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Estimating Potential Output with a Multivariate Filter

J. Benes, K. Clinton, R. Garcia-Saltos, M. Johnson, D. Laxton, P. Manchev and T. Matheson

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Estimating Potential Output with a Multivariate Filter[1]

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Abstract

This paper develops a simple model for measuring potential output that uses data on inflation, unemployment, and capacity utilization. We apply the model to 10 countries, in addition to the United States and the euro area. While there is a substantial amount of uncertainty around our estimates, we find that the financial crisis has resulted in a reduction in potential output.

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[1] The estimates contained in this paper are not official IMF estimates. The views expressed in this paper are those of the authors and do not necessarily represent those of the IMF or IMF policy. We thank Olivier Blanchard, Charles Collyns, Marcello Estevao, Koshy Mathai and David Romer for useful discussions and comments. We also thank Ioan Carabenciov, Susanna Mursula, and Kadir Tanyeri for their excellent research assistance. Kevin Clinton is a Visiting Scholar in the Research Department and Marianne Johnson is a Research Advisor at the Bank of Canada. The remaining authors work in the Fund's Economic Modeling Unit of the Research Department. The estimates of the output gap and potential output as well as the programs used to construct such estimates can be downloaded from www.douglaslaxton.org.

Contents Page

Tables

Figures

I. Introduction

Almost 5 decades ago Okun (1962) gave a definition of potential output, and provided tentative estimates for the United States. Okun anticipated most of the issues that, to this day, preoccupy research in the area. And in the aftermath of the events of 2008-09, questions about the level and growth of potential output have re-surfaced as a crucial issue for monetary and fiscal policy.

The October 2009 *World Economic Outlook* presents evidence from 88 banking crises over the past four decades (International Monetary Fund, 2009). While there is large variation across countries, the evidence suggests that output does not go back to its old trend path after a crisis, but remains permanently below it.[2] Negative factors reducing potential output include: sudden obsolescence and scrapping of existing capital stock through business failures; weak investment as a result of the unusual uncertainties and the extreme tightness of credit; discouraged worker effects on labor force participation; and erosion of skills. The issue is the size and duration of the hit to productive capacity. Central bankers would like to know the size of the gap between actual and potential GDP, so as to maintain an appropriate degree of monetary ease.[3] Over the next few years, as the recovery gets under way, reliable measures of the gap will be especially valuable to monetary policy: first, as a guideline for the appropriate withdrawal of stimulus; second, as a public communications tool to justify the interest rate stance that this will entail. Governments would be concerned about the cyclically-adjusted budget position, to help assess whether, or not, a given budget deficit implies sustainable debt growth over time.

Over the past decade economists have often used the univariate HP filter (Hodrick-Prescott, 1997) to derive empirical approximations for potential GDP. While this technique, whether applied to GDP directly, or to the inputs of a production function, can be used to develop estimates of the rate of increase in potential that vary

[2]Similar results can be found in Furceri and Mourougane, 2009, and Koopman and Székely, 2009. See also the April 2009 *Monetary Policy Report* of the Bank of Canada, Technical box 1.

[3]Mismeasurement of the output gap contributed importantly to the inflationary monetary policy errors of the 1970s. See Freedman (1989), Laxton and Tetlow (1992) and Orphanides (2001). Barnett, Kozicki, and Petrinec (2009) point out the usefulness of the output gap as a tool for policy communications.

more or less plausibly over time, estimates from the algorithm are subject to revision, as data for later dates become available, even if the national accounts themselves are not revised – see Laxton and Tetlow (1992). Revisions for the most recent quarters tend to be particularly large, creating an awkward problem for current analysis and forecasting.[4] An additional problem, with univariate filters generally, is that they ignore relevant economic information. This can create biases: e.g., for much of the estimation period used in this paper, central banks were fighting inflation; tight monetary conditions resulted in prolonged negative output gaps. Estimates of the trend of GDP, ignoring the decline in inflation throughout the period, understate potential GDP and, hence, the width of those gaps. A more current concern, following the financial crisis, is that the sharp increase in business failures has suddenly rendered obsolete a part of the economy's productive capacity.

A multivariate approach, deriving from an operational definition of potential output, can deal better with these difficulties, at least in principle. For many purposes, a useful definition is the level of output that may be sustained indefinitely without creating a tendency for inflation to rise or fall. It follows that the behavior of inflation contains crucial information on the level of potential. A period in which inflation is stable would likely be a period in which actual output is about equal to potential output, whereas increasing (decreasing) inflation would suggest that actual output is above (below) potential. Putting this idea into practice, some researchers have estimated output gaps jointly with an inflation equation.[5] There is no reason, however, to confine the additional information to the inflation rate.

This paper describes a method for measuring and updating potential output and the output gap, which incorporates relevant empirical relationships between actual and potential GDP, unemployment, core inflation and capacity utilization in manufacturing, within the framework of a small macroeconomic model. In effect, this provides a

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[4] Orphanides (2001) underlines the large margin for error involved in real-time estimates of the output gap.

[5] See, for example, Laxton and Tetlow (1992) and Kuttner (1994). A parallel strand of research, on the measurement of the equilibrium rate of unemployment, or NAIRU, has also used the behavior of inflation to identify an underlying, equilibrium variable – for example, Blanchard and Summers (1986), Fortin (1991) and Ball (2009).

multivariate (MV) filter, adaptable to many countries.[6] The approach has a flexibility, which allows the estimated growth of potential to vary with an array of recent information, while at the same time taking into account the more stable trends evident in long-run time series. We construct confidence intervals around the estimates, to give a quantitative guide to certain risks. The paper reports results for 11 countries and for the euro area.[7] They suggest that in practice, as well as in principle, the MV filter can provide useful, relatively robust, estimates of potential output.

Section II outlines the small macroeconomic model, and the techniques for estimating parameters, latent variables and confidence intervals. It also evaluates the performance of the model in terms of forecast accuracy, and robustness to new data. Section III provides an overview of the international results, focusing, for illustration, on those for the United States. Section IV highlights some conclusions.

II. The Model

A. Three Gaps

The output gap (y_t) is the log difference between actual GDP (Y_t) and potential GDP ($\overline{Y}_t$):

$$y_t = 100 * LOG(Y_t/\overline{Y}_t). \tag{1}$$

The unemployment gap (u_t) is the equilibrium unemployment rate, or NAIRU, ($\overline{U}_t$) minus the actual unemployment rate (U_t):

$$u_t = \overline{U}_t - U_t. \tag{2}$$

[6]Benes and N'Diaye (2004), Butler (1996), Julliard and others (2007), Kuttner (1994), and Laubach and Williams (2003) have developed multivariate estimates of potential output for single countries.

[7]See table 1 for a list countries for which we have estimates. A more complete, and frequently updated, set of results can be found at http://www.douglaslaxton.org/.

The capacity utilization gap (c_t) is the difference between the actual *manufacturing* capacity utilization index (C_t) and its equilibrium level ($\overline{C}_t$).

$$c_t = C_t - \overline{C}_t. \tag{3}$$

B. Three Identifying Relationships

B.1 Inflation equation

The level (y_t) and the change (y_t - y_{t-1}) in the output gap influence current core inflation ($\pi 4_t$):

$$\pi 4_t = \pi 4_{t-1} + \beta y_t + \Omega(y_t - y_{t-1}) + \varepsilon_t^{\pi 4}. \tag{4}$$

The level of the gap incorporates the standard short-run tradeoff: an increased gap implies an increased rate of inflation. The change in the gap would reflect certain rigidities in the adjustment of the economy – for example, coming out of a recession (with y_t negative and $y_t - y_{t-1}$ positive) it would capture speed-limit effects due to capacity constraints in some sectors of the economy. The lagged inflation rate, with coefficient constrained to unity, may be interpreted as a simple proxy for inflation expectations. In any event, this restriction implies no long-run tradeoff between inflation and output.

Equation (4) is by no means a state-of-the-art augmented Phillips curve, but for present purposes it performs adequately across a wide range of countries, and over lengthy time periods which include differing monetary policy regimes.[8]

[8] In future work, we envisage incorporating an explicit expectations process.

B.2 Dynamic okun's law

Okun defined a simple relationship between the current unemployment rate and the output gap.[9] However, both theory and the data indicate that there should be a lag between changes in output and the resulting changes in employment. Recognizing a lag effect, we use this equation to link the unemployment gap to the output gap:

$$u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u. \tag{5}$$

B.3 Manufacturing capacity utilization

We use a similar relationship to Okun's law to describe the capacity utilization gap. Implicitly, we assume that there is important information in capacity utilization that can help to improve our estimates of potential output and the output gap. To capture the much wider cyclical fluctuations in manufacturing than in the economy more generally, one would expect κ_2 in equation (6) to exceed unity:

$$c_t = \kappa_1 c_{t-1} + \kappa_2 y_t + \varepsilon_t^c. \tag{6}$$

C. Laws of Motion for Equilibrium Variables

C.1 Equilibrium unemployment rate or NAIRU

A stochastic process that includes transitory, level shocks ($\varepsilon_t^{\overline{U}}$) as well as more persistent shocks ($G_t^{\overline{U}}$), provides a useful empirical description of the history of equilibrium unemployment ($\overline{U}_t$):

$$\overline{U}_t = \overline{U}_{t-1} + G_t^{\overline{U}} - \frac{\omega}{100} y_{t-1} - \frac{\lambda}{100}(\overline{U}_{t-1} - U^{ss}) + \varepsilon_t^{\overline{U}}. \tag{7}$$

[9] Okun made a working assumption, which he recognized was somewhat arbitrary, that the "target" rate of unemployment was 4 percent. In modern parlance, one would say that he assumed a stable NAIRU.

The inclusion of the output gap in the NAIRU represents a partial hysteresis effect from economy-wide demand fluctuations – see Ball (2009) for a recent discussion.[10]

The persistent shocks to the NAIRU follow a damped autoregressive process:

$$G_t^{\overline{U}} = (1-\alpha)G_{t-1}^{\overline{U}} + \varepsilon_t^{G^{\overline{U}}}. \tag{8}$$

Notice that, while we allow for persistent deviations in the NAIRU, we assume a fixed steady-state level of unemployment in the long run, U^{ss}.

C.2 Potential output

Potential output ($\overline{Y}_t$) depends on the underlying trend growth rate of potential ($G_t^{\overline{Y}}$), and on changes in NAIRU:

$$\overline{Y}_t = \overline{Y}_{t-1} - \theta(\overline{U}_t - \overline{U}_{t-1}) - (1-\theta)(\overline{U}_{t-1} - \overline{U}_{t-20})/19 + G_t^{\overline{Y}}/4 + \varepsilon_t^{\overline{Y}} \tag{9}$$

In equation (9) changes in NAIRU may cause the short- and medium-term growth of potential to differ from $G_t^{\overline{Y}}$. The first difference, $\overline{U}_t - \overline{U}_{t-1}$, captures the impact of changes in the equilibrium level of unemployment on the growth of potential output, via a Cobb-Douglas production function, in which θ is the labor share. The 19-quarter difference captures the effect of induced changes in the capital stock. Thus, the one-quarter impact of a permanent 1 percentage point increase in $\overline{U}_t$ is a decline in potential of θ percent; a negative effect continues for a further 19 quarters, such that the long-run decline in the level of potential output is 1 percent.

The underlying trend growth rate ($G_t^{\overline{Y}}$) is not constant, but follows serially correlated deviations (long waves) from the steady-state growth rate, $G_{SS}^{\overline{Y}}$.

[10] Blanchard and Summers (1986) introduced the idea of hysteresis to the behavior of equilibrium unemployment. They explained the long duration of shocks to unemployment by distinguishing between insiders and outsiders in the wage bargaining process. Ball (2009) presents evidence that NAIRU remains strongly history-dependent.

$$G_t^{\overline{Y}} = \tau G_{SS}^{\overline{Y}} + (1-\tau) G_{t-1}^{\overline{Y}} + \varepsilon_t^{G^{\overline{Y}}}. \tag{10}$$

C.3 Equilibrium capacity utilization

As in the preceding equations, the stochastic process for equilibrium capacity utilization ($\overline{C}_t$) also includes pure level shocks ($\varepsilon_t^{\overline{C}}$) as well as more persistent shocks ($G_t^{\overline{C}}$).

$$\overline{C}_t = \overline{C}_{t-1} + G_t^{\overline{C}} + \varepsilon_t^{\overline{C}}, \tag{11}$$

where

$$G_t^{\overline{C}} = (1-\delta) G_{t-1}^{\overline{C}} + \varepsilon_t^{G^{\overline{C}}} \tag{12}$$

C.4 Perceived long-term inflation objectives

$$\pi 4_t^{LTE} = \pi 4_{t-1}^{LTE} + \varepsilon_t^{\pi 4^{LTE}} \tag{13}$$

The sample period contains various monetary policy regimes over countries and over time. For example, the U.S. sample period contains several policy switches, which would not always have been immediately perceived by the public. For much of the time, in most countries, the expected long-term inflation objective of the central bank would be a matter of guesswork. We postulate that the expected objective, $\pi 4_t^{LTE}$, follows an adaptive process, with revisions to last quarter's expectation embodied in the term $\varepsilon_t^{\pi 4^{LTE}}$. In the event of a regime change, or during a volatile regime, the variations in $\varepsilon_t^{\pi 4^{LTE}}$would be large. In contrast, however, the past decade has seen stable, and more or less explicit, inflation objectives. Variance in $\varepsilon_t^{\pi 4^{LTE}}$ would therefore play only a small role in our forecasts – almost negligible for the industrialized countries.

Many countries have based their monetary regime on a fixed exchange rate (some still do). In these cases, there can be no independent stable target for the long-run rate of inflation. In the model, this would be reflected in high variance of $\varepsilon_t^{\pi 4^{LTE}}$.

We use data on long-term inflation expectations from Consensus Economics (where available) to capture the history of $\pi 4_t^{LTE}$.

D. Output Gap Equation

In conventional monetary policy models, over time, an interest rate reaction function keeps inflation on target. Changes in the policy interest rate influence core inflation through a complex transmission mechanism in which the Phillips curve is a key link. In effect, monetary policy exerts its influence on the core rate of inflation through the output gap. For present purposes, it is useful to recognize this through the following equation:

$$y_t = \rho_1 y_{t-1} - \frac{\rho_2}{100}(\pi 4_{t-1} - \pi 4_{t-1}^{LTE}) + \varepsilon_t^y, \tag{14}$$

Notice that the negative effect on demand from inflation deviations from target is consistent with a broad range of monetary regimes. In the case of an inflation-targeting regime, the inflation resulting from a period of excess demand is met by a tightening in monetary conditions by the central bank, reducing the output gap. In a fixed exchange rate regime, on the other hand, the excess demand is contained by an appreciation of the real exchange rate that results from higher inflation.

Other factors (e.g. demand shocks) driving the output gap are summarized in the stochastic term ε_t^y.

III. Estimation and Testing

A. Estimation Technique

We employ Bayesian methodology – to be precise, regularized maximum likelihood (Ljung, 1999) – to estimate the model. This allows us to define prior distributions that prevent parameters from wandering into nonsensical regions, which is a non-negligible consideration in our context, as the data are uninformative about several parameters.

The estimates of the within-sample confidence intervals are derived analytically, taking the model and its parameters as the true data generating process. They incorporate the sampling uncertainty of the unobservable component estimates.

The U.S. sample period is 1967Q1 to 2010Q2. Table 2 displays the data sources for the U.S. and table 1 provides our country-specific priors regarding the steady-state values of the labor share, output growth, and the unemployment rate. The priors for these steady-state parameters are based on IMF staff estimates. Table 3 displays prior distributions and estimated posterior distributions for the U.S. [11]

B. A Prior On Steady-State Growth

To ensure that potential output growth does not deviate too far from steady state, we add the following equation to the model:

$$4 * (\overline{Y}_t - \overline{Y}_{t-1}) = G^{\overline{Y}}_{SS} + \varepsilon_t \tag{15}$$

where ε_t is a measurement error that reflects our prior beliefs about the volatility of potential output growth around its steady state, $G^{\overline{Y}}_{SS}$.

The mechanics of the 'steady-state prior' are straightforward: a prior belief that potential output growth does not deviate too far from steady state requires a lower standard deviation for ε_t than a prior belief that deviations from steady state are larger. In our baseline specification, the prior on the standard deviation of ε_t differs across countries – for each country, we set it to be the standard deviation of actual GDP growth and divide by 3.

Notice that the steady-state prior differs from the priors on the parameters because it relates *directly* to the time-series properties of an 'unobservable' (latent) variable – in this case, the volatility of potential output growth. Priors on parameters, on the other hand, are usually related to the behavior of the unobservable variables in more complicated ways.

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[11] Please see the appendix for technical details.

C. Empirical Tests

Most past work in this area has not used objective criteria for assessing the performance of alternative methods and estimates. Our approach is to go beyond subjective 'eyeball' metrics and use forecasting performance and robustness to revisions as criteria for model evaluation.

C.1 Forecasting accuracy

To gauge relative forecasting accuracy, we derive synthetic historical forecasts for year-over-year core inflation from the MV filter and a random walk, for each quarter from 1970Q2 to 2010Q2. The forecasts are synthetic in that no forecaster, in a given quarter, had access to our 2010-vintage information set. But the tests involve a level playing field, in that the models are judged on the same information. The upper panel of table 4 shows root mean squared errors (RMSEs) for horizons 1, 4, 8 and 12 quarters ahead. We find that the MV filter outperforms the random walk at all horizons. We also find relatively good forecasting results for the MV filter for the other countries.

C.2 Revision robustness

To assess the robustness of estimates for the current quarter, we look at the size of the revisions necessitated by the later arrival of new data.[12] We deem a technique that results in smaller revisions than another technique to be relatively robust. We put this idea into practice, following the recursive procedure described in the previous section, comparing the MV filter with the HP filter (with λ set at 1600).

By comparing the nowcasts from the model with estimates made using data over the entire period, we get some idea of the relative size of the revisions required once the final information set becomes available. We gauge the size of the revisions by the mean

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[12] We are concerned here simply with the arrival of data for later dates, and ignore revisions to existing data. Again, although the derived estimates are artificial, each model is assessed based on the same information.

absolute error (MAE): the average of the absolute value of the differences between nowcasts and final estimates.

The lower panel of table 4 shows the results of this exercise for the U.S. output gap. The baseline MV filter produces much smaller revisions than the HP filter. The relatively good revision properties of the filter also apply to the other countries examined.

IV. An Overview of the International Results

It is important to stress that, just as any other method for estimating latent variables, this one is not a panacea. In particular, it hinges critically on well-chosen priors. Thus, while the baseline parameterization generates plausible estimates for a wide range of countries with minimum country-specific adjustments as is demonstrated below, to produce good estimates requires the expert opinion of experienced economists.

A. Output Gap and Core Inflation

It is useful to start with the MV filter results for the output gap and inflation, jointly, as the interaction of these 2 variables captures the essential elements of the model and the derived estimates. Figures 1 and 2 provide a confidence band (+/- 2 standard deviations), as well as the central estimate, for the output gap. The inflation rate is the year-over-year percentage increase in the core CPI. The figures show quarterly numbers for the post-2000 period, through 2010Q2. [13]

Figures 1 and 2 cover the 11 economies and the euro area. The recession at the beginning of the decade, following the collapse of the high-technology bubble, was a moderate one. The central estimate of the U.S. gap troughs at about -1.5 percent in 2001; for the other industrialized countries it does not fall significantly below zero. The United States, however, subsequently experienced a stronger expansion; and in the years 2005-07 the estimated output gap is generally positive in almost all these

[13] The results on the output gap, for the United States and other countries where lengthy quarterly time series are available, are generally in accord with conventional intepretations of macroeconomic history.

economies. With the 2008-09 recession, the estimated gaps show a sharp drop, to a trough of about -5 percent for the United States.

Inflation broadly followed the direction that one would expect, given the estimated evolution of the output gap. In the United States, core inflation declined somewhat during the years of the negative gap, dipping briefly below 2 percent, but rose in the second half of the decade, in line with the emergence of a positive gap. The influence of the gap on inflation is less visible in the other countries. Even in the United States the changes in core inflation were modest: in previous decades, positive values of the gap similar to those of 2005-07 were associated with much larger increases in inflation. On these grounds one might question whether the movements in the output gap during the 2000s were as large as the MV filter suggests. There are, however, several reasons not to be surprised by a reduced sensitivity of inflation. First, the credibility of the low inflation objective was firmly established by the late 1990s, as reflected in a wide body of evidence.[14] Inflation expectations became firmly anchored, at a low level, whereas in the previous decades they were adrift. Second, globalization muted the impact of a domestic output gap on prices; in particular, the vast increase in the export capacity of China and other newly industrialized economies reduced the ability of domestic firms to raise prices in response to high demand. Third, asset prices may have absorbed inflationary pressure that would normally affect the core consumer basket.

B. Potential Output

Figures 3 and 4 show the estimated year-over-year growth rate of potential GDP, with a confidence band, as well as the growth rate of actual GDP. The estimated growth of potential is correlated with actual growth. But, as one would expect, the estimated movements of potential are relatively smooth, such that the gap in most countries accounts for most of the short-term fluctuations in GDP (an apparent exception is the United Kingdom). Moreover, there is typically a short delay in the peaks and troughs of estimated potential behind actual growth.

The forecast suggests low potential growth in 2010 for most countries before the

[14] See, for example, Mishkin (2007), for a broad survey of the evidence for the United States, and Laxton and N'Diaye (2002) for international evidence.

steady-state rate is re-established. In contrast, the forecast for China shows a mild decline from a strongly positive growth path (figure 4).

The 2-standard-deviation confidence band is about +/- 1 percent points of the central estimate of potential growth.

C. Unemployment Rate and NAIRU

The MV filter estimates suggest that movements in the NAIRU are procyclical, and can be large (see figures 5 and 6). Moreover, the estimates have quite different trends in different countries.

The estimates for the United States show a trough at about 5 percent in 2001 (following a 20-year downtrend). After this, the estimated NAIRU rises steadily for a while, to exceed the calculated steady-state value by 2005. Following the onset of the financial crisis, the estimate climbs sharply. The chart for the U.K. NAIRU has a similar profile to that of the United States, but with more gradual movements. With the exception of China, the other countries in figures 5 and 6, show a generally declining, or flat, NAIRU over the years 2000-07.

The model indicates a rise in the NAIRU for most countries, in the period 2008-10. This can be attributed to lagged labor-market effects of the steep increase in actual unemployment 2008-09 – a hysteresis phenomenon.

D. Estimates of Gaps

Figures 7 and 8 compare the MV estimates of 3 deviations from equilibrium, i.e. the estimated gaps for GDP, the unemployment rate, and capacity utilization. As one would expect, in the light of Okun's Law, the estimates indicate that the unemployment gap is strongly correlated with the current and lagged output gap; it is also apparent that the unemployment gap has smaller cyclical fluctuations. These results would reflect well known features of the labour market, such as labor hoarding and the discouraged-worker effect during recessions.[15] As regards the capacity utilization gap, pronounced volatility is evident in the amplitude of the cycles relative to those for output and unemployment gaps; the declines during recessions are especially steep.

The forecast reflects these differing cyclical characteristics. Thus, from the 2009 trough, the capacity utilization gap rebounds with a short lag behind the output gap, while the unemployment gap shows a later and more prolonged recovery.

E. Variances of Output Gap Changes and Potential Growth

Figures 9 and 10 show the variance of output gap changes relative to the variance of actual output growth for each country, over horizons stretching from 1 to 20 quarters. The conventional view – at least for industrialized economies – is that changes in output gaps dominate short-term changes in GDP, while changes in potential dominate the long-run trend. Thus, one expects the line for the relative variance of output gap changes to start above 0.5 in quarter 1, and thereafter to fall towards zero. This is confirmed in the figures.

F. Refining Estimates: Adding Information to The Model

As stated before, the baseline parameterization of the MV filter stems from the application of a generalized approach, with minimal country-specific prior information. While this approach produces plausible results across countries, the estimates can readily be refined further in our framework. More detailed information could be used to fine-tune the prior distributions in the estimation procedure or, more directly, we could use outside estimates for one or more of the unobservable variables in the model.

For example, we re-estimated the Canadian model, using the Bank of Canada estimate of the output gap in place of the baseline model estimate. Through the 1990s and 2000s, both have a rising trend, but the Bank of Canada estimate has more cyclical variance (figure 11). Moreover, the model estimate has a positive gap (excess demand) emerging in 1999 and lasting until 2008, whereas the Bank of Canada estimate indicates

[15] Okun (1962) provided a thorough discussion of these features.

a negative gap 2001-04. It turns out that imposing the central bank estimate on the model makes a material difference to the MV filter estimate of the unemployment gap for Canada (lower panel, figure 11). What had been a chronic positive gap after 1998, which may be difficult to rationalize in view of the stable behavior of core inflation, now disappears: significant positive estimates do not emerge until after 2005.

V. Conclusions

We applied the MV filter, based on a small macroeconomic model, to a wide range of economies, deriving estimates and confidence intervals for potential output and other latent variables. In ex post forecasting exercises for numerous countries, the MV filter performs well relative to a random walk. Moreover, revisions to current estimates of the output gap, implied by a later data set, with much longer time series, are substantially less with the MV filter when compared to the HP filter. This is a substantial advantage for current analysis.

The MV estimates confirm that the growth rate of potential GDP varies substantially over time. Marked changes in the growth rate are correlated with the business cycle. However, the estimated gaps between actual and potential output also show important fluctuations, consistent with the historical movements in inflation.

Future work will apply the model to a larger number of countries, and the results will be refined by tailoring the priors to each country. We will also conduct more robustness checks, such as evaluating how the model performed in previous recessions and comparing the model to more sophisticated models of potential output.

VI. Appendix: Maximum Regularized Likelihood

Let θ be a vector of parameters, let Y be the data and $L(\theta; Y)$ be the data likelihood function. Then the objective function is

$$\max_{\theta} \log L(\theta; Y) - p\Sigma_i \frac{(\theta_i - \bar{\theta}_i)^2}{\sigma^2_{\bar{\theta}_i}}$$

where $\theta_i \in [\theta_i^L, \theta_i^U]$.

This method can be interpreted as a simple Bayesian technique where the prior for each parameter is a normal distribution with mode $\bar{\theta}_i$ and variance $\frac{1}{p}\sigma^2_{\bar{\theta}_i}$, truncated at θ_i^L from below and θ_i^U from above. The parameter estimate can be seen as the mode of the posterior distribution.

Notice that the smaller p, the looser the prior for each parameter, for any given $\sigma^2_{\bar{\theta}_i}$. We use $p = 1$ so that the numbers presented in Table 3 are readily interpreted as the standard deviations of the priors. For more information on the technique, please see (Ljung, 1999).

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Table 1: Country-specific steady-state priors

Country Name	θ	G^{ss}	U^{ss}
U.S.	0.70	2.14	5.80
Germany	0.64	1.16	7.55
U.K.	0.67	2.00	5.42
France	0.63	1.36	6.95
Italy	0.61	0.70	7.40
Canada	0.70	1.96	6.10
Euro area	0.62	1.32	8.43
Australia	0.70	3.17	4.70
Norway	0.70	2.01	3.50
New Zealand	0.70	2.36	4.52
Spain	0.62	1.59	14.72
China	0.50	9.18	4.00

Table 2: Data Sources (United States)

United States	
Y	Gross Domestic Product (SAAR, Bil.Chn.2000.Dollars)
C	Capacity utilization in manufacturing sector (Haver)
$\pi 4$	Year-over-year rate of core CPI inflation (Haver)
$\pi 4^{LTE}$	Long term inflation expectations (Consensus Economics)
U	Civilian unemployment rate (SA, percent)

Table 3: Maximum Regularised Likelihood (United States)

Parameter	Prior		Posterior	
	Mode	Dispersion	Mode	Dispersion
$G^{\overline{Y}}_{SS}$	2.140	0.300	2.143	0.021
U^{ss}	5.800	0.300	5.801	0.023
θ	0.700	0.030	0.700	0.037
β	0.400	0.300	0.176	0.019
Ω	0.500	0.300	0.265	0.036
ϕ_1	0.800	0.150	0.729	0.018
ϕ_2	0.300	0.150	0.241	0.014
κ_1	0.100	0.600	0.353	0.020
κ_2	1.500	1.500	1.421	0.021
ω	3.000	1.500	2.796	0.071
λ	2.000	3.000	2.078	0.214
α	0.900	0.150	0.895	0.399
τ	0.100	0.150	0.113	0.426
δ	0.500	0.150	0.496	0.018
ρ_1	0.800	0.150	0.821	0.018
ρ_2	5.000	3.000	5.145	0.033
$\sigma_{\varepsilon^{\pi 4}}$	0.500	0.300	0.556	0.021
$\sigma_{\varepsilon^{u}}$	0.500	0.300	0.199	0.004
$\sigma_{\varepsilon^{c}}$	0.400	0.300	0.484	0.029
$\sigma_{\varepsilon^{\overline{U}}}$	0.100	0.150	0.079	0.008
$\sigma_{\varepsilon^{G^{\overline{U}}}}$	0.100	0.150	0.087	0.039
$\sigma_{\varepsilon^{\overline{Y}}}$	0.250	0.100	0.218	0.042
$\sigma_{\varepsilon^{G^{\overline{Y}}}}$	1.000	0.300	0.925	0.013
$\sigma_{\varepsilon^{\overline{C}}}$	0.250	0.150	0.290	0.024
$\sigma_{\varepsilon^{G^{\overline{C}}}}$	0.075	0.030	0.076	0.041
$\sigma_{\varepsilon^{\pi 4LTE}}$	0.300	0.300	0.081	0.042
$\sigma_{\varepsilon^{y}}$	1.000	0.300	0.892	0.004

Table 4: Forecasting and Real-Time Properties (United States)

Forecasting year-over-year core CPI inflation (RMSE)	*quarters ahead*			
	1	4	8	12
Baseline	0.500	1.338	1.870	2.186
Random walk	0.555	1.602	2.307	2.628
Estimating the output gap (MAE)	End point			
Baseline	0.365			
HP filter	1.242			

Figure 1: Output Gap Estimate with 2-Standard Deviation Confidence Interval, and Core Inflation (percent)

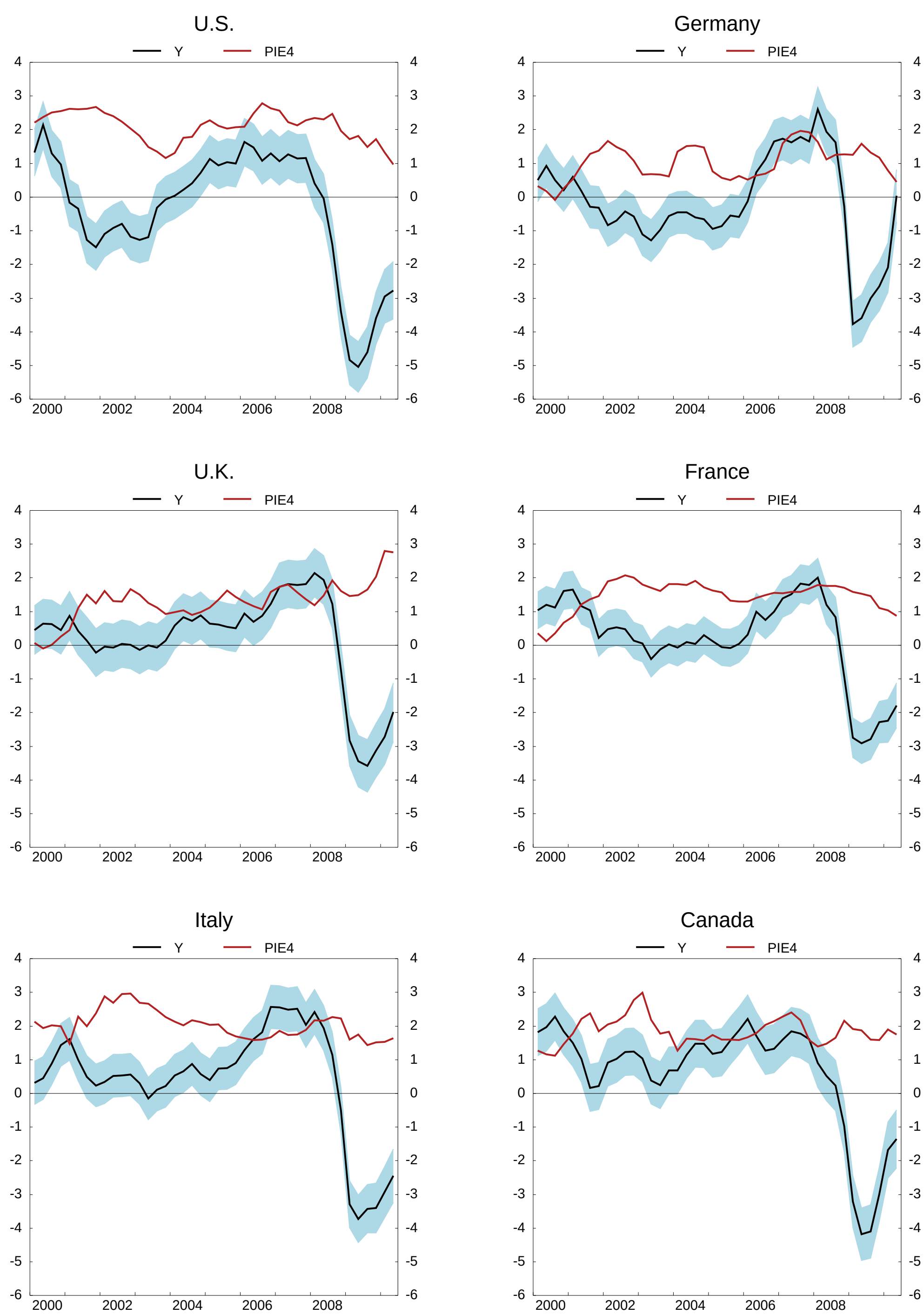

Figure 2: Output Gap Estimate with 2-Standard Deviation Confidence Interval, and Core Inflation (percent)

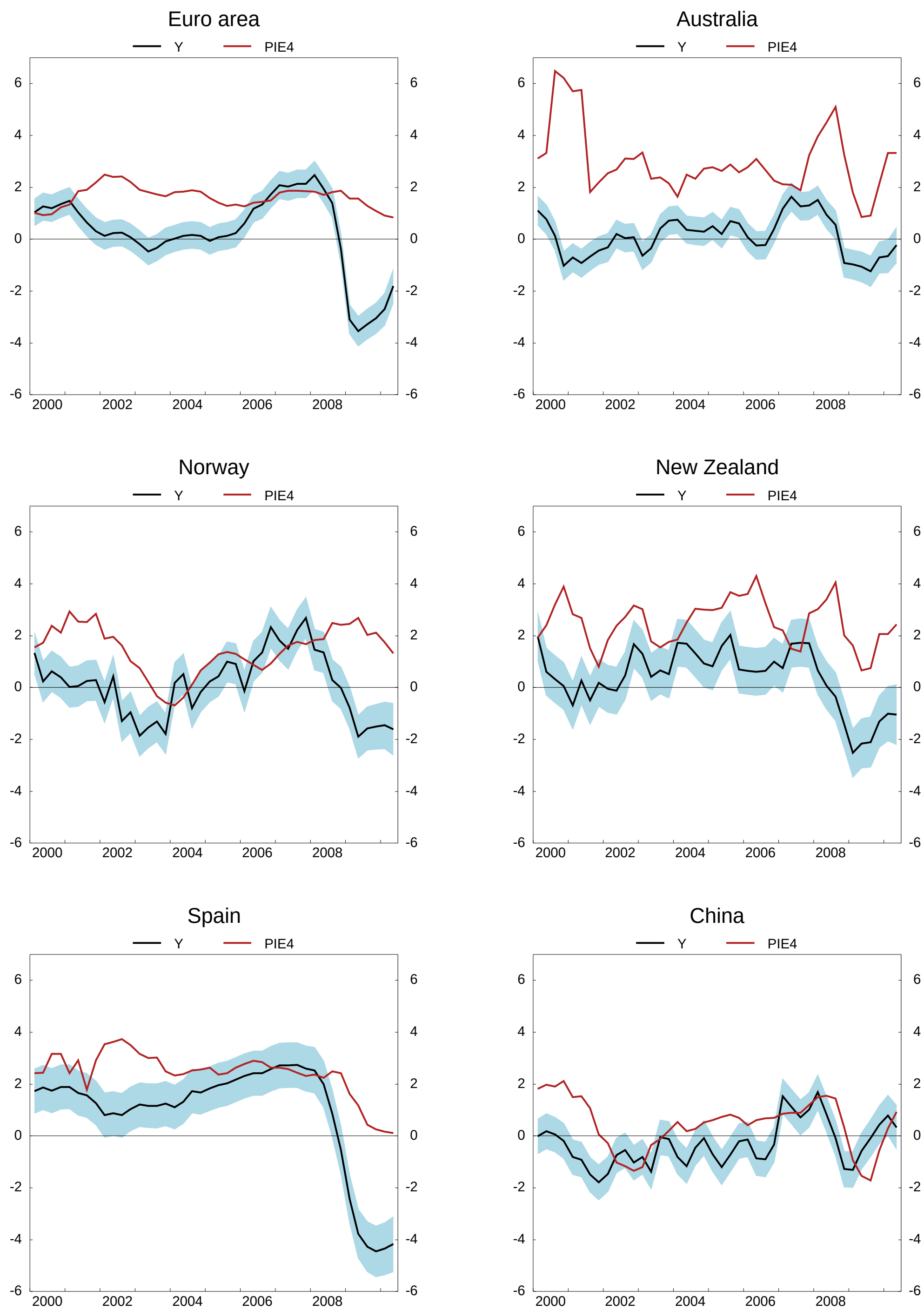

Figure 3: Year/Year Growth in Potential Output with 2-Standard Deviation Confidence Interval (percent)

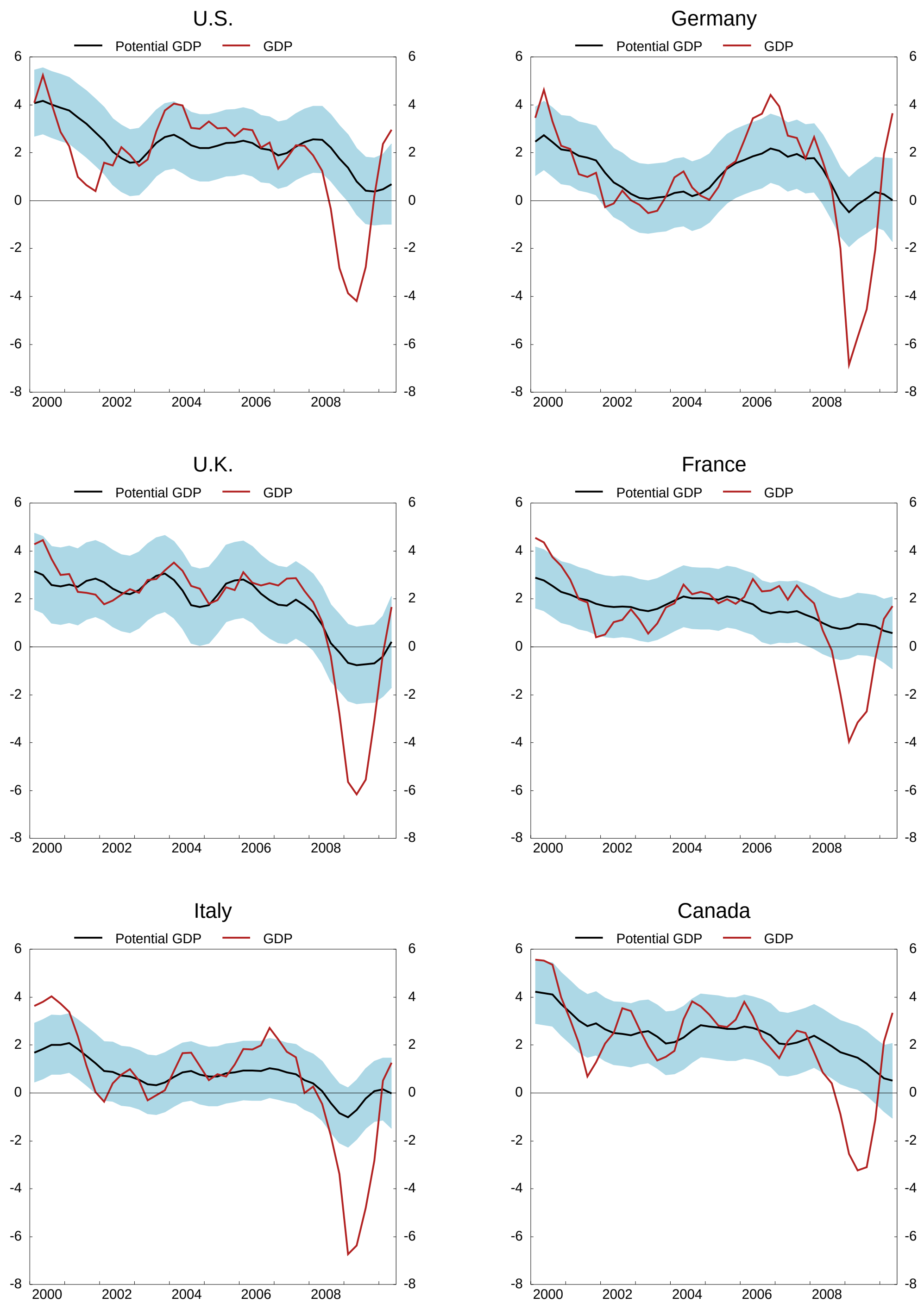

Figure 4: Year/Year Growth in Potential Output with 2-Standard Deviation Confidence Interval (percent)

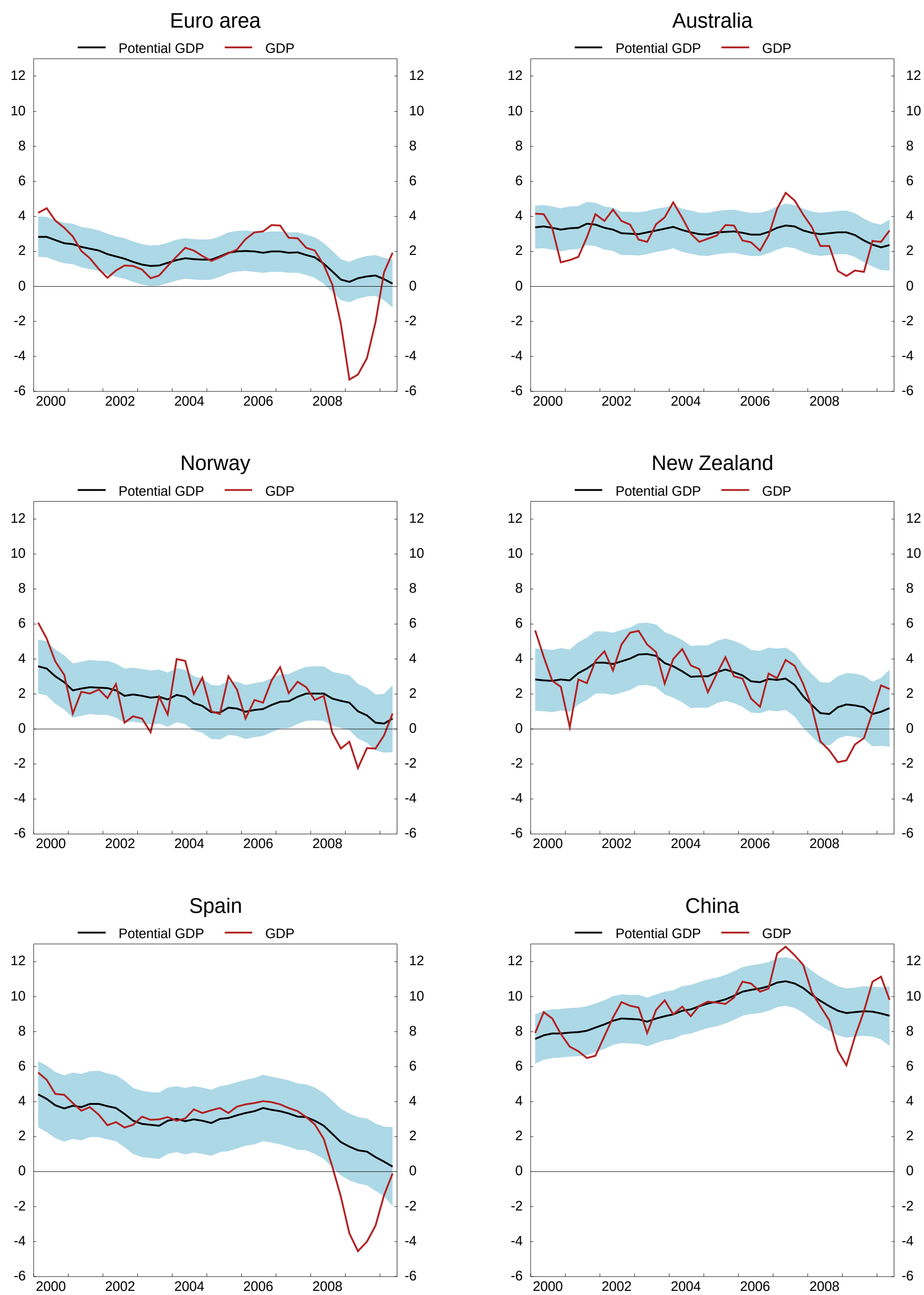

Figure 5: NAIRU Estimate with 2-Standard Deviation Confidence Interval, and Unemployment Rate (percent)

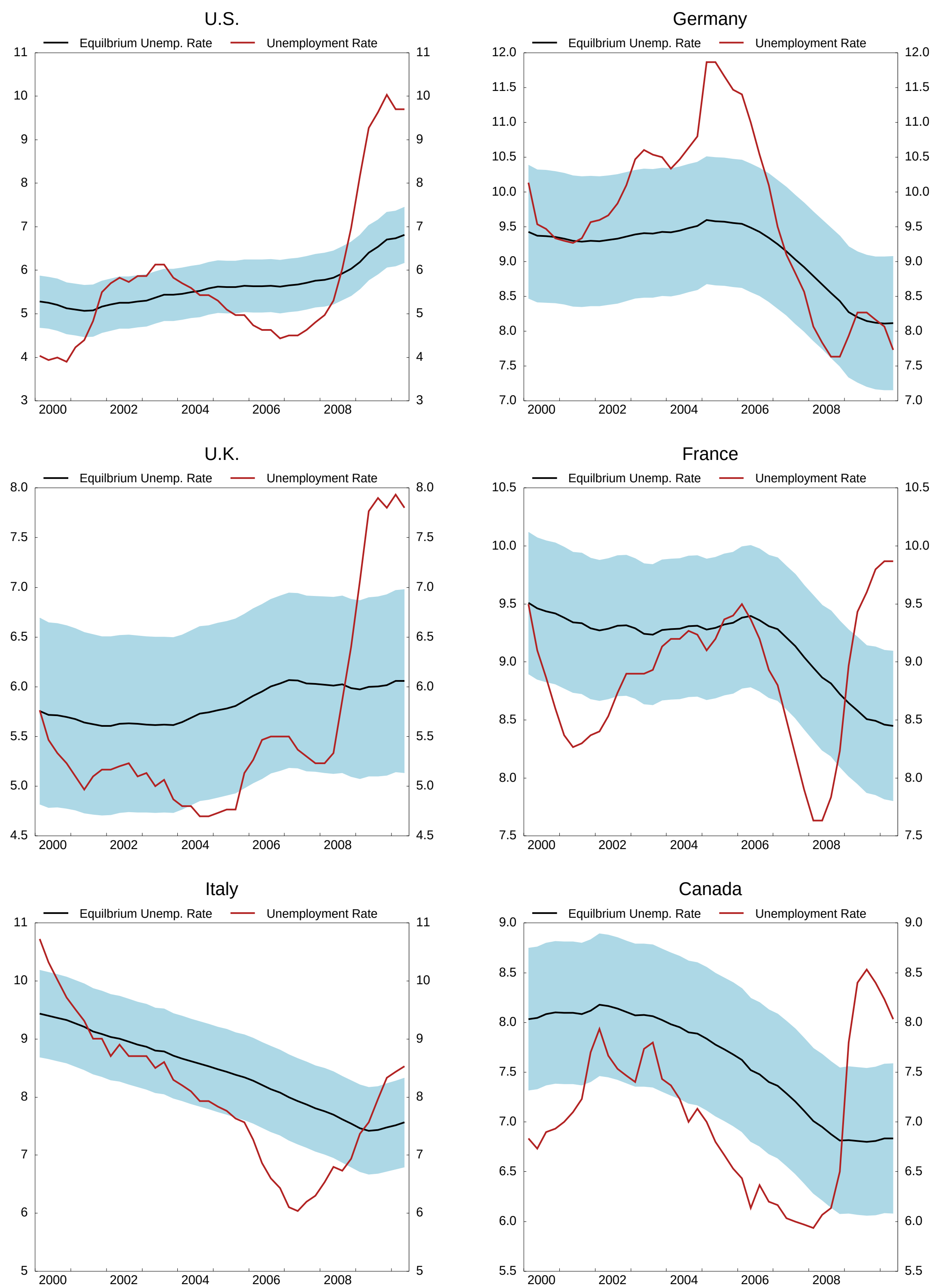

Figure 6: NAIRU Estimate with 2-Standard Deviation Confidence Interval, and Unemployment Rate (percent)

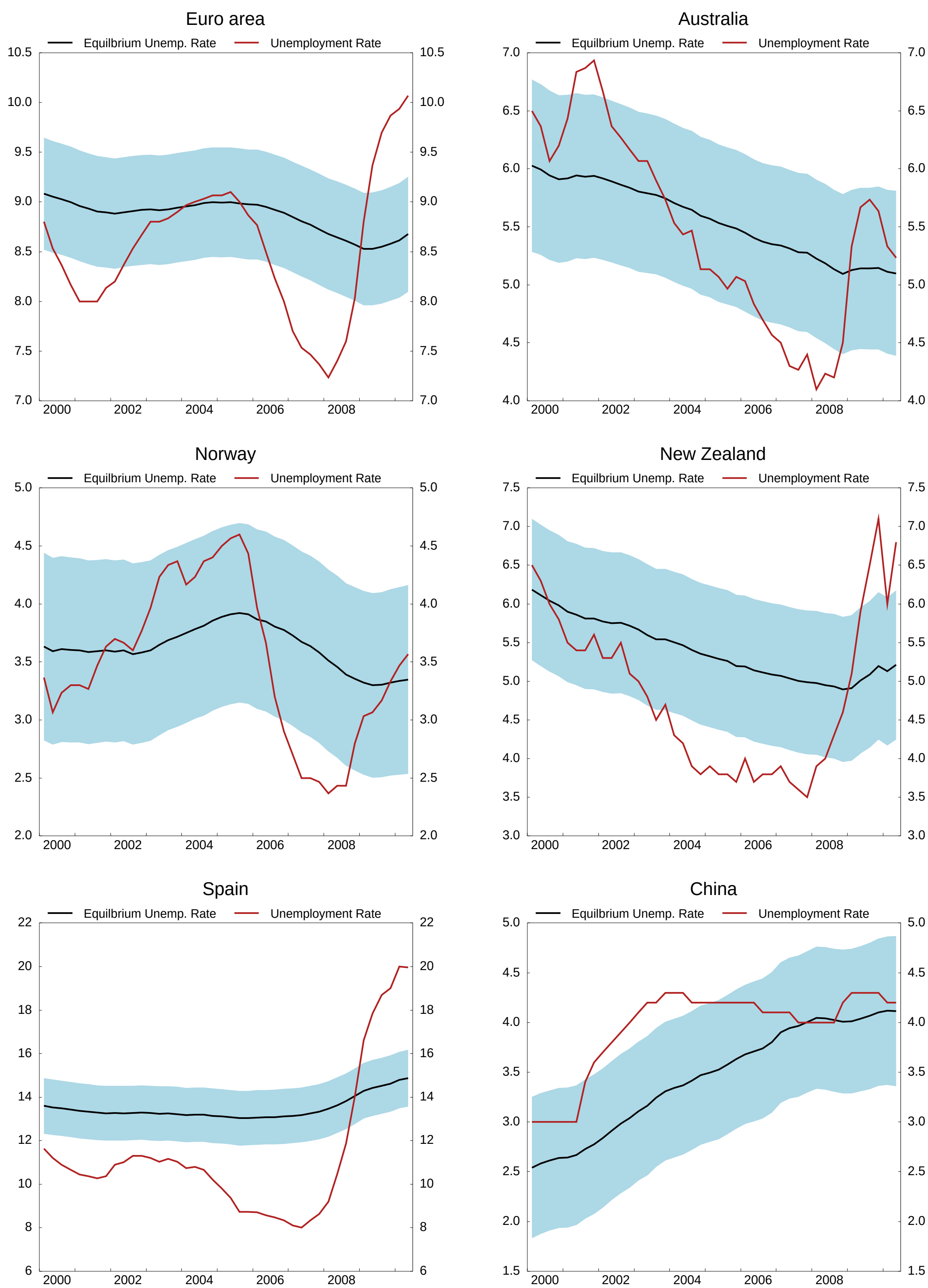

Figure 7: Output Gap, Unemployment Gap, and Capacity Utilization Gap (percent)

Figure 8: Output Gap, Unemployment Gap, and Capacity Utilization Gap (percent)

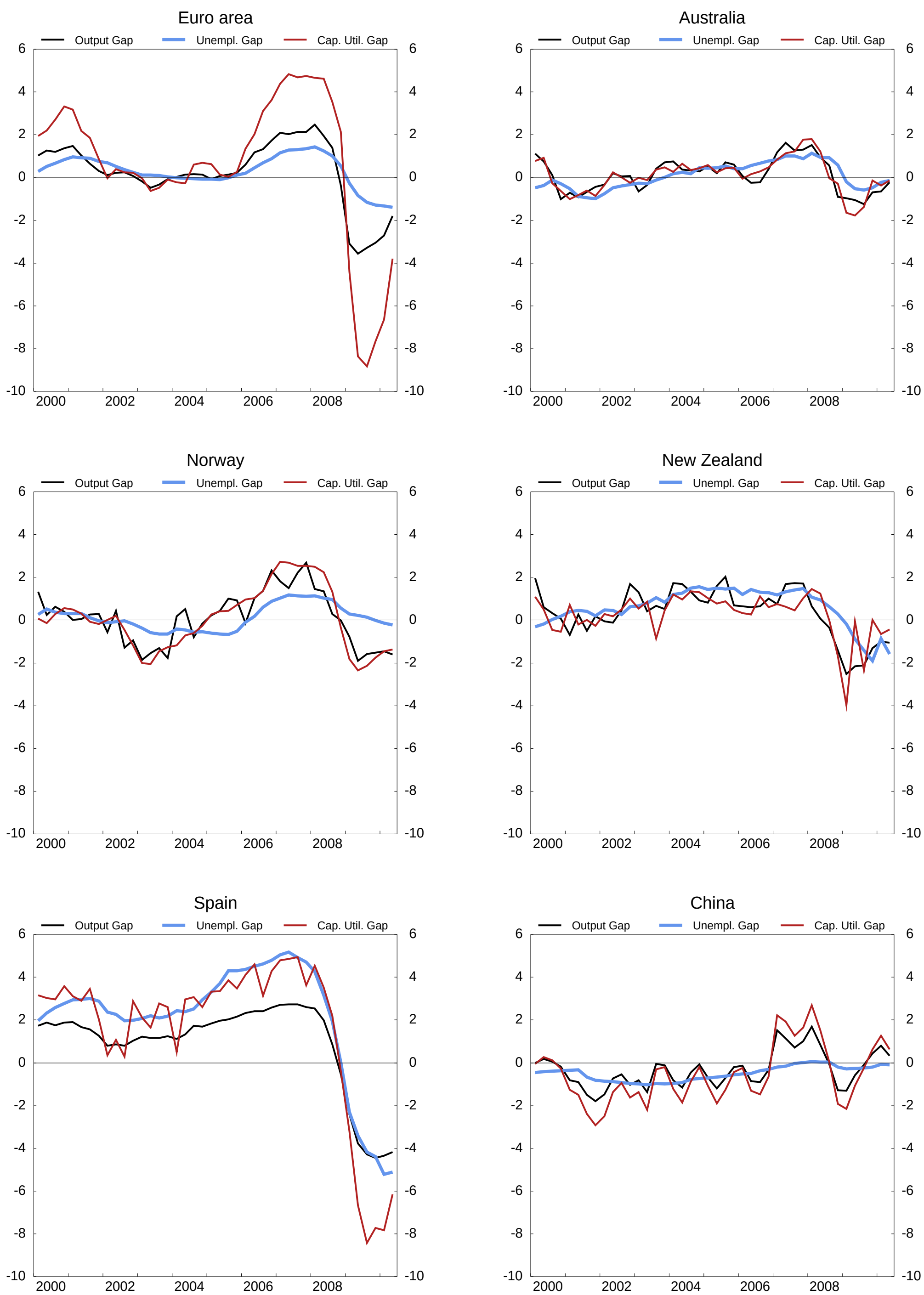

Figure 9: Variance of Output Gap Changes Relative to Variance of Actual Output Growth at Different Horizons

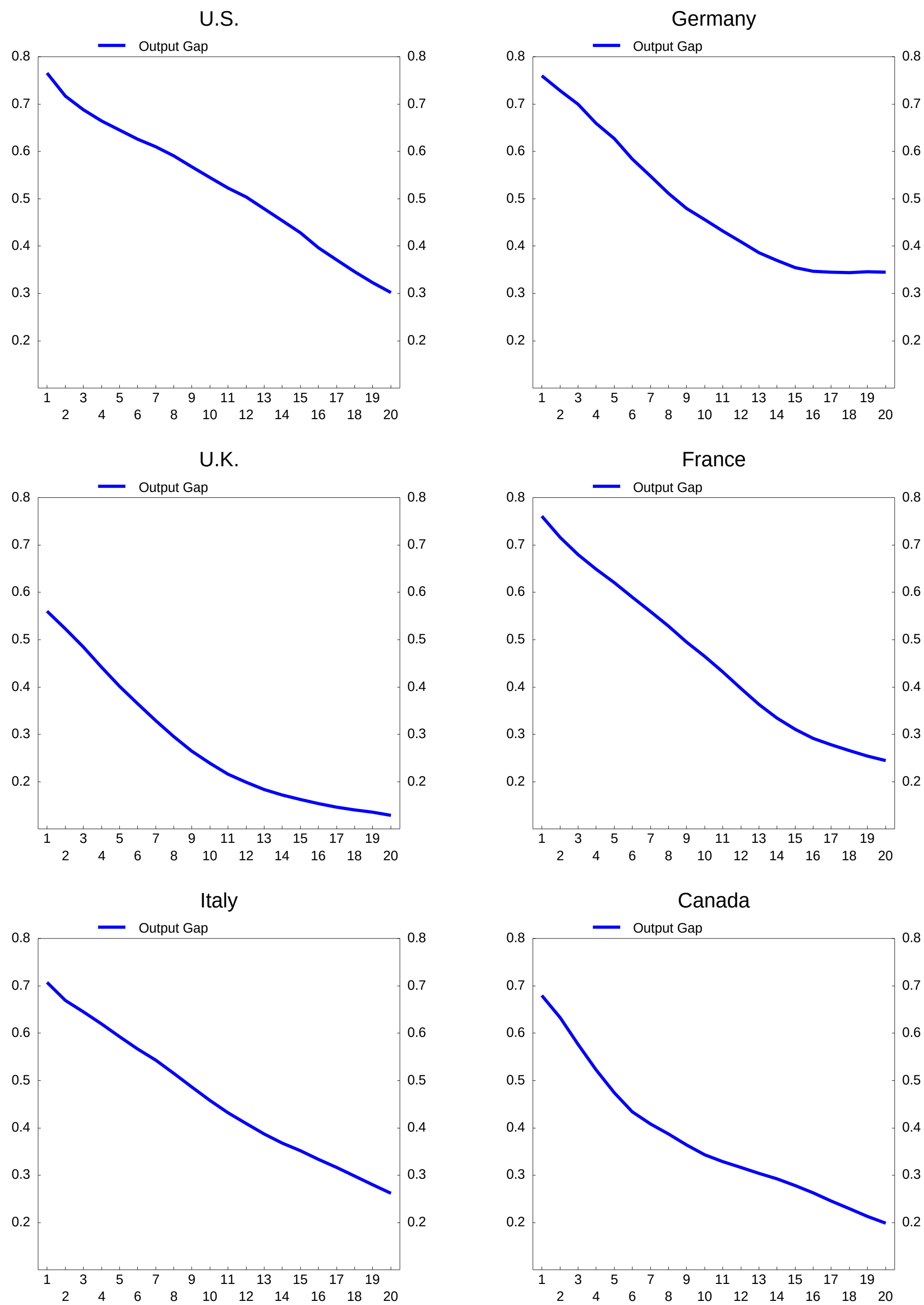

Figure 10: Variance of Output Gap Changes Relative to Variance of Actual Output Growth at Different Horizons

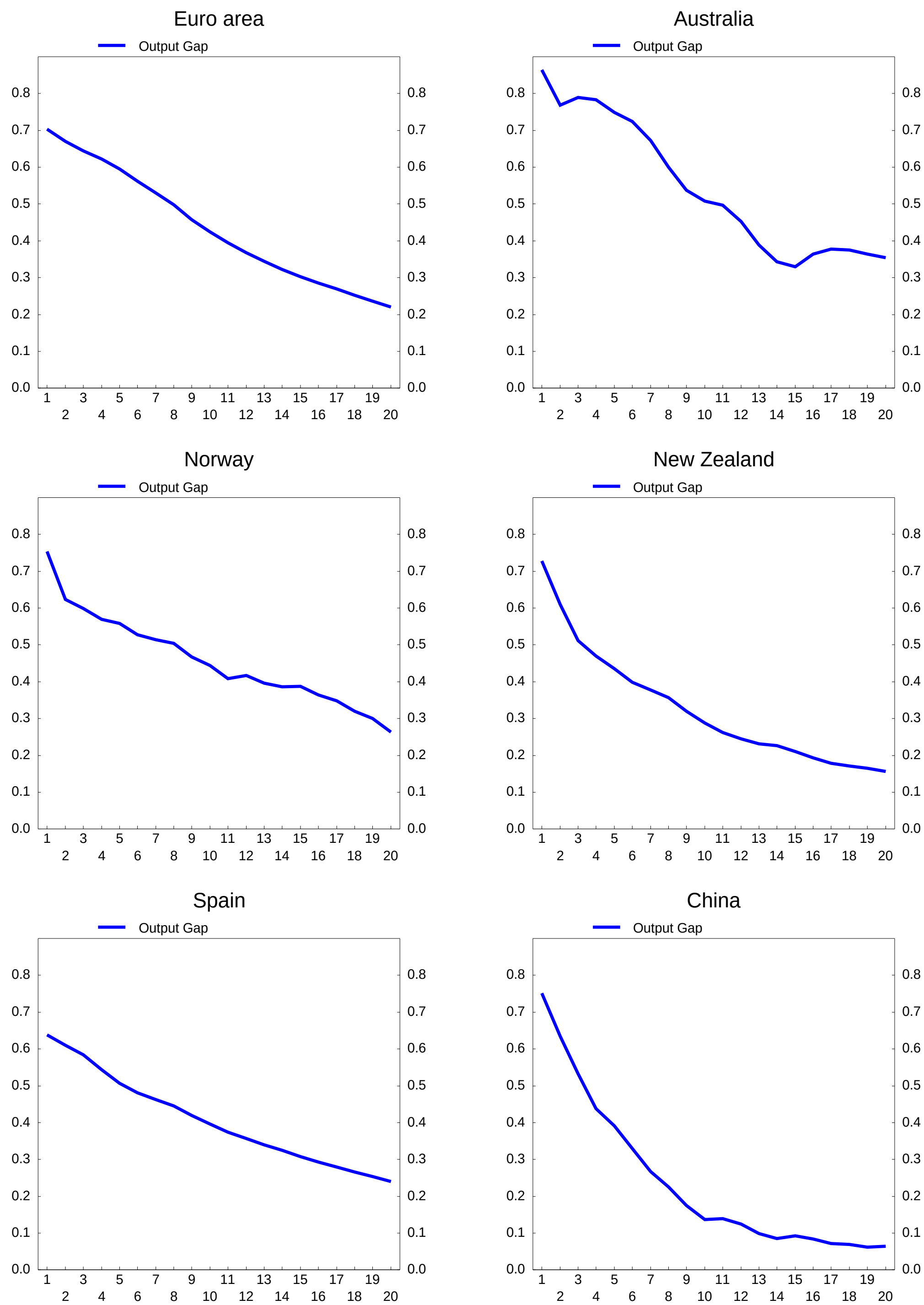

Figure 11: Adding More Information to The Canadian Model