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## Centrality-based Capital Allocations

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## **IMF Working Paper**

African Department

### **Centrality-based Capital Allocations<sup>1</sup>**

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#### **Abstract**

We look at the effect of capital rules on a banking system that is connected through correlated credit exposures and interbank lending. The rules, which combine individual bank characteristics and interconnectivity measures of interbank lending, are to minimize a measure of system-wide losses. Using the detailed German Credit Register for estimation, we find capital rules based on eigenvectors to dominate any other centrality measure, followed by closeness. Compared to the baseline case, capital reallocation based on the Adjacency Eigenvector saves about 15% in system losses as measured by expected bankruptcy costs.

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# I. Introduction

“The difficult task before market participants, policymakers, and regulators with systemic risk responsibilities such as the Federal Reserve is to find ways to preserve the benefits of interconnectedness in financial markets while managing the potentially harmful side effects.” [Yellen \(2013\)](#)

This paper examines capital requirements that mitigate the harmful side effects of interconnectedness in the context of a model of interbank contagion. Although the model is fairly classical in the way it handles contagion, it uses a very rich dataset of credit exposures of a large domestic banking system, the fifth largest system in the world. What we find is that the same nationwide amount of required capital, when distributed in part based on the interconnectedness of the system, performs better in terms of total losses to the system than the same amount of capital when allocated on the basis of banks’ individual risk-weighted assets alone. Indeed, a capital allocation based upon our best network centrality measure saves some 15% of expected bankruptcy costs, which is our preferred measure of total system losses.

The idea of tying capital charges to interbank exposures and interconnectedness in order to improve the stability of the banking system, i.e. to minimize expected social costs (arising from bailouts, growth effects, unemployment for example) is in the spirit of the regulatory assessment methodology for systemically important financial institutions (SIFIs) proposed by the [Basel Committee on Banking Supervision \(2011\)](#). In contrast to that methodology, however, our study determines an optimal rule for capital charges that is based on interconnectedness measures as well as on the portfolio risk of bank assets, and we then compare the results under the different capital allocations.

We focus on two main sources of systemic risk: correlated credit exposures and interbank connectivity. First, banks’ balance sheets can be simultaneously affected by macro or industry shocks since the credit risk of their non-bank borrowers is correlated. If losses are large, capital of the entire system is eroded, making the system less stable. Second, these shocks can trigger the default of certain financial institutions and, again, erode bank capital further. The second effect is modeled in the interbank market. Since banks are highly connected through interbank exposures, we focus on those negative tail events in which correlated losses of their portfolios trigger contagion in the interbank market.

Our model comes close to the framework proposed by [Elsinger, Lehar, and Summer \(2006\)](#) and [Gauthier, Lehar, and Souissi \(2012\)](#), which combines common credit losses with interbank network effects and externalities in the form of asset fire sales. The aim of our paper is different. We propose a tractable framework to reallocate capital for large financial systems in order to minimize contagion effects and the possible costs of a public bailout. We contrast two different capital allocations: the benchmark case, in which we allocate capital based on the risks in individual banks’ portfolios, and a comparison case, where we allocate capital based partly on some interbank network metrics (such as

*degree, eigenvector* or *betweenness*) that capture the potential contagion risk of individual banks. Literature has shown that the network structure matters. For instance, [Sachs \(2014\)](#) randomly generates interbank networks and investigates contagion effects in different setups. She finds that the distribution of interbank exposures plays a crucial role for system stability and confirms the “knife-edge” or tipping-point feature (as mentioned by [Haldane, 2009](#)), the non-monotonic completeness property of highly interconnected networks.

We compare different capital structures in which the total capital requirement to the entire banking system is constant but the capital charge varies, based on the network metric chosen and the weight we put on it. Both the choice of a metric and the weight are optimized. Among various sensible target functions to minimize we select *total expected bankruptcy costs* of defaulted banks. This measure of system losses is especially interesting as it represents a deadweight social loss and is independent of distributional considerations which would arise if we focused on the losses incurred by a certain group of bank claimants such as depositors.

We use the credit register for the German banking system. It records every bilateral lending relationship in excess of €1.5 million, including interbank lending. The richness of our data set allows us to do two things. First, we can compute centrality measures accurately. Second, we achieve a comparably high precision in exploring the implications of both the joint credit risk and the interconnected direct claims in the banking system. Using a state-of-the-art credit portfolio model, we can derive the joint distribution functions of the shocks to the banks within the system and feed the shocks into the interbank lending network, so that we can simulate how they work their way through the system.

To model the credit risk arising from exposures to the real economy, which we call *fundamental credit risk*, we generate correlated credit losses by means of the risk engine *CreditMetrics*, which is often used in bank risk management ([Bluhm, Overbeck, and Wagner, 2003](#), ch. 2). Based on a multi-factor credit risk model, the engine helps us to deal with risk concentration caused by large exposures to a single sector or highly correlated sectors. Even explicit common credit exposures, caused by firms borrowing from multiple banks, are precisely addressed. *CreditMetrics* assigns realistic, well-founded probabilities to those scenarios that have particularly large losses across the entire banking system. These bad scenarios are our main focus since capital across financial institutions is eroded simultaneously, and the banking system becomes more prone to interbank contagion.

Moreover, we model interbank contagion as in [Rogers and Veraart \(2013\)](#), which extends [Eisenberg and Noe \(2001\)](#) to include bankruptcy costs. This allows us to measure expected contagion losses and to observe the propagation process. To empirically implement our framework, we use several sources of information: the German central credit register (covering large loans), aggregated credit exposures (small loans), balance sheet data (e.g., total assets), market data (e.g., to compute sector correlations in the real economy or credit spreads), and data on rating transitions (to calibrate the *CreditMetrics* model). The approach can be applied in any country or group of countries where this type of information is available.

A major advantage of our framework is that policymakers can deal with large banking systems, making the regulation of systemic risk more tractable: while [Gauthier et al. \(2012\)](#) state that their – impressive – model requires substantial numeric effort even with the six Canadian banks considered in their paper, German regulators have to deal with more than 1,500 banking groups, which is possible when taking our approach.

This study is related to several strands of the literature including applications of network theory to economics, macro-prudential regulations and interbank contagion. [Cont, Moussa, and e Santos \(2013\)](#) find that not only banks' capitalization and interconnectedness are important for spreading contagion but also the vulnerability of the neighbors of poorly capitalized banks. [Gauthier et al. \(2012\)](#) use different holdings-based systemic risk measures (e.g. MES, CoVaR, Shapley value) to reallocate capital in the banking system and to determine macroprudential capital requirements. Using the Canadian credit register data for a system of six banks, they rely on an “Eisenberg-Noe”-type clearing mechanism extended to incorporate asset fire sale externalities. In contrast to their paper, we reallocate capital based on centrality measures extracted directly from the network topology of interbank market. [Webber and Willison \(2011\)](#) assign systemic capital requirements optimizing over the aggregated capital of the system. They find that systemic capital requirements are directly related to bank size and interbank liabilities. [Tarashev, Borio, and Tsatsaronis \(2010\)](#) claim that systemic importance is mainly driven by size and exposure to common risk factors. In order to determine risk contributions they utilize the Shapley value. In the context of network analysis, [Battiston, Puliga, Kaushik, Tasca, and Caldarelli \(2012\)](#) propose a measure closely related to eigenvector centrality to assign the systemic relevance of financial institutions based on their centrality in a financial network. Similarly, [Soramäki and Cook \(2012\)](#) try to identify systemically important financial institutions in payment systems by implementing an algorithm based on absorbing Markov chains. Employing simulation techniques, they show that the proposed centrality measure, SinkRank, highly correlates with the disruption of the entire system. In accordance with the latter two studies, we also find measures that focus on banks “being central”, especially eigenvector centrality, to dominate size as a measure of systemic importance.

As the subprime crisis has shown, banks do not have to be large to contribute to systemic risk, especially where banks are exposed to correlated risks (e.g. credit, liquidity or funding risk) via portfolios and interbank interconnectedness. Assigning risks to individual banks might be misleading. Some banks might appear healthy when viewed as single entities but they could threaten financial stability when considered jointly. [Gai and Kapadia \(2010\)](#) find that greater complexity and concentration in the network of bank connections can amplify systemic fragility. [Anand, Gai, Kapadia, Brennan, and Willison \(2013\)](#) extend their model to include asset fire sale externalities and macroeconomic feedback on top of network structures, in order to stress-test financial systems. These studies illustrate the tipping point at which the financial system breaks down based on the severity of macroeconomic shocks that affect probabilities of corporate default or asset liquidity. [Battiston, Gatti, Gallegati, Greenwald, and Stiglitz \(2012\)](#) show that interbank connectivity increases systemic risk, mainly due to a higher contagion risk. Furthermore, [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2015\)](#) claim that

financial network externalities cannot be internalized and thus, in equilibrium, financial networks are inefficient. This creates incentives for regulators to improve welfare by bailing out SIFIs.

In our analysis we keep the total amount of capital in the system constant; otherwise, optimization would be simple but silly: more capital for all, ideally 100% equity funding for banks. As a consequence, when we require some banks to hold more capital, we are willing to accept that others may hold less capital as in the benchmark case. Taken literally, there would be no lower limit to capital except zero. However, we also believe that there should be some minimum capital requirement that applies to all banks for reasons of political feasibility, irrespective of their role in the financial network. Implementing a uniform maximum default probability for all banks, as we actually do in our reallocation mechanism, might be one choice.

Finally, we realize that this is just the first step in calculating optimal capital requirements from network positions to prevent systemic risk. Clearly our results are subject to the standard critique that banks will adjust their network position in response to their new capital requirements. This is not a paper of endogenous network formation, but rather a first step in describing how the system could improve its capital allocation with a given network structure. Given the current German structure, we find that those network measures most influential in reducing total system losses are based on eigenvectors of the adjacency matrix, closeness measures, and to a lesser extent on the number of lenders a bank has, and a measure that combines this number with the indebtedness of the bank to the rest of the system. These measures will all be described in detail in [Section B.](#) At its best combination with the benchmark capital requirement, the eigenvector measure can reduce the expected systemic losses by about 15%. It works by focusing the capital requirements on a few important banks.

The rest of this paper is structured as follows. In [Section II.](#) we describe our risk engine that generates common credit losses to banks' portfolios and our interconnectedness measures. In [Section III.](#) we describe our data sources and the network topology of the German interbank market. [Section IV.](#) gives an overview of the contagion algorithm and [Section V.](#) describes how capital is optimized. In [Section VI.](#) we present our main results, and we make some final remarks in [Section VII.](#)

## II. Methodology

Our procedure can be summarized in two stages, along with our initial condition. In the initial state, we use each bank's measured portfolio, which is composed of large and small credit exposures (e.g., loans, credit lines, derivatives) to real-economy and interbank (IB) borrowers. On the liability side, banks hold capital, interbank debt and deposits. Depositors and other creditors are senior to interbank creditors.

Capital is either set to a benchmark case that is based solely on the loss distributions of their portfolios or according to other capital allocations that partly rely on network measures. Details of

how portfolio risk is mixed with network measures are explained in [Section V.](#)

In the first stage, we simulate correlated exogenous shocks to all banks' portfolios that take the form of returns on individual large loans (where loans that are shared among multiple lenders are accounted for) and aggregated small loans. Due to changes in value of borrowers' assets, their credit ratings migrate (or they default), and banks make profits or losses on their investments in the real-economy sectors. At the end of this stage, in case of portfolio losses, capital deteriorates and some banks experience negative capital, and default. Thus, we are able to generate correlated losses that affect the capital of each bank simultaneously.<sup>1</sup>

In the second stage, we model interbank contagion. To each simulation round of the first stage we apply an extended version ([Rogers and Veraart, 2013](#)) of the fictitious contagion algorithm as introduced by [Eisenberg and Noe \(2001\)](#), augmented with bankruptcy costs and a macroeconomic proxy for fire sales. Fundamental bank defaults generate losses to other interbank creditors and trigger some new defaults. Hence, bank defaults can induce domino effects in the interbank market. We refer to new bank failures from this stage as *contagious defaults*.

Finally, we repeat the previous stages for different capital allocations. We discuss the optimization procedure in [Section V.](#) Moreover, [Section B.](#) offers an overview of the interconnectedness measures calculated with the help of network analysis and utilized in the optimization process.

## A. Credit risk model

Our credit risk engine is essential to our study for two reasons. First, it leads to our initial set of bank defaults and helps us determine capital with which our banks face the contagion event in a second stage. Just as important to our model is that the risk engine establishes our benchmark capital allocation as described above. Our results turn out to be sensitive to our choice of benchmark capital, so that it is important to get the credit engine close to some realistic risk process. Second, to the best of our knowledge, this is the first paper to incorporate correlated losses and defaults in the first stage for a large banking system.<sup>2</sup> As such, it is important to work with our rich data (with whatever limitations it might have) using a risk model consistent with models that could be used by actual banking risk officers. However, we also have to make a few concessions to the data supplied by the Deutsche Bundesbank. In particular, pricing data of the loans is not available, and our study relies only on the loan portion of the bank portfolios and credit exposures arising from derivatives.

In order to model credit risk, we utilize lending information from two data sources at different

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<sup>1</sup>By incorporating credit migrations and correlated exposures, we differ from most of the literature on interbank contagion that usually studies idiosyncratic bank defaults; see [Upper \(2011\)](#), [Elsinger et al. \(2006\)](#) and [Gauthier et al. \(2012\)](#) are remarkable exceptions.

<sup>2</sup>As already mentioned, [Elsinger et al. \(2006\)](#) and [Gauthier et al. \(2012\)](#) do model correlated portfolio losses, however for the Austrian and the Canadian banking system, which both consist of much fewer banks than the German system.



levels of aggregation: large loans and small loans. These loans are given to the “real economy”. Since borrowers of large loans are explicitly known, along with various parameters such as the loan volume, probability of default and sector, we can model their credit risk with high precision. When simulating defaults and migrations of individual borrowers, we can even account for the fact that loans given by different banks to the same borrower should migrate or default synchronously.

We cannot keep this level of precision for small loans because we only know their exposures as a lump-sum to each sector. Accordingly, we simulate their credit risk on portfolio level.

### A.1. Large loans

In modeling credit portfolio risk we closely follow the ideas of CreditMetrics (Gupton, Finger, and Bhatia, 1997; Bluhm et al., 2003). In the form we use is, it is a one-period model, and all parameters are calibrated to a one-year time span. We start with a vector  $Y \sim N(0, \Sigma)$  of systematic latent factors. Each component of  $Y$  corresponds to the systematic part of credit risk in one of the *risk modeling (RM)* sectors (see Section A. for details). The random vector is normalized such that the covariance matrix  $\Sigma$  is actually a correlation matrix. In line with industry practice, we estimate correlations from co-movements of stock indices. For each borrower  $k$  in RM sector  $j$ , the systematic factor  $Y_j$  assigned to the sector is coupled with an independent idiosyncratic factor  $Z_{j,k} \sim N(0, 1)$ . Thus, the “asset return” of borrower  $(j, k)$  can be written as:

$$X_{j,k} = \sqrt{\rho}Y_j + \sqrt{1 - \rho}Z_{j,k}. \quad (1)$$

The so-called intra-sector asset correlation  $\rho$  is common to all sectors.<sup>3</sup> The word “asset return” should not be taken literally; i.e., the link between asset returns and loan losses is not established by the contingent claims analysis of a structural credit model. Rather, the latent factor  $X_{j,k}$  is mapped into rating migrations via a threshold model, and it is a rating migration matrix that the model is calibrated to. If a loan does not default, a loss on it may arise from the fact that the credit spread used in the loan pricing formula is sensitive to the credit rating.

We use 16 S&P rating classes including notches AAA, AA+, AA, . . . ,B−, plus the aggregated “junk” class CCC−C. Moreover, we treat the default state as a further rating (D) and relabel ratings as numbers from 1 (AAA) to 18 (default). Let  $R_0$  denote the initial rating of a borrower and  $R_1$  the rating one year later. A borrower migrates from  $R_0$  to rating state  $R_1$  whenever

$$X \in [\theta(R_0, R_1), \theta(R_0, R_1 - 1)],$$

where  $\theta$  is a matrix of thresholds associated with migrations between any two ratings. For one-year migration probabilities  $p(R_0, R_1)$  from  $R_0$  to  $R_1$ , which are given by empirical estimates,<sup>4</sup> the

<sup>3</sup>This assumption could be relaxed but would require the inclusion of other data sources. In the simulations we use a value of 0.20, which is very close to a value reported by Zeng and Zhang (2001). It is the average over their sub-sample of firms with the lowest number of missing observations.

<sup>4</sup>We use the 1981–2010 average one-year transition matrix for a global set of corporates (Standard and Poor’s, 2011).

thresholds are chosen such that

$$\mathbf{P}(\theta(R_0, R_1) < X_{j,k} \leq \theta(R_0, R_1 - 1)) = p(R_0, R_1),$$

which is achieved by formally setting  $\theta(R_0, 18) = -\infty$ ,  $\theta(R_0, 0) = +\infty$  and calculating

$$\theta(R_0, R_1) = \Phi^{-1} \left( \sum_{R>R_1} p(R_0, R) \right), \quad 1 \leq R_0, R_1 \leq 17.$$

The present value of each non-defaulted loan depends on notional value, rating, loan rate, and time to maturity. In this section we ignore the notional value and focus on  $D$ , the *discount factor*. A loan is assumed to pay an annual loan rate  $C$  until maturity  $T$ , at which all principal is due. We set  $T$  equal to a uniform value of 4 years, which is the digit closest to the mean maturity of 3.66 estimated from Bundesbank's borrower statistics.<sup>5</sup> Payments are discounted at a continuous rate  $r_f + s(R)$ , where  $r_f$  is the default-free interest rate and  $s(R)$  are rating-specific credit spreads. The term structure of spreads is flat. We ignore the risk related to the default-free interest rate and set  $r_f = 2\%$  throughout. The discount factor for a non-defaulted,  $R$ -rated loan at time  $t$  is

$$D(C, R, t, T) \equiv \sum_{u=t+1}^T (C + I_{\{u=T\}}) e^{-(r_f + s(R))(u-t)}. \quad (2)$$

If the loan is not in default at time 1, it is assumed to have just paid a coupon  $C$ . The remaining future cash flows are priced according to eq. (2), depending on the rating at time  $t = 1$ , so that the loan is worth  $C + D(C, R_1, 1, T)$ . If the loan has defaulted at time 1, it is worth  $(1 + C)(1 - LGD)$ , where  $LGD$  is an independent random variable drawn from a beta distribution with expectation 0.39 and standard deviation 0.34.<sup>6</sup> This means, the same relative loss is incurred on loan rates and principal. The spreads are set such that each loan is priced at par at time 0:

$$C(R_0) \equiv e^{r_f + s(R_0)} - 1, \quad D(C(R_0), R_0, 0, T) = 1.$$

Each loan generates a return equal to

$$ret(R_0, R_1) = -1 + \begin{cases} D(C(R_0), R_1, 1, T) + C(R_0) & \text{if } R_1 < 18 \\ (1 + C(R_0))(1 - LGD) & \text{if } R_1 = 18 \end{cases}.$$

Besides secure interest, the expected value of  $ret(R_0, R_1)$  incorporates credit risk premia that markets require in excess of the compensation for expected losses. We assume that the same premia are

<sup>5</sup>The borrower statistics report exposures in three maturity buckets. Exposure-weighted averages of maturities indicate only small maturity differences between BS sectors. By setting the maturity to 4 years we simplify loan pricing substantially, mainly since the calculation of sub-annual migration probabilities is avoided.

<sup>6</sup>We have chosen values reported by Davydenko and Franks (2008), who investigate LGDs of loans to German corporates, similar to Grunert and Weber (2009), who find a very similar standard deviation of 0.36 and a somewhat lower mean of 0.275.

required by banks and calibrate them to market spreads, followed by minor manipulations to achieve monotonicity in the ratings.<sup>7</sup>

Having specified migrations and the re-valuation on a single-loan basis, we return to the portfolio perspective. Assuming that the loan index  $k$  in  $(j, k)$  runs through all sector- $j$  loans of all banks, we denote by  $R_1^{j,k}$  the rating of loan  $(j, k)$ , which is the image of asset return  $X_{j,k}$  at time 1. If bank  $i$  has given a (large) loan to borrower  $(j, k)$ , the variable  $LL_{i,j,k}$  denotes the notional exposure; otherwise, it is zero. Then, the euro return on the large loans of bank  $i$  is

$$ret_{\text{large},i} = \sum_{j,k} LL_{i,j,k} ret \left( R_0^{j,k}, R_1^{j,k} \right).$$

This model does not only account for common exposures of banks to the same sector but also to individual borrowers. If several banks lend to the same borrower, they are synchronously hit by its default or rating migration.

## A.2. Small loans

As previously described, for each bank we have further information on the exposure to loans that fall short of the €1.5 mn reporting threshold of the credit register, which is the database for the large loans. However, we know the exposures only as a sum for each RM sector so that we are forced to model its risk portfolio-wise. As portfolios of small total volume tend to be less diversified than larger ones, the amount of idiosyncratic risk that is added to the systematic risk of each sector's sub-portfolio is steered by its volume.

We sketch the setup only; further details are available from the authors on request. Let us consider all small loans in a bank's portfolio that belong to the same sector  $j$ ; these are the loans that are too small to be covered by the credit register. They are commonly driven by the sector's systematic factor  $Y_j$  and idiosyncratic risk, as in eq. (1). If we knew all individual exposures and all initial ratings, we could just run the same risk model as for the large loans. It is central to notice that the returns on individual loans in portfolio  $j$  would be independent, *conditional* on  $Y_j$ . Hence, if the exposures were extremely granular, the corresponding returns would get very close to a deterministic function of  $Y_j$ , as a consequence of the conditional law of large numbers.<sup>8</sup>

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<sup>7</sup>Market spreads are derived from a daily time series of Merrill Lynch euro corporate spreads covering all maturities, from April 1999 to June 2011. The codes are ER10, ER20, ER30, ER40, HE10, HE20, and HE30. Spreads should rise monotonically for deteriorating credit. We observe that the premium does rise in general but has some humps and troughs between BB and CCC. We smooth these irregularities out as they might have substantial impact on bank profitability but lack economic reason. To do so, we fit  $Ereturn(R_0)$  by a parabola, which turns out to be monotonous, and calibrate spreads afterwards to make the expected returns fit the parabola perfectly. Spread adjustments have a magnitude of 7bp for A- and better, and 57bp for BBB+ and worse. Ultimate credit spreads for ratings without notches are: AAA: 0.47%; AA: 0.66%; A: 1.22%; BBB 2.2476%; BB: 4.10%; B: 8.35%; CCC-C: 16.40%.

<sup>8</sup>This idea is the basis of asymptotic credit risk models. The model behind Basel II is an example of this model class.

We do not go that far since small portfolios will not be very granular; instead, we utilize the central limit theorem for conditional measures, which allows us to preserve an appropriate level of idiosyncratic risk. Once  $Y_j$  is known, the total of losses on an increasing number of loans converges to a (conditionally!) normal random variable. This conditional randomness accounts for the presence of idiosyncratic risk in the portfolio.

Our simulation of losses for small loans has two steps. First, we draw the systematic factor  $Y_j$ . Second, we draw a normal random variable, where the mean and variance are functions of  $Y_j$  that match the moments of the exact  $Y_j$ -conditional distribution. The  $Y_j$ -dependency of the moments is crucial to preserve important features of the exact portfolio distribution, especially its skewness. That dependency also assures that two banks suffer correlated losses if both have lent to sector  $j$ . An exact fit of moments is not achievable for us as it would require knowledge about individual exposures and ratings of the small loans, but an approximate fit can be achieved based on the portfolio’s Hirschman-Herfindahl Index (HHI) of individual exposures. As the HHI is also unknown, we employ an additional large sample of small loans provided by a German commercial bank to estimate the relationship between portfolio size and HHI. The estimate is sector specific. It provides us with a *forecast* of the actual HHI, depending on the portfolio’s size and the sector. The HHI forecast is the second input (besides  $Y_j$ ) to the function that gives us  $Y_j$ -conditional variances of the (conditionally normal) portfolio losses. A detailed analytical calculation of the conditional moments and a description of the calibration process are available from the authors on request.

This modeling step ends up with a (euro) return on each bank’s small loans, denoted by  $ret_{\text{small},i}$ .

## B. Centrality measures

In order to assign the interconnectedness relevance/importance to each bank of the system we rely on several centrality characteristics. The descriptive statistics of our centrality measures are summarized in Table 2. The information content of an interbank network is best summarized by a matrix  $X$  in which each cell  $x_{ij}$  corresponds to the liability amount of bank  $i$  to bank  $j$ . As each positive entry represents an edge in the graph of interbank lending, an edge goes from the borrowing to the lending node. Furthermore, the adjacency matrix ( $A$ ) is just a mapping of matrix  $X$ , in which  $a_{ij} = 1$  if  $x_{ij} > 0$ , and  $a_{ij} = 0$  otherwise. In our case, the network is directed, and by  $X$  we use the full information regarding an interbank relationship, not only its existence. We do not net bilateral exposures. Our network has a *density* of only 0.7% given that it includes 1764 nodes and 22,752 links.<sup>9</sup> This sparsity is typical for interbank networks (see for example [Soramäki, Bech, Arnolda, Glass, and Beyeler, 2007](#)).

As outlined by [Newman \(2010\)](#), the notion of centrality is associated with several metrics. In

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<sup>9</sup>The *density* of a network is the ratio of the number of existing connections divided by the total number of possible links. In our case of a directed network, the total number of possible links is  $1764 \times 1763 = 3,109,932$ .

economics the most-used measures are: *out degree* (the number of links that originate from each node) and *in degree* (the number of links that end at each node), *strength* (the aggregated sum of interbank exposures), *betweenness centrality* (based on the number of shortest paths that pass through a certain node), *eigenvector centrality* (centrality of a node given by the importance of its neighbors) or *clustering coefficient* (how tightly a node is connected to its neighbors).<sup>10</sup>

The *out degree*, one of the basic indicators, is defined as the total number of direct interbank creditors that a bank borrows from:

$$k_i = \sum_j^N a_{ij} \quad (3)$$

In economic terms, for example in case of a bank default, the *out degree* defines the number of banks that will suffer losses in the interbank market, assuming equal seniority.

Similarly, we can count the number of banks that a bank lends to (*in degree*). *Degree* is the sum of *out degree* and *in degree*.

We furthermore compute each node's *strength*, that is its total amount borrowed from other banks:

$$s_i = \sum_j^N x_{ij}. \quad (4)$$

In other words, the *strength* of a node is simply a bank's total of interbank liabilities. Similarly, we calculate each bank's interbank assets, which would be labeled the strength of inbound edges in network terminology.

The empirical distribution of degrees shows a tiered interbank structure. A few nodes are connected to many banks. For example, 20 banks (around 1%) lend to more than 100 banks each. On the borrowing side, 30 banks have a liability to at least 100 banks. These banks are part of the *core* of the network as defined by [Craig and von Peter \(2014\)](#). In terms of *strength*, 158 banks have a total IB borrowed amount in excess of €1bn while only 27 banks have total interbank liabilities in excess of €10bn. On the assets side, 103 banks lend more than €1bn and 25 banks have interbank assets in excess of €10bn.

[Opsahl, Agneessens, and Skvoretz \(2010\)](#) introduce a novel centrality measure that we label *Opsahl centrality*. This measure combines the out degree (eq. (3)) with the borrowing strength (total IB liabilities, eq. (4)) of each node, using a tuning parameter  $\varphi$ :<sup>11</sup>

<sup>10</sup>For a detailed description of centrality measures related to interbank markets see [Gabrieli \(2011\)](#) and [Minoiu and Reyes \(2013\)](#).

<sup>11</sup>In our analysis we set  $\varphi = 0.5$ , leading to the geometric mean between strength and degree.

$$OC_i = k_i^{(1-\varphi)} \times s_i^\varphi$$

The intuition of Opsahl centrality is that, in the event of default, a node with a high value is able to infect many other banks with high severity. This ability is expected to translate into a higher probability of contagion (conditional on the node's default), compared with other nodes.

Before we define *closeness*, let us define a *path* from node A to B to be a consecutive sequence of edges starting from A and ending in B. Its *length* is the number of edges involved. The (directed) *distance* between A and B is the minimum length of all paths between them. For the definition of *closeness* centrality we follow [Dangalchev \(2006\)](#):

$$C_i = \sum_{j:j \neq i} 2^{-d_{ij}},$$

where  $d_{ij}$  is the distance from  $i$  to  $j$ , which is set to infinity if there is no path from  $i$  to  $j$ . This formula has a very nice intuition. If “farness” measures the sum of the distances (in a network sense) of the shortest paths from a node to all of the other nodes, then closeness is the reciprocal of the farness.

[Bonacich \(1987\)](#) proposes an *eigenvalue centrality*, based on the adjacency matrix  $A$ . If  $\kappa_1$  be the largest eigenvalue of  $A$ , then eigenvector centrality is given by the corresponding normalized eigenvector  $v$  so that  $Av = \kappa_1 v$ . Eigenvector centralities of all nodes are non-negative.

The *weighted eigenvector centrality* is defined by the eigenvector belonging to the largest eigenvalue of the liabilities matrix  $X$ .

As a third version of eigenvectors, we calculate a *weighted normalized* eigenvector based on a modification of  $X$  where each row is normalized to sum up to 1 (if it contains a nonzero entry). This normalization ignores the *amount* that a bank borrows from others, once it borrows at all, but accounts for the relative size of IB borrowings.

The *global clustering coefficient*, as in [Watts and Strogatz \(1998\)](#), refers to the property of the overall network while *local clustering coefficients* refer to individual nodes. This property is related to the mathematical concept of *transitivity*.

$$Cl_i = \frac{\text{number of pairs of neighbors of } i \text{ that are connected}}{\text{number of pairs of neighbors of } i}.$$

Here, a neighbor of  $i$  is defined as any bank that is connected to it either by lending to or borrowing from it. The local clustering coefficient can be interpreted as the “probability” that a pair of  $i$ 's neighbors is connected as well. The local clustering coefficient of a node with the degree 0 or 1 is equal to zero. Note that this clustering coefficient refers to undirected graphs where a neighbor to

a node is any other node connected to it, and where any link in either direction between two nodes means that they are connected.

The *betweenness centrality* relies on the concept of *geodesics*. A path between two nodes is called geodesic if there is not other path of shorter length. Betweenness centrality simply answers the following question: Of all the geodesics, how many of them go through a given node? More formally, if we let  $g_{ij}$  be the number of possible geodesic paths from  $i$  to  $j$  (there might be more than a single shortest path) and  $n_{ij}^q$  be the number of geodesic paths from  $i$  to  $j$  that pass through node  $q$ , then the betweenness centrality of node  $q$  is defined as

$$B_q = \sum_{\substack{i,j \\ j \neq i}} \frac{n_{ij}^q}{g_{ij}},$$

where by convention  $\frac{n_{ij}^q}{g_{ij}} = 0$  in case  $g_{ij}$  or  $n_{ij}^q$  are zero. The intuition here is that a node of high *betweenness* is more likely to lie on a shortest route that is likely to be taken between two nodes.

We also use *total assets* as a “centrality measure” on which to base the capital allocation. Finally, in addition, we measure the effect of capital allocations based on centrality measures that are summaries of all the other centrality measures: we take the first and second principal component of all of our (normalized) centrality measures.

### III. Data sources

Our model builds on several data sources. In order to construct the interbank network, we rely on the Large-Exposures Database (LED) of the Deutsche Bundesbank. Furthermore, we infer from the LED the portfolios of credit exposures (including loans, bond holdings, and derivatives) to the real economy of each bank domiciled in Germany. This data set is not enough to get the entire picture, since especially the smaller German banks hold plenty of assets falling short the reporting threshold of 1.5 million Euros for the LED. We therefore use balance sheet data and the so-called Borrower Statistics.

When calibrating the credit risk model, we rely on stock market indices to construct a sector correlation matrix and utilize a migration matrix for credit ratings from Standard and Poor’s. Rating dependent spreads are taken from the Merrill Lynch corporate spread indices.

## A. Large-Exposures Database (LED)

The Large-Exposures Database represents the German central credit register.<sup>12</sup> Banks report exposures to a single borrower or a borrower unit (e.g., a banking group) which have a notional exceeding a threshold of €1.5 mn. The definition of an exposure includes bonds, loans or the market value of derivatives and off-balance sheet items.<sup>13</sup> In this paper, we use the information available at the end of Q1 2011. The interbank market consists of 1764 active lenders. Including exposures to the real economy, they have in total around 400,000 credit exposures to more than 163,000 borrower units.<sup>14</sup>

Borrowers in the LED are assigned to 100 fine-grained sectors according to the Bundesbank’s customer classification. In order to calibrate our credit risk model, we aggregate these sectors to sectors that are more common in risk management. In our credit risk model, we use EUROSTOXX’s 19 industry sectors (and later its corresponding equity indices). Table 1 lists risk management sectors and the distribution characteristics of the PDs assigned to them. There are two additional sectors (Households, including NGOs, and Public Sector) that are not linked to equity indices.<sup>15</sup> These 21 sectors represent the *risk model (RM) sectors* of our model.

Information regarding borrowers’ default probabilities (PDs) is included as well in the LED. We report several quantiles and the mean of sector-specific PD distributions in Table 1. Since only Internal-Ratings-Based (IRB) banks and S&L banks report this kind of information, borrowers without a reported PD are assigned random PDs drawn from a sector-specific empirical distribution.

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<sup>12</sup>The Bundesbank labels this database as *Gross- und Millionenkreditstatistik*. A detailed description of the database is given by *Schmieder (2006)*.

<sup>13</sup>Loan exposures also have to be reported if they are larger than 10% of a bank’s total regulatory capital. If such an exposure falls short of €1.5 mn, it is not contained in our dataset of large exposures. Such loans represent a very small amount compared to the exposures that have to be reported when exceeding €1.5 mn; they are captured in the Borrower Statistics though and hence part of the “small loans”; see [Section B.](#)

It is also important to notice that, while the data are quarterly, the loan volume trigger is not strictly related to an effective date. Rather, a loan enters the database once its actual volume has met the criterion at some time throughout the quarter. Furthermore, the *definition of credit* triggering the obligation to report large loans is broad: besides *on-balance sheet* loans, the database conveys bond holdings as well as *off-balance sheet* debt that may arise from open trading positions, for instance. We use *total exposure* of one entity to another. Master data of borrowers contains its nationality as well as assignments to *borrower units*, when applicable, which is a proxy for the joint liability of borrowers. We have no information regarding collateral in this dataset.

<sup>14</sup>Each lender is considered at an aggregated level (i.e. as “Konzern”). At single-entity level there are more than 4.000 different lending entities who report data.

<sup>15</sup>We consider exposures to the public sector to be risk-free (and hence exclude them from our risk engine) since the federal government ultimately guarantees for all public bodies in Germany.



Table 1: Risk model (RM) sectors

No	Risk Model Sector	No of Borrowers	Volume Weight	Default probabilities (percent)					
				5%	25%	50%	75%	95%	mean
1	Chemicals	3200	0.9%	0.008	0.19	0.59	1.54	6.66	1.66
2	Basic Materials	14,419	1.5%	0	0.27	0.85	2.05	10	2.2
3	Construction and Materials	17,776	1.3%	0	0.12	0.66	1.85	7.95	1.99
4	Industrial Goods and Services	73,548	15.1%	0	0.23	0.77	2.07	13	2.57
5	Automobiles and Parts	1721	0.7%	0.001	0.31	1.05	3	14.98	2.91
6	Food and Beverage	13,682	0.8%	0.001	0.27	0.82	1.85	8	1.92
7	Personal and Household Goods	21,256	1.3%	0	0.17	0.74	1.99	14.9	2.75
8	Health Care	16,460	1.0%	0	0.03	0.12	0.86	3.84	0.98
9	Retail	25,052	1.6%	0	0.17	0.79	2.67	11.4	2.37
10	Media	2,534	0.2%	0	0.17	0.45	1.71	7.9	1.81
11	Travel and Leisure	8,660	0.7%	0	0.36	1.17	2.92	20	3.16
12	Telecommunications	299	0.8%	0	0.12	0.36	2.32	6.32	1.82
13	Utilities	15,679	3.2%	0	0.09	0.39	1.26	6.77	1.62
14	Insurance	1392	4.1%	0.029	0.03	0.09	0.66	4.82	1.15
15	Financial Services	23,634	22.5%	0.021	0.03	0.05	0.57	4.82	1.07
16	Technology	2249	0.2%	0	0.2	0.5	1.6	4.68	1.31
17	Foreign Banks	3134	22.1%	0.003	0.03	0.09	0.88	7.95	1.4
18	Real Estate	56,451	11.4%	0	0.1	0.51	1.68	8.31	1.82
19	Oil and Gas	320	0.5%	0.038	0.22	0.81	2.96	12.86	3.03
20	Households (incl. NGOs)	79,913	1.3%	0	0.05	0.35	1.24	6	1.34
21	Public Sector	1948	9.2%	0	0	0	0	0	0
TOTAL		388,327	100.0%	0	0	0	0	0	1.5

*Note:* Volume weight refers to credit exposure.

## B. Borrower and balance sheet statistics

While the LED is an unique database, the threshold of €1.5 mn of notional is still a substantial restriction. Although large loans build the majority of money lent by German banks, the portfolios of most German banks would not be well represented by them. That does not come as a surprise as the German banking system is dominated (in numbers) by rather small S&L and cooperative banks. Many banks hold only few loans large enough to enter the LED while these banks are, of course, much better diversified. For 2/3 of banks the LED covers less than 54% of the total exposures. We need to augment the LED by information on smaller loans.

Bundesbank's Borrower Statistics (BS) dataset reports lending to German borrowers by each bank on a quarterly basis. Focusing on the calculation of money supply, it reports only those loans made by banks and branches situated in Germany; e.g., a loan originated in the London office of a German bank would not enter the BS, even if the borrower is German. Corporate lending is structured in eight main industries, of which two are further split up.<sup>16</sup> Loans to households and non-profit organizations are also reported.<sup>17</sup>

While lending is disaggregated into various sectors, the level of aggregation is higher than in the LED, and sectors are different from the sectors in the risk model. We treat this mismatch by a linear mapping of exposures from BS to RM sectors. Detailed information on the mapping is available on request.

In addition to borrower statistics, we use additional figures from the monthly Balance Sheet Statistics, which is also run by the Bundesbank. These sheets contain lending to domestic insurances, households, non-profit organizations, social security funds, and so-called "other financial services" companies. Lending to foreign entities is measured by a total figure that covers all lending to non-bank companies and households. The same applies to domestic and foreign bond holdings which, if large enough, are also included in the LED.

## C. Market data

Market credit spreads are derived from a daily time series of Merrill Lynch option-adjusted euro spreads covering all maturities, from April 1999 to June 2011. The codes are ER10, ER20, ER30, ER40, HE10, HE20, and HE30.

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<sup>16</sup>The main sectors are agriculture, basic resources and utilities, manufacturing, construction, wholesale and retail trade, transportation, financial intermediation and insurance, and services.

<sup>17</sup>A financial institution has to submit BS forms if it is a monetary financial institution (MFI), which does not necessarily coincide with being obliged to report to the LED. There is one state-owned bank with substantial lending that is exempt from reporting BS data by German law. As it is backed by a government guarantee, we consider this bank neutral to interbank contagion.

Asset correlations used in the credit portfolio model are computed from EUROSTOXX weekly returns of the European sector indices for the period April 2006 – March 2011, covering most of the financial crisis. The European focus of the time series is a compromise between a sufficiently large number of index constituents and the actual exposure of the banks in our sample, which is concentrated on German borrowers but also partly European wide.

The credit ratings migration matrix is provided by [Standard and Poor's \(2011\)](#).

## D. The German interbank market

In this section we explore the German market for interbank lending in more detail. In the existing literature, [Craig and von Peter \(2014\)](#) use the German credit register to analyze the German interbank market. They find that a *core-periphery* model can be well fitted to the German interbank system: core banks build a complete sub-network (i.e., there exist direct links between any two members of the subset), while periphery banks are less connected by lending. The core-periphery structure turns out to be very stable through time. [Roukny, Georg, and Battiston \(2014\)](#) use the same data source, spanning the period 2002–2012. Providing a thorough analysis of how German interbank lending develops over time, they find most of the characteristics to be very stable, including the distributions of various centrality measures utilized in our paper. Because of these findings, complemented by our own analysis for the period 2005–2011, we neglect the time dimension completely and focus on a single point of time.<sup>18</sup>

At the end of Q1 2011, 1921 MFIs were registered in Germany, holding a total balance sheet of €8,233 bn.<sup>19</sup> The German banking system is composed of three major types of MFIs: 282 commercial banks (including four big banks and 110 branches of foreign banks) that hold approximately 36% of total assets, 439 saving banks (including 10 *Landesbanken*) that hold roughly 30% of system's assets and 1140 credit cooperatives (including 2 regional institutions) that hold around 12% of market share. Other banks (i.e. mortgage banks, building and loan associations and special purpose vehicles) are in total 60 MFIs and represent approx. 21% of the system's balance sheet.

Our interbank (IB) network consists of 1764 active banks (i.e. aggregated banking groups). These banks are actively lending and/or borrowing in the interbank market. They hold total assets worth €7,791 bn, from which 77% represent large loans and 23% small loans.

[Table 2](#) presents the descriptive statistics of the main characteristics and network measures of the German banks utilized in our analysis. The average size of a bank-individual IB exposure is around €1 bn. As figures show, there are few very large total IB exposures, since the mean is between the 90<sup>th</sup> and 95<sup>th</sup> percentile, making the distribution highly skewed. Similar properties are observed for

<sup>18</sup>Tables with these measures are available on request.

<sup>19</sup>Source: Deutsche Bundesbank's Monthly Report, March 2011.

Table 2: Interbank (IB) market and network properties

	Quantiles								mean	std dev
	5%	10%	25%	50%	75%	90%	95%			
Total IB Assets <sup>1</sup>	7591	13,089	35,553	100,450	310,624	868,299	1,647,666	990,433	7,906,565	
Total IB Liabilities <sup>1</sup>	2640	6,053	19,679	61,450	180,771	527,460	1,212,811	990,433	7,782,309	
Total Assets <sup>1</sup>	37,798	63,613	160,741	450,200	1,290,698	3,412,953	7,211,608	4,416,920	37,938,474	
Total Large Loans <sup>1</sup>	8719	17,293	60,692	208,475	675,199	2,054,840	4,208,323	3,424,445	30,221,363	
Total Small Loans <sup>1</sup>	8906	34,932	85,741	217,516	550,855	1,287,390	2,253,149	992,475	8,748,757	
Out Degree	1	1	1	2	4	9,1	16	13	82	
In Degree	1	1	4	9	14	19	25	13	37	
Total Degree	2	3	5	11	18	27	38	26	111	
Opsahl Centrality	51.5	80.6	165.8	345.9	791.6	2071.2	4090	3342	24,020	
Eigenvector Centrality	0.000003	0.000019	0.000078	0.000264	0.001092	0.004027	0.009742	0.003923	0.023491	
Weighted Betweenness	0	0	0	0	0	401.5	6225.8	10,491	82,995	
Weighted Eigenvector	0.000004	0.000012	0.000042	0.000142	0.000611	0.002217	0.004839	0.003148	0.023607	
Closeness Centrality	253.2	328.8	347.8	391.8	393.6	411.5	427.4	371.22	69.13	
Clustering Coefficient	0	0	0	0.00937	0.04166	0.12328	0.16667	0.0379	0.0677	
<b>Number of banks<sup>2</sup></b>	1764									
<b>Number of links</b>	22,752									

Note: <sup>1</sup> in thousands euro; <sup>2</sup> number of banks active in the interbank market. Data point: Q1 2011.

total assets, the total of large loans and Out Degrees, supporting the idea of a tiered system with few large banks that act as interbank broker-dealers connecting other financial institutions (see e.g., [Craig and von Peter, 2014](#)).<sup>20</sup>

## IV. Modeling contagion

As introduced in [Section II.](#), we differentiate between *fundamental defaults* and *contagious defaults* (see also [Elsinger et al. \(2006\)](#) or [Cont et al. \(2013\)](#), for instance). Fundamental defaults are related to losses from the credit risk of “real economy” exposures, while contagious defaults are related to the interbank credit portfolio (German only).<sup>21</sup>

Moreover, we construct an *interbank clearing mechanism* based on the standard assumptions of interbank contagion (see e.g., [Upper, 2011](#)): First, banks have *limited liability*. Next, interbank liabilities are senior to equity but *junior* to non-bank liabilities (e.g., deposits). Losses related to bank defaults are *proportionally* shared among interbank creditors, based on the share of their exposure to total interbank liabilities of the defaulted bank. In other words, its interbank creditors suffer the same loss-given-default.<sup>22</sup> Finally, non-bank assets of a defaulted bank are liquidated at a certain discount. This extra loss is referred to as *fire sales* and is captured by *bankruptcy costs*, defined in [Section A.](#) The clearing mechanism follows the idea of [Eisenberg and Noe \(2001\)](#) which, however, is not designed to include bankruptcy costs. To account for these costs, we follow [Rogers and Veraart \(2013\)](#) who propose a simple algorithm which converges to the fixed point with minimum losses.

When a group of banks default, they trigger losses in the interbank market. If interbank losses (plus losses on loans to the real economy) exceed the remaining capital of the banks that lent to the defaulted group, this can develop into a domino cascade. At every simulation when interbank contagion arises, we follow [Rogers and Veraart \(2013\)](#) to compute losses that take into account the above assumptions.

More formally, first recall that each bank makes a (euro) return on its large and small loans, defined in [Section A.](#) We switch the sign and define *fundamental losses* as:

$$L_i^{\text{fund}} \equiv - (ret_{\text{large},i} + ret_{\text{small},i}).$$

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<sup>20</sup>One aspect that needs to be mentioned here is that the observed IB network is not the complete picture, since interbank liabilities of German banks raised outside Germany are not reported to the LED. For example, the LED captures a loan made by Goldman Sachs to Deutsche Bank only if it is made by Goldman Sachs’ German subsidiary. This aspect might bias downwards centrality measures of big German banks that might borrow outside Germany.

<sup>21</sup>Foreign bank exposures are included in Sector 17 of the “real economy” portfolio, since we have to exclude them from the interbank network. Loans made by foreign banks to German financial entities are not reported to the LED, except they are made by a subsidiary registered in Germany. These subsidiaries are part of our interbank network.

<sup>22</sup>We do not have any information related to collateral or the seniority of claims.

Each bank incurs total portfolio losses  $L_i$  equal to its fundamental losses and losses on its interbank loans:

$$L_i = L_i^{\text{fund}} + L_i^{\text{IB}}, \quad (5)$$

where  $L_i^{\text{IB}}$  is yet to be determined. A bank defaults if its capital  $K_i$  cannot absorb the portfolio losses. We define the default indicator as

$$D_i = \begin{cases} 1 & \text{if } K_i < L_i, \\ 0 & \text{otherwise.} \end{cases}$$

Below, bankruptcy costs are modeled so that their (potential) extent  $BC_i$  is known before contagion; i.e., they are just a parameter of the contagion mechanism, but only accrue when  $D_i = 1$ .

Total portfolio losses and bankruptcy costs are now distributed to the bank's claimants. If capital is exhausted, further losses are primarily borne by interbank creditors since their claim is junior to other debt. Bank  $i$  causes its interbank creditors an aggregate loss of

$$\Lambda_i^{\text{IB}} = \min(l_i, \max(0, L_i + BC_i D_i - K_i)),$$

which is zero if the bank does not default. The Greek letter signals that  $\Lambda_i^{\text{IB}}$  is a loss *on the liability side* of bank  $i$ , which *causes* a loss on the assets of its creditors.

The term  $x_{ij}$  denotes interbank liabilities of bank  $i$  against bank  $j$ , and the row sum  $l_i = \sum_{j=i}^N x_{ij}$  defines total interbank liabilities of bank  $i$ . This gives us a *proportionality matrix*  $\pi$  to allocate losses, given by

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{l_i} & \text{if } l_i > 0; \\ 0 & \text{otherwise.} \end{cases}$$

If the loss amount  $\Lambda_i^{\text{IB}}$  is proportionally shared among the creditors, bank  $j$  incurs a loss of  $\pi_{ij} \Lambda_i^{\text{IB}}$  because of the default of  $i$ . Also the bank  $i$  we have started with may have incurred interbank losses; they amount to

$$L_i^{\text{IB}} = \sum_{k=1}^N \pi_{ki} \Lambda_k^{\text{IB}}, \quad (6)$$

which provides the missing definition in eq. (5). This completes the equation system (5)–(6), which defines our allocation of contagion losses.

Let us write the system in vector form, for which we introduce  $l = \left(\sum_{j=i}^N x_{ij}\right)_i$  as the vector of interbank liabilities,  $\wedge$  as the element-wise minimum operator, and  $\tilde{B}$  as a diagonal matrix with bankruptcy costs  $BC_i$  on the diagonal. Default dummies are subsumed in the vector  $D$  so that the product  $\tilde{B}D$  defines *actual* bankruptcy costs, i.e. the ones that become real. Losses in the IB market *caused* by bank  $i$  and incurred by the other banks, can then be written as

$$\Lambda^{\text{IB}} = l \wedge [L + \tilde{B}D - K]^+.$$

With  $\Pi$  being the proportionality matrix, interbank losses – now on the asset side – amount to

$$L^{\text{IB}} = \Pi^{\top} \Lambda^{\text{IB}}.$$

We consider the total portfolio losses  $L = L^{\text{fund}} + L^{\text{IB}}$  (not containing bankruptcy costs) as a solution.<sup>23</sup> Altogether, we have to solve the equation

$$L = \Phi(L) \equiv L^{\text{fund}} + \Pi^{\top} \left( l \wedge [L + \tilde{B}I_{[L>K]} - K]^+ \right),$$

where the indicator function and the relational operator are defined element-by-element. According to our definition of default, the operator  $\Phi$  is left-continuous on  $\mathbb{R}^N$ .

[Rogers and Veraart \(2013\)](#) show that a simple repeated application of the monotonic operator  $\Phi$  to  $L^{\text{fund}}$  generates a sequence of losses that must converge to a unique loss vector  $L^{\infty}$ , simply because the sequence is monotonic in each dimension and the solution space is compact. Since the operator  $\Phi$  is left-continuous in our setup,  $L^{\infty}$  is also a fixed point. As shown by [Rogers and Veraart \(2013\)](#), it has minimum losses among all fixed points.

## A. Bankruptcy costs

In our analysis, we are particularly interested in bankruptcy costs (henceforth BCs) since they represent a dead-weight loss to the economy. We model them as the sum of two parts. The first one is a function of a bank's total assets, because there is empirical evidence for a positive relationship between size and BCs of financial institutions; see [Altman \(1984\)](#). The second part incorporates fire sales and their effect on the value of the defaulted bank's assets. For their definition, recall that we want to model BCs such that their extent is known before contagion, which is why we make them exclusively dependent on the *fundamental* portfolio losses  $L_i^{\text{fund}}$ . If that loss of bank  $i$  exceeds capital  $K_i$ , the bank's creditors suffer a loss equal to  $\max(0, L_i^{\text{fund}} - K_i)$ . In the whole economy, fundamental losses add up to

$$\bar{L}_{\text{fund}} \equiv \sum_i \max(0, L_i^{\text{fund}} - K_i).$$

It is this total fundamental loss in the system by which we want to proxy lump-sum effects of fire sales. The larger  $\bar{L}_{\text{fund}}$ , the more assets will the creditors of defaulted banks try to sell quickly, which puts asset prices under pressure. We proxy this effect by defining a system-wide relative loss ratio  $\lambda$  that is a monotonic function of  $\bar{L}_{\text{fund}}$ . In total, if bank  $i$  defaults, we define BCs as the sum of two parts related to total assets and fire sales:

$$BC_i \equiv \phi \left( \text{TotalAssets}_i - L_i^{\text{fund}} \right) + \lambda \left( \bar{L}_{\text{fund}} \right) \max(0, L_i^{\text{fund}}). \quad (7)$$

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<sup>23</sup>We could also search for a solution for  $L^{\text{all}} \equiv L + \tilde{B}D$ , which turns out to be equivalent but more complicated.

We consider  $\phi$  to be the proportion of assets lost due to litigation and other legal costs. In our analysis we set  $\phi = 5\%$ .<sup>24</sup> It is rather for convenience than for economic reasons that we set the monotonic function  $\lambda$  equal to the cumulative distribution function of  $\bar{L}_{\text{fund}}$ . Given this choice, the larger total fundamental losses in the system are, the closer  $\lambda$  gets to 1.<sup>25</sup>

In the optimization process we minimize a measure of system losses (i.e. the target function). The mechanism of contagion proposed by us has several sets of agents, each of whom suffers separate kinds of losses. There are many conflicting arguments which agent's losses the regulator should particularly be interested in, for instance those of depositors (as a proxy for "the public" that is likely to be the party that ultimately bails banks out) or even those of bank equity holders (who are at risk while offering a valuable service to the real economy). While all of them may be relevant, our primary target function are the *expected BCs*, which is just the sum of the expected BCs of defaulted banks:

$$EBC = \mathbf{E} \sum_i BC_i D_i, \quad (8)$$

where  $D_i$  is the default indicator of bank  $i$ . BCs are a total social deadweight loss that does not include the initial portfolio loss due to the initial shock. While this is a compelling measure of social loss, there are distributional reasons that it might not be the only measure of interest.

Losses of eq.(8) are free of distributional assumptions. However some amount of BCs can be well acceptable as a side-effect of otherwise desirable phenomena (e.g., the plain existence of bank business), such that minimizing expected BCs does not necessarily lead to a "better" system in a broader sense. For example, minimum expected BCs might entail an undersupply of credit more harmful to the real economy than the benefits from low BCs.

The *expected total loss to equity holders* as well as the *expected total loss to non-bank debt holders* are therefore at least of interest, if not justifiable alternative target functions. Also their sum appears as a natural choice for a target function, this way treating the interests of bank owners and non-bank debt holders equally important. However, it can easily be shown that this sum is equivalent to our target function, the expected BCs.

It is important to note that we consider banks as institutions only, meaning that any loss hitting a bank must ultimately hit one of its *non-bank* claimants. In our model, these are simply non-bank debtors and bank equity holders, as we split bank debt into non-bank an interbank debt only. Counting

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<sup>24</sup>Our results remain robust also for other values  $\phi \in \{1\%, 3\%, 10\%\}$ . Alessandri, Gai, Kapadia, Mora, and Puh (2009) and Webber and Willison (2011) use contagious BCs as a function of total assets, and set  $\phi$  to 10%. Given the second term of our BC function that incorporates fire sales effects, we reach at a stochastic function with values between 5% and 15% of total assets.

<sup>25</sup>We acknowledge that real-world BCs would probably be sensitive to the amount of interbank credit losses, which we ignore. This simplification, however, allows us to calculate potential BCs before we know which bank will default through contagion, such that we do not have to update BCs in the contagion algorithm. If we did, it would be difficult to preserve proportional loss sharing in the Eisenberg-Noe allocation.



total losses on the asset side of banks' balance sheets (or just their credit losses) makes little sense anyway, as they involve interbank losses and therefore a danger of double counting.

We considered all of the above losses in our simulations, but we report only those related to eq. (8) with the purpose of exposition in this paper.

## V. Optimization

In this section, we define the way we reallocate capital. Several points should be made about it.

First, the rules themselves are subject to a variety of restrictions. These include the fact that the rules must be simple and easily computed from observable characteristics, and they should preferably be smooth to avoid cliff effects. Simplicity is important not just because of computational concerns. Simple formal rules are necessary to limit discretion on the ultimate outcome. Too many model and estimation parameters set strong incentives for banks to lobby for a design in their particular interest. While this is not special to potential systemic risk charges, it is clear that those banks who will most likely be confronted with increased capital requirements are the ones with the most influence on politics. Vice versa, simplicity can also help to avoid arbitrary punitive restrictions imposed upon individual banks. In this sense, the paper cannot offer deliberately fancy first-best solutions for capital requirements.

Second, as noted in the introduction, the rules must keep the total capital requirement the same so that we do not mix the effects of capital reallocation with the effect of increasing the amount of capital in the entire system.

Finally, for reasons of exposition, we focus on capital reallocations based on a single centrality measure. While we did explore more complicated reallocation rules that were optimized over combinations of centrality measures, these reallocations gave only marginal improvements and so are not reported.

We introduce a range of simple capital rules over which our chosen loss measure, the expected BCs, is minimized. We minimize over two dimensions, the first being the choice of a centrality measure and the second being represented by a gradual deviation from a VaR based benchmark allocation towards an allocation based on the chosen centrality measure.

In our benchmark case, capital requirements focus on a bank's individual portfolio risk (and not on network structure). For our analysis we require banks to hold capital equal to its portfolio VaR on a high security level of  $\alpha = 99.9\%$ , which is in line with the level used in Basel II rules for the banking book.<sup>26</sup> There is one specialty of this VaR, however. In line with Basel II again, the benchmark capital

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<sup>26</sup>To check the numerical stability of our results, we re-ran several times the computation of VaR measures at quantile

requirement treats interbank loans just as other loans. For the determination of bank  $i$ 's benchmark capital  $K_{\alpha,i}$  (and *only* for this exercise), each bank's German interbank loans (on the asset side of the balance sheet) are merged with loans to foreign banks into portfolio sector no. 17 where they contribute to losses just as other loans.<sup>27</sup>

In the whole system, total required capital adds up to  $TK_{\alpha} \equiv \sum_i K_{\alpha,i}$ . To establish a "level playing field" for the capital allocation rules,  $TK_{\alpha}$  is to be the same for the various allocations tested.

The basic idea is that we give banks a – hypothetical – proportional capital relief from their benchmark capital, which we then redistribute according to a rule in which capital is scaled up or down by a centrality measure.

Given  $C_i$  to be one of the centrality measures introduced in Section B., we subtract a fraction  $\beta$  from each bank's benchmark capital  $K_{\alpha,i}$  for redistribution. Some required capital is added back, again as a fraction of  $K_{\alpha,i}$ , which is proportional to the centrality measure:

$$K_{\text{simple},i}(\beta) \equiv K_{\alpha,i} (1 - \beta + \beta a C_i). \quad (9)$$

The parameter  $a$  is chosen such that total system capital remains the same as in the benchmark case, which immediately leads to

$$a = \frac{\sum_j K_{\alpha,j}}{\sum_j K_{\alpha,j} C_j}, \quad \text{for all } \beta.$$

This simple capital rule is not yet our final allocation because it has a flaw, although it is almost exactly the one we use in the end. Among those banks for which capital decreases in  $\beta$  (these are the banks with the lowest  $C_i$ ), some banks' capital buffer may get so small that its probability of default (PD) rises to an unacceptable level. We want to limit the PDs (in an imperfect way) to one percent, which we assume to be politically acceptable. To this end, we require each bank to hold at least capital equal to bank  $i$ 's VaR at  $\alpha = 99\%$ , which we denote by  $K_{\min,i}$ . In other words, we set a floor on  $K_{\text{simple},i}(\beta)$ . The VaR is again obtained from the model used for benchmark capital, which treats IB loans as ordinary loans and hence makes the upper PD limit imperfect.<sup>28</sup>

If we plainly applied the floor to  $K_{\text{simple},i}(\beta)$  (and if the floor was binding somewhere), we would require more capital to be held in the system than in the benchmark case. We therefore introduce a

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$\alpha = 99.9\%$ . For this computation we employed a new set of one million simulations, and kept the same PDs for loans where unreported values had been reported by random choices. Results are very similar with an average variance of under 2%. VaR measures at 99% have a variance of under 0.5%.

<sup>27</sup>Default probabilities for these loans are taken from the Large Exposure Database in the same way as for loans to the real economy.

<sup>28</sup>If banks held exactly  $K_{\min,i}$  as capital, actual bank PDs after contagion could be below or above 1 percent, depending on whether quantiles of portfolio losses in a risk model where interbank loans are directly driven by systematic factors are larger than in presence of contagion (but without direct impact of systematic factors). However, the probability of bank defaults through *fundamental* losses cannot exceed 1 percent, given that asset correlations in the factor model are positive.

tuning factor  $\tau(\beta)$  to reestablish  $TK_\alpha$ . The final capital rule is:

$$K_{\text{centr},i}(\beta) \equiv \max(K_{\text{min},i}, K_{\alpha,i} [1 - \beta + \beta \tau(\beta) a C_i]). \quad (10)$$

For a given  $\beta$ , the tuning factor  $\tau(\beta)$  is set numerically by root finding. To anticipate our results, tuning is virtually obsolete. Even if 30% of benchmark capital is redistributed – which is by far more than in the optimum, as will turn out – we find  $0.999 < \tau(\beta) \leq 1$  throughout, and there is a maximum of only 14 banks for which the floor becomes binding.

In general, the approach is not limited to a single centrality measure. As regards the formal approach, we could easily replace  $C_i$  in eq. (10) by a linear combination of centrality measures  $\sum_k a_k C_i^k$  and optimize the  $a_k$  along with  $\beta$ ; the degree of freedom would be equal to the number of centrality measures included. However, optimization quickly becomes prohibitively expensive as every step requires its own extensive simulation. We carry out a couple of bivariate optimizations, focusing on the centrality measures found most powerful in one-dimensional optimizations. Improvements over the one-dimensional case are negligible and therefore not reported.

In this section we have dealt only with capital *requirements*. To assess their consequences on actual systemic risk, we also have to specify in which way banks intend to obey these requirements. In practice, banks hold a buffer on top of required capital in order to avoid regulatory distress. However, as our model runs over one period only, we do not lose much generality when we abstract from additional buffers and assume banks to hold exactly the amount of capital they are required to hold.

## VI. Results

Our main results from the reallocation of capital are depicted in [Figure 1](#). [Table 3](#) presents the corresponding optimum values in numbers. As defined in (8), the target function that we compare across centrality measures is the total expected bankruptcy costs. Our benchmark allocation (based only on  $\text{VaR}(\alpha = 99.9\%)$ ) is represented by the point where  $\beta = 0$ . As  $\beta$  increases, more of the capital is allocated to an allocation rule based upon the centrality measure as described in eq. (10). The extreme right end of the diagram still allocates 75% of the capital based on the original  $\text{VaR}(\alpha = 99.9\%)$ . To allocate less would probably be difficult to support politically and, as can be seen in the figure, covers the area for the minimum losses in all cases but one. Even in this case, the minimum for the closeness centrality is very close to the minimum in the plotted range.

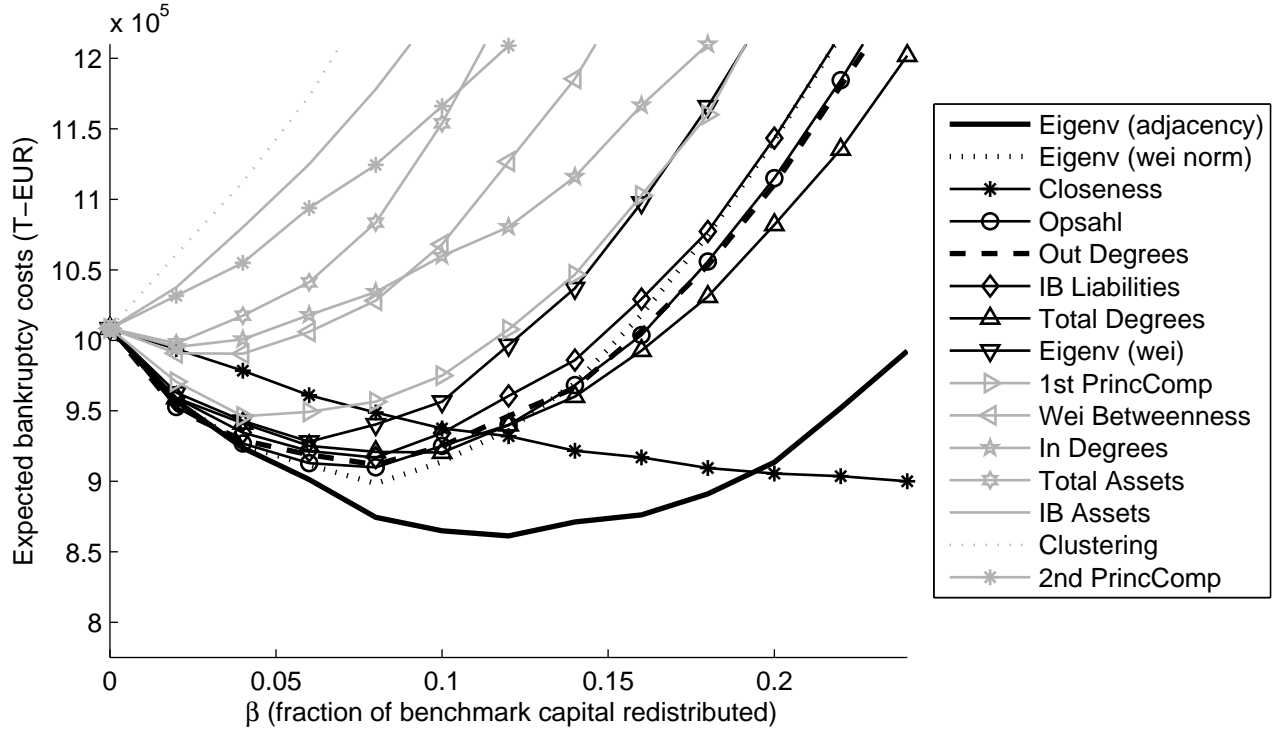
Several patterns are clearly evident in the diagram and supported by findings in [Table 3](#). The first observation in [Figure 1](#) is that some centrality measures are completely dominated by others. Over the region of  $0 \leq \beta \leq 24\%$ , capital reallocations based on three of the centrality measures were destructive to the system over the entire region: Clustering, IB Assets, and the Second Principal Component.

Table 3: Total expected bankruptcy costs after contagion, applying optimized centrality based capital allocation

Centrality measure	Expected BCs (T-EUR)	Saving (percent)	Optimal beta (percent)
Eigenvector (adjacency)	861,000	14.6	12
Eigenvector (weighted normalized)	898,000	10.9	8
Closeness	900,000	10.7	24
Opsahl Centrality	910,000	9.8	8
Out Degree	912,000	9.6	8
IB Liabilities	917,000	9.1	8
Degree	920,000	8.7	10
Eigenvector (weighted)	928,000	8.0	6
1st Principal Component	946,000	6.2	4
Weighted Betweenness	991,000	1.8	4
In Degree	995,000	1.3	2
Total Assets	998,000	1.0	2
IB Assets	1,008,000	0.0	0
Clustering	1,008,000	0.0	0
2nd Principal Component	1,008,000	0.0	0
(Benchmark, purely VaR based)	1008,000	—	—

*Note:* Expected BCs include fundamental and contagious defaults. Saving in percent refers to the benchmark case.  $\beta$  has been optimized in bins of two percent size.

Figure 1: Comparison of centrality based capital allocations

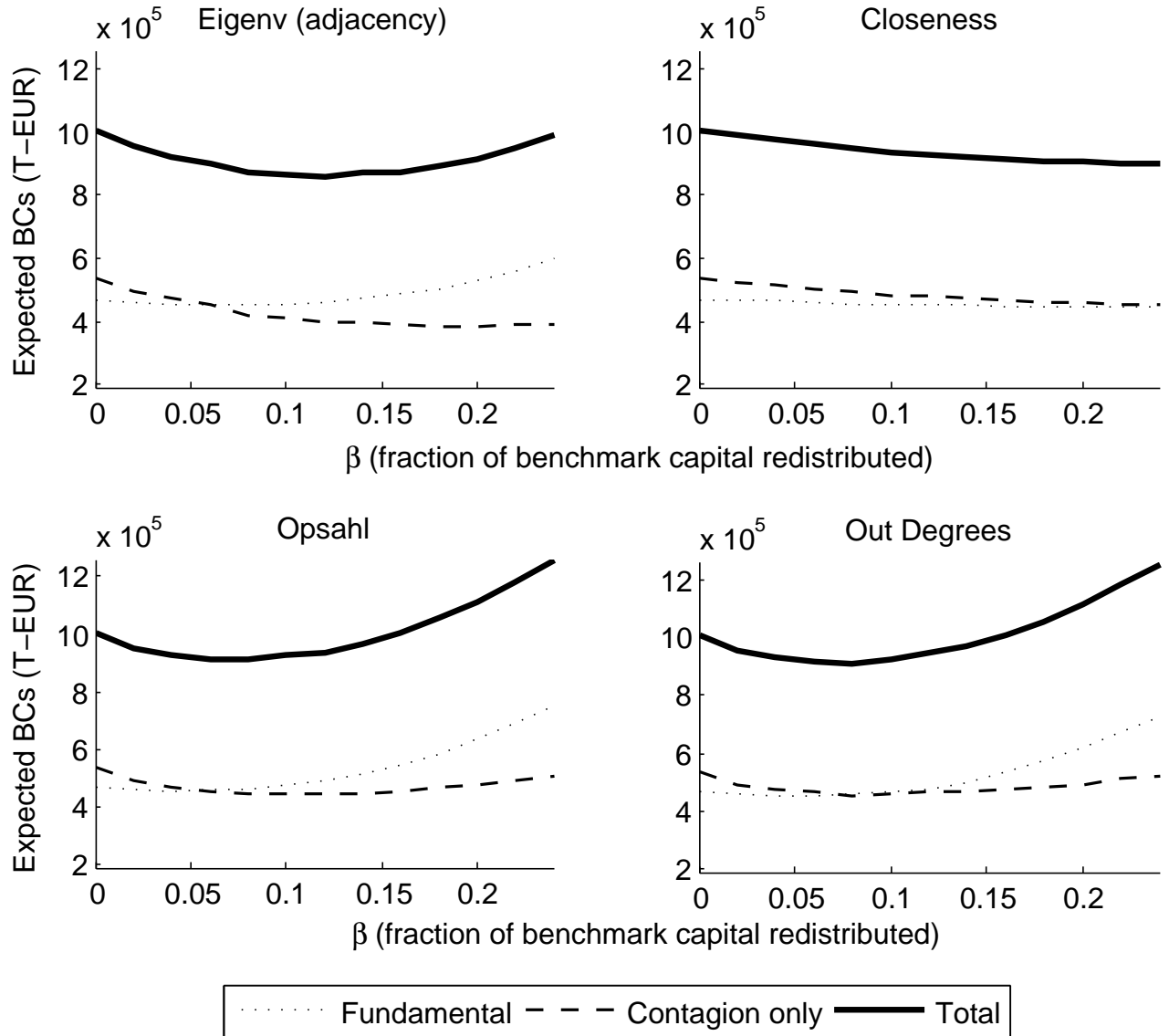


Capital allocations based on all other centrality measures help to improve the stability of the banking system in terms of expected total bankruptcy costs over at least some of the region of  $\beta$ .

One measure stands out among all of them: the Adjacency Eigenvector. With about a 12% re-allocation of capital from the baseline capital to one based on the Adjacency Eigenvector saves the system 14.6% in system losses as measured by expected BCs. Radically giving 24% of the allocation to one based on Closeness saves the system less (10.7%), but these two capital allocations behave differently from the rest in two senses. First, they are more effective at reducing losses, meaning that the next most effective group of centrality measures, Opsahl, Weighted Eigenvector, Out-Degree, Total Degree, and Weighted Normalized Eigenvector, stabilize the system so that it reduces losses between 9% and 10%, in contrast to 14.6%. Second, they all do so at a level of capital reallocation that involves only 8% being reallocated from the benchmark to the centrality based allocation. The two most successful reallocations redistribute more. Finally the rest of the centrality measures generate even smaller savings at even smaller amounts redistributed. Because the allocations based on Closeness and Adjacency Eigenvector perform best, we focus on them in the discussion that follows (although the Weighted Normalized Eigenvector actually performs second-best, we skip it from further analyzes because of its similarity to the Adjacency Eigenvector). To a lesser extent we also discuss the effect of the next-best performing allocations, Opsahl Centrality and Out-Degree.

Figure 2 presents expected BCs under the best-performing centrality measures for both pre- and post-contagion losses. The case of Closeness stands out from the other three. For Closeness, the

Figure 2: Expected bankruptcy costs: All defaults, fundamental defaults and contagious defaults



*Note:* Y-axes represent expected Bankruptcy Costs (as measured by Equation 7) from fundamental defaults, contagious defaults, and all defaults, under different capital allocations. On the X-axis,  $\beta$  represents the redistributed fraction of benchmark capital,  $\text{VaR}(\alpha = 99.9\%)$ .

fundamental defaults mimic the baseline case throughout the range of capital reallocation. They barely increase despite changes in the distribution of bank defaults. As  $\beta$  increases, the decline in losses from contagion are not matched by an increase in pre-contagion losses, and the total losses decrease. The best performing allocation, based on the Adjacency Eigenvector, for the smaller range of  $\beta$ , does not increase the fundamental losses much over the benchmark and gives striking savings in post-contagion losses compared to all other measures. It does exactly what one would expect from a capital reallocation based on a wider set of information: strongly reduce the cascading losses at a small expense of fundamental ones. Once the fundamental losses start increasing steeply, at an inflection point of about  $\beta = 15\%$ , the total costs rise rapidly as well. Below this value, the increase in fundamental costs is relatively smooth, just as the fall-off in benefits from the post-contagion savings are smooth. This reflects the large range of  $\beta$  where the capital reallocation is very effective. The other two measures are similar to the behavior of the allocation based on Adjacency Eigenvector. As before, the inflection points of fundamental losses lead to a minimum, but featuring some differences. First, the post-contagion losses stop decreasing at around the same value of  $\beta$ ; second, the inflection points are sharper; and finally, all of this happens at smaller values of  $\beta$ , and with smaller declines in post-contagion bankruptcy costs. These are less effective measures upon which to base capital reallocations.

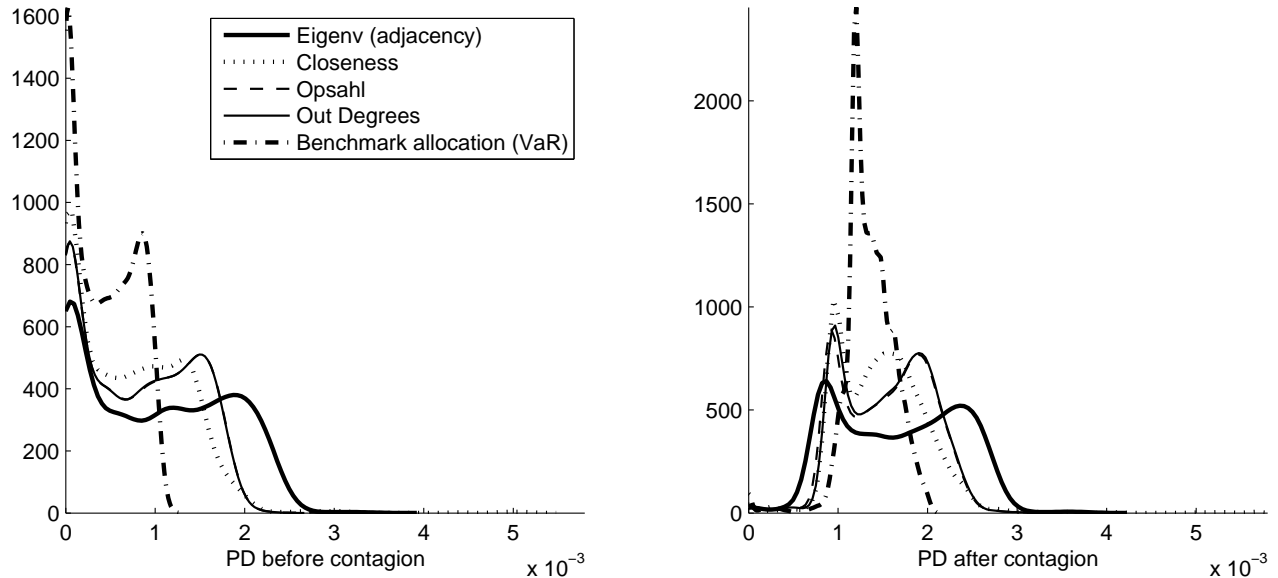
Figure 3 displays frequency distributions of all banks' default probabilities before and after contagion in the benchmark case (VaR based capital)<sup>29</sup> and compares them with distributions generated by the best centrality based allocations. These four capital reallocations are taken at their optimal  $\beta$ , according to the values presented in Table 3. All densities show a similar picture for the fundamental defaults ("PD before contagion"). The reallocations all spread the distribution to the right on fundamental defaults. Indeed, fundamental defaults for the best basis, the Adjacency Eigenvector, spread the fundamental defaults most to the right. We know from the costs of the fundamental defaults that total bankruptcy costs remain the same. They were redistributed such that more (presumably smaller) banks defaulted to save a few whose default triggers potentially larger bankruptcy costs. This is most pronounced for the Adjacency Eigenvector based capital allocation, but it is true for all the other three measures as well: pre-contagion default frequencies shift to the right with a longer and fatter tail than in the benchmark case. This is not surprising as the PDs before contagion are limited to 0.1% by construction (cf. footnote 28); any exceedance is caused by simulation noise. The best allocations based on the other three centrality measures lie somewhere in between the fat-tailed Adjacency Eigenvector based distribution and the thin-tailed benchmark. Still the lesson from the left-hand side of Figure 3 is that the capital allocations based on each of our centrality measures in the initial set of defaults sacrifice a large number of less relevant banks by shifting capital to the relevant banks. There is also, of course, a relationship to size, but we cannot present related graphs for confidentiality reasons.

Post-contagion probabilities of default on the right-hand side of Figure 3 show further interesting patterns with respect to the benchmark case. The capital reallocation based on each of our centrality

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<sup>29</sup>Recall that the VaR used for capital is not identical with the actual loss quantile in the model that includes contagion, be it before or after contagion; see Section V.

Figure 3: Frequency distributions of individual bank PDs



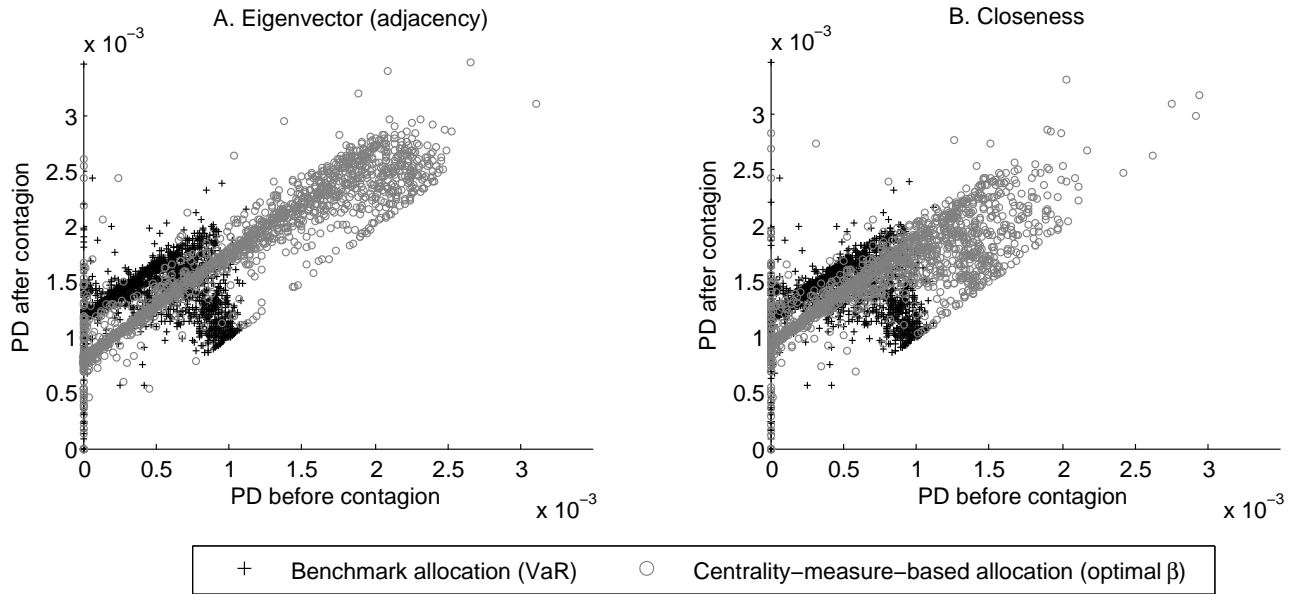
*Note:* On Y-axis is presented the estimated density of the distribution of bank PDs. On X-axis, are represented PDs (per bank). Results obtained with 500,000 simulations.

measures continues to sacrifice more of the (presumably) smaller banks in order to reduce the probability of default, post contagion, for the larger banks. The end result of this is a distribution that is considerably wider in its default probability both pre- and post-contagion. The cost in terms of the banks that are more likely to fail is made up through a few banks (otherwise suffering – or elsewhere causing – large losses, in expectation) that are less likely to fail during the contagion phase of the default cycle. It is remarkable that each of our reallocations behave in this way. The extra capital gained from the reduction in the benchmark capital rule which focuses on pre-contagion default has not reduced post-contagion defaults across-the-board for any tested reallocation. Instead, in contrast to benchmark allocation our centrality based capital rules perform less consistently (at least in this sense), sacrificing many defaults to save those which matter in terms of our loss function.

Figure 3 can also be used as a validation of the “traditional” way of measuring the risk of interbank loans, that is, by treating them as in the benchmark case; cf. Section V. In the latter case they are part of an ordinary industry sector and driven by a common systematic factor. This treatment is very much in line with the approach taken in the Basel III framework. The bank PDs sampled in Figure 3 are default probabilities from the model including contagion. If the bank individual loss distributions generated under the “traditional” treatment were a perfect proxy of the ones after contagion, we should observe the histogram of PDs after contagion in the VaR based benchmark case to be strongly concentrated around 0.1%. Instead, the PDs are widely distributed and even on average 35 percent higher than that possibly concluded from the label “99.9-percent VaR”. While the link to the Basel framework is rather of methodological nature, our analysis clearly documents that interbank loans are



Figure 4: Relationship between default probabilities of individual banks before and after contagion



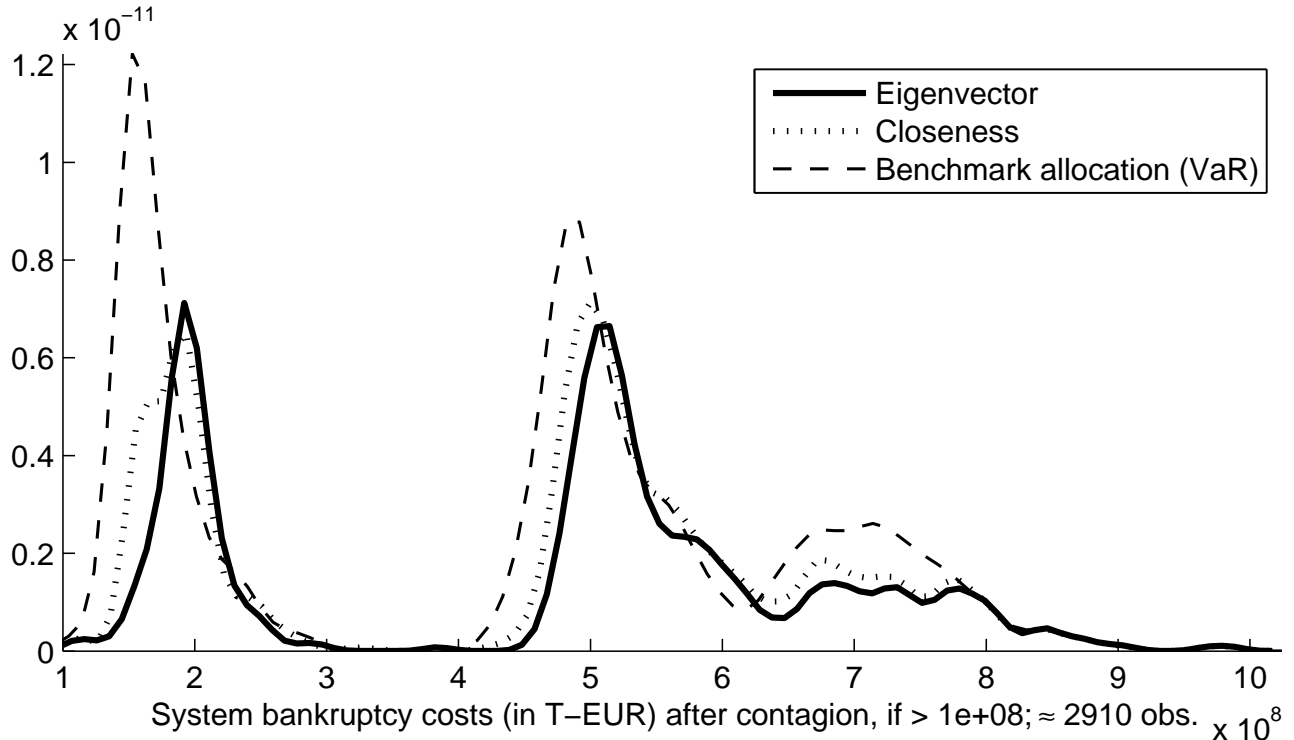
*Note:* PDs are not to be interpreted as real; e.g., actual capital holdings of German banks have not found entry into their calculation.

special and that correlating their defaults by Gaussian common factors may easily fail to capture the true risk. As there are, however, also good reasons to remain with rather simple “traditional” models, such as those behind the Basel rules, our modeling framework lends itself to a validation of the capital rules for interbank credit. This exercise is beyond the scope of this paper.

In [Figure 4](#), each observation represents a bank for which we calculate the probability of fundamental defaults ( $x$ -axis) and the PD including both fundamental and contagious defaults ( $y$ -axis). (Obviously there is nothing lower than the 45 degree line in these diagrams.) In the case of the VaR based benchmark allocation (black markers), the relationship is clearly non-linear. Most banks, although their fundamental PD is effectively limited to 0.1%, experience much higher rates of default due to contagion. Suggested by the graph, there seems to be a set of around 30 events where the system fully breaks down, leading to the default of most banks, irrespectively of their default propensity for fundamental reasons. The benchmark capital requirement does a very good job in limiting the probability of fundamental defaults to 0.1%, which is what causes the relationship between fundamental and total default probabilities to be nonlinear.

In contrast to the benchmark capital, the optimized capital allocation based on our best-performing measure, the Adjacency Eigenvector, has a stronger linear relationship between fundamental and total PD although with heteroskedastic variance that increases with fundamental default probability. Indeed, it looks as if the probability of total default could be approximated by the probability of fundamental default plus a constant. This leads us to a conclusion that imposing a capital requirement based on the Adjacency Eigenvector causes many banks to have a higher probability of default than in

Figure 5: Kernel density of BCs greater than 100 Billion euros



*Note:* The figure zooms into a detail of *unconditional* densities. Densities for BCs smaller than 100 Billion euros are almost identical.

the baseline case, but because these tend to be smaller banks, they impose smaller bankruptcy costs on the system as a whole. The reallocation based on the Closeness does this less. There are more banks with a lower default probability in the post-contagion world than with the Adjacency Eigenvector based allocation. The Closeness allocation induces less of a linear relationship between fundamental and total default probabilities. The lesson here is much the lesson of [Figure 3](#): the sacrifice of a lot of small banks to reduce the system losses caused by default of the few banks which are most costly. The additional information of these diagrams is that the same banks sacrificed in the pre-contagion phase are those banks sacrificed in the contagion phase. This relationship is strongest for our best-performing allocation basis, the Adjacency Eigenvector.

In order to get the full picture, we now focus on the distribution of bankruptcy costs. We provide in [Figure 5](#) the tail distribution of BCs for benchmark capital and for capital based on the Adjacency Eigenvector and Closeness, plotted for BCs exceeding 100 Billion euros (note that the lines are ordinary densities plotted in the tail; they are not tail-conditional). One is cautioned not to place too much emphasis on the BCs being multi-modal. The default of an important bank might trigger the default of smaller banks which are heavily exposed to it, creating a cluster.

Several things are apparent from [Figure 5](#). Both Closeness and Adjacency Eigenvector perform better than the benchmark in precisely those catastrophic meltdowns where we would hope they would

help. The benchmark is dominated by both reallocations at each of the three modes exhibited by the benchmark allocation. Second, the optimal Adjacency Eigenvector reallocation dominates Closeness although to a lesser extent than the savings from benchmark allocation. Our optimal capital rule, based on the Adjacency Eigenvector, shows the best performance in the area where the banking authority would like capital requirements to have teeth. Finally, all rules perform equally poorly for extreme BCs of greater than 780 Billion. When the catastrophe is total, allocating an amount of capital that is too small does not make much difference. Almost the entire capital is used and plenty of banks fail.

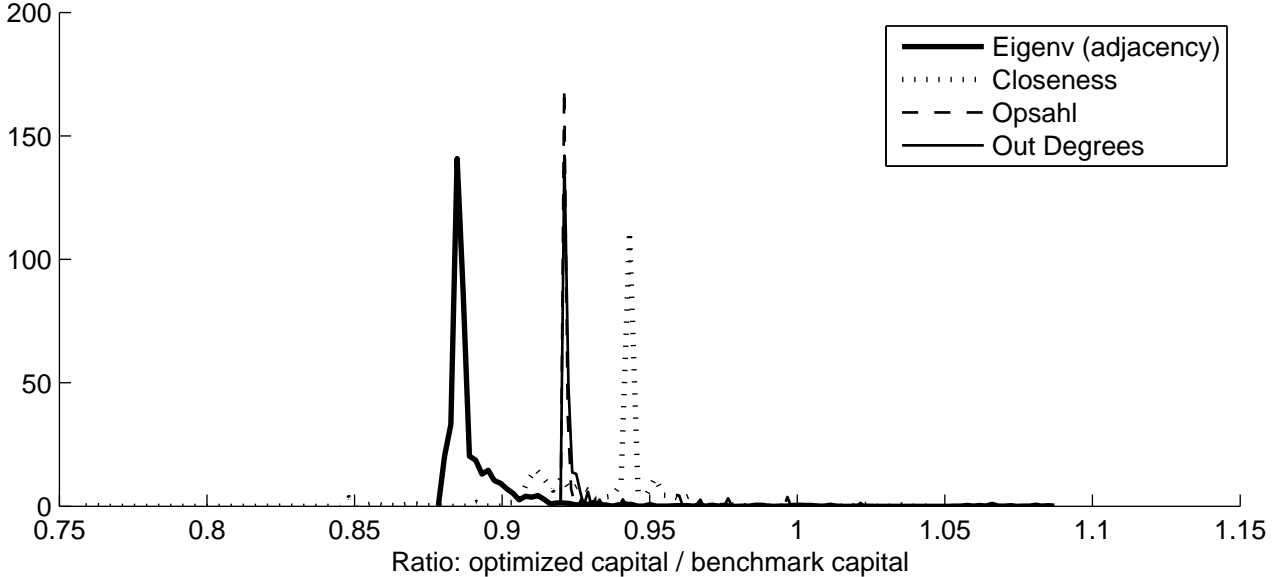
At this point the question arises whether a capital reallocation combining two centrality measures might perform better than one based on a single measure. Indeed, it could not perform worse, by definition. However, [Figure 5](#) shows one of the problems with this logic. Although the two centrality measures perform differently in the end, and despite their low correlation compared with other combinations of centrality measures, the way in which they reshape the loss tail in [Figure 5](#) is rather similar though; in a sense, there is a lack of “orthogonality” in their impact on the loss distribution. As a consequence, the resulting optimum from the combined allocation rule leads only to a 0.2% improvement in expected BCs. We also combined other centrality measures and never observed an improvement; all optimal choices were boundary solutions that put full weight on only one of the two centrality measures.

Finally, [Figure 6](#) shows, under various capital allocations, bank individual ratios of centrality based capital divided by VaR based benchmark capital. The first observation for all of the measures is that ratios involve a cut to the capital requirement for the vast majority of banks, and the cut is highly concentrated at the same amount within the centrality measure. Each centrality measure cuts a different amount for the majority of the banks, except that Opsahl and Out-Degree have very similar effects in the distribution of their capital requirements. The Adjacency Eigenvector cuts most drastically by 12% of benchmark capital and the Closeness is least drastic at a 5% decrease for the vast majority of banks.

To conclude, we infer for each allocation that the capital reallocation is a comparably “mild” modification of the benchmark case for almost all of the banks because the extra capital allocated to some banks is not excessive. Another intuitive inference is that the other centrality measures might mis-reallocate capital. For example, let’s say that benchmark capital for a bank represents 8% of the total assets. Forcing this bank to hold 13 times more capital under the new allocation rule would mean it would have to hold more capital than total assets. Due to this mis-allocation likelihood, we infer that more constraints should be set in order to achieve better performing allocations using centrality measures. We leave this extra mile for future research. Clearly, our general approach to capital reallocation is focused on a few particular banks for each of the centrality measures, simply because the measures focus on them. Beyond that, we cannot say much without revealing confidential information about individual banks.

For most of our centrality-based capital reallocations, there is a reduction of the expected BCs in the whole system, at least over a range of capital redistributed. In the allocations we examined most

Figure 6: Relative changes of required capital for different capital allocations



*Note:* The horizontal axis is capital under the optimal capital allocations (on the various centrality measures) divided by capital in the VaR based benchmark case. The maximum is truncated at the 99.5% quantiles.

closely, these new requirements function on the same principle of shifting capital away from those banks that were presumably small and had little effect on contagion towards a few banks that had a large effect. The shift has the outcome one might expect: more banks are likely to fail in the fundamental defaults phase of our experiment, even in those instances where total costs due to bankruptcy do not increase during this phase. The less expected outcome is that even in the post-contagion phase, more banks fail under the capital reallocation. However, failed banks have a lower systemic cost in terms of bankruptcy, on average, than those banks that are saved through the reallocation. Under some circumstances, this is true even in the pre-contagion phase of defaults. Our two most effective reallocations, based on Adjacency Eigenvector and Closeness, give considerable savings in terms of total bankruptcy costs: 14.6% and 10.7%, respectively. The reallocations based solely on these measures perform just about as well as when the capital requirement is based on a combination of both.

## VII. Conclusion

In this paper we present a tractable framework that allows us to analyze the impact of different capital allocations on the financial stability of large banking systems. Furthermore, we attempt to provide some empirical evidence of the usefulness of network-based centrality measures. Combining simulation techniques with confidential bilateral lending data, we test our framework for different capital reallocations. Our aim is twofold. First, to provide regulators and policymakers with a stylized frame-

work to assess capital for systemically important financial institutions. Second, to give a new direction to future research in the field of financial stability using network analysis.

Our main results show that there are certain capital allocations that improve financial stability, as defined in this paper. Focusing on the system as a whole and defining capital allocations based on network metrics produces results that outperform the benchmark capital allocation which is based solely on the portfolio risk of individual banks. Our findings come with no surprise when considering a stylized contagion algorithm. The improvement in our capital allocations comes from the fact that they take into account the “big picture” of the entire system where interconnectedness and centrality play a major role in triggering and amplifying contagious defaults. As for the best network measure, we find that the capital allocation based on Adjacency Eigenvector dominates any other centrality measure tested. These results strengthen the claim that systemic capital requirements should depend also on interconnectedness measures that take into account not only individual bank centrality but also the importance of their neighbors. In the optimal case, by reallocating 12% of capital based on Eigenvector centrality, expected system losses (measured by expected bankruptcy costs) decrease by almost 15% from the baseline case.

As shown by [Löffler and Raupach \(2013\)](#), market-based systemic risk measures can be unreliable when they assign capital surcharges for systemically important institutions (as can other alternatives such as the systemic risk tax proposed by [Acharya, Pedersen, Philippon, and Richardson \(2012\)](#)). What we propose in this paper is a novel tractable framework to improve system stability based on network and balance sheet measures. Our study complements the methodology proposed by [Gauthier et al. \(2012\)](#). As in that paper, we take into account the fact that market data for all financial intermediaries does not exist when dealing with large financial systems. Instead we propose a method that relies mainly on the information extracted from a central credit register.

We are not providing further details on how capital reallocation in the system could be implemented by policymakers. This aspect is complex, and the practical application involves legal and political issues. Further, we assume the network structure to be exogenous. In reality, banks will surely react to any capital requirement that is based on measurable characteristics by adjusting the characteristics. Such considerations are beyond the scope of the paper. Future research is needed to measure the determinants of endogenous network formation in order to make a more precise statement about the effects of a capital regulation as suggested in this paper.

Further future research could go in several directions. For example, bailout rules used for decisions on aid for banks that have suffered critical losses could be based on information gained from centrality measures. How this bailout mechanism would be funded and the insurance premium assessed to each bank are central to this line of research. Finally, the method can be extended by including information on other systemically important institutions (e.g., insurance companies or shadow banking institutions), although further reporting requirements for those institutions would be necessary.

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