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# Clearing, Counterparty Risk and Aggregate Risk

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# Clearing, counterparty risk and aggregate risk*

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**Abstract**

We study the optimal design of clearing systems, focusing on counterparty risk insurance and prevention. We study whether decentralized clearing improves on no clearing, and whether centralized clearing generates further improvement. We analyze how counterparty risk should be allocated, whether traders should be fully insured against that risk, and how moral-hazard affects the optimal allocation of risk. The main advantage of centralized clearing is the mutualization of counterparty risk. We show, however, that the improved risk-sharing brought about by centralized clearing can induce greater risk-taking, even in the first-best. Furthermore, while mutualization is useful to share idiosyncratic risk, it cannot provide insurance against aggregate risk. When the latter is significant, it is necessary that protection buyers exert effort to find robust counterparties. With moral hazard, this requires that they retain some exposure to counterparty risk.

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# 1 Introduction

Counterparty risk is the risk to each party of a contract that his or her counterparty will not live up to its contractual obligations. As vividly illustrated by the failure of Lehman Brothers and the near failures of AIG and Bear Stearns, counterparty risk is a real issue for investors. These episodes also suggest that some institutions should have paid more attention to the risk that counterparties such as AIG or Lehman Brothers could default. This, in turn, underscores that institutions should monitor the risk of their counterparties, and ensure that they contract with creditworthy counterparties.

Clearing entities, and in particular Centralized Clearing Platforms (hereafter CCPs) can offer insurance against counterparty risk. The clearing entity interposes between the two parties. If one of them were unable to meet its obligations to the other, the clearing entity would make the payment on behalf of the defaulting party. Would the development of CCPs improve the working of markets? In September 2009, the G20 leaders, followed by the Dodd-Frank Wall Street Reform & Consumer Protection Act, and then the European Commission, answered yes. They proposed that all standardized OTC derivatives contracts be centrally cleared.[1]

Was this the right move? More generally, how, and to what extent, can clearing improve the allocation of risk and how should it be designed? Should it be decentralized or centralized? Should it provide full insurance against counterparty default? Is it likely to decrease or increase risk-taking? Is clearing enough to cope with counterparty risk, or should it be complemented by other risk-mitigation tools? We take an optimal contracting approach to analyze these issues and offer policy implications. We focus on the market for Credit Derivative Swaps (hereafter CDS). Because of the multiplicity of counterparties in this OTC market, and the relatively long delay between the time at which contracts are struck and the time at which all payments are completed, counterparty risk is a key issue for CDS. While the large size of the CDS market warrants an investigation, many of the economic mechanisms we uncover are also at play in other markets.

We consider a simple model in which a population of risk-averse agents who hold risky assets (protection buyers) faces a population of risk-neutral limited-liability agents (protection sellers). For example, they can be financial institutions holding a portfolio of loans. For

[1] The G20 meeting in September 2009 chose December 2012 as deadline for this change. It is not clear this deadline will be met.

simplicity we assume that the asset held by each protection buyer can take only two values (high and low) with equal probability. The protection sellers can insure the protection buyers against the risk of a low value of the asset, but protection sellers themselves may default. This creates counterparty risk for protection buyers and reduces the extent to which they can hedge their own risk. At some cost, protection buyers can exert effort to search for good counterparties with low default risk. When deciding whether to do so, protection buyers trade off the benefits of better insurance (granted by good counterparties) and the cost of effort. If the protection buyers are sufficiently risk-averse, or the search cost is low enough, then it is optimal to exert effort to find a creditworthy counterparty.

Even when they exert effort, protection buyers remain exposed to some counterparty risk, as the default probability of the protection seller remains strictly positive. In this context, how can a clearing entity offer insurance against counterparty risk and improve welfare? To clarify the economic drivers underlying this issue, we distinguish three cases: The most favorable case arises when the risk exposures of the protection buyers are independent and their search effort is observable and contractible. The intermediate case is when the risk to which protection buyers are exposed has a systematic component. The most difficult case arises when there is both aggregate risk and moral hazard. The optimal design as well as the usefulness of clearing arrangements vary across these three cases.

First consider the case in which there is no moral hazard and no aggregate risk, as the values of the assets held by the protection sellers are i.i.d. In this case, compare bilateral clearing with centralized clearing:

- With bilateral clearing, there is a clearing agent interposing between one protection buyer and one protection seller. For a fee, the clearing agent insures (fully or partially) the protection buyer against the default of the protection seller. The clearing agent chooses a portfolio of liquid, low-return assets (cash) and illiquid, higher-return assets that can, however, not be used to pay insurance. To ensure that resources are available to pay the protection buyer, the clearing agent must keep some cash aside. This has an opportunity cost since the return on cash is lower than the return on the illiquid assets. Because of this cost, it is optimal to have only partial insurance against counterparty risk. Because there remains counterparty risk, it is optimal to exert effort to search for a creditworthy counterparty if the protection buyer is sufficiently risk-averse.

- With centralized clearing, the CCP interposes between all protection buyers and all protection sellers. Hence, the total insurance payment by the CCP is the sum of all the individual payments to protection buyers. Since the individual risks are independent, the law of large numbers applies and the sum of all payments is deterministic. Correspondingly, the fees levied by the CCP are exactly equal to the amount needed to insure all the protection buyers against counterparty risk. Hence, it is no longer necessary to set aside cash. Thus, the *first* benefit of mutualization in a CCP is that it avoids the opportunity cost associated with early liquidation. Now, since this cost is not incurred, full insurance against counterparty risk is optimal. This is the *second* benefit of mutualization. Also, with mutualization the protection buyers effectively insure one another. Hence, they are not affected by the default of the protection sellers. Consequently there is no point in searching for good counterparties. Thus, the *third* benefit of mutualization is that it avoids the search cost. Note that an apparently paradoxical implication of this result is that, since protection buyers won't exert search effort, the aggregate default rate of counterparties will be greater with centralized clearing than with decentralized clearing. And yet, this is not a sign of market failure, but a feature of the first-best outcome.

Second, turn to the case in which there is aggregate risk - but still no moral hazard.[2] To model aggregate risk, we assume there are two equiprobable macro-states, referred to as good and bad. In the good state, the probability that each individual protection buyer's asset's value is high is greater than one half. In the bad state, it is lower than one half.[3] Conditional on the realization of the macro-state, the values of the protection buyers' assets are i.i.d. In this context, the aggregate value of the protection buyers' assets is larger in the good state than in the bad state. While mutualization among protection buyers continues to be useful with respect to the idiosyncratic risk component, it cannot provide any insurance against the macro-risk. In this context, the protection sellers become valuable again, even with centralized clearing, because the value of the assets they bring to the table is useful to insure the protection buyers against macro-risk. Correspondingly, the effort to search for good counterparties is also valuable. If the protection buyers are sufficiently risk-averse, the

[2] In the first case, we analyzed why centralized clearing dominated decentralized clearing. For brevity, in the second and third cases we consider only centralized clearing. This is without loss of generality. In our optimal contracting framework, the optimal centralized mechanism dominates decentralized clearing by construction.

[3] On average, ex–ante, the probability that the value of the asset is good is exactly one half. The model with aggregate risk nests the model without aggregate risk as a particular case.

optimal contract involves i) effort to locate good counterparties, and ii) full insurance of the protection buyers, thanks to the mutualization of their idiosyncratic risk and transfers from the protection sellers in the bad state.

Third, consider the most challenging case, in which there is both aggregate risk and moral hazard. In this case, the CCP cannot observe whether protection buyers exert search effort or not, or whether the protection seller with whom they contract is creditworthy or not. In this context, should the CCP continue to promise full insurance against counterparty risk as in the second case above? If it did, then the protection buyers would not have any incentives to incur the cost associated with the search for creditworthy counterparties. Consequently, the average amount of resources brought to the table by protection sellers would be relatively low. In the bad macro-state their default rate would be high and the CCP would have to pay a lot of insurance. This liability could exceed the resources of the CCP, and push it into bankruptcy. To avoid this, when there is moral hazard, the CCP should not offer full insurance against counterparty risk. The protection sellers should remain partially exposed to the risk of default of their counterparty. This risk exposure, while suboptimal in the first-best, is needed in the second-best, to maintain the incentives of the protection buyers to exert effort.

Thus, our analysis yields the following implications:

- Centralized clearing is superior to decentralized clearing, since it enables the mutualization of risk. Policy makers are therefore right to promote centralized clearing. But, at the same time, they should keep in mind the limitations of centralized clearing.

- While the mutualization delivered by centralized clearing reduces the exposure to *idiosyncratic* risk, it does not reduce the exposure to *aggregate* risk. Minimizing that exposure requires exerting effort to find creditworthy counterparties, robust to macro-shocks.

- Centralizing clearing can reduce both the social value and the private incentives to exert the search effort. Hence, while improving the *allocation of counterparty risk*, the centralization of clearing might increase the *aggregate counterparty default rate*.

- Under the plausible assumption that the effort to find creditworthy counterparties is unobservable, there is a moral hazard problem and the CCP must be designed to maintain the incentives of protection buyers. This precludes full insurance against counterparty default. And this incentive constraint is especially important when aggregate risk is significant. Thus, when *aggregate* risk is large, incentive compatibility requires that protection buyers retain some exposure to some of the *idiosyncratic* component of counterparty default risk.

Thus, our analysis contributes to the micro-prudential and the macro-prudential study of clearing mechanisms. Micro-prudential analyses focus on one financial institution, studying how to regulate that institution, e.g., to avoid excessive risk-taking. Macro-prudential analyses consider a *population* of financial institutions and focus on the *equilibrium interactions* between these institutions as well as on *aggregate outcomes* generated by these interactions. All of these features are present in our analysis. This is because, by construction, CCPs raise macro-prudential issues, since they clear the trades of a population of financial institutions. Furthermore, our analysis emphasizes the interaction between the design of CCPs and the presence of aggregate risk. It underscores that, when aggregate risk is significant, CCPs are useful but should not provide full insurance against counterparty risk, lest this would jeopardize the incentives of market participants to search for creditworthy counterparties.

The next section presents the institutional background. Section 3 reviews the literature. Section 4 presents the model. Section 5 analyzes the case with no aggregate risk and no moral hazard. Section 6 turns to the case with aggregate risk and no moral hazard. Section 7 examines the situation in which there is both aggregate risk and moral hazard. Section 8 concludes, summarizes our implications and sketches avenues for further research. Proofs not given in the text are in the appendix.

# 2 Institutional background

**Definition of clearing:** After a transaction is agreed upon, it needs to be implemented. This typically involves the following actions:

- Determining the positions of the different counterparties (how many securities or contracts have been bought and sold and by whom, how much money should they receive or pay). This is the narrow sense of the word "clearing."

- Transferring securities or assets (to custodians, which are financial warehouses) and settling payments. This activity is referred to as "settlement."

- Reporting to regulators, calling margin and deposits, netting.

- Handling counterparty failures.

Understood in a broad sense, clearing refers to this whole process. The market-wide system used for clearing operations is often referred to as the "market infrastructure."

**The basic mechanism of clearing and counterparty risk:** Clearing in spot markets differs somewhat from its counterpart in derivative markets. First consider the case in which $A$ and $B$ agree on a spot trade: $B$ buys an asset (stock, bond, commodity) from $A$, against the payment of price $P$. The clearing entity receives the asset from $A$ and transfers it to $B$ (or his custodian or storage facility). The clearing entity also receives the payment of $P$ dollars from $B$ and transfers it to the account of $A$.[4]

In derivative markets, things are a bit more complicated, because contracts are typically written over a longer maturity, and are often contingent on certain events. Consider for example the case of a CDS. $A$ sells protection to $B$ against the default of a given bond. Before the maturity of the contract, as long as the underlying bond does not default, $B$ must pay an insurance premium to $A$. Just like the payment of the price for the purchase of an asset, this payment can take place via the clearing agent. If, before the maturity of the contract, the underlying bond defaults, $A$ must pay the face value of the bond to $B$, while $B$ must transfer the bond to $A$. Thus, the clearing entity receives the bond from $B$ and transfers it to $A$, and it receives the cash payment of the face value from $A$ and transfers this to the account of $B$.

Clearing entities also typically provide insurance against the default of trading counterparties. For example, in the CDS trade described above, if the underlying bond defaults *and* $A$ is bankrupt, then the clearing entity can provide the insurance instead of $A$: In this case, it is the clearing entity that receives the bond and pays cash to $B$. Such insurance is more significant in derivative markets than in spot markets: other things equal, the risk of default of one of the counterparties is greater over the long maturity of derivative contracts than during the few days or hours it takes to clear and settle a spot trade. To meet the default costs, the clearing entity must have capital and reserves.

[4] In practice, this process might involve additional intermediaries, such as the brokers of $A$ and $B$. For simplicity, these are not discussed here.

**Bilateral versus centralized clearing:** The clearing process can be bilateral and operated in a decentralized manner. In this case the trade between $A$ and $B$ is cleared by a "clearing broker" or "prime broker." If on the same day there is a trade between two other institutions, $C$ & $D$, it can be cleared by a different broker. In contrast, with Central Counterparty Clearing (hereafter CCC) the clearing process for several trades (between $A$ & $B$ as well as between $C$ & $D$) is realized within a single entity, referred to as the Central Clearing Platform (hereafter CCP). In this centralized clearing system, the CCP takes on the counterparty risk of all the trades. This implies that the CCP can be exposed to a large amount of counterparty default risk. To cope with such risk, the CCP needs relatively large capital and reserves. Such reserves can be built up by levying a fee on the brokers using its services (possibly contingent on activity levels). The CCP can also issue equity capital subscribed by the brokers and financial institutions using its services. To the extent that the counterparty loss on a given trade is paid for by the capital and reserves of the CCP, provided by all the members of the CCP, centralizing clearing leads to the mutualization of counterparty default risk.

CCC has been the prevailing model for futures and stock exchanges. A polar case is the Deutsche Börse, where the trading platform and the clearing platform are vertically integrated. In contrast, decentralized clearing is most frequent when trades are conducted in OTC markets. Up to now, a large fraction of the Credit Derivative Swaps market has been OTC and cleared in a decentralized way. Note, however, that trading mechanisms and clearing mechanisms are distinct. Thus, it is possible to have OTC trading and CCC. In that case, the search for counterparties and the determination of the terms of trade is decentralized, while, after the deal would have been struck, the two parties clear the trade in a CCP.[5]

# 3 Literature

Our paper is related to the analyses of the costs and benefits of centralized and bilateral clearing by Acharya and Bisin (2010), Duffie and Zhu (2009), and Pirrong (2011).

[5]This can be the case, e.g., for swap deals struck on the OTC market, and then cleared through LCH.Clearnet or SwapClear. In that case, the original swap is transformed into two deals: between the swap buyer and the CCP, and between the CCP and the swap seller.

**Acharya and Bisin (2010):** Acharya and Bisin (2010) study the inefficiency arising when each protection seller can contract with several protection buyers in an OTC market. In such a market no protection buyer can control the trades of his counterparty seller with the other buyers. Yet, when the protection seller contracts with an additional protection buyer, this exerts a negative externality on his preexisting counterparties, since it increases the counterparty default risk they all incur. Because of this externality, the equilibrium arising in a decentralized market is not Pareto efficient. Acharya and Bisin (2011) show how, with centralized clearing and trading, such externality can be avoided, by implementing price schedules penalizing the creation of counterparty risk.

The focus of our analysis differs from that of Acharya and Bisin (2010). In Acharya and Bisin (2010), with centralized clearing, all trades become *observable* and contractible, externalities can be internalized thanks to appropriate *pricing mechanisms*, and the *first-best obtains*. In our analysis, even with centralized clearing, effort remains *unobservable*, moral hazard continues to be an issue, prices are not sufficient to implement the optimal mechanism, *quantity constraints* must be imposed to make effort incentive-compatible, and the *first-best is not reached*.

**Duffie and Zhu (2009):** Duffie and Zhu (2009) compare two types of netting systems: *i)* bilateral netting between pairs of dealers across different underlying assets, and *ii)* multilateral netting among many dealers across a single class of underlying assets, such as credit default swaps (CDS). Although it relies on different economic forces, our analysis of the mutualization benefits of CCPs echoes their analysis of the effectiveness of CCPs. For example, they write, on page 2 of the paper:

> "The introduction of a CCP for CDS can ... be effective when there are extensive opportunities for multi-lateral netting. For example, if Dealer A is exposed by $100 million to Dealer B through a CDS, while Dealer B is exposed to Dealer C for $100 million on the same CDS, and Dealer C is simultaneously exposed to Dealer A for the same amount on the same CDS, then a CCP eliminates this unnecessary circle of exposures."

But the focus of Duffie and Zhu (2009) is different than ours. Taking deposit constraints in different systems as given they study which system is more economical in terms of collateral requirements. This is motivated by their observation that collateral is costly. Thus, the objective in their analysis is the *netting efficiency* of the system. In contrast, while we also take into account the cost of cash deposits, the objective in our analysis is the information constrained *risk-sharing efficiency* of the system. Thus, while the risk aversion of agents and their incentive-compatibility constraints play an important role in our analysis, they are absent from Duffie and Zhu (2009).

Also, while we consider only one market, Duffie and Zhu (2009) analyze netting efficiency in a multi-market setting. They point out that when traders intervene in different markets, having separate CCPs in different markets can raise the amount which must be deposited as collateral. For example, they write, on page 2 of the paper:

> "For instance, if Dealer A is exposed to Dealer B by \$100 million on CDS, while at the same time Dealer B is exposed to Dealer A by \$150 million on interest-rate swaps, then the introduction of central clearing for CDS (only) increases the maximum loss between these two dealers, before collateral and after netting, from \$50 million to \$150 million. In addition to any collateral posted by Dealer A to the CCP for CDS, Dealer A would need to post a significant amount of additional collateral to Dealer B."

**Pirrong (2009):** Pirrong (2009) discusses risk-sharing and moral hazard issues associated with a CCP. He makes three important points with which our analysis is in line:

- When the clearing entity insures both parties to a trade against counterparty risk, it does not eliminate this risk. It just transfers it from the trading parties to itself. Therefore, one must analyze how this transfer affects risk-sharing and risk-taking decisions.
- Without information asymmetry, centralized clearing can improve risk-sharing by mutualizing risk.
- With information asymmetry, such mutualization can undermine incentives.

Pirrong (2009) concludes (page 4) that "with asymmetric information, it is not necessarily the case that the formation of a CCP is efficient." Our analysis differs from his because we model i) the effort to find creditworthy counterparties, and ii) the difference between idiosyncratic and aggregate risk. Furthermore, our optimal contracting approach enables us to characterize the optimal incentive-compatible centralized clearing mechanism (rather than focusing on an ad-hoc and therefore suboptimal mechanism.) We show that this optimally designed CCP is welfare-improving relative to bilateral clearing. Thus, our optimal contracting approach leads us to reach an opposite conclusion to that of Pirrong (2009).

# 4 The model

There are five dates, $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ and 1. A unit mass continuum of risk-averse protection buyers face a large population of risk-neutral protection sellers.[6] The discount rate of all market participants is normalized to one and the risk-free rate to 0.

Each protection seller $i$ is endowed with one unit of asset in place with an uncertain return $\tilde{R}_i$ at $t = 1$. The protection sellers are heterogeneous. Some of them are solid creditworthy institutions, which we hereafter refer to as "good": They generate $\tilde{R}_i = R > 1$ with probability $p$ and 0 otherwise, with $pR > 1$. Others are more fragile, less creditworthy institutions, hereafter referred to as "bad." They generate $\tilde{R}_i = R$ with probability $p - \delta$ only. The protection sellers' positions are completely illiquid, i.e., their liquidation value at $t = 0$ is zero. All protection sellers are risk-neutral, have no initial endowment apart from the asset generating $R$ or 0, and have limited liability.

Each protection buyer $j$ is endowed with an asset whose random final value $\tilde{\theta}_j$ realizes at time $t = 1$. We assume that $\tilde{\theta}_j$ can take on two values: $\bar{\theta}$ with probability $\frac{1}{2}$ and $\underline{\theta}$ otherwise. The asset owned by the protection buyer can be thought of as a loan, and $\underline{\theta}$ can be interpreted as occurring when the borrower partially defaults on the loan. We assume that $R > \bar{\theta} - \underline{\theta} = \Delta\theta$ and that all exogenous random variables are independent.

The sequence of play is the following:

- At time $t = 0$, the market infrastructure (no clearing, decentralized clearing, or CCP) is put in place.

- At time $t = \frac{1}{4}$, protection buyers can exert effort ($e = 1$) at cost $B$, in which case they are matched with a good protection seller for sure, or no effort ($e = 0$), in which case they are always matched with a bad protection seller. The preferences of the protection buyers are quasi linear, that is, there exists a concave utility function $u$ such that the

[6] Concavity of the objective function of the protection buyer can reflect institutional, financial or regulatory constraints, such as leverage constraints or risk-weighted capital requirements. For an explicit modeling of hedging motives see Froot, Scharfstein and Stein (1993) and Froot and Stein (1998).

utility of a protection buyer with consumption $x$ is $u(x) - B$ under effort and $u(x)$ without effort.

- At time $\frac{1}{2}$, each protection buyer is matched with a protection seller. For simplicity, but without affecting the results qualitatively, we assume that, when matched with a protection seller, the protection buyer has all the bargaining power. Thus, the protection buyer offers a contract maximizing his expected utility, subject to the participation, feasibility and incentive constraints (spelled out below).

- At time $\frac{3}{4}$, aggregate macroeconomic uncertainty $\tilde{\gamma}$ is resolved. For simplicity and clarity, until Section 6, there is no aggregate risk, i.e., $\tilde{\gamma}$ is constant. In Section 6, we study how the introduction of macro-risk alters the economics of clearing and risk-sharing in our framework.

The realizations of the random variables ($\tilde{R}_i$, $\tilde{\theta}_j$, and $\tilde{\gamma}$) are observable and contractible. Until Section 7, for simplicity and clarity, we also assume that the effort of the protection buyers is observable and contractible. Thus, the optimal clearing arrangement we characterize implements the first-best. In Section 7, we study how the introduction of moral hazard alters the situation and we analyze the information-constrained optimal clearing arrangement implementing the second-best.

# 5 Idiosyncratic risk and observable effort

In this section, we study optimal risk-sharing contracts when effort of the protection buyers is observable and the risk protection buyers are exposed to is purely idiosyncratic. We first characterize the optimal contract between a protection buyer and a protection seller, without clearing. We then consider bilateral contracting with a single clearing agent. We conclude the section with the analysis of the multilateral contract with a CCP.

## 5.1 Bilateral contracting without clearing

As a benchmark, consider the simple case without clearing, i.e., there is direct contracting between the protection buyer and the protection seller, and no clearing agent. The contract involves a premium $\pi$ paid by the protection buyer when $\bar{\theta}$ occurs, and a transfer $\tau$ paid by the protection seller when $\underline{\theta}$ occurs and the protection seller does not default. We first characterize the optimal contract when the protection buyer chooses to exert the search effort. We then derive the condition under which it is optimal for him to do so.

The protection buyer chooses $\pi$ and $\tau$ to maximize her expected utility subject to the participation constraint of the protection seller. The optimal contract with effort solves

$$\max_{\pi,\tau} \frac{1}{2}u\left(\bar{\theta} - \pi^{e=1}\right) + \frac{1}{2}pu\left(\underline{\theta} + \tau^{e=1}\right) + \frac{1}{2}\left(1-p\right)u(\underline{\theta}) - B \tag{1}$$

subject to the participation constraint of the good protection seller

$$\frac{1}{2}\pi^{e=1} + \frac{1}{2}p\left(-\tau^{e=1}\right) \geq 0. \tag{2}$$

Let $\Delta\theta$ denote $\bar{\theta} - \underline{\theta}$. Proposition 1 presents the solution to the optimization problem above.

**Proposition 1** *In the bilateral contract with effort, there is full risk-sharing as long as the protection seller does not default and the seller's participation constraint binds, i.e.,* $\tau^{e=1} + \pi^{e=1} = \Delta\theta$ *and* $\tau^{e=1} = \frac{\Delta\theta}{1+p}$*. However, the protection buyer is exposed to counterparty risk with probability* $\frac{1}{2}\left(1-p\right)$*.*

If the protection buyer does not exert effort, he is matched with a bad protection seller for sure. Then, the optimal contract without effort solves

$$\max_{\pi,\tau} \frac{1}{2}u\left(\bar{\theta} - \pi^{e=0}\right) + \frac{1}{2}\left(p-\delta\right)u\left(\underline{\theta} + \tau^{e=0}\right) + \frac{1}{2}\left(1-p+\delta\right)u(\underline{\theta}) \tag{3}$$

subject to the participation constraint of the bad protection seller

$$\frac{1}{2}\pi^{e=0} + \frac{1}{2}\left(p-\delta\right)\left(-\tau^{e=0}\right) \geq 0. \tag{4}$$

Proposition 2 presents the solution to the optimization problem above.

**Proposition 2** *In the bilateral contract without effort, there is full risk-sharing as long as the protection seller does not default and the seller's participation constraint binds, i.e.,* $\tau^{e=0} + \pi^{e=0} = \Delta\theta$ *and* $\tau^{e=0} = \frac{\Delta\theta}{1+p-\delta}$*. The protection buyer is exposed to counterparty risk with probability* $\frac{1}{2}\left(1-p+\delta\right)$*.*

In the contract without effort, the protection buyer receives a higher transfer in $\underline{\theta}$ state in which the protection seller does not default compared to the contract with effort, $\tau^{e=0} > \tau^{e=1}$. At the same time, he pays a lower premium, $\pi^{e=0} < \pi^{e=1}$. He is, however, left without any insurance more often. In the next proposition, we state when effort is preferred to no effort.

**Proposition 3** *Without clearing, the protection buyer prefers to exert the search effort if and only if*

$$\frac{u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right)-u(\underline{\theta})}{\frac{\Delta\theta}{1+p-\delta}}-\frac{u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right)-u\left(\underline{\theta}+\frac{\Delta\theta}{1+p}\right)}{\frac{\Delta\theta}{1+p-\delta}-\frac{\Delta\theta}{1+p}}\geq\frac{2B}{\delta}\frac{1+p-\delta}{\Delta\theta}. \tag{5}$$

Irrespective of whether the protection buyer exerts effort or not, the optimal contract states that his consumption is the same when $\bar{\theta}$ occurs and when $\underline{\theta}$ occurs but the protection seller does not default. More precisely, if the protection buyer exerts effort, his final consumption is $\underline{\theta}+\frac{\Delta\theta}{1+p}$ when the protection seller does not default, and $\underline{\theta}$ otherwise, while, if the protection buyer exerts effort, his final consumption is $\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}$ when the protection seller does not default, and $\underline{\theta}$ otherwise. The cost of effort is $B$; the benefit of effort is that it reduces the probability of the low consumption state by $\delta$. The right-hand-side of (5) involves the ratio of the cost $B$ to the benefit $\delta$. The left-hand-side of (5) is the difference between two slopes in the consumption-utility space.

$$s_1=\frac{u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right)-u(\underline{\theta})}{\frac{\Delta\theta}{1+p-\delta}},$$

is the slope of the line from $(\underline{\theta},u(\underline{\theta}))$ to $(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta},u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right))$, while

$$s_2=\frac{u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right)-u\left(\underline{\theta}+\frac{\Delta\theta}{1+p}\right)}{\frac{\Delta\theta}{1+p-\delta}-\frac{\Delta\theta}{1+p}},$$

is the slope of the line from $(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta},u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right))$ to $(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta},u\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}\right))$. By concavity of $u$, $s_1>s_2$. The more concave is $u$, the greater is the difference $s_1-s_2$. Thus, Proposition 3 implies that exerting the search effort is more attractive: 1) the more concave the utility function $u$ is; and 2) the lower the search cost $B$ is.

## 5.2 Bilateral contracting with a single clearing agent

Now turn to the case in which, in addition to the protection buyer and the protection seller, there is a clearing agent. Suppose he is risk-neutral, has limited liability and is endowed with initial cash $c$. The clearing agent can invest $c$ in an illiquid asset earning a per-unit return $\rho > 1$, and in a liquid asset earning the risk-free return (cash). We assume that the illiquid asset matures at time $t = 2$, too late to be used for insurance payments in case of protection seller's default. Let $\alpha$ denote the fraction of the clearing agent's assets invested in the liquid asset. The rest is invested in the illiquid asset.

Providing insurance to the protection buyer will entail an opportunity cost for the clearing agent since he has to forego the return on the illiquid asset. Since the protection seller faces no such opportunity cost, he will continue to insure the protection buyer against the $\tilde{\theta}$ risk, as long as he is not in default. The clearing agent could provide (costly) insurance to the protection buyer against counterparty risk. Hence, in addition to a premium $\pi$ and a transfer $\tau$ between the protection buyer and seller, the contract also specifies a fee $\pi^C$ paid by the protection buyer to the clearing agent when the protection seller does not default, and a transfer $\tau^C$ paid by the clearing agent to the protection buyer when $\underline{\theta}$ occurs and the protection seller defaults.[7]

When the protection buyer exerts the search effort, the participation constraint of the clearing agent is

$$\rho c \leq \alpha^{e=1} c + \left(1 - \alpha^{e=1}\right) \rho c + \frac{1}{2}\left(1+p\right) \pi^{C,e=1} - \frac{1}{2}\left(1-p\right) \tau^{C,e=1},$$

or, equivalently,

$$\frac{1}{2}\left(1-p\right) \tau^{C,e=1} + \alpha^{e=1} c \left(\rho - 1\right) \leq \frac{1}{2}\left(1+p\right) \pi^{C,e=1} \tag{6}$$

Note that $\alpha^{e=1} c \geq \tau^{C,e=1}$ must hold since the illiquid asset cannot be used to make insurance payments. The left-hand side of (6) represents the cost of providing insurance to the protection buyer. When the protection seller defaults, the clearing agent must make the insurance payment $\tau^{C,e=1}$. To be able to make the payment, the clearing agent must set aside some cash and hence forego a net return $(\rho - 1)$. The right-hand side of (6) represents the benefit of providing insurance, which is given by the fee that the clearing agent collects.

[7] It is without loss of generality that we postulate that the protection buyer makes the same payment to the clearing agent as long as the protection seller does not default. What matters are the final consumption allocations across states, which are given by the sum of payments and transfers between the three parties.

When the protection seller does not exert effort, the participation constraint of the clearing agent is modified as follows

$$\frac{1}{2}(1-p+\delta)\,\tau^{C,e=0}+\alpha^{e=0}c\,(\rho-1)\leq\frac{1}{2}(1+p-\delta)\,\pi^{C,e=0} \tag{7}$$

Under no effort, the clearing agent has to pay out $\tau^{C,e=0}$ more often, while it receives the fee $\pi^{C,e=0}$ less often.

In the two propositions below, we characterize the optimal contract with effort. The optimal contract with effort solves

$$\max_{\alpha,\pi,\tau,\pi^{C},\tau^{C}}\ \frac{1}{2}u(\bar{\theta}-\pi^{e=1}-\pi^{C,e=1})+\frac{1}{2}pu(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1})+\frac{1}{2}(1-p)\,u(\underline{\theta}+\tau^{C,e=1})-B \tag{8}$$

subject to the participation constraints of the protection seller (2) and the clearing agent (6).

**Proposition 4** *When there is no moral hazard and no aggregate risk, in the contract with effort, the clearing agent invests fraction $\alpha^{e=1}=\frac{\tau^{C,e=1}}{c}$ in the safe asset and uses these funds to provide partial insurance ($\tau^{C,e=1}<\tau^{e=1}-\pi^{C,e=1}$) to the protection buyer when the protection seller defaults. As long as the protection seller does not default, there is full risk-sharing, i.e., $\tau^{e=1}+\pi^{e=1}=\Delta\theta$.*

Insurance against counterparty risk is only partial since its provision is costly. The clearing agent must be compensated for the opportunity cost such insurance entails, $\rho-1$. To minimize this cost, the fraction of funds invested in the safe asset is just equal to the amount to be paid to the protection buyer in case the protection seller defaults, $\alpha^{e=1}c=\tau^{C,e=1}$. We also have

$$\pi^{C,e=1}=\tau^{C,e=1}\frac{1-p+2(\rho-1)}{1+p}\text{ and }\tau^{e=1}=\frac{\Delta\theta}{1+p}.$$

To quantify the cost and benefit of insurance against counterparty risk, it is useful to introduce the function $\varphi^{e=1}$, defined as

$$\varphi^{e=1}\left(\tau^{C}\right)\equiv\frac{u'(\underline{\theta}+\tau^{C})}{u'(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1})}=\frac{u'(\underline{\theta}+\tau^{C})}{u'\left(\underline{\theta}+\frac{\Delta\theta}{1+p}-\tau^{C}\frac{1-p+2(\rho-1)}{1+p}\right)}.$$

Function $\varphi^{e=1}$ is the ratio of the marginal utility of the protection buyer in the state $\underline{\theta}$ when the protection seller defaults and when he does not default. It is a known function of exogenous variables and $\tau^C$, and it is decreasing in $\tau^C$. Since insurance against counterparty risk is only partial, $\varphi$ is greater than one. Higher insurance against counterparty default (higher $\tau^C$) reduces $\varphi$, moving the marginal utilities closer to one, i.e., closer to full insurance. The following proposition characterizes the optimal degree of counterparty risk insurance in the contract with effort.

**Proposition 5** *When there is no moral hazard and no aggregate risk, if $\varphi^{e=1}(0) \leq 1+\frac{2(\rho-1)}{1-p}$, then the clearing agent is not used. Otherwise, if*

$$\varphi^{e=1}(c) > 1 + \frac{2(\rho-1)}{1-p}, \tag{9}$$

*then $\alpha^* = 1$, while if (9) does not hold, $\tau^C$ is given by*

$$\varphi^{e=1}\left(\tau^C\right) = 1 + \frac{2(\rho-1)}{1-p}. \tag{10}$$

The optimal degree of insurance balances benefits of insurance against counterparty risk (left-hand side of (10)) with the opportunity cost of holding cash reserves (right-hand side of (10)). If the opportunity cost of cash reserves is very high ($\rho - 1$ is high), there will be less insurance. As the probability of counterparty default $(1-p)$ rises, the transfer is the event of default increases. Indeed, when counterparty default is frequent, it is worthy to put a high amount of assets in cash reserves to pay $\tau^{C,e=1}$.

The optimal contract without effort solves

$$\max_{\alpha,\pi,\tau,\pi^C,\tau^C} \frac{1}{2}u(\bar{\theta}-\pi^{e=0}-\pi^{C,e=0}) + \frac{1}{2}(p-\delta)\,u(\underline{\theta}+\tau^{e=0}-\pi^{C,e=0}) + \frac{1}{2}(1-p+\delta)\,u(\underline{\theta}+\tau^{C,e=0}) \tag{11}$$

subject to the participation constraints of the protection seller (4) and the clearing agent (7). Next, we state two propositions characterizing the optimal contract without effort.

**Proposition 6** *When there is no moral hazard and no aggregate risk, in the contract without effort, the clearing agent provides partial insurance ($\tau^{C,e=0} < \tau^{e=0} - \pi^{C,e=0}$) to the protection buyer when the protection seller defaults. As long as the protection seller does not default, there is full risk-sharing, i.e., $\tau^{e=0} + \pi^{e=0} = \Delta\theta$.*

Let

$$\varphi^{e=0}\left(\tau^{C}\right) \equiv \frac{u'(\underline{\theta}+\tau^{C})}{u'\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}-\tau^{C}\frac{1-p+\delta+2(\rho-1)}{1+p-\delta}\right)}$$

We have the following counterpart of Proposition 5.

**Proposition 7** *When there is no moral hazard and no aggregate risk, in the contract without effort, if $\varphi^{e=0}(0) \leq 1+\frac{2(\rho-1)}{1-p+\delta}$, then the clearing agent is not used. Otherwise, if*

$$\varphi^{e=0}(c) > 1+\frac{2(\rho-1)}{1-p+\delta}, \tag{12}$$

*then $\alpha^{*}=1$, while if (9) does not hold, $\tau^{C}$ is given by*

$$\varphi^{e=0}\left(\tau^{C}\right) = 1+\frac{2(\rho-1)}{1-p+\delta}. \tag{13}$$

In the contract without effort, the clearing agent provides more insurance to the protection buyer compared to the contract with effort, $\tau^{C,e=0} \geq \tau^{C,e=1}$. This is in line with the above remark that the amount put in the safe asset is increasing in the probability of counterparty default. But, since it is costly, the insurance remains partial. And, since the amount paid to the protection buyer in case of default is higher when there is no effort, so is fee paid to the clearing, i.e., $\pi^{C,e=0} > \pi^{C,e=1}$. The protection buyer prefers to exert search effort is and only if

$$\begin{aligned}(1+p)\,u\left(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1}\right)+(1-p)\,u(\underline{\theta}+\tau^{C,e=1})-2B \geq \\ (1+p-\delta)\,u\left(\underline{\theta}+\tau^{e=0}-\pi^{C,e=0}\right)+(1-p+\delta)\,u(\underline{\theta}+\tau^{C,e=0}),\end{aligned}$$

which holds if the cost of effort ($B$) is low and the increase in default risk due to lack of effort ($\delta$) is high.

## 5.3 Multilateral contracting with a CCP

Now consider the case in which there is a CCP interposing between all the protection buyer-protection seller pairs who want to contract. We maintain our assumption that effort is observable, so the CCP can request it without facing incentive constraints. In this context, since the link between the effort of the protection buyer and the type of the protection seller is one to one, there is no need for the contracts to be contingent on the type of the protection seller.

Because all random variables are $i.i.d$, the aggregate mass of defaults is deterministic. For the sake of comparability with the single clearing agent case, we assume that the CCP is initially endowed with wealth $c$, which it can invest in an illiquid project with time 2 return $\rho > 1$. The total amount of insurance transfers to be paid by the CCP at time $t = 1$ is

$$\int_{i=0}^{1} \mathbf{1}(\theta_i = \underline{\theta})\mathbf{1}(R_i = 0)\tau^C di, \tag{14}$$

where $\mathbf{1}$ denotes the indicator variable, and $i$ indexes the protection buyer-protection seller pairs. Because the $\tilde{\theta}_i$ and $\tilde{R}_i$ are independent, by the law of large numbers (14) is equal to

$$\frac{1}{2}(1-p)\tau^C$$

if protection buyers exert search effort and it is equal to

$$\frac{1}{2}(1-p+\delta)\tau^C$$

if protection buyers do not exert effort.

The participation constraint of the CCP is

$$\frac{1}{2}(1-p)\tau^C + \alpha c(\rho-1) \leq \frac{1}{2}(1+p)\pi^C \tag{15}$$

if effort is exerted and

$$\frac{1}{2}(1-p+\delta)\tau^C + \alpha c(\rho-1) \leq \frac{1}{2}(1+p-\delta)\pi^C \tag{16}$$

otherwise.

The participation constraints of the CCP (15) and (16) bind (by the same logic as in the previous subsection). Note that any $\alpha > 0$ tightens the participation constraint due to the opportunity cost of cash reserves. This yields the following proposition:

**Proposition 8** *When there is no moral hazard and no aggregate risk, the CCP optimally sets $\alpha = 0$. The fees are given by*

$$\pi^{C,e=1} = \frac{1-p}{1+p}\tau^{C,e=1}$$

*if the search effort is exerted,and*

$$\pi^{C,e=0} = \frac{1-p+\delta}{1+p-\delta}\tau^{C,e=0}$$

*otherwise.*

Proposition 8 states that the CCP can collect the fees such that they exactly cover the insurance payments. Hence, unlike the single clearing agent, the CCP does not need to set aside cash reserves and incur the corresponding opportunity cost. Since there is no opportunity cost of providing insurance, the next proposition shows that it is optimal for the CCP to provide full insurance to the protection buyers, regardless of whether or not the effort is exerted.

**Proposition 9** *When there is no moral hazard and no aggregate risk, the CCP provides full insurance against counterparty default risk, regardless of whether or not the search effort is exerted. It is hence optimal for the protection buyers to economize on search costs and never exert effort. In this first-best allocation, the aggregate default rate is higher than without the CCP.*

With the CCP, all idiosyncratic risk can be mutualized. Consequently, when there is no aggregate risk, protection buyers are fully insured. Hence, there is no need to exert effort to find creditworthy counterparties and no-effort is optimal. This underscores that, in spite of formal similarities, the setup we consider is very different from that analyzed by Holmström and Tirole (1998).

- In Holmström and Tirole (1998), at a cost $B$, the agent can reduce the probability of low output. Thus, effort increases the average output in the economy. Even with risk-neutrality this can be valuable, if $B$ is low enough.

- In the present model, at a cost $B$, the agent can find a protection seller with low probability of default. But, effort does not increase the average output in the economy: the overall output of the protection sellers is exogenous and unaffected by the protection buyers' efforts. Therefore, the effort of the protection buyers can be useful only if it increases their risk-sharing ability. However, with mutualization and no aggregate-risk, full risk-sharing can be achieved even if the protection sellers are bad. Hence, effort is not optimal.

Thus, while the CCP achieves the first-best, it can at the same time increase aggregate default risk. This is not a symptom of a market failure, however. Rather, it is a feature of the first-best.

Finally, note that with all idiosyncratic risk mutualized by the CCP, it is no longer strictly necessary for protection sellers to be a part of the risk-sharing transaction. Risk-mutualization among protection buyers is enough. This differs from the environment with aggregate risk, in which risk-mutualization by protection buyers is not enough to deliver full insurance. In that case, as we show in the next section, sellers play an important role even with the CCP mutualizing the idiosyncratic risk. Therefore, the effort of the protection buyers to search for good protection sellers can be beneficial as it increases the resources available to insure against aggregate risk.

# 6 Aggregate risk and observable effort

## 6.1 Aggregate risk

We now study the optimal design of the CCP when there is aggregate risk, but still no moral hazard. To extend our analysis to the case of aggregate risk, our set-up is modified as follows. There are two aggregate states in the economy, high and low, each occurring with equal probability. In the high state, the probability of $\bar{\theta}$ is $\frac{1}{2}+\gamma$, while the probability of $\underline{\theta}$ is $\frac{1}{2}-\gamma$. In the low state, the probability of $\bar{\theta}$ is $\frac{1}{2}-\gamma$, while the probability of $\underline{\theta}$ is $\frac{1}{2}+\gamma$. Conditional on the macro-state, the realizations of the $\tilde{\theta}_j$ are i.i.d. Note that, ex-ante, the two values $\underline{\theta}$ and $\bar{\theta}$ are equiprobable, as in the analysis of the previous sections. But, ex-post, after observing the macro-state, one of them becomes more likely. When $\gamma = 0$ there is no macro-risk and agents are only exposed to idiosyncratic risk. At the other extreme, when $\gamma = \frac{1}{2}$, there is no idiosyncratic risk, and, in each of the two macro-states, the values of all the assets of all the protection sellers are the same. We now consider the case in which $\gamma \in (0, \frac{1}{2})$ and there is combination of aggregate and idiosyncratic risk. We also assume that

$$pR > \frac{\Delta\theta}{2} > (p-\delta)\,R, \tag{17}$$

which implies that, if all protection buyers contract with creditworthy protection sellers, the total resources brought to the table by all the parties are sufficient to fully insure the protection buyers, while, if the protection buyers fail to exert effort, full insurance may be infeasible. Finally, to focus on the effects of aggregate risk in the simplest possible set-up, we assume that the CCP does not have any cash endowment, $c = 0$.

The macro-state is realized at time $\frac{3}{4}$, after the market infrastructure and the contracts have been designed and effort has been exerted. The realization of the macro-state is publicly observable, and contracts are contingent upon it. Let $\left(\bar{\pi}, \bar{\tau}, \bar{\pi}^C, \bar{\tau}^C\right)$ denote fees and transfers in the high state and let $\left(\underline{\pi}, \underline{\tau}, \underline{\pi}^C, \underline{\tau}^C\right)$ denote fees and transfers in the low state.

## 6.2 Optimal clearing and contracting with effort

Consider first the contract with effort. The objective function of the protection buyer under effort is given by

$$\frac{1}{2}\left[\left(\frac{1}{2}+\gamma\right)u(\bar{\theta}-\bar{\pi}^{e=1}-\bar{\pi}^{C,e=1})+\left(\frac{1}{2}-\gamma\right)pu(\underline{\theta}+\bar{\tau}^{e=1}-\bar{\pi}^{C,e=1})+\right.$$
$$\left.\left(\frac{1}{2}-\gamma\right)(1-p)\,u(\underline{\theta}+\bar{\tau}^{C,e=1})-B\right]+\frac{1}{2}\left[\left(\frac{1}{2}-\gamma\right)u(\bar{\theta}-\underline{\pi}^{e=1}-\underline{\pi}^{C,e=1})+\right.$$
$$\left.\left(\frac{1}{2}+\gamma\right)pu(\underline{\theta}+\underline{\tau}^{e=1}-\underline{\pi}^{C,e=1})+\left(\frac{1}{2}+\gamma\right)(1-p)\,u(\underline{\theta}+\underline{\tau}^{C,e=1})-B\right] \tag{18}$$

The participation constraint of the protection seller is

$$\left(\frac{1}{2}+\gamma\right)\bar{\pi}^{e=1}+\left(\frac{1}{2}-\gamma\right)p\left(-\bar{\tau}^{e=1}\right)+\left(\frac{1}{2}-\gamma\right)\underline{\pi}^{e=1}+\left(\frac{1}{2}+\gamma\right)p\left(-\underline{\tau}^{e=1}\right)\geq 0 \tag{19}$$

As before, the CCP will collect the fees such that they exactly cover the insurance payments in each state so that

$$\left(\frac{1}{2}-\gamma\right)(1-p)\,\bar{\tau}^{C,e=1}=\left[\frac{1}{2}+\gamma+\left(\frac{1}{2}-\gamma\right)p\right]\bar{\pi}^{C,e=1}$$

in the high state and

$$\left(\frac{1}{2}+\gamma\right)(1-p)\,\underline{\tau}^{C,e=1}=\left[\frac{1}{2}-\gamma+\left(\frac{1}{2}+\gamma\right)p\right]\underline{\pi}^{C,e=1}$$

in the low state. Hence, we have that

$$\bar{\pi}^{C,e=1}=\frac{\left(\frac{1}{2}-\gamma\right)(1-p)\,\bar{\tau}^{C,e=1}}{\frac{1}{2}(1+p)+\gamma\,(1-p)}\text{ and }\underline{\pi}^{C,e=1}=\frac{\left(\frac{1}{2}+\gamma\right)(1-p)\,\underline{\tau}^{C,e=1}}{\frac{1}{2}(1+p)-\gamma\,(1-p)} \tag{20}$$

As shown in the appendix, this yields the following proposition.

**Proposition 10** *With aggregate risk but no moral hazard, the optimal contract with centralized clearing and effort provides full insurance to the protection buyers, and their expected utility is*

$$u\left(E\left[\tilde{\theta}\right]\right)-B. \tag{21}$$

*When the protection seller does not default, transfers are*

$$\underline{\tau}^{e=1} = \frac{1}{\frac{1}{2}(1+p) - \gamma(1-p)} \frac{\Delta\theta}{2} > \frac{1}{\frac{1}{2}(1+p) + \gamma(1-p)} \frac{\Delta\theta}{2} = \bar{\tau}^{e=1}, \tag{22}$$

*while in case of default they are*

$$\bar{\tau}^{C,e=1} = \underline{\tau}^{C,e=1} = \frac{\Delta\theta}{2}. \tag{23}$$

(23) is a direct implication of full risk sharing. (22) is a direct implication of (23), (20) and the fact that the consumption of the protection buyer is the same whether the protection seller defaults or not. Furthermore, feasibility requires that the transfer paid by the protection seller when he does not default ($\underline{\tau}^{e=1}$) be lower than his resources ($R$). Thus, in the optimal contract we have

$$R > \underline{\tau}^{e=1} = \frac{1}{\frac{1}{2}(1+p) - \gamma(1-p)} \frac{1}{2}\Delta\theta. \tag{24}$$

Since the right-hand side of (24) is increasing in $\gamma$, and maximum $\gamma$ is $\frac{1}{2}$, a sufficient condition for feasibility is $pR > \frac{\Delta\theta}{2}$ which holds by (17).

(21) is identical to its counterpart without aggregate risk. (17) implies that, under effort, the population of protection sellers brings enough resources to the table to provide full insurance to the protection buyers. Since they are competitive and risk-neutral, the protection sellers are willing to do that as long as they break even on average. Thus, insurance comes at no cost, except the search cost $B$. Otherwise stated, if aggregate risk is relatively small, and when protection buyers exert effort, optimal risk sharing can be provided within the CCP. The main difference between this result and that of the previous section is that, without aggregate risk, effort and protection sellers were not needed for full risk-sharing. This is no longer the case with aggregate risk.

## 6.3 Optimal clearing and contracting without effort

Now consider the contract without effort. The objective function of the protection buyer without effort is given by

$$\begin{aligned} &\frac{1}{2}\left[\left(\frac{1}{2}+\gamma\right) u(\bar{\theta} - \bar{\pi}^{e=0} - \bar{\pi}^{C,e=0}) + \left(\frac{1}{2}-\gamma\right)(p-\delta)\, u(\underline{\theta} + \bar{\tau}^{e=0} - \bar{\pi}^{C,e=0}) + \right. \\ &\left.\left(\frac{1}{2}-\gamma\right)(1-p+\delta)\, u(\underline{\theta} + \bar{\tau}^{C,e=0})\right] + \frac{1}{2}\left[\left(\frac{1}{2}-\gamma\right) u(\bar{\theta} - \underline{\pi}^{e=0} - \underline{\pi}^{C,e=0}) + \right. \\ &\left.\left(\frac{1}{2}+\gamma\right)(p-\delta)\, u(\underline{\theta} + \underline{\tau}^{e=0} - \underline{\pi}^{C,e=0}) + \left(\frac{1}{2}+\gamma\right)(1-p+\delta)\, u(\underline{\theta} + \underline{\tau}^{C,e=0})\right] \end{aligned} \tag{25}$$

The participation constraint of the protection seller is

$$\left(\frac{1}{2}+\gamma\right)\bar{\pi}^{e=0}+\left(\frac{1}{2}-\gamma\right)(p-\delta)\left(-\bar{\tau}^{e=0}\right)+\left(\frac{1}{2}-\gamma\right)\underline{\pi}^{e=0}+\left(\frac{1}{2}+\gamma\right)(p-\delta)\left(-\underline{\tau}^{e=0}\right)\geq 0 \tag{26}$$

As before, the CCP will collect the fees such that they exactly cover the insurance payments in each state so that

$$\left(\frac{1}{2}-\gamma\right)(1-p+\delta)\,\bar{\tau}^{C,e=0}=\left[\frac{1}{2}+\gamma+\left(\frac{1}{2}-\gamma\right)(p-\delta)\right]\bar{\pi}^{C,e=0}$$

in the high state and

$$\left(\frac{1}{2}+\gamma\right)(1-p+\delta)\,\underline{\tau}^{C,e=0}=\left[\frac{1}{2}-\gamma+\left(\frac{1}{2}+\gamma\right)(p-\delta)\right]\underline{\pi}^{C,e=0}$$

in the low state. Hence, we have that

$$\bar{\pi}^{C,e=0}=\frac{\left(\frac{1}{2}-\gamma\right)(1-p+\delta)\,\bar{\tau}^{C,e=0}}{\frac{1}{2}(1+p-\delta)+\gamma(1-p+\delta)}\text{ and }\underline{\pi}^{C,e=0}=\frac{\left(\frac{1}{2}+\gamma\right)(1-p+\delta)\,\underline{\tau}^{C,e=0}}{\frac{1}{2}(1+p-\delta)-\gamma(1-p+\delta)} \tag{27}$$

These equations are the same as those we had with effort, except that here the cost $B$ is not incurred and the probability of counterparty default is raised by $\delta$.

Thus, we can state the following proposition.

**Proposition 11** *With aggregate risk but no moral hazard, if*

$$R\geq\frac{1}{\frac{1}{2}(1+p-\delta)-\gamma(1-p+\delta)}\frac{\Delta\theta}{2} \tag{28}$$

*holds, then the optimal contract with the CCP and without effort provides full insurance to the protection buyers, with transfers in the event of default* $\bar{\tau}^{C,e=0}=\underline{\tau}^{C,e=0}=\frac{\Delta\theta}{2}$. *If (28) does not hold, then there is full risk-sharing in the high aggregate state. But in the low aggregate state, the amount of insurance is limited by the resource constraint, so that* $\bar{\tau}^{C,e=0}>\underline{\tau}^{C,e=0}$.

(28) is the counterpart of (24) in the case with no effort. To understand the economic forces driving the result in Proposition 11, it is useful to first consider the extreme case with only macro risk ($\gamma=\frac{1}{2}$). In that case, (28) becomes

$$p-\delta>\frac{\Delta\theta}{2},$$

which, by (17), does not hold. For less extreme cases, the right-hand-side of (28) is increasing in $\gamma$, which measures the extent of aggregate risk. When aggregate risk is large (i.e., when

$\gamma$ is large), if the protection buyers don't exert effort, the amount of resources in the bad macro-state is insufficient to provide full risk-sharing, even with a CCP.

If (28) holds so that full risk-sharing is feasible, the expected utility in the contract without effort is

$$u\left(E\left[\tilde{\theta}\right]\right) \tag{29}$$

Comparing (21) and (29), it follows that whenever (28) holds, protection buyers will choose not to exert search effort since they can get full insurance while saving on the search cost $B$.

Note that if $\gamma = 0$ and there is no aggregate risk, condition (28) is equivalent to $R \geq \frac{\Delta\theta}{1+p-\delta}$ which is always satisfied since we assume $R > \Delta\theta$. That is, for $\gamma = 0$, full risk-sharing is feasible when effort is not exerted. Hence, exerting effort is not optimal (see Proposition 9).

As mentioned above, for $\gamma = \frac{1}{2}$, condition (28) is equivalent to $(p-\delta)\,R \geq \frac{\Delta\theta}{2}$ which cannot hold due to (17). That is, when effort is not exerted and $\gamma = \frac{1}{2}$, full risk-sharing is not feasible. We can state the following proposition for this special case.

**Proposition 12** *With aggregate risk, no moral hazard and no idiosyncratic risk, $\gamma = \frac{1}{2}$, the optimal contract has $\bar{\pi}^{e=0} = (p-\delta)\,R$, $\underline{\tau}^{e=0} = R$ and $\underline{\tau}^{C,e=0} = (p-\delta)\,R$. Protection buyers prefer to exert the search effort if and only if*

$$u\left(E\left[\tilde{\theta}\right]\right) - B \geq \frac{1}{2} u\left(\bar{\theta} - (p-\delta)\,R\right) + \frac{1}{2} u\left(\underline{\theta} + (p-\delta)\,R\right). \tag{30}$$

Condition (30) is more likely to hold: 1) the smaller $B$; 2) the smaller $p-\delta$; 3) the more concave $u$. Since the right-hand side of (28) is increasing in $\gamma$, there exists $\gamma^*$ such that for all $\gamma > \gamma^*$, condition (28) does not hold. We have that

$$\gamma^* = \frac{1 + p - \delta - \frac{\Delta\theta}{R}}{2\,(1-p+\delta)} \tag{31}$$

If aggregate risk is limited, in the sense that $\gamma \leq \gamma^*$, exerting effort is not optimal, since full risk-sharing can be obtained without effort. But if aggregate risk is large, in the sense that $\gamma > \gamma^*$, then exerting effort is preferred to not exerting effort if the expected utility under effort (21) is higher than the expected utility in the contract without effort and with partial insurance. The latter is given by:

$$\frac{1}{2} u(\underline{\theta} + \bar{\tau}^{C,e=0}) \left[1 + \left(\frac{1}{2} - \gamma\right)\right] + \frac{1}{2}\left[\left(\frac{1}{2} + \gamma\right)(p-\delta)\, u(\underline{\theta} + \underline{\tau}^{e=0} - \underline{\pi}^{C,e=0}) + \right.$$
$$\left. \left(\frac{1}{2} + \gamma\right)(1-p+\delta)\, u(\underline{\theta} + \underline{\tau}^{C,e=0})\right].$$

Suppose $\gamma > \gamma^*$ and protection buyers choose not to exert effort ex ante. One might wonder why, if the low state is realized, protection buyers do not search for good protection sellers ex post? This is because, after the low state is realized, it's too late to share the macro-risk. Effort in our set-up enhances the risk-sharing capacity, but this can only be done ex ante, before the resolution of uncertainty.

# 7 Aggregate risk and moral hazard

We now turn to the case in which effort is unobservable. We first assume that a benevolent CCP can optimally design all the transfers and fees $\left(\tilde{\pi}, \tilde{\tau}, \tilde{\pi}^C, \tilde{\tau}^C\right)$ to maximize the expected utility of protection buyers. We then consider the case in which the CCP sets $\tilde{\pi}^C$ and $\tilde{\tau}^C$, while the protection buyers set $\tilde{\pi}$ and $\tilde{\tau}$.

## 7.1 When the benevolent CCP sets the transfers

Suppose that a benevolent CCP can optimally design all the state-contingent transfers and fees $\left(\tilde{\pi}, \tilde{\tau}, \tilde{\pi}^C, \tilde{\tau}^C\right)$ to maximize the expected utility of protection buyers. Consider first the contract with effort. Focus on the values of $\gamma$ such that it would be optimal to exert effort if it were observable (as spelled out in the previous section). If, for a given $\left(\tilde{\pi}, \tilde{\tau}, \tilde{\pi}^C, \tilde{\tau}^C\right)$, a protection buyer exerts effort, his payoff is given by

$$\begin{aligned}&\frac{1}{2}\left[\left(\frac{1}{2}+\gamma\right)u(\bar{\theta}-\bar{\pi}-\bar{\pi}^C)+\left(\frac{1}{2}-\gamma\right)pu(\underline{\theta}+\bar{\tau}-\bar{\pi}^C)+\right.\\&\left.\left(\frac{1}{2}-\gamma\right)(1-p)\,u(\underline{\theta}+\bar{\tau}^C)-B\right]+\frac{1}{2}\left[\left(\frac{1}{2}-\gamma\right)u(\bar{\theta}-\underline{\pi}-\underline{\pi}^C)+\right.\\&\left.\left(\frac{1}{2}+\gamma\right)pu(\underline{\theta}+\underline{\tau}-\underline{\pi}^C)+\left(\frac{1}{2}+\gamma\right)(1-p)\,u(\underline{\theta}+\underline{\tau}^C)-B\right]\end{aligned} \tag{32}$$

If a protection buyer does not exert effort, his payoff is given by

$$\begin{aligned}&\frac{1}{2}\left[\left(\frac{1}{2}+\gamma\right)u(\bar{\theta}-\bar{\pi}-\bar{\pi}^C)+\left(\frac{1}{2}-\gamma\right)(p-\delta)\,u(\underline{\theta}+\bar{\tau}-\bar{\pi}^C)+\right.\\&\left.\left(\frac{1}{2}-\gamma\right)(1-p+\delta)\,u(\underline{\theta}+\bar{\tau}^C)\right]+\frac{1}{2}\left[\left(\frac{1}{2}-\gamma\right)u(\bar{\theta}-\underline{\pi}-\underline{\pi}^C)+\right.\\&\left.\left(\frac{1}{2}+\gamma\right)(p-\delta)\,u(\underline{\theta}+\underline{\tau}-\underline{\pi}^C)+\left(\frac{1}{2}+\gamma\right)(1-p+\delta)\,u(\underline{\theta}+\underline{\tau}^C)\right]\end{aligned} \tag{33}$$

Note that the contract cannot be contingent on effort, since the latter is unobservable. Comparing (32) and (33), a protection buyer will prefer to exert unobservable effort if

$$\left(\frac{1}{2}-\gamma\right)\left[u(\underline{\theta}+\bar{\tau}-\bar{\pi}^C)-u(\underline{\theta}+\bar{\tau}^C)\right]+\left(\frac{1}{2}+\gamma\right)\left[u(\underline{\theta}+\underline{\tau}-\underline{\pi}^C)-u(\underline{\theta}+\underline{\tau}^C)\right]\geq\frac{2B}{\delta} \tag{34}$$

Suppose the CCP offers the same contract as that characterized in Proposition 10, so that a protection buyer obtains full insurance. Then, $\bar{\tau} - \bar{\pi}^C = \bar{\tau}^C$ and $\underline{\tau} - \underline{\pi}^C = \underline{\tau}^C$, and the protection buyer will prefer to save on the search cost $B$ and not exert effort. This will lead to high (and socially suboptimal) default rates. Hence, we can state the following lemma.

**Lemma 1** *When the CCP offers full insurance against counterparty risk, unobservable effort is not exerted. Hence, to induce effort, the incentive constraint (34) must bind and the CCP provides only partial insurance against counterparty default risk.*

We now solve for the second-best contract with effort. The classic tradeoff between incentives and insurance arises. Incentive-compatibility requires that the protection buyer remain exposed to some protection seller default risk.

The program of a benevolent CCP is to maximize (32) subject to the incentive constraint (34), the participation constraint of the protection seller (19) and the CCP fees (20). We can now state the following proposition.

**Proposition 13** *With aggregate risk and moral hazard, consumption is no longer equalized across states in the optimal contract inducing effort. Thus,*

$$\underline{\theta} + \bar{\tau} - \bar{\pi}^C = \underline{\theta} + \underline{\tau} - \underline{\pi}^C > \bar{\theta} - \bar{\pi} - \bar{\pi}^C = \bar{\theta} - \underline{\pi} - \underline{\pi}^C > \underline{\theta} + \bar{\tau}^C = \underline{\theta} + \underline{\tau}^C, \tag{35}$$

*and the optimal transfers and fees $\underline{\pi}, \underline{\tau}, \underline{\pi}^C$, $\bar{\pi}^C$ and $\underline{\tau}^C$ are given by:*

$$u(\underline{\theta} + \underline{\tau} - \underline{\pi}^C) - u(\underline{\theta} + \underline{\tau}^C) = \frac{2B}{\delta},$$

$$\underline{\pi} = p\underline{\tau} - \frac{2\gamma\,(1-p)\,\underline{\tau}^C}{\frac{1}{2}\,(1+p) - \gamma\,(1-p)},$$

$$\frac{u'(\underline{\theta} + \underline{\tau}^C)}{u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^C)} = \frac{(1-p)\,u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C)}{u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^C) - p u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C)},$$

$$\bar{\pi}^C = \frac{\left(\frac{1}{2} - \gamma\right)(1-p)}{\frac{1}{2}\,(1+p) + \gamma\,(1-p)}\underline{\tau}^C \text{ and } \underline{\pi}^C = \frac{\left(\frac{1}{2} + \gamma\right)(1-p)}{\frac{1}{2}\,(1+p) - \gamma\,(1-p)}\underline{\tau}^C.$$

In the first-best, the protection buyer gets full insurance when he exerts search effort so that $\underline{\theta} + \tilde{\tau} - \tilde{\pi}^C = \bar{\theta} - \tilde{\pi} - \tilde{\pi}^C = \underline{\theta} + \tilde{\tau}^C$. By (35), this is no longer the case with moral hazard. The intuition is as follows. As stated in Lemma 1, to ensure that the protection buyer exerts effort, he must be exposed to some counterparty risk so that there must be a wedge between $\underline{\theta}+\tilde{\tau}-\tilde{\pi}^C$ and $\underline{\theta}+\tilde{\tau}^C$. Such a wedge can be obtained either by raising $\underline{\theta}+\tilde{\tau}-\tilde{\pi}^C$ or by lowering $\underline{\theta}+\tilde{\tau}^C$. The latter is more costly since the consumption of the protection buyer is minimal when $\underline{\theta}$ is realized and the protection seller defaults. Hence, the former is preferred. Since raising $\tilde{\tau}$ requires an increase in $\tilde{\pi}$, we have $\underline{\theta}+\tilde{\tau}-\tilde{\pi}^C > \bar{\theta}-\tilde{\pi}-\tilde{\pi}^C$.

The optimal contract without effort is as characterized in Proposition 11. Protection buyers prefer the contract with effort if and only if their expected utility under effort is higher than their expected utility without effort. Since the expected utility in the optimal contract with effort is necessarily lower under moral hazard compared to the first-best, while the expected utility without effort is the same as in the first-best, we have the following proposition.

**Proposition 14** *The set of parameters for which the optimal contract mandates effort is smaller under moral hazard than when effort is observable.*

For completeness, we state a condition under which the benevolent CCP chooses to offer the contract inducing search effort for the special case when $\gamma=\frac{1}{2}$.

**Proposition 15** *With moral hazard, aggregate risk and no idiosyncratic risk, $\gamma=\frac{1}{2}$, the CCP offers the contract inducing search effort by the protection buyers if and only if*

$$\frac{1}{2}u(\bar{\theta}-\bar{\pi})+\frac{1}{2}u(\underline{\theta}+\underline{\tau}^C)+\frac{p-\delta}{\delta}B \geq \frac{1}{2}u\left(\bar{\theta}-(p-\delta)\,R\right)+\frac{1}{2}u\left(\underline{\theta}+(p-\delta)\,R\right). \tag{36}$$

## 7.2 When the parties set the terms of their contracts

Having analyzed the optimal contract designed by the CCP, we now consider the case in which the CCP sets $\tilde{\pi}^C$ and $\tilde{\tau}^C$, while the protection buyers set $\tilde{\pi}$ and $\tilde{\tau}$. That is, expecting state-contingent premia $\tilde{\pi}^e$ and transfers $\tilde{\tau}^e$, the benevolent CCP chooses fees $\tilde{\pi}^C$ and transfers $\tilde{\tau}^C$ to maximize the expected utility of the protection buyer subject to relevant constraints. Rational expectations require that the protection buyer does choose the premia and transfers expected by the CCP. We now argue that the rational expectations condition will be satisfied in the optimal contracts designed by the CCP.

Consider first the case when the benevolent CCP chooses to offer the contract without effort since it yields a higher expected utility than the contract with effort. Given $\tilde{\pi}^{C,e=0}$ and $\tilde{\tau}^{C,e=0}$, if the protection buyer decides not to do effort, he will choose the same $\tilde{\pi}^{e=0}$ and $\tilde{\tau}^{e=0}$ as the ones chosen by the CCP since this is the best contract without effort. Would the protection buyer choose to do effort, given $\tilde{\pi}^{C,e=0}$ and $\tilde{\tau}^{C,e=0}$? No, since even the best contract with effort is dominated by the contract without effort in this case. Hence, the rational expectations condition is satisfied.

Now consider the case when the benevolent CCP chooses to offer the contract inducing unobservable effort since it yields a higher expected utility than the contract without effort. By the same logic as in the previous case, a protection buyer cannot do better by deviating from $\tilde{\pi}$ and $\tilde{\tau}$ from the optimal contract with effort.

Thus, to implement the second-best, it is enough that the CCP sets optimally its fees and the amount of insurance against counterparty default. Once it has done that, the best response of the contracting parties will yield the optimal contract.

# 8 Conclusion

We analyze three ways in which counterparty risk can be mitigated.

- Trading parties can deposit resources in safe assets, and these resources can be used to make promised payments in case the counterparty defaults. This is comparable to self-insurance, whereby an agent saves to insure against future negative shocks.

- Trading parties can exert effort to find creditworthy counterparties, whose counterparty default risk is low. This is comparable to self-protection, whereby an agent exerts effort to reduce damage probabilities.

- Finally, trading parties can mutualize their risk.

We show that an appropriately designed centralized clearing mechanism enables the trading parties to benefit from the mutualization of (the idiosyncratic component of) risk, and therefore dominates no-clearing or decentralized clearing. But we also warn that such an arrangement has limitations:

- First, risk-mutualization is effective only to deal with idiosyncratic risk. It leaves the trading parties exposed to aggregate risk. Dealing with that risk can require that agents exert effort to search for solid, creditworthy counterparties.

- Second, risk-mutualization can weaken the incentives of the trading parties to exert such search effort. When effort is unobservable, i.e., when there is moral hazard, the CCP must be designed to maintain the incentives to search for solid counterparties. This requires that the agents keep some exposure to the risk that their counterparty will default.

We thus uncover a tradeoff between i) the ability of the system to withstand aggregate shocks (which requires that incentives be maintained), and ii) the extent to which risk can be mutualized in a CCP. To complement these insights and offer a more complete set of policy implications, it would interesting to explore the following avenues for further research:

**Governance:** The analysis above spells out the optimal design of the CCP, maximizing the expected utility of protection buyers, subject to the participation, incentive and feasibility constraints of the different parties. In practice, who would perform and implement this design, and thus set up a socially optimal CCP?

CCPs are, in a sense, utilities, providing services to financial institutions. Such utilities are often structured as cooperatives, or mutuals, whose members are both owners and users. Consider a mutual CCP, whose members would be the protection buyers. Its objective and constraints would be those analyzed above, and it would therefore implement the optimal CCP we characterized. In contrast, consider a for-profit, shareholder-owned CCP. Its objective, the maximization of profit, would not necessarily coincide with the maximization of social welfare. We have shown above that, in presence of moral hazard, the CCP should expose its members to some counterparty risk, to maintain their incentives. Suppose that the for-profit CCP did not do that and offered full insurance. Then, if the protection buyers believed the CCP would indeed deliver full insurance, they would not exert search effort, and they would also be willing to pay large fees. In the good macro-state, the CCP would use part of these fees to pay insurance, and the remaining part would be profits. In the bad macro-state, the rate of counterparty failures would be large and the CCP would go bankrupt. But, to the extent that a large population of protection buyers would operate within the CCP, this bankruptcy would be a systemic event. The government would have to step in and bail out the protection buyers, thus confirming their initial expectation that full insurance would actually be provided. Thus, CCPs managed as for-profit organizations may be prone to gambling and generate systemic risk. To avoid this outcome, they should be regulated. In particular their capital should be large enough to absorb counterparty defaults, so that government bail outs would not be needed.

**Risk control and management by the CCP:** In the analysis above, the CCP controls counterparty risk only indirectly, via the incentives of the protection buyers. But the CCP could also exert effort to directly gauge the creditworthiness of the protection sellers. This could mitigate the moral hazard problem analyzed above and improve risk-sharing.

**Competition:** Consider the case in which a benevolent CCP would seek to maintain the incentives of the protection buyers by exposing them to some idiosyncratic risk. Could the protection buyers undo this by acquiring the remaining insurance from another clearing entity? This suggests that exclusivity could be needed to implement the second-best. In that sense the CCP would be a natural monopoly. Of course, in that case, the CCP should be regulated, to avoid excessively high fees. Again, the mutual structure, which gives ownership and control rights to users, could prove useful in this context.

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# Appendix

**Proof of Proposition 1** Let $\mu$ denote the Lagrange multiplier on the protection seller's participation constraint. The first-order conditions yield:

$$u'\left(\bar{\theta} - \pi^{e=1}\right) = \mu = u'\left(\underline{\theta} + \tau^{e=1}\right).$$

Thus, the participation constraint binds, so that $\pi^{e=1} = p\tau^{e=1}$. Also, there is full risk-sharing as long as the protection seller does not default, and $\tau^{e=1} + \pi^{e=1} = \bar{\theta} - \underline{\theta} \equiv \Delta\theta$. But, the protection buyer remains exposed to counterparty risk. QED

**Proof of Proposition 2** Let $\mu$ denote the Lagrange multiplier on the protection seller's participation constraint. The first-order conditions yield:

$$u'\left(\bar{\theta} - \pi^{e=0}\right) = \mu = u'\left(\underline{\theta} + \tau^{e=0}\right).$$

Thus, the participation constraint binds, so that $\pi^{e=0} = (p - \delta)\,\tau^{e=0}$. Also, there is full risk-sharing as long as the protection seller does not default, and $\tau^{e=0} + \pi^{e=0} = \Delta\theta$. QED

**Proof of Proposition 3** Expected utility of the protection buyer under effort is higher than his expected utility under no effort if and only if

$$\begin{aligned}\frac{1}{2}u\left(\bar{\theta} - \pi^{e=1}\right) + \frac{1}{2}pu\left(\underline{\theta} + \tau^{e=1}\right) + \frac{1}{2}(1-p)\,u(\underline{\theta}) - B \geq \\ \frac{1}{2}u\left(\bar{\theta} - \pi^{e=0}\right) + \frac{1}{2}(p-\delta)\,u\left(\underline{\theta} + \tau^{e=0}\right) + \frac{1}{2}(1-p+\delta)\,u(\underline{\theta})\end{aligned}$$

or, equivalently,

$$(1+p)\,u\left(\underline{\theta} + \tau^{e=1}\right) - 2B \geq (1+p-\delta)\,u\left(\underline{\theta} + \tau^{e=0}\right) + \delta u(\underline{\theta})$$

since there is full risk-sharing as long as the protection seller does not default.

Substituting for $\tau^{e=1}$ and $\tau^{e=0}$ from Propositions (1) and (2), respectively, we get

$$(1+p)\,u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right) - 2B \geq (1+p-\delta)\,u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) + \delta u(\underline{\theta})$$

or, after collecting terms and re-arranging,

$$\frac{u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) - u(\underline{\theta})}{\frac{\Delta\theta}{1+p-\delta}} - \frac{u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right) - u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right)}{\frac{\Delta\theta}{1+p-\delta}\frac{\delta}{1+p}} \geq \frac{2B}{\frac{\delta\Delta\theta}{1+p-\delta}}$$

Note that the first terms is the slope of a line between $u(\underline{\theta})$ and $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right)$, while the second term is the slope of a line between $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p}\right)$ and $u\left(\underline{\theta} + \frac{\Delta\theta}{1+p-\delta}\right)$. QED

**Proof of Proposition 4** Let $\mu$ and $\mu^C$ denote the Lagrange multipliers on the protection seller's and clearing agent's participation constraints, respectively. Let $\mu_0$ and $\mu_1$ denote the Lagrange multipliers on the feasibility constraints $0 \leq \alpha \leq 1$ and let $\mu_2$ be the Lagrange multiplier on the feasibility constraint $\alpha c \geq \tau^C$. The first-order conditions with respect to $\pi$, $\tau$, $\pi^C$, $\tau^C$ and $\alpha$ (where we omit indexing by $e = 1$ everywhere) yield

$$u'(\bar{\theta} - \pi - \pi^C) = \mu \tag{37}$$

$$u'(\underline{\theta} + \tau - \pi^C) = \mu \tag{38}$$

$$u\acute{}(\bar{\theta} - \pi - \pi^C) = \mu^C \tag{39}$$

$$(1-p)\, u'(\underline{\theta} + \tau^C) = \mu^C (1-p) + 2\mu_2 \tag{40}$$

$$\mu^C (\rho - 1)\, c = \mu_0 - \mu_1 + \mu_2 c \tag{41}$$

By (37) and (38), the participation constraint of the protection seller binds, $\pi = p\tau$, and there is full risk-sharing as long as the protection seller does not default, with $\tau = \frac{\Delta\theta}{1+p}$.

By (39), $\mu^C > 0$ and the participation constraint of the clearing agent binds. By (37) and (39), we have that $\mu = \mu^C$. Substituting into (40), we get

$$u'(\underline{\theta} + \tau^C) = \mu + \frac{2\mu_2}{1-p}$$

which, combined with (38), yields

$$\underline{\theta} + \tau^C < \underline{\theta} + \tau - \pi^C$$

Hence, the clearing agent only provides a partial insurance against counterparty default.

Moreover, $\alpha > 0$ must hold since otherwise the clearing agent cannot provide any insurance against counterparty risk (as $\tau^C = 0$ in this case). Hence, $\mu_0 = 0$. It follows from (41) that $\mu_2 > 0$ must hold and $\alpha = \frac{\tau^C}{c}$. Since the participation constraint of the clearing agent binds, we have that $\pi^C = \tau^C \frac{1-p+2(\rho-1)}{1+p}$. QED

**Proof of Proposition 5** Combining (40) and (41), we get:

$$\frac{u'(\underline{\theta} + \tau^C)}{u'(\underline{\theta} + \tau - \pi^C)} = \frac{2(\rho-1) + (1-p)}{(1-p)} + \frac{2\mu_1}{(1-p)\, cu'(\underline{\theta} + \tau - \pi^C)}$$

Substituting for $\tau$ and $\pi^C$, we get

$$\frac{u'(\underline{\theta} + \tau^C)}{u'\left(\underline{\theta} + \frac{\Delta\theta}{1+p} - \tau^C \frac{1-p+2(\rho-1)}{1+p}\right)} = 1 + \frac{2(\rho-1)}{1-p} + \frac{2\mu_1}{(1-p)\, cu'(\underline{\theta} + \tau - \pi^C)} \tag{42}$$

where $\frac{2(\rho-1)}{1-p} > 0$ since $\rho > 1$.

Let

$$\varphi\left(\tau^C\right) \equiv \frac{u'(\underline{\theta} + \tau^C)}{u'\left(\underline{\theta} + \frac{\Delta\theta}{1+p} - \tau^C \frac{1-p+2(\rho-1)}{1+p}\right)} \tag{43}$$

Note that $\varphi\left(\tau^C\right)$ is decreasing in $\tau^C$ (since the numerator of $\varphi\left(\tau^C\right)$ is decreasing in $\tau^C$, while the denominator of $\varphi\left(\tau^C\right)$ is increasing in $\tau^C$). If $\varphi\left(\tau^C\right) > 1 + \frac{2(\rho-1)}{1-p}$ at $\tau^C = c$ (which is the maximum $\tau^C$ as $\alpha = 1$), then $\varphi\left(\tau^C\right) > 1 + \frac{2(\rho-1)}{1-p}$ for all smaller $\tau^C$ and hence $\mu_1 > 0$ and $\alpha = 1$. Then, $\tau^C = c$. If $\varphi(0) \leq 1 + \frac{2(\rho-1)}{1-p}$, then (42) cannot hold for any $\tau^C > 0$ and the clearing agent is not used. Otherwise, $\tau^C$ is given by $\varphi\left(\tau^C\right) = 1 + \frac{2(\rho-1)}{1-p}$. QED

**Proof of Proposition 6** The proof proceeds as in Proposition 4. The first-order conditions with respect to $\pi$, $\tau$, $\pi^C$, $\tau^C$ and $\alpha$ (where we omit indexing by $e = 0$ everywhere) yield

$$\begin{aligned}
u'(\bar{\theta} - \pi - \pi^C) &= \mu && (44)\\
u'(\underline{\theta} + \tau - \pi^C) &= \mu && (45)\\
u'(\bar{\theta} - \pi - \pi^C) &= \mu^C && (46)\\
(1-p+\delta)\, u'(\underline{\theta} + \tau^C) &= \mu^C (1-p+\delta) + 2\mu_2 && (47)\\
\mu^C (\rho - 1)\, c &= \mu_0 - \mu_1 + \mu_2 c && (48)
\end{aligned}$$

By (44) and (45), the participation constraint of the protection seller binds, $\pi = (p-\delta)\,\tau$, and there is full risk-sharing as long as the protection seller does not default, with $\tau = \frac{\Delta\theta}{1+p-\delta}$.

By (46), $\mu^C > 0$ and the participation constraint of the clearing agent binds. By (44) and (46), we have that $\mu = \mu^C$. Substituting into (47), we get

$$u'(\underline{\theta} + \tau^C) = \mu + \frac{2\mu_2}{1-p+\delta}$$

which, combined with (45), yields

$$\underline{\theta} + \tau^C < \underline{\theta} + \tau - \pi^C$$

Hence, the clearing agent only provides a partial insurance against counterparty default.

Moreover, $\alpha > 0$ must hold since otherwise the clearing agent cannot provide any insurance against counterparty risk (as $\tau^C = 0$ in this case). Hence, $\mu_0 = 0$. It follows from (41) that $\mu_2 > 0$ must hold and $\alpha = \frac{\tau^C}{c}$. Since the participation constraint of the clearing agent binds, we have that $\pi^C = \tau^C \frac{1-p+\delta+2(\rho-1)}{1+p-\delta}$. QED

**Proof of Proposition 7** Combining (47) and (48), and substituting for $\tau$ and $\pi^C$, we get:

$$\frac{u'(\underline{\theta}+\tau^C)}{u'\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}-\tau^C\frac{1-p+\delta+2(\rho-1)}{1+p-\delta}\right)}=1+\frac{2(\rho-1)}{1-p+\delta}+\frac{2\mu_1}{(1-p+\delta)\,cu'(\underline{\theta}+\tau-\pi^C)} \tag{49}$$

Let

$$\varphi^{e=0}\left(\tau^C\right)\equiv\frac{u'(\underline{\theta}+\tau^C)}{u'\left(\underline{\theta}+\frac{\Delta\theta}{1+p-\delta}-\tau^C\frac{1-p+\delta+2(\rho-1)}{1+p-\delta}\right)} \tag{50}$$

Note that $\varphi^{e=0}\left(\tau^C\right)$ is decreasing in $\tau$. If $\varphi^{e=0}\left(\tau^C\right)>1+\frac{2(\rho-1)}{1-p+\delta}$ at $\tau^C=c$ (which is the maximum $\tau^C$ as $\alpha=1$), then $\varphi^{e=0}\left(\tau^C\right)>1+\frac{2(\rho-1)}{1-p+\delta}$ for all smaller $\tau^C$ and hence $\mu_1>0$ and $\alpha=1$. Then, $\tau^C=c$. If $\varphi^{e=0}(0)\le 1+\frac{2(\rho-1)}{1-p+\delta}$, then (42) cannot hold for any $\tau^C>0$ and the clearing agent is not used. Otherwise, $\tau^C$ is given by $\varphi^{e=0}\left(\tau^C\right)=1+\frac{2(\rho-1)}{1-p+\delta}$.

Finally, note that the right-hand side of (49) is lower than the right-hand side of (42). Also, $\frac{1-p+\delta+2(\rho-1)}{1+p-\delta}>\frac{1-p+2(\rho-1)}{1+p}$. It follows that $\tau^C$ satisfying (49) is at least as high as $\tau^C$ satisfying (42), and strictly higher for $\tau^C<c$. QED

**Proof of Proposition 9** Consider first the optimal contract with effort. Substituting $\pi^{C,e=1}$ in the objective function (8), the first-order condition with respect to $\tau^{C,e=1}$ is

$$\frac{1}{2}u'\left(\bar{\theta}-\pi^{e=1}-\pi^{C,e=1}\right)+\frac{1}{2}pu'\left(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1}\right)=\frac{1}{2}(1+p)\,u'\left(\underline{\theta}+\tau^{C,e=1}\right) \tag{51}$$

As in the previous subsections, there is full risk-sharing as long as the protection seller does not default. Hence,

$$u'\left(\bar{\theta}-\pi^{e=1}-\pi^{C,e=1}\right)=u'\left(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1}\right).$$

Equation (51) simplifies to

$$u'\left(\underline{\theta}+\tau^{e=1}-\pi^{C,e=1}\right)=u'\left(\underline{\theta}+\tau^{C,e=1}\right)$$

implying $\tau^{e=1}-\pi^{C,e=1}=\tau^{C,e=1}$. Using full risk-sharing across states and the binding participation constraint of the protection seller, we get $\tau^{e=1}=\frac{\Delta\theta}{1+p}$ and $\tau^{C,e=1}=\frac{\Delta\theta}{2}$. The expected utility under effort, $EU^{e=1}$, is thus

$$EU^{e=1}=u\left(\underline{\theta}+\frac{\Delta\theta}{2}\right)-B=u\left(E\left[\tilde{\theta}\right]\right)-B. \tag{52}$$

Similarly, in the optimal contract without effort, the first-order condition with respect to $\tau^{C,e=0}$ is

$$\frac{1}{2}u'\left(\bar{\theta}-\pi^{e=0}-\pi^{C,e=0}\right)+\frac{1}{2}(p-\delta)\,u'\left(\underline{\theta}+\tau^{e=0}-\pi^{C,e=0}\right)=\frac{1}{2}(1+p-\delta)\,u'\left(\underline{\theta}+\tau^{C,e=0}\right)$$

which yields, using full risk-sharing as long as the protection seller does not default, $\tau^{e=0} - \pi^{C,e=0} = \tau^{C,e=0}$. Using the binding participation constraint of the protection seller, we get $\tau^{e=0} = \frac{\Delta\theta}{1+p-\delta}$ and $\tau^{C,e=0} = \frac{\Delta\theta}{2}$. The expected utility without effort, $EU^{e=0}$, is thus

$$EU^{e=0} = u\left(\underline{\theta} + \frac{\Delta\theta}{2}\right) = u\left(E\left[\tilde{\theta}\right]\right). \tag{53}$$

Comparing expected utility of the protection buyer under effort and without effort, we have

$$EU^{e=1} = u\left(E\left[\tilde{\theta}\right]\right) - B < u\left(E\left[\tilde{\theta}\right]\right) = EU^{e=0}.$$

QED

**Proof of Proposition 10** Let $\mu$ denote the Lagrange multiplier on the protection seller's participation constraint. First-order conditions with respect to $\bar{\pi}^{e=1}$, $\bar{\tau}^{e=1}$, $\bar{\tau}^{C,e=1}$, $\underline{\pi}^{e=1}$, $\underline{\tau}^{e=1}$, and $\underline{\tau}^{C,e=1}$ yield

$$u'(\bar{\theta} - \bar{\pi}^{e=1} - \bar{\pi}^{C,e=1}) = \mu \tag{54}$$

$$u'(\underline{\theta} + \bar{\tau}^{e=1} - \bar{\pi}^{C,e=1}) = \mu \tag{55}$$

$$\frac{\left(\frac{1}{2}+\gamma\right) u'(\bar{\theta} - \bar{\pi}^{e=1} - \bar{\pi}^{C,e=1})}{\frac{1}{2}(1+p) + \gamma(1-p)} + \frac{\left(\frac{1}{2}-\gamma\right) pu'(\underline{\theta} + \bar{\tau}^{e=1} - \bar{\pi}^{C,e=1})}{\frac{1}{2}(1+p) + \gamma(1-p)} = u'(\underline{\theta} + \bar{\tau}^{C,e=1}) \tag{56}$$

$$u'(\bar{\theta} - \underline{\pi}^{e=1} - \underline{\pi}^{C,e=1}) = \mu \tag{57}$$

$$u'(\underline{\theta} + \underline{\tau}^{e=1} - \underline{\pi}^{C,e=1}) = \mu \tag{58}$$

$$\frac{\left(\frac{1}{2}-\gamma\right) u'(\bar{\theta} - \underline{\pi}^{e=1} - \underline{\pi}^{C,e=1})}{\frac{1}{2}(1+p) - \gamma(1-p)} + \frac{\left(\frac{1}{2}+\gamma\right) pu'(\underline{\theta} + \underline{\tau}^{e=1} - \underline{\pi}^{C,e=1})}{\frac{1}{2}(1+p) - \gamma(1-p)} = u'(\underline{\theta} + \underline{\tau}^{C,e=1}) \tag{59}$$

By (54), the participation constraint of the protection seller binds. Using (54) and (55) in (56), and simplifying yields:

$$u'(\underline{\theta} + \bar{\tau}^{C,e=1}) = \mu$$

Similarly, using (57) and (58) in (59), and simplifying yields

$$u'(\underline{\theta} + \underline{\tau}^{C,e=1}) = \mu$$

Using full risk-sharing across states, the binding participation constraint of the protection seller and (20), we get

$$\bar{\tau}^{C,e=1} = \underline{\tau}^{C,e=1} = \frac{\Delta\theta}{2}$$

Furthermore,

$$\bar{\tau}^{e=1} = \frac{1}{\frac{1}{2}(1+p) + \gamma(1-p)} \bar{\tau}^{C,e=1}, \; \underline{\tau}^{e=1} = \frac{1}{\frac{1}{2}(1+p) - \gamma(1-p)} \underline{\tau}^{C,e=1},$$
$$\bar{\pi}^{e=1} = \Delta\theta - \bar{\tau}^{e=1}, \; \underline{\pi}^{e=1} = \Delta\theta - \underline{\tau}^{e=1}$$

and $\bar{\pi}^{C,e=1}$, $\underline{\pi}^{C,e=1}$ are given by (20).

QED

**Proof of Proposition 11** Let $\mu$ denote the Lagrange multiplier on the protection seller's participation constraint. Let $\psi$ denote the Lagrange multiplier on the feasibility constraint $R \geq \underline{\tau}^{e=0}$. First-order conditions with respect to $\bar{\pi}^{e=0}$, $\bar{\tau}^{e=0}$, $\bar{\tau}^{C,e=0}$, $\underline{\pi}^{e=0}$, $\underline{\tau}^{e=0}$, and $\underline{\tau}^{C,e=0}$ yield

$$u'(\bar{\theta} - \bar{\pi}^{e=0} - \bar{\pi}^{C,e=0}) = \mu \tag{60}$$

$$u'(\underline{\theta} + \bar{\tau}^{e=0} - \bar{\pi}^{C,e=0}) = \mu \tag{61}$$

$$\frac{\left(\frac{1}{2}+\gamma\right) u'(\bar{\theta} - \bar{\pi}^{e=0} - \bar{\pi}^{C,e=0})}{\frac{1}{2}(1+p-\delta) + \gamma(1-p+\delta)} + \frac{\left(\frac{1}{2}-\gamma\right)(p-\delta)\, u'(\underline{\theta} + \bar{\tau}^{e=0} - \bar{\pi}^{C,e=0})}{\frac{1}{2}(1+p-\delta) + \gamma(1-p+\delta)} = u'(\underline{\theta} + \bar{\tau}^{C,e=0}) \tag{62}$$

$$u'(\bar{\theta} - \underline{\pi}^{e=0} - \underline{\pi}^{C,e=0}) = \mu \tag{63}$$

$$u'(\underline{\theta} + \underline{\tau}^{e=0} - \underline{\pi}^{C,e=0}) = \mu + \frac{\psi}{\left(\frac{1}{2}+\gamma\right)(p-\delta)} \tag{64}$$

$$\frac{\left(\frac{1}{2}-\gamma\right) u'(\bar{\theta} - \underline{\pi}^{e=0} - \underline{\pi}^{C,e=0})}{\frac{1}{2}(1+p-\delta) - \gamma(1-p+\delta)} + \frac{\left(\frac{1}{2}+\gamma\right)(p-\delta)\, u'(\underline{\theta} + \underline{\tau}^{e=0} - \underline{\pi}^{C,e=0})}{\frac{1}{2}(1+p-\delta) - \gamma(1-p+\delta)} = u'(\underline{\theta} + \underline{\tau}^{C,e=0}) \tag{65}$$

By (60), the participation constraint of the protection seller binds. Using (60) and (61) in (62), and simplifying yields:

$$u'(\underline{\theta} + \bar{\tau}^{C,e=0}) = \mu$$

Similarly, using (63) and (64) in (65), and simplifying yields

$$u'(\underline{\theta} + \underline{\tau}^{C,e=0}) = \mu + \frac{\psi}{\frac{1}{2}(1+p-\delta) - \gamma(1-p+\delta)} \tag{66}$$

There are two cases: 1) $\psi = 0$ and 2) $\psi > 0$. In the first case, $R \geq \underline{\tau}^{e=0}$ and, since $\psi = 0$, we have (using full risk-sharing across states, the binding participation constraint of the protection seller and (27):

$$\bar{\tau}^{C,e=0} = \underline{\tau}^{C,e=0} = \frac{\Delta\theta}{2}$$

Furthermore,

$$\bar{\tau}^{e=0}=\frac{1}{\frac{1}{2}\left(1+p-\delta\right)+\gamma\left(1-p+\delta\right)}\bar{\tau}^{C,e=0},\ \underline{\tau}^{e=0}=\frac{1}{\frac{1}{2}\left(1+p-\delta\right)-\gamma\left(1-p+\delta\right)}\underline{\tau}^{C,e=0},$$
$$\bar{\pi}^{e=0}=\Delta\theta-\bar{\tau}^{e=0},\ \underline{\pi}^{e=0}=\Delta\theta-\underline{\tau}^{e=0}$$

and $\bar{\pi}^{C,e=0}$, $\underline{\pi}^{C,e=0}$ are given by (27).

If

$$R<\frac{1}{\frac{1}{2}\left(1+p-\delta\right)-\gamma\left(1-p+\delta\right)}\frac{\Delta\theta}{2}$$

we are in the second case, with $\underline{\tau}^{e=0}=R$. Since $\psi>0$ and

$$\frac{1}{\frac{1}{2}\left(1+p-\delta\right)-\gamma\left(1-p+\delta\right)}<\frac{1}{\left(\frac{1}{2}+\gamma\right)\left(p-\delta\right)},$$

we have that:

$$\begin{aligned}\underline{\theta}+\underline{\tau}^{C,e=0} &\leq \underline{\theta}+\underline{\tau}^{e=0}-\underline{\pi}^{C,e=0}<\bar{\theta}-\underline{\pi}^{e=0}-\underline{\pi}^{C,e=0}\\ \underline{\theta}+\underline{\tau}^{C,e=0} &\leq \underline{\theta}+\underline{\tau}^{e=0}-\underline{\pi}^{C,e=0}<\underline{\theta}+\bar{\tau}^{C,e=0}\end{aligned}$$

holds.

QED

**Proof of Proposition 12** For the special case of $\gamma=\frac{1}{2}$, the first-order conditions with respect to $\bar{\pi}^{e=0}$, $\underline{\tau}^{e=0}$, and $\underline{\tau}^{C,e=0}$ are given by (54), (58), and (59). (The other three variables drop out of the objective function and constraints.) Substituting $\gamma=\frac{1}{2}$, we get,

$$u'(\underline{\theta}+\underline{\tau}^{e=0}-\underline{\pi}^{C,e=0})=\mu+\frac{\psi}{p-\delta}=u'(\underline{\theta}+\underline{\tau}^{C,e=0})$$

so that $\underline{\tau}^{e=0}-\underline{\pi}^{C,e=0}=\underline{\tau}^{C,e=0}$ and, using $\underline{\tau}^{e=0}=R$ and $\underline{\pi}^{C,e=0}$ from (27),

$$\underline{\tau}^{C,e=0}=\left(p-\delta\right)R$$

Using the binding participation constraint of the protection seller, we have

$$\bar{\pi}^{e=0}=\left(p-\delta\right)R.$$

Hence, for $\gamma=\frac{1}{2}$, the expected utility of the protection buyer without effort is given by:

$$\frac{1}{2}\left[u(\bar{\theta}-\bar{\pi}^{e=0})+u(\underline{\theta}+\underline{\tau}^{C,e=0})\right]=\frac{1}{2}\left[u\left(\bar{\theta}-\left(p-\delta\right)R\right)+u\left(\underline{\theta}+\left(p-\delta\right)R\right)\right]$$

The protection buyer prefers to exert search effort if and only if

$$u\left(E\left[\tilde{\theta}\right]\right) - B \geq \frac{1}{2}u\left(\bar{\theta} - (p-\delta)\,R\right) + \frac{1}{2}u\left(\underline{\theta} + (p-\delta)\,R\right)$$

By concavity of $u$, $u\left(E\left[\tilde{\theta}\right]\right) > \frac{1}{2}u\left(\bar{\theta} - (p-\delta)\,R\right) + \frac{1}{2}u\left(\underline{\theta} + (p-\delta)\,R\right)$. The difference between the two increases as 1) $p-\delta$ decreases and 2) $u$ is more concave. In sum, for a sufficiently small $B$, effort is preferred to no effort.

QED

**Proof of Proposition 13** Let $\mu$ and $\nu$ denote the Lagrange multipliers on the protection seller's participation constraint and on the protection buyer's incentive constraint, respectively. The first-order conditions with respect to $\bar{\pi}^{e=1}$, $\bar{\tau}^{e=1}$, $\bar{\tau}^{C,e=1}$, $\underline{\pi}^{e=1}$, $\underline{\tau}^{e=1}$, and $\underline{\tau}^{C,e=1}$ are:

$$\begin{aligned}
u'(\bar{\theta} - \bar{\pi} - \bar{\pi}^{C}) &= \mu & (67)\\
p\,(1+\nu)\,u'(\underline{\theta} + \bar{\tau} - \bar{\pi}^{C}) &= p\mu & (68)\\
\frac{(1-p)\left[\left(\frac{1}{2}+\gamma\right)u'(\bar{\theta} - \bar{\pi} - \bar{\pi}^{C}) + p\,(1+v)\left(\frac{1}{2}-\gamma\right)u'(\underline{\theta} + \bar{\tau} - \bar{\pi}^{C})\right]}{\left[\frac{1}{2}\,(1+p) + \gamma\,(1-p)\right]\left[1-p\,(1+\nu)\right]} &= u'(\underline{\theta} + \bar{\tau}^{C}) & (69)\\
u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^{C}) &= \mu & (70)\\
p\,(1+\nu)\,u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^{C}) &= p\mu & (71)\\
\frac{(1-p)\left[\left(\frac{1}{2}-\gamma\right)u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^{C}) + p\,(1+\nu)\left(\frac{1}{2}+\gamma\right)u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^{C})\right]}{\left[\frac{1}{2}\,(1+p) - \gamma\,(1-p)\right]\left[1-p\,(1+\nu)\right]} &= u'(\underline{\theta} + \underline{\tau}^{C}) & (72)
\end{aligned}$$

By (67), $\mu > 0$ and hence the participation constraint of the protection seller binds. Using (67) and (68) in (69), we get:

$$u'(\underline{\theta} + \bar{\tau}^{C}) = \frac{(1-p)\,\mu}{1-p\,(1+\nu)} \tag{73}$$

Similarly, using (70) and (71) in (72), we get:

$$u'(\underline{\theta} + \underline{\tau}^{C}) = \frac{(1-p)\,\mu}{1-p\,(1+\nu)} \tag{74}$$

It follows that $\bar{\tau}^{C} = \underline{\tau}^{C}$. By (67) and (70), $\bar{\pi}+\bar{\pi}^{C} = \underline{\pi}+\underline{\pi}^{C}$. By (68) and (71), $\bar{\tau}-\bar{\pi}^{C} = \underline{\tau}-\underline{\pi}^{C}$. Using these equalities and (20) to substitute in the binding participation constraint, we get

$$\underline{\pi} = p\underline{\tau} - \frac{2\gamma\,(1-p)\,\underline{\tau}^{C}}{\frac{1}{2}\,(1+p) - \gamma\,(1-p)}$$

which yields $\underline{\pi}$ as a function of $\underline{\tau}$ and $\underline{\tau}^C$.

Furthermore, it must be that $\nu > 0$ in the optimum so that the incentive constraint binds. Suppose not, i.e., $\nu = 0$. Then, marginal utilities are equalized across all states, implying that $\bar{\tau} - \bar{\pi}^C = \bar{\tau}^C$ and $\underline{\tau} - \underline{\pi}^C = \underline{\tau}^C$. But then, the incentive constraint cannot hold as $B > 0$. A contradiction. Since the incentive constraint binds, we have, using $\bar{\tau} - \bar{\pi}^C = \underline{\tau} - \underline{\pi}^C$ and $\bar{\tau}^C = \underline{\tau}^C$,

$$u(\underline{\theta} + \underline{\tau} - \underline{\pi}^C) - u(\underline{\theta} + \underline{\tau}^C) = \frac{2B}{\delta}$$

This equation yields $\underline{\tau}$ as a function of $\underline{\tau}^C$.

Next, note that by (70) and (71),

$$\frac{u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C)}{u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^C)} = 1 + \nu > 1$$

since $\nu > 0$. It follows that $\underline{\theta} + \underline{\tau} - \underline{\pi}^C > \bar{\theta} - \underline{\pi} - \underline{\pi}^C$. Furthermore, by (74), we have

$$u'(\underline{\theta} + \underline{\tau}^C) = \frac{(1-p)}{1 - p\,(1+\nu)} u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C)$$

Since $1 + \nu > 1$, we have that $\bar{\theta} - \underline{\pi} - \underline{\pi}^C > \underline{\theta} + \underline{\tau}^C$. The optimal transfer $\underline{\tau}^C$ is given by:

$$u'(\underline{\theta} + \underline{\tau}^C) = \frac{(1-p)\, u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C) u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^C)}{u'(\underline{\theta} + \underline{\tau} - \underline{\pi}^C) - p u'(\bar{\theta} - \underline{\pi} - \underline{\pi}^C)}$$

In sum, protection buyer's utilities are no longer equalized, even when the protection seller does not default, and

$$\underline{\theta} + \underline{\tau} - \underline{\pi}^C = \underline{\theta} + \bar{\tau} - \bar{\pi}^C > \bar{\theta} - \underline{\pi} - \underline{\pi}^C = \bar{\theta} - \bar{\pi} - \bar{\pi}^C > \underline{\theta} + \underline{\tau}^C = \underline{\theta} + \bar{\tau}^C$$

holds in the optimum. QED

**Proof of Proposition 15** For $\gamma = \frac{1}{2}$, the expected utility under no effort is given by

$$\frac{1}{2} u\left(\bar{\theta} - (p - \delta)\, R\right) + \frac{1}{2} u\left(\underline{\theta} + (p - \delta)\, R\right)$$

Since the incentive-compatibility constraint binds in the optimal contract with effort, i.e.,

$$u(\underline{\theta} + \underline{\tau} - \underline{\pi}^C) - u(\underline{\theta} + \underline{\tau}^C) = \frac{2B}{\delta},$$

we have that the expected utility under effort is

$$\begin{aligned}
&\frac{1}{2}u(\bar{\theta}-\bar{\pi})+\frac{1}{2}\left[pu(\underline{\theta}+\underline{\tau}-\underline{\pi}^{C})+(1-p)\,u(\underline{\theta}+\underline{\tau}^{C})\right]-B \\
&=\frac{1}{2}u(\bar{\theta}-\bar{\pi})+\frac{1}{2}u(\underline{\theta}+\underline{\tau}^{C})+\frac{1}{2}p\left[u(\underline{\theta}+\underline{\tau}-\underline{\pi}^{C})-u(\underline{\theta}+\underline{\tau}^{C})\right]-B \\
&=\frac{1}{2}u(\bar{\theta}-\bar{\pi})+\frac{1}{2}u(\underline{\theta}+\underline{\tau}^{C})+\frac{p-\delta}{\delta}B
\end{aligned}$$

where

$$\bar{\pi}=p\underline{\tau}$$

$$\frac{u'(\underline{\theta}+\underline{\tau}^{C})}{u'\left(\underline{\theta}+\underline{\tau}-\frac{1-p}{p}\underline{\tau}^{C}\right)}=\frac{(1-p)\,u'(\bar{\theta}-\bar{\pi})}{u'\left(\underline{\theta}+\underline{\tau}-\frac{1-p}{p}\underline{\tau}^{C}\right)-pu'(\bar{\theta}-\bar{\pi})}$$

Hence, for $\gamma=\frac{1}{2}$, effort is preferred to no effort under moral hazard if and only if

$$\frac{1}{2}u(\bar{\theta}-\bar{\pi})+\frac{1}{2}u(\underline{\theta}+\underline{\tau}^{C})+\frac{p-\delta}{\delta}B\geq\frac{1}{2}u\left(\bar{\theta}-(p-\delta)\,R\right)+\frac{1}{2}u\left(\underline{\theta}+(p-\delta)\,R\right)$$

QED