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# Time-Varying Neutral Interest Rate—The Case of Brazil

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### Time-Varying Neutral Interest Rate—The Case of Brazil

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### Abstract

Emerging markets have experienced a sizeable decline in their neutral real interest rates until recently. In this paper we try to identify the main factors that contributed to it, with a focus on Brazil. We estimate an interval for Brazil's time-varying neutral rate based on a range of structural and econometric models. We assess the implications of incorrectly estimating a time-varying neutral rate using a small structural model with a simple monetary policy instrument rule. We find that policy prescriptions are very different when facing uncertainty of neutral rate and of output gap. Our result contrasts sharply with Orphanides (2002), suggesting that the best response to neutral rate uncertainty is to ensure policy remains highly sensitive to inflation and output variations.

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## I. INTRODUCTION

Knut Wicksell (1898) was probably the first economist to try and define the neutral real interest rate (henceforth the "neutral rate"). His interpretation was that "[it] is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor lower them." The profession still thinks of the neutral rate in a similar way as the interest rate consistent with output at potential and inflation remaining stable. This is not just a theoretical curiosity; the neutral rate is an important operational benchmark against which to measure the stance of monetary policy, playing a pivotal function in some rules-based regimes (Laubach and Williams, 2003).

Policymakers that use some concept of the neutral rate in setting policies face considerable challenges, however. First, the neutral rate cannot be seen nor measured and instead has to be estimated, often with a low degree of confidence. Second, while often thought to reflect slow moving structural factors, notably saving and investment behavior over the medium- to long-run, neutral rates can and do change. In particular, emerging market economies undergoing rapid structural change may experience large shifts in the neutral rate, further complicating their use as a benchmark for monetary policy.

Our definition of the neutral rate in this paper is based on the nominal policy rate or the targeted interest rate—for some countries, the overnight interbank rate—deflated by expected inflation. In other words, we typically focus on the neutral "ex-ante" short-term real rate, recognising that that there is likely to be something of a maturity mismatch between an overnight interest rate and inflation expectations that are often expressed for the coming 12 months.[2] It is possible to think of the neutral rate as a short-, medium- or long-run concept (Archibald and Hunter, 2001). We define the neutral rate here to be invariant to both the cycle and factors that do not alter long-term saving and investment patterns. As such, we consider it to be a slow moving benchmark against which to set monetary policy through the business cycle.

In this paper, we infer some stylized facts about neutral rates across a large sample of emerging markets and identify factors that may have caused their decline. Our analysis has a broad focus, but we highlight the challenges facing Brazil, which has experienced one of the largest decreases in the neutral real interest rate over the last decade—due to both domestic and external reasons—but continues to present one of the highest rates of interest among emerging market economies. In particular, we identify an interval for Brazil's neutral rate based on a range of structural and econometric models. Our innovation is to contrast these approaches and discuss their suitability for an economy undergoing rapid structural changes.

A second contribution is to assess the implications of incorrectly estimating a time-varying neutral rate using a small structural model with a simple monetary policy instrument rule. This is a topic that has attracted scant attention in the literature, with much of the focus previously turned to uncertainty regarding potential output (e.g. Erhmann and Smets, 2003)

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[2] For a few emerging markets, the paucity of data requires the use of actual (instead of expected) inflation.

or the natural rate of unemployment. We recognise that neutral rates and potential output growth are linked, but we find that the policy prescriptions are very different when facing uncertainty of either variety.

The plan of this paper is as follows: Section II highlights some stylized facts about the declining neutral rates in emerging markets. Section III offers a concise review of the relevant literature. Section IV presents estimates of the neutral real interest rate from consumption-based models calibrated for Brazil. Section V explores several econometric techniques used to generate time-varying estimates of Brazil's neutral real interest rate. To assess whether these estimates are consistent with theory, section VI compares them to those from Section IV and discusses the implications for monetary policy based on a small macro model calibrated for Brazil. Concluding remarks are provided in section VII.

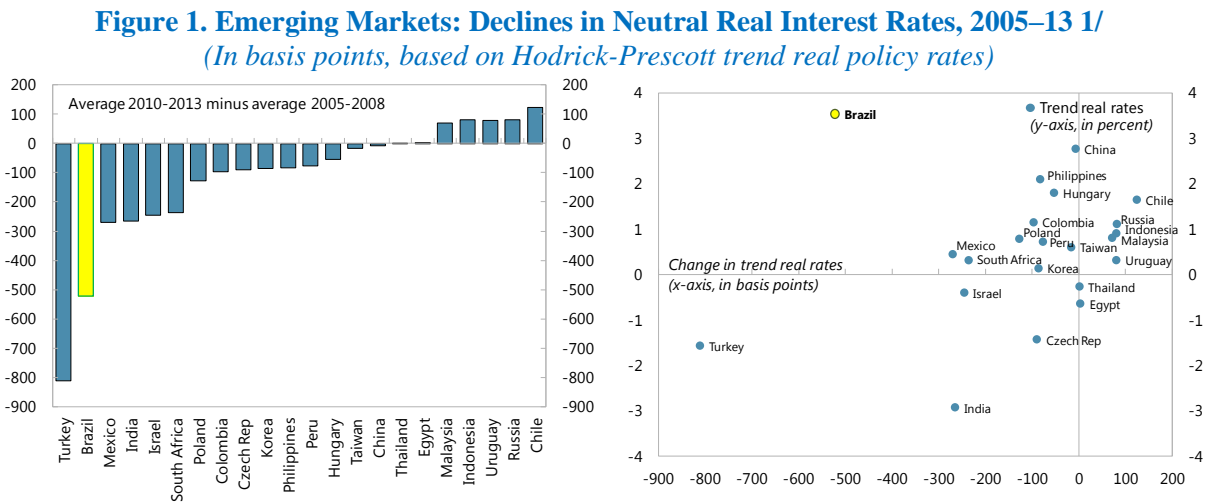## II.   STYLIZED FACTS ABOUT NEUTRAL RATES IN EMERGING MARKETS

Actual real interest rates are typically thought to fluctuate around the unobservable neutral rate—i.e., the real interest rate compatible with a closed output gap and with stable inflation. In this section, we use this assumption to infer some facts about neutral rates in emerging markets, in terms of how they might have changed, how they co-move, and what might be the common factors that explain such comovement. We use the terms "trend rate" and "neutral rate" interchangeably in this section.

Our first observation is that neutral rates appeared to decline across most emerging markets (henceforth "EMs") between 2002 and 2011. This is suggested by the steady drop in trend real policy rates (henceforth "trend rates") obtained from the application of statistical filters. In Figure 1 (left panel), the trend is filtered from nominal policy interest rates deflated by prevailing consumer price inflation—hence, this is by construction an "ex post" proxy for the neutral rate that allows us to use a larger sample of countries (some of which do not have a long history of inflation expectation surveys). Brazil's trend rate has dropped near 500 basis points (bps) between 2010 and 2013 compared with the average of the previous 4-year period. The only country with a larger drop in the sample was Turkey (around 800 bps).

After reaching near 10 percent during the currency crisis of 2002–03, Brazil's strong track record of macroeconomic stabilization, underpinned by successful inflation targeting and fiscal policy frameworks, have surely played a key part in allowing trend rates to fall substantially, including by reducing sovereign risk premia and raising domestic savings. Notwithstanding this sizeable decline, Brazil's trend real rate remains the highest in our EM sample. This is illustrated in the right panel of Figure 1, which plots the level of the trend real rate (2010–13 average, in percent) against the change in the trend real rate between the two four-year periods surrounding the global financial crisis (2005–08 vs. 2010–13).

Assuming less than perfect global financial integration, trend real rates in EMs should be determined, in part, by domestic factors, including saving and investment demand,  as well as the level of the "world neutral rate" which reflects common factors like "excess" global

savings and unconventional monetary policies in the US and Europe.[3] To disentangle these effects, we carried out a principal components decomposition of actual (i.e., unfiltered) de-meaned real short-term interest rates for the countries in our sample and find that two factors explain about 45 percent of the common variation in real rates.

**Figure 1. Emerging Markets: Declines in Neutral Real Interest Rates, 2005–13 1/**
*(In basis points, based on Hodrick-Prescott trend real policy rates)*



Sources: Thomson Datastream, Haver, and authors' calculations
1/ For Turkey, the neutral rate is based on overnight rates until May 2010, and on 1-week repo rates afterwards.

Figure 2 shows the extracted principal components (PCs), with the first PC—accounting for about ¼ of the common variance and labeled as the "common trend"—showing a clear declining tendency between 2002–10 indicative of global factors exerting persisting downward pressure on emerging market interest rates. This common trend component has been broadly stable since 2010. The second PC accounted for a further $\frac{1}{5}$ of variance and appears to be cyclical as it shows a tendency to fluctuate around zero. This "common cycle" component declined during 2011–12 but started rising during Q2 2013 at about the same time that markets began to price-in an unwinding by the the U.S. Federal Reserve of quantitative easing.[4] This suggests that, while domestic factors surely play a role in neutral rate formation in emerging markets, common global factors, both trend and cyclical, are likely to have exerted an important influence in recent years.

We conclude from this statistical overview that neutral real interest rates did indeed decline substantially, particularly between 2002–11. Country-specific explanations may account for declines in many cases, but common global factors have also contributed importantly to this outcome. These common factors have been both persistent and transient, with cyclical factors

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[3] Gourinchas and Jeanne (2012) argue that the excess demand for safe assets has resulted in a decline in the world neutral real interest rate.

[4] Perrelli, Roache, and Góes (2014) discuss the impact of the tapering tantrum on the slope of the yield curves of emerging market economies.

appearing to have been more important recently. However, these mechanical approaches can only bring us so far in helping us understand how and why neutral rates may have changed and what, as a consequence, might be the implications for monetary policy. In the next section, we briefly survey the existing literature to provide the foundation for a more comprehensive analytical framework.

**Figure 2. Emerging Markets: PCA Decomposition of Real Policy Rates, 2002–13 1/**
*(normalized with standard deviation = 1)*
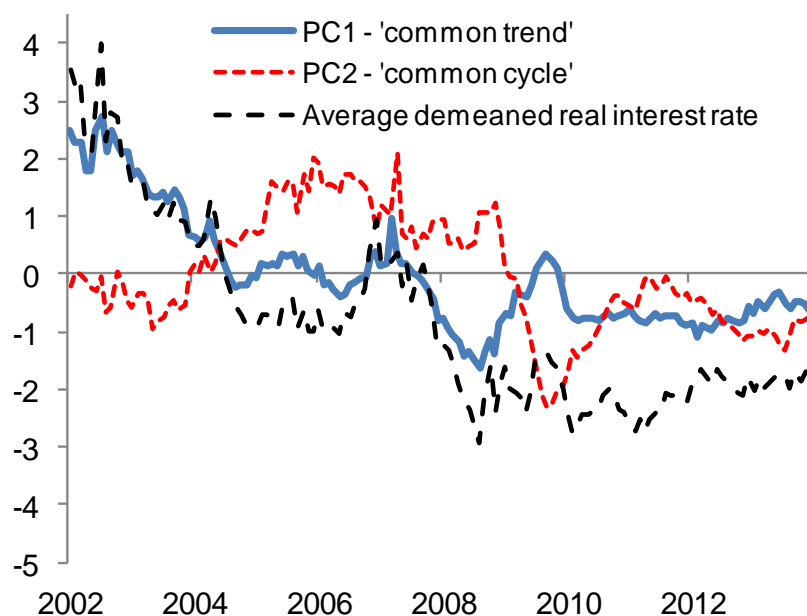


Sources: Thomson Datastream, Haver, and authors' calculations
[1]/ Calculated from a sample of 17 emerging markets that had available real ex-post overnight interbank interest rate data from January 2001-December 2013.

## III. A BRIEF REVIEW OF THE LITERATURE ON ESTIMATING NEUTRAL RATES

Notwithstanding Wicksell's (1898) seminal theoretical work and the pivotal role played by the neutral rate in Taylor's (1993) celebrated monetary policy rule, the econometric literature on this topic has flourished only recently.

Early estimates of the U.S. neutral rate date back to Bonfim (1997, 2001), and Kozicki and Tinsley (2001)—the latter based on forward rates from U.S. Treasury bonds. Laubach and Williams' (2003) Kalman filtered estimates of the U.S. neutral rate spurred widespread applications of statistical filters and state space models among central banks, while highlighting the inherent statistical uncertaintites from these approaches. Tao (2005) provided real time estimates for the U.S. neutral rate. An early review of these methods is presented in Giammarioli Valla (2004). Basdevant, Björksten, and Karagedikli's (2004) insightful approach combines the slope of the yield curve with a state space framework to estimate New Zealand's neutral interest rate. Bernhardsen and Gerdrup (2007) discusses the

estimation of short- and long-run equilibrium interest rates on the determination of the Norwegian neutral rate.[5]

For Brazil, Miranda and Muinhos (2003) and Muinhos and Nakane (2006) provide early estimates of the neutral rate, while Neto and Portugal (2009) consider the intial years of the inflation target regime (1999–2005) —a period marked by elevated economic and financial volatility, very high nominal interest rates, widespread indexation of wages and contracts, and developing inflation expectations surveys. Since then the Brazilian economy has changed substantially, and so did its neutral interest rate. Perrelli (2012) provides an early attempt to model Brazil's neutral real interest rate in the post-global financial crisis era.

Several academic works have attempted to explain Brazilian persistently high levels of interest rates. Arida, Bacha, and Resende (2005), Gonçalves, Holland, and Spacov (2007), and Bacha, Holland, and Gonçalves (2009), worked with the hypothesis that jurisdictional uncertainty on long term contracts has led to high interest rates—to certain extent, as a substitute for dollarization—in the Brazilian economy. Barbosa (2006) highlights the role of public indebtedness, while Segura-Ubiergo (2012) discusses the low level of domestic savings as one of the key factors behind Brazil's high interest rates.

## IV.  Structural Estimates of the Neutral Real Interest Rate in Brazil

In this section, we use structural models to assess Brazil's neutral real interest rate. Our estimates are based on the calibration of the first-order condition for optimal inter-temporal consumption of a representative agent. Our calibrations use Brazil's actual growth rate of per capita income and its volatility, along with a range of behavioral parameters typically adopted in this literature. One important source is Issler and Piqueira (2000) which provides estimates of behavioral parameters in the problem of consumption optimization in Brazil, using different types of utility functions.

We begin by assuming that the representative individual has a constant relative risk aversion (CRRA) utility function given by:

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad \text{with} \quad \gamma > 0 \tag{1}$$

where $C$ is per capita consumption, $\gamma$ is the coefficient of relative risk aversion, and $t$ is a time indicator. The consumer maximizes her lifetime utility discounting future consumption by $\beta$, subject to a budget constraint where financial assets $B$ grow as a function of per capita income ($Y$), consumption ($C$), and the real return on assets ($1+r$):

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[5] Applications to Latin American countries include Fuentes and Gredig (2008) and Magud and Tsounta (2012).

$$\max E_t \sum_{t=0}^{\infty} \beta^t u(C_t) \quad \text{subject to} \quad B_{t+1} = (1 + r_t)(B_t + Y_t - C_t) \tag{2}$$

In the steady state, $Y_t = C_t$ for all $t$ such that the first order condition can be rewritten simply as

$$(1 + r) = E_t \left[ \frac{\beta Y_{t+1}^{-\gamma}}{Y_t^{-\gamma}} \right]^{-1} \tag{3}$$

As equation (3) suggests, the first order condition can be written in terms of the steady state growth rate of per capita income ($g$), since $Y_{t+1} = (1 + g)Y_t$. Assuming log normal $g$, the Euler equation can be approximated as follows:

$$\ln(1 + r^*) \cong -\ln \beta + \gamma E_t(g) - \frac{1}{2}\gamma^2 var_t(g) \tag{4}$$

where $r^*$ is the neutral rate and $var(g)$ is the variance of per capita growth.

Equation (4) indicates a positive relationship between the neutral rate and the expected rate of growth of per capita income, consistent with the stylized fact that trend interest rates in advanced economies are lower than in faster growing emerging markets. But while estimating $E_t(g)$ and $var_t(g)$ is a relatively easy task, pinning down values for behavioral parameters like $\beta$ and $\gamma$ in rapidly changing emerging markets is more challenging and subject to considerable uncertainty.

In the case of Brazil, we consider the average growth rate of per capita income in two sub-periods (2005-2008 versus 2010-2013), estimated respectively at 3.5 percent and 2.5 percent per annum. We also compute the standard deviation of the per capita income growth for the two sub-periods, which printed 1.4 percent and 2.8 percent, respectively, highlighting the higher volatility of the recent period. For the discount factor $\beta$ we use values typically adopted in the calibration of Ramsey models—in the range of 0.97 to 0.99—and also used in the literature of other emerging markets (e.g., Fuentes and Gredig (2008) uses a similar approach for the case of Chile). For risk aversion $\gamma$, the literature suggests a value in the range of 1 to 2. This exercise then suggests a neutral rate in a wide percent range, which averaged across parameters yields a NRIR of about 7¼ percent for the period 2005-2008 and 5¾ percent for the 2010–13 years (Text Tables).

| Brazil: NRIR from consumption model, 2005-2008 | | | | | Brazil: NRIR from consumption model, 2010-2013 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | \gamma | | | | | \gamma | | | |
| \beta | 1.0 | 1.5 | 2.0 | Avg | \beta | 1.0 | 1.5 | 2.0 | Avg |
| 0.970 | 6.5 | 8.3 | 10.0 | 8.3 | 0.970 | 5.5 | 6.7 | 7.9 | 6.7 |
| 0.975 | 6.0 | 7.8 | 9.5 | 7.8 | 0.975 | 5.0 | 6.2 | 7.4 | 6.2 |
| 0.980 | 5.5 | 7.2 | 9.0 | 7.2 | 0.980 | 4.5 | 5.7 | 6.9 | 5.7 |
| 0.985 | 5.0 | 6.7 | 8.5 | 6.7 | 0.985 | 4.0 | 5.2 | 6.4 | 5.2 |
| 0.990 | 4.5 | 6.2 | 8.0 | 6.2 | 0.990 | 3.5 | 4.7 | 5.8 | 4.7 |
| Avg | 5.5 | 7.3 | 9.0 | 7.2 | Avg | 4.5 | 5.7 | 6.9 | 5.7 |
| Per capita growth (2005-2008): | | | 3.5% | | Per capita growth (2010-2013): | | | 2.5% | |
| Std dev of pc growth (2005-2008): | | | 1.4% | | Std dev of pc growth (2010-2013): | | | 2.8% | |

If consumer behavior is subject to habit persistence, then our estimates of the neutral rate may need to be lowered. The intuition is that consumption today is determined, in part, by consumption in past periods; in other words, consumption may be a slow-moving process (see Fuhrer (2000)). To satisfy (4), which predicts rising consumption in line with per capita income growth, the real interest rate needs to be higher to encourage consumers to save more in the current period and consume more in the future. This consideration may be particularly relevant for Brazil where historically high real interest rates would imply implausibly impatient (small $\beta$) and/or very risk averse households (large $\gamma$) in equation (4). Hence we incorporate habit persistence through the parameter $\theta$ in the first-order condition:

$$\ln(1 + r^*) \cong -\ln\beta + \gamma E_t(g) - \frac{1}{2}\gamma(1 - \theta) \tag{5}$$

We estimate (5) for Brazil using the same parameters as above and calibrate the coefficient of habit formation $\theta$ to roughly approximate the estimates of the trend real interest rate from the statistical filters discussed above, as a proxy for the neutral real interest rate. This results in a habit parameter ($\theta$) of about 0.97 for the period 2010-2013, compared to a habit parameter close to one for the former period.[6] This model suggests that a decline in habit persistence may have played some role in lowering neutral rate, although it cannot fully explain the decline in Brazil's neutral real interest rate between these two periods. Under such framework, the neutral real interest rate ranges between 3 and 7 percent, with an average of 5 percent—very close to the empirical findings discussed next (Text Tables).

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[6] Note that when $\theta$ approaches one, the impact of habit formation on the CRRA utility function vanishes, and this model delivers a NRIR of approximately 7¼ percent—similar to the case we presented earlier. Therefore, the trend rate experienced by Brazil in 2005-2008 (8¾ percent) cannot be fully explained by habit persistence.

| Brazil: NRIR from consumption with habit, 2005-2008 | | | | | Brazil: NRIR from consumption with habit, 2010-2013 | | | |
|---|---|---|---|---|---|---|---|---|
| | \gamma | | | | | \gamma | | |
| \beta | 1.0 | 1.5 | 2.0 | Avg | \beta | 1.0 | 1.5 | 2.0 | Avg |
| 0.970 | 6.5 | 8.2 | 9.9 | 8.2 | 0.970 | 4.0 | 4.5 | 5.0 | 4.5 |
| 0.975 | 6.0 | 7.7 | 9.4 | 7.7 | 0.975 | 3.5 | 4.0 | 4.5 | 4.0 |
| 0.980 | 5.5 | 7.2 | 8.9 | 7.2 | 0.980 | 3.0 | 3.5 | 4.0 | 3.5 |
| 0.985 | 5.0 | 6.7 | 8.4 | 6.7 | 0.985 | 2.5 | 3.0 | 3.5 | 3.0 |
| 0.990 | 4.5 | 6.2 | 7.9 | 6.2 | 0.990 | 2.0 | 2.5 | 3.0 | 2.5 |
| Avg | 5.5 | 7.2 | 8.9 | 7.2 | Avg | 3.0 | 3.5 | 4.0 | 3.5 |

Per capita growth (2005-2008): 3.5% Per capita growth (2010-2013): 2.5%
Coefficient of habit formation: 0.999 Coefficient of habit formation: 0.970

Structural consumption models are appealing in the sense that they provide a solid bridge between theory and practice and deliver somewhat plausible results for Brazil. These models can also inform important policy discussions regarding economic growth in emerging economies—faster growth is expected to be matched by higher neutral rates unless behavioral changes provide offseting effects. In turn, this is related to the patterns of saving and investment— neutral rates in low-saving countries (such as Brazil, Turkey, and South Africa) are typically higher than in high-saving peers (like Indonesia and Malaysia). Therefore, increasing household savings in Brazil would contribute to a lower neutral rate.

**Table 1. Brazil: Estimates of the Neutral Rate Based on Consumption Models**

| Model | Average 2005–08 | Average 2010–13 |
|---|---|---|
| Basic CRRA utility function (Ramsey) | 7¼ percent | 5¾ percent |
| CRRA utility with habit formation | 7¼ percent | 3½ percent |

Source: Authors' calculations

Last, but not least, the careful calibration of consumption models can deliver results close to those found empirically through the application of model-free methods, such as statistical filters, as we discuss in the next section.

## V.   ECONOMETRIC ESTIMATES OF NEUTRAL RATE IN BRAZIL

One important drawback of the structural models explored in the previous section is that their key parameters are not directly observable, while estimates are often subject to considerable uncertainty. An alternative approach is to apply a range of econometric techniques to evaluate the likely range for neutral rates across a sample of emerging markets and over time, with a particular focus on Brazil.

### A.   Statistical Filters

Standard filters are useful for disentangling the real interest rate cycle from its secular trend. We apply a range of filters—including the Hodrick-Prescott, Ravn-Uhlig, and Christiano-Fitzgerald time-varying filters—to the ex-post real interest rates in a sample of 24 emerging

markets. Table 2 presents the average from these filters for each country, and contrasts the rates across different sub-periods.

A simple average of the neutral real interest rates for this sample suggests a drop of more than 200 basis points, from about 2.9 percent early in the previous decade to approximately ¾ percent in the four-year period after the global financial crisis. For Brazil, specifically, neutral rates have declined significantly during the sample period, from an average 9¼ percent to an average of 3½ percent—a 575 basis points decline. In spite of this remarkable progress towards international standards, Brazil so far remains with the highest neutral real interest rate in our sample of emerging markets (Figure 3).

**Table 2. Emerging Markets: Estimates of the Ex-Post Neutral Rate from Statistical Filters**

| Country | 2002-2004 | 2005-2008 | 2010-2013 | Change from 2002-2004 to 2010-2013 (in basis points) |
|---|---|---|---|---|
| Brazil | 9.3 | 8.8 | 3.5 | -575 |
| Chile | 0.8 | 0.4 | 1.7 | 89 |
| China | 4.3 | 2.8 | 2.8 | -150 |
| Colombia | 0.1 | 2.1 | 1.2 | 101 |
| Czech Rep | 1.2 | -0.5 | -1.4 | -259 |
| Egypt | 4.9 | -0.7 | -0.6 | -552 |
| Hungary | 3.8 | 2.3 | 1.8 | -204 |
| India | n.a. | -0.3 | -2.9 | n.a. |
| Indonesia | 2.0 | 0.1 | 0.9 | -113 |
| Israel | 3.9 | 2.1 | -0.4 | -425 |
| Korea | 0.7 | 1.0 | 0.1 | -57 |
| Malaysia | 0.6 | 0.1 | 0.8 | 19 |
| Mexico | 2.2 | 3.2 | 0.5 | -176 |
| Pakistan | 3.2 | -0.3 | 1.1 | -212 |
| Peru | -0.1 | 1.5 | 0.7 | 80 |
| Philippines | n.a. | 2.9 | 2.1 | n.a. |
| Poland | 4.7 | 2.1 | 0.8 | -386 |
| Russia | 3.7 | 0.3 | 1.1 | -255 |
| South Africa | 4.5 | 2.7 | 0.3 | -422 |
| Taiwan | 1.3 | 0.8 | 0.6 | -69 |
| Thailand | -0.1 | -0.3 | -0.3 | -16 |
| Turkey | 6.8 | 6.5 | -1.6 | -840 |
| Ukraine | 3.2 | -4.4 | 3.1 | -5 |
| Uruguay | n.a. | -0.5 | 0.3 | n.a. |
| **Simple EM Average** | **2.9** | **1.4** | **0.7** | **-211** |

Sources: Haver and authors' calculations

**Figure 3. Brazil: Filter-based Real Policy Rate and Real Interest Gap, 2003–13**
*(In percent; gap equals actual minus trend real rate; negative gap implies monetary easing)*
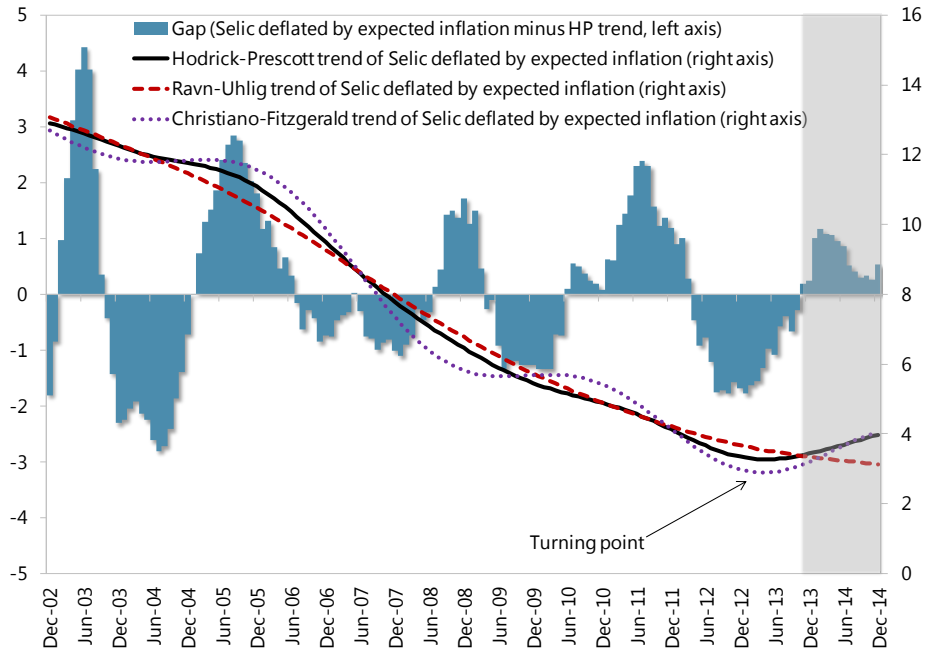


Sources: BCB Focus Survey, Thomson Datastream, Haver, and authors' calculations

There are two main drawbacks to estimating neutral rates using statistical filters. Most filters are unable to reliably disentangle permanent (or very persistent) and temporary fluctuations. In the case of interest rates, the filtered trend may be identifying extended cyclical variation that may not be driven by an underlying structural shift in saving-investment balances, potential output growth, or consumer preferences. Additional information and structure may be required to identify such cases, including, for example, the behavior of inflation. A second issue is that many two-sided filters suffer from end-point bias, which serves only to reinforce the effect of the current period's actual interest rate—which may be strongly affected by cyclical conditions—on estimates of the neutral. For these reasons we present a range of alternative econometric methods in the following sub-sections.

## B.  Yield Curve Models

Interest rates at different maturities across the yield curve add more information to the estimation process, particularly by incorporating market expectations of the path of short-term rates. With "well-anchored" inflation expectations, the average slope of the yield curve should mainly reflect the term premium. When computed over a sufficiently long horizon and between cyclically-equivalent end-points, the average term premium can be considered neutral with respect to the business cycle.[7]

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[7] For further discussion, see for example Basdevant et al, 2004.

Yields on Brazil's long-term 10-year local currency sovereign bonds and short-term 3-month interest rates (extracted from swap contracts) have trended lower during most of the sample period. We estimate the average term premium in Brazil during 2006–13, i.e. the difference in yields between long-term bonds and short-term swaps—to be about 140 basis points with significant cyclical variation (Figure 4).

In this framework, a proxy for the NRIR can be backed out by adding the "excess" term premium—defined as the current term premium minus the sample average—to the trend real short-term interest rate. In the second half of 2013, for example, the trend real interest rate in Brazil floated around 3.4 percent, while its "excess" term premium averaged 110 basis points. Hence, this approach suggests an estimate of Brazil's neutral real interest rate around 4½ percent during that reference period.

**Figure 4. Brazil: Slope of the Yield Curve, 2006–13**
*(Yields in percent; slope of the yield curve in basis points)*
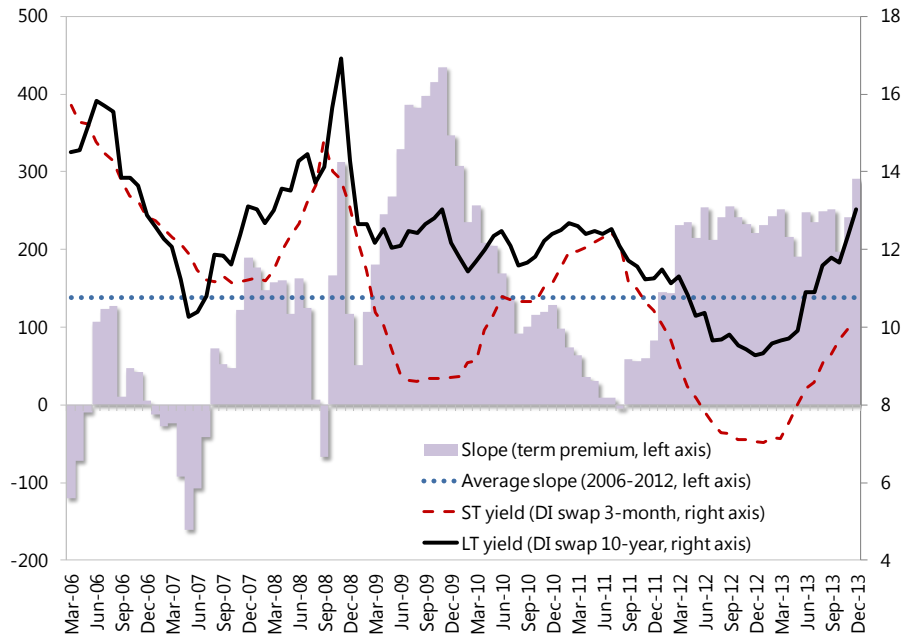


Sources: BCB, Thomson Datastream, Haver, and authors' calculations

This methodology relies on the identification of cyclically-neutral positions to anchor the computation of the average term premium, which may be difficult to verify in such a short sample period. Given the the limited experience of emerging market economies with the duration of business cycles under inflation target regimes, the results are subject to elevated uncertainty.[8] Another complicating factor is the possibly sizeable divergence between market participants' expectations for the path of interest rates extracted from surveys (e.g. Brazil's central bank's Focus survey) and the path priced in futures contracts.

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[8] Moreover, there is also some potential circularity in this approach as the term structure of the interest rate could, in principle, reflect market expectations about future monetary policy decisions—for instance, when futures contracts on interest rate swaps "price in" hikes on policy rates throughout the medium term.

## C. State Space Models

The state space framework is well-suited to estimating neutral rates since they explicitly take account of the existence of latent variables. In the most simple application, it is assumed that short-term and long-term nominal interest rates are determined by a common unobservable time-varying neutral rate, inflation expectations, and idiosyncratic shocks. The long-term rate is also explained by a term premium.

We apply this approach to Brazil by estimating a system of four equations: two signal (observable) equations, and two state (latent) equations. The signal equations for the short-term rate (6) and the long-term rate (7) are estimated as functions of the 12-month ahead expected inflation $(E_t(\pi_{t+12}))$, Brazil's output gap $(Ygap_t)$, the 3-month $(US3m_t)$ and 10-year $(US10y_t)$ yields on U.S. Treasury securities (to capture interest rate parity issues), the Brazilian 5-year CDS $(BRCDS_t)$ spreads (as a gauge for shifts in sovereign risks), and the neutral real interest rate $r_t^*$. The output gap is calculated as as the difference between the central bank's Economic Activity Index (IBC-Br)—a high-frequency real-time gauge of GDP—and an estimate of its trend using a standard filter. We use the yields on Brazil's 10-year local currency sovereign bonds for the long-term interest rate and the 3-month swap rate for the short-term rate. We also control for the time-varying term premium $\alpha_t$ in the long-term rate equation:

$$r_t^{short} = E_t(\pi_{t+12}) + Ygap_t + US3m_t + r_t^* \tag{6}$$

$$r_t^{long} = E_t(\pi_{t+12}) + US10y_t + BRCDS_t + \alpha_t + r_t^* \tag{7}$$

State equation (8) models $r^*$ as a random walk, while state equiation (9) assumes the term premium follows a stationary AR(1) process with a drift:

$$r_t^* = r_{t-1}^* + \varepsilon_{1,t} \tag{8}$$

$$\alpha_t = \gamma_0 + \gamma_1 \alpha_{t-1} + \varepsilon_{2,t} \tag{9}$$

A Kalman filtered neutral rate calculated from the estimates of this model (Table 3 and Figure 5) has declined over the past decade, albeit with significant cyclical variation, ranging between 3 and 5 percent over the last two years. The model estimates are determined mainly by expected inflation, long-term U.S. Treasury yields, and Brazil's CDS spreads:

**Table 3. Brazil: State Space Estimates of the Neutral Real Interest Rate, 2006–13**

| Signal equation: | Expected inflation | Output gap | U.S. 3-month yields | U.S. 10-year yields | CDS spreads |
|---|---|---|---|---|---|
| Short-term rate | 0.721 | 0.028 | 0.112 | | |
| | (0.24)*** | (0.04) | (0.14) | | |
| Long-term rate | 1.421 | | | 0.779 | 0.490 |
| | (0.56)*** | | | (0.22)*** | (0.17)*** |

Sources: BCB, Thomson Datastream, Haver, and authors' calculations
1/ Standard errors in parenthesis: * significant at 10 percent; ** significant at 5 percent; and *** significant at 1 percent.

**Figure 5. Brazil: Neutral Real Interest Rate based on State Space Model, 2006–13**
*(In percent)*
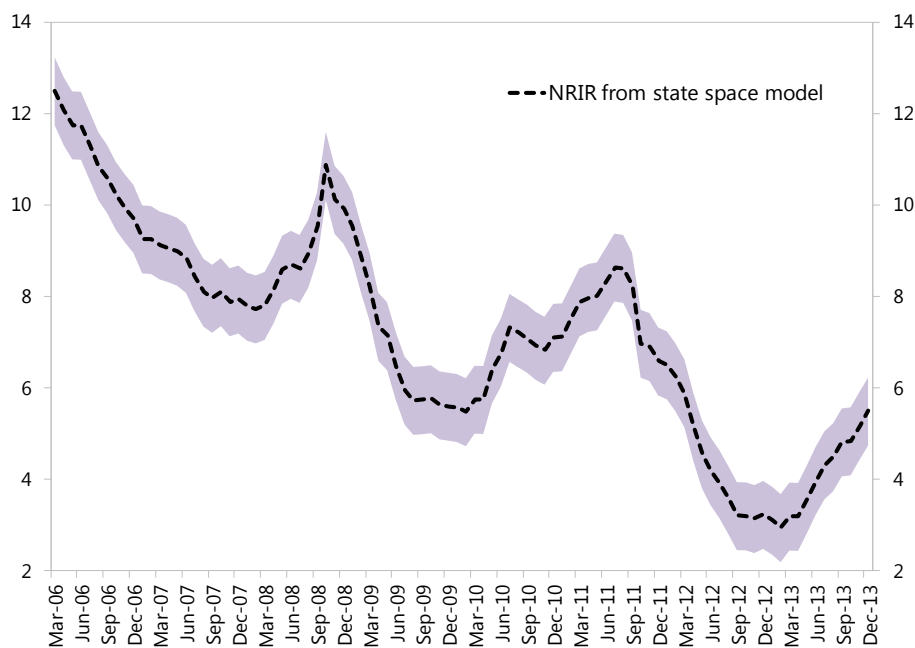


Sources: BCB, Thomson Datastream, Haver, and authors' calculations

## D.    Fundamentals-based Regression Models

Stronger macroeconomic fundamentals may help to explain the decline in Brazil's neutral real interest rate over the past decade suggested by various statistical methods. In this section we estimate two parsimonoius models that try to decompose changes in the neutral rate by fundamental determinants. Our approach follows Bernhardsen and Gerdrup (2007) and Goldfajn and Bicalho (2011), and constitutes a natural extension to the state space system presented in the previous section.

**Long-run Equilibrium Real Interest Rate**

The long-run equilibrium real interest rate is estimated as a function of productivity, inflation outcomes, external vulnerability, sovereign creditworthness, and financial deepening. Specifically, the control variables are:

| | |
|---|---|
| Productivity growth: | Annual growth rate of output per employed person; |
| Inflation gap: | Difference between the actual 12-month consumer's inflation rate (IPCA) and the center of the Brazilian Central Bank's (BCB) inflation target band; |
| Inflation surprise: | Difference between the one-year ahead expected inflation (from the BCB *Focus* survey) and the actual inflation outcome; |

| Sovereign risk: | 5-year credit default swap (CDS) spreads; |
|---|---|
| Public indebtedness: | Ratio of net public debt to GDP; |
| Financial deepening: | Domestic credit to the private sector as a percent of GDP; |
| Global interest rates: | 3-month and 10-year yields on U.S. Treasury securities. |

The coefficient on private credit-to-GDP ratio should be positive because financial deepening that reflects increased supply of savings (whether foreign or domestic) should lower the neutral rate. The coefficients on sovereign risk and public indebtedness are expected to be negative. The interest parity condition implies a positive coefficient on global interest rates. Better inflation performance—smaller inflation gaps and inflation surprises—should lower the neutral rate. Higher productivity growth should increase the demand for funds and raise the neutral rate. Our econometric estimates are largely consistent with these priors, fit the data well, and pass the usual diagnostic tests (Table 4).

We find three main domestic reasons for the fall in the neutral rate: lower public debt, reduced sovereign risk, and an increased supply of saving that has translated into financial deepening (together, the green bars in Figure 6). In contrast, inflation performance has not made a major contribution to a lower neutral rate, at least since 2007. This reflects, in part, that the inflation gap has not narrowed over recent years, while inflation surprises have been largely asymmetric (on the upside), as shown in Figure 7.[9]

Lower global interest rate contributed about 200 basis points to the decline in Brazil's neutral rate during 2010–13 relative to their average contribution in 2005–08. According to this approach, the long-run equilibrium real interest rate in Brazil was running between 2½ and 3 percent by mid-2013. The limited sample size and inherent model uncertainty suggest a wide confidence interval around these figures.

---

[9] This asymmetry has also been highlighted in Roache (2014).

**Figure 6. Brazil: Determinants of Long-Run Equilibrium Real Interest Rate, 2005–13**
*(Contributions to the equilibrium real interest rate, in percentage points, excluding the intercept)*
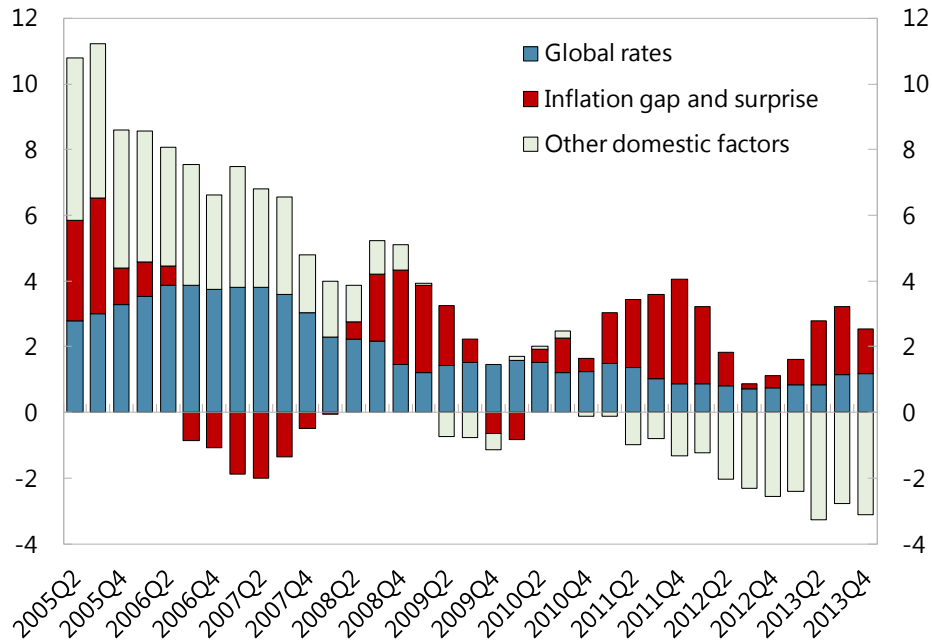


Sources: BCB, Thomson Datastream, Haver, and authors' calculations

**Figure 7. Brazil: Inflation Surprise and Inflation Gap, 2005–13**
*(Period average in percent; surprise is actual inflation minus 12-month ahead expected inflation; gap is actual inflation minus the mid-point of the inflation target band)*
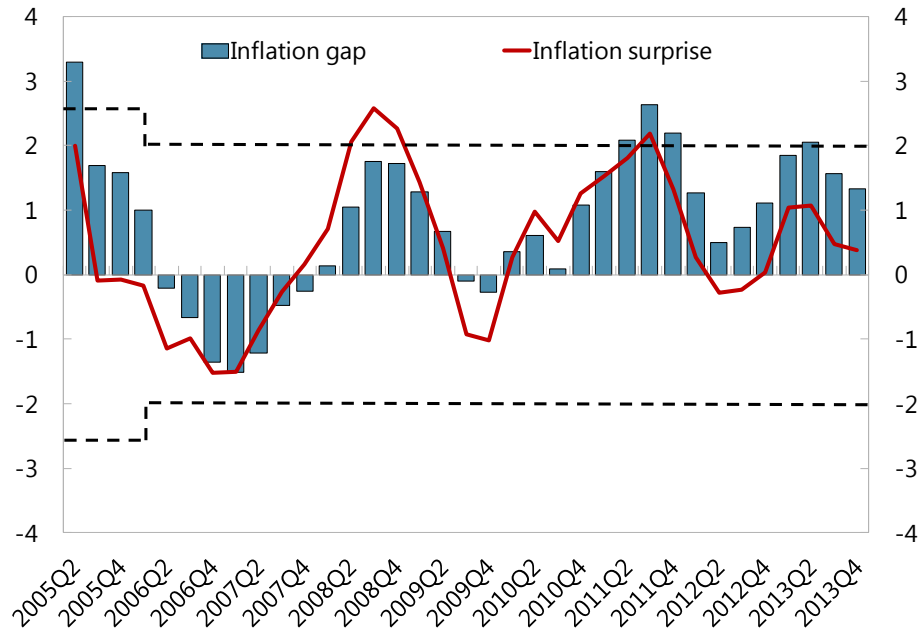


Sources: BCB, Thomson Datastream, Haver, and authors' calculations

**Table 4. Brazil: Determinants of Long-Run Equilibrium Real Interest Rate, 2003–13**

Table: Brazil: Fundamentals-based Cointegrating Regressions, 2003-2013 1/

| | Dependent variable: Real policy rate deflated by 12-month ahead expected inflation 2/ 3/ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (A) | (B) | (C) | (D) | (E) | (F) | (G) | (H) | (I) | (J) | (K) | (L) |
| Linear trend | -0.274 | | | -0.227 | -0.223 | -0.097 | -0.134 | -0.288 | -0.228 | | | |
| s.e. | (0.03)*** | | | (0.02)*** | (0.02)*** | (0.11) | (0.05)** | (0.03)*** | (0.03)*** | | | |
| Global short-term interest rate_t | | 1.486 | | | | | | | | 0.481 | 0.503 | 0.365 |
| s.e. | | (0.41)*** | | | | | | | | (0.02)*** | (0.00)*** | (0.00)*** |
| Global long-term interest rate_t | | | 3.011 | | | | | | | 0.724 | 0.910 | 0.413 |
| s.e. | | | (0.63)*** | | | | | | | (0.07)*** | (0.01)*** | (0.01)*** |
| Inflation gap_t-1 | | | | 0.026 | | | | | | 0.575 | 1.045 | 0.678 |
| s.e. | | | | (0.07) | | | | | | (0.03)*** | (0.00)*** | (0.01)*** |
| Inflation surprise_t-1 | | | | | 0.094 | | | | | 0.524 | 0.431 | 0.645 |
| s.e. | | | | | (0.110) | | | | | (0.03)*** | (0.01)*** | (0.01)*** |
| Private credit/GDP_t-1 | | | | | | -0.210 | | | | -0.134 | -0.086 | -0.178 |
| s.e. | | | | | | (0.15) | | | | (0.01)*** | (0.00)*** | (0.00)*** |
| Public debt/GDP_t-4 | | | | | | | 0.235 | | | 0.111 | 0.141 | 0.163 |
| s.e. | | | | | | | (0.09)*** | | | (0.01)*** | (0.00)*** | (0.00)*** |
| Productivity growth_t-4 | | | | | | | | -0.106 | | | 0.245 | |
| s.e. | | | | | | | | (0.18) | | | (0.00)*** | |
| Sovereign risk_t-4 | | | | | | | | | 0.731 | | | 0.038 |
| s.e. | | | | | | | | | (0.20)*** | | | (0.00)*** |
| Constant | 13.549 | 5.354 | -3.030 | 12.240 | 12.035 | 16.704 | -0.330 | 14.157 | 10.988 | 2.637 | -1.790 | 3.092 |
| s.e. | (0.76)*** | (0.96)*** | (2.31) | (0.46)*** | (0.61)*** | (2.62)*** | (5.08) | (0.91)*** | (1.08)*** | (1.02)*** | (0.13)*** | (0.07)*** |
| Adjusted R2 | 0.83 | 0.28 | 0.51 | 0.79 | 0.80 | 0.82 | 0.82 | 0.82 | 0.84 | 0.94 | 0.95 | 0.97 |
| Long-run variance | n.a. | 22.77 | 16.30 | 1.44 | 3.29 | 2.28 | 1.60 | 3.68 | 1.96 | 0.02 | 0.00 | 0.00 |
| Engle-Granger coint. test (prob) | n.a. | 0.01 | 0.00 | 0.02 | 0.11 | 0.00 | 0.00 | 0.12 | 0.03 | 0.03 | 0.08 | 0.03 |
| Sample size | 44 | 43 | 43 | 42 | 42 | 42 | 39 | 36 | 35 | 39 | 36 | 35 |

1/ Standard errors in parenthesis: * significant at 10%; ** significant at 5%; and *** significant at 1%.

2/ Models B thru L are cointegrating regressions (Fully Modified Least Squares).

3/ Long-run covariance estimates use prewhitening with lags based on AIC, Bartlett kernel, and Newey-West automatic bandwidth selection.

4/ Probability of rejecting the null hypothesis of no cointegration when the null hypothesis is true.

Source: BCB, Thomson Datastream, Haver, and authors' calculations

## Short-Run Equilibrium Real Interest Rate

The short-run equilibrium rate can diverge substantially from our notion of the long-run neutral rate, factor that clearly should be taken into account by policymakers. Following seminal work by Rudebusch and Svensson (1999), adapted by Bernhardsen and Gerdrup (2007) for Norway, and by Goldfajn and Bicalho (2011) for Brazil, we estimate a simplified IS model that helps to disentagle short-term interest rate movements from its long-term (neutral) trajectory.

Our model is novel in the sense that it incorporates the impact of the public sector on the financial deepening process—via the the Brazilian Development Bank (Banco Nacional de Desenvolvimento Econômico e Social, BNDES)—in the gap equation:

$$Y_t - \tilde{Y}_t = \alpha(Y_{t-1} - \tilde{Y}_{t-1}) + \beta(Z_t - \tilde{Z}_t) + \gamma(r_{t-2} - \tilde{r}_{t-2}) +$$

$$\delta(e_t - \tilde{e}_t) + \rho(G_t - \tilde{G}_t) + \theta(L_t - \tilde{L}_t) + \varepsilon_t$$

(10)

where:

- $Y_t - \tilde{Y}_t$ is the Brazilian output gap (actual minus potential output);

- $Z_t - \tilde{Z}_t$ is the world's output gap (actual minus potential output);

- $r_{t-2} - \tilde{r}_{t-2}$ is the real interest rate gap (actual minus the long-run equilibrium interest rate estimated by the authors using model J in Table 4);

- $e_t - \tilde{e}_t$ is the exchange rate gap (actual minus its long-term trend estimated by the authors using statistical filters);

- $G_t - \tilde{G}_t$ is the gap of the central government consumption (actual minus its long-term trend estimated by the authors using statistical filters); and

- $L_t - \tilde{L}_t$ is the gap of BNDES lending (actual minus its long-term trend estimated by the authors using statistical filters).

The estimated coefficient on the global output gap is positive and the coefficients on the real interest rate gap and the exchange rate gap are both negative, as expected (Table 5). The government consumption gap is not statistically significant but the effect of BNDES lending is positive and significant, suggesting a procyclical role of public sector lending during the sample period considered in this paper.

**Table 5. Brazil: Determinants of Short-Run Equilibrium Real Interest Rate, 2005–13**

| Dep. variable: output gap | Lagged output gap | World output gap | Interest rate gap | Exchange rate gap | BNDES lending gap | Government consumption gap |
|---|---|---|---|---|---|---|
| Adj. $R^2$=0.74 | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}$ | $\hat{\delta}$ | $\hat{\rho}$ | $\hat{\theta}$ |
| Point estimate | 0.358 | 0.208 | -0.453 | -4.885 | 0.060 | 0.006 |
| Std. error | (0.02)*** | (0.07)*** | (0.14)*** | (1.55)*** | (0.03)** | (0.06) |

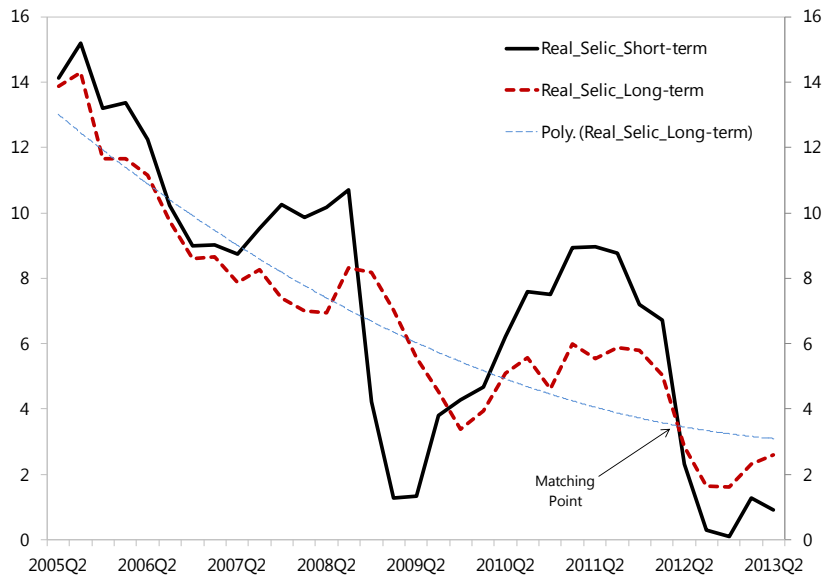Sources: BCB, Thomson Datastream, Haver, and authors' calculations
1/ Standard errors in parenthesis: * significant at 10 percent; ** significant at 5%; and *** significant at 1 percent.
3/ HAC standard errors and covariance estimates use prewhitening with 4 lags, Bartlett kernel, and Newey-West automatic bandwidth selection.

Using the estimates from Table 5, we solve equation (10) for $r_t$ at each period of time to obtain a path for the short-run equilibrium real interest rate.[10] At the steady state, all gaps close. As a corollary, the short-run and the long-run equilibrium real interest rates are equal, suggesting a locus for the neutral rate. This situation is illustrated in Figure 8, which shows that early in 2012 both rates met around 4 percent in Brazil, and after that the short-run equilibrium rate was lower than the long-run equilibrium rate until late 2013.

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[10] In order to do it, we rewrite the IS model such that the short-run equilibrium real interest rate equals the long-run equilibrium real interest rate plus the sum of all other gap variables at each point in time.

**Figure 8. Brazil: Short- and Long-Run Equilibrium Real Interest Rates, 2005–13**
*(In percent)*



Sources: BCB, Thomson Datastream, Haver, and authors' calculations

These estimates provide very strong evidence that the short-run and long-run equilibrium real rates can diverge substantially, reflecting both domestic and external factors and the stance of monetary policy.[11]

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[11] A possible topic of discussion in this context is the impact of macroprudential (MaPs) measures on estimates of the IS curve for Brazil. While MaPs are not expected to affect the steady state of the model variables, they may influence their actual prints (and thus their gaps). MaPs may also interfere on the transmission mechanism of the monetary policy, expediting or delaying the convergence of short-run rates towards a neutral stance, like it happened in the segment of car loans in Brazil recently. For a deeper examination of Brazil's experience with MaPs, see Pereira da Silva and Harris (2012), and Tovar, Garcia-Escribano, and Vera-Martin (2012).

## VI. INSIGHTS FROM A SMALL MONETARY MODEL

In this section we use a small rational expectations model to better understand the policy implications of a changing neutral rate. The aim is to identify how monetary policy settings might respond to a changing neutral rate, but also to assess what happens if policymakers "get it wrong"; in other words, the implications of a policy error made on the basis of an incorrect assessment of the neutral rate.

Previous research has focused more on incorrect estimates of potential output. Orphanides (2002) suggests that when policymakers make systematic errors estimating potential output, activist policy rules can lead to worse outcomes, notably higher output gap and inflation variability. Uncertain neutral rates have also been shown to affect macroeconomic outcomes and optimal monetary policy design. Orphanides and Williams (2007) show that, like potential output, uncertainty about neutral rates and resulting (unintentional) policy errors using Taylor-type rules increase growth and inflation variability. They suggest that the optimal policy rule, in the face of such uncertainty, should exhibit more sensitivity to inflation and output than if the neutral rate was known with certainty. They also suggest that a difference rule in which policy responds to changes in inflation and output (or unemployment) can remove the feedback of policymaker misperceptions of the neutral rate to the economy. Taylor and Williams (2010) also advocate a rule that removes the influence of the estimated neutral rate on policy rate settings. Specifically, they suggest that optimal policy should show greater inertia with the interest rate from the previous period serving as the "steady state" benchmark (in place of an estimate of the neutral rate), while still responding to inflation and output gaps.

Our model is based on the framework described by Berg, Karam, and Laxton (2006) and includes four structural equations that link output, inflation, and policy interest rates. Specifically, the four equations include: (1) an aggregate demand or IS curve that relates the level of real activity to expected and past real activity, the real interest rate, and the real exchange rate; (2) a price-setting or Phillips curve that relates inflation to past and expected inflation, the output gap, and the exchange rate; (3) an uncovered interest parity condition for the exchange rate, with some allowance for backward-looking expectations; and (4) a rule for setting the policy interest rate as a function of the output gap and expected inflation.

$$y_t - y_t^* = \beta_{ld} ygap_{t+1} + \beta_{lag}\left(y_{t-1} - y_{t-1}^*\right) - \beta_r\left(r_{t-1} - r_{t-1}^*\right) + \beta_z\left(z_{t-1} - z_{t-1}^*\right) + \varepsilon_t^y \tag{11}$$

$$\pi_t = \alpha_{ld}\pi 4_{t+1} + \left(1 - \alpha_{ld}\right)\pi 4_{t-1} + \alpha_{ygap}\left(y_{t-1} - y_{t-1}^*\right) + \alpha_z\left(z_t - z_{t-1}\right) + \varepsilon_t^\pi \tag{12}$$

$$z_t = z_{t+1}^e - \tfrac{1}{4}\left(r_t - r_t^{US} - \rho_t\right) + \varepsilon_t^z \tag{13a}$$

$$z_{t+1}^e = \delta_z z_{t+1} + \left(1 - \delta_z\right)z_{t-1} \tag{13b}$$

$$i_t = \gamma_{lag} i_{t-1} + \left(1 - \gamma_{lag}\right)\left[r_t^{**} + \pi 4_t + \gamma_\pi\left(\pi 4_{t+4} - \pi_{t+4}^*\right) + \gamma_{ygap}\left(y_t - y_t^*\right)\right] + \varepsilon_t^i \tag{14}$$

Where:

$y_t \equiv$ log output in period $t$

$y_t{}^* \equiv$ log potential output in period $t$

$y_t - y_t{}^* \equiv$ output gap in period $t$

$\pi_t \equiv$ annualized quarter-on-quarter change in the log price index in period $t$

$\pi_{t+4}{}^* \equiv$ inflation target in period $t+4$

$\pi 4_t \equiv$ year-on-year change in the log price index in period $t$

$z_t \equiv$ log real bilateral exchange rate versus the U.S. dollar (+ = depreciation) in period $t$

$z_t{}^* \equiv$ log equilibrium real bilateral exchange rate versus the U.S. dollar in period $t$

$z^e{}_{t+1} \equiv$ expected (model consistent) real exchange rate in period $t+1$

$r_t^{US} \equiv$ United States short-term real interest rate in period $t$

$\rho_t \equiv$ country-specific risk premium in period $t$

$r_t{}^* \equiv$ "true" neutral real interest rate in period $t$

$r_t{}^{**} \equiv$ policymakers estimate of the neutral real interest rate in period $t$

$i_t \equiv$ nominal policy rate in period $t$

$\varepsilon_t^y \equiv$ idiosyncratic output shock in period $t$

$\varepsilon_t^\pi \equiv$ idiosyncratic inflation shock in period $t$

$\varepsilon_t^z \equiv$ idiosyncratic exchange rate shock in period $t$

$\varepsilon_t^z \equiv$ idiosyncratic policy interest rate shock in period $t$

We start the model in its steady state, when policymakers accurately identify the constant neutral rate (i.e., $r^{**} = r^*$), the output gap is closed ($y = y^*$), and inflation is at the target ($\pi = \pi^*$). The parameters for the model $\beta_{lead}$, $\beta_{lag}$, $\beta_r$, $\beta_z$, $\alpha_{lead}$, $\alpha_{ygap}$, $\alpha_z$, $\delta_z$, $\gamma_{lag}$, $\gamma_\pi$, and $\gamma_{ygap}$ have been approximately calibrated to Brazil over the 2000Q1 to 2012Q2 period and are provided in the appendix. The inflation target is equal to the mid-point of the Central Bank of Brazil's target range of 4½ percent ± 2 percent.

Our focus is on a changing neutral rate and our first scenario considers an unanticipated decline in the neutral rate $r^*$ that is correctly identified by policymakers. Throughout this period, all other shocks are set to zero to isolate the impact of the change in $r^*$. In particular, we consider a gradual 3½ percentage point decline in the neutral rate to 3 percent over a period of eight years, keeping $r^*=r^{**}$. We contrast this with two alternative scenarios in which policymakers "get it wrong". We implement a change in the neutral rate without changing any of the other steady-state values in the model, including potential output. As the Euler condition shows, there is a strong link between the neutral rate and potential output growth. We justify our focus only on a changing neutral rate in two ways: $r^*$ can change for a constant potential growth rate due to consumer preferences; and we want to ease the comparison with the previous literature (e.g., on potential output uncertainty) which largely kept the assumed neutral rate unchanged.

In the second scenario, we assume that policymaker beliefs are lagging. Specifically, we assume that policymakers believe the neutral rate is following the same downward trajectory as the true neutral rate, but with a two year lag. This seems a reasonable assumption.

Policymakers will often change their beliefs about important structural variables such as the neutral rate and subsequent policy settings only slowly over time (see for example Sack and Wieland, 2000). At the same time, the neutral rate itself is likely to change only gradually, suggesting long lags in learning. We do not model policymaker beliefs explicitly—as done in Orphanides and Williams, 2007—and instead adopt the heuristic approach to their formation of simply assuming that they decline gradually over time.

In our third and final scenario, we assume that policymakers believe that the neutral rate is falling at a faster rate than is actually occurring. At some point, at least in the simple model above, this error will be recognized—it is easy to show that at a steady state with $r^{**} < r^*$ inflation will be persistently above target by the quantity $\gamma_\pi(\pi - \pi^*)$, implying that a rational inflation-targeting central bank will eventually fully learn about the neutral rate.

**Figure 9. Small Model: Scenarios for Neutral Rates**
*(percent)*



Years before/ after start of NIR decline
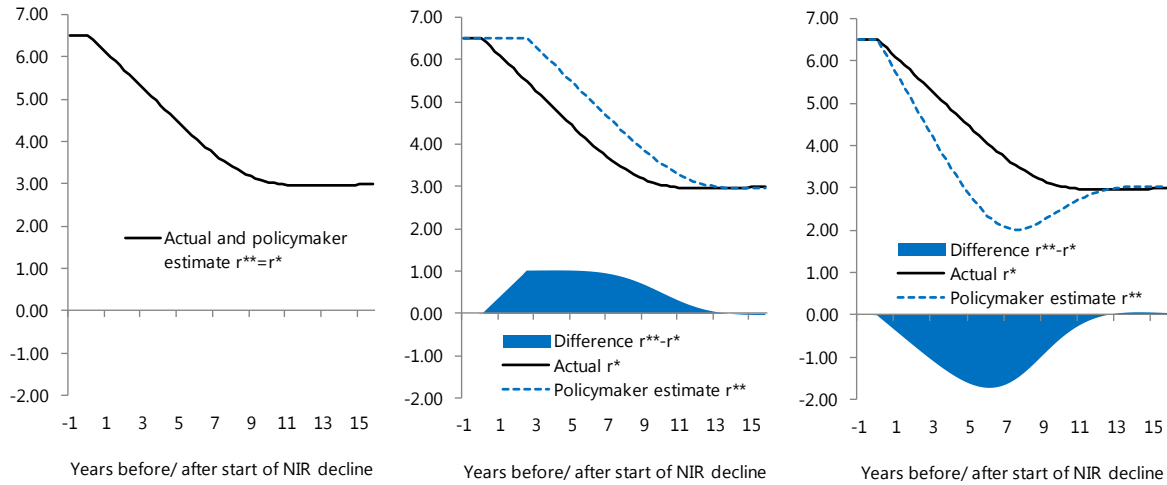
Source: Authors' calculations

To implement the two scenarios in which policymakers hold mistaken beliefs, we need to break the link between the policymaker's expected inflation ($\pi 4_{t+4}$) that is determined by (11) and (12) and current policy settings in (14). The standard model (11)-(14) would imply inconsistent behavior on the part of policymakers as they set policy on the basis of $r^{**}$ and project future inflation on the basis of $r^*$, where $r^{**}>r^*$. In other words, they would recognize, yet ignore, their own mistake. One way to achieve this is to assume that the policymakers' horizon, in an environment of higher neutral rate uncertainty, shrinks to the point that they act only on the basis of observable variables. We implement this assumption by setting the policy rule to target the gap between current (observable) inflation and the inflation target. This means the policy rule (14) becomes:

$$i_t = \gamma_{lag} i_{t-1} + \left(1 - \gamma_{lag}\right)\left[ r_t^{**} + \pi 4_t + \gamma_\pi \left(\pi 4_t - \pi_{t+4}^*\right) + \gamma_{ygap}\left(y_t - y_t^*\right)\right] + \varepsilon_t^i \qquad (15)$$

The results of these simulations are shown in Figure 10. In the first scenario, where $r^{**} = r^*$, a small negative output gap emerges and inflation falls slightly below the target mid-point. This outcome is due to policy inertia causing the policy interest rate to move sluggishly, with the speed of policy adjustment reflected in the value of $\gamma_{lag}$ in (15).

Outcomes are worse when policymakers make mistakes. In the second scenario, the immediate effective tightening in monetary policy caused by inertia is now compounded by policy being set on the basis of an incorrect neutral rate $r^{**} > r^*$. Over the first year, the real interest rate gap $r - r^*$ rises by about 50 basis points and this inadvertent monetary tightening results in a modestly negative output gap. The interest rate gap increases by a further 50 basis points in the second year, while the output gap continues to widen modestly. At the same time, inflation falls persistently, but modestly, below the target. The interest rate gap peaks at about 110 basis points during the third year of the mistake, before $r^{**}$ begins to converge to $r^*$.

In the third scenario, policy loosens as policymakers mistakenly set $r^{**} < r^*$. Over the first year of the mistake, the interest rate gap declines by about 20 basis points and the output gap rises, but very modestly. The interest rate gap peaks during the fifth year of the mistake at about 130 basis points reflecting, in part, the very gradual nature of the change in neutral rates.

**Figure 10. Falling Neutral Real Interest Rates in a Small Monetary Model: Scenarios 1/**
*(percent)*
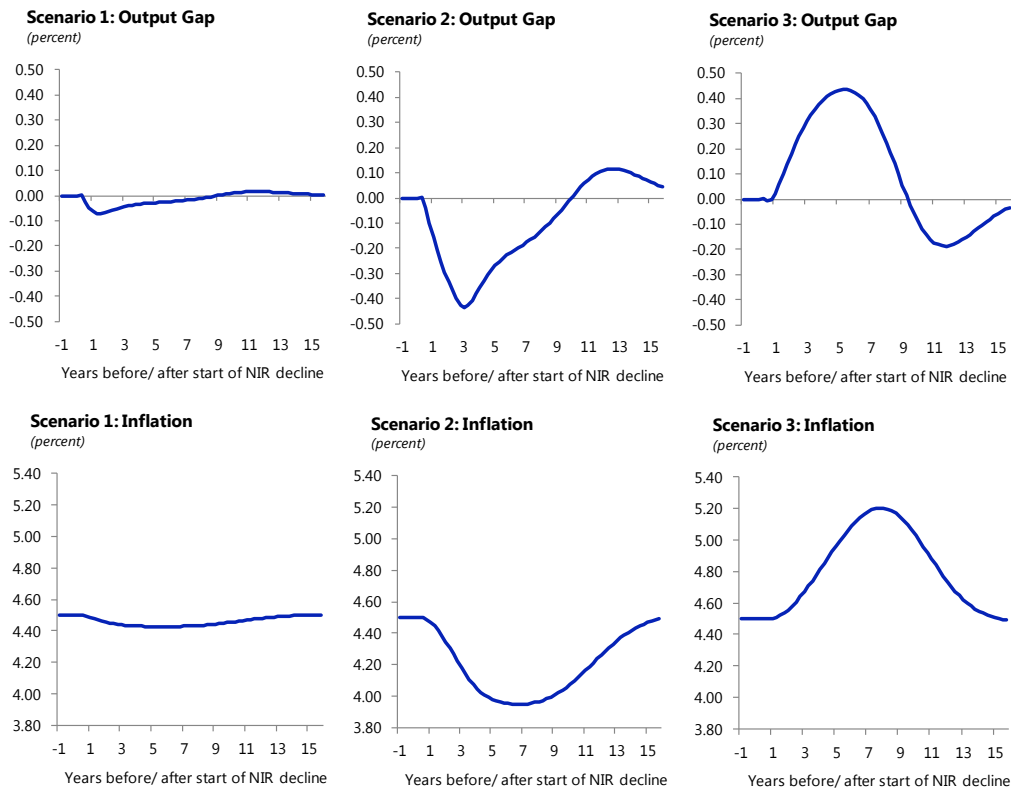


Source: Authors' calculations.1/

The striking aspect of these results is that the adverse outcomes are modest given the size of the mistake (measured as $r^{**}$ - $r^*$). In scenarios two and three, the output gap peaks at about -0.5 and 0.5 percentage points, respectively. Similarly, inflation diverges from the target by about 0.7 percentage points (in opposite directions) in both scenarios. The reason is that policymakers are continuing to follow their own rules at the same time as mistakenly estimating the neutral rate. This means that policymakers are partly correcting their own mistake in each quarter.

To illustrate, consider the third scenario where $r^{**} < r^*$. As the monetary policy becomes stimulative, the output gap widens, and inflation rises above target; policymakers will perceive this to be a positive output shock (i.e., in equation (10), $\varepsilon_t^y > 0$). In response, they will partially undo the policy rate reductions triggered by their downward revision of $r^*$, with the strength of the policy reaction condition by the parameters in the policy rule (15). Over time, policymakers will likely learn that their estimate $r^{**}$ is incorrect. So long as they retain their initial estimate $r^{**}$, the ongoing policy response to perceived output shocks will prevent the actual interest rate gap (defined as the actual real interest rate $r$ less the true neutral rate $r^*$ or $r - r^*$) from diverging too far away from zero.

Figure 11 shows real interest rate gaps from policy rules with different parameters. Policymakers with loss functions that are highly sensitive to both output and inflation gaps (in Figure 11 this is policy rule 1 with very high values for $\gamma_\pi$ and $\gamma_{ygap}$) will react sharply to the outcomes of their mistaken estimate $r^{**}$. In turn, this results in an interest rate gap that hardly diverges from zero. In contrast, policymakers that attach a much lower weight to output and inflation gaps (in Figure 11 this is policy rule 2 with low values for $\gamma_\pi$ and $\gamma_{ygap}$) instead implicitly attach a much higher weight to their estimate of the neutral. (Note that the interest rate gap $r - r^*$ in this case is somewhat different from the neutral rate estimate gap $r^{**}$ - $r^*$ because actual real rates are calculated using one quarter ahead inflation.)

**Figure 11. Scenario 3: Actual Real Interest Rate Gaps Under Different Policy Rules 1/**



Years before/ after start of NIR decline
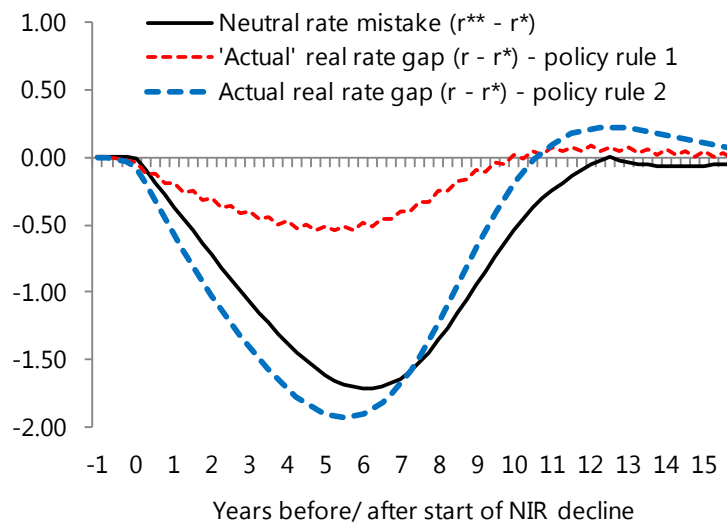
Source: Authors' calculations.
1/ Policy rule 1 assigns coefficients $\gamma_{lag}$=0, $\gamma_\pi$=3 and $\gamma_{ygap}$=3. Policy rule 2 assigns coefficients $\gamma_{lag}$=0, $\gamma_\pi$=0.01 and $\gamma_{ygap}$=0.01.

This analysis clearly highlights the inherent risks for policymakers when the neutral rate appears to be shifting significantly. The most important risk is that policymakers attach a higher weight to anchoring current policy settings on the basis of their estimate of the neutral and less weight on output and inflation gap outcomes. This is consistent with the findings of Orphanides and Williams (2007) and Taylor and Williams (2010). It is well known that uncertainty about state variables (in this case the neutral rate) does not affect the optimal policy feedback rule—see Smets (2002) for an exposition.

However, a simple linear decision rule of the form $i_t = -Fx_t$, where $x_t$ is a vector of state variables (such as output gaps and inflation) is not certainty equivalent as shown by Söderlind (1999). This means that the efficient policy rule coefficients will depend on the covariance of the predetermined variables in the model, including $r^*$. In other words, if uncertainty about $r^*$ increases during its transition to a new steady state, policymakers should maintain their current rule or even increase their activism and respond more forcefully to output and inflation gaps, thereby reinforcing the self-correcting mechanism.

# VII. CONCLUDING REMARKS

A battery of empirical and semi-structural techniques suggest that neutral rates in emerging markets have declined over the past decade. In Brazil, where real interest rates have traditionally been (and remain) high, there is particularly strong evidence of a large fall in the neutral rate.

The secular decline in Brazil's neutral rate is deeply rooted into its experience as an inflation-tergeting economy. As highlighted in Mishkin and Savastano (2001), Brazil's move to targeting inflation in July 1999 was "clearly the first comprehensive attempt to establish a regime of this type in Latin America". Also impressive were the speed of implementation and the degree of transparency and accountability embedded in the framework, with immediate impact on inflation expectations and the neutral rate.

For the sample period covered in this paper, our econometric results suggest that global real interest rates have pushed down Brazil's neutral rate but domestic factors have also been important, particularly: an increase in the supply of saving (reflected in swift financial deepening); declining public debt; and lower sovereign risk. At the same time, we find that there has been substantial cyclical variation in the estimates of equilibrium (or neutral) short-term interest rates, reflecting factors such as domestic and world output gaps.

The good news is that high levels of uncertainty regarding the true neutral rate need not complicate policymaking too much. On the basis of small macromodel simulations calibrated for Brazil, we find that the costs of incorrectly estimating the neutral rate, in isolation, are likely to be low. This is because standard monetary policy instrument rules provide a self-correcting mechanism that ensures mistakes are partly corrected. The intuition is clear—an activist policy rule will ensure that policymakers respond symmetrically to observable outcomes, whether they are due to output and inflation shocks or incorrectly estimated neutral rates. This guarantees that actual real interest rates are never too far away from the "true" neutral rate, thereby minimizing the variability of output and inflation gaps.

Our result contrasts sharply with Orphanides (2002) who showed that, in the presence of an incorrectly estimated level of potential output, activist policy rules can exacerbate output and inflation gap variability. However, it is consistent with Orphanides and Williams (2007) and Taylor and Williams (2010) in that the best response to neutral rate uncertainty is to ensure policy remains highly sensitive to inflation and output variations. The main risk in updating assumptions about the neutral rate is that policy becomes less activist and more focused on "targeting the neutral rate," which would amplify the costs of mistakes.

## REFERENCES

Archibald, Joanne and Leni Hunter, 2001, "What is the Neutral Real Interest Rate, and How Can We Use It?" *Reserve Bank of New Zealand Bulletin*, Vol. 64, pp. 15-28.

Arida, P., E. L. Bacha, and A. L. Resende, 2005, "Credit, Interest, and Jurisdictional Uncertainty: Conjectures on the Case of Brazil," in *Inflation Targeting, Debt, and the Brazilian Experience, 1999 to 2003*, edited by F. Giavazzi, I. Goldfajn, and S. Herrera. Cambridge, MA: MIT Press.

Bacha. E. L., M. Holland, and F. M. Gonçalves, 2009, "A Panel-Data Analysis of Interest Rates and Dollarization in Brazil," *Revista Brasileira de Economia*, Vol. 63, n. 4, pp. 341–360.

Barbosa, F. de H., 2006, "The Contagion Effect of Public Debt on Monetary Policy: The Brazilian Experience," *Brazilian Journal of Political Economy*, Vol. 26, n. 2, pp. 231-238.

Basdevant, Olivier, Nils Björksten, and Özer Karagedikli (2004), "Estimating a Time Varying Neutral Real Interest Rate for New Zealand," *Reserve Bank of New Zealand Discussion Paper* 2004/01.

Berg, Andrew, Philippe Karam, and Douglas Laxton, 2006, "A Practical Model-Based Approach to Monetary Policy Analysis—Overview," *IMF Working Paper* 06/80.

Bernhardsen, Tom , and Karsten Gerdrup, 2007, "The Neutral Real Interest Rate," *Norges Bank Economic Bulletin* 2, Vol. 78, pp. 52-64.

Bomfim, Antulio, 1997, "The Equilibrium Fed Funds Rate and the Indicator Properties of Term-Structure Spreads," *Economic Inquiry* 35:4 (1997), 830–846.

_____, 2001, "Measuring Equilibrium Real Interest Rates: What Can We Learn from Yields on Indexed Bonds?" *Board of Governors of the Federal Reserve System FEDS* 2001-53.

Erhmann, Michael, and Frank Smets, 2003, "Uncertain Potential Output: Implications for Monetary Policy," *Journal of Economic Dynamics and Control*, 27, pp. 1611-1638.

Fuentes S., Rodrigo, and Fabián Gredig U. (2008), "La Tasa de Interés Neutral: Estimaciones para Chile," *Economía Chilena*, Vol. 11, n. 2.

Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models", *American Economic Review*, Vol. 90, no. 3 June.

Giammarioli, Nicola, and Natacha Valla (2004), "The natural real interest rate and monetary policy: a review," *Journal of Policy Modeling* n. 26, 641–660.

Goldfajn, I., and A. Bicalho, 2011, "A Longa Travessia para a Normalidade: Os Juros Reais no Brasil," in *Novos Dilemas da Política Econômica - Ensaios em Homenagem a Dionisio Dias Carneiro*, edited by E. L. Bacha e M. B. de Bolle. Rio de Janeiro: Grupo Editorial Nacional.

Gonçalves, F. M., M. Holland, and A. D. Spacov, 2007, "Can Jurisdictional Uncertainty and Capital Controls Explain the High Level of Real Interest Rates in Brazil? Evidence from Panel Data," *Revista Brasileira de Economia*, Vol. 61, n. 1, p.p. 49–75.

Gourinchas, Pierre-Olivier, and Olivier Jeanne, 2012, "Global safe assets," *BIS Working Papers* n. 399.

Issler, J. V., and N. S. Piqueira, 2000, "Estimating Relative Risk Aversion, the Discount Rate, and the Intertemporal Elasticity of Substitution in Consumption for Brazil Using Three Types of Utility Function," *Brazilian Review of Econometrics*, v.20, n. 2, pp. 201–239.

Kozicki, S., and P. A. Tinsley, "Shifting Endpoints in the Term Structure of Interest Rates," *Journal of Monetary Economics* 47:3, 613–652.

Laubach, T., and J. Williams, 2003, "Measuring the Natural Rate of Interest," *The Review of Economics and Statistics*, Vol. 83, p.p. 218–231.

Magud, Nicolas E., and Evridiki Tsounta, 2012, "To Cut or Not to Cut? That is the (Central Bank's) Question—In Search of the Neutral Interest Rate in Latin America," *IMF Working Paper 12/243*.

Miranda, P., and M. Muinhos, 2003, "A Taxa de Juros de Equilíbrio: Uma Abordagem Múltipla," *Brazilian Central Bank Working Paper* n. 66.

Mishkin, Frederic S., and Miguel A. Savastano, 2001, "Monetary Policy Strategies for Latin America," *Journal of Development Economics,* Vol. 66, p.p. 415–444.

Muinhos, M., and M. Nakane, 2006, "Comparing Equilibrium Real Interest Rates: Different Approaches to Understand Brazilian Rates," *Brazilian Central Bank Working Paper* n. 101.

Neto, P. C. F. de B., and M. S. Portugal, 2009, "The Natural Rate of Interest in Brazil between 1999 and 2005," *Revista Brasileira de Economia*, Vol. 63, n. 2, pp. 103–118.

Orphanides, A., 2002, "Monetary-Policy Rules and the Great Inflation," *American Economic Review*, Vol. 92, No. 2, pp. 115-120.

_____ and J. C. Williams, 2007, "Robust Monetary Policy with Imperfect Knowledge," *Journal of Monetary Economics,* 54, 1406-1435.

Pereira da Silva, Luiz Awazu, and Ricardo Eyer Harris, 2012, "Sailing Through the Global Financial Storm: Brazil's Recent Experience with Monetary and Macroprudential Policies to Lean Against the Financial Cycle and Deal With Systemic Risks," *Brazilian Central Bank Working Paper* n. 290.

Perrelli, Roberto, 2012, "The Neutral Real Interest Rate in Brazil," *Brazil—Article IV Staff Report*, IMF Country Report No. 12/191 (Washington: International Monetary Fund).

Perrelli, Roberto, Shaun K. Roache, and Carlos Góes, 2014, "The Impact of the Tapering Tantrum on the Slope of the Yield Curve of Emerging Market Economies," *IMF Working Paper* (forthcoming).

Roache, Shaun K., 2014, "Inflation Persistence in Brazil—A Cross Country Comparison," *IMF Working Paper* 14/55.

Rudebusch, G. D., and L. E. O. Svensson, 1999, "Policy Rules for Inflation Targeting", in *Monetary Policy Rules*, edited by J. B. Taylor. Chicago: University of Chicago Press.

Sack, Brian, and Volker Wieland, 2000, "Interest-Rate Smoothing and Optimal Monetary Policy: A Review of Recent Empirical Evidence," *Journal of Economics and Business* vol. 52, n. 1-2, pp. 205-228.

Segura-Ubiergo, Alex (2012), "The Puzzle of Brazil's High Interest Rates," *IMF Working Paper* 12/62.

Smets, Frank, 2002, "Output Gap Uncertainty: Does it Matter for the Taylor Rule?" *Empirical Economics*, 27, pp. 113-129.

Söderlind, Paul, 1999, "Solution and Estimation of RE Macromodels with Optimal Policy," *European Economic Review*, 43, pp. 813-823.

Taylor, John B., 1993, "Discretion versus Policy Rules in Practice," *Carnegie Rochester Conference Series on Public Policy* 39, 195–214.

_____, and John C. Willliams, 2010, "Simple and Robust Rules for Monetary Policy," in Benjamin M. Friedman & Michael Woodford (ed.), *Handbook of Monetary Economics*, edition 1, volume 3, chapter 15, pp. 829-859 (Elsevier).

Tovar, Camilo E., Mercedes Garcia-Escribano, and Mercedes Vera Martin (2012), "Credit Growth and the Effectiveness of Reserve Requirements and Other Macroprudential Instruments in Latin America," *IMF Working Paper* 12/142.

Wicksell, Knut, 1898, *Interest and Prices,* translated by R. F. Kahn in 1936 (New York: Sentry Press).

Wu, Tao, 2005, "Estimating the 'Neutral' Real Interest Rate in Real Time," *Federal Reserve Bank of San Francisco Economic Letter*, Number 2005–27.