, \gamma$ , and decreasing in  $D_A$ .*

From the last proposition, we can see that the region where the unified regulator is less forbearing than the multiple regulators is increasing in the liquidation value, bankruptcy costs, penalty rate, systemic risk, and level of deposits of bank  $B$ , and decreasing in the level of deposits of bank  $A$ . The results regarding  $L_A, c_A, P$ , and  $D_A$  parallel those for non-systemic bank. The other results concerning the attributes of bank  $B$  follow from the link between the two banks due to systemic risk. The intuition for the other results is as follows. Higher values of  $c_B$  increase the expected costs associated with the closure of bank  $B$  — either due to regulatory action or bank failure. This also increases the losses resulting from either closure or failure of bank  $A$  as both outcomes increase the probability that bank  $B$  is closed (due to systemic risk). Closure of bank  $A$  increases the insolvency risk of bank  $B$  with certainty while allowing bank  $A$  to continue results in the same outcome only if bank  $A$  fails. Therefore, increases in  $c_B$  make regulators more forbearing towards the systemic bank. Because the unified regulator has to fully guarantee the deposits (while the *LoLR* only loses the liquidity injection) the reduction in the liquidation threshold associated with the bank  $A$  for the unified regulator is lower. As a result, higher  $c_B$  increases the dominance region, i.e., the region for which the unified regulator setting is less forbearing than the unified regulator arrangement.

Table 1 provides a summary of how the the region for which the unified regulator setting is less forbearing than the unified regulator changes as we vary the parameters.

## III. The Model with Private Information

In the previous section we assumed that the degree of systemic risk imposed by bank  $A$  (that is, the parameter  $\gamma$ ) was publicly known. In this section we extend our analysis to an environment where there is incomplete information regarding the value of  $\gamma$ . Thus, assume that  $\gamma$  has publicly known distribution  $Z(\gamma)$  with support  $[\underline{\delta}, \bar{\delta}]$  and expected value  $\underline{\gamma} \equiv E[\tilde{\gamma}]$ . The *DI* can always choose to observe, costlessly, the realized  $\gamma$ . If the *DI* chooses to observe  $\gamma$ , then the *LoLR* automatically observes a signal from a coarser information set. The *DI* can then supplement the *LoLR*'s information with a report of its own observation. We will assume that the *LoLR*'s information set takes the following form.

$$\{\{\gamma\} | \gamma < \gamma_*\} \cup \{\{\gamma_*, 1\}\}$$

**Table 1. Change in the region for which the multiregulator arrangement is less forbearing than the unified regulator arrangement**

$L_i$  is the liquidation value,  $c_i$  is the bankruptcy cost, and  $D_i$  is the amount of deposits, where  $i = A, B$ .  $P$  is the penalty rate and  $\gamma$  is the degree of systemic risk.

	Bank $B$ (non-systemic)	Bank $A$ (systemic)
$\Delta L_B$	–	–
$\Delta c_B$	–	–
$\Delta D_B$	+	–
$\Delta P$	–	–
$\Delta \gamma$		–
$\Delta L_A$		–
$\Delta c_A$		–
$\Delta D_A$		+

In other words, there exists a value  $\gamma_*$  such that, if the  $DI$  receives information  $\gamma < \gamma_*$  then the  $LoLR$  receives identical information. If  $\gamma \geq \gamma_*$  then the  $LoLR$  does not observe  $\gamma$ ; and instead only knows that it lies in the interval  $[\gamma_*, 1]$ . Let  $\bar{\gamma} \equiv E[\tilde{\gamma} | \gamma \geq \gamma_*]$ .

## A. Information sharing

We begin by assuming that the  $DI$  elects to receive the information. For  $\gamma < \gamma_*$ , there are no strategic information sharing considerations and our former results go through. The same is true for values  $\gamma \geq \gamma_*$  such that  $u_A^{DI}(\gamma) > u_A$ . In this case, bank  $A$  is liquidated, which is exactly the action desired by the  $DI$ .

### 1. The $DI$ has incentive to act strategically

Consider the case in which  $\gamma \geq \gamma_*$  and  $u_A^{DI}(\gamma) \leq u_A$ , i.e., the  $DI$  does not want to liquidate institution  $A$ . When  $u_A^{LoLR}(\bar{\gamma}) \leq u_A$ , the  $LoLR$  would have incentive to provide liquidity support to this institution  $A$ . A report sent by the  $DI$  to the  $LoLR$  saying  $\gamma$  is such that  $u_A^{LoLR}(\gamma) > u_A$ , if believed, would cause the  $LoLR$  to refuse to provide liquidity support to bank  $A$ . The resulting outcome is the opposite of that desired by the  $DI$ .

When  $u_A < u_A^{LoLR}(\bar{\gamma})$ , a talking strategy from the  $DI$  stating  $\gamma$  is such that  $u_A^{LoLR}(\gamma) \leq u_A$ , if believed, would cause the  $LoLR$  to provide liquidity assistance to bank  $A$ , which is the outcome wished by the  $DI$ . As a result, information sharing is strategically important and suggests the  $DI$  has an incentive to draft a report in such a way as to induce  $LoLR$  to act according to  $DI$ 's preferences. In what follows, we assume  $\gamma \geq \gamma_*$  and  $u_A^{DI}(\gamma) \leq u_A$  unless stated otherwise.

Let the  $DI$  send a message  $m : [0, 1] \rightarrow M$  to the  $LoLR$ , where without loss of generality it can be assumed that  $M$  is a finite set. The  $LoLR$  observes the message, forms belief  $\bar{\gamma}(m) = E[\tilde{\gamma} | m]$ , and takes action  $l_A \in \{0, 1\}$ . The decision of the  $LoLR$  will be based on the  $u$  threshold associated with  $\bar{\gamma}(m)$ . We denote this threshold by  $u_A^{LoLR}(\bar{\gamma}(m))$ . Let  $\gamma^*$  be such that  $u_A^{LoLR}(\gamma^*) = u_A$ .

### 2. Communication is ineffective

If  $\gamma \geq \gamma_* \geq \gamma^*$ , then for any message  $m \in M$  we have  $\bar{\gamma}(m) \geq \gamma^*$ , which implies  $u_A = u_A^{LoLR}(\gamma^*) \geq u_A^{LoLR}(\bar{\gamma}(m))$ . Therefore, bank  $A$  is not liquidated. Consider now the cases in which  $\gamma_* \leq \gamma < \gamma^*$  and  $\gamma_* < \gamma^* \leq \gamma$ . If  $\gamma_* < \gamma^* \leq \gamma$ , which is a situation described by the lower dashed blue line

in Figure 6, the *DI* has incentives to fully reveal information on the degree of *A*'s systemic importance. This follows from the fact that, if the *LoLR* believes the report, he provides liquidity support to bank *A*. However, the *DI* does not have incentive to reveal his information to the *LoLR* if  $\gamma_* \leq \gamma < \gamma^*$ , which is the case represented by the upper dashed blue line in Figure 6. The reason is that, if the *LoLR* believes the report, he does not provide liquidity assistance to bank *A*.

One must note that if  $\gamma_* < \gamma^* \leq \gamma$ , then any message  $m$  such that  $\bar{\gamma}(m) \geq \gamma^*$  implies  $u_A^{LoLR}(\bar{\gamma}(m)) \leq u_A^{LoLR}(\gamma^*) = u_A$ . In this case, the *LoLR* provides liquidity to bank *A* and the bank is not liquidated. This would also be the action of the *LoLR* if he observed  $\gamma$ , which implies the he would be better off after communication. However, if  $\gamma_* \leq \gamma < \gamma^*$ , then any message  $m$  such that  $\bar{\gamma}(m) \geq \gamma^*$  implies  $u_A^{LoLR}(\bar{\gamma}(m)) \leq u_A^{LoLR}(\gamma^*) = u_A < u_A^{LoLR}(\gamma)$ . As a result, the *LoLR* would not liquidate the bank. The *LoLR* is worse off with communication since his decision would be to liquidate the bank if he knew  $\gamma$ . This analysis suggests that fully separating equilibria do not exist. The proposition below confirms this intuition.

**Proposition 7** *If  $\gamma \geq \gamma_*$  and  $u_A^{DI}(\gamma) \leq u_A$ , then babbling equilibria exist, i.e., equilibria in which  $m(\gamma) = m \forall \gamma \geq \gamma_*$ . In addition, every equilibrium has the same outcome as that of a babbling equilibrium, i.e., communication is ineffective.*

The result of the last proposition parallel those of Kahn-Santos in that regulators with private information fail to share information. In our model, the *DI* fails to share information that induces the *LoLR* to take an action different than he would have taken in the absence of communication. Therefore, if  $\gamma \geq \gamma_*$ , then bank *A* will not be liquidated if  $\gamma_*$  is large enough and will be liquidated if otherwise.

**Corollary 1** *If  $\gamma \geq \gamma_*$ , then bank *A* is not liquidated if  $\exists \hat{\gamma} \in [0, 1]$  such that  $E[\tilde{\gamma} | \gamma_* \leq \tilde{\gamma} \leq \hat{\gamma}] = \gamma^*$  and is liquidated if otherwise.*

The above corollary states the following. Given that the posterior belief regarding the degree of systemic risk is higher than the initial conjecture, the better the bank's financial condition — lower liquidity shocks ( $\omega$ ) and higher solvency signals ( $u$ ) — the more likely the *LoLR* revises the decision to not provide liquidity injections.

## B. Information gathering

We now investigate the incentive of the *DI* to gather information given  $\gamma_*$ . Although a higher revealed degree of systemic risk results in better outcomes from the *DI*'s point of view, it is not always the case that the *DI* wants to learn about  $\gamma$ . The reason is that, if the revealed  $\gamma$  is low, the *LoLR* might liquidate the bank while the *DI* wants its continuation. Our next result states that the *DI* might not gather information even if it is free to do so.

**Proposition 8** *Let  $\underline{\gamma}$  be such that  $u_A^{DI}(\underline{\gamma}) = u_A$ . If*

1.  $\max\{u_A^{LoLR}(\underline{\gamma}), u_A^{DI}(\underline{\gamma})\} \leq u_A$ ,
2.  $\underline{\gamma} \leq \underline{\delta}$ , i.e.,  $\underline{\gamma}$  is below the lower bound of the support of  $Z(\gamma)$ .
3.  $\nexists \hat{\gamma} \in [0, 1]$  such that  $E[\tilde{\gamma} | \gamma_* \leq \tilde{\gamma} \leq \hat{\gamma}] = \gamma^*$ , and
4. If  $\gamma_* \leq \gamma^*$ ,

*then in equilibrium the *DI* chooses not to observe  $\gamma$ .*

This result shows that a multiregulator setting decreases the amount of information gathered even if information can be collected at no cost. The reason is that, if the expected outcome of

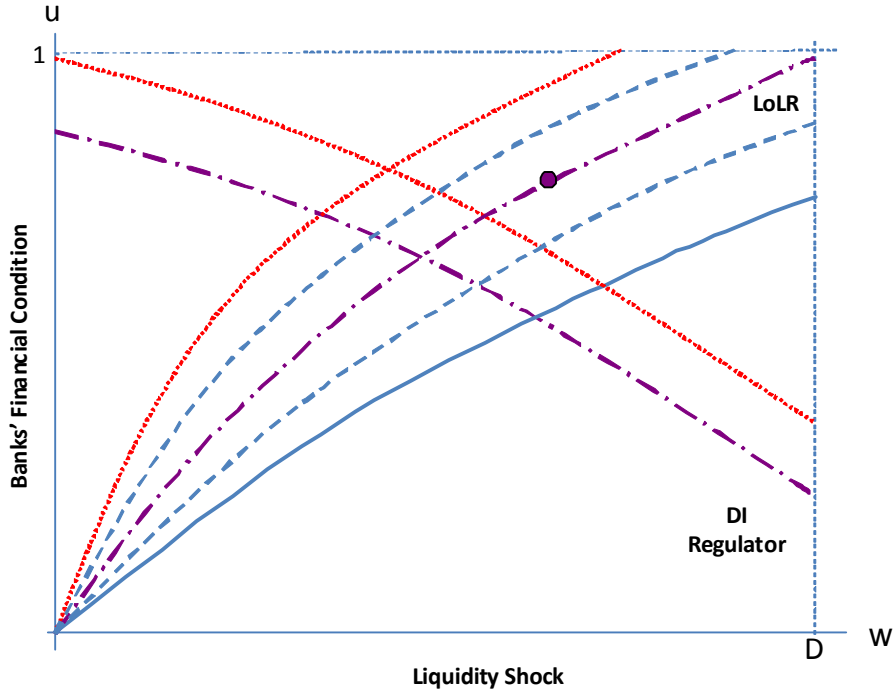


Figure 6: *DI*'s incentive to reveal information. The purple dot represents a realization of  $(\omega_A, \omega_B, u_A)$ . The dashed-dotted purple lines represent the *LoLR* and *DI* liquidation cutoffs given  $\gamma^*$ , which is the value of gamma such that the *LoLR*'s cutoff goes through the realization  $(\omega_A, \omega_B, u_A)$ . The solid blue line represents the *LoLR*'s liquidation cutoff given  $\bar{\gamma}$ . The dotted red lines represent the *LoLR* and *DI* liquidation cutoffs given  $\gamma_*$ . The dashed blue lines represent the *LoLR* liquidation cutoffs for realized  $\gamma$  above and below  $\gamma^*$ . The *DI* has (does not have) incentive to report truthfully for realized values of gamma above (below)  $\gamma^*$ .

revealing information makes a regulator worse off, then that regulator has less incentive to gather the information in the first place. Therefore, as a basic result, there is better gathering and using of information by a unified regulator than by a divided one.

#### IV. Concluding Remarks

An important ingredient missing from most recent reform proposals mandating the oversight of systemic risk is the analysis of regulators' incentives. This includes "regulatory forbearance" for the incentive to keep institutions afloat when they should be unwound—which will likely vary across the alternative ways the regulatory functions could be allocated.

We show how adding a systemic risk monitoring mandate to the regulatory mix without a set of associated policy tools does not alter the basic regulator's incentives at the heart of some of the regulatory shortcomings leading to this crisis. Regulators often have the incentive to keep an institution afloat, even when insolvent, because regulators strongly dislike closing institutions under their watch, especially because in some cases, given enough time, an institution *may* be back on its feet. Therefore, in the absence of concrete methods formally to limit financial institutions' ability to become systemically important in the first place—regardless of how regulatory functions



are allocated—regulators may well be more forgiving with systemically important institutions compared to those that are not. This is because the systemically important institutions will have a more damaging effect on other institutions under the regulators’ purview. Moreover, competing regulators are less likely to gather information and may have an incentive not to share information once gathered.

While we acknowledge the limitations of our model, we see its main value in the illustration of the importance of analyzing strategic considerations in the optimal allocation of regulatory functions—particularly as a counterweight to analyses which ignore regulatory incentives.<sup>6</sup>

## Appendix

**Proof of Proposition 1.** Let  $s_j \in \{l_B, d_B\}$  be the strategy of regulator  $j$  in the game starting at  $t = 2$  and define  $s_B \equiv (s_j, s_{-j})$ . The utility function of regulator  $j$  is given by

$$\begin{aligned} U_j(s_j, s_{-j}) \equiv & E \left[ I_j(s_B, \tilde{R}_B) - \alpha_{CB} \chi(s_B, \tilde{R}_B) \theta_j(s_B) \mid \chi(s_A, \tilde{R}_A), \tilde{u}_B \right] = \\ & u_B [I_j(s_B, R_B) - \alpha_{CB} \chi(s_B, R_B) \theta_j(s_B)] + (1 - u_B) [I_j(s_B, 0) - \alpha_{CB} \theta_j(s_B)] \\ & - \chi(s_A, \tilde{R}_A) \gamma [I_j(s_B, R_B) - I_j(s_B, 0) - \alpha_{CB} \chi(s_B, R_B) \theta_j(s_B) + \alpha_{CB} \theta_j(s_B)]. \end{aligned}$$

In order to find the best response of regulator  $j$  given the strategy of regulator  $-j$  we calculate

$$\begin{aligned} U_j(1, s_{-j}) - U_j(0, s_{-j}) = & \\ & u_B \left[ \begin{aligned} & I_j((1, s_{-j}), R_B) - I_j((0, s_{-j}), R_B) - \alpha_{CB} \chi((1, s_{-j}), R_B) \theta_j(1, s_{-j}) \\ & + \alpha_{CB} \chi((0, s_{-j}), R_B) \theta_j(0, s_{-j}) \end{aligned} \right] \\ & + (1 - u_B) [I_j((1, s_{-j}), 0) - I_j((0, s_{-j}), 0) - \alpha_{CB} \theta_j(1, s_{-j}) + \alpha_{CB} \theta_j(0, s_{-j})] \\ & - \chi(s_A, \tilde{R}_A) \gamma \left[ \begin{aligned} & I_j((1, s_{-j}), R_B) - I_j((0, s_{-j}), R_B) \\ & - \alpha_{CB} \chi((1, s_{-j}), R_B) \theta_j(1, s_{-j}) + \alpha_{CB} \chi((0, s_{-j}), R_B) \theta_j(0, s_{-j}) \\ & - I_j((1, s_{-j}), 0) + I_j((0, s_{-j}), 0) + \alpha_{CB} \theta_j(1, s_{-j}) - \alpha_{CB} \theta_j(0, s_{-j}) \end{aligned} \right] \end{aligned}$$

If a bank is closed by any regulator, the income accruing to each regulator is the same regardless of the regulator responsible for the closure. This translates into  $I_j((0, 0), \cdot) = I_j((1, 0), \cdot) = I_j((0, 1), \cdot)$ , which we use to find that  $U_j(1, 0) - U_j(0, 0) = \alpha_{CB} > 0$ . As a result, regulator  $j$  always choose  $s_j = 1$  given  $s_{-j} = 0$ . We also find that

$$U_j(1, 1) - U_j(0, 1) = u_B - \chi(s_A, \tilde{R}_A) \gamma - \frac{I_j((0, 1), 0) - I_j((1, 1), 0)}{I_j((1, 1), R_B) - I_j((1, 1), 0) + \alpha_{CB}}.$$

It follows that, given  $s_{-j} = 1$ , regulator  $j$  chooses  $s_j = 1$  for

$$u_B \geq u_B^j(\chi) \equiv \frac{I_j((0, 1), 0) - I_j((1, 1), 0)}{I_j((1, 1), R_B) - I_j((1, 1), 0) + \alpha_{CB}} + \chi(s_A, \tilde{R}_A) \gamma$$

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<sup>6</sup>Repeated interactions between banks and regulators will, over time, cause adjustments both in the riskiness of the investments chosen and in the size of the capital and liquidity buffers adopted by the banks; for the purposes of this exposition all of these are held constant, although they can be incorporated into the analysis. Repeated interactions between depositors and banks and regulators will endogenize liquidity demand, that is, that a fear of a closure or failure will trigger increased depositors’ withdrawals. However, it is worth noting that, although repeated interactions among independent regulatory authorities could lessen the problems of the externalities they impose on one another, they would not eliminate them.

and chooses  $s_j = 0$  if otherwise. Therefore, we have the following Nash equilibria:  $(s_j^*, s_{-j}^*) = (0, 1)$  for  $u_B < u_B^j(\chi)$ ,  $(s_j^*, s_{-j}^*) = (1, 0)$  for  $u_B < u_B^{-j}(\chi)$ , and  $(s_j^*, s_{-j}^*) = (1, 1)$  for  $u_B \geq u_B^m(\chi) \equiv \max\{u_B^j(\chi), u_B^{-j}(\chi)\}$ . The result from the proposition now follows from the observation that  $I_{LoLR}((0, 1), 0) = 0$ ,  $I_{LoLR}((1, 1), 0) = -\omega_B$ ,  $I_{LoLR}((1, 1), R_B) = P\omega_B$ ,  $I_{DI}((0, 1), 0) = L_B Y_B - D_B$ ,  $I_{DI}((1, 1), 0) = -(D_B - \max\{0, \omega_B\})$ ,  $I_{DI}((1, 1), R_B) = 0$ . ■

**Proof of Proposition 2.** Let  $s_j \in \{l_A, d_A\}$  be the strategy of regulator  $j$  in the game starting at  $t = 1$  and define  $s_A \equiv (s_j, s_{-j})$ . The utility function of regulator  $j$  is given by

$$\begin{aligned} & U_j(s_j, s_{-j}) \equiv \\ & E \left[ I(s_A, \tilde{R}_A) - \alpha c_A \chi(s_A, \tilde{R}_A) \theta(s_A) + E \left[ I(s_B^*, \tilde{R}_B) - \alpha c_B \chi(s_B^*, \tilde{R}_B) \theta(s_B^*) \mid \chi(s_A, \tilde{R}_A), \tilde{u}_B \right] \mid \tilde{u}_A \right] \\ & = \int \left\{ \begin{array}{l} u_A \left[ \begin{array}{l} I_j(s_A, R_A) - \alpha c_A \chi(s_A, R_A) \theta(s_A) + u_B (I_j(s_B^*, R_B) - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*)) \\ \quad + (1 - u_B) (I_j(s_B^*, 0) - \alpha c_B \theta_B(s_B^*)) \\ - \chi(s_A, R_A) \gamma (I_j(s_B^*, R_B) - I_j(s_B^*, 0) - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*) + \alpha c_B \theta_B(s_B^*)) \end{array} \right] \\ + (1 - u_A) \left[ \begin{array}{l} I_j(s_A, 0) - \alpha c_A \theta(s_A) + (u_B - \gamma) (I_j(s_B^*, R_B) - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*)) \\ \quad + (1 - u_B + \gamma) (I_j(s_B^*, 0) - \alpha c_B \theta_B(s_B^*)) \end{array} \right] \end{array} \right\} dF_B. \end{aligned}$$

In order to find the best response of regulator  $j$  given the strategy of regulator  $-j$  we calculate

$$\begin{aligned} & U_j(1, s_{-j}) - U_j(0, s_{-j}) = \\ & \int \left\{ \begin{array}{l} u_A \left[ \begin{array}{l} I_j((1, s_{-j}), R_A) - \alpha c_A \chi((1, s_{-j}), R_A) \theta(1, s_{-j}) \\ - \chi((1, s_{-j}), R_A) \gamma \left( \begin{array}{l} I_j(s_B^*, R_B) - I_j(s_B^*, 0) \\ - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*) + \alpha c_B \theta_B(s_B^*) \end{array} \right) \\ + (1 - u_A) [I_j((1, s_{-j}), 0) - \alpha c_A \theta(1, s_{-j})] \end{array} \right] \end{array} \right\} dF_B \\ & - \int \left\{ \begin{array}{l} u_A \left[ \begin{array}{l} I_j((0, s_{-j}), R_A) - \alpha c_A \chi((0, s_{-j}), R_A) \theta(0, s_{-j}) \\ - \chi((0, s_{-j}), R_A) \gamma \left( \begin{array}{l} I_j(s_B^*, R_B) - I_j(s_B^*, 0) \\ - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*) + \alpha c_B \theta_B(s_B^*) \end{array} \right) \\ (1 - u_A) [I_j((0, s_{-j}), 0) - \alpha c_A \theta(0, s_{-j})] \end{array} \right] \end{array} \right\} dF_B \end{aligned}$$

If a bank is closed by any regulator, the income accruing to each regulator is the same regardless of the regulator responsible for the closure. This translates into  $I_j((0, 0), \cdot) = I_j((1, 0), \cdot) = I_j((0, 1), \cdot)$ , which we use to find that  $U_j(1, 0) - U_j(0, 0) = \alpha c_A > 0$ . As a result, regulator always choose  $s_j = 1$  given  $s_{-j} = 0$ . We also find that

$$\begin{aligned} & U_j(1, 1) - U_j(0, 1) = \\ & u_A [I_j((1, 1), R_A) - I_j((0, 1), R_A) + \alpha c_A] + (1 - u_A) [I_j((1, 1), 0) - I_j((0, 1), 0)] \\ & + \int u_A \gamma [I_j(s_B^*, R_B) - I_j(s_B^*, 0) - \alpha c_B \chi(s_B^*, R_B) \theta_B(s_B^*) + \alpha c_B \theta_B(s_B^*)] dF_B \\ & = u_A [I_j((1, 1), R_A) - I_j((0, 1), R_A) + \alpha c_A] + (1 - u_A) [I_j((1, 1), 0) - I_j((0, 1), 0)] \\ & \quad + \int_{u_B^m(\chi=0)} u_A \gamma [I_j((1, 1), R_B) - I_j((1, 1), 0) + \alpha c_B] dF_B \end{aligned}$$

It follows that, given  $s_{-j} = 1$ , regulator  $j$  chooses  $s_j = 1$  for

$$u_A \geq u_A^j(\gamma) \equiv \frac{I_j((0, 1), 0) - I_j((1, 1), 0)}{I_j((1, 1), R_A) - I_j((1, 1), 0) + \alpha c_A + \gamma [I_j((1, 1), R_B) - I_j((1, 1), 0) + \alpha c_B] \Pr [u_B \geq u_B^m(\chi = 0)]}$$

and chooses  $s_j = 0$  if otherwise. Therefore, we have the following Nash equilibria:  $(s_j^*, s_{-j}^*) = (0, 1)$  for  $u_A < u_A^j(\gamma)$ ,  $(s_j^*, s_{-j}^*) = (1, 0)$  for  $u_A < u_B^{-j}(\gamma)$ , and  $(s_j^*, s_{-j}^*) = (1, 1)$  for  $u_A \geq u_A^m(\gamma) \equiv \max \{u_A^j(\gamma), u_A^{-j}(\gamma)\}$ . The result from the proposition now follows from the observation that  $I_{LoLR}((0, 1), 0) = 0$ ,  $I_{LoLR}((1, 1), 0) = -\omega_A$ ,  $I_{LoLR}((1, 1), R_A) = P\omega_A$ ,  $I_{DI}((0, 1), 0) = L_A Y_A - D_A$ ,  $I_{DI}((1, 1), 0) = -(D_A - \max\{0, \omega_A\})$ ,  $I_{DI}((1, 1), R_A) = 0$ . ■

**Proof of Proposition 3.** The problem of the unified regulator is analogous to that of a regulator in the multiregulator setting assuming that the other regulator always choose not to close the bank. If we let  $s \in \{0, 1\}$  be the action of the unified regulator starting at  $t = 2$  and  $U(s)$  be the utility function, we find that the unified regulator does not close bank  $B$  if:

$$U(1) - U(0) = u_B - \chi(s_A, \tilde{R}_A) \gamma - \frac{I(0, 0) - I(1, 0)}{I(1, R_B) - I(1, 0) + \alpha c_B} \geq 0.$$

It follows that the unified regulator does not close bank  $B$  if

$$u_B \geq u_B^u(\chi) \equiv \frac{I(0, 0) - I(1, 0)}{I(1, R_B) - I(1, 0) + \alpha c_B} + \chi(s_A, \tilde{R}_A) \gamma$$

The result from the proposition now follows from the observation that  $I(0, 0) = LY_B - D_B$ ,  $I(1, 0) = -D_B$ ,  $I(1, R_B) = P \max\{0, \omega_B\}$ . ■

**Proof of Proposition 4.** The problem of the unified regulator is analogous to that of a regulator in the multiregulator setting assuming that the other regulator always choose not to close the bank. If we let  $s \in \{0, 1\}$  be the action of the unified regulator starting at  $t = 1$  and  $U(s)$  be the utility function, we find that the unified regulator does not close bank  $A$  if:

$$U(1) - U(0) = u_A [I(1, R_A) - I(0, R_A)] + (1 - u_A) [I(1, 0) - I(0, 0) - \alpha c_A] + \int_{u_B^u} u_A \gamma [I((1, R_B) - I((1, 0) + \alpha c_B)] dF_B \geq 0$$

It follows that the unified regulator does not close bank  $B$  if

$$u_A \geq u_A^u(\gamma) \equiv \frac{I(0, 0) - I(1, 0)}{I(1, R_A) - I(1, 0) + \alpha c_A + \gamma [I(1, R_B) - I(1, 0) + \alpha c_B] \Pr [u_B \geq u_B^u(\chi = 0)]}$$

The result from the proposition now follows from the observation that  $I(0, 0) = LY_A - D_A$ ,  $I(1, 0) = -D_A$ , and  $I(1, R_A) = P \max\{0, \omega_A\}$ . ■

**Proof of Proposition 5.** (i) If  $\omega_B = 0$  then we have  $u_B^u(\chi = 1) = \frac{L_B Y_B}{\alpha c_B + D_B} + \gamma = \max \{u_B^{DI}(\chi = 1), u_B^{LoLR}(\chi = 1)\} = u_B^m(\chi = 1)$

(ii) If  $\omega_B > 0$  we need to compare

$$u_B^u(\chi = 1) = \frac{L_B Y_B}{P\omega_B + \alpha c_B + D_B} + \gamma \text{ and } u_B^m(\chi = 1) = \max \{u_B^{LoLR}(\chi = 1), u_B^{DI}(\chi = 1)\},$$

where

$$u_B^{LoLR}(\chi = 1) = \frac{\omega_B}{(P+1)\omega_B + \alpha c_B} + \gamma,$$

$$u_B^{DI}(\chi = 1) = \frac{L_B Y_B - \omega_B}{\alpha c_B + D_B - \omega_B} + \gamma.$$

One must note that

$$\frac{\partial u_B^u}{\partial \omega_B} < 0, \frac{\partial u_B^{LoLR}}{\partial \omega_B} > 0, \text{ and } \frac{\partial u_B^{DI}}{\partial \omega_B} < 0 \text{ for } \alpha \text{ big enough.}$$

If  $u_B^u > u_B^{LoLR} \forall \omega_B \in (0, D_B]$ , then the unified regulator arrangement is always less forbearing than the multiregulator arrangement. To see this, note that for  $\omega_B = 0$ ,  $u_B^{DI} = u_B^u$ , and if  $\alpha$  is big enough,  $u_B^{DI} < u_B^u$  for  $\omega_B > 0$ . Therefore  $u_B^u > u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\}$  for  $\alpha$  sufficiently large.

Suppose  $\exists \omega_B^* \in (0, D_B]$  such that  $u_B^u = u_B^{LoLR}$ . Since  $u_B^{DI} < u_B^u$  for  $\omega_B > 0$  and  $\alpha$  big enough,  $u_B^{LoLR}(\omega^*) = u_B^u(\omega^*) > u_B^{DI}(\omega^*)$ . In addition,  $u_B^{DI} > u_B^{LoLR}$  for  $\omega_B = 0$ . Therefore,  $\exists \omega_B^{**} \in (0, \omega_B^*)$  such that  $u_B^{DI}(\omega_B^{**}) = u_B^{LoLR}(\omega_B^{**})$ . Because  $u_B^{DI}$  is decreasing and  $u_B^{LoLR}$  is increasing we have  $u_B^u > u_B^{DI} = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^m$  for  $\omega_B \in (0, \omega_B^{**})$ . Since  $u_B^u$  is decreasing and  $u_B^{LoLR}$  is increasing, we have  $u_B^u > u_B^{LoLR} = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^m$  for  $\omega_B \in [\omega_B^{**}, \omega_B^*)$ , and  $u_B^m = \max \{u_B^{LoLR}, u_B^{DI}\} = u_B^{LoLR} > u_B^u$  for  $\omega_B \in [\omega_B^*, D_B]$ .

The critical point  $\omega_B^*$  solves  $u_B^{LoLR} = u_B^u$ . It is straightforward to see that  $\gamma$  is cancelled out, which makes  $\omega_B^*$  independent of  $\gamma$ . Differentiating both sides with respect to  $L_B$  we get

$$\frac{\partial \omega_B^*}{\partial L_B} = \frac{Y_B [(P+1)\omega_B^* + \alpha c_B]}{P(\omega_B^* - L_B Y_B) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

which is clearly positive for  $\alpha$  sufficiently large.

Differentiating both sides of  $u_B^{LoLR} = u_B^u$  with respect to  $D_B$  we get

$$\frac{\partial \omega_B^*}{\partial D_B} = \frac{-\omega_B^*}{P(\omega_B^* - L_B Y_B) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

which is clearly negative if  $\alpha$  is big enough.

Differentiating both sides of  $u_B^{LoLR} = u_B^u$  with respect to  $c_B$  and  $P$  we get:

$$\frac{\partial \omega_B^*}{\partial c} = \frac{\alpha(L_B Y_B - \omega_B^*)}{P(\omega_B^* - L_B Y_B) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

$$\frac{\partial \omega_B^*}{\partial P} = \frac{\omega_B^*(L_B Y_B - \omega_B^*)}{P(\omega_B^* - L_B Y_B) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B}.$$

If  $\varphi(L_B) \equiv L_B Y - \omega_B^* < 0$ , then we have  $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} < 0$  for  $\alpha$  big enough. If  $\varphi(L_B) > 0$ , then we have that  $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} > 0$  for  $\alpha$  sufficiently large. Therefore, we need to investigate  $\varphi(L_B)$ . One

can easily check that  $\varphi(0) = 0$ . In addition,  $\varphi(L_B) = 0$  and  $u_B^{LoLR} = u^u$  can be simultaneously satisfied for  $\bar{L}_B = \frac{D_B}{Y_B}$ . If we differentiate  $\frac{\partial \omega_B^*}{\partial L_B}$  with respect to  $L_B$  we get

$$\frac{\partial^2 \omega_B^*}{\partial L_B^2} = \frac{-\frac{\partial \omega_B^*}{\partial L_B} 2 \left( P \frac{\partial \omega_B^*}{\partial L_B} - Y_B (P+1) \right)}{P(\omega_B^* - L_B Y_B) + P\omega_B^* - L_B Y_B + \alpha c_B + D_B},$$

which is positive for  $\alpha$  big enough. In this case  $\varphi''(L_B) < 0$ . By the Mean Value Theorem  $\exists L_B^* \in (0, \bar{L}_B)$  such that  $\varphi'(L_B^*) = 0$ , which implies that  $\varphi(L_B)$  achieves its maximum at  $L_B^*$ . Since  $\varphi'(L_B)$  is decreasing,  $\varphi'(L_B) > 0$  for  $L_B < L_B^*$  and  $\varphi'(L_B) < 0$  for  $L_B > L_B^*$ .

We now claim that  $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} > 0$  for all  $L_B \in (0, \bar{L}_B)$ . We first show that the claim holds for all  $L_B \in [L_B^*, \bar{L}_B]$ . Because  $\varphi'(L_B) < 0$  for  $L_B \in (L_B^*, 1)$  and  $\varphi(\bar{L}_B) = 0$ , it follows that  $\varphi(L_B) > 0$  for  $L_B \in [L_B^*, \bar{L}_B)$ . Therefore,  $\frac{\partial \omega_B^*}{\partial c_B}, \frac{\partial \omega_B^*}{\partial P} > 0$  for  $L_B \in [L_B^*, \bar{L}_B)$ . We now show that the claim holds for all  $L_B \in (0, L_B^*)$ . This follows from the fact that  $\varphi(L_B) > 0$  for  $L_B \in (0, L_B^*)$ . Suppose not, i.e.,  $\varphi(L_B) = 0$  for some  $L'_B \in (0, L_B^*)$ . Because  $\varphi'(L_B) > 0$  for  $L_B \in (0, L_B^*)$ , it follows that  $\varphi(L_B) < 0 \forall L_B \in [0, L'_B)$ . But this contradicts the fact that  $\varphi(0) = 0$ . ■

**Proof of Proposition 6.** (i) If  $\omega_A = 0$ , then

$$u_A^u(\gamma) = \frac{L_A Y_A}{D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr[u_B \geq u_B^u(\chi = 0)]} =$$

$$\max \left\{ 0, \frac{L_A Y_A}{D_A + \alpha c_A + \gamma(D_B + \alpha c_B) \Pr[u_B \geq u_B^m(\chi = 0)]} \right\} = u_A^m(\gamma)$$

(ii) If  $\omega_A > 0$  we compare

$$u_A^u(\gamma) = \frac{L_A Y_A}{P\omega_A + D_A + \alpha c_A + \gamma[\alpha c_B + D_B] \Pr[u_B \geq u_B^u(\chi = 0)]} \text{ and}$$

$$u_A^m(\gamma) = \max \{ u_A^{DI}(\gamma), u_A^{LoLR}(\gamma) \}$$

where

$$u_A^{LoLR}(\gamma) = \frac{\omega_A}{(P+1)\omega_A + \alpha c_A + \gamma \alpha c_B \Pr[u_B \geq u_B^m(\chi = 0)]},$$

$$u_A^{DI}(\gamma) = \frac{L_A Y_A - \omega_A}{D_A - \omega_A + \alpha c_A + \gamma(D_B + \alpha c_B) \Pr[u_B \geq u_B^m(\chi = 0)]}.$$

It is straightforward to check that

$$\frac{\partial u_A^u}{\partial \omega_A} < 0, \frac{\partial u_A^{LoLR}}{\partial \omega_A} > 0, \text{ and } \frac{\partial u_A^{DI}}{\partial \omega_A} < 0 \text{ for } \alpha \text{ big enough.}$$

If  $u_A^u > u_A^{LoLR} \forall \omega_A \in (0, D_A]$ , then the unified regulator arrangement is always less forbearing than the multiregulator arrangement. To see this, note that for  $\omega_A = 0$ ,  $u_A^{DI} = u_A^u$ , and if  $\alpha$  is big enough,  $u_A^{DI} < u_A^u$  for  $\omega_A > 0$ . Therefore  $u_A^u > u_A^m = \max \{ u_A^{LoLR}, u_A^{DI} \}$  for  $\alpha$  sufficiently large.

Suppose  $\exists \omega_A^* \in (0, D_A]$  such that  $u_A^u = u_A^{LoLR}$ . Since  $u_A^{DI} < u_A^u$  for  $\omega_A > 0$  and  $\alpha$  big enough,  $u_A^{LoLR}(\omega_A^*) = u_A^u(\omega_A^*) > u_A^{DI}(\omega_A^*)$ . In addition,  $u_A^{DI} > u_A^{LoLR}$  for  $\omega = 0$ . Therefore,  $\exists$

$\omega_A^{**} \in (0, \omega_A^*)$  such that  $u_A^{DI}(\omega_A^{**}) = u_A^{LoLR}(\omega_A^{**})$ . Because  $u_A^{DI}$  is decreasing and  $u_A^{LoLR}$  is increasing we have  $u_A^u > u_A^{DI} = \max\{u_A^{LoLR}, u_A^{DI}\} = u_A^m$  for  $\omega_A \in (0, \omega_A^{**})$ . Since  $u_A^u$  is decreasing and  $u_A^{LoLR}$  is increasing, we have  $u_A^u > u_A^{LoLR} = \max\{u_A^{LoLR}, u_A^{DI}\} = u_A^m$  for  $\omega_A \in [\omega_A^{**}, \omega_A^*)$ , and  $u_A^m = \max\{u_A^{LoLR}, u_A^{DI}\} = u_A^{LoLR} > u_A^u$  for  $\omega_A \in [\omega_A^*, D_A]$ .

The critical point  $\omega_A^*$  solves  $u_A^{LoLR}(\dots, \omega_A^*, \gamma) = u_A^u(\dots, \omega_A^*, \gamma)$ . Differentiating both sides with respect to  $L$  we get

$$\frac{\partial \omega_A^*}{\partial L_A} = \frac{Y_A [\alpha (c_A + \gamma c_B \Pr(\cdot)) + (P+1) \omega_A^*]}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)}.$$

Because  $\omega_A^*$  is bounded, this is clearly positive for  $\alpha$  big enough.

Differentiating both sides of  $u_A^{LoLR} = u_A^u$  with respect to  $D_A$  we get

$$\frac{\partial \omega_A^*}{\partial D_A} = \frac{-\omega_A^*}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)},$$

which is clearly negative for  $\alpha$  sufficiently large.

Differentiating both sides of  $u_A^{LoLR} = u_A^u$  with respect to  $c_B$ ,  $\gamma$ ,  $c_A$ ,  $D_B$ , and  $P$  we get:

$$\begin{aligned} \frac{\partial \omega_A^*}{\partial c_B} &= \frac{\gamma \alpha \Pr(\cdot) (L_A Y_A - \omega_A^*) + \gamma \alpha c_B \frac{\partial \Pr(\cdot)}{\partial c_B} (L_A Y_A - \omega_A^*) - \gamma D_B \frac{\partial \Pr(\cdot)}{\partial c_B} \omega_A^*}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)} \\ \frac{\partial \omega_A^*}{\partial \gamma} &= \frac{\Pr(\cdot) \alpha c_B (L_A Y_A - \omega_A^*) - D_B \Pr(\cdot) \omega_A^*}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)} \\ \frac{\partial \omega_A^*}{\partial c_A} &= \frac{\alpha c_B (L_A Y_A - \omega_A^*)}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)} \\ \frac{\partial \omega_A^*}{\partial D_B} &= \frac{\gamma \alpha c_B \frac{\partial \Pr(\cdot)}{\partial D_B} (L_A Y_A - \omega_A^*) - \gamma \omega_A^* \left( \Pr(\cdot) + D_B \frac{\partial \Pr(\cdot)}{\partial D_B} \right)}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)} \\ \frac{\partial \omega_A^*}{\partial P} &= \frac{\gamma \alpha c_B \frac{\partial \Pr(\cdot)}{\partial P} (L_A Y_A - \omega_A^*) + \omega_A^* \left( L_A Y_A - \gamma D_B \frac{\partial \Pr(\cdot)}{\partial P} - \omega_A^* \right) - \gamma \omega_A^* \left( \Pr(\cdot) + D_B \frac{\partial \Pr(\cdot)}{\partial P} \right)}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)} \end{aligned}$$

If  $\varphi(L_A) \equiv L_A Y_A - \omega_A^* < 0$ , then we have  $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} < 0$  for  $\alpha$  big enough. If  $\varphi(L_A) > 0$ , then we have that  $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$  for  $\alpha$  sufficiently large. Therefore, we need to investigate  $\varphi(L_A)$ . One can easily check that  $\varphi(0) = 0$ . In addition,  $\varphi(L_A) = 0$  and  $u_A^{LoLR} = u_A^u$  can be simultaneously satisfied for  $\bar{L}_A = \frac{D_A + \gamma D_B \Pr(\cdot)}{Y_A}$ . If we differentiate  $\frac{\partial \omega_A^*}{\partial L_A}$  with respect to  $L_A$  we get

$$\frac{\partial^2 \omega_A^*}{\partial L_A^2} = \frac{-\frac{\partial \omega_A^*}{\partial L_A} 2 \left( P \frac{\partial \omega_A^*}{\partial L_A} - Y_A (P+1) \right)}{P(\omega_A^* - L_A Y_A) + P\omega_A^* - L_A Y_A + D_A + \alpha c_A + \gamma(\alpha c_B + D_B) \Pr(\cdot)},$$

which is positive for  $\alpha$  big enough. In this case  $\varphi''(L_A) < 0$ . By the Mean Value Theorem  $\exists L_A^* \in (0, \bar{L}_A)$  such that  $\varphi'(L_A^*) = 0$ , which implies that  $\varphi(L_A)$  achieves its maximum at  $L_A^*$ . Since  $\varphi'(L_A)$  is decreasing,  $\varphi'(L_A) > 0$  for  $L_A < L_A^*$  and  $\varphi'(L_A) < 0$  for  $L_A > L_A^*$ .

We now claim that  $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$  for all  $L_A \in (0, \bar{L}_A)$ . We first show that the claim holds for  $[L_A^*, \bar{L}_A]$ . Because  $\varphi'(L_A) < 0$  for  $L_A \in (L_A^*, 1)$  and  $\varphi(\bar{L}_A) = 0$ , it follows that

$\varphi(L_A) > 0$  for  $L_A \in [L_A^*, \bar{L}_A)$ . Therefore,  $\frac{\partial \omega_A^*}{\partial c_B}, \frac{\partial \omega_A^*}{\partial \gamma}, \frac{\partial \omega_A^*}{\partial c_A}, \frac{\partial \omega_A^*}{\partial D_B}, \frac{\partial \omega_A^*}{\partial P} > 0$  for  $L_A \in [L_A^*, \bar{L}_A)$ . We now show that the claim holds for  $L_A \in (0, L_A^*)$ . This follows from the fact that  $\varphi(L_A) > 0$  for all  $L_A \in (0, L_A^*)$ . Suppose not, i.e.,  $\varphi(L_A) = 0$  for some  $L'_A \in (0, L_A^*)$ . Because  $\varphi'(L_A) > 0$  for  $L_A \in (0, L_A^*)$ , it follows that  $\varphi(L_A) < 0 \forall L_A \in [0, L'_A)$ . But this contradicts the fact that  $\varphi(0) = 0$ .

■

**Proof of Proposition 7.** The result that babbling equilibria exist comes from the observation that if the *LoLR*'s belief treats all messages as uninformative, i.e.,  $\bar{\gamma}(m) = \bar{\gamma}$ , and if messages are independent of types,  $m(\gamma) = m \forall \gamma \geq \gamma_*$ , then the optimal action taken by the *LoLR* given  $\bar{\gamma}(m) = \bar{\gamma}$  and the message strategy adopted by the *DI* are best responses to each other.

Consider the case in which  $\exists \hat{\gamma} \in [0, 1]$  such that  $E[\tilde{\gamma} | \gamma_* \leq \tilde{\gamma} \leq \hat{\gamma}] = \gamma^*$ . As a result,  $\bar{\gamma} \geq \gamma^*$  and  $u_A^{LoLR}(\bar{\gamma}) \leq u_A^{DI}(\gamma^*) = u$ , which implies that the outcome of a babbling equilibrium has the *LoLR* not liquidating bank *A* for all  $\gamma \geq \gamma_*$ .

Suppose there is an equilibrium in which the *LoLR* liquidates bank *A* for some  $\gamma'' \geq \gamma_*$ . If  $\forall \gamma' \geq \gamma_*$  we have  $m(\gamma') = m(\gamma'') = m$ , then the equilibrium is a babbling equilibrium, in which case consistency of beliefs requires  $\bar{\gamma}(m) = \bar{\gamma}$  and the *LoLR* does not liquidate bank *A* for all  $\gamma \geq \gamma_*$ , which is a contradiction. Let  $m(\gamma') \neq m(\gamma'')$  for some  $\gamma'$  and let  $M'$  be the set of different messages sent in equilibrium. For every  $m' \in M'$  we define the set  $\Gamma(m') = \{m^{-1}(m')\}$ . Clearly,  $\cup \Gamma'(m') = [\gamma_*, 1]$ .

Suppose the *LoLR* liquidates bank *A* for all  $m' \in M'$ . Let  $\mu$  be the measure associated with the distribution  $Z$ . Optimality requires  $\bar{\gamma}(m') < \gamma^* \forall m' \in M$ . However, one must note that  $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \int_{\Gamma(m')} \gamma \frac{z(\gamma)}{\mu(\Gamma(m'))} d\gamma = \bar{\gamma}$  must hold. But since  $\bar{\gamma}(m') < \gamma^*$ , we have a contradiction as  $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \bar{\gamma}(m') < \sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \gamma^* = \gamma^* \leq \bar{\gamma}$ . Therefore, there must  $\exists m' \in M'$  such that  $\bar{\gamma}(m') \geq \gamma^*$ , i.e., the *LoLR* does not liquidate bank *A* upon seeing message  $m'$ . But by assumption the *LoLR* liquidates bank *A* upon receiving  $m(\gamma'') \neq m'$ . Therefore, the *DI* that observes  $\gamma \in \Gamma(m(\gamma''))$  has an incentive to deviate and report  $m'$ , which contradicts the assumption that there is an equilibrium in which the *LoLR* liquidates bank *A* for some  $\gamma'' \geq \gamma_*$ .

Now consider the case  $\nexists \hat{\gamma} \in [0, 1]$  such that  $E[\tilde{\gamma} | \gamma_* \leq \tilde{\gamma} \leq \hat{\gamma}] = \gamma^*$ . In this case  $\bar{\gamma} < \gamma^*$  and  $u_A^{LoLR}(\bar{\gamma}) > u_A^{DI}(\gamma^*) = u$ , which implies that the outcome of a babbling equilibrium has the *LoLR* liquidating bank *A* for all  $\gamma \geq \gamma_*$ .

Suppose there is an equilibrium in which the *LoLR* does not liquidate bank *A* for some  $\gamma'' \geq \gamma_*$ . If  $\forall \gamma' \geq \gamma_*$  we have  $m(\gamma') = m(\gamma'') = m$ , then the equilibrium is a babbling equilibrium, in which case consistency of beliefs requires  $\bar{\gamma}(m) = \bar{\gamma}$  and the *LoLR* liquidates bank *A* for all  $\gamma \geq \gamma_*$ , which is a contradiction. Let  $m(\gamma') \neq m(\gamma'')$  for some  $\gamma'$  and let  $M'$  be the set of different messages sent in equilibrium. For every  $m' \in M'$  we define the set  $\Gamma(m') = \{m^{-1}(m')\}$ . Clearly,  $\cup \Gamma'(m') = [\gamma_*, 1]$ .

Suppose the *LoLR* does not liquidate bank *A* for all  $m' \in M'$ . Optimality requires  $\bar{\gamma}(m') \geq \gamma^* \forall m' \in M$ . However, one must note that  $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \int_{\Gamma(m')} \gamma \frac{z(\gamma)}{\mu(\Gamma(m'))} d\gamma = \bar{\gamma}$  must hold. But since  $\bar{\gamma}(m') \geq \gamma^*$ , we have  $\sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \bar{\gamma}(m') \geq \sum_{m' \in M'} \frac{\mu(\Gamma(m'))}{1-Z(\gamma_*)} \gamma^* = \gamma^* > \bar{\gamma}$ . Therefore, we have a contradiction. As a consequence,  $\exists m' \in M'$  such that  $\bar{\gamma}(m') < \gamma^*$ , i.e., the *LoLR* liquidates bank *A* upon seeing message  $m'$ . But by assumption, the *LoLR* does not liquidate bank *A* upon

receiving  $m(\gamma'') \neq m'$ . Therefore, the *DI* that observes  $\gamma \in \Gamma(m')$  has an incentive to deviate and report  $m''$ , which contradicts the assumption that there is an equilibrium in which the *LoLR* does not liquidate bank *A* for some  $\gamma'' \geq \gamma_*$ . ■

**Proof of Proposition 8.** If the *DI* chooses not to learn about  $\gamma$ , then bank *A* is not liquidated since  $\max\{u_A^{LoLR}(\underline{\gamma}), u_A^{DI}(\underline{\gamma})\} \leq u_A$ . Suppose the *DI* chooses to learn about  $\gamma$ . Because  $\underline{\underline{\gamma}}$  is below the lower bound of the support of  $Z(\gamma)$ , for any observed  $\gamma$  we have  $u_A^{DI}(\gamma) \leq u_A^{DI}(\underline{\underline{\gamma}}) = u_A$ , which implies that the *DI* prefers bank *A* not to be liquidated. If  $\gamma \geq \gamma_*$ , then because  $\exists \hat{\gamma} \in [0, 1]$  such that  $E[\tilde{\gamma} | \gamma_* \leq \tilde{\gamma} \leq \hat{\gamma}] = \gamma^*$ , bank *A* is liquidated and the *DI* is worse off. Let  $\gamma < \gamma_*$ . Since  $\gamma_* \leq \gamma^*$ , we have that  $u_A^{LoLR}(\gamma) > u_A^{LoLR}(\gamma_*) \geq u_A^{LoLR}(\gamma^*) = u_A$ , which implies that the *LoLR* always liquidate bank *A*. Therefore, the *DI* is worse off if he chooses to learn  $\gamma$ . ■



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