WP/11/69

IMF Working Paper

Fiscal sustainability and the fiscal reaction function for South Africa

Philippe Burger, Ian Stuart, Charl Jooste, and Alfredo Cuevas

INTERNATIONAL MONETARY FUND

 WP/11/69

IMF Working Paper

AFR

Fiscal sustainability and the fiscal reaction function for South Africa

Prepared by Philippe Burger, Ian Stuart, Charl Jooste, and Alfredo Cuevas

Authorized for distribution by Abebe Aemro Selassie

March 2011

Abstract

How does the South African government react to changes in its debt position? In investigating the question, this paper estimates fiscal reaction functions using various methods (OLS, VAR, TAR, GMM, State-Space modelling and VECM). The paper finds that since 1946 the South African government has ran a sustainable fiscal policy, by reducing the primary deficit or increasing the surplus in response to rising debt. Looking ahead, the paper considers the use of fiscal reaction functions to forecast the debt/GDP ratio and gauging the likelihood of achieving policy goals with the aid of probabilistic simulations and fan charts.

JEL Classification Numbers:H62; H63

Keywords: Fiscal reaction function; public debt; deficits.

Author's E-Mail Address:acuevas@imf.org

Contents Page

Annexes

Fiscal sustainability and the fiscal reaction function for South Africa:

Assessment of the past and future policy applications

Philippe Burger[1], Ian C. Stuart[2], Charl Jooste[2] and Alfredo Cuevas[3]

I. Introduction

Having reduced its debt burden over the past thirteen years, the South African government again finds itself facing rising debt. This increase resulted mainly from falling tax revenues brought on by the 2008/9 global financial crisis. Understanding how the South African government has in the past reacted to the variation in public debt aids the assessment of the sustainability of fiscal policy and the ambitiousness of government's proposed fiscal consolidation.

This paper first considers South Africa's past debt trajectory and the theoretical underpinnings of fiscal reaction functions, followed by estimates of the fiscal reaction function. The fiscal reaction functions are estimated using a variety of methods, including Ordinary Least Squares (OLS), Vector Autoregression (VAR), General Method of Moments (GMM) and Vector Error-Correction (VECM). To deal with non-linearities, fiscal reaction functions are also estimated using State-Space and Threshold Autoregressive (TAR) modelling. Looking ahead, the paper considers the use of fiscal reaction functions to forecast debt and estimate the probability of achieving the medium term targets outlined in the 2010 budget. The paper concludes that fiscal policy will remain sustainable in the next few years, with limited risk of significant upward pressure on public debt. The projected budget deficit reduction outlined in recent budget documentation is quite feasible by historical standards.

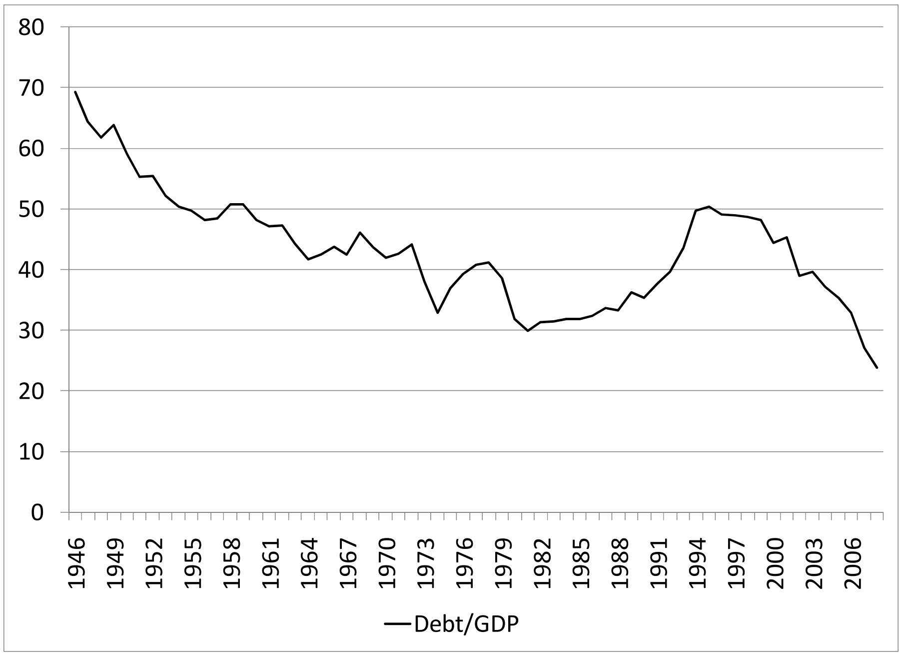
II. South Africa's past debt trajectory

Unlike that of many OECD countries, the South African public debt/GDP ratio has not exceeded the 50 percent mark since 1960 (see Figure 1).[4] However, the public debt/GDP ratio did increase significantly in the early 1990s, immediately prior to the first democratic election. Weak economic growth resulting from low investment and a lack of investor confidence contributed to lower revenue collection. Political tension, combined with domestic and international recession

[1] University of the Free State.
[2] National Treasury of South Africa.
[3] International Monetary Fund.
[4] In the late 1990 it was widely believed that the debt/GDP ratio had reached 60 percent in the mid-1990s. However, there was also an increasing awareness that the GDP figures (i.e. the denominator of the debt/GDP ratio) did not capture the informal sector of the economy. Thus, when in 1999 the GDP figures were revised to include the informal sector more fully, the mid-1990s debt/GDP ratio fell from approximately 60 percent to 50 percent.

meant that the government could not introduce expenditure cuts. The large deficits of the early 1990s, averaging about 4.5 percent per year, caused the debt/GDP ratio to increase from 35 percent in 1990 to 50 percent in 1995.

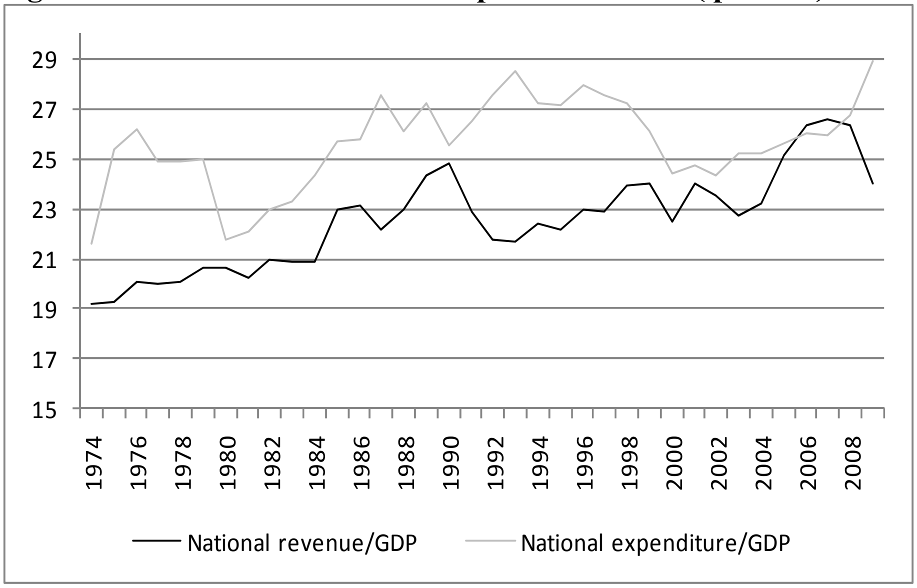
Figure 1 – Public debt/GDP in South Africa (percent)



Source: Statistics South Africa, National Treasury and author's calculations.

With the debt/GDP ratio at its post-apartheid peak in 1996/7, debt service costs amounted to 15 percent of revenue, making it one of the largest expenditure items on the government budget. Through its Growth, Employment and Redistribution (GEAR) strategy, the new democratic government aimed to reduce the conventional budget deficit/GDP ratio to below 3 percent per year. Specifically, the government reversed some of the increases in expenditure as a percentage of GDP, while improved growth and better tax administration resulted in revenue growth. The government continued pursuing a prudent fiscal policy well through the following decade. Indeed, in 2006 and 2007 government registered a small budget surplus taking advantage of the boom of the middle of the last decade (see Figure 2) and the public debt/GDP ratio decreased to 23.8 percent by 2008. Public debt started rising again as government ran larger fiscal deficits to provide countercyclical fiscal stimulus to combat the 2008/09 recession. Debt will continue to grow for a few more years, as the exit from stimulus is set to take place gradually. The 2010 Budget projected that it would rise somewhat above 40 percent of GDP by 2013.

Figure 2 – National revenue and expenditure/GDP (percent)



Source: Statistics South Africa, National Treasury and authors' calculations.

III. Fiscal reaction functions and debt sustainability – the basics

Establishing how government reacts to its debt burden can be done through the estimation of a fiscal reaction function. Fiscal reaction functions usually specify, for annual data, the reaction of the primary balance/GDP ratio to changes in the one-period lagged public debt/GDP ratio, controlling for other influences. According to Bohn (1995, 2007) this represents an error correction mechanism: If the public debt/GDP ratio increases, government should respond by improving the primary balance, to arrest and even reverse the rise in the public debt/GDP ratio. The rationale behind this is rooted in the budget constraint of government (cf. Bohn 1998, 2007, Gali and Perotti 2003, De Mello 2005). In simplified terms, this constraint can be written as:

$$D_t = D_{t-1} + iD_{t-1} - B_t \qquad (1)$$

where:
D: Public debt
i: Nominal interest rate on government bonds
B: Primary balance (+ surplus; - deficit)

Using Equation (1), substituting forward, and taking expectations yields the expression:

$$D_t = \sum_{j=1}^{\infty} \rho_j E_t(B_{t+j}) + \; lim_{j\to\infty} \rho_t E_t[D_{t+j}] \qquad (2)$$

where: $\rho_j = \prod_{s=1}^{j} \beta_s$ and $\beta_s = 1/(1 + i_s)$

Upon imposing the standard transversality condition, the second term in Equation (2) falls away, which then means that in a sustainable equilibrium debt equals the sum of the discounted value of all future primary surpluses.

One can also use Equation (1) to derive Equation (3) for the change in the level of indebtedness measured against the yardstick of GDP (where we have omitted the time index from the parameters r and g to prevent clutter):

$$\Delta(D/Y)_t = ((r - g)/(1 + g))(D/Y)_{t-1} - (B/Y)_t \qquad (3)$$

where:
r: real interest rate
g: real economic growth rate
Y: nominal GDP

Equation (3) leads immediately to the well known expression for the primary balance that will ensure the debt/GDP ratio remains unchanged:

$$(B/Y)_t = ((r - g)/(1 + g))(D/Y)_{t-1} \qquad (4)$$

If we start from a position in which debt levels are considered acceptable, Equation (4) can be interpreted as a fiscal rule, with the rule defining the primary balance/GDP ratio required to keep to such a debt/GDP target. To study the actual behaviour of government, one can estimate a fiscal reaction function of the analogous form:

$$(B/Y)_t^{Act} = \alpha^*(D/Y)_{t-1}^{Act} + \varepsilon_t \qquad (5)$$

where superscript 'Act' indicates the actual time series, as opposed to the required primary balance/GDP ratio or the target debt/GDP ratio, and where one might conjecture that the coefficient α^* should be on average equivalent to $(r - g)/(1 + g)$.

To allow for inertia in government behaviour, a lag of the primary balance, $(B/Y)_{t-1}^{Act}$, can be added to the right-hand side of Equation (5) (De Mello 2005:10). The output gap, $\hat{y}$, can also be added to the right-hand side as a control variable to allow for the possibility that government pursues short-run demand stabilisation (Bohn 1998:951; De Mello 2005:10; cf. Taylor 2000 for a rule based on the output gap). The basic fiscal reaction function is then specified as:

$$(B/Y)_t^{Act} = \alpha_1 + \alpha_2(B/Y)_{t-1}^{Act} + \alpha_3(D/Y)_{t-1}^{Act} + \alpha_4(\hat{y})_t + \varepsilon_t \qquad (6)$$

Before estimating it, a question can be raised regarding the stationarity of the variables in Equation (6). Note that if the change that government brings about in the primary balance in reaction to changes in the level of debt, i.e. $\alpha_3/(1 - \alpha_2)$ from Equation (6),[5] is close to α^* in

[5] Where $\alpha_3/(1 - \alpha_2)$ is the long-term reaction taking into consideration the short-run reaction, α_3, and the level of inertia, α_2.

Equation (5), it would mean that government is attempting to stabilize its debt ratio at the realised level in the previous period. Indeed, if $\alpha_3/(1 - \alpha_2) = \alpha^* = (r - g)/(1 + g)$ the debt/GDP ratio and the primary balance will be first-difference stationary, while if $\alpha_3/(1 - \alpha_2) > \alpha^* = (r - g)/(1 + g)$ the debt/GDP ratio and the primary balance will be stationary. That said, Equation (3) can be rewritten thus:

$$(D/Y)_t = ((1 + r)/(1 + g))(D/Y)_{t-1} - (B/Y)_t \qquad (7)$$

Equation (7) indicates that the debt/GDP ratio depends on its own lag, the interest rate, the economic growth rate and the primary balance. Even though the debt/GDP series may be stationary, standard stationarity tests could have difficulty rejecting the null hypothesis of a unit root (Bohn 1998:955). For instance, if r = 2 percent and g = 4 percent, then $((1 + r)/(1 + g)) = 0.98$, which is very close to a unit root. So, we have both a potential issue with the data, and a difficult case for testing.

The paper addresses the problem of possible non-stationarity in the data from three angles. It first estimates Equation (6) with OLS, VAR and GMM as if the data were stationary. Secondly it estimates Equation (6) as State-Space and TAR models assuming non-linearities in behaviour. Thirdly it treats the debt/GDP ratio and the primary balance/GDP data as if the data are non-stationary and estimates a Vector Error-Correction model (VECM). In the case of a VECM one does not estimate Equation (6) directly, but rather a model containing Equations (8) and (9) (assume for reasons of exposition that there is one lag in the short-run dynamics of the model), with Equation (6) then derived from Equation (8).

$$\Delta(B/Y)_t^{Act} = c_{11} + \alpha_{12}((B/Y)_{t-1}^{Act} - \beta_{12}(D/Y)_{t-1}^{Act} - \beta_{13}) + \Gamma_{11}\Delta(B/Y)_{t-1}^{Act} + \Gamma_{12}\,\Delta(D/Y)_{t-1}^{Act} + \varphi_4(\hat{y})_t + \varepsilon_{11t} \qquad (8)$$

$$\Delta(D/Y)_t^{Act} = c_{21} + \alpha_{13}((B/Y)_{t-1}^{Act} - \beta_{12}(D/Y)_{t-1}^{Act} - \beta_{13}) + \Gamma_{21}\,\Delta(B/Y)_{t-1}^{Act} + \Gamma_{22}\,\Delta(D/Y)_{t-1}^{Act} + \varphi_4(\hat{y})_t + \varepsilon_{21t} \qquad (9)$$

where $(B/Y)_{t-1}^{Act} - \beta_{12}(D/Y)_{t-1}^{Act} - \beta_{13}$ in both Equations (8) and (9) represents the deviation from the long-run relationship given by:

$$(B/Y)_{t-1}^{Act} = \beta_{12}(D/Y)_{t-1}^{Act} + \beta_{13}. \qquad (10)$$

Equations (8) and (9) include the output gap in the short-run dynamics of the model, catering for the possibility that fiscal policy might react to the business cycle. The fiscal reaction to the debt/GDP position is captured by α_{12} in Equation (8), which represents the error correction term, i.e. the response of B/Y to deviations from the long-run relationship captured in Equation (10). Equation (8) can be rewritten as a VAR in levels, which in turn can be used to obtain a VECM equivalent of Equation (6):

$$(B/Y)_t^{Act} = c_{11} - \alpha_{12}\beta_{13} + (1+\alpha_{12}+\Gamma_{11})(B/Y)_{t-1}^{Act} - \Gamma_{11}(B/Y)_{t-2}^{Act} + (-\alpha_{12}\beta_{12}+\Gamma_{12})\,(D/Y)_{t-1}^{Act} - \Gamma_{12}(D/Y)_{t-2}^{Act} + \varphi_{11}(\hat{y})_t + \varepsilon_{11t} \qquad (8a)$$

Parameter α_1, α_2 and α_3 in Equation (6) can then be arrived at by summing parameter values over the lags in Equation (13), so that $\alpha_1 = c_{11} - \alpha_{12}\beta_{13}$, $\alpha_2 = (1+\alpha_{12})$ and $\alpha_3 = -\alpha_{12}\beta_{12}$.

IV. Data and methods

The fiscal reaction function as specified in Equation (6) is used for modelling purposes below. Public debt/GDP data for South Africa can be found in the South African Reserve Bank (SARB) database and represents national government debt (provinces and local authorities have no or negligible power to issue securities). The annual data series exists from 1946. The South African government published primary balance data for a while in the late 1990s in its Budget Review, but ceased its publication subsequently. Thus, primary balance data for national government based on Government Finance Statistics (GFS) was obtained from the South African Reserve Bank for the period 1974 to 2008. To obtain a time series that is even longer, the paper uses System of National Accounts (SNA) data, available since 1946, to reconstruct the general government primary balance.[6] Furthermore, Equation (6) also includes the output gap, which is constructed in two ways, first with a Hodrick-Prescott filter[7] and secondly with a Kalman filter.[8] Equation (6) was estimated for both fiscal and calendar years. The GFS data pertain to fiscal years, while SNA data pertain to calendar years. When using SNA data for the primary balance, the debt/GDP ratio is in calendar years, while in the case of GFS data the ratio is in fiscal years.

The paper uses a variety of modelling techniques to ensure robustness and explore various aspects of the data. Initially we estimate simple OLS models. Because there might be concern about simultaneity, non-linearity and more complex interactions between variables, the paper also presents fiscal reaction functions estimated with VAR, GMM, and TAR models. The VAR captures multiple interactions between the variables, while the GMM estimations address the concern that the explanatory variables and the error term might be correlated due to non-linearity, measurement error or simultaneity (*cf.* Pindyck and Rubinfeld 1998:295). In a fiscal reaction function, one would expect simultaneity given the presence of lagged debt/GDP and the output gap. For the GMM estimation, the lags of the explanatory variables are taken as

---

[6] Until recently this was straightforward given that the SARB tables reported current income and current expenditure of general government. Using interest payments, depreciation and investment by general government, one could reconstruct the primary balance. However, the production, distribution and accumulation accounts of the national accounts introduced in 2006 cover the detail of sectors differently. Therefore, with no current expenditure and revenue reported explicitly, the balances had to be reconstructed from the production, distribution and accumulation account for general government. This reconstruction in addition to earlier data yielded an annual series available since 1946.

[7] To tackle the end-point problem in calculating the HP trend (see Mise et al, 2005a,b), an AR(n) model (with n set at 12 to eliminate serial correlation) was used. The AR model was used to forecast two additional years that were then added to each of the series before applying the HP filter.

[8] Signal extraction using a Kalman filter allows for separating out an unobservable component from an observable component that contains noise. The model is $x_t = E_t x_t + \varepsilon_t$ and $E_t x_t = E_{t-1} x_{t-1} + v_t$ where x_t and $E_t x_t$ are respectively the observable variable and unobservable component (*cf.* Valente 2003:526).

instruments. A TAR model considers differentiated reactions of the primary balance/GDP ratio to positive and negative output gaps.

The paper also estimates Equation (6) using State-Space modelling over the longest sample to investigate parameter changes. More specifically Equation (6) becomes the signal equation in which the debt/GDP parameter, α_3, is specified as a state variable that is allowed to vary over time. The other parameters are specified as fixed parameters, while α_3 is specified as a random walk (see Rapach and Weber 2004). If the government's choice of primary balance value depends on the primary balance needed to stabilise the debt/GDP ratio (i.e. if the size of α_3 is influenced by $(r - g)/(1 + g)$), then the data generating process underlying r and g, will also influence α_3. The log of GDP usually is an I(1) variable, so g is I(0). Rapach and Weber (2004), however, showed that real interest rates in many countries are non-stationary variables. A non-stationary r will also cause $(r - g)/(1 + g)$ to be non-stationary. Thus, if α_3 depends on $(r - g)/(1 + g)$, then α_3 can be expected to be non-stationary – hence the decision to specify α_3 as a random walk. The fixed parameter models can only be estimated for 1974 to 2008 because structural changes imply that those specifications would not be adequate for longer sample periods. The State-Space model, with its variable parameter, is estimated for the period 1946 to 2008.

The paper also estimates the vector error-correction model contained in Equations (8) and (9). The model is estimated below using the Johansen procedure, where Equations (8) and (9) are modelled as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{t=i}^{k} \Gamma_i \Delta X_{t-i} + \sum_{t=j}^{n} \varphi_j Z_{t-j} + c_t + \varepsilon_{kt} \qquad (11)$$

where $X_t = (B/Y, D/Y, Constant)$ is a 3x1 vector that includes the endogenous I(1) variables), $Z_t = (\hat{y})$ is a 1x1 vector that includes the exogenous I(0) variables, Γ_i are 2x2 short-run coefficient matrices; φ_j is a 2x1 vector containing coefficients of the exogenous variable, c_t is a vector containing the constants, and ε_{kt} are normally and independently distributed error terms. The trace test is used to determine the number of cointegrating vectors. Π in Equation (11) can be decomposed as the following α and β' matrices:

$$\Pi X_{t-1} = \alpha\beta' X_{t-1} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} 1 & -\beta_{12} & -\beta_{13} \end{bmatrix} \begin{bmatrix} (B/Y)_{t-1} \\ (D/Y)_{t-1} \\ 1 \end{bmatrix} \qquad (12)$$

where α is a 2x1 matrix (two variables and one cointegrating relationship) that contains the error-correction (adjustment) parameters, and β' is a 1x3 matrix that contains the long-run parameters (including a constant).

V. **Estimation results**

How has the South African government reacted to its debt position and the cycle? Did its reaction ensure the sustainability of fiscal policy? Has the reaction function evolved over time? This section seeks to answer these and other related questions. We first explore some time series properties of the data with a view to inform our work on estimating the fiscal reaction function, which we then proceed to do. As Bohn (1998) has warned, however, the time series properties of the data should not themselves be taken as indicators of fiscal sustainability.

A. The stationarity of the data

A first statistical question to consider is whether or not the debt/GDP ratio is stationary. The top two panels of Table 1 therefore report ADF test and KPSS test results for the debt/GDP ratio. The results from various tests do not reveal a consistent pattern. As Table 1 shows, according to the ADF test the debt/GDP ratio is an I(0) variable for the period 1946-2008, but an I(1) variable for the period 1974-2008. The KPSS results, though, are virtually the opposite: for the period 1946-2008 the KPSS test rejects the null hypothesis of stationarity at a 5 percent level, but not at a 1 percent level, while for the period 1974-2008 the debt/GDP ratio is an I(0) variable.

Table 1 – Stationarity test for the Debt/GDP ratio and the primary balance/GDP ratio

1946-2008		**1974-2008**	
The Debt/GDP ratio			
ADF test (OLS)		**ADF test (OLS)**	
H_0 : D/Y is I(1)	**H_0 : D/Y is I(2)**	**H_0 : D/Y is I(1)**	**H_0 : D/Y is I(2)**
$(D/Y)_{t-1}$	**$d(D/Y)_{t-1}$**	**$(D/Y)_{t-1}$**	**$d(D/Y)_{t-1}$**
-0.019**		-0.13	-0.447**
[-2.51]		[-1.062]	[-2.178]
KPSS test		**KPSS test**	
H_0 : D/Y is I(0) (Test statistic)	**H_0 : D/Y is I(1) (Test statistic)**	**H_0 : D/Y is I(0) (Test statistic)**	**H_0 : D/Y is I(1) (Test statistic)**
0.58^{++}	0.18	0.16	?
ADF-type regression (GMM) (H_0 : D/Y is I(1))		**ADF-type regression (GMM) (H_0 : D/Y is I(1))**	
Without control variable		**Without control variable**	
$(D/Y)_{t-1}$		**$(D/Y)_{t-1}$**	
-0.024$^{\#\#\#}$		-0.021	
[-3.46]		[-1.351]	
With control variable		**With control variable**	
$(D/Y)_{t-1}$	**$(\hat{y})$**	**$(D/Y)_{t-1}$**	**$(\hat{y})$**
-0.015$^{\#}$	-0.446	-0.035$^{\#\#\#}$	-1.04
[-1.767]	(0.042)	[-3.34]	(0.000)

	The primary balance/GDP ratio (SNA data)				**The primary balance/GDP ratio (GFS data)**	
	1946-2008		**1974-2008**		**1974-2008**	
	Level	**1st diff**	**Level**	**1st diff**	**Level**	**1st diff**
ADF test t and p value	-2.642*** (0.009)		-2.203** (0.029)		-3.55*** (0.003)	
KPSS test value	0.87^{+++}	0.22	0.58^{++}	0.28	0.50^{++}	0.26

Values in [ ] are t values, values in ( ) are p values

ADF test critical t values (1 percent, 5 percent and 10 percent): -2.60; -1.95; -1.61 (indicated by ***; ** and *) for 1946-2008 and -2.63; -1.95; -1.61 (indicated by ***; ** and *) for 1974-2008.

KPSS critical values (1 percent, 5 percent and 10 percent): 0.73; 0.46; 0.35 (indicated by $^{+++}$; $^{++}$ and $^{+}$).

ADF-type regression (GMM) critical t values (1 percent, 5 percent and 10 percent): -2.64; -1.99; -1.65 (indicated by $^{\#\#\#}$; $^{\#\#}$ and $^{\#}$).

The behaviour of the debt/GDP ratio may be close to a random walk, and standard stationary tests such as the ADF test might have difficulty rejecting the null hypothesis of a unit root, even though the series might be stationary (Bohn 1998:955). The KPSS test (with its null hypothesis of stationarity), might also have difficulty distinguishing between stationary and non-stationary series when the behaviour of the series, even when stationary, is very close to being non-stationary. To take a different approach to this problem, we follow Bohn (1998) and present in the third panel of Table 1 what in essence is a Dickey-Fuller test, but one that also includes a control variable. In addition, this test is estimated with GMM with critical values that were generated in a Monte Carlo simulation (the Monte Carlo results are reported in Appendix 1). The test first regresses the change in the debt/GDP ratio on the level of the debt/GDP ratio. Since GMM already corrects for autocorrelation, there is no need to include lags of the dependent variable. Like Bohn, it then re-estimates this relationship, but also includes a control variable; in the case of this paper the output gap is taken as control. Running the Bohn ADF-type regressions shows that debt is mean reverting, and thus, stationary, irrespective of the sample period (1946-2000 or 1974-2008). However, because of the weakness of all these tests, the analysis will consider the possibility that the debt/GDP series is non-stationary.

We also examined the stationarity of the primary balance series, also with dissonant results (bottom panel of table 1). The primary balance over 1946-2008 is an I(0) variable according to the ADF test, but an I(1) variable according to the KPSS test. For the 1974-2008 period the tests are performed on both the SNA and the GFS data for the primary balance. The ADF test still indicates that the primary balance/GDP ratio is an I(0) variable, but the KPSS test indicates that the null hypothesis of stationarity cannot be rejected at a 1 percent level of significance, though it can be rejected at a 5 percent level of significance. As will be argued below, the longer 1946-2008 period was characterised by a change in the primary balance needed to keep debt/GDP stationary – which is an issue that will be explored below in the non-linear analysis. The non-stationarity that exists according to the KPSS test might result from the non-linearity in behaviour during this longer sample period. With respect to the period 1974-2008, it seems that it would be safe to treat the debt/GDP ratio and the primary balance/GDP ratio as stationary. Again, because of the weakness of these tests and the nature of the primary balance/GDP series, the analysis will consider the possibility that the primary balance/GDP series, together with the debt/GDP series, is non-stationary.

B. Estimation results: OLS, TAR, VAR and GMM models

In view of the inconclusiveness of the tests for the stationarity of the data, results from a variety of estimation techniques will be reported. The results of the estimates using the various methods are presented in Table 2 and refer to data for the period 1974 to 2008. All the regressions include a lag of the output gap. Because the intercept was usually statistically insignificant, it was omitted in some of the cases. In all the regressions α_3 is statistically significant, indicating that government does react to the level of its debt/GDP ratio. The output gap parameters are statistically significant in most models and are always positive, pointing to countercyclical behaviour by government. The majority of the estimates for the parameter of the lag of the primary balance/GDP are larger than 0.5. In general, these results point to a high degree of inertia present in government behaviour when it sets its primary balance.

Table 2 - Fiscal reaction functions for South Africa

	OLS#	OLS##	VAR#	VAR##	TAR#	TAR##	GMM#	GMM##	GMM#	GMM##
Data	**GFS**	**GFS**	**GFS**	**GFS**	**GFS**	**GFS**	**GFS**	**GFS**	**SNA**	**SNA**
$(B/Y)_{t-1}$	0.75	0.77	0.53	0.57	0.68	0.73	0.64	0.48	0.63	0.28
	[2.87]	[8.19]			[6.89]	[8.11]	[5.06]	[3.65]	[5.51]	[2.10]
$(D/Y)_{t-1}$	0.04	0.02	0.05	0.03	0.04	0.02	0.01	0.02	0.01	0.03
	[2.76]	[1.66]			[2.80]	[1.61]	[2.09]	[2.10]	[2.43]	[2.38]
$(\hat{y})_{t-1}$	0.25	0.22	0.18	0.29			0.28	0.33	0.35	0.67
	[2.24]	[2.26]					[1.72]	[1.30]	[2.32]	[2.52]
$(\hat{y})_{t-1}$					0.21	0.10				
Positive					[1.10]	[0.77]				
$(\hat{y})_{t-1}$					0.38	0.33				
Negative					[1.72	[2.63]				
C	0.04	0.03	0.05	0.04	0.02	0.03				
	[7.67]	[1.97]			[3.11]	[2.10]				
Adj R-sq	0.71	0.72	0.60	0.74	0.74	0.74	0.63	0.63	0.58	0.25

Estimated with HP-generated output gap; ## Estimated with a Kalman filter-generated output gap.
'Data' refers to source of the primary balance data, which is either GFS or SNA data.

To address possible concerns about the non-stationarity of the time series, Appendix 2 presents a Monte Carlo experiment done with 500 trials with hypothesised parameter values that are representative of those estimated above. Appendix 3 presents the Kernel plots of the parameters. The parameters show a stable and normal distribution around the hypothesised parameters, suggesting that the equations are free from misspecification and coefficients are stable.

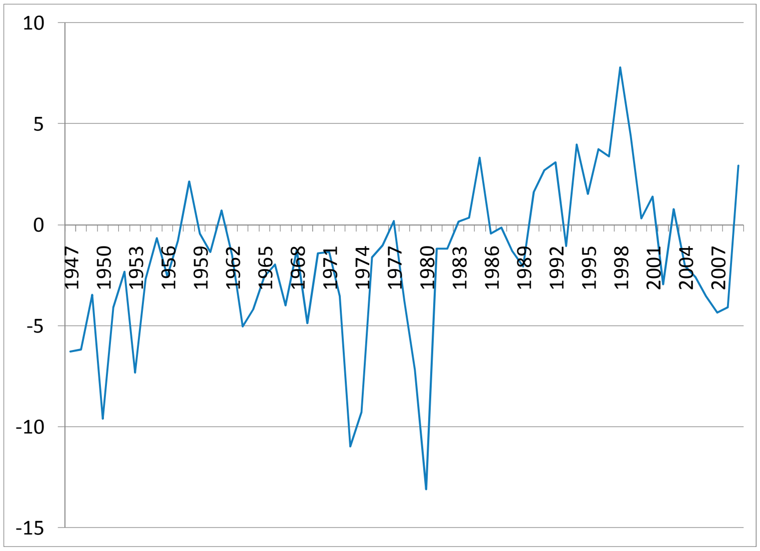
The TAR model presents the results for government's reaction during different phases of the business cycle – thus, the output gap serves as transition variable. The threshold was set at zero. The coefficient on the output gap given an expansion/contraction is 0.21/0.38, with only that of the contraction statistically significant at a 10 percent level. This points to countercyclical policy, particularly during slowdowns and recessions.

But what do the main parameter estimates shown in table 2 tell us about fiscal sustainability? The discussion from earlier sections indicated that fiscal policy will be sustainable if $\alpha_3/(1 - \alpha_2) > \alpha^* = (r - g)/(1 + g)$. Figure 3 presents an empirical estimate of $(r - g)/(1 + g)$, calculated with the 10-year government bond rate, the real GDP growth rate and the GDP deflator for the period 1947-2008.[9] As can be seen using the values for α_3 that can be derived from the estimates in

[9] This is a rough indicator. An alternative indicator could use the effective interest rate on debt, which is total interest payments divided by total debt, and can be understood as a weighted average of the rates on the various instruments outstanding. (This, however, is not the same as the historical cost of financing, particularly on bonds carrying a coupon but also issued at a discount). The effective rate, being a weighted interest rate, would be expected to be somewhat smoother than the actual 10-year government bond rate.

Table 2, for most of the period 1974-2008, $\alpha_3/(1 - \alpha_2) > \alpha^* = (r - g)/(1 + g)$, except possibly for a period in the 1990s (peaking in 1998 when interest rates peaked following the South African Reserve Bank's reaction to the Asian Crisis). This may also help explain why except for the 1990s, the public debt/GDP ratio in South Africa in general remained stable or decreased. Figure 3 also indicates that for most of our 60-year sample period the cost of funding for government was moderate enough to allow it to control the dynamics of its debt with relative ease.

Figure 3 – The (r – g)/(1+g) ratio (percent)



C. The VECM estimates

In view of the remaining concerns regarding the stationarity of the debt/GDP ratio and the primary balance/GDP ratio, we estimated a VECM for the period 1974-2008. The analysis was done with the SNA primary balance data (the SNA data provide a slightly better fit than the GFS data). The primary balance/GDP ratio and the debt/GDP ratio are included in the long-run component of the model, while the output gap, given its stationarity, is included in the short-run dynamics of the model. The information criteria indicate that no lags should be included in the short-run component of the model. The analysis nevertheless includes one lag to discern possible short-run effects in a VECM-Granger (the model was also run with no lags, and the results do not differ in any substantial way – even the parameters are approximately the same). The trace tests (done for one and zero lags) indicate the presence of one cointegrating equation (see Table 3 for the model with one lag).

Table 3 – Unrestricted Cointegration Rank (Trace) Test

National accounts primary balance and national debt

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.
None *	0.402	17.47	15.5	0.025
At most 1	0.015	0.51	3.84	0.475

* Rejection of null hypothesis: Trace test indicates one cointegrating equation at the 5 percent level

The VECM estimates are presented in Table 4. The estimations do not suffer from serial correlation, while the impulse-response functions (Appendix 3) are all stable. Note that in the cointegration equation panel of Table 4, containing the long-run component, a *minus* in front of a parameter means a *positive relationship* between the variable to which the parameter applies and the variable on which the vector is normalized. The long-run component indicates that for every 1 percent increase in the debt/GDP ratio, the primary balance/GDP will increase by 0.131 percent. With a constant of -4.6 percent, a debt/GDP ratio of 40 percent translates into a primary surplus in the long run of 0.6 percent, while a 50 percent ratio translates into a long-run primary surplus of 1.95 percent.

Table 4 – VECM results

Cointegrating Equation:		
$(B/Y)_{t-1}$	1	
$(D/Y)_{t-1}$	-0.131	
	[-2.144]	
C	0.046	
Error Correction Equation:	**D(B/Y)**	**D(D/Y)**
Cointegrating Equation	-0.445	-0.552
	[-3.285]	[-2.232]
$D(B/Y)_{t-1}$	0.253	-0.177
	[1.639]	[-0.627]
$D(D/Y)_{t-1}$	-0.074	-0.063
	[-0.69]	[-0.323]
C	0.002	-0.009
	[0.768]	[-2.055]
($\hat{y}$)	0.096	-0.512
	[0.920]	[-2.687]
Adj. R-sq	0.28	0.39
Weak exogeneity test χ^2 (prob)	0.001	0.023
$D(D/Y)_{t-1} \nRightarrow D(B/Y)_{t-1}$ (prob)*	0.49	
$D(B/Y)_{t-1} \nRightarrow D(D/Y)_{t-1}$ (prob)*	0.53	
Serial corr LM (1)(lag 1) (prob)	0.19	
Serial corr LM (1)(lag 2) (prob)	0.39	
Serial corr LM (1)(lag 3) (prob)	0.20	

Values in [ ] represent t values
* Probability of the VECM Grange causality test

The parameter estimates from Table 4 confirm the findings from the previous section. As can be seen in Figure 3, the *(r – g)/(1 + g)* ratio was mostly negative during the 1970s, most of the 1980s and 2000s, with only the 1990s positive. Given a continuously positive debt/GDP ratio during the sample period, the estimates of the primary balance/GDP ratio calculated with Equation (10) imply that the long-run primary balance/GDP ratio that the government ran was positive. Therefore, with the possible exception of part of the 1990s, the long-run primary balance/GDP ratio that the government ran exceeded the primary balance/GDP ratio required to ensure sustainability.

The error-correction term for the primary balance/GDP ratio equation indicates a fiscal response to deviations from the long-run relationship equal to -0.445. Thus, a little under a half of the deviation is corrected in the first period after the deviation occurs. The weak exogeneity tests conducted on the error-correction terms indicate that the null hypothesis of weak exogeneity can be rejected in all cases. A VECM Granger causality test was also conducted on the short-run component of the model, but it showed no evidence of Granger causality. This is borne out by the initial finding that this model could also be estimated without lags of the dependent variables in the short-run dynamics. Though positive (indicating countercyclical policy), the output gap coefficient is statistically *in*significant in the short-run dynamics of the primary balance/GDP ratio. However, the output gap coefficient is negative and statistically significant in the short-run dynamics of the debt/GDP ratio, indicating that a positive output gap lowers the change in the debt/GDP ratio, while a negative gap increases the change. This may be a sign of differentiated countercyclical fiscal policy. Using the results from Table 4, Table 5 (left-hand panel) rewrites the VECM as a VAR in levels – i.e. it presents Equation (8a) discussed above, while Table 5 (right-hand panel), presents the sum of parameters and hence the VECM equivalent of Equation (6). The results from Table 5 are very much in line with those in Table 2, with $\alpha_3 = 0.056$ and $\alpha_2 = 0.555$.

Table 5 – VAR in levels version of VECM results

Parameters		Sum of parameters	
$(B/Y)_{t-1}$	0.808	(B/Y)	0.555
$(D/Y)_{t-1}$	-0.016	(D/Y)	0.058
$(B/Y)_{t-2}$	-0.253	$(\hat{y})$	0.096
$(D/Y)_{t-2}$	0.074	C	-0.020
$(\hat{y})_{t-1}$	0.096		
C	-0.020		

D. The State-Space estimates

In a bid to study the evolution and stability of the reaction function itself, we estimated a state-space model of the fiscal reaction function. Table 6 presents the State-Space estimates of Equation (6) for the period 1946-2008. Again, since its inclusion caused the results to be weaker and because it was statistically insignificant, the intercept was excluded. The model was also estimated with the contemporaneous value of the output gap, as it yields the best results. As noted above, the parameter of the debt/GDP ratio, α_3, is specified as a random walk, while the lag of the primary balance/GDP ratio and the output gap have constant coefficients. At 0.28, inertia is lower than in most of the models estimated above. The output gap parameter is positive and statistically significant, again indicating countercyclical behaviour by the South African government.

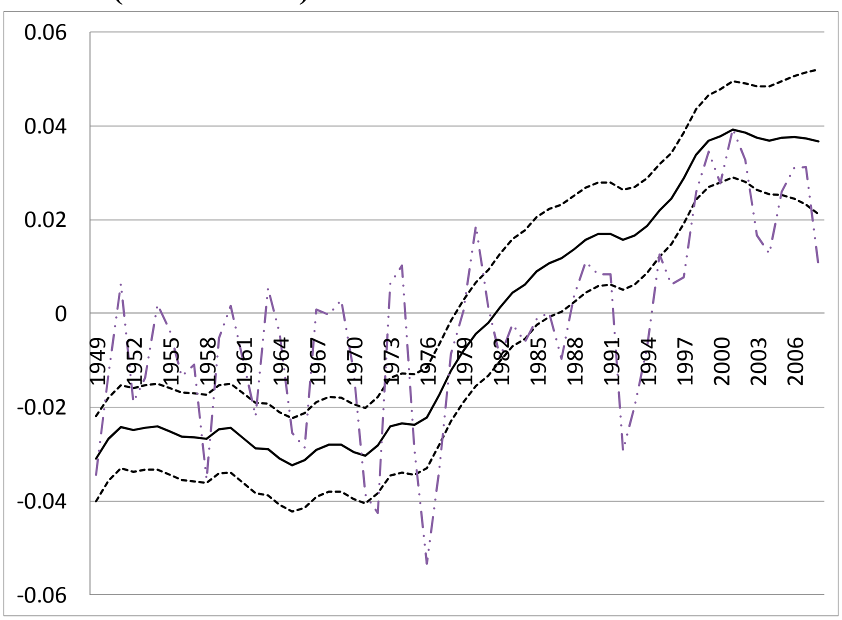
The end state of the state variable for the debt/GDP ratio, α_3, is 0.037. Figure 4 presents this parameter's development over time. It shows the point estimate of α_3 to be negative up to the 1970s, with its confidence interval including zero. As Figure 4 indicates, $\alpha^* = (r - g)/(1 + g)$

was also mostly negative up to the 1970s. Except for the late 1950s and early 1960s, α_3 nevertheless exceeded $(r - g)/(1 + g)$ (so $\alpha_3/(1 - \alpha_2) > \alpha^*$ will also hold), which explains why the debt/GDP ratio still decreased up to the 1970s. In a sense, it would seem that over that early period policy reaction was muted as debt dynamics did not require a strong reaction to ensure sustainability.

Table 6 - Fiscal reaction function (State-space End states)

	Fixed Coefficient
$(B/Y)_{t-1}$	0.286 (0.071)
$(\hat{y})$	0.311 (0.001)
	End State
$(D/Y)_{t-1}$	0.037 (0.03)

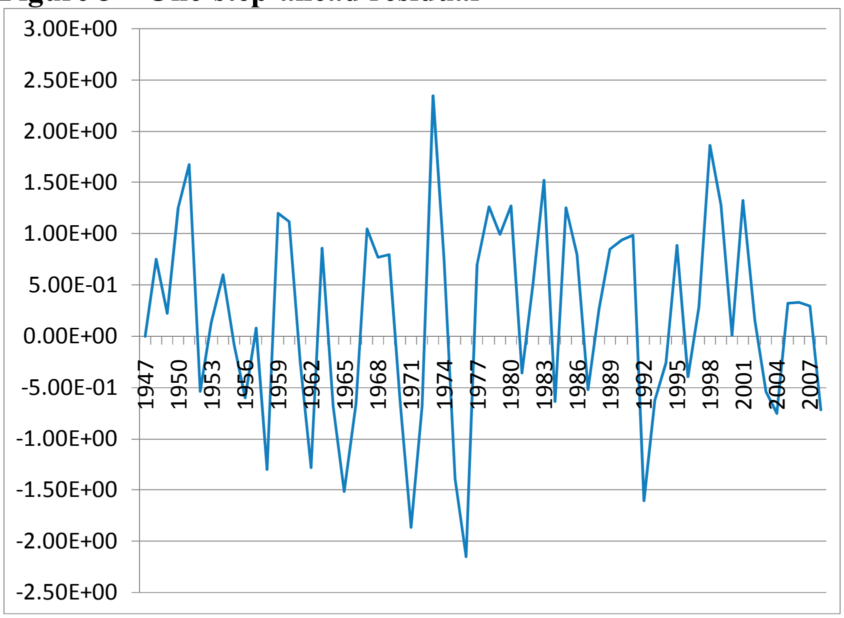
Figure 4 – Public debt/GDP state variable (line with confidence interval) and primary balance (dot-dash line)



Since the early 1970s, estimates of α_3 show a rising trajectory, indicating a stronger reaction to the debt/GDP ratio. As can be observed in the higher level for $(r - g)/(1 + g)$ in the 1980s and 1990s, the increase in α_3 also reflects the changing environment and hence the need since the 1980s for a stronger reaction to debt levels to ensure sustainability. The significant change observed in α_3 during the 1970s and 1980s also explains why none of the fixed parameter models estimated for samples stretching further back than the 1970s was successful in estimating a statistically significant α_3. The estimated fiscal reaction to the debt/GDP ratio is consistently positive from the late seventies onward, and appears remarkably stable at high levels in the period after 1998, even though $(r - g)/(1 + g)$ turned negative. A relatively high α_3 and negative *(r – g)/(1 + g)* explain the halving of the debt/GDP ratio during this period, from roughly 50 percent to 23 percent. Lastly, Figure 5 presents the one-step-ahead forecast standardised residual (calculated as the difference between the actual and one-step-ahead predicted series) showing it to be well behaved and clearly stationary.

A main conclusion from the foregoing analysis is that fiscal policy has, during most of South Africa's modern history, shown a degree of responsiveness to public debt commensurate with what was needed to preserve debt sustainability. The specific magnitude of the responsiveness of the primary balance to debt has varied, but it has done so in such a way to stay ahead of the changes in the parameters governing the evolution of the required primary balance.

Figure 5 – One-step-ahead residual



VI. Uses of the fiscal reaction for debt forecasting and policy design

Understanding how the South African government has reacted to changes in its debt/GDP ratio provides a basis for investigating the fiscal consolidation path set out in the 2010 Budget Review. Following Equation (7), forward estimates of the primary balance combined with projections of the real interest and growth rates allow one to estimate the probable changes to the debt/GDP ratio – a key measure of fiscal sustainability. In addition, the method described here allows for the consideration of the probability of achieving policy goals, defined as future debt levels.

A. The reaction function as basis for probabilistic debt modelling

Deterministic scenario testing typically involves projecting future debt paths by choosing exogenous values for growth and interest rates. This method assumes a static interplay of variables, and it produces relatively few outcomes (e.g. high-growth vs. low-growth scenarios), to which one cannot ascribe probabilities. By producing a distribution (represented by a fan chart) of a thousand possible debt/GDP outcomes, the method used here captures the inherently probabilistic nature of risk analysis. To do so, this section extends the original method of Celasun *et al.* (2006) to project debt service costs (and by extension the budget balance) using a calibrated, symmetrical fiscal reaction function with parameter values based on the models estimated above (Table 7). The section extends Celasun *et al.* (2006) further by producing a fan chart with an asymmetrical reaction of the primary balance to the output gap. (The reader is referred to that article for a more detailed description of this methodology.) This is done with the TAR model parameters.

Table 7 – Parameters for the fan-chart model

	Symmetrical model	**Asymmetrical model**
$(B/Y)_{t-1}$	0.75	0.68
$(\hat{y})_{t-1}$	0.25	
$(\hat{y})_{t-1}$ Positive		0.21
$(\hat{y})_{t-1}$ Negative		0.38
$(D/Y)_{t-1}$	0.04	0.03
C	0.04	0.04

Following Celasun *et al.* (2006), this analysis separately simulates paths for real GDP and real interest rates using a VAR to extract the statistical properties of the innovations to these series, which are needed for the construction of the fan chart. This analysis, however, also includes a GDP deflator in the VAR to produce jointly a distribution of real interest rates, growth and inflation outcomes. The VAR sample period is from 1995Q1 to 2010Q1.

$$Z_t = u_0 + A(L)Z_{t-l} + u_t \qquad (13)$$

where Z = (y, r, py), referring to real GDP (y), the real interest rate (r) and the GDP deflator (py). A(L) is the lag polynomial where the number of lags (2) was selected using the Akaike Information Criterion.[10] The residual variance-covariance matrix is unique since the forecast residuals are affected neither by the ordering of the VAR nor by restrictions imposed (Enders 2004:292). Instead of assuming normality, bootstrapped draws on the residuals were taken.

For the forward looking part of the exercise, the bootstrapped draws, together with the parameters estimated in equation (13), were used to forecast a thousand possible combinations of

[10] A 'de-meaned' real interest rate series was used in the statistical estimation of the VAR, allowing for a change in that series' mean identified in the data; the out-of-sample forecasts were "grossed up" using the mean from the last part of the estimation sample period.

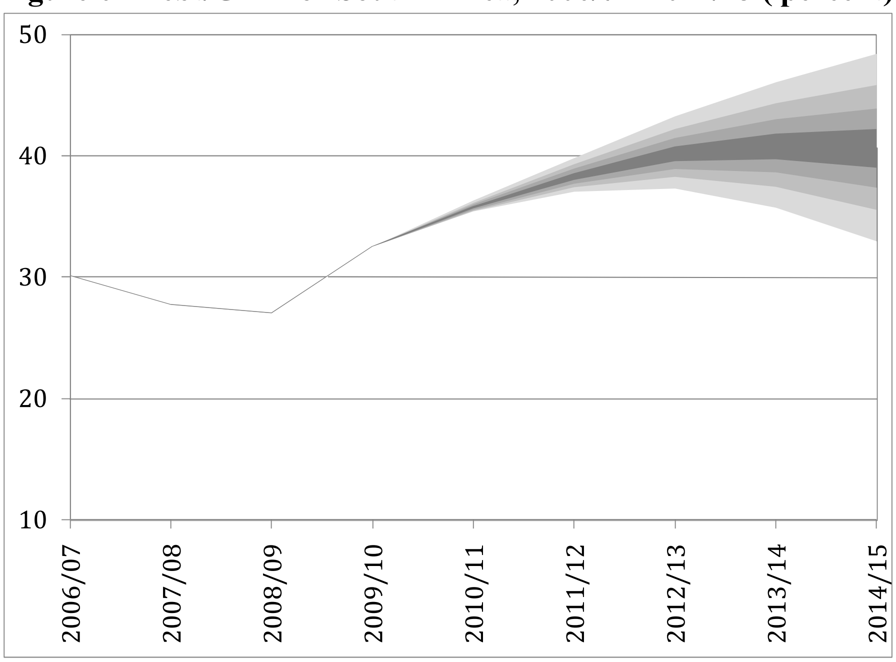
economic growth rates and interest rates for the next five fiscal years. The nominal interest rate, calculated as the sum of the real interest rate and the change in the GDP deflator, is used to calculate a plausible distribution of debt service costs.[11]

Finally, the Hodrick-Prescott filter is run on the median outcomes of the 1000 bootstrapped outcomes for real GDP to estimate potential GDP. To overcome the end-period bias in HP filter estimates of the output gap, the sample period is extended by 12 quarters. The thousand output gaps are fed into the fiscal reaction function from Table 7, to forecast a distribution of primary balance outcomes. Combining the distribution of debt service costs with the primary balance estimates produces a distribution of overall budget balances that can be compared against point projections. The combination of the projected fiscal and real variables, along with the initial value of public debt, yields a distribution of future ratios of the public debt/GDP ratio.

B. The "Fan chart"

A useful way of presenting the large number of possible outcomes is with a fan chart for debt/GDP. The thousand projected paths of debt produced by the model can be grouped into deciles to draw a fan-chart distribution. Grey-scaled colour bands represent each of these deciles; deciles shown in darker colours are closer to the central or median projection. Lighter colours indicate more extreme outcomes. The most extreme outcomes (i.e. those falling outside of the 80 percent confidence interval) are not shown on the chart. In a symmetric, unimodal distribution, such as a "bell curve," the decile bands near the centre would be narrower, that is, the density of outcomes would be higher in neighbourhoods of a given size close to the central projection.

Figure 6 –Debt/GDP for South Africa, 2006/7 – 2014/15 (percent)

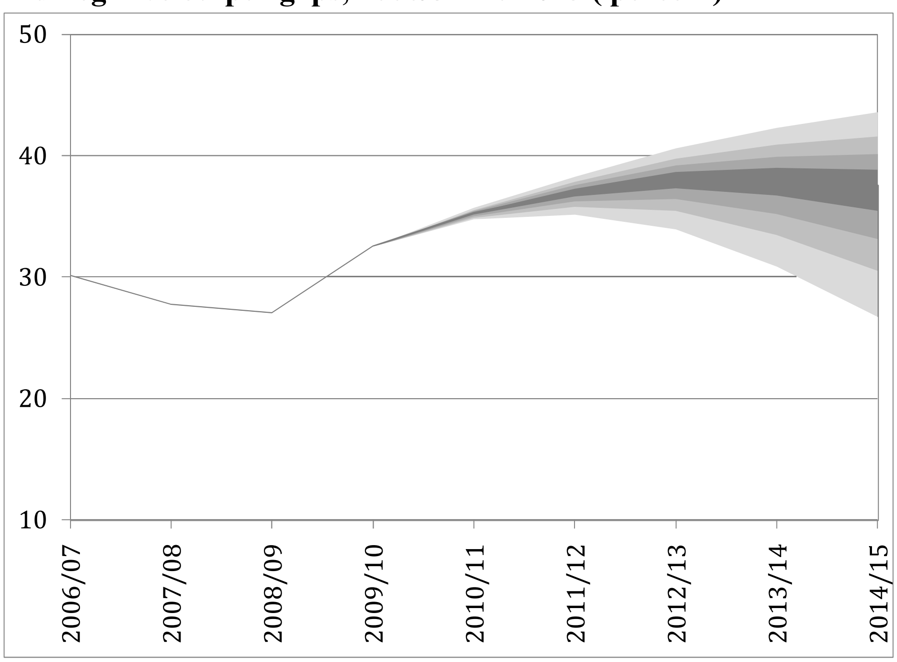


[11] The debt service costs (dsc) are estimated as follows: $dsc_t = dsc_{t-1} + 1/3(\Delta debt_{t-1} \times i_{t-1}) + 2/3(\Delta debt_t \times i_t)$.

Figure 6 shows the median forecast for debt/GDP increasing from 28 percent of GDP in 2009/10 to 41 percent of GDP in 2014/15. By these forecasts, following 15 years of declining national debt/GDP, South Africa is likely to approach its 2000/1 level in the five years following 2009/10. The fan chart also suggests that the probability that government debt-to-GDP ratio will stay below the 50 percent mark is over 90 percent.

The debt fan chart model can be extended by introducing a business cycle-dependent asymmetric reaction by government to the output gap, as indicated in the second column of Table 7. Calibrating the debt model according to the asymmetric fiscal reaction function produces a median debt outcome of around 38 percent of GDP and debt ratio stabilisation in 2014/15 (see Figure 7). The fan chart is asymmetrically distributed around the mean with greater dispersion below the median. This indicates that there is greater 'downside' risk to the projections, with the potential for a large positive surprise, understood as a result well below a 40 percent debt/GDP ratio, exceeding the potential for a large adverse surprise.

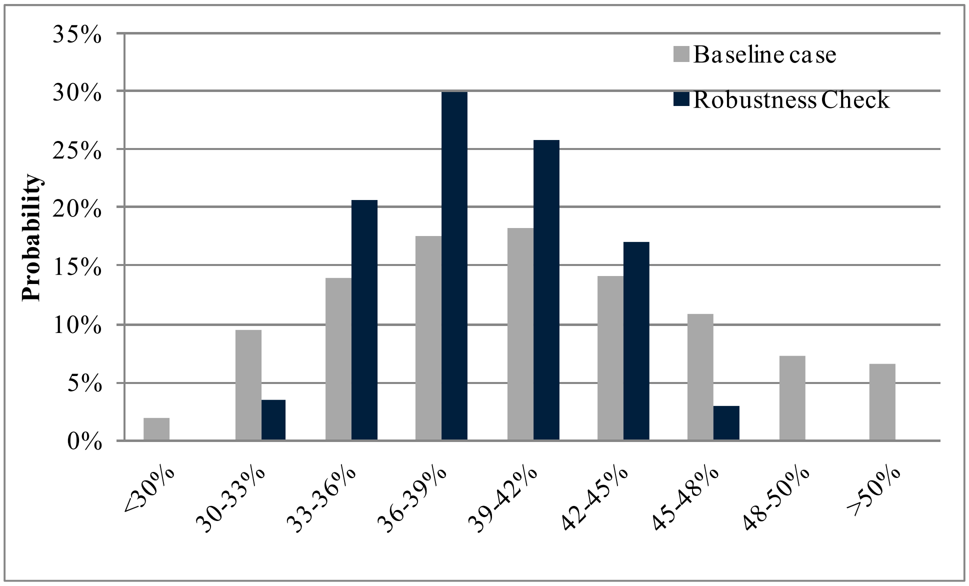
Figure 7 – Debt/GDP for South Africa calibrated for differentiated reactions to positive and negative output gaps, 2006/7 – 2014/15 (percent)



The South African government has projected gross debt levels over the next three fiscal years of 37.1 percent, 40.9 percent and 43.1 percent debt/GDP, based on projected budget balances of -6.5 percent, -5.6 percent and -4.7 percent of GDP over the same period (Budget Review, 2010). Although these are not in any formal sense targets, they can be considered to represent an acceptable trajectory for public debt from the perspective of government. These published forecasts, and especially the forecast for the outer year, can be compared against the distribution of likely budget balances and debt outcomes produced by the model.

Even in the most conservative simulation (Figure 6), the fan-chart suggests that there is a fairly low chance that debt will breach 50 percent of GDP by fiscal year 2014/15. More specifically, the analysis, as summarised in the "baseline" bars of Figure 8, shows that the most probable outcome is a debt/GDP ratio in a neighbourhood around 40 percent by 2015/16, with less than a 7 percent chance that the debt/GDP ratio will exceed 50 percent. As shown in Figure 8, the probability that the debt to GDP ratio will be as projected by government or better is about 75 percent.

Figure 8 – Distribution of possible debt to GDP outcomes, 2015/16 (percent)



Our baseline simulation uses a policy reaction function based on estimates from regressions run on data post 1973. Nevertheless, as Figure 4 presented earlier suggests, the weight given to debt in the policy reaction function, while positive throughout that period, had a tendency to increase in more recent times. If we were instead to produce stochastic simulations using estimates from more recent data, they would tend to weigh debt stabilisation more heavily. Thus, as a robustness check, the simulation exercise was also performed using a fiscal reaction function derived from estimates of the VAR (third column of Table 2), which is the most debt-focused of our models. Using these estimates, the policy responsiveness to debt is sharper, while the reaction to the output gap is more muted. In this simulation, as the "robustness-check" columns in Figure 8 show, the density of debt by the end of the projection horizon is more concentrated about its central values of about 40 percent of GDP. Comparing our baseline to this alternative simulation, it is thus apparent that the variance of possible debt outcomes becomes larger when the policy reaction function gives more weight to economic stabilization and less to debt.

VII. Conclusion

This paper estimated fiscal reaction functions to study how the South African government has historically reacted to its debt position by. The paper remained agnostic about the statistical properties of the debt/GDP ratio and primary balance/GDP ratio, by catering for the possibility of stationary data, non-linear data and non-stationary data. This was done by employing a variety of techniques, including OLS, VAR, GMM, TAR, State-Space modelling and VECM. From all these models the same message emerges: The South African government indeed did, during the sample periods explored, tighten fiscal policies in the face of shocks to the debt/GDP position. Furthermore, from the State-Space estimations it is clear that this reaction has increased over time as circumstances have required it: the fiscal response was rather muted during the period prior to the 1970s, when a favourable combination of real interest and economic growth rates created very favourable debt dynamics; but it became more energetic in the following decades, when the combination of economic growth and real interest rates produced an environment that demanded more fiscal restraint to ensure sustainability.

We also found that, with the possible exception of a few short periods (the late 1950s, early 1960s and the mid 1990s) the South African government tended to run fiscal balances in excess of those required simply to stabilise debt (in the absence of other shocks). These were periods, especially the last one, during which slow economic growth or elevated funding costs put upward pressure on the debt trajectory. Indeed, concern over rising interest costs is often cited as the main reason for the debt reduction programme of the late 1990s. The containment of interest cost will likely remain an important consideration going forward.

If the past is a guide to the future, fiscal policy is such that there is little risk that public debt will become too high. Projected debt and budget balance distributions indicate that the published fiscal targets are not unduly ambitious by historical standards. Our simulations also confirm that the variability of potential debt outcomes rises the more fiscal policy focuses on stabilizing output, and falls the more policy focuses on debt. Thus, as it is natural to expect, there is some tension between the objectives of stabilizing debt and output. Looking forward, it could be useful to complement point forecasts and policy targets such as those in the government's Budget Review with a broad probabilistic assessment of the risks around a central projection. These assessments can be weaved around a model-generated central projection (derived from the estimated fiscal policy reaction function), as was done in the text, or around an official forecast. Either way, such an assessment would help ensure that policy objectives and strategies are robust to shocks.

References

BOHN, H. 2007. Are stationary and cointegration restrictions really necessary for the intertemporal budget constraint? *Journal of Monetary Economics*, 54, pp. 1837-47

_______. 1998. The behaviour of US public debt and deficits. *The Quarterly Journal of Economics*, 113 (3), pp. 949-63.

_______. 1995. The Sustainability of Budget Deficits in a Stochastic Economy. *Journal of Money, Credit and Banking*, 27(1), pp. 257-271

CELASUN, O., DEBRUN, X. and OSTRY, J.D. 2006. Primary Surplus Behavior and Risks to Fiscal Sustainability in Emerging Market Countries: A “Fan-Chart” Approach. *IMF Working Paper 06/67*. Washington

DE MELLO, L. 2005. Estimating a fiscal reaction function: The case of debt sustainability in Brazil. *OECD Economics Department Working Paper No. 423.* OECD, Paris.

DEBRUN, L. 2005. Estimating a fiscal reaction function: The case of debt sustainability in Brazil. *OECD Economics Department Working Paper No. 423.* OECD, Paris.

GALI, J. and PEROTTI, R. 2003. Fiscal Policy and Monetary Integration in Europe. *CEPR Discussion Paper No. 3933*, Centre for Economic Policy Research, London.

HAKKIO, G.S. and RUSH, M. 1991. Is the budget deficit too large? *Economic Inquiry*, XXIX, pp.429-45

MISE, E., KIM, T. and NEWBOLD, P. 2005. On suboptimality of the Hodrick-Prescott filter at time series endpoints. *Journal of Macroeconomics,* 27(2005):53-67

NATIONAL TREASURY OF SOUTH AFRICA. 2010. Budget Review. National Treasury. Pretoria

PINDYCK, R.S. and RUBINFELD, D.L. 1998. *Econometric models and econometric forecasts.* 4th ed. McGraw-Hill International Edition, Boston.

RAPACH, D.E. and WEBER, C.E. 2004. Are real interest rates really nonstationary? New evidence from tests with good size and power. *Journal of Macroeconomics*, 26(2006), pp. 409-30

SOUTH AFRICAN RESERVE BANK (SARB). 2010. *Online data*: http://www.reservebank.co.za

TAYLOR, J. B. (2000). Reassessing discretionary fiscal policy. *Journal of Economic Perspectives*, 14(3): 21-36.

TERäSVIRTA, T. 2004. Smooth transition regression modelling. IN LüTKEPOHL, H. and KRäTZIG, M. (Ed). 2004. Applied time series econometrics. Cambridge University Press, Cambridge.

TONG, H. and LIM, K.S. 1980. Threshold autoregressions, limit cycles and data. *Journal of the royal statistical society.* B42:245-292.

VALENTE, G. 2003. Monetary policy rules and regime shifts. Applied Financial Economics, 13, pp. 525-35

Appendix 1 – Dickey-Fuller t values for GMM

	OLS DF (Y)*	OLS ADF (Y)**	GMM DF (Y)***
No trend, no intercept (1 percent; 5 percent and 10 percent)	-2.58; -1.95; -1.62	-2.63; -1.95; -1.61	-2.64; -1.99; -1.65
No trend, intercept	-3.43; -2.86; -2.57	-3.63; -2.95; -2.61	-3.53; -2.92; -2.61
Trend and intercept	-3.96; -3.41; -3.12	-4.24; -3.54; -3.20	-4.02; -3.45; -3.15

*From Brooks (2008:623)
** From Eviews7 for ADF with zero lags
*** Estimated in a Monte Carlo process with a 1000 observations and 50 000 repetitions.

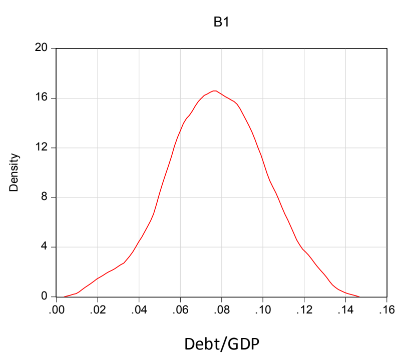
Appendix 2 - Kernel density estimates of the fiscal reaction parameters

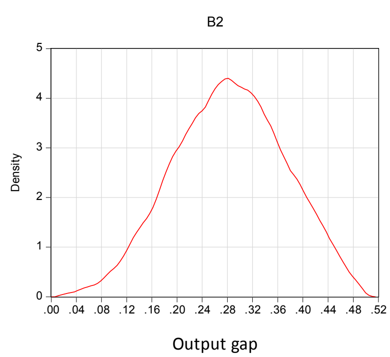
Heuristically, the Kernel density estimator is an adjusted histogram where the edges are smoothed (Silverman, 1986). The observations that are further from the point being evaluated are assigned a smaller weight. The Kernel is defined where the series X at point x is estimated by:

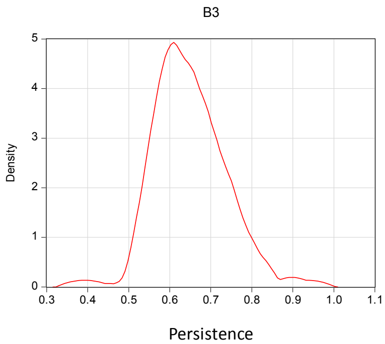
$$f(x) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\left(\frac{x - X_i}{h}\right)\right)$$

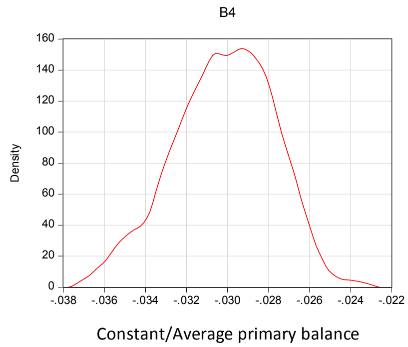
where N is the number of observations and h is the bandwidth and K is the weighting function that integrates to one.

Kernel density estimates of the fiscal reaction function parameters









The distribution of the parameters is presented in the accompanying figure. The parameters seem to follow closely a normal distribution. The centred mean is close to the hypothesised parameters.

Appendix 3 – Impulse-response functions

Cholesky one standard deviation innovations with 90 percent bootstrapped confidence bands.