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Measuring Systemic Risk-Adjusted Liquidity (SRL) — A Model Approach

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Measuring Systemic Risk-Adjusted Liquidity (SRL) — A Model Approach

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Abstract

Little progress has been made so far in addressing—in a comprehensive way—the externalities caused by impact of the interconnectedness within institutions and markets on funding and market liquidity risk within financial systems. The Systemic Risk-adjusted Liquidity (SRL) model combines option pricing with market information and balance sheet data to generate a probabilistic measure of the frequency and severity of multiple entities experiencing a joint liquidity event. It links a firm's maturity mismatch between assets and liabilities impacting the stability of its funding with those characteristics of other firms, subject to individual changes in risk profiles and common changes in market conditions. This approach can then be used (i) to quantify an individual institution's time-varying contribution to system-wide liquidity shortfalls and (ii) to price liquidity risk within a macroprudential framework that, if used to motivate a capital charge or insurance premia, provides incentives for liquidity managers to internalize the systemic risk of their decisions. The model can also accommodate a stress testing approach for institution-specific and/or general funding shocks that generate estimates of systemic liquidity risk (and associated charges) under adverse scenarios.

JEL Classification Numbers: C46, C51, G01, G13, G21, G28, G58.

Keywords: systemic risk, liquidity risk, Net Stable Funding Ratio (NSFR), extreme value theory, financial contagion, macroprudential regulation.

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[1] Corresponding author: Andreas Jobst, Chief Economist, Bermuda Monetary Authority (BMA), 43 Victoria Street, Hamilton HM 12, Bermuda, e-mail: ajobst@bma.bm. This chapter builds on analytical work completed while the author was an economist at the IMF and co-author of the Global Financial Stability Report (GFSR). Technical elements of this model have been applied as part of the Systemic CCA stress testing framework in the Financial Sector Assessment Programs (FSAPs) for Germany, Spain, Sweden, the United Kingdom, and the United States between 2010 and 2012.

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I. INTRODUCTION

A defining characteristic of the recent financial crisis was the simultaneous and widespread dislocation in funding markets, which can adversely affect financial stability in absence of suitable liquidity risk management and policy responses. In particular, banks' common asset exposures and their increased reliance on short-term wholesale funding in tandem with high leverage levels helped propagate rising counterparty risk due to greater interdependence within the financial system. The implications from liquidity risk management decisions made by some institutions spilled over to other markets and other institutions, contributing to others' losses, amplifying solvency concerns, and exacerbating overall liquidity stress as a result of these negative dynamics. Thus, private sector liquidity (as opposed to monetary liquidity), which is created largely through banks and other financial institutions via bilateral arrangements and organized trading venues, is invariably influenced by common channels of market pricing that can amplify cyclical movements in system-wide financial conditions with the potential of negative externalities resulting from individual actions (CGFS, 2011).

The opportunity cost of holding liquidity is invariably cyclical, resulting in a notorious underpricing of liquidity risk, which tends to perpetuate a disregard for the potential inability of markets to sustain sufficient liquidity transformation under stress. Banks have an incentive to minimize liquidity (and mitigate the opportunity cost of holding excess liquidity in lieu of return-generating assets) in anticipation that central banks will almost certainly intervene in times of stress as lenders-of-last-resort. Even without central bank support, liquidity risk is most expensive when it is needed most while generating little if any additional return in good times. While central banks can halt a deterioration of funding conditions in order to maintain the efficient operation of funding markets (see Figure 1), prevent financial firms from failing, and, thus, limit the impact of liquidity shortfalls on the real economy, their implicit subsidization of bank funding accentuates the magnitude of liquidity risks under stress. Central bank measures during the credit crisis have further reinforced this perception of contingent liquidity support, giving financial institutions an incentive to hold less liquidity than needed (IMF, 2010a).

Current systemic risk analysis—as a fundamental pillar of macroprudential surveillance and policy—is mostly focused on solvency conditions. Disruptions to the flow of financial services become systemic if there is the potential of financial instability to trigger serious negative spillovers to the real economy.[2] Macroprudential policy aims to limit,

[2] Impairment to the flow of financial services occurs where certain financial services are temporarily unavailable, as well as situations where the cost of obtaining the financial services is sharply increased. It would include disruptions due to shocks originating outside the financial system that have an impact on it, as well as shocks originating from within the financial system.

mitigate or reduce systemic risk, thereby minimizing the incidence and impact of disruptions in the provision of key financial services that can have adverse consequences for the real economy (and broader implications for economic growth).[3] Substantial work is underway to develop enhanced analytical tools that can help to identify and measure systemic risk in a forward-looking way, and, thus, support improved policy judgments. While *systemic solvency risk* has already entered the prudential debate in the form of additional capital rules that apply to systemically important financial institutions (SIFIs),[4] little progress has been made so far in addressing systemic liquidity risk.

In contrast, proposals aimed at measuring and regulating *systemic liquidity risk* caused by the interconnectedness across financial institutions and financial markets have been few and far between. Systemic liquidity risk is associated with the possibility that maturity transformation in systemically important institutions and markets is disrupted by common shocks that overwhelm the capacity to fulfill all planned payment obligations as and when they come due. For instance, multiple institutions may face simultaneous difficulties in rolling over their short-term debts or in obtaining new short-term funding (much less long-term funding). However, progress in developing a systemic liquidity risk framework have been hampered by the rarity of system-wide liquidity risk events, the multiplicity of interactions between institutions and funding markets, and the conceptual challenges in modeling liquidity conditions affecting institutions and transactions separately or jointly.[5]

The policy objective of such efforts would be to minimize the possibility of systemic risk from liquidity disruptions that necessitate costly public sector support. While a financial institution's failure can cause an impairment of all or parts of the financial system, firms are not charged for the possibility that their risk-taking affects the operation of the financial

[3] In a recent progress report to the G-20 (FSB/IMF/BIS, 2011b), which follows an earlier update on macroprudential policies (FSB/IMF/BIS, 2011a), the FSB takes stock of the development of governance structures that facilitate the identification and monitoring of systemic financial risk as well as the designation and calibration of instruments for macroprudential purposes aimed at limiting systemic risk. While the report acknowledges considerable progress in the conduct of macroprudential policy, the report finds that there is still much scope for systemic risk regulation and institutional arrangements for the conduct of policy. In fact, there is still no consistent theory of macroprudential surveillance, but rather several conceptual and methodological proposals that coexist in a loose manner.

[4] In July 2011, the Basel Committee on Banking Supervision (BCBS, 2011) published its draft guidelines for assessing the loss absorbency capital requirement of systemically important banks ("G-SIBs").

[5] The issue of systemic liquidity is also closely related to infrastructural resilience to liquidity shocks. In this case, systemic crises are assumed to stem from the inadequate risk-proofing of elements of the infrastructure that are critical to the functioning of the financial system in absence of close substitutes. The assessment of the systemic importance of markets, however, presents more conceptual challenges than that of institutions—which might explain current supervisory reluctance to move quickly.

system as a whole. In fact, individual actions might cause losses elsewhere in the system through direct credit exposures and financial guarantees, forced asset sales, and greater uncertainty regarding mutual exposures (possibly in combination with greater risk aversion of investors), which increases the cost of funding for all financial institutions. These "negative externalities" impose costs to the system, which increases the greater the importance of a single institution to the system ("too-important-to-fail") and the higher the level of asymmetric information as coordination failures accentuate the impact of common shocks.[6] Thus, more stringent prudential liquidity requirements, much like higher capital levels, might be beneficial *ex ante* by creating incentives of shareholders to limit excessive risk-taking, which would otherwise increase the potential loss in case of failure (Jensen and Meckling, 1976; Holmstrom and Tirole, 1997). However, certain liquidity standards might also encourage greater concentrations in assets that receive a more favorable regulatory treatment based on their liquidity characteristics during normal times (which remains to be tested during times of stress).

A number of prudential reforms and initiatives are underway to address shortcomings in financial institutions' liquidity practices, which have resulted in more stringent supervisory liquidity requirements. Under the post-crisis revisions of the existing Basel Accord, known as Basel III, the Basel Committee on Banking Supervision (BCBS, 2010a, 2010b and 2009) has proposed two quantitative liquidity standards to be applied at a global level and published a qualitative guidance to strengthen liquidity risk management practices in banks. Under this proposal, individual banks are expected to maintain a stable funding structure, reduce maturity transformation, and hold a sufficient stock of assets that should be available to meet its funding needs in times of stress—as measured by two standardized ratios:

- *Liquidity Coverage Ratio* (LCR). This ratio is intended to promote short-term resilience to potential liquidity disruptions by requiring banks to hold sufficient high-quality liquid assets to withstand the run-off of liabilities over a stressed 30-day scenario specified by supervisors. More specifically, "the LCR numerator consists of a stock of unencumbered, high-quality liquid assets that must be available to cover any net [cash] outflow, while the denominator is comprised of cash outflows less cash inflows (subject to a cap at 75 [percent] of total outflows) that are expected to occur in a severe stress scenario (BCBS, 2011 and 2012b)."[7]

[6] As long as markets remain stable and prove robust, more reliance is placed on the resilience of the financial system. In times of stress, however, moral hazard intensifies potential systemic vulnerabilities to liquidity risk that extend across institutions and national boundaries.

[7] On January 8, 2012, the Basel Committee confirmed its commitment to the LCR (BCBS, 2012a) as a reflection of the central principle that a bank is expected to have sufficient liquid assets to withstand plausible

(continued…)

- *Net Stable Funding Ratio* (NSFR). This structural ratio limits the stock of unstable funding by encouraging longer term borrowing in order to restrict liquidity mismatches from excessive maturity transformation. It requires banks to establish a stable funding profile over the short term (i.e., the use of stable (long-term and/or stress-resilient) sources to continuously fund short-term cash flow obligations that arise from lending and investment activities). The NSFR reflects the proportion of long-term assets that are funded by stable sources of funding with maturities of more than one year (except deposits), which includes customer deposits, long-term wholesale funding, and equity (but excludes short-term funding).[8] A value of this ratio of less than 100 percent indicates a shortfall in stable funding based on "the difference between balance sheet positions after the application of available stable funding factors and the application of required stable funding factors for banks where the former is less than the latter (BCBS, 2011 and 2012b)."[9]

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shocks to net cash flows. During periods of stress, banks would be allowed to temporarily fall below the minimum requirement of holding an amount of liquid assets (which can be readily converted into cash) equal to the expected net cash outflows (over a 30-day period). The Basel Committee will provide guidance on the accumulation and circumstances justifying the utilization of these assets for liquidity management and examine possible conflicts between the compliance with the LCR and central bank policies during periods of stress. It is expected to finalize key aspects of the LCR in the response to specific concerns regarding the scope of liquid assets and some adjustments to the calibration of net cash outflows with a view to subsequently publishing its recommendations by the end of 2012. For instance, the abolition of the two-tier system of distinguishing liquid assets and their ratings-based adjustment for inclusion in the LCR calculation are currently under discussion (Watt, 2012).

[8] These sources and uses of funds are not equally weighted but enter as risk-adjusted components into the calculation of the NSFR.

[9] On April 12, 2012, the Basel Committee published the results of its latest Basel III monitoring exercise based on rigorous reporting processes to periodically review the implications of revised regulatory standards. Out of a global sample 212 banks that participated in the study (including 103 "Group 1 banks" (i.e., those that have Tier 1 capital in excess of EUR3 billion and are internationally active) and 109 "Group 2 banks" (i.e., all other banks)), 205 firms submitted data for the analysis of the two liquidity measures. One week earlier, the European Banking Authority (EBA) (2012) published the aggregate results of all participating banks within the EU as a follow-up to the comprehensive European quantitative impact study (EU-QIS) completed in 2010. With regard to the liquidity measures, 45 percent of the participating firms met or exceeded the minimum LCR level of 100 percent, and 60 percent reported a LCR ratio at or above 75 percent as of end-June 2011. For the NSFR, 46 percent of sample banks meet or exceed the minimum NSFR requirement of 100 percent and 75 percent of firms had an NSFR of 85 percent or higher (with a sample-weighted average of 94 percent) (BCBS, 2012b; EBA, 2012). However, the macro-financial implications of the proposed liquidity standards might not have been sufficiently explored in this monitoring exercise. Pengelly (2012) reports that the compliance with the NSFR, which emphasizes the availability of long-term sources of funding, could conflict with plans to make senior bondholders absorb bank losses under so-called "bail-in" clauses. Banks might find it difficult to lengthen the maturity of their balance sheet by issuing additional debt if mandatory bail-in clauses were attached to them, especially given that investors are likely to demand additional spread to accept bail-in risk.

However, these prudential measures do not directly targeting system-wide implications. The current approach assumes that sufficient institutional liquidity would reduce the likelihood of knock-on effects on solvency conditions in distress situations and complement the risk absorption role of capital—but without considering system-wide effects.[10] Larger liquidity buffers at each bank should lower the risk that multiple institutions will simultaneously face liquidity shortfalls, which would ensure that central banks are asked to perform only as lenders of last resort—and not as lenders of first resort. However, this rationale underpinning the Basel liquidity standards ignores the impact of the interconnectedness of various institutions and their diverse funding structures across a host of financial markets and jurisdictions on the probability of such simultaneous shortfalls.[11] Moreover, in light of the protracted adoption of both the LCR and the NSFR (whose implementation is envisaged in 2015 and 2018, respectively) and the associated risk of undermining timely adjustment of industry standards, Perotti (2012) argues for strong transitional tools in the form of "prudential risk surcharges." These would be imposed on the gap between current liquidity positions of banks and the envisaged minimum liquid standards at a level high enough to compensate for and discourage the creation of systemic risk in order to ensure early adoption of safer standards while offering sufficient flexibility of banks to chart their own path towards compliance.

An effective macroprudential approach that targets systemic liquidity risk presupposes the use of objective and meaningful measures that can be applied in a consistent and transparent fashion (and the attendant design of appropriate policy instruments). Ideally, any such methodology would need to allow for extensive back-testing and should benefit from straightforward application (and avoid complex modeling (or stress-testing)). While it should not be too data intensive to compute and implement, enough data would need to be collected to ensure the greatest possible coverage of financial intermediaries in order to accommodate different financial sector characteristics and supervisory regimes across national boundaries. In addition, the underlying measure of systemic risk should be time-varying, and, if possible, it should offset the procyclical tendencies of liquidity risk and account for changes to an institution's risk contribution, which might not necessarily follow cyclical patterns. Finally, it would also motivate a risk-adjusted pricing scheme so that

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[10] In addition, national authorities have begun implementing their own stringent liquidity regulations ahead of the phase-in schedule agreed internationally. For instance, the U.K. Financial Services Authority (FSA) initiated a monthly *Liquidity Reporting Profile* (LRP) at the end of 2010 (IMF, 2011d), which includes market-wide liquidity risk trends.

[11] There is also the unaddressed question whether measures targeting system-wide risk directly can be more effective than addressing this risk through prudential control at individual institutions.

institutions that contribute to systemic liquidity risk are assigned a proportionately higher charge (while the opposite would hold true for firms that help absorb system-wide shocks from sudden increases in liquidity risk).

In this regard, several proposals are currently under discussion (see Table 1), including the internalization of public sector cost of liquidity risk via insurance schemes (Goodhart, 2009; Gorton and Metrick, 2009; Perotti and Suarez, 2009 and 2011), capital charges (Brunnermeier and Pedersen, 2009), taxation (Acharya and others, 2010a and 2010b), investment requirements (Cao and Illing, 2009; Farhi and others, 2009), as well as arrangements aimed at mitigating the system-wide effects from the fire sale liquidation of assets in via collateral haircuts (Valderrama, 2010) and modifications of resolution regimes (Roe, 2009; Acharya and Oncu, 2010). In particular, Gorton (2009) advocates a systemic liquidity risk insurance guarantee fee that explicitly recognizes the public sector cost of supporting secured funding markets if fragility were to materialize. Roe (2009) argues that the internalization of such cost would ideally be achieved by exposing the lenders to credit risk of the counterparty (and not just that of the collateral) by disallowing unrestricted access to collateral even in case of default of the counterparty. In this way, lenders would exercise greater effort in discriminating ex ante between safer and riskier borrowers.[12] Such incentives could be supported by time-varying increase in liquidity requirements, which also curb credit expansion fueled by short-term and volatile wholesale funding and reduce dangerous reliance on such funding (Jácome and Nier, 2012).

In this paper, we propose a structural approach—the systemic risk-adjusted liquidity (SRL) model—for the structural assessment and stress testing of systemic liquidity risk. Although macroprudential surveillance relies primarily on prudential regulation and supervision, calibrated and used to limit systemic risk, additional measures and instruments are needed to directly address systemic liquidity risk. This paper underscores why more needs to be done to develop macroprudential techniques to measure and mitigate such risks arising from individual or collective financial arrangements—both institutional and market-based—that could either lead directly to system-wide distress of institutions and/or significantly amplify its consequences. The SRL model complements the current Basel III liquidity framework by extending the prudential assessment of stable funding (based on the NSFR) to a system-wide approach, which can help substantiate different types of

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[12] Acharya and Oncu (2010) refine this proposition by excluding very liquid and safe collateral, such as U.S. Treasury securities, and perhaps agencies (assuming the agencies are effectively government-backed), from "stays" in bankruptcy proceedings.

macroprudential tools, such as a capital surcharge, a fee, a tax, or an insurance premium that can be used to price contingent liquidity access.[13]

The SRL model quantifies how the size and interconnectedness of individual institutions (with varying degrees of leverage and maturity mismatches defining their risk profile) can create short-term liquidity risk on a system-wide level and under distress conditions.[14] The model combines quantity-based indicators of institution-specific funding liquidity (conditional on maturity mismatches and leverage), while adverse shocks to various market rates are used to alter the price-based measures of monetary and funding liquidity that, in turn, form the stress scenarios for systemic liquidity risk within the model (see Table 2 and Box 2). In this way, the SRL model fosters a better understanding of institutional vulnerabilities to the liquidity cycle and related build-ups of risks based on market rates that are available at high frequencies and which lend themselves to the identification of periods of heightened systemic liquidity risk (CGFS, 2011).

This approach forms the basis for a possible capital charge or an insurance premium—a pre-payment for the contingent (official) liquidity support that financial institutions eventually receive in times of joint distress—by identifying and measuring ways in which they contribute to aggregate risk over the short-term.[15] Such a liquidity charge should reflect the marginal contribution of short-term funding decisions by institutions to the generation of systemic risk from the simultaneous realization of liquidity shortfalls. Proper pricing of the opportunity cost of holding insufficient liquidity—especially for very adverse funding situations—would help lower the scale of contingent liquidity support from the public sector (or collective burden sharing mechanisms). The charge needs to be risk-based, should be increasing in a common maturity mismatch of assets and liabilities, and would be applicable to all institutions with access to safety net guarantees. Since liquidity runs are present in the escalating phase of all systemic crises, our focus is on short-term wholesale liabilities, properly weighted by the bank's maturity mismatch.

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[13] Usable macroprudential stress tests at the present stage do not sufficiently heed the interactions between solvency and liquidity as well as the system-wide impact of funding conditions.

[14] This model is based on previous analytical work in the context of the October 2009 and April 2011 issues of the Global Financial Stability Report (IMF, 2009 and 2011b; Jobst, 2011). See Davis (2011) for a summary of the main considerations of the model.

[15] This contrasts with Perotti and Suarez (2009), who propose a charge per unit of refinancing risk-weighted liabilities based on a vector of systemic additional factors (such as size and interconnectedness) rather than the contribution of each institution to the overall liquidity risk (and how it might be influenced by joint changes in asset prices and interest rates).

The remainder of the paper is organized as follows. After presenting some macroprudential considerations affecting the conceptualization of systemic liquidity risk in the next section, we provide an overview of the SRL model and its contribution to the systemic risk measurement and modeling literature. We then present the technical specification and its application for stress testing in Section III. Section IV illustrates the empirical case of measuring systemic liquidity risk of the largest U.S. commercial and investment banks. Section V concludes the paper.

Table 1. Selected Regulatory Proposals for Managing Systemic Liquidity Risk.

Author	Goodhart (2009)	Perotti and Suarez (2009 and 2011)	Brunnermeier and Pedersen (2009)	Acharya and others (2010a, 2010b, and 2012)	Cao and Illing (2009), Farhi and others (2009)	Valderrama (2010)
Proposal	Liquidity insurance: charge breakeven insurance premium (collected including during good times), monitor risk and sanction on excessive risk-taking.	Mandatory liquidity insurance financed by taxing short-term wholesale funding.	Capital charge for maturity mismatch.	Impose incentive-compatible tax (paid including during good times) to access government guarantee (including for loan guarantees and liquidity facilities).	Minimum investment in liquid assets or reserve requirement.	Mandatory haircut for repo collaterals.
Treatment of systemic risk aspects	It depends on the way premiums are calibrated. Premiums could include add on factors reflecting the systemic importance of each institution.	Each institution pays different charges according to their contribution to negative externalities, reflecting systemic risks.	Calibrating charges to reflect externality measures (e.g., “CoVaR”) for each institution.	Calibrating tax to reflect each institution’s contribution to systemic risks.	If all the relevant institutions hold more liquidity, the system will be more resilient on aggregate. Furthermore, one could potentially introduce add-on requirements for systemically important institutions.	Delink the interaction between market and funding liquidity through cycle. Would affect a wide range of market participants in addition to banks.
Drawbacks	No concrete examples how to calculate the premium.	No concrete example provided how to measure the systemic risk to the whole sale funding structure.	Not clear whether a solvency-oriented CoVaR can be used for liquidity charge calculation.	No concrete examples how to implement the proposed tax implementation strategy. Refers to difficulties to measure externality or contributions to externality.	Additional analysis needed to fully incorporate systemic aspects due to interconnectedness and other externalities.	No concrete examples given on how to implement.

Source: IMF (2011b).

II. Macroprudential Considerations of Systemic Liquidity

The recent financial crisis demonstrated the devastating impact of systemic liquidity events. Under normal circumstances, regulation and supervision (both capital adequacy and appropriate liquidity risk management) ensure, as far as possible, that the maturity transformation in the banking sector in conducted safely with the necessary access to central bank lending facilities and depositor protection preventing sudden run-offs of liabilities that could deplete the availability of sufficient funding under stress. However, higher perceived counterparty risk can result in funding constraints and can depress asset prices to a point where they eventually overwhelm individual liquidity buffers despite these mitigating factors. In addition, market failure derived from ill-designed mechanisms for coordinating financial intermediaries and investors in different funding markets (see Figure 1) can exacerbate these problems (Franke and Krahnen, 2008).

During the recent crisis, several aspects contributed to systemic liquidity risk that escaped microprudential oversight:

- *Banks were allowed to enter improvident off-balance sheet commitments* (such as liquidity back-up facilities for special investment vehicles (SIVs)) without appropriate liquidity management, which appeared efficient and profitable as liquidity risk was not priced in (Haldane, 2010).

- *Microprudential supervision failed to recognize vulnerabilities from currency-specific maturity transformation in offshore financial centers.* In particular, European banks used short-term wholesale U.S. dollar markets to fund long-term U.S. dollar-denominated assets. When funding was no longer available, this double mismatch required liquidity support via mutually agreed U.S. dollar swap agreements between the U.S. Federal Reserve and other central banks, illustrating the significant size and systemic nature of offshore dollar markets.[16]

- *Non-banks engaged in maturity transformation*, creating a "shadow banking sector" without safeguards imposed—at least in principle—on the banking sector. A number of short-term money market investments, such as mutual funds, asset-backed commercial paper (ABCP), and repo placements, were used to fund longer-term

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[16] Many institutions are active in both on- and off-shore markets that are linked via interbank lending, foreign exchange swaps, and securities trading. Addressing these systemic vulnerabilities cannot be dealt with by national policymakers alone and will require closer international cooperation to collect data and information on these markets (IMF/FSB, 2009 and 2010).

assets while being treated by cash providers as if they were deposits (i.e., they expected to be able to redeem them at par on demand) while in fact they were not.

These shortcomings in the liquidity regulation of depository institutions increased aggregate vulnerabilities of funding arrangements to shocks from counterparty risk. One case in point is the wholesale funding market, where network externalities and the structural complexity across financial and non-financial institutions amplified these vulnerabilities. As the economic environment and asset allocation behavior changed, lenders were more likely to increase haircuts on repo financing, limit eligibility of collateral, or stop rolling over short-term funding altogether in order to offset an asset shock by means of de-leveraging their balance sheets (Shin, 2009; Shleifer and Vishny, 2010).[17] As such a behavior occurred collectively, it caused liquidation of assets under fire sale conditions (Coval and Stafford, 2007), which resulted in a negative confidence-induced downward liquidity spiral, increasing funding pressures as deteriorating counterparty risk eventually weighed on solvency and required official liquidity support (CGFS, 2011).

Table 2. Overview of Liquidity Indicators.

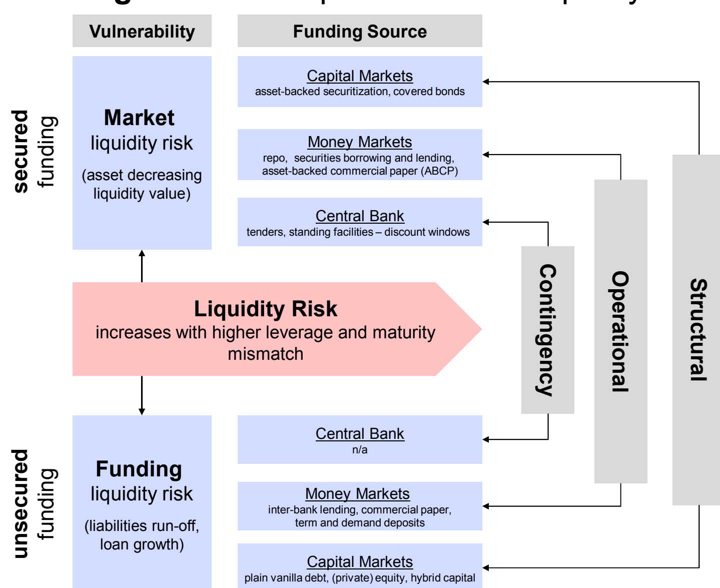
	Quantities	Prices
Monetary liquidity	Base money and broader monetary aggregates	Policy and money market interest rates
	Access to central bank liquidity facility *(e.g., bidding volume)*	Monetary conditions indices
	Foreign exchange reserves	
Funding liquidity	Bank liquidity ratios	Unsecured interbank lending *(Libor-OIS spreads)*
	Bank net cash flow estimates	Secured interbank lending *(repo rates)*
	Maturity mismatch measures	Margins and haircuts on repo collateral
	Commercial paper market volumes	FX swap basis
		Violation of arbitrage conditions *(bond-CDS basis, covered interest rate parity)*
		Spreads between assets with similar credit characteristics
		Qualitative surveys of funding conditions
Market liquidity	Transaction volumes	Bid-ask spreads on selected global assets
		Qualitative fund manager surveys

[17] Moreover, liquidity transformation outside the banking sector is heavily reliant on trading protocols, informal rules of conduct, and collective burden sharing arrangements, whose limited capacity to absorb extreme shocks affects the way coordination failures can perpetuate market paralysis.

Sources: CGFS (2011), IMF (2011b), and author. *Note:* The solid line around the quantity-based liquidity indicators reflects the model focus on funding liquidity; however, in the stress testing application (see Box 2), price-based measures of monetary and funding liquidity are taken into account by way of various market rates.

The proper identification, monitoring and mitigating of systemic liquidity risk requires a profoundly macroprudential view.[18] The traditional approach to financial stability analysis concentrates analytical efforts on the identification of vulnerabilities prior to stress from individual failures, assuming that the financial system is in equilibrium and adjusts when it experiences a shock. From a more technical perspective on liquidity risk, functional discussions have tended to focus on market liquidity, which can be measured by spreads, turnover ratios, or other price impact measures, while the supervisory focus was on funding liquidity of institutions based on liquidity ratios. As opposed to this conventional approach, the potential build-up of systemic vulnerabilities warrants comprehensive monitoring of on-going developments in areas where the impact of disruptions to financial stability is deemed most severe and wide-spread—and especially in areas of economic significance to both the financial sector and the real economy. Although the crisis has shifted attention of supervisors from market liquidity to funding risk of individual institutions (see Table 2), microprudential liquidity requirements still remain largely devoid of systemic risk considerations.[19]

Figure 1. Conceptualization of Liquidity Risk.



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[18] See BIS (2011) for a survey of ongoing theoretical and empirical work on macroprudential policy and regulation.

[19] The stability of both secured and unsecured funding markets, such as repos (and their role in conducting liquidity transformation), however, is determined by the resilience of other institutions to ensure that markets for assets held predominantly by the financial sector remain liquid.

Macroprudential regulation would discourage the underpricing of liquidity risk while limiting system-wide vulnerabilities of funding sources to common asset price shocks. Any macroprudential approach would need to encourage the fair pricing of liquidity risk in good times while acknowledging the adverse impact of market stress on funding conditions, which, in turn, can affect perceived solvency. Such conjunctural assessment of liquidity risk conditional on both time and the cross-sectional variability of funding would be motivated by the belief that systemic liquidity risk results from (i) contingent claims on other institutions and markets (via off-balance sheet activities), (ii) the cyclicality of volatility-based margin requirements and haircuts in wholesale funding markets that amplify exposures to common asset price shocks, and (iii) the negative correlation between asset returns and funding costs during times of stress.

Aligning private incentives to mitigate liquidity risk beyond individual institutions requires methodological and empirical approaches aimed at the identification and measurement of systemic risk. While there is still no consistent theory of regulating systemically important activity in the financial sector, existing approaches can be broadly distinguished based on their conceptual underpinnings regarding several core principles used to determine systemic relevance. In general, there are two general approaches: (i) a particular activity causes a firm to fail, whose importance to the system imposes marginal distress on the system ("contribution approach"),[20] or (ii) a firm experiences losses from a single (or multiple) large shock(s) due a significant exposure to the commonly affected sector, country and/or currency ("concentration of activity"), which are large relative to overall losses ("participation approach"). In the case of the former, the contribution to systemic risk arises from the initial effect of direct and indirect exposures to the failing institution (e.g., defaults on liabilities and/or asset fire sales), which escalates to spillover effects to previously unrelated institutions and markets as a result of greater uncertainty or the reassessment of financial risk (i.e., changes in risk appetite and/or the market price of risk). In contrast, the participation in systemic risk occurs via the an institution's credit and market risk exposure to other financial institutions and market risks, which result in expected losses that exceed the loss bearing capacity of bank creditors. Table 3 below shows the distinguishing features of both approaches.

[20] Drehmann and Tarashev (2011) refer to this as a "bottom-up approach", whereas as a "top-down approach" would be predicated on the quantification of expected losses of the system, with and without a particular institution being part of it.

Table 3. General Systemic Risk Measurement Approaches.

	Contribution approach ("Risk Agitation")	**Participation approach** ("Risk Amplification")
Concept	systemic resilience to individual failure	individual reliance to common shock
Description	a contribution to systemic risk conditional on individual failure due to knock-on effect	expected loss from systemic event due to common exposure and risk concentration
Risk transmission	"institution-to-institution"	"institution-to-aggregate"
Risk indicators	economic significance of asset holdings ("size")	claims on other financial sector participants (credit exposure)
	intra- and inter-system liabilities ("connectedness")	market risk exposure (interest rates, credit spreads, currencies)
	degree of transparency and resolvability ("complexity")	risk-bearing capacity (solvency and liquidity buffers, leverage)
	participation in system-critical function/service, e.g., payment and settlement system ("substitutability")	economic significance of asset holdings, maturity mismatches debt pressure ("asset liquidation")
Policy objectives	avoid/mitigate contagion effect (by containing systemic impact upon failure)	maintain overall functioning of system and maximize survivorship
	avoid moral hazard	preserve mechanisms of collective burden sharing

Sources: Drehmann and Tarashev (2011), FSB (2011), Weistroffer (2011), and author. *Note:* The policy objectives and different indicators to measure systemic risk under both contribution and participation approaches are not exclusive to each concept. Moreover, the availability of certain types of balance sheet information and/or market data underpinning the various risk indicators varies between different groups of financial institutions, which requires a certain degree of customization of the measurement approach to the distinct characteristics of a particular group of financial institutions, such as insurance companies.

The distinction of measurement approaches also reflects varying channels of risk transmission, with the contribution of banks via common exposures and asset liquidation causing most systemic liquidity risk. In general, banks are prone to contribute to systemic risk from individual failures that propagate material financial distress or activities via intra-and inter-sectoral linkages to other institutions and markets ("contribution approach", see Table 2). Thus, material financial distress at such a bank, or the nature, scope, size, scale, concentration, or connectedness with other financial institutions, including through either their reliance on the same providers of funding and large common exposures to similar types of assets or their substitutability as providers of liquidity in critical payment and settlement systems could pose a threat to financial stability in the absence of close substitutes (FSOC, 2011). Among the main channels that facilitate the transmission of the negative effects caused by individual distress—substitutability, complexity of operations, connectedness via common exposures, and asset liquidation—especially the latter two are of greatest relevance from a technical perspective in this paper:

- *Exposures within the sector and/or financial system.* Claims by creditors, counterparties, investors, or other market participants, as well as common exposures to certain asset classes, industry sectors, and markets establish relationships that can affect both the probability but also the magnitude of systemic risk if these exposures are significant enough to cause either material impairment of other significant financial institutions (by threatening their financial condition and/or competitive position) or disruptions to critical functions of the sector and/or financial system.

- *Timing of payments and asset liquidation.* Liquidity risk and maturity mismatches are important criteria for assessing the potential of material financial distress to pose systemic risk. The sudden disposal of large asset positions of an institution in distress could significantly disrupt trading and/or cause significant losses for other firms with similar holdings due to increases in asset and funding liquidity risk (which could perpetuate and possibly accentuate the impact of individual failure on financial stability.

III. Methodology

A. Overview

The SRL model is based on an options pricing concept to gauge the general level of liquidity risk for a portfolio of institutions based on the current regulatory proposal aimed at limiting term structure transformation—the Net Stable Funding Ratio (NSFR). The NSFR defines individual liquidity risk as the effective maturity mismatch between short-term liabilities ("available stable funding", or ASF), weighted by their susceptibility to market fragilities (rollover risk), and short-term assets ("required stable funding", or RSF), weighted by their market liquidity value (funding risk), after controlling for the maturity of off-balance sheet hedging transactions, such as FX swaps.[21, 22] Using an "expected loss"

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[21] Funding risk is reflected in the liquidity of the market, i.e., the ability to trade an asset without affecting the price.

[22] The NSFR is defined as the ratio of "available amount of stable funding" of a bank divided by its "required amount of stable funding." Generally described, the numerator in the ratio, "available stable funding" (or ASF), is calculated by applying to items that are sources of funding a so-called "ASF factor," which ranges from 0 to 100 percent depending upon the stability of funding associated with the particular equity or liability component. Analogously, the denominator in the ratio, the "required stable funding" (or RSF), is calculated by applying to each asset and certain off-balance sheet commitments (i.e., items requiring funding) a so-called "RSF factor,"

(continued…)

notion to evaluate any level of liquidity shortfall to meet the regulatory minimum standard of stable funding, the model combines balance sheet information and market data (equity and equity options) in order to generate a stochastic measure of the NSFR . In this way, the probability of falling below the lower boundary of this structural ratio translates into a risk-adjusted analog of stable funding. Given the historical variation of the market data underpinning the components of the NSFR, and the extent to which this measure links institutions implicitly to common changes in market prices, it is then possible to derive a joint distribution of expected losses and determine when firms simultaneously fall below a funding threshold defined by the RSF. The joint probability function is then used to identify an individual institution's contribution to systemic liquidity risk to price liquidity risk within a macroprudential framework that can provide incentives for liquidity managers to internalize the systemic risk of their decisions. The contribution to systemic liquidity risk depends on an institution's funding and asset structure and its interconnectedness with other institutions, which informs the calculation of a firm-specific capital charge or insurance premium. Capital could be assigned based on own-liquidity risk plus some marginal contribution to joint liquidity risk, if larger. The sensitivity of these estimates can then be examined within a stress testing approach using shocks to asset values and the sources of funding.

The innovation of the SRL approach is its use of *contingent claims analysis* (CCA) to measure individual liquidity risk consistent with proposed prudential standards in order to quantify its system-wide effects. So far, the CCA methodology has been widely applied to measure and evaluate solvency risk and credit risk at financial institutions (see Appendix 1).[23] In the SRL model, however, CCA is used to derive a forward-looking measure of liquidity risk that helps determine the probability of an individual institution experiencing a liquidity shortfall and the associated expected loss when the shortfall indeed occurs. For a sample of financial institutions, these individual estimates are aggregated to a joint probability of expected losses from simultaneous liquidity shortfall, which also quantifies the marginal contribution of an institution to systemic liquidity risk.[24]

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which reflects the amount of the particular item that supervisors believe should be supported with stable funding.

[23] The CCA is a generalization of option pricing theory pioneered by Black and Scholes (1973) and Merton (1973 and 1974). It is based on three principles that are applied in this chapter: (i) the values of liabilities are derived from assets; (ii) assets follow a stochastic process; and (iii) liabilities have different priorities (senior and junior claims). Equity can be modelled as an implicit call option, while risky debt can be modelled as the default-free value of debt less an implicit put option that captures expected losses. In the SRL model, advanced option pricing is applied to account for biases in the Black-Scholes-Merton specification.

[24] This method uses publicly available information. Although the focus here is on banks given data limitations, the methodology is sufficiently flexible to be used for nonbank institutions that contribute to systemic liquidity

(continued…)

The SRL model accounts for changes in common factors determining individual funding conditions, their implications for the market-implied linkages between financial institutions, and the resulting impact on systemic liquidity risk. Thus, it accomplishes two essential goals of risk measures in this area: (i) measure the extent to which an institution contributes to systemic liquidity risk, and (ii) use this to indirectly price the liquidity assistance that an institution would need to receive in cases of severe funding problems in order to head off further escalation towards insolvency. The systemic dimension of the model is captured by the following three properties (see Table 4):

- *drawing on the market's evaluation of a firm's risk profile (including the liquidity risk that the institution will be unable to offset continuous cash outflows conditional on the regulatory expectation of stable funding).* That evaluation, in turn, is based on perceived riskiness of liquidity shortfall—i.e., the likelihood that the amount of available stable funding for each institution falls below the amount of required stable funding as defined by the NSFR—as implied by the institution's equity and equity options in the context of the economic and financial environment present at the time of measurement;

- *controlling for the firm's sources of stable funding.* The sources are modeled as being sensitive to the same markets as the funding sources of every other institution but by varying degrees. Changes in common funding conditions (and its impact on the perceived risk profile of each firm) establish market-induced linkages among institutions. The proposed framework thus combines market prices and balance sheet information to inform a risk-adjusted measure of systemic liquidity risk. That measure links institutions implicitly to the markets in which they obtain equity capital and funding; and

- *quantifying the chance of simultaneous liquidity shortfalls via joint probability distributions.* After obtaining a market-based measure of individual liquidity risk, the probability that two or more banks will experience liquidity shortfall simultaneously is made explicit by computing joint probability distributions (which also accounts for differences in the magnitude of an individual firm's liquidity shortfall). Hence, the liquidity risk resulting from a particular funding configuration is assessed not only for individual institutions but for all firms within a system in order to generate estimates of systemic risk. Because the SLR model takes into account the joint dynamics between the ASF and RSF via their covariance structure (and the impact of potential stresses on

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risk. Indeed, the proposal builds on several strands of recent research that focus on the interactions between financial institutions and markets in the context of systemic liquidity risk.

it), it provides a far deeper analysis of the liquidity risk to which a firm is exposed than does looking at them separately or with only accounting data.

Table 4. Main Features of the SRL Model.

Treatment of funding and market liquidity risks	Market and funding risks are embedded in equity prices and of funding rates and their volatility.
Treatment of solvency-liquidity feedbacks	There is no explicit treatment of the impact of solvency risk on liquidity risk. However, the derived risk-adjusted NSFR embeds a recognition that banks are vulnerable to solvency risks.
Treatment of channels of systemic risk	Estimates the non-linear, non-parametric dependence structure between sample firms so linkages are endogenous to the model and change dynamically.
Data requirements	Minimal use of supervisory data; approach relies on pre-defined prudential specification of liquidity risk (e.g., NSFR) to assess the impact of maturity mismatches but can be directly linked to non-diversifiable liquidity risk.

B. Model Specification and Estimation Steps

The SRL model follows a three-step estimation process. First, the components of the NSFR are valued at market prices in order to generate a time-varying measure of funding risk in keeping with the proposed prudential liquidity standards aimed at limiting maturity transformation. Second, the aggregate cash flow implications of changes to liquidity risk causing a bank to fail the market-based NSFR are modelled as a put option in order to estimate expected losses arising from insufficient stable funding. Finally, these individually estimated net exposures from liquidity risk are aggregated via a multivariate distribution that determines the probabilistic measure of joint liquidity shortfalls on a system-wide level.

1. Step 1 – Calculating the market-based measure of stable funding ("market-based NSFR")

First, the SRL model transposes the proposed Basel III liquidity standard aimed at limiting maturity transformation—the Net Stable Funding Ratio (NSFR)—into a market-based measure of individual liquidity risk. This regulatory benchmark serves as the starting point for the quantification of individual liquidity risk.[25] The components of the NSFR—the ASF in the numerator and the RSF in the denominator—are transposed into a time-varying measure of the NSFR at market prices, where the RSF and ASF values reflect differences between the balance sheet and actual market values of total assets and liabilities of each firm (see Figure 2).[26] The actual balance sheet measures of the ASF and RSF values are re-scaled by (i) the ratio of the book value of total liabilities to the present value of total liabilities *B* (which can be observed) and (ii) the ratio of the book value of total assets to market-implied value of total assets *A* (which is obtained as a risk-neutral density from equity option prices with maturities between three and 12 months, see below), respectively.[27] Doing so generates the values A_{RSF} and B_{ASF}, which transform the NSFR ratio into a marked-based measure of an institution's liquidity risk ("market-implied NSFR").

Since the asset value *A* cannot be observed, it is estimated directly from option prices. More specifically, the state-price density (SPD) of the implied total asset value underpinning A_{RSF} is estimated from equity option prices without any assumptions on the underlying diffusion process. This avoids the calibration error of using the "two-equations-two unknowns" approach in the traditional Merton model, which contains empirical irregularities that can influence solving both implied asset value and asset volatility simultaneously.[28] Using equity option prices, we can derive the risk-neutral probability distribution of the underlying asset price at the maturity date of the options. We determine the implied asset value as the expectation over the empirical SPD by adapting the Breeden and Litzenberger

[25] The NSFR measures the amount of longer-term, stable sources of funding employed relative to the liquidity profiles of the assets funded and the potential for contingent calls on funding liquidity arising from off-balance sheet commitments and obligations.

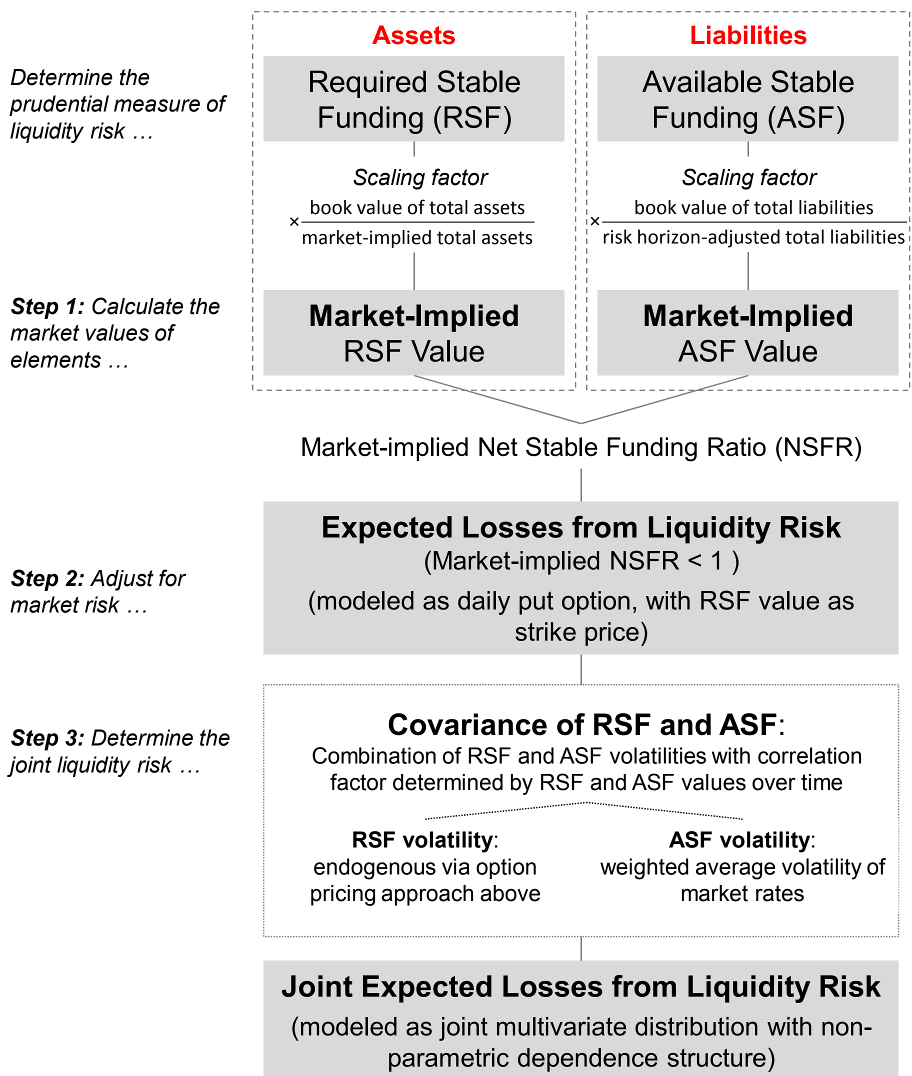
[26] This approach represents a significant improvement over concept of systemic liquidity insurance presented in Perotti and Suarez (2009).

[27] Estimations of these scaling factors, and the subsequent covariance and the joint expected losses, are computed over a rolling window of τ observations with periodic updating to reflect their changing characteristics.

[28] Even though the Merton model contains simplifying assumptions, such as constant volatility as well as a lognormal and continuous asset process, its empirical irregularities are more pronounced the lower the intrinsic value of the put option (and the further away asset values are from the default barrier). In other words, alternative (and more accurate) option pricing methods would generate expected losses *similar* to the ones under the Merton model in distress situations while differences would emerge as distress abates.

(1978) method (see Appendix 2), together with a semi-parametric specification of the Black-Scholes option pricing formula (Aït-Sahalia and Lo, 1998). More specifically, this approach uses the second derivative of the call pricing function (on European options) with respect to the strike price (rather than option prices as identifying conditions). Estimates are based on option contracts with identical time to maturity, assuming a continuum of strike prices. Since available strike prices are always discretely spaced on a finite range around the actual price of the underlying asset, interpolation of the call pricing function inside this range and extrapolation outside this range are performed by means nonparametric (local polynomial) regression of the implied volatility surface (Rookley, 1997).

Figure 2. Methodology to Compute Systemic Liquidity under the Systemic Risk-adjusted Liquidity (SRL) Model.



2. Step 2 – Calculating expected losses from funding liquidity risk ("risk-adjusted NSFR")

Second, the net exposure arising from individual liquidity risk is modelled as a put option in order to derive a risk-adjusted version of the market-based NSFR (see Figure 3). The aggregate cash flow implications of changes to liquidity risk can then be quantified by viewing liquidity risk as if it were a put option written on the NSFR, where the present value of the RSF represents the "strike price," with the short-term volatility of all assets underpinning the RSF determined by the implied volatility derived from equity options prices.[29] The value of the ASF is assumed to follow a random walk with intermittent jumps that create sudden and large changes in the valuation of the liabilities. The volatility of these liabilities included in the ASF is computed as a weighted average of the observed volatilities of latent factors derived from a set of market funding rates deemed relevant for banks. These two time-varying elements, the adjusted RSF and ASF values, provide the basis for computing a put option, which has intrinsic value (i.e., is "in-the-money") when the discounted value of the ASF falls below that of the RSF over the same time horizon, constituting an expected loss due to liquidity shortfall. The value of this derived put option can be shown to result in significant hypothetical cash losses for an individual firm as the risk-adjusted NSFR declines.

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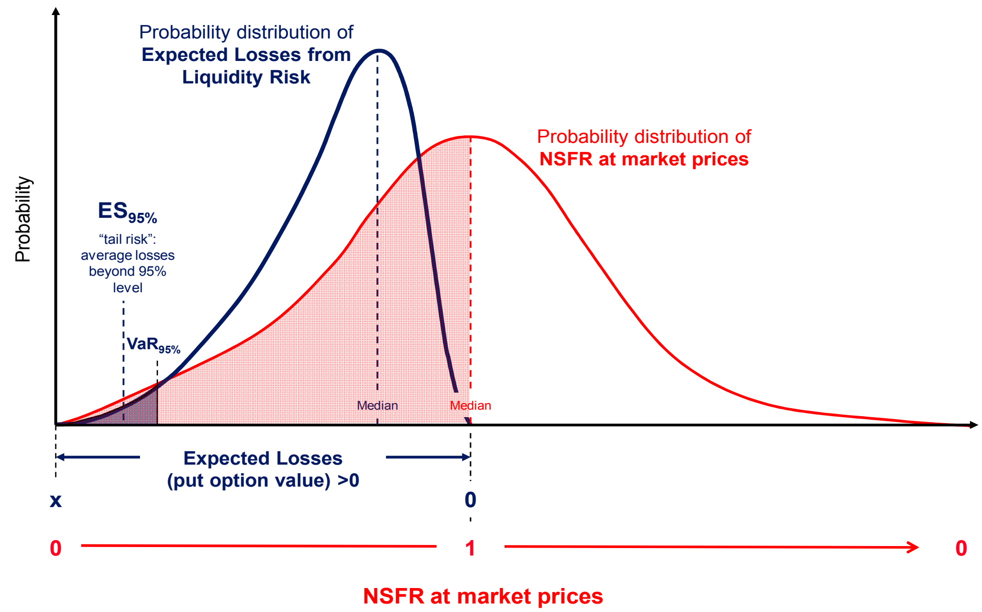
[29] The NSFR reflects the impact of funding shocks as an exposure to changes in market prices in times of stress. The procedure can also be applied to other measures of an individual firm's liquidity risk.

Figure 3. Conceptual Relation between the Net Stable Funding Ratio at Market Prices and Expected Losses from Liquidity Risk.



Note: This figure illustrates the relation between the net exposure from liquidity risk and the NSFR at market prices distribution functions (based on multiple observations of each over a certain period of time). Expected losses, which are modeled as a put option that approximates the cash flow profile over a given risk horizon, arise once there is some probability that the NSFR drops below the regulatory requirement to be greater than one. The greater the potential for funding distress projected by a declining NSFR, the greater are these losses. The tail risk of individual expected losses from a liquidity shortfall is represented by the expected shortfall (ES) at the 95th percentile, which is the area under the curve beyond the threshold value set by the Value-at-Risk (VaR).

The value of the put option increases the higher the probability of the ASF falling below the RSF over a one-year risk horizon (so that the NSFR is smaller than one, which would breach the lower boundary that banks will be mandated to maintain under the current Basel III proposal). Such probability is influenced by changes to the firm's funding pattern, risk profile, and market perceptions of risk, which can be derived from changes to the implied asset value and volatility reflected in the institution's equity option prices and from its asset and liability structure. Thus, the present value of market-implied expected losses associated with the current liquidity position of a single institution can be valued as a modified implicit put option of B_{ASF} based on the present value of A_{RSF} as the strike price in keeping with the traditional Black-Scholes-Merton (BSM) model (see Appendix 1):

$$P_{NSFR}(t) = A_{RSF} e^{-r(T-t)} \Phi\left(-d_{NSFR} - \sigma_{A_{RSF},B_{ASF}} \sqrt{T-t}\right) - B_{ASF} \Phi\left(-d_{NSFR}\right), \tag{1}$$

over time horizon T–t at the general risk-free rate, r, and asset growth rate, $r_{A_{RSF}}$, where

$$d_{NSFR} = \ln\left(\frac{B_{ASF}}{A_{RSF}(t)}\right) + \left(\left(r_{A_{RSF}} + \frac{\sigma^2_{A_{RSF},B_{ASF}}}{2}\right)(T-t)\right) \Big/ \sigma_{A_{RSF},B_{ASF}}\sqrt{T-t}. \tag{2}$$

Note the dependence on the duration of liabilities (or debt claims) *B* underpinning the ASF, the total funding mismatch of the firm $A_{RSF}(t)/B_{ASF}$ using the definition underpinning the NSFR assumptions, and the joint asset-liability volatility given by

$$\sigma_{A_{RSF},B_{ASF}} = \sqrt{\sigma^2_{A_{RSF}} + \sigma^2_{B_{ASF}} - 2\rho_{A_{RSF},B_{ASF}}\sigma_{A_{RSF}}\sigma_{B_{ASF}}}. \tag{3}$$

$\Phi(\cdot)$ in equation (1) denotes the standard normal distribution. The volatility of the RSF, $\sigma_{A_{RSF}}$, draws on the *general* asset volatility (which is "embedded" in the equity option formula)

$$\sigma_{A_{RSF}} \equiv \sigma_A = \frac{E(t)}{A(t)\Phi(d)}\sigma_E = \left(1 - \frac{Be^{-r(T-t)}\Phi\left(d-\sigma_A\sqrt{T-t}\right)}{A(t)\Phi(d)}\right)\sigma_E, \tag{4}$$

similar to that employed in the CCA approach (see Appendix 1) and conditional on changes in leverage $A(t)/B$, where

$$d = \ln\left(\frac{A(t)}{B}\right) + \left(\left(r_A + \frac{\sigma^2_A}{2}\right)(T-t)\right) \Big/ \sigma_A\sqrt{T-t}. \tag{5}$$

$E(t)$ represents the equity value at time *t*, σ_E is the equity volatility, and r_A is the risk-free asset rate.[30] The volatility of short-term liabilities in the ASF, $\sigma_{B_{ASF}}$, is derived as a weighted average of the observed volatilities of latent factors of significant market interest rates identified by a dynamic factor model (DFM)[31] (see Box 2). The correlation $\rho_{A_{RSF},B_{ASF}}$ between

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[30] Note that in the empirical section of this paper, the approximation technique using Moody's KMV is used to derive $\sigma_{A_{RSF}}$ (see Box 1).

[31] A DFM of the ASF is specified based on one principal component extracted from each group of observed market rates (at different maturities) as explanatory variables: (i) short-term sovereign rates (with maturities ranging from three to nine months); long-term sovereign rates (with maturity ranging from three to ten years); (ii) total equity market returns (domestic market and Morgan Stanley Composite Index (MSCI)); (iii) financial bond rates (investment grade, both medium- and long-term); (iv) domestic currency LIBOR rates (ranging from

(continued…)

both volatilities can be constant or estimated using the historical relation between the ASF and the RSF (at levels) over a rolling time window.

This specification of option price-based expected losses, however, does not incorporate skewness and kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices (Backus and others, 2004). Thus, we mitigate the shortcomings of the BSM approach and enhance equation (1), without altering its general analytical form, by means of a jump diffusion that follows a standard Poisson process

$$\Pr\left(N\left(t\right)=k\right)=\frac{\left(\lambda t\right)^{k}}{k!\,e^{-\lambda t}}, \tag{6}$$

where λ is the average number of jumps per unit time (i.e., the number of jump events up to time t). The jump size follows a log-normal distribution

$$J \sim \varphi \exp\left[-\frac{\nu^{2}}{2}+\nu\Phi\left(0,1\right)\right] \tag{7}$$

with average jump size φ and volatility v of the jump size calibrated over an estimation time period.[32] The k^{th} term in this series corresponds to the scenario where k jumps occur over a specific time horizon. By conditioning the present value of A_{RSF} on the expected jump process over a rolling window of τ observations with periodic updating, the put option value can be written as

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three to nine months); and the domestic currency overnight index swap (OIS) rates (ranging from three to nine months). The volatility of the ASF is calculated as the average volatility of these daily market rates weighted by the regression coefficient of each principal component, estimated over a five-year period from January 3, 2005 to December 14, 2010.

[32] Further refinements of this option pricing model are possible, including various simulation approaches, which might come at the expense of losing analytical tractability. The ad hoc model of Dumas and others (1998) is designed to accommodate the implied volatility smile and is easy to implement, but requires a large number of market option prices. The pricing models by Heston (1993) and Heston and Nandi (2000) allow for stochastic volatility, but the parameters driving these models can be difficult to estimate. Many other models have been proposed, to incorporate stochastic volatility, jumps, and stochastic interest rates. Bakshi and others (1997), however, suggest that most of the improvement in pricing comes from introducing stochastic volatility. Introducing jumps in asset prices leads to small improvements in the accuracy of option prices. Other option pricing models include those based on copulas, Levy processes, neural networks, GARCH models, and non-parametric methods. Finally, the binomial tree proposed by Cox and others (1979) spurred the development of lattices, which are discrete-time models that can be used to price any type of option—European or American, plain-vanilla or exotic.

$$P_{NSFR_k}(t)=\sum_{k=0}^{\infty}\frac{\exp(-\varphi\lambda t)(\varphi\lambda t)^k}{k!}A_{RSF}e^{-r(T-t)}\Phi\left(-d_{NSFR_k}-\sigma_{A_{RSF},B_{ASF}}\sqrt{t}\right)-B_{ASF}\Phi\left(-d_{NSFR_k}\right). \quad (8)$$

The asset volatility of the RSF with jumps $\sigma_{A_{RSF_k}}=\sqrt{\sigma^2_{A_{RSF}}+k\nu^2/t}$ as well as the risk-free interest rate $r_{A_{RSF_k}}=r_{A_{RSF}}-\lambda(\varphi-1)+k\ln(\varphi)/t$ are updated accordingly,[33] so that equations (2) and (3) can be re-written as

$$d_{NSFR_k}=\ln\left(\frac{B_{ASF}}{A_{RSF}(t)}\right)+\left(\left(r_{A_{RSF_k}}+\frac{\sigma^2_{A_{RSF_k},B_{ASF}}}{2}\right)(T-t)\right)\Big/\sigma_{A_{RSF_k},B_{ASF}}\sqrt{T-t} \quad (9)$$

and

$$\sigma_{A_{RSF_k},B_{ASF}}=\sqrt{\sigma^2_{A_{RSF_k}}+\sigma^2_{B_{ASF}}-2\rho_{A_{RSF_k},B_{ASF}}\sigma_{A_{RSF_k}}\sigma_{B_{ASF}}}\,. \quad (10)$$

Analogous to equation (4) under the BSM model, the volatility of the RSF, $\sigma_{A_{RSF_k}}$, is set equal to the revised general asset volatility σ_{A_k} (consistent with the above specification of a jump diffusion process),[34] which is derived by solving

$$\sigma_{A_{RSF_k}}\equiv\sigma_{A_k}=\sqrt{\left(\frac{E(t)}{A(t)\Phi(d_k)}\sigma_E\right)^2+\frac{k\nu^2}{t}}\,, \quad (11)$$

where

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[33] The value of derivatives with convex payoffs (which includes this put option specification) increases when jumps are present (i.e., when $\lambda>0$)—regardless of the average jump direction.

[34] Given that the implied asset value is derived separately, this approach avoids the traditional "two-equations-two-unknowns" approach to derive implied assets and asset volatility based on Jones and others (1984), which was subsequently extended by Ronn and Verma (1986) to a single equation to solve two simultaneous equations for asset value and volatility as two unknowns. Duan (1994) shows that the volatility relation between implied assets and equity is redundant if equity volatility is stochastic. An alternative estimation technique for asset volatility introduces a maximum likelihood approach (Ericsson and Reneby, 2004 and 2005) which generates good prediction results.

$$d_k = \ln\left(\frac{A(t)}{B}\right) + \left(\left(r_{A_k} + \frac{\sigma^2_{A_k}}{2}\right)(T-t)\right) \Big/ \sigma_{A_k}\sqrt{T-t}\ , \tag{12}$$

with general asset volatility with jumps $\sigma_{A_k} = \sqrt{\sigma_A^2 + k v^2 / t}$ and the risk-free interest rate $r_{A_k} = r_A - \lambda(\varphi - 1) + k\ln(\varphi)/t$.

Box 1. Derivation of the Implied Asset Volatility Using the Moody's KMV Model.

Default risk in the global Moody's KMV model (Crosbie and Bohn, 2003) is defined using a Merton-style option pricing definition (see Appendix 1). But under the physical measure, the asset volatility under risk-neutral valuation can also be derived explicitly in absence of information about equity volatility—in lieu of equation (4) above.

We can recover a measure of the *adjusted* asset volatility (see Appendix 4)

$$\sigma_A^* = \left(\Phi^{-1}\left(EDF_{A_{MKMV}}\right) + \rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M}\right)$$
$$\pm\sqrt{\left(\Phi^{-1}\left(EDF_{A_{MKMV}}\right) + \rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M}\right)^2 + 2\ln\left(\frac{A_{MKMV}(t)}{B_{MKMV}}\right) + r_{A_{MKMV}}}$$

to account for the market price of risk, given the modified estimated default frequency (EDF) after adjustment for risk-neutrality based on

$$EDF_{A_{MKMV}} = \Phi\left(d_{MKMV} - \sigma_A^*\sqrt{T-t}\right) =$$
$$\Phi\left(\left(\ln\left(\frac{A_{MKMV}(t)}{B_{MKMV}}\right) + \left(r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^*\frac{\mu_M - r}{\sigma_M} - \frac{\left(\sigma_A^*\right)^2}{2}\right)(T-t)\right)\Big/\sigma_A^*\sqrt{T-t}\right),$$

which satisfies equation (4) above, and uses the correlation between the asset and market return $\rho_{A_{MKMV},M}$, asset return $r_{A_{MKMV}}$, the "Market Sharpe Ratio" $(\mu_M - r)/\sigma_M$ (which changes daily but is assumed to be the same for all firms and financial institutions, consisting of the general risk-free rate r, average market return μ_M, and market volatility σ_M).

Thus, the actual (but unobservable) asset volatility, σ_A, as risk-neutral model input, is derived by solving the optimization constraint

$$\frac{1}{\tau}\sum_{t=1}^{\tau}\left(\sigma_A^*\sqrt{\frac{A(t)}{E(t)}\frac{\gamma(t)}{\eta^*}}\right) = \frac{1}{\tau}\sum_{t=1}^{\tau}\sigma_A^*$$

over a rolling time window of τ number of estimation days (consistent with the maximum-likelihood estimation of the multivariate distribution below) on η^* subject to the intertemporal error correction term

$$\gamma(t) = \frac{\sigma_{A(t)}}{\sigma_{A(t)_{MKMV}}} \frac{\sigma_{A(t+\tau)_{MKMV}}}{\sigma_{A(t+\tau)}}$$

over the same estimation window (with $\sigma_{A(t)_{MKMV}}$ and $\sigma_{A(t+\tau)_{MKMV}}$ denoting the asset volatility at time t and $t+\tau$ days underpinning the observed "real" default probability $EDF_{A_{MKMV}}$ reported by Moody's KMV for an individual entity). Thus, the general asset volatility under risk-neutrality can be written as

$$\sigma_A = \sigma_A^* \frac{\gamma(t)}{\eta^*} \sqrt{\frac{A(t)}{E(t)}}\,.$$

3. *Step 3 – Estimating the joint expected losses from liquidity risk ("joint, risk-adjusted NSFR")*

Finally, the individually estimated net exposures to liquidity risk are aggregated to determine the magnitude of liquidity shortfalls on a system-wide level (see Figure 4). The expected losses arising from the variation of each individual firm's risk-adjusted NSFR over time are treated as a portfolio for which we calculate the joint probability of all firms experiencing a liquidity shortfall simultaneously. In this way, each firm's maturity mismatch between assets and liabilities, its implication on the market-based assessment of its risk profile, and the stability of its funding are linked with those characteristics at other firms that are subject to common changes in market conditions.

We adapt the Systemic CCA framework (Gray and Jobst, 2011a, 2011b, and forthcoming) to formally capture the realizations of joint liquidity shortfalls by controlling for the realization of tail events.[35] As part of a four-step process, we define a non-parametric dependence function of individual expected losses, which is combined with the marginal distributions of these individual loss estimates, in order to generate a joint distribution that defines an aggregate measure of liquidity risk. Using estimates of the joint

[35] See also Gray and Jobst (2009, 2010a, and 2010b) as well as Gray and others (2010) for a more general application of this approach to the integrated balance sheets of an entire economy. An application of the Systemic CCA approach in the context of FSAP stress tests and spillover analysis can be found at IMF (2010b, 2010c, 2011a, 2011c, 2011d, 2011e, 2011f, and 2012). See also IMF (2008).

tail risk of this multivariate distribution, such as the *conditional Value-at-Risk* (VaR) (or *expected shortfall* (ES)), we can gauge systemic liquidity risk in times of stress at a statistical confidence level of choice.

(i) Estimating the marginal distributions of individual expected losses from liquidity risk

We first specify the individual asymptotic tail behavior of individual expected losses in keeping with extreme value theory (EVT). EVT is as a general statistical concept of deriving a limit law for sample maxima, where the Fisher-Tippett-Gnedenko theorem (Fisher and Tippett, 1928; Gnedenko, 1943) defines the attribution of a given distribution of normalized maxima (or minima) to be of extremal type (assuming that the underlying function is continuous on a closed interval). Let the vector-valued series

$$\mathbf{X}_j^n = P_{NSFR_k,1}^n(t),\ldots,P_{NSFR_k,m}^n(t) \tag{13}$$

denote i.i.d. random observations of expected losses (i.e., a total of *n*-number of daily put option values $P_{NSFR_k,m}^n(t)$ up to time *t*), each estimated over a rolling window of τ observations with periodic updating for $j \in m$ firms in the sample. Given $X = \max\left(P_{NSFR_k,j}^1(t),\ldots,P_{NSFR_k,j}^n(t)\right)$, there exists a choice of normalizing constants $\beta_j^n > 0$ and α_j^n, such that the probability of each ordered *n*-sequence of normalized sample maxima $\left(X-\alpha^n\right)/\beta^n$ converges to the non-degenerate limit distribution $G(x)$ as $n \to \infty$ and $x \in \mathbb{R}$, so that

$$F_n^{\left[\beta^n x+\alpha^n\right]}(x) = \lim_{n\to\infty} \Pr\left(\frac{X-\alpha^n}{\beta^n} \le y\right) = \left[F\left(\beta^n y+\alpha^n\right)\right]^n \to G(x), \tag{14}$$

falls within the maximum domain of attraction (MDA) of the generalized extreme value (GEV) distribution and conforms to one of three distinct types of extremal behavior, Gumbel (EV0), Fréchet (EV1) or negative Weibull (EV2)) as limiting distributions of maxima of dependent random variables:

$$\text{EV0: } G_0(x) = \exp\left(-\exp(-x)\right) \quad \mathit{if}\ x \ge 0, \xi = 0, \tag{15}$$

$$\text{EV1: } G_{1,\xi}(x) = \exp\left(-x^{-1/\xi}\right) \quad \mathit{if}\ x \in \left[\mu - \sigma/\xi, \infty\right), \xi > 0, \text{ and} \tag{16}$$

$$\text{EV2: } G_{2,\xi}(x) = \exp\left(-(-x)^{-1/\xi}\right) \quad \textit{if } x \in (-\infty, \mu - \sigma/\xi], \xi < 0 .\text{[36]} \tag{17}$$

The limit distributions above are combined into a *unified* parametric family of the GEV probability distribution function with the shape parameter determined by the type of sub-model (EV0, EV1 or EV2), so that

$$g_{\mu,\sigma,\xi}(x) = \begin{cases} \dfrac{1}{\sigma}\left(\left(1 + \dfrac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi}\right)^{\xi+1} \exp\left(-\left(1 + \dfrac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi}\right) & \textit{if } 1 + \dfrac{\xi(x-\mu)}{\sigma} > 0, \xi \neq 0 \\ \dfrac{1}{\sigma}\left(\left(1 + \dfrac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi}\right)^{\xi+1} \exp\left(-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right) & \textit{if } x \in (-\infty, +\infty), \xi = 0 \end{cases} \tag{18}$$

or simply

$$g_{\mu,\sigma,\xi}(x) = \frac{1}{\sigma}\left(\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi}\right)^{\xi+1} \exp\left(-\left(1 + \frac{\xi(x-\mu)}{\sigma}\right)_+^{-1/\xi}\right), \tag{19}$$

and the cumulative distribution function

$$G_{\mu,\sigma,\xi}(x) = \begin{cases} \exp\left(-\left(1 + \dfrac{\xi(x-\mu)}{\sigma}\right)^{-1/\xi}\right) & \textit{if } 1 + \dfrac{\xi(x-\mu)}{\sigma} \geq 0 \\ \exp\left(-\exp(-x)\right) & \textit{if } x \in \mathbb{R}, \xi = 0. \end{cases} \tag{20}$$

Thus, in the context of multiple series, the j^{th} univariate marginal density function based on GEV is defined as

$$y_j(x) = \left(1 + \frac{\hat{\xi}_j\left(x - \hat{\mu}_j\right)}{\hat{\sigma}_j}\right)_+^{-1/\hat{\xi}_j} \quad (\text{for } j = 1, \ldots, m) \tag{21}$$

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[36] See Embrechts and others (1997) as well as Vandewalle and others (2004) for additional information on the definition of extreme value theory. For an application of extreme value theory in the insurance sector, see Thérond and Ribereau (2012) and Thérond and Planchet (2007).

where $1+\xi_j\left(x-\mu_j\right)/\sigma_j>0$, the scale parameter is $\sigma_j>0$, location parameter is μ_j, and shape parameter is $\xi_j\neq 0$.[37,38] The moments of the univariate density function in equation (21) above are estimated via the *linear combinations of ratios of spacings* (LRS) method (see Appendix 3), which determines how quickly the probability of extreme observations converges to zero), which identifies possible limiting laws of asymptotic tail behavior, i.e., the likelihood of even larger extremes as the level of statistical confidence approaches certainty (Coles and others, 1999; Poon and others, 2003; Jobst, 2007).

(ii) Estimating the dependence structure of individual expected losses from liquidity risk

Second, we define the co-movement of expected losses as a non-parametric, multivariate dependence function by expanding the bivariate logistic method proposed by Pickands (1981) to the multivariate case and adjusting the margins according to Hall and Tajvidi (2000) so that

$$\Upsilon(\omega)=\min\left\{1,\max\left\{n\left(\sum_{i=1}^{n}\bigwedge_{j=1}^{m}\frac{y_{i,j}/\hat{y}_{\bullet j}}{\omega_j}\right)^{-1},\omega,1-\omega\right\}\right\},\qquad(22)$$

where $\hat{y}_{\bullet j}=\sum_{i=1}^{n}y_{i,j}/n$ reflects the average marginal density of all $i\in n$ put option values and $0\leq\max\left(\omega_1,...,\omega_{m-1}\right)\leq\Upsilon\left(\omega_j\right)\leq 1$ for all $0\leq\omega_j\leq 1$.[39] $\Upsilon(\cdot)$ represents a convex function on $[0,1]$ with $\Upsilon(0)=\Upsilon(1)=1$, i.e., the upper and lower limits of $\Upsilon(\cdot)$ are obtained under complete dependence and mutual independence, respectively. It is estimated iteratively (and

[37] The shape parameter also indicates the number of moments of the distribution, e.g., if $\xi=1/2$, the first moment (mean) and the second moment (variance) exist, but higher moments have an infinite value. This is of practical importance since many results for asset pricing in finance rely on the existence of several moments. A positive shape parameter also implies that the moments of order $n\geq 1/\xi$ are unbounded, i.e., $1/\xi$ indicates the highest bounded moment for the distribution.

[38] The upper tails of most (conventional) limit distributions (weakly) converge to this parametric specification of asymptotic behavior, irrespective of the original distribution of observed maxima (unlike parametric VaR models). The higher the absolute value of shape parameter, the larger the weight of the tail and the slower the speed at which the tail approaches its limit.

[39] Note that the marginal density of a given extreme relative to the average marginal density of all extremes is minimized ("$\wedge$") across all firms $j\in m$, subject to the choice of factor ω_j.

over a rolling window of τ observations with periodic updating (e.g., a daily sliding window of 120 days)) subject to the optimization of the (m-1)-dimensional unit simplex

$$S_m = \left\{ \left(\omega_1, ..., \omega_{m-1}\right) \in \mathbb{R}_+^m : \omega_j \geq 0, 1 \leq j \leq m-1; \sum_{j=1}^{m-1} \omega_j \leq 1 \text{ and } \omega_m = 1 - \sum_{j=1}^{m-1} \omega_j \right\}, \tag{23}$$

which establishes the degree of coincidence of multiple series of cross-classified random variables similar to a χ-statistic that measures the statistical likelihood that observed values differ from their expected distribution (Jobst and Kamil, 2008). This specification stands in contrast to a general copula function that links the marginal distributions using only a single (and time-invariant) dependence parameter.

(iii)Estimating the joint distribution of expected losses from liquidity risk

We then combine the marginal distributions of these individual expected losses with their dependence structure to generate a *multivariate extreme value distribution* (MGEV) over the a rolling window of τ number of days with daily updating by following the aggregation mechanism of the Systemic CCA approach.[40] The resultant multivariate cumulative distribution function is specified as

$$G_{t,m}\left(x\right) = \exp\left\{ -\left(\sum_{j=1}^{m} y_{t,j} \right) \Upsilon_t\left(\omega\right) \right\} \tag{24}$$

with corresponding probability density function

$$g_{t,m}\left(x\right) = \hat{\sigma}_{t,m}^{-1} \left(\left(\sum_{j=1}^{m} y_{t,j} \right) \Upsilon_t\left(\omega\right) \right)^{\hat{\xi}_{t,m}+1} \exp\left\{ -\left(\sum_{j=1}^{m} y_{t,j} \right) \Upsilon_t\left(\omega\right) \right\} \tag{25}$$

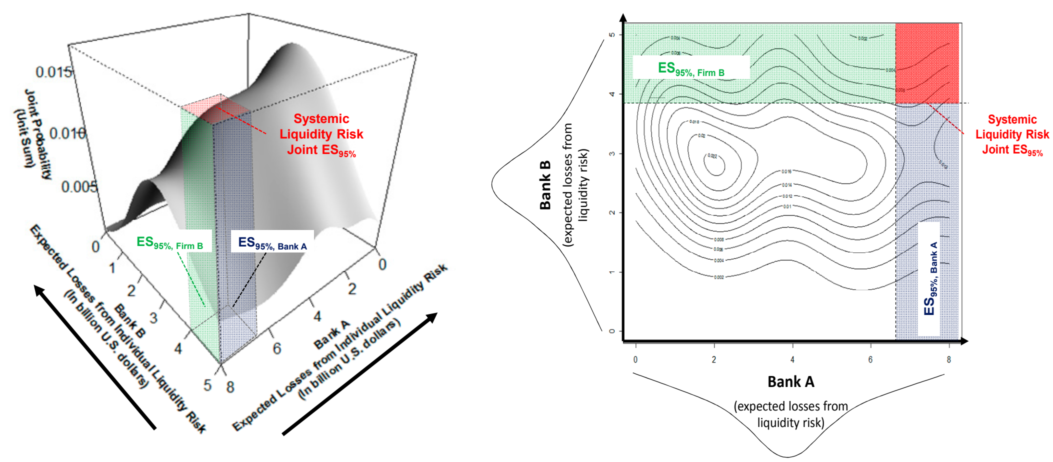
at time $t=\tau+1$ by maximizing the likelihood $\prod_{j=1}^{m} g_{t,m}\left(x|\theta\right)$ over all three parameters $\theta=\left(\mu,\sigma,\xi\right)$ simultaneously. Equation (24) represents the functional form of the daily series

[40] The analysis of dependence is completed separately from the analysis of marginal distributions, and, thus, differs from the classical approach, where multivariate analysis is performed jointly for marginal distributions and their dependence structure by considering the complete variance-covariance matrix, such as the MGARCH approach. This approach is fundamental to the specification of copula functions, and is also applied to the aggregation methodology.

of expected losses from liquidity shortfall for all sample firms based on the empirical panel of observations $\mathbf{X}_j^n$ in equation (13) above. Since the logarithm is a continuously increasing function over the range of likelihood, the parameter values that maximize the likelihood will also maximize the logarithm as a global maximum. Thus, we can write the lognormal likelihood function as $\sum_{j=1}^{m} \ln g\left(x|\theta\right)$ so that the maximum likelihood estimate (MLE) of the true values θ_0 is

$$\hat{\theta}_{MLE} = \underset{\theta\in\Theta}{\arg\max}\, \hat{\ell}\left(\theta|x\right) \to \theta_0. \tag{26}$$

Figure 4. Conceptual Relation between Expected Losses from Liquidity Risk: Two-Firm (Bivariate) Case.



Note: This figure illustrates the bivariate case of aggregating expected losses in order to determine the joint probability of two sample firms experiencing a liquidity shortfall at the same time, using the estimation results for individual institutions. The left panel of Figure 4 shows the kernel density function of two firms (Bank A and Bank B). The probability of systemic liquidity risk is captured by combining the individual bank estimates (depicted by the green and blue panels), which generates the joint expected losses at a defined level of statistical confidence, such as the 95th percentile (red cube). The left panel can also be shown in two-dimensions as a so-called "contour plot" (see right panel of Figure 4).

(iv) <u>Estimating a tail risk measure of joint expected losses from liquidity risk</u>

Finally, we obtain the general expression of the joint *expected shortfall* (ES) (or conditional *Value-at-Risk* (VaR)) as the probability-weighted residual density beyond a pre-specified statistical confidence level (say, a=0.95) of maximum losses. ES defines the conditional average value z of aggregate potential losses in excess of the statistical confidence limit ("severity threshold") based on all observations over estimation days τ. Thus, we can obtain continuous densities from $G(\cdot)$ and compute ES as

$$ES_{t,\tau,m,a} = -\frac{1}{a}\int_0^a G_{t,\tau,m}^{\leftarrow}(x)\,dx = -\frac{1}{a}\int_0^a G_{t,\tau,m}^{-1}(x)\,dx = -\mathrm{E}\left[z_\tau \middle| z_\tau \geq G_{t,\tau,m}^{-1}(a) = VaR_{t,a}\right], \qquad (27)$$

where $G^{-1}(a)$ is the quantile corresponding to probability *a* and $G^{-1}(a) = G^{\leftarrow}(a)$ with $G^{\leftarrow}(a) \equiv \inf\left(x \middle| G(x \geq a)\right)$ and threshold quantile value

$$VaR_{t,a} = \sup\left\{G_{t,\tau,m}^{-1}(\square) \middle| \Pr\left[z_\tau > G_{t,\tau,m}^{-1}(\square)\right] \geq a = 0.95\right\}, \qquad (28)$$

with the point estimate of joint potential losses of *m* firms at time *t* defined as[41]

$$G_{t,\tau,m}^{-1}(a) = \hat{\mu}_{t,m} + \hat{\sigma}_{t,m} / \hat{\xi}_{t,m}\left(\left(-\frac{\ln(a)}{\Upsilon_t(\omega)}\right)^{-\hat{\xi}_{t,m}} - 1\right). \qquad (29)$$

C. Pricing the cost of liquidity support

The aggregate measure of expected losses can then be used to base practical macroprudential tools that help mitigate liquidity risk from both an individual and system-wide perspective. We extract the time-varying contribution of each individual institution as the discounted capital loss resulting from a possible funding shortfall or an approximation of a fair value risk premium for systemic liquidity risk affecting the likelihood of individual funding shortfall.

The contribution of each individual firm is determined by calculating the cross-partial derivative of the joint distribution of expected losses.[42] The joint *expected shortfall* (ES) can also be written as a linear combination of individual ES values, $ES_{t,\tau,j,a}$, where the relative weights $\psi_{\tau,m,a}$ (in the weighted sum) are given by the second order cross-partial

[41] Expected shortfall (ES) is an improvement over Value-at-Risk (VaR), which, in addition to being a pure frequency measure, is "incoherent," i.e., it violates several axioms of convexity, homogeneity, and sub-additivity found in coherent risk measures. For example, sub-additivity, which is a mathematical way to say that diversification leads to less risk, is not satisfied by VaR.

[42] Note that this approach could also be used to identify the effectiveness of closer supervisory monitoring of identified liquidity problems of a particular bank. If remedial actions are effective, they would decrease the bank's contribution to overall systemic liquidity risk to a level that closely matches the individual liquidity risk.

derivative of the inverse of the joint probability density function $G^{-1}_{t,\tau,m}(\cdot)$ to changes in both the dependence function $\Upsilon_t(\cdot)$ and the individual marginal severity $y_{t,j}$ of expected losses. Thus, by re-writing $ES_{t,\tau,m,a}$ in equation (27) above, we obtain

$$ES_{t,\tau,m,a} = -\sum_{j}^{m} \psi_{\tau,m,a} \mathrm{E}\left[z_{\tau,j} \middle| z_{\tau,m} \geq G^{-1}_{t,\tau,m}(a) = VaR_{t,a} \right], \tag{30}$$

where the relative weight of institution *j* at statistical confidence level *a* is defined as the marginal contribution

$$\psi_{\tau,m,a} = \frac{\partial^2 G^{-1}_{t,\tau,m}(a)}{\partial y_{t,j} \partial \Upsilon_t(\omega)} \quad \text{s.t.} \sum_{j}^{m} \psi_{\tau,j,a} = 1 \text{ and } \psi_{\tau,j,a} z_{\tau,j} \leq z_{\tau,m}, \tag{31}$$

attributable to the joint effect of both the marginal density and the change of the dependence function due to the presence of institution $j \in m$ in the sample.

We can use this amount to develop two price-based risk mitigation mechanisms, a capital surcharge and an insurance premium, which take into account the contingent support that banks would receive from a central bank in times of systemic liquidity stress, and, thus, represent the potential public sector cost arising from two or more institutions experiencing a significant liquidity shortfall (see Appendix 5):

(i) a *capital surcharge* would need to offset expected losses from liquidity risk at any given point in time during the envisaged risk horizon and be based on an institution's own liquidity risk (reflected in the expected losses derived from the risk-based and market-implied NSFR) or on its marginal contribution to joint liquidity risk, whichever of the two is higher.

(ii) an *insurance premium* would reflect the cost of covering all expected losses from liquidity risk subject to the chance that an institution, in concert with other banks, fails to meet required funding needs and falls below the minimum required market-implied NSFR of one.

The capital charge would be the present value of the sum of money (in billions and as a percent of total capital), which would be needed by each firm to offset expected losses from liquidity shortfalls when the marked-implied NSFR of one is breached either individually or jointly with a predetermined probability consistent with prudential standards for market risk. Basing the capital surcharge on the higher of two indicators—the maximum capital that offsets the amount of individual expected losses or the contribution of an institution to overall expected losses) under both normal and stressed conditions—is motivated by the fact that sometimes the individual component is higher and sometimes the contribution to the systemic risk is higher depending on the state of distress of the system, which varies in its impact on firms, which, in turn, also changes their susceptibility to funding shocks. Such a macroprudential measure, if adopted, could encourage de-leveraging assuming all else equal (e.g., the potential of the offsetting effect of higher returns from holding less liquid assets is ignored).

By contrast, the fair value insurance premium would compensate for the liquidity support that would be needed to maintain a market-implied NSFR above one even during stressful times (occurring with a likelihood of five percent, for instance). The fair value insurance premium is derived as the actuarial value associated with the ASF exceeding the present value of the RSF over a risk horizon of one year, which is modeled based on the hazard rate of expected losses from system-wide liquidity shortfall given system-wide funding needs. This premium is multiplied by all short-term uninsured liabilities, i.e., the portion of deposits that is not covered by the insurance scheme. This reflects the cost of insuring against the downside risk that no cash inflows are available to cover debt service obligations in times of stress. As with any insurance premium, the primary objective would not be the collection of fees for the *ex post* funding of such a guarantee but the change in behavior it would hope to elicit to internalize an institution's own liquidity risks and/or the impact of its own funding choices on system-wide liquidity risk. Given that the pricing would be based on the aggregate incidence of liquidity shortfall, the risk is "pooled" with others in the sample that made up the joint probability—making it less expensive.

D. Stress testing

The SRL approach can also be used within a stress testing framework to examine the vulnerabilities of individual institutions—and the system as a whole—to shocks affecting the valuation of assets and liabilities that underpin the NSFR. Volatility shocks to both asset returns and funding costs as well as the joint dynamics (i.e., dependence structure) between them can significantly alter the net exposure to liquidity shortfalls reflected in a market-based measure of liquidity risk. Thus, stress scenarios can be determined based on the historical calibration of market factors affecting the valuation of both the ASF and RSF, the constituent components of NSFR—as an approximation of net cash flows in stress situations. The stresses can be based on firm-specific shocks, common shocks or both:

- *Firm-specific shocks* are modeled by modifying the jump diffusion process (with the frequency, average size, and volatility of jumps calibrated to past stress scenarios) of assets underpinning the RSF, which could result in unbounded and random liabilities (Eberlein and Madan, 2010).

- *Common shocks* are modelled by means of stressing the historical variance-covariance matrix of latent factors of several market interest rates that impact the valuation of both the ASF and RSF (see Box 2). In adverse conditions, we observe higher volatilities of all market interest rates but lower correlations between those rates that are more indicative of changes in short-term asset returns and those rates that better explain short-term funding costs.

Box 2. Stress Testing Within the SRL Model Framework.

In adverse conditions, shocks to the volatilities of market funding rates and the correlation between asset returns and funding rates can be mechanically imposed in the model to better examine short-term funding vulnerabilities. The following five-step process offers a useful way to apply market-informed common shocks to the liquidity position of sample firms:[43]

1. Estimation of latent variables from market rates affecting the ASF and RSF. We derive latent variables of multiple market rates by extracting the principal component of each of the following six general categories: (i) short-term government bond yields (at maturities of 3, 6 and 12 months), (ii) long-term government bond yields (at maturities of 3, 5, 7 and 10 years), (iii) total equity market returns (total return of domestic stock market, MSCI total return index), (iv) home currency LIBOR rates (at maturities of 3, 6 and 12 months), (v) home currency OIS rates (at maturities of 3, 6 and 12 months), and (vi) financial bond yields (1-5 years, 15+ years, and rated "AAA").

2. Definition of baseline ASF volatility as a composite measure. Under baseline conditions, the volatility of the ASF is defined as a weighted average volatility of these latent variables whose explanatory power on the ASF—on levels—defines the relative size of these weights. The weights are derived as dynamic factor model (DFM) regression coefficients of the ASF on these latent variables, and—once re-scaled to unity—define the individual contribution of each latent variable volatility (measured as the standard deviation over a rolling window of τ observations with periodic updating (e.g., a daily sliding window of 120 days)) to the composite ASF volatility. Note that the volatility of the RSF, in contrast, is endogenous to the model specification, while the covariance between the RSF and the ASF is based on the correlation of their levels over the same rolling window time period.

3. Definition of shocks to volatility—stressed volatility and covariance of the ASF and the RSF. The historical return series of each latent variable are multiplied by the product of the empirical variance-covariance matrix and a "shocked" variance-covariance matrix[44] (defined by an ad hoc change to the

[43] Also various liquidity stress testing methodologies could be incorporated in this approach, such as van den End (2008), Wong and Hui (2009), and Aikman and others (2009).

[44] The shock to the variance-covariance matrix does not need to be uniform, such as a 20 percentage point shock to volatility and/or correlation, but can also accommodate asymmetric changes based on each particular market rate, with some experiencing greater shocks than others.

pairwise correlation and volatility of all elements according to a historical or synthetic scenario) in order to derive stressed series of all latent variables and their corresponding volatilities.[45]

4. Estimation of stressed ASF and RSF volatility. Like in the baseline case, the stressed ASF volatility is a composite measure derived as a weighted average of the stressed individual volatility series of each latent variable. But this time, the stressed volatility series are multiplied by corresponding coefficient values estimated in a dynamic factor regression of the baseline composite ASF volatility (see above) defined by the baseline volatilities of these latent variables. The same process is repeated for the stressed RSF volatility based on the endogenously generated RSF volatility and the "shocked" variance-covariance matrix of the same latent variables. The stressed covariance between the ASF and RSF is an explicit result of the "shocked" variance-covariance matrix.

5. Update the individual and joint measure of liquidity risk using the stressed input values. The joint probability distribution of liquidity shortfalls is re-estimated after re-calculating individual expected losses from the liquidity shortfall using the stressed ASF and RSF volatility measure and aggregating these results in the same way as in the baseline case.

IV. EMPIRICAL APPLICATION – U.S. BANKING SECTOR

The SRL model was applied to 14 largest commercial and investment banks in the United States based on firm-level data obtained from financial statements and markets covering a five-year time period from January 1, 2005 to December 14, 2010 (1,442 observations).[46,47] The variations in the components of the NSFR—that is, in the ASF and RSF as weighted sums of their constituent liabilities and asset positions (see Table 5)—were used to compute the expected losses due to liquidity shortfalls under extreme conditions.[48,49]

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[45] This is done technically by multiplying the inverted, lower-triangular variance-covariance matrix of historical market returns with an inverted variance-covariance matrix defined by "shocked" assumptions on correlation and individual volatility of latent variables.

[46] The sample covers Bank of America, J.P. Morgan, Citigroup, Morgan Stanley, Goldman Sachs, Merrill Lynch, Wells Fargo, US Bancorp, PNC Financial Services, Bank of New York Mellon, Sun Trust, BB&T, State Street, and Regions Bank.

[47] For an examination of the NFSR in the context of the European banking sector, see Oliver Wyman (2012).

[48] Extreme conditions were defined to be those that occur with a probability of five percent or less.

[49] Further specifications of the model include: three-month equity call prices for sample banks, put option time horizon of t=1, risk-free rate=0.03, and an estimation window for jump parameters over 120 days. Price changes would qualify as jumps if equity daily returns were outside an acceptable range of ±10 percent and represented a deviation of more than 50 percent on a normalized scale over a 120-day rolling window (i.e., observations were in the top decile over a half-year observation period).

The results suggest that the current liquidity standards in Basel III (whether as an accounting measure or a risk-adjusted measure) are not able to capture the potential liquidity shortfall under stressed conditions. The median of the marked-based NSFR for the 14 banks stays above one and has continued to improve since the credit crisis (see Figure 5). In contrast, the median expected losses generated by the SRL model would suggest that banks have become more vulnerable to extreme liquidity shocks and that their expected losses arising from potential liquidity shortfalls were higher during some time frames, namely in the run-up to the March 14, 2008, Bear Stearns rescue and around year-end 2008. Those results apply especially to firms dependent on funding sources that are more susceptible to short-term (and more volatile) market interest rates; that dependency, in combination with their relatively higher exposure to maturity mismatches, accentuates their vulnerability to liquidity risk.

Table 5. Weighting Factors for the Calculation of Available and Required Stable Funding.

Available Stable Funding		Required Stable Funding	
Equity	1.00	Cash	0.00
Tier 2	1.00	*Advances to banks*	*0.00*
Subordinated debt maturing after one year	1.00	*Customer loans*	*0.65*
Demand deposits	*0.85*	*Commercial loans*	*0.75*
Long-term deposits	1.00	*Other commercial and retail loans*	*0.90*
Bank deposits	0.00	*Other loans*	*1.00*
Other deposits and short-term borrowing	*0.00*	*Derivative assets*	*0.25*
Derivatives liabilities	*0.00*	Trading securities	0.20
Trading liabilities	*0.00*	*Available for sale securities*	*0.00*
Senior debt maturing after one year	1.00	Held-to-maturity securities	1.00
Other long-term funding	1.00	Investment in associates	1.00
Other interest-bearing liabilities	0.00	Other earning assets	1.00
Other reserves	0.00	Insurance assets	1.00
		Residual assets	1.00
		Reserves for non-performing assets	1.00
		Contingent funding	0.05

Source: Bankscope. *Note:* The figures in italics are based on assumptions published in IMF (2011b), which reflect the availability of relevant prudential data available in Bankscope.

Using the results for individual banks, we estimate system-wide liquidity risk under extreme stress. We derive the joint expected shortfall (ES) at the 95th percentile for all sample banks and find that interlinkages in banks' funding positions and their common exposure to the risk of funding shocks raise joint liquidity risk beyond the simple sum of individual net exposures if liquidity shortfalls happen simultaneously. The contagion risk from this interdependence gets accentuated during times of extreme stress if estimates are derived at a high level of statistical confidence (at percentile levels far removed from average outcomes). Failing to take into account this systemic component by only adding up expected losses associated with individual banks' risk-adjusted NSFR would have resulted in an *underestimation of system-wide liquidity* shortfall for the period between mid-2009 and mid-2010, where the green line exceeds the red line in Figure 6.[50,51] Conversely, the sum of individual expected losses would have overestimated system-wide liquidity shortfall under extreme stress conditions during the first half of 2008 when the probability of several banks experiencing a liquidity shortfall was low; however, this changes in the run-up to the collapse of Lehman Brothers in September 2008 when a general increase of expected losses and their sensitivity to extreme shocks generated higher systemic liquidity risk.

The results suggest that, accounting for their interdependence is imperative for a more accurate representation of systemic liquidity risk. The joint expected losses may be easier to discern by looking at averages over specified periods (see Table 6). During the crisis period from late 2008 to 2009, the joint expected losses were largest, as one would surmise. Thus, the joint tail risk of the expected shortfalls takes into account nonlinear dependence and the probability of extreme changes in funding costs affecting the valuation of firms.

The results imply that some banks contribute to systemic liquidity risk beyond their individual exposure to liquidity shortfalls in times of distress, which underscores the usefulness of a system-wide assessment of liquidity risk. During the height of the credit crisis, the average contribution of the largest U.S. banks to the tail risk from simultaneous liquidity shortfalls was higher than their individual liquidity risk (with the later accounting only for the average net exposure to liquidity risk) (see Table 3, first half). During 2010, expected losses from the likelihood of rising systemic liquidity risk as measured by the 95 percent ES (based on the multivariate distribution of expected losses) exceeded $31 billion on average (see Table 6), which is considerable, but far lower than during the peak of the financial crisis (at $150 billion). Appreciable differences among banks indicate varying sensitivity to common changes in funding conditions and the extent to which these translate into externalities affecting general liquidity risk in the overall sample.

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[50] The increasing relevance of accounting for the interdependence of individual liquidity risk profiles during the height of the credit crisis was also attributable to a disproportionate contribution of a few banks, which resulted in median value of 8.3 percent, just above the average (or expected) value of 7.6 percent for a sample of 13 institutions (see Table 6).

[51] In Figure 6, the red line represents the daily sum of individual, market-implied expected losses and the green line indicates the joint tail risk of these individual expected losses. Both tail risks are measured so that the chances of such events are five percent or less.

We also calculate both a capital surcharge and an insurance premium, which take into account the support that U.S. banks would likely to receive in times of systemic liquidity stress, and, thus, represent the potential public cost of marginal expected losses when two or more institutions face significant liquidity shortfalls. Table 7 presents the distribution of the capital charges over selected U.S. banks, and Table 8 does so for the value of the fair value insurance premium that would compensate for the contribution to joint expected losses caused by each bank. Both capital surcharges and the cost of insurance have been calculated based on the methodology presented in Appendix 5. After application of the SRL model we find that:

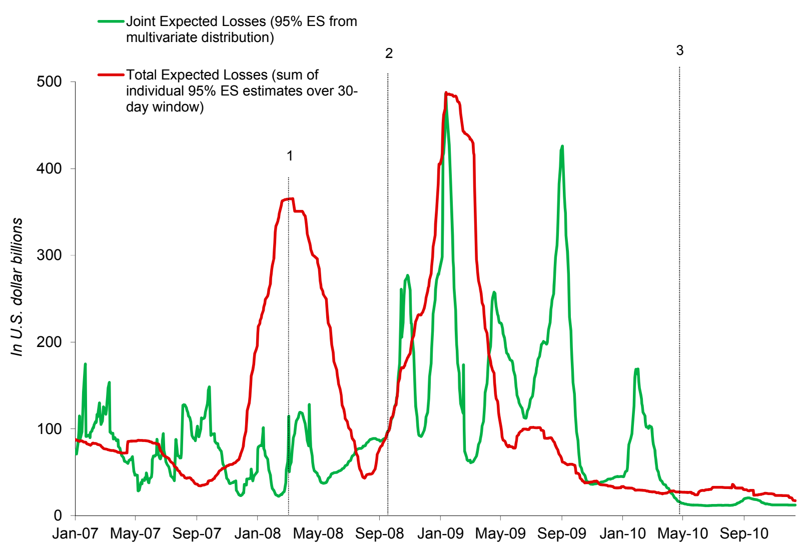
- *most banks would need to increase total capital by at least 4.82 percent* to offset expected losses of $2.05 billion *(median)* arising from their individual contributions to systemic liquidity risk (as of end-Q4 2010). This number, however, would reach $9.86 billion for the worst bank in the sample (if only individual liquidity risk were considered given the bank's lower contribution to systemic liquidity risk of a maximum of $5.96 billion). After considering stress periods, like the recent credit crisis, in the assessment methodology, the additional capital need to mitigate systemic liquidity risk could more than triple on average to 15.09 percent of total capital (see Table 7).

- *alternatively, the commensurate insurance premium would amount to $0.8 billion*, or less than half the additional capital needed to offset expected losses from liquidity risk and compensates for the liquidity support that would be needed to maintain the NSFR above one over a one-year risk horizon (see Table 8).

Figure 5. Individual Market-based NSFR and Associated Expected Losses Using Option Pricing *(sample median)*.

Sources: Bloomberg and Bankscope. *Note:* Dates of vertical lines are as follows: (1) March 1, 2008—Bear Stearns Rescue; (2) September 14, 2008—Lehman Brothers failure; and (3) April 27, 2010—Greek debt crisis. Expected losses are at the 95percent confidence level.

Figure 6. Joint Expected Losses from Systemic Liquidity Risk Using Option Pricing *(expected shortfall at the 95th percentile, In billions of U.S. dollars)*.



Sources: Bloomberg and Bankscope. *Note:* Dates of vertical lines are as follows: (1) March 1, 2008—Bear Stearns Rescue; (2) September 14, 2008—Lehman Brothers failure; and (3) April 27, 2010—Greek debt crisis. Expected losses are at the 95 percent confidence level. 1/ expected shortfall at the 95th percentile level based on the multivariate distribution of individual expected losses. 2/ sum of individual estimates of expected shortfall at the 95th percentile level over 30-day rolling window.

Table 6. Joint Expected Losses from Systemic Liquidity Risk *(expected shortfall at the 95th percentile, In billions of U.S. dollars).*

	Pre-Crisis: end-June, 2006—end-June, 2007	**Subprime Crisis**: July 1, 2007—Sept. 14, 2008	**Credit Crisis**: Sept. 14, 2008—Dec. 31, 2009	**Sovereign Crisis**: Jan. 1—Dec. 31, 2010
Minimum	14.8	22.4	36.1	17.4
Median	**65.4**	**68.9**	**150.3**	**31.4**
Maximum	191.8	148.5	486.2	60.2
Standard error	*18.9*	*26.6*	*56.9*	*8.9*

Sources: Bloomberg and Bankscope. *Note:* This exercise was run on a set of 14 large U.S. commercial and investment banks. The expected shortfall measure of systemic liquidity risk was estimated at the 95th percentile level.

Table 7. Capital Charge for Individual Liquidity Risk and Contributions to Systemic Liquidity Risk *(In billions of U.S. dollars).*

	Individual Liquidity Risk			**Contribution to Systemic Liquidity Risk**				**Economic Significance**	
	Stress Period: Sept. 14, 2008—Dec. 31, 2009	Last quarter (2010 Q4) (1)	Average of 2010 Q1-Q4 (2)	*Stress Period:* Sept. 14, 2008—Dec. 31, 2009	Last quarter (2010 Q4) (3)	Average of 2010 Q1-Q4 (4)	Capital charge *(maximum of (1)-(4))*	Share of total capital (In percent)	Share of total assets (In percent)
Minimum	0.00	0.00	0.03	1.55	0.08	0.27	0.27	1.57	0.20
Median	**1.46**	**0.74**	**1.18**	**6.42**	**0.66**	**2.05**	**2.05**	**4.82**	**0.73**
Maximum	33.32	8.53	9.86	13.51	3.09	5.96	9.86	3.25	0.44

Sources: Bloomberg and Bankscope. *Note:* This exercise was run on a selected set of 14 large U.S. commercial and investment banks. The last column matches the distributions of the individual capital charges and reported total capital of all sample institutions. In this case, the maximum capital charge for the worst bank in 2010 coincides with a disproportionately higher total capital amount, which reduces the percentage share of the capital add-on for systemic liquidity from 4.82 percent (median) to 3.25 percent (maximum). The expected losses during the stress period (covering the height of the recent credit crisis) are listed here for illustrative purposes (indicating a potential extension of the presented approach consistent with the revised market risk estimation under the new regulatory framework for banks (BCBS, 2010b). For details about the calculation of the capital charge, see Appendix 5.

Table 8. Summary Statistics of Banks' Contributions to Systemic Liquidity Risk and Associated Fair Value Insurance Premium.

	Pre-Crisis: end-June, 2006—end-June, 2007	**Subprime Crisis:** July 1, 2007—Sept. 14, 2008	**Credit Crisis:** Sept. 14, 2008—Dec. 31, 2009	**Sovereign Crisis:** Jan. 1—Dec. 31, 2010
	Individual contribution to systemic liquidity risk[1] *(at 95th percentile; in percent)*			
Minimum	1.2	0.6	1.0	1.7
Median	**6.8**	**4.5**	**8.3**	**7.6**
Maximum	13.4	35.1	16.7	14.5
Total	100.0	100.0	100.0	100.0
	Insurance cost based on reported exposure *(fair value insurance premium * uninsured short-term liabilities (In billions of U.S. dollars))*			
Minimum	0.7	0.1	0.7	0.1
Median	**1.9**	**1.4**	**3.9**	**0.8**
Maximum	7.8	17.2	11.3	1.9

Sources: Bloomberg and Bankscope. *Note:* This exercise was run on a set of 14 large U.S. commercial and investment banks. Insured deposits here are defined as 10 percent of demand deposits reported by sample banks. Note that the share of deposits covered by guarantees varies by country and could include time and savings deposits. Robustness checks reveal that reducing the amount of uninsured short-term liabilities does not materially affect the median and maximum. For details of the calculation see Appendix 5. 1/ Each bank's percentage share reflects its contribution to total expected losses from systemic liquidity risk.

V. Conclusion

In the wake of the global financial crisis, there has been a concerted effort to establish a regulatory framework to address systemic risk, which has resulted in a multi-faceted approach comprising complementary measures in areas of regulatory policies, supervisory scope, and resolution arrangements as part of a sustainable solution towards a more resilient financial sector while avoiding impairment to efficient activities that do not cause and/or amplify stress in any meaningful manner. However, systemic liquidity risk from a macroprudential perspective remains largely unaddressed.

The SRL model seems better suited than existing prudential approaches to identify, quantify and mitigate systemic liquidity risk since it (i) measures the marginal contribution of each institution to total systemic liquidity risk, and (ii) and can be used to construct a supervisory charge for the institution's contribution to systemic liquidity risk that provides incentives for the internalization of the cost of contingent liquidity support in times of stress. In particular, it offers several potential benefits by generating an objective and meaningful measure of systemic risk in areas that are absent in the current Basel III liquidity framework:

- *Liquidity risk is treated as a dynamic exposure via a risk-adjusted value of the net stable funding ratio* (rather than a combination of discrete accounting identities as in the current Basel III framework) by drawing on a market-based evaluation of the riskiness of a firm. This approach generates a forward-looking measure of liquidity risk (subject to different degrees of leverage and maturity mismatches defining the risk profile of institutions), which helps determine the probability of an individual institution experiencing a liquidity shortfall and incurring an associated expected loss (as an approximation of the economic cost of being unable to service on-going debt payments, resulting in net cash outflows); and

- *The impact of a particular funding configuration is not assessed individually but in concert with all banks to generate estimates of systemic liquidity risk.* As such, it takes the systemic components of liquidity risk into account by estimating the joint sensitivity of assets and liabilities (and their volatilities) of multiple entities to common changes in market prices. In this way, the SRL model helps assess (and quantify consistently) how the size and interconnectedness of individual institutions can create short-term, system-wide vulnerabilities to cases of considerable liquidity risk shared by several entities.

However, there are also distinct drawbacks to this approach, which are worthwhile bearing in mind. These constitute key limitations that need to be acknowledged and reflected appropriately as caveats in the discussion of findings. The presented findings are derived from valuation models that are subject to varying degrees of estimation uncertainty and assumptions, which need to be taken into account when drawing policy conclusions. For instance, the chosen option pricing model might fail to capture some relevant economics, especially during times when the validity of valuation models driving market prices might be undermined by rare and non-recurring events, and not by repeated realizations of predictable outcomes generated by a process of random events that exhibits stochastic stability (in violation of its steady state assumptions). Since the statistical apparatus underlying conventional asset pricing theory fails to capture sudden and unexpected realizations beyond historical precedent, exploring different alternative approaches, such as cash flow models, can help assess any pricing distortions aberrant price dynamics might impose on the option pricing method underpinning the SLR model.[52]

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[52] Also note that the sources of stable funding are modelled as being sensitive to common funding conditions (and its impact on the perceived risk profile of each firm), which establish market-induced linkages among institutions. These linkages might experience erratic changes in times of considerable market uncertainty that are removed from the structural definition of the model.

Going forward, several technical improvements to the SLR model could be considered. For instance, it could be usefully extended to include a dynamic configuration of weighting factors of both the ASF and RSF (possibly informed by empirical calibration), which would provide insights about the sensitivity of prudential measures of liquidity risk to exogenous assumptions.

Overall, the SLR model provides a tractable framework for the assessment of system-wide valuation effects arising from joint liquidity risk. However, much like any other efforts aimed at the design of macroprudential tools for managing systemic liquidity risk, its effectiveness and expedient use are contingent on data availability given the increasing sophistication of financial products and transactions, the diversity of financial institutions, as well as the growing interdependence of capital markets and their impact on liquidity risk management of financial institutions. New sources of systemic liquidity risk are likely to be found in areas where financial activities are loosely organized and governed by incentive structures that encourage greater-risk taking in a benign economic environment but entail more adverse consequences when stress occurs.

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Appendix 1—Standard Definition of Contingent Claims Analysis (CCA)

CCA is used to construct risk-adjusted balance sheets, based on three principles: (i) the values of liabilities (equity and debt) are derived from assets; (ii) liabilities have different priority (i.e., senior and junior claims); and (iii) assets follow a stochastic process. Assets (present value of income flows, proceeds from assets sales, etc.) are stochastic and over a horizon period may be above or below promised payments on debt which constitute a default barrier. Uncertain changes in future asset value, relative to the default barrier, are the driver of default risk which occurs when assets decline below the barrier. When there is a chance of default, the repayment of debt is considered "risky," to the extent that it is not guaranteed in the event of default (risky debt = risk-free debt minus guarantee against default). The guarantee can be held by the debt holder, in which case it can be thought of as the expected loss from possible default or by a third party guarantor, such as the government.

In the first structural specification, commonly referred to as the Black-Scholes-Merton (BSM) framework (or short "Merton model") of capital structure-based *option pricing theory* (OPT), the total value of firm's assets follows a stochastic process and may fall below the value of outstanding liabilities.[53] Thus, the asset value *A(t)* at time *t* describes a continuous asset process so that the physical probability distribution of the end-of-period value is:

$$A(T-t) \sim A(t)\exp\left\{\left(r_A+\sigma_A^2/2\right)(T-t)+\sigma_A\sqrt{T-t}z\right\}, \tag{A1.1}$$

for time to maturity *T-t*. More specifically, *A(t)* is equal to the sum of its equity market value, *E(t)*, and its risky debt, *D(t)*, so that $A(t)=E(t)+D(t)$. Default occurs if *A(t)* is insufficient to meet the amount of debt owed to creditors at maturity, which constitute the bankruptcy level ("default threshold" or "distress barrier"). The equity value *E(t)* is the value of an implicit call option on the assets, with an exercise price equal to default barrier. It can be computed as the value of a call option

$$E(t)=A(t)\Phi(d)-Be^{-r(T-t)}\Phi\left(d-\sigma_A\sqrt{T-t}\right), \tag{A1.2}$$

with

[53] See Black and Scholes (1973) and Merton (1973 and 1974).

$$d=\frac{\ln\left(\frac{A(t)}{B}\right)+\left(r+\frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A\sqrt{T-t}}, \tag{A1.3}$$

asset return volatility σ_A, and the cumulative probability $\Phi(\cdot)$ of the standard normal density function. Both the asset, *A(t)*, and asset volatility, σ_A, are valued *after* the dividend payouts. The value of risky debt is equal to default-free debt minus the present value of expected loss due to default,

$$D(t)=Be^{-r(T-t)}-P_E(t). \tag{A1.4}$$

Thus, the present value of market-implied expected losses associated with outstanding liabilities can be valued as an implicit put option, which is calculated with the default threshold *B* as strike price on the asset value *A(t)* of each institution. Thus, the present value of market-implied expected loss can be computed as:

$$P_E(t)=Be^{-r(T-t)}\Phi\left(-d-\sigma_A\sqrt{T-t}\right)-A(t)\Phi(-d), \tag{A1.5}$$

over time horizon *T–t* at risk-free discount rate *r*, subject to the duration of debt claims, the leverage of the firm, and asset volatility. Note that the above option pricing method for $P_E(t)$ does not incorporate skewness, kurtosis, and stochastic volatility, which can account for implied volatility smiles of equity prices.

Since the implicit put option $P_E(t)$ can be decomposed,

$$P_E=\Phi\left(-d-\sigma_A\sqrt{T-t}\right)\left(1-\frac{\Phi(-d)}{\Phi\left(-d-\sigma_A\sqrt{T-t}\right)}\frac{A(t)}{Be^{-rT}}\right)Be^{-r(T-t)}=PD\times LGD, \tag{A1.6}$$

there is no need to introduce a potential inaccuracy of assuming a certain LGD. As a consequence of the assumptions on the underlying asset price process, the risk-neutral probability distribution (or *state price density*, SPD) of $A(t)$ is a log-normal density

$$f_t^*\left(A(T-t)\right)=e^{-r_{t,T-t}T-t}\frac{\partial^2 E(t)}{\partial B^2}\bigg|_{B=A(T-t)}$$
$$=\frac{1}{A(T-t)\sqrt{2\pi\sigma_A^2(T-t)}}\exp\left[-\frac{\left[\ln\left(A(T-t)/A(t)\right)-\left(r_{t,T-t}-\sigma_A^2/2\right)(T-t)\right]^2}{2\sigma_A^2(T-t)}\right] \tag{A1.7}$$

with mean $\left(r-\sigma_A^2/2\right)(T-t)$ and variance $\sigma_A^2(T-t)$ for $\ln\left(A(T-t)/A(t)\right)$, where $r_{t,T-t}$ and $f^*(\cdot)$ denote the risk-free interest rate and the risk-neutral probability density function (or SPD) at time *t*, with risk measures:

$$\Delta\overset{def}{=}\frac{\partial E(t)}{\partial A(t)}=\Phi(d) \text{ and } \Gamma\overset{def}{=}\frac{\partial^2 E(t)}{\partial A(t)^2}=\frac{\Phi(d)}{A(t)\sigma_A\sqrt{T-t}}. \tag{A1.8}$$

Appendix 2—Estimation of the Empirical State Price Density (SPD)

Breeden and Litzenberger (1978) show Arrow-Debreu prices can be replicated via the concept of the *butterfly spread* on European call options as basis for extracting the *state price density* (SPD) from observable derivatives prices. This spread entails selling two call options at strike price *K* and buying two call options with adjacent strike prices $K^{-} = K - \Delta K$ and $K^{+} = K + \Delta K$ respectively, with the stepsize ΔK between the two call strikes. If the terminal underlying asset value $A(T) = K$ then the payoff $Z(\cdot)$ of $1/\Delta K$ of such butterfly spreads at time $T-\tau$ (with time to maturity τ and maturity term *T*) is defined as

$$Z\left(A(T),K;\Delta K\right) = Price\left(A(T-\tau),\tau,K;\Delta K\right)\Big|_{\tau=0} = \frac{u_1 - u_2}{\Delta K}\Big|_{A(T)=K,\tau=0} = 1, \tag{A2.1}$$

with

$$u_1 = C\left(A(T-\tau),\tau,K+\Delta K\right) - C\left(A(T-\tau),\tau,K\right), \tag{A2.2}$$

and

$$u_2 = C\left(A(T-\tau),\tau,K\right) - C\left(A(T-\tau),\tau,K-\Delta K\right). \tag{A2.3}$$

$C(A,\tau,K)$ denotes the price of a European call option with an underlying asset price *A*, time to maturity τ and strike price *K*. As $\Delta K \to 0$, $Price\left(A(T-\tau),\tau,K;\Delta K\right)$ of the position value of the butterfly spread becomes an Arrow-Debreu security paying one if $A(T) = K$ and zero in other states. If $A(T) \in \mathbb{R}^{+}$ is continuous, we can obtain a security price

$$\lim_{\Delta K \to 0}\left(\frac{Price\left(A(t),\tau,K;\Delta K\right)}{\Delta K}\right)\Bigg|_{K=A(T)} = f^{*}\left(A(T)\right)e^{-r_{t,\tau}}, \tag{A2.4}$$

where $r_{t,\tau}$ and $f^{*}(\cdot)$ denote the risk-free rate and the risk-neutral probability density function (or SPD) at time *t*. On a continuum of states *K* at infinitely small ΔK a complete state pricing function can be defined. Moreover, as $\Delta K \to 0$, this price

$$\lim_{\Delta K \to 0}\left(\frac{Price\left(A(t),\tau,K;\Delta K\right)}{\Delta K}\right) = \lim_{\Delta K \to 0}\frac{u_1 - u_2}{(\Delta K)^2} = \frac{\partial^2 C_t(\cdot)}{\partial K^2} \tag{A2.5}$$

will tend to the second derivative of the call pricing function with respect to the strike price evaluated at *K*, provided that $C(\cdot)$ is twice differentiable. Thus, we can write

$$\frac{\partial^2 C_t(\cdot)}{\partial K^2}\Big|_{K=A(T)} = f_t^*\left(A(T)\right)e^{-r_{t,\tau}\tau}, \tag{A2.6}$$

across all states, which yields the SPD

$$f_t^*\left(A(T)\right) = e^{r_{t,\tau}\tau}\frac{\partial^2 C_t(\cdot)}{\partial K^2}\Big|_{K=A(T)}, \tag{A2.7}$$

under no-arbitrage conditions and without assumptions on the underlying asset dynamics. Preferences are not restricted since no-arbitrage conditions only assume risk-neutrality with respect to the underlying asset. The only requirements for this method are that markets are perfect, i.e., there are no transactions costs or restrictions on sales, and agents are able to borrow and lend at the risk-free interest rate.

Appendix 3—Moments of the GEV Distribution and Estimation of the Shape Parameter Using the Linear Combination of Ratios of Spacings (LRS) Method

Since all raw moments of $G(\cdot)$ are defined contingent on the tail shape, the natural estimator of $\hat{\xi}_j$ is derived by means of the LRS method using the linear combination

$$\hat{\xi} = (n/4)^{-1} \sum_{i=1}^{(n/4)} \left(\log(\hat{v}_i) / -\log(c) \right) \tag{A3.1}$$

for n observations, where

$$\hat{v}_i = \left(x_{n(1-a):n} - x_{na^c:n} \right) / \left(x_{na^c:n} - x_{na:n} \right) \tag{A3.2}$$

and

$$c = \sqrt{\log(1-a)/\log(a)} \tag{A3.3}$$

for quantile $a = i/n$. Since $x_{na:n} = G_{\hat{\xi},\hat{\mu},\hat{\sigma}}^{-1}(a)$ at statistical significance a, the approximation

$$\hat{v}_i \approx \frac{G_{\xi}^{-1}(1-a) - G_{\xi}^{-1}(a^c)}{G_{\xi}^{-1}(a^c) - G_{\xi}^{-1}(a)} = c^{-1+\xi} \tag{A3.4}$$

holds. The simple statistics are defined as

$$\textit{mean}: \ \mu - \frac{\sigma}{\xi} + \frac{\sigma}{\xi} g_1 \text{ given } \begin{cases} \mu + \sigma\left(\Gamma(1-\xi) - 1\right)/\xi & \textit{if } \xi \neq 0, \xi < 1 \\ \mu + \sigma\gamma & \textit{if } \xi = 0 \\ \infty & \textit{if } \xi \geq 1 \end{cases}$$

$$\textit{variance}: \ \frac{\sigma^2\left(g_2 - g_1^2\right)}{\xi^2} \text{ given } \begin{cases} \sigma^2\left(g_1 - g_1^2\right)/\xi^2 & \textit{if } \xi \neq 0, \xi < 1/2 \\ \sigma^2 \pi^2/6 & \textit{if } \xi = 0 \\ \infty & \textit{if } \xi \geq 1/2 \end{cases}$$

$$\textit{skewness}: \ \frac{g_3 - 3g_1g_2 + 2g_1^3}{\left(g_2 - g_1^2\right)^{3/2}} \text{ given } \begin{cases} \dfrac{g_3 - 3g_1g_2 + 2g_1^3}{\left(g_2 - g_1^2\right)^{3/2}} & \textit{if } \xi \neq 0 \\ \dfrac{12\sqrt{6}\zeta(3)}{\pi^3} & \textit{if } \xi = 0 \end{cases}$$

$$\textit{kurtosis}: \frac{g_4 - 4g_1g_3 + 6g_2g_1^2 - 3g_1^4}{\left(g_2 - g_1^2\right)^2} - 3 \text{ given } \begin{cases} \dfrac{g_4 - 4g_1g_3 + 6g_2g_1^2 - 2g_1^4}{\left(g_2 - g_1^2\right)^2} & \textit{if } \xi \neq 0 \\ \dfrac{12}{5} & \textit{if } \xi = 0, \end{cases}$$

where $g_p = \Gamma(1 - p\xi)$ for $p = 1, \ldots, 4$, Euler's constant γ (Sondow, 1998) and Riemann zeta function $\zeta(t)$ (Borwein and others, 2000) and gamma probability density function $\Gamma(t)$.

Appendix 4—Derivation of the Asset Volatility Under Risk-neutrality Using the Moody's KMV Model-based EDF Value

$$EDF_{A(t)_{MKMV}} = \Phi\left(d_{MKMV} - \sigma_{A_{MKMV}}\sqrt{T-t}\right) \tag{A4.1}$$

$$EDF_{A(t)_{MKMV}} = \Phi\left(\frac{\ln\left(\frac{A_{MKMV}(t)}{B_{KMV}}\right) + \left(r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^* \frac{\mu_M - r}{\sigma_M} - \frac{\left(\sigma_A^*\right)^2}{2}\right)(T-t)}{\sigma_A^*\sqrt{T-t}}\right) \tag{A4.2}$$

$$\Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right) = \frac{\ln\left(\frac{A_{MKMV}(t)}{B_{KMV}}\right) + \left(r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^* \frac{\mu_M - r}{\sigma_M} - \frac{\left(\sigma_A^*\right)^2}{2}\right)(T-t)}{\sigma_A^*\sqrt{T-t}} \tag{A4.3}$$

Using T-t=1, and rearranging gives

$$\ln\left(\frac{A_{MKMV}(t)}{B_{KMV}}\right) + r_{A_{MKMV}} + \rho_{A_{MKMV},M}\sigma_A^* \frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right)\sigma_A^* - \frac{\left(\sigma_A^*\right)^2}{2} = 0, \tag{A4.4}$$

so the quadratic formula can be used to solve for

$$\sigma_A^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\left(\rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right)\right) \pm \sqrt{\left(\rho_{A_{MKMV},M}\frac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right)\right)^2 - 4\left(\ln\left(\frac{A_{MKMV}(t)}{B_{KMV}}\right) + r_{A_{MKMV}}\right)\left(-\frac{1}{2}\right)}}{2\left(-\frac{1}{2}\right)} \tag{A4.5}$$

$$\sigma_A^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\begin{pmatrix} \rho_{A_{MKMV},M} \dfrac{\mu_M - r}{\sigma_M} \\ -\Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right) \end{pmatrix} \pm \sqrt{\begin{gathered}\left(\rho_{A_{MKMV},M} \dfrac{\mu_M - r}{\sigma_M} - \Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right)\right)^2 \\ -4\left(\ln\left(\dfrac{A_{MKMV}(t)}{B_{KMV}}\right) + r_{A_{MKMV}}\right)\left(-\dfrac{1}{2}\right)\end{gathered}}}{2\left(-\dfrac{1}{2}\right)} \tag{A4.6}$$

$$= \begin{pmatrix} \rho_{A_{MKMV},M} \dfrac{\mu_M - r}{\sigma_M} \\ +\Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right) \end{pmatrix} \pm \sqrt{\begin{gathered}\left(\rho_{A_{MKMV},M} \dfrac{\mu_M - r}{\sigma_M} + \Phi^{-1}\left(EDF_{A(t)_{MKMV}}\right)\right)^2 \\ +2\left(\ln\left(\dfrac{A_{MKMV}(t)}{B_{KMV}}\right) + r_{A_{MKMV}}\right)\end{gathered}}$$

Appendix 5—Technical Description of Price-based Macroprudential Measures for Systemic Liquidity using the Systemic Risk-Adjusted Liquidity (SRL) Model

The SRL model, if applied to a banking system, can be used to calibrate two price-based measures, a capital surcharge and an insurance premium, either of which could be used as a macroprudential tool to help mitigate systemic liquidity risk:

- a *capital surcharge* would need to offset expected losses from liquidity risk at any given point in time based on a firm's own liquidity risk (highest risk-based NSFR over some pre-specified period, such as one quarter) or its marginal contribution to joint liquidity risk, whichever is higher; and

- an *insurance* premium would be an actuarial fee imposed on firms, which would compensate them for expected losses from liquidity risk in a systemic event when they fall below the minimum required NSFR of one in concert with other institutions.

Implicitly, these two measures proxy for the amount of contingent support that banks would receive from a central bank in times of systemic liquidity stress. Numerical examples of these two approaches are in the main text of the chapter (see Tables 6 and 7), and their calculations are explained below. Unlike the capital surcharge, which is meant to absorb losses at any point in time, the insurance premium is measured over time (in this case, one year ahead) and, thus, spreads out the probability of the firm's experiencing a liquidity shortfall over a risk horizon.

For the capital surcharge, the method follows the current bank supervisory guidelines for market risk capital requirements (BCBS, 2009), in which the Value-at-Risk (VaR) is calculated each day and compared to three times the average quarterly VaRs over the last four quarters. The maximum of these two numbers becomes the required amount of regulatory capital for market risk. In a similar way, each firm would need to meet an additional capital requirement, c_{SLR} (in currency units), at time *t*, to offset the net exposure from either its individual liquidity risk or its contribution to systemic liquidity risk at a statistical confidence level of *a*=0.95. First, we choose the higher of (i) the previous quarter's expected shortfall at percentile *a*, $\overline{ES}_{t-1,\tau,j,a}$, averaged over daily observations associated with individual liquidity risk of firm *j*, and (ii) the average of this quarterly measure over the preceding four quarters, multiplied by an individual multiplication factor κ_j. This amount would be compared to greater of (i) the last available quarterly marginal contribution, $\overline{\Psi}_{t-1,\tau,j,a}$, multiplied by the total expected shortfall, $\overline{ES}_{t,\tau,a}$, over the same time period, and (ii) the average of this quarterly measure over the preceding four quarters, multiplied by a general multiplication factor κ. The higher of the two maximums would then be the surcharge. Therefore, based on an estimation window of τ days in each quarter t, the capital surcharge c_{SLR} would be

$$c_{SLR}=\max\left\{\begin{array}{l}\max\left\{\overline{ES}_{t-1,\tau,j,a};\kappa_j\times\frac{1}{4}\sum_{t=-4}^{0}\overline{ES}_{t,\tau,j,a}\right\};\\ \max\left\{\overline{\psi}_{t-1,\tau,j,a}\times\overline{ES}_{t-1,\tau,a};\kappa\times\frac{1}{4}\sum_{t=-4}^{0}\left(\overline{\psi}_{t,\tau,j,a}\times\overline{ES}_{t,\tau,a}\right)\right\}\end{array}\right\} \tag{A5.1}$$

The comparison of the two maximums is motivated by Figure 6, whereby a firm's individual liquidity risk is not commensurate to its contribution to systemic liquidity risk depending on existing interlinkages between firms and how they influence the probability of joint liquidity shortfalls. Note that the amount of capital to be withheld under this measure is exactly the present value of funds needed to offset the expected losses that would be incurred when the requirement of NSFR greater than one is violated either jointly or individually for a given level of statistical confidence.

An alternative method is to require individual firms to pay an insurance premium for liquidity support based on their likelihood of experiencing expected loss from liquidity risk that is shared by other firms as well. The conditional probability of expected losses from individual shortfall that raises system-wide liquidity risk, i.e., the marginal contribution of an institution to system-wide liquidity risk, can be used to calculate a fair value price for the necessary insurance coverage specific to each firm. To illustrate this, we calculate the ratio of the potential systemically-based expected losses of each institution to the total amount of required stable funding—the probabilistic proportion of underfunding (relative to the existing funding level), akin to a probability of distress for a certain risk horizon. More specifically, we estimate the average marginal contribution of each firm to the expected shortfall (with statistical significance a) as a measure of systemic liquidity, averaged over the previous four quarters, which is then divided by the average of the discounted present value of the asset value underpinning RSF over the previous four quarters.

The fair value of a risk-based insurance premium can be obtained as the natural logarithm of 1 minus the ratio between the sum of the marginal contribution to the joint expected shortfall at percentile a (over the preceding four quarters) and the sum of the present values of the quarterly RSFs of all sample firms, multiplied by the negative inverse of the time period under consideration (see equation (A5.4) below). This assumes that the *conditional* probability of expected losses from liquidity shortfall under the risk-neutral measure is constant over time and can be expressed as an exponential function, given a survival probability

$$\exp\left(-\int_0^t h(u)\,du\right)=\exp(-ht) \tag{A5.2}$$

at time t with a constant hazard rate

$$h \approx \frac{\sum_{t=-4}^{0}\left(\overline{\psi}_{t,\tau,j,a} \times \overline{ES}_{t,\tau,a}\right)}{\sum_{j}^{m}\sum_{t=-4}^{0} RSF_{t,j} \times \exp\left(-r\left(T-t\right)\right)} \tag{A5.3}$$

over one period t=1. Thus, the cost f_{SLR} of insuring stable funding over the short-term against possible shortfalls can be calculated by multiplying the insurance premium (in basis points) by the value of average uncovered short-term liabilities $\overline{L}_{t,j}^{ST}$ (i.e., excluding secured deposits and investments) over the previous four quarters as a nominal base.

$$f_{SLR} = \underbrace{-\frac{1}{T}\ln\left(1-h\right)}_{insurance\ premium} \times 10,000 \times \frac{1}{4}\sum_{t=-4}^{0} \overline{L}_{t,j}^{ST}, \tag{A5.4}$$

where r is the risk-free rate and T-t is the time horizon (i.e., residual maturity). This amount would compensate for the individual firm's cost of future systemic liquidity support.

Because they take into account a single firm's contribution to systemic liquidity risk, either the capital surcharge or the insurance premium could be used as price-based macroprudential tool to instill incentives for more resilient and diversified funding structures. Based on estimates during times of stress, both measures could be refined to avoid procyclical tendencies. For instance, in the context of the capital surcharge, the multiplication factor κ_j could be calibrated on data obtained during times of stress and set such that minimum prudential levels of capital charges are maintained "through the cycle" with suitable latency of time-varying adjustment. Equally, the maximization of the capital surcharge (and the determination of the conditional probability of individual underfunding for the insurance premium calculation) could be directly conditioned on an estimate of the marginal contribution to systemic liquidity risk based on data obtained during times of stress.