LONG-RUN TRENDS IN LONG-MATURITY REAL RATES

1311-2021 *

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Abstract

Taking advantage of key recent advances in long-run financial and economic data, this paper analyzes the statistical properties of global long-maturity real interest rates over the past seven centuries. In contrast to existing consensus, which has overwhelmingly concentrated on short samples for short-maturity rates, we find that long-maturity real interest rates across advanced economies are in fact trend stationary, and exhibit a persistent downward trend since the Renaissance. We investigate structural breaks in real interest rates over time using multiple statistical approaches, and find that only the Black Death and the "Trinity default" of 1557 appear as consistent inflection points in capital markets on both global and country levels. While a 1914 break is also suggested in multiple series (though less robust than existing literature would lead one to expect), the evidence for an inflection point in 1981 appears much weaker. We further examine trends in persistence, as well as commonly-invoked drivers of global real rates: exploiting significant data advances, we argue that historically, demographic and productivity factors appear to show no promising causal role, and in fact diverge from real interest rates over the long run.

JEL Codes: E4, F3, N20.

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1 Introduction

More than a decade after the Global Financial Crisis (GFC), and following a sharp rebound in economic activity from a worldwide pandemic shock, global real interest rates remain at historically depressed levels. Although the fall in real interest rates has been ongoing since the peaks of the 1980s disinflation, the drop was particularly acute after the GFC, leading to intense academic and policy discussion over what factors might best explain it. Despite disagreements over specific drivers, many researchers have posited that current real interest rate dynamics can be traced to a key inflection point during the 1970s or 1980s, when the transition from fixed exchange rates coincided with a deregulation spurt in international finance and an acceleration of globalization, culminating in the "great moderation" phase (Rachel and Summers, 2019). Other contributions emphasize structural global factors, including financial flows Bernanke (2005), demographics (Goodhart and Pradhan, 2021), trend growth (Holston et al., 2017), or monetary policy (Garcia and Perron, 1996; Bernanke, 2004). Summaries are provided by Blanchard (2022) and Lunsford and West (2019), with the latter finding the strongest positive association between demographic factors and the safe rate between 1890-2016. Overall, however, the literature has generally taken as given that most of the post-GFC plummet in real rates reflects latent secular forces, and is likely to be permanent.

This paper will take advantage of recent advances in long-run financial and economic data to approach the debate from a fundamentally different angle - investigating real interest rate trends from a multi-century perspective. Beyond contextualizing trends and drivers for the recent decades, this paper is motivated by a literature that sought to characterize the fundamental statistical properties of (real) interest rates, and their relation to other macroeconomic variables. For both areas of interest, (very) long-run datasets can potentially offer far superior results compared to an existing literature that has typically focused on relatively short sample periods, typically 75 to 150 years at most. First, it is clear that a multi-century overview encompasses a much richer tail event, monetary, and fiscal variation over time than an exclusive post-War, or post-Bretton Woods assessment - and this longer perspective is the obvious desideratum to assess "secular" forces in the global economy, and the structural impact of generational "macro shocks" like the GFC or COVID. However, such investigations were until recently impossible given a lack of adequate data.

Second, there are key econometric reasons that recommend a very long-run examination of the statistical properties of real rates: most importantly, it is known that the power of unit root and other statistical tests improves significantly over longer sample periods; the advantages remain, even relative to increasing the number of observations in a given (shorter) sample using hier frequency data (Perron, 1989). Frankel (1986) in his exposition on PPP demonstrated that the rejection of the random walk for U.S.-U.K. exchange rate time series crucially depended on the time horizon: the statistical power significantly improved once horizons of over 100 years were chosen, finally allowing for a rejection of the unit root. Therefore, even though recent contributions have

¹For a practical example, see Lothian and Taylor (1996). The authors there show that the existence of a unit root for real exchange rates cannot be rejected over the sub-periods of 1946-1990, and 1974-1990 - but this result disappears

expanded the time span of statistical real interest rate assessments into the 19th century (Hamilton et al., 2016), ideally longer datasets should be used.

Third, existing analyses of real interest rate dynamics overwhelmingly focus on short-maturity interest rates, with very few exceptions. In fact, all key reference contributions have reached generalizations on interest rate properties on the basis of central bank policy or bill rates. Much, however, suggests that using long-maturity interest rates, which reflect fundamental growth and inflation expectations, is more appropriate. In any event, different ends of the term structure need not necessarily exhibit identical structural statistical features. Not least, there are crucial data reasons that recommend the use of long-maturity data: long-maturity financial instruments - perpetual consols, for instance - have been in existence far longer than either short-term consolidated debt or rate-setting policy institutions, with the issuance of consolidated sovereign long-maturity debt dating back at least to the first Venetian issuance of negotiable Monte Vecchio bonds in 1262 (Pezzolo, 2003; Stasavage, 2011; Eichengreen et al., 2021). In addition, long-maturity data properly reflect structural trends and expectations by market participants: long-maturity rates incorporate structural growth, rare disaster, inflation, demographic, and capital flow expectations as opposed to short-maturity rates, which are much more likely to be subject to noise or random business cycle shocks. For the assessment of "secular", "generational" forces affecting large parts of the advanced international system over long investment horizons, therefore, it is not clear that the existing literature has been operating with a plausible empirical basis.² Not least, historical studies have repeatedly demonstrated that as early as the 14th century, households and institutions treated sovereign long-maturity assets as a relatively "safe asset", to which they increased financial exposure in times of crisis (de Moor and Zuijderduijn, 2013).3

With recourse to a new multi-century real interest rate dataset constructed by Schmelzing (2022) and based on long-maturity market rates, this paper will seek to analyze more fundamental dynamics underpinning both the secular evolution, as well as the present features of the global interest rate environment.⁴ Following a literature review, section three of this paper will describe the new long-run interest rate data, with section four proceeding with an analysis of the general

when the sample length is extended to 1803-1990, for which the series can clearly reject the unit root. The authors use a range of other random 45-year intervals, for which the rejection of unit root is also impossible: this strongly underscores the role of sample length, rather than sub-sample-specific features involved in the post-1945 episodes. We test our basic conclusions for different sample lengths in the appendix in an analogous way, an exercise which confirms the importance of sample length for real interest rates.

²These dynamics are underpinned by asset pricing theory: as risk-averse investors move from single-periods to infinite horizons, the riskless asset is no longer a real Treasury bill, but a real perpetuity (Campbell, 2018, chapter 9).

³For instance, de Moor and Zuijderduijn (2013) recently showed that Dutch households systematically increased exposure to long-maturity annuities in times of heightened pandemic, natural disaster, or war frequencies - at the expense of exposure to livestock, houses (though not land), and cash. Mulho (1971) and Mueller (1997) showed separately that not insignificant shares of Venetian and Florentine debt in the 14th and 15th centuries was foreign-owned, bought by European investors in search for financial safety.

⁴The data carefully isolates sovereign long-maturity interest rates that were based on voluntary debt transactions averaging just over 10-years average maturity over time, with coerced and other involuntary transactions identified separately and excluded. For further details see the discussion in section 3.1 and appendix section 1.

statistical properties of global real rates since the birth of sovereign debt in 1311.

There, we will present three key statistical results. First, we show that - in contrast to the existing consensus - we are able to reject non-stationarity at conventional significance levels once one allows for a deterministic trend and long-maturity real interest rate data are employed: for both headline global real rates, as well as for all advanced economy country-level series, this result is shown to hold for long samples, and confirmed for a range of test specifications. Global real rates exhibit a gentle but firm downward trend (averaging about 1.6% every 100 years), and the evidence appears strongly consistent with the hypothesis that they are stationary around this trend. We confirm this result in both global and individual country data, using both ADF-GLS tests and the Zivot and Andrews (1992) test, which is robust to the presence of instabilities. Nevertheless, we can replicate the non-stationarity findings for the much shorter data series available for short-maturity rates even when a trend is included. However, we show also that evidence from long-maturity corporate and mortgage rate trends over time suggests that trend stationarity may be present more broadly in long-maturity debt.

Second, using Chow tests Chow (1960), we investigate whether structural breaks are present in our data, focusing on a list of widely-discussed break dates (including 1981), consistent with traditional and recent scholarship. Among our posited structural breaks, only the Black Death, as well as 1557 - the onset of a largely forgotten global financial crisis that "shook Europe to its foundation" (Ehrenberg, 1928, 114) and saw the concurrent default of three of the largest sovereign creditors, together with the impending collapse of perhaps the largest "systemically important financial institution" ever to have existed - are consistent statistically significant periods of structural change on both the global and country levels. The two key 20th century inflections - 1914 and 1981 - are also confirmed for a variety of subseries, but overall appear to be less significant than one would expect. We suggest in our interpretation that structural breaks can be plausibly linked to major combined political-economic shocks - but not per se to institutional innovations (such as the founding of central banks), sovereign default events alone, or foreign exchange (FX) regime changes - and generally fail to be associated with major monetary policy events (whether defined institutionally, or measured by dramatic shifts in policy).

Third, on the basis of our new data, we also present results on real rate persistence; when real rates drop, how long do they take to return to trend? Existing knowledge about long-run trends in the persistence of financial variables is thus far limited to scattered evidence on FX rates, and deviations from purchasing power parity. We present the first analysis on real interest rate persistence over the very long-run: the estimated half life deviations from the long run trend hover around two years over the first six centuries of the sample, showing a moderate upwards trend. Notably, therefore the early modern period (defined in this paper as the full period prior to the Congress of Vienna, 1300-1815) already experienced adjustment speeds in capital markets comparable to those during the late classical gold standard. After 1914, we find that half-lives for global real rates meaningfully increased, before moderating once more since the 1980s. Such evidence can contextualize the persistence of contemporary deviations of global real rates from deeper trends, and indicates that while mean-reversion from the sharp falls in global real rates

induced by the GFC and the pandemic shock is to be expected, directionally the historical trend does suggests that a downward tendency in global rates will continue to prevail: the expected mean-reversion takes places in the context of a falling trend, rather than in the context of a pre-GFC, or pre-1980s constant value.

Finally, part five assesses the historical and economic context of our statistical results on real interest rate dynamics, in particular evaluating the very long-run relationship with demographics and productivity: we take advantage of the significant advances over recent years in the reconstruction of output growth series by economic historians (e.g. Broadberry and Fouquet (2015); Ridolfi and Nuvolari (2021); Pfister (2021); de la Escosura et al. (2022)) to examine the recent literature on the relationship between real interest rates and both output growth and population growth in a long-run perspective: the evidence leads us to question arguments that posit that the "recent" decline in global real rates can be primarily explained by productivity or demographic trends (as opposed, for example, to the way that rising global wealth might affect rates of time preference or the intertemporal elasticity of substitution). It appears that for most of history, real interest rates have trended counter to growth and demographics. Taken together, these results suggests far stronger continuities in the present, post-GFC international financial system with (pre-)20th century dynamics than commonly acknowledged: not just regarding the downward trend in real interest rates, but also regarding levels of variation and adjustment speeds. There has indeed been an especially sharp downward shift of real interest rates in the period after the global financial crisis. As we discuss in section 5.2 however, there have been four previous long-lasting eras of below trend rates (generally periods of relative geopolitical calm, though there are additional factors, all of which we briefly assess). What we do know is that all of them ended, and, based on our evidence, the leading theory has to be that the post-financial-crisis real long-term interest rate, having declined far faster than the slightly downward long-term trend would imply, should be expected to eventually correct over a sufficiently long period.

2 Literature

Repeatedly, the macroeconomic literature has posited that, on purely *a priori* grounds, real interest rates should exhibit mean-reverting, stationary properties. Foundational asset pricing frameworks such as the Black-Scholes relationship or the consumption CAPM assume constancy in real interest rates. Standard finance no arbitrage models of the yield curve also assume that interest rates are stationary (see survey in Neely and Rapach (2008) and discussion in, e.g., Bauer and Rudebusch (2020)).

However, finance and macro literature actually employing statistical tests has had significant difficulties to confirm these a priori assumptions. Instead, they have overwhelmingly been unable to reject non-stationarity in real interest rates.

The earliest contributions focused on nominal rates: Fama (1975) instigated an intense (and broadly dismissive) debate after suggesting U.S. short-maturity bill rates during 1953-1971 were able to

predict subsequent inflation rates. Shiller and Siegel (1977) investigated both long-maturity and short-maturity U.K. interest rates' correlation to price level changes - but were not interested in statistical analysis.⁵ Afterwards, attempting a falsification of expectations theory based on data over 1890-1979, Mankiw and Miron (1986) narratively defined four interest rate sub-samples and suggested that the founding of the Federal Reserve led to a key change in the statistical properties of short-maturity nominal interest rates: over 1915-1979, short-maturity U.S. nominal interest rates followed a random walk; while they exhibited predictable behavior prior to this inflection date.

A key contribution was made afterwards by Rose (1988), who showed that U.S. real interest rates appear to exhibit a unit root: on the annual level, Rose used an ADF test for high-grade long-maturity corporate bonds, and for short-term commercial paper rates, for two periods spanning 1892-1970, and 1901-1950. Later, Garcia and Perron (1996) analyzed a regime-based framework of U.S. ex post real bill rates between 1961-1986: the authors conclude that U.S. real rates exhibit an "essentially random" process, with shifting means and variances in the post-war period, though without a clear predictable patterns. Ang and Bekaert (2002) studied U.S., German, and U.K. bill rates and propose that a regime-switching framework best captures out-of-sample observations in post-war real rate data. Later, Rapach and Weber (2004) used an extended unit-root test (Ng and Perron, 2001) to re-examine Rose (1988)'s findings - and confirmed non-stationarity across a range of countries. While the clear majority of existing studies tested short-maturity (real) interest rates, Nelson and Plosser (1982), Perron (1989) and Rapach and Weber (2004) tested U.S. long-maturity bond yields, albeit for a relatively short time period (1900-1973, or in the case of Rapach and Weber (2004) for 12 countries beyond the U.S.).

More recently, Hamilton et al. (2016) confirmed real interest rate non-stationarity, partly using policy and discount rates on the basis of U.S. annual data beginning in the late 19th century, and quarterly data beginning in the 1940s. On the annual level, the authors splice together short-maturity rates for various periods, relying on a mix of commercial paper rates and Fed fund rates for the U.S., for instance. A heteroskedasticity-robust test and KPSS results lead the authors to reject the null hypothesis of stationarity for U.S. short-maturity data at the 5% level: Bai-Perron break tests identify 1915 and 1921 as U.S. break dates. The only systematic relationship in the

⁵Specifically, Shiller and Siegel (1977) use British consol yields between 1727-1973 for long-maturity rates, and they splice a variety of 3-month bill rates together for short-maturity rates, beginning with Overend and Gurney corporate rates over 1824-1844, followed by bank bill rates over 1845-1938. Despite the paper title, the authors do not estimate real rates directly. Siegel (1992) relies on the same nominal basis, adding inflation data to construct ex post measures for the U.S. and U.K.

⁶We replicate Rose (1988)'s approach in the appendix (tables A.8.1-A.8.3 and discussion), applying both alternative inflation expectation approaches, for his two periods, and two assets, but also allowing for a trend: we fail to reject a unit root for all 16 variations of the data. Rose sourced historical U.S. inflation and interest rate data primarily from Friedman and Schwartz (1963) and Nelson and Plosser (1982). Non-U.S. data in his analysis begins in 1957 and is tested for quarterly three-month bill rate equivalents, using IMF International Financial Statistics. An earlier paper by Huizinga and Mishkin (1986) used short-maturity U.S. data between 1916-1927, and 1953-1984, and posited a higher degree of (ex post) real rate stability, but did not utilize any ADF, KPSS or related tests.

⁷Perron (1989) relies on Nelson and Plosser (1982)'s data, who fail to reject non-stationarity for U.S. nominal bond yields over 1900-1970.

authors' identification relates to a stationary gap between U.S. and world real rates.

DelNegro et al. (2019) investigated both global long-maturity and short-maturity interest rates over 1870-2016, finding evidence that interest rates "fluctuated between 1% and 2% for about a century, rose in the late seventies, and [have] been on a steady decline ever since". The authors rely empirically on the "Jorda-Schularick-Taylor" (JST) database - but do not undertake formal statistical tests, are not interested in linking the contribution to literature on general interest rate properties, and omit longer periods of time. Finally, Bauer and Rudebusch (2020) propose a time-varying model with shifting endpoints, which the authors suggest is superior in accounting for evidence of the "extreme" interest rate persistence. For ten-year treasury yields on a quarterly basis between 1971:IV - 2018:I, the authors reject stationarity using ADF and Phillips-Perron (PP) tests.

In sum, despite strong a priori propositions that real interest rates should exhibit stationary properties, existing literature has overwhelmingly failed to reject non-stationarity of real interest rates - however, such literature has focused almost exclusively on a combination of spliced short-maturity nominal interest rate data fusing different issuers, and realized inflation rates. And despite multiple studies using timespans of more than one century, their statistical power to examine "secular" properties still remains limited, as evidenced by the apparent spuriousness of the previously-highlighted 1970s and 1980s "inflections". In sum, substantial extensions of sample spans and a clear focus on long-maturity rates could in fundamental ways increase both the quantitative power and qualitative insights in the context of the extensive debates on real interest rate properties: we turn to precisely such an exercise in the next section.

3 Long-run empirics in global real rates, 1311 - 2021

3.1 A short history of short-maturity debt

To understand existing biases arising from the predominant use of short-maturity data in the literature on the properties of real interest rates, it is useful to revisit the historical background to the evolution of short-maturity and long-maturity debt instruments. We have observed in the literature review above that the vast majority of existing contributions assessing real interest rate properties and trends focus on multi-decade empirics for short-maturity yields, or even outright policy discount rates - often splicing short-term discount and market rates. Perhaps one reason for this existing bias is an implicit assumption that an "expectations hypothesis" (EH) holds - which systematically links all maturities in nominal and real terms across the term structure: under this framework, long-maturity yields are a direct function of continuously rolled-over short-maturity assets, and cannot therefore unduly deviate from the properties of the short end: effectively, expected excess returns on long-maturity bonds over short-maturity bonds are constant over time. In other words, "the risk premium of an *n*-period bond over a one-period bond (the term premium)

⁸As we shall discuss in section 3, the JST database only covers long rates from 1870-2016 and, because it draws quite heavily on Homer and Sylla (2005), it inherits many of the its limitations, as discussed in Schmelzing (2022), which uses primary sources, fills in many significant gaps, and covers a much longer time span.

is constant" (Campbell et al., 2018, chapter 8.2).9

Of course, another major reason for the focus on short-rates is that, in modern times, the (very) short rate is the variable controlled by central banks. This does not, however, seem like a compelling reason to prefer it to longer rates that have a much bigger impact on the economy. An additional reason - though rarely stated explicitly - is the exclusive focus of recent "natural" interest rate debates on the short-maturity component, typically referencing Wicksell (1936). For instance, in Laubach and Williams (2003) the natural rate is treated as identical to the real short-term rate: "Since Wicksell (1936), the natural rate of interest—the real short-term interest rate consistent with output equaling its natural rate and constant inflation—has played a central role in macroeconomic and monetary theory". Later in the same paper (p.1064), the authors justify the focus on short-term rates with the inability to measure long-term inflation expectations adequately. This is of course an important issue, and we devote an appendix section to several alternative approaches offered in existing literature.¹⁰

Hence, however plausible the choice to focus on short-maturities might at first appear - such a choice crucially restricts the insights that can be derived: as is known among economic historians, consolidated sovereign short-maturity assets are historically a decidedly "recent" innovation, only emerging as a regularly-used public debt instrument towards the end of the 19th century. Accordingly, this time is also the point of inception for existing "long-run" investigations on general real rate trends (Hamilton et al., 2016; DelNegro et al., 2019). Short-maturity datapoints, in other words, were only able to cover a fraction of the full empirical horizon of advanced economy "sovereign debt" trends: hence, the statistical power of short-maturity tests has thus far been dramatically limited. However, even key reference works, meanwhile, never reconstructed long-maturity data points on a sufficiently high frequency to allow extensions into longer samples. Only recently, with a new contribution (Schmelzing, 2022) did such an extension become possible for the first time - a development we exploit in the subsequent sections. But before we turn to the new data, we provide the key historical context.

On the back of a decree by its Grand Council in 1262 (the *ligatio pecuniae*), the Venetian Republic is typically regarded as the first issuer of perpetual consolidated sovereign debt, following a century of experimentation across Northern Italian city states with creditor consortia, who were given

⁹Mainstream affine term structure models equally assume that the expectations hypothesis (EH) holds, see (Campbell et al., 2018, chapter 8.3).

¹⁰It is perhaps relevant to observe that Wicksell (1936)'s definition of the concept not once specifically referred to a short-maturity dimension - in fact, the author has in mind an economy-wide average concept encompassing both short-maturity and long-maturity interest rates. Refer to Wicksell's following original summaries: "At any moment and in every economic situation there is a certain level of the average rate of interest which is such that the general level of prices has no tendency to move either upwards or downwards. This we call the normal rate of interest. Its magnitude is determined by the current level of the natural capital rate, and rises and falls with it". And further: "No statistics of the natural rate of interest are available. A precise investigation would necessitate an ad hoc enquiry, and for the past this is as good as impossible. While the general trend of the natural rate over decades can be observed in the movement of the money rate itself, of the banks' discount rates and the so called open-market rate, and of the prices of debentures and Government securities, this can be done only by assuming that on the average the two rates of interest are equal to one another" (Wicksell, 1936, 120, 168).

multi-year rights on tax revenues in return for emergency loans. Swiftly, other city states and cities north of the Alps follow suit and consolidate individual *rentes* contracts, so that over the course of the 14th century, a pan-European public debt market in long-maturity assets emerges – see Epstein (2000); Munro (2003); Eichengreen et al. (2021). And the evidence suggests that these markets were remarkably liquid: for instance, by the mid-15th century, the face value of annually-traded long-maturity public debt in hubs such as Venice reaches up to 29% of GDP.¹¹

By contrast - though they often contracted ad hoc personal loans on short-maturity bases - no sovereign on either side of the Alps issued consolidated short-maturity debt prior to the 18th century. The only outlier may be said to have been the Spanish Empire, which famously experimented with large-scale *Asientos* issuance during the 16th century. The monarchy sharply distinguished this type of issuance from the regular long-maturity *Juros* instruments (on which it never defaulted), and even at the peak of Spanish geopolitical dominance towards the end of the century, the *Asientos* share in total public debt stood at less than 10% (Alvarez-Nogal, 2014).

Regular German, French, Italian, and Dutch consolidated debt issuance up until the 19th century takes place exclusively in long-maturity assets, even though instruments with maturities of more than five years are occasionally referred to as "bills". ¹² In the U.S., the first Treasury Bill is auctioned no earlier than December 1929 (Garbade, 2008). Prior, several war financing operations see the intermittent issuance of U.S. "certificates of indebtedness", which are documented to have been sold over 1812-5, in 1837, during the Mexican War, in 1857, during the Civil War, and in 1907. For many decades in between these dates, the U.S. government does not issue any negotiable short-maturity debt instruments. ¹³ The situation is analogous in the other leading advanced economies: first U.K.

¹¹A figure based on Mueller (1997)'s information that in 1434, 300,000 ducats of *Monte Vecchio* debt in face value traded in Venice.

¹²For the United Provinces, which "occasionally" issued unfunded short-maturity obligaties during the 16th century, which "tended to become long-term" debt since they were continually rolled-over, see Gelderblom and Jonker (2011); for the key Italian States, see Gelabert (1999); Pezzolo (2008). The Crown of Aragon similarly issues Prestechs under Alfons V. and John II., which - while de jure short-term debts - are de facto long-maturity obligations, see Kuechler (1983). For the Holy Roman Empire - whose debt over centuries is synonymous with the Habsburg personal domain ("Erblande") debts, as there is no centralized Treasury - Huber (1893) provides a breakdown, by debt type, for the year 1564: 69.9% of Ferdinand I.'s debt is recorded in the form of long-maturity "pledge" debts, de facto consolidated and transferable instruments, with 30% in "unverwiesene" Schulden - often, but not exclusively short-maturity de jure unconsolidated claims (many of which are treated by all involved sides as de facto long-maturity negotiable claims). Much of sovereign debt activity in the Holy Roman Empire takes place on the level of the sovereign prince-electors: for detailed financial studies, including debt breakdowns, see for instance Schirmer (2006) for Saxony; France intensifies short-maturity issuance under State Minister Jean-Baptiste Colbert and during the 18th century, mainly in the form of billets and later by erecting facilities such as the designated Caisse d'emprunts: such developments lag regular annuities issuance by two centuries, however, and do not assume comparable volumes, regularity, and credibility: at the eve of the French Revolution, only 7.9% of French public debt outstanding exists in short-maturity form (Legay, 2011, 244). Schmelzing (2022) provides further documentation and an overview of more than 330 short-maturity issuances between 1312-1850, together with nominal interest rates, and more detailed discussion.

¹³For the U.S. history, see in particular Hollander (1919): prior to the first irregular issuance of short-maturity certificates in 1812, the Treasury under Alexander Hamilton actually relies on bank overdrafts and loans to satisfy the short-maturity spectrum.

Treasury bill issuance is legislated in the 1877 Treasury Bills Act, but only formalized in detail in 1889, and only issued in meaningful volumes from the First World War (BoE, 1963). In dynamics similar to U.S. certificates, between 1707 and 1853, various issuances of U.K. "Exchequer Bills" take place; in effect, revenue stream conversions by the Bank of England, with maturities typically of one year and with peak issuance during wartime (Richards, 1936). Prior to the founding of the Bank of England, "tallies" are regularly issued - intended as short-maturity IOUs funded upon anticipated tax revenues: de facto, the instruments are negotiable, but like Dutch "obligaties" they are in practice assuming effective long-maturity, through automatic roll-overs upon expiry at the prevailing long-maturity rate.

Overall, historically, long-maturity debt thus not only constitutes the "original" sovereign debtit is the sole historically-consistent basis enabling a multi-century assessment of the time-series
properties of sovereign interest rates. In this sense, a consistent and longer long-maturity time
series allows much higher statistical power, particularly for example to discriminate between unit
root and near unit root hypotheses. Short-maturity interest rate series are by construction far more
restricted (a limitation amplified further for *policy* rates, where time series are even shorter). At
the same time, we stress that we do not claim that our results will speak directly to the multitude
of existing studies focusing on short-maturity rates: different time-series properties may affect
different parts of the term structure.

3.2 Long-maturity data

We turn to the new data that now allows a critical expansion of interest rate samples, and potentially new insights into statistical interest rate properties. We draw upon Schmelzing (2022), who constructed a comprehensive sample of advanced economy long-maturity liquid interest rates since the "birth of sovereign debt" in the Renaissance. The series covers over 80% of advanced economy GDP on an annual level, across eight countries, and incorporates both marketable and relevant personal loan transactions. The inception year of 1311 for the series is not accidental: from this date onwards, both voluntary long-maturity nominal data points *and* suitable price data is available, allowing the calculation of inflation-adjusted, real interest rates.¹⁴

On the inflation side, our headline series utilize mainly the reference urban price series constructed by Allen (2001), as well as subsequent country-level improvements (e.g. Alvarez-Nogal and de la Escosura (2013) for Spain; Ridolfi (2019) for France). All data points on the inflation side refer to realized inflation rates of representative consumption baskets for workers denominated in silver equivalents - there exists as of yet no convincing approach to assess inflation expectations in the early modern period, but lagged measures of realized inflation rates are widely regarded as a

¹⁴A version of key long-term interest rate and inflation data we use was posted by Schmelzing (2020) via the Bank of England in 2020 (https://www.bankofengland.co.uk/working-paper/2020/eight-centuries-of-global-real-interest-rates-r-g-and-the-suprasecular-decline-1311-2018), and earlier long-dated price inflation data can also be found in Reinhart and Rogoff (2009). We briefly elaborate on data construction details in appendix section 1 below - for an extensive discussion see Schmelzing (2020) and Schmelzing (2022).

close approximation of inflation expectations, including in the literature of particular relevance to our results.¹⁵ We opt for a construction of an ex ante real rate measure, and follow the approach in Homer and Sylla (2005), who deflate the nominal rate in year t with a seven-year lagged inflation measure to approximate inflation expectations (over t-7 to t-1, not including the current year t), using progressive weights. We also smooth a few extreme inflation outliers, mainly associated with the 20th century, post-fixed exchange rate system, to avoid the severe measurement issues for these years.¹⁶ Given its relevance, in the appendix we test two alternative approaches to construct inflation expectations, adopting methodologies presented in Eichengreen (2015) and Hamilton et al. (2016). The confirmation of our ADF-GLS results in these variations suggest that the particular inflation expectations construct does not crucially change the properties we posit below. Of course, a virtually unlimited menu of alternative expectations constructions are possible, and the issue merits investigation, but the ones we use are well established in the literature and certainly form a plausible benchmark.

The effective average maturity of the assets over time closely approximates modern ten-year long-maturity Treasury bonds, and draws upon a wide sample of primary, printed primary and secondary sources. A large sample of non-voluntary sovereign debt operations - including intragovernmental lending, de facto confiscations, war indemnities, and office sales - undertaken by all advanced economies in the early modern period is separately identified and excluded from headline series. On average, the dataset covers more than four long-maturity data points per country-year between 1311-1870 on the consolidated sovereign debt side alone, in total over 12,000 long-maturity data points pre-1870.¹⁷ It thus represents a significant empirical leap over previous reference works such as Homer and Sylla (2005). Focusing only on the most recent period of 1870-2016, a related undertaking is represented by "JST" (Jorda et al., 2017). As noted earlier, a substantial share of observations in "JST" ultimately traces back to Homer and Sylla (2005), with its associated gaps and inconsistent definitions. Besides covering a much shorter time span than

¹⁵Recall that most previous studies - including Rose (1988) - treat realized past inflation as a proxy for inflation expectations. A range of robustness tests on the inflation side are presented in Schmelzing (2022), including on local currency denominations, "elite" consumption baskets, or long-run country-level inflation series, available via https://scholar.harvard.edu/pfschmelzing/jmp-dataset.

¹⁶In particular, we follow Homer and Sylla (2005) by attaching a 33% weight to the *t-1* CPI annual figure, and decreasing this weight successively over *t-2* to *t-7*. For the severe inflation shocks, we interpolate the last "pre- shock" CPI figure to the first "post-shock" CPI figure for the hyperinflation years in Germany (1916-1923), Spain (1937-1939), Italy (1943-1945), and Japan (1944-1947).

¹⁷No assets with a contracted maturity below two years are included at any point in the dataset. The all-time average effective maturity of non-perpetual assets in the sample is stated as 13.2 years: for full details, see in particular the appendix of Schmelzing (2022), available via https://scholar.harvard.edu/pfschmelzing/jmp-dataset. Schmelzing (2022) also attempts to isolate a "safe" interest rate over time, which splices successive country-level long-maturity real rates and traces the ex ante contemporary "dominant" economy, a series which aligns with secondary literature and features not a single default event on principal over seven centuries. Other "safety" features associated with the interest rate data are tested in (ibid.), including the frequency of debasement operations, military and geopolitical power of the issuing sovereign, and the absolute economic size (output) of the issuing sovereign. While of importance for the eventual identification of positive real rate drivers, neither the existence or the particular level of risk premia over time represent relevant issues for our endeavors in this paper.

the Schmelzing (2022) data, there are issues of consistency for example where JST/Homer and Sylla fuse yields for both medium- and long-maturity sovereign instruments to create a "long-term bond yield"; moreover data for key countries is not observed directly, but based on estimated risk premia. See Schmelzing (2022), whose newer data work takes advantage of recent advances in economic history to address these limitations, and is based largely on primary sources.¹⁸

Our data therefore traces the only consistent and high frequency definition of sovereign real interest rates over centuries, allowing us to take full advantage of the significantly higher statistical power associated with a multi-century sample length. Figure 1 displays the resulting headline global real rate series on this basis, together with its structural trend over time: this GDP-weighted basis, weighting each of the eight economies according to its rolling GDP share, is our default basis to report "global" results, unless otherwise noted (this basis is abbreviated "Global GW" in the following).

The blue structural trend line already suggests visually that the series appears to exhibit a downward trend over time: apparently, the oft-invoked low interest rate environment of recent decades therefore has deep historical roots. Specifically, the series displays a linear downward trend slope that declines by just over 1.6 basis points (1/100th of one percent - thus amounting to a decline of 1.6% every century) per annum over the observation period (1311-2021). But the next section will proceed beyond "visual" impressions and confirm statistical properties in more detail.

4 Statistical Properties

We undertake an Augmented Dickey-Fuller test (ADF-GLS), as modified by Elliott-Rothenberg-Stock (Elliott et al., 1996), and also test our series for structural breaks over a set of historically well-known break dates following Chow (1960). Finally, we conclude this section with an investigation of long-run persistence (half-lives) of global real rates. A range of alternative specifications of the ADF-GLS (including a specification assuming no time trend, and using two alternative inflation expectations constructs from existing literature) and related robustness tests (including results from a Monte-Carlo simulation, and a Bai-Perron test) are to be found in the appendix, broadly confirming our results. Not least, we present the evidence for short-maturity real rates, using the shorter time period available - we are able to essentially corroborate previous studies using the same approach that we utilize to demonstrate long-maturity trend-stationarity over long

¹⁸For instance, in Jorda et al. (2017) German, Japanese, and Italian government long-maturity bond yields prior to 1913 are based on an unpublished estimate of risk premia over British consols; later periods for Italy splice together evidence from "medium-term" and "long-term" observations. Similarly, the source for Dutch and German yields between 1880-1913 in "JST" is given as Flandreau and Zumer (2000) - but these authors in turn provide as their source for German and Dutch yields Homer and Sylla (2005) - France over 1870-1947 is also fully based on the latter.

¹⁹See appendix table A.1 for ADF-GLS without time trend, and sections 3.2 (Bai-Perron) and 3.4 (Monte Carlo). A companion paper, provisionally titled "Uncovered Interest Rate Parity Across Long Term Bond Markets in the Very Long Run", is set to examine very long-run properties of the separate nominal rate and inflation components in detail.

Global real rate of the control of t

Figure 1: Headline global real rates, and trend, 1311-2021

Notes: Data based on Schmelzing (2022)'s GDP-weighted global real rate data. Long-maturity ex ante basis, deflated by using seven-year progressively-lagged inflation, excluding current year *t*. Structural trend (in blue) displayed as lowess function: the lowess (locally weighted average) function fits a non-parametric model, which is the "best fit" given the time series data.

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4.1 An analysis of trend-stationarity properties of real interest rates

We start our analysis by testing the stationarity properties of real interest rates using unit root tests. We find unambiguous evidence in favor of trend stationarity for global real rates when tested via the ADF-GLS test. The null hypothesis of a unit root can be rejected at the 5 percent significance level for the headline real series (using GDP-weights in any given year, "Global GW" - this is the default basis on which we report all "global" results unless otherwise noted), for all headline real rate country series, and for an arithmetically-weighted global series ("Global AW"), which gives equal weight to each of the eight constituent countries in any given year, regardless of absolute size of GDP. Specifically, we also note that the evidence in favor of trend stationarity is confirmed for U.S. and U.K. country-level data, with which some researchers might be more familiar: the fact that the results hold up for these variations, and a variety of different inflation and weighting bases, gives us a high degree of confidence in the robustness of the above methodology.

²⁰Rose (1988) uses short-maturity U.S. commercial paper rates, among other rates - we can replicate his failure to reject the unit root for all 8 possible variations on short-maturity real rates (in addition to replicating all other failures to reject), using multiple inflation approaches, see appendix tables A.8.2 and A.8.3.

	Table 1: ADF-GLS Te	est with Time Trend	
	Real Rat	e Series	
Region	Number of lags	ADF-GLS test statistic	Optimal lag
Global GW	3	-6.420	MAIC
	2	-6.776	Seq, SIC
	1	-9.499	
Global AW	3	-6.549	
	2	-6.751	Seq, SIC, MAIC
	1	-9.119	
Italy	3	-9.937	MAIC
	2	-10.886	Seq, SIC
	1	-14.422	
UK	3	-6.763	
	2	-6.792	Seq, SIC, MAIC
	1	-8.956	
Dutch	3	-8.812	Seq, MAIC
	2	-10.356	SIC
	1	-13.192	
France	3	-6.017	MAIC
	2	-6.456	Seq, SIC
	1	-8.308	
Germany	3	-8.244	
	2	-8.184	Seq, SIC, MAIC
	1	-11.577	
Spain	3	-5.722	
	2	-5.742	Seq, SIC, MAIC
	1	-7.846	
U.S.	3	-5.420	
	2	-6.011	
	1	-6.188	Seq, SIC, MAIC
Japan	3	-4.034	
	2	-3.953	Seq, SIC, MAIC
	1	-5.258	

Note: The table reports the ADF-GLS test statistic for several choices of the number of lags (with a maximum of 3 lags). The regression includes a constant and a deterministic time trend. The critical values at the 1, 5 and 10 percent significance levels are the following for all observations: -3.48 (1%); -2.89 (5%); -2.57 (10%). "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the Bayesian Information Criterion (SIC), or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value. "Global GW" = GDP-weighted global real rate basis; "Global AW" = arithmetically-weighted global real rate basis, taking equal weights for each of the eight countries.

Specifically, the fact that we reject a unit root using both the ADF-GLS test specification without a time trend as well as the specification with a time trend - in addition to identical results when adopting the inflation expectation constructs proposed by Eichengreen (2015) and Hamilton et al. (2016), in the appendix (tables A.2 and A.3) - makes us confident that these results are not a function of the particular model used: the results, instead, appear to be firmly a function of the extended sample length, the fact that we exclusively focus on long-maturity rates, or a combination of both factors.

Further, we test a range of alternative asset price series separately for the most recent period in the appendix (section 2.3), including testing private real long-maturity rates, real long sample policy rates, and U.K. real rates. The results broadly confirm our propositions above, and appear to indicate that the ability to reject non-stationarity appears to extend to other fixed income series once long samples are employed. The intuition that our powerful sample length (rather than idiosyncrasies of our data sources, maturities, or geographies) are driving our results is also suggested by Monte Carlo simulations where we generate time series with the same properties (mean, trend, variance and serial correlation) as our global real rate series and, by varying the sample length, we demonstrate that the power of the ADF-GLS test significantly falls when sample sizes shorten. That is, researchers working with samples of progressively smaller sizes are significantly less likely to reject non-stationarity (a more detailed discussion is available in appendix section 3.4).

Table 2: Zivot-Andrews Test Results				
	Test statistic			
Global Real GW	-9.727			
Global Real AW	-8.948			
Italy Real	-12.581			
UK Real	-10.164			
Dutch Real	-9.497			
France Real	-8.466			
Germany Real	-10.769			
Spain Real	-8.235			
US Real	-6.610			
Japan Real	-4.701			

Note: The table reports the Zivot and Andrews (1992) unit root test statistic, which allows for a break in both the mean and the trend under the alternative. The critical values at the 1, 5 and 10 percent significance levels are the following for all observations: -5.57 (1%); -5.08 (5%); -4.82 (10%). The trimming parameter is 0.10. The test rejects when the test statistic is negative and larger (in absolute value) than the critical value. When the test rejects the unit root, the column labeled "break date" reports the estimated break date. "Global GW" = GDP-weighted global real rate basis; "Global AW" = arithmetically-weighted global real rate basis.

Next, we report the test by Zivot and Andrews (1992), which tests for non-stationarity while being

robust to the presence of structural breaks in the trend. Per table 2, the unit root is rejected once more for both global real rates, as well as for all country-series. While the Zivot-Andrews test is robust to instabilities, it is not a test for structural breaks. Therefore, the next section will discuss more closely the possibility that our series may have undergone structural changes and we will draw from the historical literature for a set of likely break dates to gain deeper insights about the long-term patterns of real interest rates over time.²¹

4.2 An analysis of instabilities in real interest rates

We present Chow tests in table 3, choosing break dates in years where the existing literature would suggest or recognize high probabilities of expecting a trend break. The Chow test is implemented in a model that includes a constant and a deterministic trend (as a fraction of the total sample size), using Newey and West's (1987) Heteroskedasticity and Autocorrelation consistent (HAC) standard errors with a maximum lag length of 4. Again, as with all other exercises in this main body, we use the progressively-lagged realized inflation measure to construct real rates, excluding the current year *t*, following the methodology in Homer and Sylla (2005).²²

We choose five annual dates (1349, 1557, 1694, 1914, 1981) to perform the Chow test: these dates are chosen on the basis of long-standing debates in the economic and financial history literature - literature that suggests to us a high degree of ex ante plausibility for each of these dates, as they all represent events of deep structural changes. It is important to note that, in the implementation of the Chow test, at any given break date we simultaneously allow for breaks at all the other potential break dates.²³

Per table 3, the Chow test results indicate that for the GDP-weighted global real rate ("Global GW") we find empirical evidence in favor of breaks in the trend or the mean in 1349 or 1557. For the equal-weighted global real rate ("Global AW", not separately shown here), we find empirical evidence of breaks in either the trend or the mean in the years 1349, 1557, or 1981.²⁴

²¹While not a test for structural breaks, as part of the Zivot-Andrews procedure it is possible to estimate the most likely break date in the sample: we interpret the full historical-economic context for these dates in appendix section 3.1. We also discuss Bai-Perron's (Bai and Perron, 1998) structural break test in the appendix (see table A.7 and the discussion there).

 $^{^{22}}$ An alternative inflation expectations construct for the Chow test, using seven-year equally-lagged inflation including current year t, in the spirit of Eichengreen (2015), is presented in the appendix, section 2.2, table A.4.

²³That is, we do not tests for breaks one-at-a-time.

²⁴On the country level, the empirical evidence points towards the existence of a break at the following times: for the Italian real rate in 1694 in either the trend or the mean; for the UK real rate in 1349 and 1557 in either the trend or the mean; for the French real rate in all the dates in either the trend or the mean; for the German real rate in 1981 in either the trend or the mean (and marginally in 1557); for the Spanish real rate in 1694 and 1914 in either the trend or the mean , and in the mean in 1557 as well; for the Dutch real rate in 1981 in either the trend or the mean; for the US real rate in 1981 in either the trend or the mean as well. There is no evidence of breaks in the Japanese real rate.

Table 3: Chow Test Results - Global Real Rate GW - progressively-lagged inflation							
	Coefficient	Std error	t-statistic	P-value	95% Confidence interva		
trendbreak 1349	1.649739	.3034669	5.44	0.000	1.053923	2.245555	
trendbreak 1557	1732261	.0720339	-2.40	0.016	3146549	0317974	
trendbrak 1694	026058	.0671763	-0.39	0.698	1579495	.1058334	
trendbreak 1914	1531424	.3122241	0.49	0.624	459867	7661518	
trendbreak 1981	.4594748	.4091601	1.12	0.262	3438552	1.262805	
meanbreak 1349	0890055	.0141602	-6.29	0.000	1168072	.0612038	
meanbreak 1557	.0690446	.0275366	2.51	0.012	0149802	.123109	
meanbreak 1694	0229655	.0350931	0.65	0.513	045935	.091866	
meanbreak 1914	0869152	.2854017	-0.30	0.761	6472625	.4734321	
meanbreak 1981	4911894	.3866844	-1.27	0.204	-1.250392	.2680126	
trend	6291718	.2674622	-2.35	0.019	-1.154297	1040463	
constant	.6330557	.2629259	2.41	0.016	.1168366	1.149275	

Note: The model includes a constant and a deterministic trend (as a fraction of the total sample size). The standard errors are based on Newey and West's (1987) Heteroskedasticity and Autocorrelation consistent (HAC) procedure with a maximum lag length of 4. The coefficient associated with "trendbreak 1349" denotes the difference of the estimated trend coefficients before and after a break in 1349 (allowing for breaks in all the other potential break dates); hence, the associated t-statistic is the Chow test for the absence of a structural break in 1349. Similarly, "meanbreak 1349" refers to a break in the mean in 1349.

We observe that the years 1349 and 1557 are confirmed as breaks - a result broadly echoed in the country level results, but on the other hand, we find that the years 1694, 1914, and 1981 cannot be confirmed on the "Global GW" Chow basis: all of which are years where existing literature has posited strong claims that would expect one to find such breaks. While finding outright strong regularities for these confirmations and rejections, respectively, may not be straightforward, the most promising approach remains a better understanding of the exact historical and economic context over time, in the context of existing research. The following list thus asks: what is the particular logic of selecting these dates? And what do our results suggest for the economic-financial context of the five break years?

- 1349. This year was chosen against the backdrop of a long literature positing a major financial trend break associated with the Black Death (Epstein, 2000; Pamuk, 2007; Clark, 2016). The death of one-third to one-half of the population is said to have created a boost in capital-per-capita, and a substantial increase in real wages resulting from labor scarcity. As we detail below, we confirm the epochal role of the Black Death on credit markets: however, the directional context pre- and post-1349 appears more idiosyncratic than recognized thus far.
- 1557. While little progress on our understanding on the subject has been made since an authoritative study appeared in German in 1896 (Ehrenberg (1896), only partially translated

via Ehrenberg (1928)) - which regarded the initial wave of shocks over 1557-62 as events "that shook the finance and trade of Europe to its foundation" - a variety of case-study and piecemeal literature appearing since confirmed that an epochal crisis afflicted the international economy during the second half of the 16th century. Indications are that the "triple sovereign default of 1557-8" (France, Spain, and the States General) could have been either a consequence or the actual inception point of a very deep-seated reversal in financial markets lasting at least to the early 17th century.²⁵ In direct consequence of the sovereign defaults, by far the largest bank of its day - perhaps the most significant "systemically important financial institution" ever to have existed - the Fuggers, slid to the brink of default. Narrowly escaping themselves, a seemingly unending string of international merchant and merchant-bank defaults can afterwards be traced in the respective sources. Recurring chaos at the largest financial fairs over decades - from Seville over Lyon, to Rome - is reported in equal measure (Ehrenberg, 1928; Lapeyre, 1955; Delumeau, 1959; Kindleberger, 1998).²⁶ We chose this date mainly to test for a major early modern standalone sovereign default-cum-private financial crisis event: our affirmative results on a break date here indicate that secularly, though financial turmoil on its own may still not be a sufficient driver of structural inflections - at least when accompanied by significant political volatility (the French Wars of Religion; the near-uninterrupted Habsburg military campaigns in the 16th century) these combined forces can culminate into deep structural change.

• 1694. This year was chosen against the backdrop of the prominent "North Weingast" thesis, which associated the Glorious Revolution with a revolution in "credible commitments" (North and Weingast, 1989). It appears particularly relevant that the authors used (nominal) interest rate data to prove their seminal thesis. Yet, this event is widely rejected in our Chow tests, on both the global levels and all country levels. Of course, the original statement of the North-Weingast thesis received extensive criticism over the years (Sussman and Yafeh, 2006) - but has not been contextualized in the setting of a comprehensive (real) interest rate dataset covering centuries. The failure to identify 1694 with a structural break is part of more comprehensive evidence that fails to associate all other central bank inceptions with structural breaks, too.²⁷

²⁵The Netherlands are not a formally recognized independent nation state until the Peace of Westphalia 1648: de facto, however, the States are characterized by a high degree of autonomy, with the Receiver General of the States General, and the individual provinces, issuing their own debt secured by land and other taxes, the *bede* Fritschy (2017, chapter 2). Ehrenberg (1928, 113f.) puts the Fugger exposure alone at 600,000fl., which the Receiver General refused to honor in 1557: it is not specified in the source whether the debt was denominated fully in Dutch guilders or Ducats. Hauser (1930) dates the French default to September 1, 1557, with other sources dating the default to the year 1558.

²⁶Ehrenberg (1928, 114ff.) details the uncertainty over the Fugger's survival in 1557-8, and also presents balance sheets of the bank: he indicates that the Fuggers had formally written off more than 620,000fl from the Dutch and Spanish defaults by 1563, against a balance sheet size of 5.6M fl. Ehrenberg regards 1557 as the beginning of the end of the Fugger empire. Significant parts of his work remain untranslated; the contemporary account of Hauser (1930) relies mainly on him.

²⁷The same failure to associate central bank inceptions with structural breaks is observed when undertaking Bai-Perron tests, see appendix table A.7 and discussion.

- 1914. This year was chosen against the backdrop of the strong narrative of an institutional inflection arising from the founding of the Federal Reserve (Barsky et al., 1988; Bernstein et al., 2010), coupled with the major monetary inflection related to the (de facto) departure from the centuries-old bullion standard. In addition, the economies deal with the geopolitical shock of the First World War the first industrial war far outstripping the human and financial costs of earlier inter-state conflicts. And indeed, across our series, the years in and around the First World War constitute strong (though not fully consistent) breaks across global and country-levels: ironically, however, the U.S. is not one of these. As in 1694, we therefore fail to confirm any obvious monetary channel influencing real rates.
- 1981. This year was chosen against the backdrop of prevalent narratives intensifying after 2008 of a key inflection point in advanced economies and financial markets during the early 1980s, though there is no clear consensus on a single driver of this alleged inflection (Rachel and Summers, 2019; Mian et al., 2021; Goodhart and Pradhan, 2021). Antedating the 2008 crisis, a sizable literature found evidence of structural changes in inflation and interest rates during the late 1970s or early 1980s (Garcia and Perron, 1996; Ang and Bekaert, 2002; Neely and Rapach, 2008) coupled with sharply declining general macroeconomic volatility (Bernanke, 2004) invoking monetary policy dynamics as the driving causal force.

4.3 Persistence

We measure persistence and half-life next. To the extent that the existing literature has investigated whether the post-1980s behavior of global real rates is "transitory" - whether rates will eventually return to "a more normal level" (Laubach and Williams, 2016), long-run estimates of such adjustment speeds can provide an indication of how quickly global real rates typically return to their structural trend after shocks. As per table 4, which uses our Chow test dates to delineate sub-samples, we observe that the half-life of real interest rate deviations for the entire sample period ranges between 1 and 10 years across individual sub-samples, with one sharp outlier in the case of France. Breaking down this figure for sub-periods reveals that half-lives stood below two years during the fourteenth and fifteenth centuries, a figure that moderately rises over the remainder of the bullion regime era, after which it sharply increases over the late gold standard and early gold-exchange standard period, only to moderate again towards the end of the 20th century, to levels slightly (but not sharply) above long-run averages seen during the early modern period. This general curve is equally visible if alternative weighting methods are employed ("Global AW"), and if country-level trends are investigated separately. The "safe asset providers" over time - Italy pre-1600, the U.K. after 1557, the U.S. in the 20th century - appear to record consistently faster adjustment speeds. It is important to note that our relatively short half lives (around two years) do not contradict the fact that we need a relatively large sample in order to have sufficient power to reject non-stationarity, as we show in our Monte Carlo simulations in

the appendix.²⁸

In terms of levels, our data suggests that early modern persistence in global real rates appears to be broadly comparable to respective persistence in real exchange rates - but generally higher than identified persistence in real exchange rates after 1914, from which half-lives of 3-5 years have traditionally been posited (Rogoff, 1996; Lothian and Taylor, 1996; Itskhoki, 2021).²⁹ A decomposition of aggregate persistence measures at the country level does not reveal notable cross-country differences: while aggregation biases similar to those posited for real exchange rate persistence measurements are therefore not apparent.³⁰

Generally, our persistence levels are consistent with related financial evidence from earlier studies, but they also suggest intriguing differences. Volckart and Wolf (2006), focusing on FX bullion content deviations between Flanders, Lübeck, and Prussia over 1385-1450 to assess money market integration found that arbitrage dynamics generated a half-life of eight months for the correction of silver price deviations between Flanders-Lübeck, and a half-life of 21 months for deviations between Lübeck-Prussia. Volckart and Wolf (2006) further point towards a secular trend towards sharply lower half-lives, with the classical gold standard period reporting FX arbitrage speeds (half-lives) of six days between London and New York in demand bill markets.

On the price side, several studies have investigated long-run trends in price dispersion and persistence: most recently, Federico et al. (2021) presented new evidence on both price co-movement, and price convergence in European grain markets over 1350-1910. Trend-wise, the authors find a slow but steady improvement of market efficiency and integration, reflected by increasing co-movement of agricultural prices and fewer price deviations, including intra-national deviations - dating the onset of price convergence in advanced economies to the fifteenth century.³¹

²⁸In fact, our Monte Carlo simulations, tailored to our estimated global real rate series, show that a researcher would reject non-stationarity only 77 percent of the times in a sample of 100 observations, and only 37 percent of the times in a sample of 50 observations, while he/she would reject it with probability almost equal to one in our sample of 711 observations. See appendix section 3.4 for details.

²⁹The uncertainty around our estimated half-lives is much lower than the substantial uncertainty surrounding estimates of the half-lives of real exchange rates, due to the high persistence of the latter.

³⁰We intend to discuss nominal and inflation persistence breakdowns separately in a companion paper provisionally titled "Uncovered Interest Rate Parity Across Long Term Bond Markets in the Very Long Run".

³¹Previously, Froot et al. (1995), Persson (1999) and Bateman (2011) have calculated persistence for European grain markets over time, with the former positing half-lives of 2.8 - 6.2 years for Dutch and English food price deviations (law of one price (LOR) basis) over a 700-year period, with no pronounced change in the modern or float periods; and Persson (1999) finding 1.5 - 3 years in the early modern period, with a secular trend towards faster adjustments. Using European wheat prices, Bateman (2011) found lower half-lives, on average 6.34 months, over the period of 1550-1750, with no pronounced trend.

Table 4: Half-Lives of Real Rates, by Sub-samples								
	Period	Half-life	Confide	nce interval				
Global Real GW	1311-1349	1.2068	0.8219	1.7101				
	1350-1557	1.4674	1.2171	1.7141				
	1558-2021	2.7947	2.3746	3.7538				
Global Real AW	1311-1349	1.4057	1.0381	1.8739				
	1350-1557	1.4684	1.2141	1.7878				
	1558-1981	2.9377	2.3892	5.1246				
	1982-2021	3.1947	1.7151	8.9091				
Italy Real	1311-1694	1.3487	1.1968	1.5028				
	1695-2021	2.9033	2.3573	4.1928				
U.K. Real	1311-1349	1.4533	1.1059	1.8816				
	1350-1557	1.4208	1.1253	1.7764				
	1558-2021	1.6769	1.4417	1.9141				
Dutch Real	1367-1981	1.4007	1.2499	1.5634				
	1982-2021	3.5355	2.0216	7.6561				
France Real	1311-1349	0.6484	0.4825	1.0242				
	1350-1557	1.4689	1.2555	1.7063				
	1558-1694	1.9536	1.3281	3.0483				
	1695-1914	2.3102	1.9086	2.7919				
	1915-1981	7.9032	3.6772	38.5262				
	1982-2021	6.1940	1.7688	N.A.				
Germany Real	1311-1981	2.3343	2.0484	2.6317				
	1982-2021	3.1061	1.4449	10.1295				
Spain Real	1332-1557	1.4091	1.1292	1.7348				
	1558-1694	1.9593	1.3370	3.0222				
	1695-1914	1.9299	1.6413	2.7374				
	1915-2021	5.3997	3.4453	8.8593				
U.S. Real	1791-1981	3.2676	2.5545	4.2219				
	1982-2021	0.7855	0.5515	1.2806				
Japan Real	1870-2021	2.8812	2.1358	5.4815				

Note: The half-life is estimated as the first horizon at which the impulse response equals one-half of the initial impact effect. The impulse response is estimated using a deterministically detrended linear autoregressive model (with the number of lags chosen by the Bayesian Information Criterion) using Kilian (1999)'s bootstrap; the point estimate is based on the median unbiased response while the confidence interval is based on the 5-th and 95-th quantiles of the bootstrap distribution. The half-lives are separately estimated in each sub-sample identified by the statistically significant break dates (according to the Chow test). "Global GW" = GDP-weighted global real rate basis; "Global AW" = arithmetically-weighted global real rate basis. NA denotes situations where the half-life is infinity.

Against this evidence, our levels of persistence generally fall in line with the lower end of the range of estimates for related financial variables - however, we are unable to confirm any evidence of secularly faster adjustment speeds. On the contrary, real rate persistence on our basis appears to exhibit a wave-like trend over the early modern era, with some evidence of a "hump shape" over the 17th century. A sharp upwards break in half-lives then coincides with the end of the fixed regime in 1914, followed by a moderation after 1981 (to a present level above the early 20th century). This finding is relevant in the context of existing literature that associated the eve of the First World War - coinciding closely with the founding of the Federal Reserve and the departure from fixed exchange rate regimes - with a decline in financial volatility and seasonality in interest rates (Barsky et al. (1988); Bernstein et al. (2010)): while the management of seasonality, and general predictability in financial markets may both have improved, this did not necessarily imply faster adjustment speeds. Overall, the figures indeed suggest that, at least until the end of 2021, post-GFC global real rates have "under-corrected" relative to historical adjustments (given that they remained visibly below structural trend levels and persistence confidence bands after well over a decade as of today) - however, as of yet this multi-year deviation does not register as a structural break: and it cannot necessarily be expected to do so even in the event of a further downward entrenchment of rates, given the rare occurrence of structural breaks we identified (occurring on average every one-and-a-half centuries). An ultimate "normalization" to the structural downward trending level would therefore be the default expectation on the basis of our data.

To summarize: this section suggests that testing long horizon samples of real interest rates strongly confirms their stationary properties around a downward deterministic trend - in contrast to existing literature that focused on short-maturity rates, using short samples. Our results are robust to a range of specifications, including alternative inflation expectation constructs and the existence of potential structural breaks, all of which are described in detail in the appendix. Importantly, we also present ADF-GLS tests for short sample and short-maturity series in the appendix (appendix tables A.8.1, A.8.2, and A.8.3): we can replicate results in previous literature via this exercise, demonstrating that it is indeed impossible to reject a unit root using such shorter horizons (keeping other attributes constant). Our results on the persistence are novel, too, with the finding of a slight increase in half-lives in global real rates presenting somewhat of a challenge in the context of existing literature. Taken together, these results rationalize why the existing consensus on real interest rate properties has arisen: but they equally demonstrate that previous results do not necessarily hold once a higher statistical power is achieved by using longer samples - albeit we emphasize that if the term premium is non-stationary, it is perfectly possible for the long-term rate to be stationary and the short-rate not, at least over a prolonged period. The long-maturity real interest rate appears to exhibit quite distinct properties when compared to other financial variables - notably real exchange rates (Itskhoki, 2021).

5 The continuity of real rate trends, Twentieth Century breaks, and low interest rate eras over time

From the discussion of statistical properties, we now proceed to an attempt at broader interpretations of historical patterns and possible economic drivers. How do the statistical results accord with economic and historical evidence, and to what extent do structural breaks and general trends fit the empirics of other macroeconomic variables?

5.1 The 1914 break, and assessing the (alleged) 1981 inflection

We concentrate here on a discussion of the much-discussed significance of "recent" trend breaks in global real rates, particularly in 1914 and the 1980s. We recall that in our (unbalanced) panel above, we found limited evidence of both 1914 and 1981 as breaks - neither assumed the constancy of 1349 or 1557 as relevant inflection points.

In tables 5.1 and 5.2. we now test for structural breaks in a balanced panel, to take account of the fact that the U.S. and Japanese series begin notably later than the European observations (in 1790 and 1870, respectively). In this exercise, we initiate all series in 1870 and end in 2016: below, the results are shown for the GDP-weighted global real series ("Global GW"), and for the arithmetically-weighted global real series ("Global AW"). We test the 20th century break points (1914 and 1981) once more in this set-up via the Chow test.

Table 5.1: Chow Test Results - balanced panel Global GW - progressively-lagged inflation							
Regressor	Coefficient	Std error	t-statistic	p-value	95% Confidence interva		
trendbreak 1914	1450128	.0756641	-1.92	0.057	2945955	0045699	
trendbreak 1981	.0678054	.090859	0.75	0.457	1118166	.2474274	
meanbreak 1914	.0619555	.0423217	1.46	0.145	0217116	.1456225	
meanbreak 1981	1069321	.0725561	-1.47	0.143	2503705	.0365063	
trend	1028904	.0637047	-1.62	0.109	2288303	0230494	
constant	.1153798	.0595556	1.94	0.055	0023576	.2331171	

Note: The model includes a constant and a deterministic trend (as a fraction of the total sample size). The standard errors are based on Newey and West's (1987) HAC estimator with a maximum lag length of 4. The coefficient associated with "trendbreak 1981" denotes the difference of the estimated trend coefficients before and after a break in 1981 (allowing for breaks in all the other potential break dates); hence, the associated t-statistic is the Chow test for the absence of a structural break in trend in 1981. Similarly, "meanbreak 1981" refers to a break in the mean in 1981.

Once we focus on this balanced panel, both 1914 and 1981 emerge as more relevant break dates, though still not as quite a unanimous one. Specifically, for Global AW, we now find breaks in both 1914 and 1981. For Germany, France, Holland, and the U.S., we now record 1981 as a trend

break.32

Overall, the idea that existing narratives around both the 1914 and the 1980s "interest rate inflections" are a function of a restricted sample length that existing research had thus far limited itself to, remains consistent with our data.³³ Our intuition therefore remains that narratives of a "recent", a 1980s inflection point in global real rates, are partly a function of too short a sample length that existing research was able to utilize (overall, we interpret our results to be a function of three factors: our approach of allowing for a trend in the data, of our much longer sample length, and of the fact that we focus on long-maturity rates).

Table 5.2: Chow Test Results - balanced panel Global AW - progressively-lagged inflation							
Regressor	Coefficient	Std error	t-statistic	p-value	95% Confidence interva		
trendbreak 1914	1947143	.0790839	-2.46	0.015	3510578	0383709	
trendbreak 1981	.1678928	.0946711	1.77	0.078	0192654	.355051	
meanbreak 1914	0832306	.0480721	1.73	0.086	0118046	.1782658	
meanbreak 1981	1669644	.0721482	-2.31	0.022	3095965	0243323	
trend	1364817	.0583981	-2.34	0.021	2519307	0210327	
constant	.1463777	.0541504	2.70	0.008	.039326	.2534294	

Note: The model includes a constant and a deterministic trend (as a fraction of the total sample size). The standard errors are based on Newey and West's (1987) HAC estimator with a maximum lag length of 4. The coefficient associated with "trendbreak 1981" denotes the difference of the estimated trend coefficients before and after a break in 1981 (allowing for breaks in all the other potential break dates); hence, the associated t-statistic is the Chow test for the absence of a structural break in trend in 1981. Similarly, "meanbreak 1981" refers to a break in the mean in 1981.

5.2 Low real rate eras over time

An alternative to pinpointing particular break points is to identify secular departures from the structural (downward) trend of real rates in the context of historical evidence and literature - in other words identifying narrative "low real interest rate periods". We undertake such an approach in addition to the econometric approaches, cognizant of the qualifications such judgments must necessarily involve. We stress that we view these episodes as akin to "long-lived transitory shocks", not associated with break dates according to our structural change tests: importantly, at the conclusion of these respective eras, the structural trend in each case returned to the long-run regularities. Put differently, the crucial observation is that these low rate eras all ended.

³²Given space, we do not report country results here. In addition to the above, we also find a break in the mean for the U.K. in 1914, for U.S., Italy, Spain, and France in the trend in 1914. For Japan we find no break.

³³In addition, a Bai-Perron exercise allowing for multiple breaks only identifies a single structural break across all series after the year 1917, in the case of the U.S. (1983, see appendix table A.7 and discussion).

The resulting periods are visualized in Figure 2 as shaded areas. They include the following four episodes:

- The (pre-) Black Death era (1311-1353). A long consensual literature viewed the most devastating demographic shock in recorded history as a key inflection point in financial markets (Homer and Sylla, 2005; Pamuk, 2007; Clark, 2016): and indeed, our Chow test indicated 1349 as a relevant break date, for the global series and for country-levels. In representative fashion, Stephen Epstein posited that "the Black Death saw a major change of trend in European interest rates which set in motion a gradual decline in the real cost of capital that lasted up to the eighteenth century... the fall in the expected rates of return and cost of capital for individuals was nearly as impressive" (Epstein, 2000, 61f.). However, our evidence suggests that the pre-Black Death era saw comparatively *low* real interest rates - only the early 1500s once more returned to rates prevailing two centuries prior. While we cannot be conclusive, we note that recent scholarship associates the years immediately preceding the Black Death with the protracted "Fourteenth Century Crisis": stagnant productivity growth and incomes, seigniorial and rural impoverishment, indeed "international commercial recession" - perhaps, as recent literature argued, phenomena ultimately caused by sudden climatic changes in Europe: Campbell (2016) posited that the end of the "Medieval Climate Anomaly" (MCA) led to an abrupt end of mild and wet winters in Central and Northern Europe, and to a sudden cooling, causing the severe economic adjustment problems in the half-century from the 1270s.34
- Post-Bullion Famine (1483-1541). A long literature has evolved on the "Bullion famine" affecting all our advanced economies through the first half of the 15th century, following the classic theses in Day (1978), with recent discussions in Munro (2003) and Aerts (2006). But towards the end of the century, the re-capture of the Balkan mines by European armies, and the ability to resume coining output familiar from 14th century levels led to an economic reversal, a boom in money supply and output, and particularly a surge in maritime trade after the discovery of the New World. In this sense, we are dealing almost with a mirror image of the "stagnant" 14th century low rate era: this contrast may suggest that low real rate eras can in principle be associated with both low-growth or high-growth environments.
- The 1732-1810 boom. This second period broadly spans the time from the South Sea Bubble and the French "Law" Crash, to the Congress of Vienna. It is a period of relative geopolitical calm (to the 1790s), demographic and economic advancement, and institutional security. Evidence also abounds for the existence of a prolonged capital market boom spanning several decades, leaving traces in both sharply rising sovereign debt/GDP figures, and a boom in private credit (Neal, 2015, chapter 6), Hoffman et al. (2019).

³⁴In economic terms, according to Campbell (2005), "the marginal productivity of male agricultural labour fell by 60 per cent between 1209 and 1309 and was lower between 1270 and 1329 than at any other time between 1209 and 1869...it was the massive and fortuitous loss of life in the Black Death of 1348-9 that took the demographic tension out of the equation and thereby broke the deadlock on the land".

• The great FX transition era (1937-1985). Beginning with the prominent 1937 break, this period broadly spans the Bretton Woods regime and the early years of the successive global float (Eichengreen, 2019; Ilzetzki et al., 2022). In geopolitical terms, it almost exactly covers the Cold War era (covering the unprecedented reduction in actual inter-state military conflicts after the first use of nuclear weapons). It ends amid the onset of the great moderation, the introduction of inflation targeting by advanced economy central banks, and the dissolution phase of the Soviet Union.

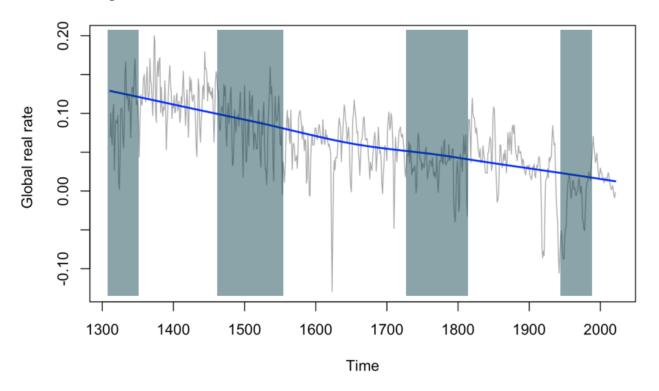


Figure 2: Global real rates, structural trend, and low-rate eras, 1311-2021

Notes: Global real rates displayed as nominal long-maturity rates deflated by progressively-lagged seven-year realized inflation. The four low-rate episodes (shaded) are 1311-1353, 1483-1541, 1732-1810, and 1937-1985. For details, see text above. Periods are narratively defined.

Are there any regularities that would unite these four eras? Without developing formal models or claiming to be conclusive, we note, for one, that all four periods are marked by relative geopolitical calm: not necessarily in terms of conflict incidence, but certainly in terms of interstate conflict *intensity*. This is even more apparent when we isolate the "Dominant" economy in respective eras, in other words the equivalent "safe asset provider": Northern Italy, prior to the arrival of the marauding "mercenary companies" from the mid-14th century, but after the peak of the Guelph-Ghibelline clashes, enjoys a phase of relative political idleness in the early 1300s (Caferro, 1998); Habsburg Spain under Charles V is almost uninterruptedly engaged in the struggle against the rival Valois, and takes an uncompromising stance against Protestant dissidents in the Germany prior to the Peace of Augsburg - however, these are low-intensity conflicts, nowhere near comparable with the internal religious wars in France, or the geopolitical and religious escalations

of the 17th century (Parker, 2019); for most of the eighteenth century, and only interrupted by the reluctant participation in the War of the Austrian Succession, the safe asset provider Britain under the dominating influence of the Duke of Newcastle retreated militarily into a "pose of studied insularity" (Scott, 2002). And of course, the second half of the 20th century - while marked by the intense ideological competition against communism - saw actual conflict intensity reach unprecedentedly low levels: perhaps not since the War of Saint Sabas in 1256 did the leading economy experience a longer stretch of abstention from direct inter-state conflict. While these periodizations can be underpinned by more quantitative evidence - including reference conflict chronologies by political scientists - as we reiterate later in this paper, all this does not imply that we see geopolitical risk premia as the driving force of secular real rate trends per se (or, more broadly, necessarily subscribe to a "rare disaster" framework that would be applicable here).³⁵

While we are able to go well beyond anecdotal associations - notable geopolitical events clearly appear to be a regular occurrence immediately before, after, and during our interest rate breaks - we are cautious to propose sweeping generalizations about the drivers of the structural breaks over time here. Whereas politico-financial crises appear to be a *necessary* component for a structural real interest rate break, they appear to be far from *sufficient*. What appears to be certain, at least, is that the evidence for a primary role for "monetary regime changes", in the form of either policy rule shifts or key institutional events - long a focal point of economic and econometric literature - appears to be surprisingly poorly supported in the long context. As flagged, not least, the foundational dates of central banks (both de facto and de jure), such as 1609 in Holland, 1694 in Britain, 1876 in Germany, 1893 in Italy, or 1914 in the U.S. all fall outside of our structural break samples.

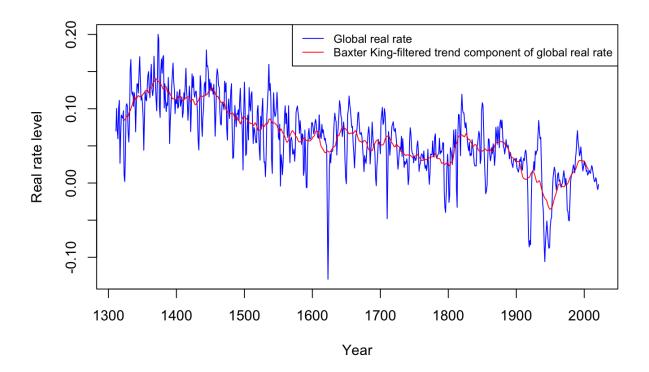
6 Real interest rates in the context of growth and demographic variables

The relationship between real interest rates and other macroeconomic variables has been of key interest in the literature for many years, but very long-run empirical investigations were hampered for the same reasons that prevented the study of very long-run interest rate trends: a lack of satisfactory data. Fortunately, over the most recent years in particular, significant advances have been made in the improvement of long-run output and population growth series - two variables of particular interest in the real interest rate literature. These advances allow us to relate our new evidence on real interest rates more granularly to output growth trends, and population growth over centuries. Table 6 reports simple period averages of our three key macro variables over time. All figures are based on an identically-weighted eight country sample. The early phase of the demographic sample still contains a non-negligible share of (decadal- or semi-centennial-level) interpolations - but this does not influence subsequent statements or calculated averages. It is evident from the table already that real interest rates and both population growth on the one hand, and real output growth rate on the other, move in divergent directions secularly. Afterwards,

³⁵Two such reference conflict chronologies include Clodfelter (2017) and Levy (1983), from the latter of which we can derive long-run statistical evidence on conflict intensity.

we discuss both variables in detail and undertake a few more formal exercises involving the use of a Baxter-King bandpass filter (Baxter and King, 1999) that removes short- and medium-run fluctuations (shorter than 100 years) from all variables. Figure 3 displays the global real rate together with its Baxter-King filtered long-run component. Beginning with long-run output growth





Notes: The figure displays the progressively-lagged global real rate series (raw data, blue line), together with the Baxter and King (1999) filtered long run trend of the same series, extracting fluctuations with cycles of length more than 100 years (red line).

trends, recent years have seen fundamental updates to the seminal work of Maddison (2010). While "big assumptions" (mainly involving interpolation) still have to be made in the underlying construction, new series have significantly improved granularity. New multi-century output reconstructions have been undertaken for Northern Italy (Malanima, 2011), France (Ridolfi and Nuvolari, 2021), Spain (de la Escosura et al., 2022), Germany (Pfister, 2021), Holland (van Zanden and van Leeuwen, 2012), and the U.K. (Broadberry and Fouquet, 2015). These typically rely on (wage-based) demand-side constructions, or more granular reconstructions of sectoral outputs.³⁶ The new developments are fortunate as they now allow a nuanced comparison both on the country-

³⁶In "demand-side approaches", the GDP series are constructed on the basis of (combinations of urban and rural) real wage data - assuming that they reflect aggregate income trends when combined with estimates of urbanization rates, and of trends in days worked: the series for France (Ridolfi and Nuvolari, 2021), Germany (Pfister, 2021), and Northern Italy Malanima (2011) fall into this group. English and Dutch data (van Zanden and van Leeuwen, 2012; Broadberry and Fouquet, 2015), meanwhile, rely on sectoral output reconstructions. For a recent alternative approach using wealth data to construct Italian GDP benchmarks, see van Zanden and Felice (2022).

level and on the aggregate level of our real interest rate trends, and associated trends in output. To arrive at aggregate real output growth rates, we GDP-weighted the country constituent output growth rates based on the aggregate total GDP share for the benchmark years (creating GDP shares based on Maddison's benchmark GDP figures, and interpolating the calculated benchmarks): Figure 4 displays the resulting series over the period 1311-2021, together with our respective global real rate series. We report the long-run trend in these variables, extracted using Baxter and King's bandpass filter tailored to retain only fluctuations at frequencies longer than 100 years. We observe modest - almost static - output dynamics over the first half of the series, with aggregate growth driven mainly by (moderate) levels of population growth prior to the 18th century, though with some more vigorous dynamics on the regional level: individual growth spurts do take place in the early modern period, with Dutch real per capita incomes rising by 82% between 1500-1600, for instance, and a more general 29% real per capita output increase for our entire sample over 1500-1700. A sharp acceleration is detectable across all country levels during the 18th and 19th centuries, giving rise to the familiar Industrial Revolution dynamics. During the second half of the 20th century, a deceleration is notable, with current (post-2008) rates of growth meaningfully below trend lines: yet, absolute rates of growth in the early 21st century remain highly elevated compared to very long-run averages.

Table 6: Macro Variables: Averages by Period (in percent per annum)								
1311-2021 1311-1500 1500-1800 1800-1914 1914-2021								
Real aggregate output growth rate	0.93	-0.03	0.41	2.04	2.88			
Aggregate population growth rate 0.30 -0.21 0.24					0.000			
Real interest rate 6.02 10.85 5.61 4.61								

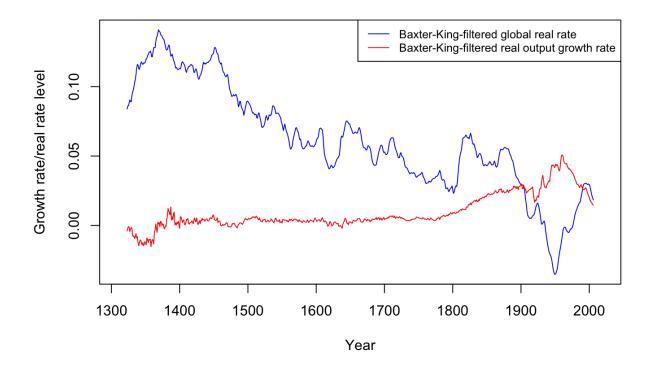
Note: The table reports the average value for three variables real aggregate output growth rate; aggregate population growth rate; real interest rate. All three variables are based on an identically-weighted sample of our eight advanced economies over time. For full underlying output and population sources, see text and notes to figures 3 and 4.

Crucially - and as is clearly visible from the chart itself - the output growth series overall displays a monotonic *upwards* trend.³⁷ The positive correlation between output growth and real rates appears to be a rather unique phenomenon of the very most recent part of the data series, starting in the second half of the 20th century. Generally speaking, therefore, real interest rates and aggregate output growth rates exhibit trend dynamics in directly *opposite* directions over the secular term.³⁸ Taken together, these observations imply that output growth trends and real interest rate trends do not have a correlation in the direction commonly assumed. In fact, over extended periods of time over the long-run, both measures in fact go in opposite directions.

³⁷This upwards trend is readily confirmed visually, but can also be obtained via standard tests such as Mann-Kendall (Mann, 1945; Kendall, 1955).

³⁸Of course, historically, countries that have accumulated very high debt will often experience extended periods of slower growth and higher real interest rates, as suggested by Reinhart et al. (2012), and now the central conclusion of a large literature as surveyed in Abbas et al. (2019). This would not, however, explain the long-term opposing trends.

Figure 4: A comparison of Baxter-King-filtered real aggregate output growth rates and real interest rates, 1311-2021.



Notes: The aggregate output growth rate covers identical eight country sample, with country constituent series weighted according to GDP-weights of the sum of country outputs. Sources include Malanima (2011) for Northern Italy; Pfister (2021) for Germany; de la Escosura et al. (2022) for Spain; Ridolfi and Nuvolari (2021) for France; van Zanden and van Leeuwen (2012) for Holland; Broadberry and Fouquet (2015) for the U.K. The red line displays the (filtered) long-run trend of the real aggregate output growth rate. The blue line displays the (filtered) long-run trend of the global real rate.

Demographic factors have been among the most frequently-invoked factors in the assessment of real interest rate dynamics in recent decades. Lunsford and West (2019) for instance consider demographic variables as the most plausible explanatory variables compared to growth and productivity variables - in particular, the authors find statistically significant roles for the share of 40-64-year olds in the population, as well as the dependency ratio, over the period 1890-2016, relying on U.S. data.³⁹ Gagnon et al. (2021) find that "demographic factors account for much, if not all of the actual permanent decline in the equilibrium real rate [since the 1980s]" - highlighting in particular falling fertility rates and mortality in the 1960s and 1970s as causal variables. But what does the (very) long-run data suggest for demographic context? A first-order reconstruction of population growth rates since the Renaissance can equally utilize Maddison (2010)'s data. But advances in output growth empirics since then have been mirrored by advances in demographic data for advanced economies, often undertaken by the same authors. New population estimates from which we derive aggregate population growth rates over the long-run - are equally provided

³⁹In Lunsford and West (2019), the dependency ratio is positively correlated with real rates, while the share of 40-64-year olds is negatively correlated; the authors find no statistically significant role for life expectancy.

by Malanima (2011), van Zanden and van Leeuwen (2012), Broadberry and Fouquet (2015), Ridolfi (2019, building on Dupaquier (1988)), Pfister (2021), and de la Escosura et al. (2022).

Figure 5 now displays long-run fluctuations in the aggregate population growth rate; as before, we extract fluctuations of 100 years and more using Baxter and King's bandpass filter. The series covers the sample 1311-2021 for the identical weighted country-sample that we use for the global real rate series. As is visible from the chart, and can be confirmed by the statistical exercise below, aggregate population growth rates and real interest rates have structurally trended in *opposite* directions, except for the most recent decades. During major population growth shocks, the two series are positively correlated, such as during sharp plunge in aggregate population growth is observed during the Black Death of 1348-1351, the most dramatic demographic "rare disaster" striking advanced economies since the Renaissance - a period coinciding with a sharp plunge in global real rates: but these episodes resulting in positive correlations remain a clear exception over the long-run.⁴⁰

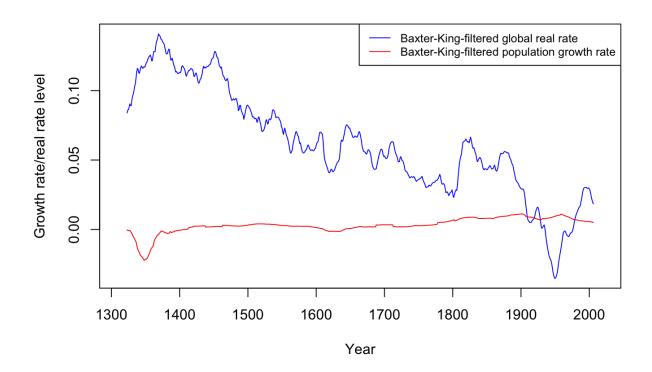
The finding that a secular upwards trend in population growth coincides with a downwards trend in real rates is not necessarily inconsistent with some recent literature - even though most contributions assume a positive correlation between the two variables (Rachel and Smith, 2016). But Carvalho et al. (2016) stress that population growth impacts (short-maturity) real rates through two divergent channels: a "supply effect" implying a rising (falling) population growth rate lowers (raises) capital per-worker, thus putting upwards (downwards) pressure on real rates; on the opposite, a (lagged) "demand effect" in which a rise (fall) in population growth should eventually lead to a lower (higher) dependency ratio, which generates downwards (upwards) pressure on real rates. Both channels are of course posited in the context of modern welfare states and the existence of substantial retirement periods: it is not obvious ex ante whether and to what extent prior to the introduction of modern welfare states and prolonged life expectancies, the balance between the two channels changes significantly.⁴¹ However, note that even Carvalho et al. (2016) posit that the forces generating a positive correlation between population growth and real rates dominate: therefore, similar to the evidence from output dynamics, the long-run evidence from our new data appears to run counter to recent causal demographic propositions of real rate dynamics.

Table 7 now reports the estimated correlation between the long-run filtered component of global real rates versus those of aggregate output and population growth. The table confirms a strong negative and statistically significant correlation.

⁴⁰Of course, we are studying here global sovereign borrowing rates - this does not necessarily imply that the rate of return on, say, land moved in the same way.

⁴¹Empirically, economy-wide life expectancy at birth for advanced economies accelerates sharply from the mid-19th century, but shows no decisive trend prior to this (thus also displaying an apparent disconnect with real interest rate dynamics). For a comprehensive recent empirical contribution to elite lifespan dynamics see Cummins (2017): though his sample over-weights English and Welsh data, his finding that European "elite" longevity increased sharply around two particular points, around 1400 and again around 1650, seems somewhat more consistent with our real rate evidence than data from either life expectancy at birth or population.

Figure 5: A comparison of Baxter-King-filtered aggregate population growth rates and real interest rates, 1311-2021.



Notes: The aggregate population growth rate covers identical eight country sample, with country constituent series weighted according to GDP-weights of the sum of country outputs. Sources include Malanima (2011) for Northern Italy; Pfister (2021) for Germany; de la Escosura et al. (2022) for Spain ("compromise estimate"); Dupaquier (1988) and Ridolfi (2019) for France; van Zanden and van Leeuwen (2012) for Holland; Broadberry and Fouquet (2015) for the U.K. The red line displays the (filtered) long-run trend of the aggregate population growth rate. The blue line displays the (filtered) long-run trend of the global real rate.

Table 7: Global Real Rates Vs. Aggregate Real Output and Population Growth									
Correlation 95% CI t-statistic p-value									
Baxter-King-filtered Long-run Compo-	-0.6425	(803,	-0.482)	-7.85	0.0000				
nent of Aggregate Population Growth									
Baxter-King-filtered Long-run Compo-	-0.7399	(-0.818,	-0.661)	-18.50	0.000				
nent of Aggregate Real Output Growth									

Note: The table reports the correlation of each series in the first column with Baxter and King's bandpass filtered long-run component of the global real rate, together with its 95 percent coverage confidence interval (CI). The bandpass filter is tailored to retain fluctuations larger than 100 years. The t-statistic is obtained using Newey and West (1987) standard errors with a maximum of 4 lags.

In sum, the long-run evidence puts doubts on both productivity-centered and demographics-centered explanations of the post-1980s trend fall in real interest rates, as represented for instance

by Gordon (2016), Holston et al. (2017), or Goodhart and Pradhan (2021). Several contributions have already voiced skepticism about productivity channels, including Hamilton et al. (2016) and Lunsford and West (2019) - we can confirm these from a much longer-term perspective. But importantly, we can qualitatively go beyond these existing facts, and find strong evidence in favor a long-run *negative* prediction of aggregate population growth rates and aggregate real output growth rates for real rates. The very most recent decades - where for both variables the correlation with real rates is positive - in fact appear to be a unique exception in a multi-century perspective.

Since demographics and productivity therefore cannot explain the 700-year trend decline in longterm real interests, is there a compelling alternative explanation? This important question is, unfortunately, beyond the scope of this paper: we can only offer a couple of suggestions for future research. Stefanski and Trew (2022), using the evidence in Schmelzing (2020) as motivation, offer a theoretical explanation based on trend rising social patience using a dynamic heterogenous agent model of fertility, in which more patient economic and social groups have more progeny. Changes in war are another possibility. Post-war Germany and Japan both (partially) defaulted outright on their pre-Second World War debt, Germany inflated away its First World War debt. Nevertheless, over time, the odds that any of the eight countries in our data set will outright default on any portion of their debt may have faded over time, even if the risk of inflating down the real value of the debt remains very much present.⁴² Relatedly, although fluctuations are massive, it is almost surely the case that trend liquidity in major government bond markets has been positive over the centuries and continues to be.⁴³ Whatever the answer(s), we can only say that it needs to be consistent with the evidence here on the fact that long-term real interest rates series appear to be trend stationary: and regardless of the underlying cause, the analysis does suggest that when real interest rates fall far below the (declining) long-term trend, there is a high chance of mean reversion unless of course there has been a rare structural break.

7 Conclusion

The sharp drop in the real interest rate in the 21st century, particularly in the years after the global financial crisis, has been arguably the most important macroeconomic development in modern

⁴²Alternatively, the gradual change in the *relative* financial risks between the private and public sectors may have induced a faster decline of sovereign risk premia, a hypothesis that would be consistent with secularly rising measures of bank stress in our eight economies since at least the 17th century (Metrick and Schmelzing, 2021).

⁴³One would of course expect substantial increases in market liquidity over our sample length. But to what extent this development is driven by financial broadening rather than deepening is not established. Interestingly, in a number of polities at at the relative forefront of innovation and market depth, market turnover in the early modern period is highly elevated: as expounded above, Mueller (1997) reports a turnover figure for Venetian Monte Vecchio sovereign debt in the year 1434 equivalent to over 28% of Venetian nominal GDP for the year. Hammel (1988), who assembled one of the most nuanced housing market datasets for multiple centuries (1284-1700) in a leading Imperial city found turnover rates of 3.4% - 6.8% of the housing stock in a given year, overwhelmingly debt-financed (the U.S. turnover equivalent in 2003-2018 stood at 5.4% for single family homes). At least for the U.K., evidence also suggests that liquidity factors may have been a more relevant factor for equity markets, rather than for bond markets (Campbell et al., 2018). As of now, we lack high-frequency bid-ask spread data which would be one variable able to provide particularly nuanced insights.

times. The main goal of our paper is to investigate to what extent the fall should be regarded as permanent (or even continuing), and to what extent can it be regarded as temporary, with expected eventual reversion to a longer-term trend. This paper has offered a novel approach to this problem by taking advantage of recent advances in quantitative financial history to get a better handle on the econometric properties of real interest rates than has previously been possible. Importantly, we focus on long-maturity sovereign debt (comparable to modern 10-year Treasury bonds) over eight centuries, from 1311-2021. This is in contrast to earlier research that has mainly focused on short rates, and uniformly with much shorter time periods. Importantly, prior to this paper, virtually all of the literature has been unable to reject the hypothesis of a unit root in real interest rates. The results here, based on much longer data sets, and on eight countries, are markedly different.

Overall, we do not find compelling evidence of a break in the long-run trend in global real interest rates since the 16th century, including across wars and changes in monetary regimes, although 1914 and 1981 show up in multiple variations of individual series. Not only do we strongly reject unit roots in almost every case (whether one allows for a trend or does not allow for a trend), we also find that little robust evidence of any recent structural break, and indeed most of very long-run series yield only a couple significant breaks with any consistency. We find no evidence for several prominent "global" dates where existing literature would lead one to expect such breaks with high confidence: multiple advanced economy series have in fact not seen robust structural breaks since the 16th century, and the same is true when isolating the "modern" safe long-dated assets, including U.S. Treasury bonds (from 1787) and U.K. Treasury bonds. Importantly, this does not seem to be a manifestation of how we apply our (very standard) methodology, as we are able to replicate the non-stationarity results for short rates found in earlier studies over their shorter time periods. This, in turn, suggests that it is a premium for short-term debt - potentially the particular safety and liquidity premia inherent in short term debt (Krishnamurthy and Vissing-Jorgensen, 2012) - rather than the real interest rate itself, that is non-stationary: at least this is an issue to be explored. It is unlikely that the reason our results are so different from the literature has to do with some anomalous feature of long-term sovereign bonds (not short-term debt); in fact, the appendix shows that we can equally reject the unit root hypothesis for long-maturity mortgage and corporate real rates, even though these series are much shorter than our headline sovereign rate data.

Regarding historical triggers of such structural breaks - factors prompting generational structural changes - many of the key inflection points are associated with deep combined politico-financial crises, such as 1557 on the global level (against the backdrop of the "Trinity default" coupled with political turmoil in major economies), the Spanish political-cum-fiscal breakdown of 1814, associated with double state default and the restoration of the monarchy. More broadly, however, financial stress - while repeatedly a prominent component - does not appear as a *sufficient* cause to prompt such historical breaks. On the other hand, severe geopolitical stress by itself appears sufficient, at least in several sub-periods and geographies, to trigger structural breaks at times - as, for instance, during the 15th century. What appears generally surprising in the context of prior literature is that monetary factors alone constitute much less of an obvious source of radical change

in very long samples: our approach raised doubts on multiple obvious monetary "inflections" including those previously associated with the establishment of central banks (such as 1694 or 1914), or major policy shifts - notably associated with 1981.

We are aware of the debate in the econometrics literature on the difficulties in distinguishing trend breaks from unit roots. However, the glaring downward trend in our 700-year real interest rates series is so consistent and clear it is hard to imagine not allowing for a trend, and in fact, and even for the series where we detect a trend break, the stationarity results still hold without allowing for one. Of course, further research is needed, though we are doubtful alternative approaches would radically change our novel finding that long-term real interest rates are stationary. We are also keenly aware that extrapolating our negative trend line another two centuries even at the glacial pace of 1.6 basis points per annum, would imply that the global trend real interest rate proceeds linearly, and ultimately reaches levels well below zero. A more likely possibility, perhaps, is that it will asymptote to a lower bound (positive or negative), though we do not see evidence of that yet and leave this issue to further study.

Finally, the recent literature has given a great deal of attention to demographics and productivity as drivers of today's real interest rates. Our research in no way rules this out, since it is possible there has been a modern structural break that has not yet lasted long enough to be statistically significant in the 700-year time series. Nevertheless, again making use of substantial recent advances in quantitative economic history, we can employ much-improved long-term demographic and real output data, which over most of our sample have been rising, not falling, yet the sovereign long-term real interest rate has trended down consistently. Other explanations for the very slow trend decline, including decreasing default risk and increasing liquidity, merit exploration in future research, though we do not presume an answer here. Hence, although we absolutely cannot conclude that the post-2008 epoch of seemingly very low real rates will eventually unwind back to a (gently declining) trend, we would argue that our data gives a valuable new perspective that may have important implications for understanding a broad range of modern macroeconomics and asset pricing issues.

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APPENDIX

1. Data construction - real rates

Here we elaborate on the construction of the "global real rate" time series in more detail: for an extensive discussion, readers should refer to the discussions in Schmelzing (2020) and Schmelzing (2022).

The country-level data focuses on a collection of long-maturity nominal sovereign interest rate datapoints. "Long-maturity" is defined as sovereign debt transactions with either a de jure (contracted) or de facto contract length of at least two years (e.g. perpetual instruments in practice redeemed well after two years, such as life or heritable annuities). A detailed data overview of both contracted and de facto maturities of hundreds of contracts included in the series yields an average maturity of the assets used over the period 1311-1830 of 13.2 years. The lowest average maturity for a 50-year subperiod is 6.1 years (1510-1560), the highest average maturity for any subperiod is 24 (1409-1459).

An innovative feature of the series in Schmelzing (2022) is that it features separate collections of "unconsolidated" and "consolidated" long-maturity debt transactions for the eight economies over time. "Unconsolidated" transactions are all transactions that were not formally structured on the basis of standardized, subscription-based issuance that involved a multitude of creditors. In the early modern period, sovereigns for a variety of purposes chose to engage in "unconsolidated" debt transactions: these transactions - to the extent they are included in the series - were fully voluntary, and secured, but de facto created an individualized one-to-one contract between a creditor (a wealthy merchant, a member of the nobility, a religious order) and the sovereign. Such transactions were quick to conclude when funds were needed swiftly, and they allowed tailored contract terms - but often they were conducted in parallel to standardized, consolidated issuance of debt. 935 such loans have been identified over the period of 1311-1866 across the eight economies. The majority of these datapoints are based on new primary and printed primary sources: among primary sources, the extensive Habsburg collections in the Austrian State Archives (OeStA) have been used, as well as the Prussian (GStaPK) and Dutch (StAAM) counterparts; examples in the printed primary space are the German Imperial Registries ("Regesta Imperii"), a source collection covering more than 145,000 medieval and early modern documents from the court and administration of the Holy Roman Emperors;44 In the British realm, the equivalent Calendar of State Papers spanning the reign of the Tudors to the Stuarts (1509-1714) were analyzed. Dozens of complementary registry collections contain datapoints on transactions involving ducal, regional, or merchant creditor counterparties. All these collections also contain relevant new data on consolidated debt transactions, for which new primary material was equally sourced: for instance, 19th century datapoints are based on daily newspaper and stock exchange reports (so-called Börsenkursblätter, held from 1832 in the Bethmann-Bank archive in Frankfurt); from 1866 for Spain, Italy, Germany, Holland, and the U.S. in the Frankfurter Zeitung). But for these latter assets, recent years have also seen advances in the secondary literature, and such recent contributions, including de Luca (2015, for new rates of the Duchy of Milan), Huang et al. (2019, for a new set of urban annuities), Gelderblom and Jonker (2020, for new Dutch market yields over 1596-1794), and Sussman (2022, for new 17th century British rates) have equally been integrated. In addition, separate samples of "short-maturity", intra-governmental, and "involuntary" debt transactions have been identified in the data collection process, which may be utilized in future exercises but are fully excluded from the present headline series.⁴⁵

⁴⁴The current stage of this long-running project is set to conclude in the year 2033 under the auspices of the "Deutsche Kommission für die Bearbeitung der Regesta Imperii e. V.", for an introduction see Schulz (2017).

⁴⁵In this category, we find also yields on "office sales", loans from quasi central banks (such as the Venetian *Banco Giro*

Methodologically, separate country series are then first constructed on the basis of these unconsolidated interest rates. Next, equivalent country series are constructed for consolidated interest rates - rates, hence, which refer exclusively to funded, standardized debt issuance transactions, as long as they are equally in the long-maturity spectrum. Finally, the respective unconsolidated and consolidated country series are fused for each country, by averaging the two observations for each year (one unconsolidated country average, one consolidated country average): this step yields the eight benchmark country series which respectively incorporate unconsolidated as well as consolidated debt operations.

Overwhelmingly, the debt transactions involved are denominated in gold currency: the most frequently used currencies are the Venetian ducat, the Florentine florin, the German Rheinflorin, the French écu, the Dutch guilder, and the Spanish ducat. With the weights and finesse of these international currencies increasingly being standardized in the early modern period, it was relatively difficult for the sovereign to engage in quasi-default on gold-denominated liabilities by way of a debasement operation: in practice, moreover, a significant share of debt contracts under consideration contained explicit clauses about the exact weight to be used in the coins for the repayment. Hence, while debasement represented an easy and frequently-used - recourse to diminish silver-denominated domestic liabilities, such as wage payments to mercenaries and other court employees - we should not assume the pricing of excessive interest rate premia contained in our datapoints related to debasement risk.⁴⁶ Similarly, most long-run inflation series are denominated in silver-price equivalents: the alternative is to base series on local-currency price changes, which tend to be more volatile in the short-run. Key reference works such as Allen (2001) report both bases: this allows a comparison of long-run trends, which have been conducted: while temporary divergences are notable over short time-spans, no meaningful long-run differences are observed for resulting real rate series, which is intuitive given that arbitrage dynamics and sufficient commodity market integration existed: whatever gaps between local currency and silver currency prices opened up tended to converge relatively swiftly, certainly over multi-year horizons (Federico et al., 2021).

The Global GW basis: when reference is made to "global real rates" in Schmelzing (2022) or in this paper, we refer - unless explicitly stated otherwise - to the series obtained by weighting the eight country series according to their respective rolling GDP shares over time. These GDP shares are obtained using the consistent definitions of population and per capita real GDP figures in Maddison (2010): the sum of the eight country-level aggregate GDP figures (population x per capita GDP, for each country, linearly interpolating between Maddison's benchmark years) represents the "global GDP" figure, with the U.S. and Japan entering the sample in 1790 and 1870, respectively. Alternative weighting approaches are of course possible, and while very minor, the recent advances in country-level accounting (as discussed above) have also slightly shifted the *relative* GDP shares over Maddison's figures - hence, our addition of the "Global AW" weighting approach in all exercises, which simply weights all eight countries with equal weights regardless of aggregate output. The fact that our results hold across these different approaches suggests that the particular weighting methodology is ultimately not decisive.

or the *Bank of Amsterdam*), loans from higher officials employed in the sovereign courts, "withholding taxes" (disguised forced loans), and war indemnities (also often disguised as loans).

⁴⁶For recent summaries and advances in this lengthy literature, see the introduction in Denzel (2010, LVIII-CX), and Karaman et al. (2020).

2. Robustness

We present robustness checks and a variety of additional results following from our benchmark results in the main body of the text. We begin by testing the data in using the ADF-GLS test in a specification *without* a time-trend, via table A.1.

Tables A.2 and A.3 in section 2.1 then proceed with variations on the inflation expectations construct used to calculate real interest rates: recall that our exercises in the main text follow the methodology in Homer and Sylla (2005), and use a progressively lagged seven-year realized inflation basis, not including the current year, in order to infer inflation expectations. Table A.2 alternatively follows the approach in Hamilton et al. (2016), and estimates inflation expectations via an autoregressive model: per table A.2 we can confirm all our ADF-GLS results in this approach, too. Second, we follow Eichengreen (2015), who uses an equal-weighted seven-year realized inflation basis, including the current year, to adjust U.S. nominal yields in order to obtain real rates. For all except one series (Global GW), we can equally confirm the ADF-GLS results from the main body.

In section 2.2, we undertake an analogous exercise varying inflation expectations approach for our Chow test, applying an equal-lagged inflation expectation approach (again in the spirit of Eichengreen (2015)) to the five break dates: we can confirm all our main results in this alternative setting (table A.4). Afterwards, in section 2.3, we test our basic ADF-GLS approach for a variety of alternative long-sample fixed income series, to assess whether there are indications that peculiarities inherent in our long-sample sovereign debt series may be driving results: in fact, however, as reported via tables A.5 and A.6, we find that stationarity can be confirmed for a number of alternative long-run datasets which mainly focus on British variables starting in the 18th and 19th centuries.

In the subsequent online appendix section (3.1 - 3.4), we first report the historical context for the single structural breaks suggested by the Zivot-Andrews (Zivot and Andrews, 1992) test, followed by Bai-Perron (Bai and Perron, 1998) results (table A.7) for all our series, and afterwards go on to show in particular that we can fully replicate the failure to reject unit roots in real rates when we apply our two alternative inflation expectations approaches for Rose (1988)'s shorter sample data (tables A.8.1-A.8.3). We confirm this failure to reject a unit root uniformly at the 1% level for all 16 possible real rate variations, including for his long-maturity corporate bond rate basis, as well as for his short-maturity commercial paper rate basis. This underscores that our headline results do not appear to be a function of how we specifically combine the individual nominal rate and inflation series. Finally, part 3.4 reports our Monte Carlo simulation results.

Table A.1 now reports stationarity test results based on the ADF-GLS test without including a time trend. The results confirm our main results in the body of the paper.

		LS Test without Time Trer	nd
	Rea	l Rate Series	
Region	Number of lags	ADF-GLS test	Optimal lag
Global GW	3	-5.064	MAIC
	2	-5.411	Seq, SIC
	1	- 7·735	
Global AW	3	-3.924	MAIC
	2	-4.134	Seq, SIC
	1	-5.848	
Italy	3	-9.480	MAIC
	2	-10.443	Seq, SIC
	1	-13.920	
UK	3	-3.006	
	2	-3.102	Seq, SIC, MAIC
	1	-4.281	
Dutch	3	-6.971	Seq, SIC, MAIC
	2	-8.385	
	1	-10.977	
France	3	-5.534	MAIC
	2	-5.963	Seq, SIC
	1	- 7.721	
Germany	3	-8.129	
	2	-8.078	Seq, SIC, MAIC
	1	-11.444	
Spain	3	-3.090	
	2	-3.155	Seq, SIC, MAIC
	1	-4.477	
U.S.	3	-4.910	
	2	-5.507	
	1	-5.730	Seq, SIC, MAIC
Japan	3	-3.256	
	2	-3.241	Seq, SIC, MAIC
	1	- 4.389	

Note: the table reports the test statistic for several number of lags (for a maximum of three lags). The regression includes a constant. For all series except for U.S. and Japan, the critical values at 1, 5, 10 percent significance levels are the following for all observations: -2.58 (1%); -1.95 (5%); -1.62 (10%). For the U.S., the critical values are -2.582; -1.95; -1.619. For Japan, they are -2.593; -1.95; -1.613. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

.1 Variations of inflation expectations

Hamilton et al. (2016) use an autoregressive inflation expectations approach. We replicate their approach for our multi-century data in table A.2 below. We calculate one-period-ahead inflation expectations following the same procedure as in Hamilton et al. That is, the evolution of inflation is recursively estimated in rolling windows using an autoregressive model with one lag; then, the estimated model is used to predict one-year-ahead inflation in an out-of-sample fashion. The rolling window estimation procedure guards against instabilities by using only past observations in the latest window of data. We use a window size equal to 30 years. As shown, we are able to reject a unit root with this inflation basis, too, for both the global (GW and AW), and all country series.

Second, Eichengreen (2015) uses a seven-year equal-lagged realized inflation expectation construct to adjust nominal yields, and includes current-year inflation (year t). We replicate his approach in table A.3 below. As shown, we are equally able to reject a unit root with this inflation basis, for all series, including Global AW - with the only exception of the Global GW series.

Ta	able A.2: ADF-GLS Test for the	e Hamilton et al. (2016)) inflation basis
	Real	Rate Series	
Region	Number of lags	ADF-GLS test	Optimal lag
Global GW	3	-6.137	Seq, MAIC, SIC
	2	-7.041	
	1	-9.338	
Global AW	3	-6.549	
	2	-6.751	Seq, SIC, MAIC
	1	-9.199	
Italy	3	-9.158	MAIC
	2	-10.013	Seq, SIC
	1	-13.428	
UK	3	-11.775	MAIC
	2	-13.740	
	1	-17.098	SIC, Seq (o)
Dutch	3	-3.663	Seq, MAIC
	2	-3.912	SIC
	1	-4.496	
France	3	-4.791	MAIC
	2	-5.143	Seq, SIC
	1	-6.385	
Germany	3	-5.117	Seq, MAIC, SIC
	2	-6.033	
	1	-9.357	
Spain	3	-6.684	MAIC
	2	-7.264	Seq, SIC
	1	-10.144	
U.S.	3	-4.159	
	2	-5.899	
	1	-6.968	SIC, MAIC, Seq
Japan	3	-3.633	
	2	-3.681	Seq, MAIC
	1	-4.840	SIC

Note: the table reports the test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. For all series except for Japan, the critical values at 1, 5, 10 percent significance levels are the following for all observations: -3.48 (1%); -2.89 (5%); -2.57 (10%). For Japan, the critical values are -3.518; -2.978; -2.688. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

	Table A.3: ADF-GLS Test for the Eichengreen (2015) inflation basis						
	Real	Rate Series					
Series	Number of lags	ADF-GLS test	Optimal lag				
Global GW	3	-2.505					
	2	-2.480	Seq, MAIC, SIC				
	1	-4.488					
Global AW	3	-3.365	Seq, MAIC, SIC				
	2	-3.405					
	1	-6.120					
Italy	3	-3.163					
	2	-3.109	Seq, MAIC, SIC				
	1	-5.749					
UK	3	-10.069	Seq, SIC				
	2	-9.100	MAIC				
	1	-15.852					
Dutch	3	-9.617					
	2	-10.232	Seq, SIC, MAIC				
	1	-18.259					
France	3	-5.585					
	2	-5.617	Seq, MAIC, SIC				
	1	-10.220					
Germany	3	-11.135	Seq, SIC				
	2	-9.651	MAIC				
	1	-18.133					
Spain	3	-6.946	Seq				
	2	-6.603	SIC, MAIC				
	1	-10.719					
U.S.	3	-6.964	Seq				
	2	-6.719	SIC, MAIC				
	1	-10.070					
Japan	3	-4.153					
	2	-3.983	Seq, SIC, MAIC				
	1	-6.973					

Note: the table reports the test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. For all series except for Japan, the critical values at 1, 5, 10 percent significance levels are the following for all observations: -3.48 (1%); -2.89 (5%); -2.57 (10%). For Japan, the critical values are -3.518; -2.978; -2.688. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

.2 Inflation expectation variation for Chow test

We now present the Chow test familiar from the main body via table A.4, but applying an equal-lagged inflation basis - analogous to the Eichengreen (2015) approach (which includes current year inflation) discussed for table A.3. We observe that all the main results from the progressively-lagged approach are confirmed, in particular the relevance of the 1349 and 1914 break points.

Table A	.4: Chow Test R	lesults - Glo	bal Real Rat	te GW - eg	ual-lagged i	nflation
	Coefficient	Std error	t-statistic	p-value	95%	Confidence interval
trendbreak 1349	.2992621	.3542793	0.84	0.399	3963187	.9948429
trendbreak 1557	0040828	.0131019	-0.31	0.755	0298066	.021641
trendbrak 1694	.0165211	.0162744	1.02	0.310	0154316	.0484739
trendbreak 1914	1537574	.0781571	-1.97	0.050	3072086	0003063
trendbreak 1981	.3392345	.3263694	1.04	0.299	301549	.9800179
meanbreak 1349	0023604	.0120145	-0.20	0.844	0259492	.0212284
meanbreak 1557	.0027016	.005392	0.50	0.616	0078848	.013288
meanbreak 1694	0099685	.0089481	-1.11	0.266	0275369	.0075999
meanbreak 1914	.1332954	.0687009	1.94	0.053	0015899	.2681806
meanbreak 1981	3078964	.3157292	-0.98	0.330	9277891	.3119964
trend	1948497	.2854704	-0.68	0.495	7553332	.3656338
constant	.1810158	.2802269	0.65	0.519	3691728	.7312044

Note: The Chow test is implemented using Newey and West (1987) standard errors with a maximum lag length of 4. The coefficient associated with "trendbreak 1981" denotes the difference of the estimated trend coefficients before and after a break in 1981 (and allowing for breaks in all the other potential break dates); hence, the associated t-statistic is the Chow test for the absence of a structural break in trend in 1981. Similarly, "meanbreak 1981" refers to a break in the mean in 1981.

.3 Testing alternative long-run rates series

In this subsection, we test a number of alternative long-sample fixed income datasets offered over recent years, to determine more precisely if our stationarity results could possibly be driven by particularities in our multi-century dataset, or whether a confirmation of such results for other series suggests that an extended sample length could result in common statistical properties for additional assets and geographies. For one, Dimsdale and Thomas (2016) constructed a series of long-maturity U.K. real interest rates on an annual basis over the period of 1703-2016. The authors construct a measure of inflation expectations to adjust nominal yields, describing their approach as follows: "Long term inflation expectations are based on a simple HP filter with lambda of 100 starting in the year 1600". To our knowledge, no existing literature has tested the statistical properties of this particular series, but to further investigate whether our conclusions on stationarity are robust to a variety of methodological approaches, we have conducted an equivalent ADF-GLS test, with time trend, which strongly confirm stationarity at the 5% and and 10% levels. We take these results as further indication that our conclusions are robust across a variety of inflation and data construction methodologies. Table A.5 contains details.

Next, table A.6 tests the statistical properties for an additional set of alternative fixed income series over the period of 1695-2021, 1853/4-2016, and 1913-2021, respectively. Our intention is to assess whether our result of trend stationarity for global sovereign real rates, as discussed in the main text above, extends to other asset classes. At least for selected countries and asset classes, this question can be addressed.

	Tal	ole A.5: UK rea	al rates 1	703-2016	1	
		ADF-GLS	with trer	nd		
Rates	N. of lags	ADF-GLS	Cri	itical val	ues	Optimal lag
		test statistic	1%	5%	10%	
		1703-	-2016			
U.K. Long-maturity rates	3	-3.356	-3.480	-2.890	-2.570	
	2	-3.477	-3.480	-2.890	-2.570	Seq, SIC, MAIC
	1	-2.842	-3.480	-2.890	-2.570	

Note: Using U.K. real rate series constructed by Dimsdale and Thomas (2016). The table reports the ADF-GLS test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

We find that, indeed, trend stationarity is confirmed for a number of additional asset bases, using equivalent real ex post rates, including British corporate bond real rates (long-maturity), and U.K. real mortgage rates (long-maturity) at the 1% critical level. Trend stationarity is equally confirmed for Bank of England real policy rates (short-maturity). The shorter U.S. real interest rate sample as collected by Shiller (2015), covering long-maturity government bonds over 1872-2015, (which we deflate here with current year year-on-year CPI change provided in the same database) on the other hand, only fails to reject non-stationarity at the 10% level.

Overall, these results suggest that stationarity properties of real asset yields and returns are not restricted to sovereign assets, at least in the post-1695 period. This cast doubts on the assertion that interest rates should be modeled as non-stationary - that they, essentially, follow a random walk over the long run.

Table A.6: ADF-GLS - Other Rate Variations								
Series Test statistic Critical values								
	(t-statistic for α_1)	1%	5%	10%				
1 - Shiller US Real Rate, 1872-2015	-2.6567	-3.46	-2.93	-2.64				
2 - BOE Bank Real Series, 1694-2016	-10.431	-3.48	-2.89	-2.57				
3 - UK Mortgage Real Series, 1853-2016	-4.2927	-3.46	-2.93	-2.64				
4 - UK Corporate Bond Real Series, 1854-2016	-4.4664	-3.46	-2.93	-2.64				

The estimated model is an autoregression with three lags, a constant and a deterministic trend. The test rejects a unit root when the t-statistic is negative and, in absolute value, larger than the critical value.

Notes: [1], Shiller real rate, uses series "U.S. long government bond rate" data in Shiller (2015), deflated by seven-year progressively lagged change in "consumer prince index" series, both over 1913-2015, and both available from the author's homepage (http://www.econ.yale.edu/ shiller/data.htm); [2-4] use British interest rate series in the BoE's Millennium dataset, via Dimsdale and Thomas (2016). Deflation is throughout undertaken via seven-year progressively lagged headline U.K. consumer price index change, via the same source. The corporate bond series combines industrial and railway debentures and is long-maturity. The BoE bank rate is defined as the minimum lending rate between 1695-1972, the minimum band 1 dealing rate for 1981-1997, the repo rate for 1997-2006, and the bank rate for 2006-2016.

3. Online Only Appendix

.1 Zivot-Andrews break context

The Zivot and Andrews (Zivot and Andrews, 1992) test is a test for unit roots robust to instabilities. As part of the procedure, it is possible to estimate the most likely break date in the sample. The break dates identified are the following. For the global real rate series: 1937 for the Global Real GW rate and 1803 for the Global Real AW rate. For the individual real rate series: 1916 for Italy; 1553 for the UK; 1437 for Dutch; 1937 for France; 1628 for Germany; 1814 for Spain, 1850 for the US; and 1938 for Japan (although the latter is not significant). We here discuss the historical context of these break dates. While the break dates indeed have historical plausibility, we reiterate that we have a strong conviction in our choice of the five Chow test dates based on our read of the long-run historical context and literature.

- Three of the Zivot-Andrews dates, including the headline global real series, are obtained for years immediately before the outbreak of the Second World War:
 - First, the interwar period is a period of general political volatility. Economically, it is identified with the transition from the U.K. to the U.S. as the global "safe asset provider", as well as the chaotic experimentation with the gold-exchange standard (Chitu et al., 2014), and major tail events such as the German hyperinflation and the Great Depression. The latter two events, however, are not specifically overlapping with real rate breaks.⁴⁷
 - The global break appears to be driven by country-level breaks in or around 1937 in multiple countries, including France (1937), and Japan (1938), on the Zivot-Andrews basis. What does specific country-level context suggest for possible drivers of structural real rate breaks?
 - In French monetary history, 1937 equally represents a pivotal year, following the country's belated and rocky departure from the gold standard, the conclusion of the Tripartite Agreement, and the de facto nationalization of the Banque de France (Mouré, 2002; Harris, 2021).
 - In Japan, 1938 follows the outbreak of the Second Sino-Japanese War (Japan's de facto entering of the Second World War). Financial and monetary historians associate the years 1937-8 with the country's full-scale adoption of financial repression and debt monetization, including a loss of BoJ independence, and price and capital controls (Shizume, 2017).
- Germany's break date (1628) is set during the upheavals of the Thirty Years' War, and just after the unprecedented "Kipper- and Wipper" debasement crisis which plunged entire Central and Western Europe into a currency chaos. Politically, the year 1626 records several major geopolitical engagements in the country's North, with favorable outcomes for the Catholic side under Albrecht Wallenstein. While the full fiscal disintegration of the Holy Roman Empire and its constituent polities is typically dated to the late phase of the war (1635-1648), one can plausibly posit a combined politico-financial collapse as the driver for this break (Münkler, 2017).
- The year 1916 stands out in Italian monetary and political history. After the belated entry into the First World War in 1915, Italian public finances began to deteriorate sharply, with deficits reaching 60% of national income by 1918; by late 1916, the volume of government notes in circulation had already almost tripled compared to the pre-war volume (Fratianni and Spinelli, 1997, 108ff.).⁴⁸

⁴⁷As noted above, the German, Italian, and Japanese hyperinflation years are excluded from the Schmelzing (2022) data, but war years generally are included.

⁴⁸As detailed above, we exclude the Italian 'hyperinflation' years of 1943-5, associated with the double invasion of

- The British break date (1553) immediately follows the Great Debasement begun by Henry VIII, the largest known currency debasement in the country's history (Munro, 2010; Karaman et al., 2020).
- The Spanish break (1814) coincides with the liberation of Spain from Napoleonic occupation and royal restoration, following a disastrous inflation surge, a double sovereign default (first under the Cortes de Cadiz in 1812-3, then under Fernando VI in 1814), and institutional and economic breakdown (Comin, 2012; Pousada, 2015; Simal, 2017).
- In his classic study on Burgundy, Herman van der Wee called the years 1437-8 "the final blow" to the region's medieval economy (van der Wee, 1963, 61). Our Dutch break (1437) coincides with a period of deep state crisis in the Low Countries: popular revolts break out in that year in the major merchant hubs of Ghent and Bruges, both the English Crown and the Emperor Sigismund are engaged in military invasions to remove the Burgundian ruler Philip the Good from power, and in 1438, a full-scale trade war with the Hanse breaks out, halting Dutch Eastern maritime trade. On top of it all, a severe famine and extreme cold during 1437-8 aggravated widespread death and subsequent epidemics, as well as contributing to a further surge in inflation rates (reaching more than 36% for the year 1437) Bolton and Bruscoli (2008). "The perspective that emerges is one of turbulence and disorder, of danger and crisis, in the history of the Burgundian state", in the words of Vaughan (1970).
- The U.S. break (1850) coincides with major institutional reform in public finance and corporate governance, as reconstructed by Wallis (2005). Wallis argues that the period of 1842 to 1852 following recurrent state defaults by 1840 significantly strengthened secure private property rights in the U.S. Particular macroeconomic shocks, including the financial crises of 1838-40, and later 1857, are somewhat removed from our structural break, as is the major geopolitical shock of 1861-5.

To us, this list underscores our read of the historical context discussed in the main body, namely that major political-cum-economic upheavals are plausible contenders to explain structural breaks in real interest rates over time: it appears that political shocks can be accompanied by monetary shocks, but that the latter do not typically cause a break on their own. In particular, we do not read the evidence as necessarily suggesting that major FX transitions are themselves a sufficient factor in real interest rate inflections: we note that individual countries which departed the gold standard well before 1914 (namely Spain) still appear to have shown no trend break at this earlier date (Obstfeld et al., 2005). Neither do FX transitions without accompanying political escalations result in positive Chow identifications when tested (e.g. 1971).

.2 Bai-Perron break dates

To begin, Table A.7 displays Bai-Perron structural breaks in the global real rate series, following Bai and Perron (1998). The Bai-Perron test - here the test assumes a deterministic trend of the underlying series, given our ADF-GLS results - does not "impose" structural breaks: none, one, or multiple structural breaks are possible, up to a maximum of five breaks. The results yield an optimal partition of six periods for global real rates (GW and AW), and equally six periods for all country series, with most showing a break in the first half of the 15th century, the mid-16th century, the second half of the 17th century, the early 19th century, and during the First World War. The U.K., France and Spain display a break during the early or late 18th century, respectively, and only the U.S. and Japan display breaks after the First World War.

Italy from the Nazis in the North ("Fall Achse") and Allies in the South ("Operation Avalanche"): economically, the years are associated with a major plunge in industrial output and exceptional deficit financing; the Lire exchange rate with the dollar which stood at 19 in the year 1940, was fixed at 100 in 1943 (Fratianni and Spinelli, 1997, 166f.). If these years were included, the year 1943 would mark the Zivot-Andrews break point.

Like the breaks considered by the Chow test in the main body, the break dates thus obtained retain a high degree of plausibility. For instance, the French and U.K. breaks occur during crucial phases of the Hundred Years' War; the U.K. does not record a break during the Napoleonic Wars, during which it is the only country not to experience war on its own soil; the U.S. does not record a break during World War One, when it is the last entrant and only peripherally involved.

	Table A.7: Bai-Perron Test Results				
Series	N. breaks	Estimated break dates			
Global Real GW	5	1416 1524 1630 1806 1915			
Global Real AW	5	1428 1552 1670 1806 1915			
Italy Real	5	1443 1557 1696 1802 1915			
UK Real	5	1416 1552 1658 1775 1892			
Dutch Real	5	1465 1564 1663 1802 1921			
France Real	5	1423 1565 1672 1793 1915			
Germany Real	5	1417 1524 1630 1809 1915			
Spain Real	5	1436 1594 1711 1814 1917			
US Real	5	1825 1869 1907 1941 1983			
Japan Real	5	1891 1926 1952 1974 1996			

Note: the table reports the results of the sequential Bai and Perron's test (Bai and Perron, 1998) for an unbalanced panel that uses the period of 1311-2021. The test is implemented in Stata using the xtbreak routine by Ditzen et al. (2021). The model includes a constant and a deterministic trend (as a fraction of the total sample size) and we test for breaks in both the constant as well as the slope of the deterministic trend. The test is implemented using a HAC variance estimator, the trimming parameter is 15 percent and the maximum number of break points is 5 for all series.

.3 Replicating Rose (1988) with two inflation expectation variations

Tables A.8.1-A.8.3 replicate Rose (1988)'s results for U.S. shorter sample unit root tests, including long-maturity corporate bond yields (CB) and short-maturity commercial paper rates (CP), using his two alternative observation periods, 1892-1970 and 1901-1950, and using annual nominal (table A.8.1) and real rate (tables A.8.2 and A.8.3) data. We use the same data for both nominal rates and inflation, sourcing from Friedman and Schwartz (1963) and Nelson and Plosser (1982): the latter data is taken from Peter C.B. Phillips data page (http://korora.econ.yale.edu/phillips/data/npenp.dat). Table A.8.2 uses the progressively-lagged inflation expectation approach as in Homer and Sylla (2005); table A.8.3 uses the equal-lagged approach as in Eichengreen (2015)).

We observe that once we apply our two alternative inflation expectation approaches to his underlying data - which for the underlying price data is based on a GNP deflator series, and alternatively a CPI index, both of which we test - and construct equivalent real rate samples we are unable to reject a unit root for all progressively-lagged real rate variations (table A.8.2), as well as for equal-lagged variations (table A.8.3), with no exceptions. In particular, we emphasize that we replicate Rose's results when allowing for a deterministic trend in ADF-GLS.

		11 40 7	(00)			
	'l'a	ble A.8.1: Rose			n	
		Nominal	rate seri	ies		
Rates	N. of lags	ADF-GLS	Cr	Critical values		Optimal lag
		test statistic	1%	5%	10%	
		1892	2-1970			
Corporate bond yields	3	-0.793	-3.660	-3.097	-2.803	
	2	-0.523	-3.660	-3.097	-2.803	
	1	-0.246	-3.660	-3.097	-2.803	Seq, SIC, MAIC
Commercial paper rates	3	-1.429	-3.660	-3.097	-2.803	Seq, SIC, MAIC
	2	-0.785	-3.660	-3.097	-2.803	
	1	-1.763	-3.660	-3.097	-2.803	
		190:	1-1950			
Corporate bond yields	3	-1.494	-3.770	-3.190	-2.890	
	2	-1.212	-3.770	-3.190	- 2.890	
	1	-1.355	-3.770	-3.190	-2.890	Seq, SIC, MAIC
Corporate bond yields	3	-2.164	-3.770	-3.190	-2.890	
	2	-1.766	-3.770	-3.190	-2.890	MAIC
	1	-2.405	-3.770	-3.190	-2.890	Seq(o), SIC

Note: The table reports the ADF-GLS test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). Seq(o) denotes cases where the sequential lag length procedure selects zero lags. The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

	Ta	ble A.8.2: Rose	e (1988) r	eplicatio	n	
	Real rate	e series - progi	ressively	lagged i	nflation	
Rates	N. of lags	N. of lags ADF-GLS Critical values			ues	Optimal lag
		test statistic	1%	5%	10%	
		GNP	Deflator			
		1892	2-1970			
Corporate bond yields	3	-2.669	-3.660	-3.097	-2.803	
	2	-2.757	-3.660	-3.097	-2.803	
	1	-3.026	-3.660	-3.097	-2.803	Seq, SIC, MAIC
Commercial paper rates	3	-2.452	-3.660	-3.097	-2.803	Seq
	2	-2.048	-3.660	-3.097	-2.803	MAIC
	1	-2.570	-3.660	-3.097	-2.803	SIC
		190:	1-1950			
Corporate bond yields	3	-2.257	-3.770	-3.190	-2.890	
	2	-2.340	-3.770	-3.190	-2.890	
	1	-2.614	-3.770	-3.190	-2.890	Seq, SIC, MAIC
Commercial paper rates	3	-2.261	-3.770	-3.190	-2.890	Seq, SIC, MAIC
	2	-2.084	-3.770	-3.190	-2.890	
	1	-2.41 0	-3.770	-3.190	-2.890	
		СРІ	Index			
		1892	2-1970			
Corporate bond yields	3	-2.751	-3.660	-3.097	-2.803	
	2	-2.655	-3.660	-3.097	-2.803	Seq, MAIC
	1	-3.533	-3.660	-3.097	-2.803	SIC
Commercial paper rates	3	-2.571	-3.660	-3.097	-2.803	Seq, SIC
	2	-2.054	-3.660	-3.097	-2.803	MAIC
	1	-2.873	-3.660	-3.097	-2.803	
		190:	1-1950			
Corporate bond yields	3	-2.196	-3.770	-3.190	-2.890	
	2	-2.189	-3.770	-3.190	-2.890	MAIC
	1	-3.005	-3.770	-3.190	-2.890	Seq, SIC
Commercial paper rates	3	-2.246	-3.770	-3.190	-2.890	
	2	-2.029	-3.770	-3.190	-2.890	MAIC
	3	-2.677	-3.770	-3.190	-2.890	Seq(o), SIC

Note: The inflation approach follows Homer and Sylla (2005), using seven-year progressively lagged inflation (*t*-6 to *t*, excluding the current-year inflation *t*). The GNP deflator is from Nelson and Plosser (1982); the CPI index is from Nelson and Plosser (1982). The table reports the ADF-GLS test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). Seq(o) denotes cases where the sequential procedure selects zero lags. The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

	Та	ble A.8.3: Rose	e (1988) 1	eplicatio	n	
	Real	rate series - e	qual lagg	ged inflat	tion	
Rates	N. of lags ADF-GLS Critical values		ues	Optimal lag		
		test statistic	1%	5%	10%	
		GNP	Deflator			
		189:	2-1970			
Corporate bond yields	3	-2.303	-3.660	-3.097	-2.803	
	2	-2.220	-3.660	-3.097	-2.803	
	1	-2.078	-3.660	-3.097	-2.803	Seq, SIC, MAIC
Commercial paper rates	3	-2.080	-3.660	-3.097	-2.803	Seq
	2	-1.428	-3.660	-3.097	-2.803	
	1	-1.659	-3.660	-3.097	-2.803	SIC, MAIC
		190	1-1950			
Corporate bond yields	3	-2.214	-3.770	-3.190	-2.890	
	2	-2.13 0	-3.770	-3.190	-2.890	
	1	-1.945	-3.770	-3.190	-2.890	Seq, SIC, MAIC
Commercial paper rates	3	-2.135	-3.770	-3.190	-2.890	
	2	-1.886	-3.770	-3.190	-2.890	
	1	-1.969	-3.770	-3.190	-2.890	Seq, SIC, MAIC
		CDI				
			Index			
			2-1970			
Corporate bond yields	3	-2.657	-3.660	-3.097	-2.803	
	2	-2.438	-3.660	-3.097	-2.803	MAIC
	1	-2.793	-3.660	-3.097	-2.803	Seq, SIC
Commercial paper rates	3	-2.857	-3.660	-3.097	-2.803	Seq, SIC, MAIC
	2	-1.653	-3.660	-3.097	-2.803	
	1	-2.080	-3.660	-3.097	-2.803	
		190	1-1950			
Corporate bond yields	3	-2.196	-3.770	-3.190	-2 .890	
	2	-2.137	-3.770	-3.190	- 2.890	
	1	-2.339	-3.770	-3.190	- 2.890	Seq, SIC, MAIC
Commercial paper rates	3	-2.486	-3.770	-3.190	-2.890	Seq
	2	-1.870	-3.770	-3.190	-2.890	
	3	-2.152	-3.770	-3.190	-2.890	SIC, MAIC

Note: The inflation approach follows Eichengreen (2015), using seven-year equal lagged inflation (*t*-7 to *t*-1, including the current-year inflation *t*. The GNP deflator is from Nelson and Plosser (1982); the CPI index is from Nelson and Plosser (1982). The table reports the ADF-GLS test statistic for several number of lags (for a maximum of three lags). The regression includes a constant and a time trend. "Optimal lag" indicates the optimal number of lags according to the sequential procedure ("Seq"), the SIC, or the Modified Information Criterion (MAIC). The test rejects when the test statistic is negative and larger (in absolute value) than the critical value.

.4 Monte Carlo simulations on the power of the ADF-GLS test for the global real rate

To investigate the use of long versus small sample sizes in testing for unit roots in our data, we performed a small Monte Carlo simulation exercise. We generated 5000 time series that have the same features as our (stationary) global real rate (such as the mean, the deterministic trend and the standard error): $y_t = 0.7437y_{t-1} + 0.31 - 0.0000436t + 0.02313\varepsilon_t$, where ε_t is an independent, zero mean and unit variance Gaussian random variable. For each generated series, we calculated: (a) the ADF-GLS test statistic for a sample of 711 observations (the same size as our full sample); (b) the ADF-GLS test statistic for the sub-sample including the first 100 observations; (c) the ADF-GLS test statistic for the sub-sample including the first 50 observations. We then report the number of times the ADF-GLS test rejects a unit root at the 5 percent significance level across Monte Carlo simulations.

The simulation results indicate that the ADF-GLS test statistic rejects a unit root 100 percent of the times in the full sample, whereas it does so only 77 percent of the time in the sample with 100 observations and 37 percent of the time in the sample with 50 observations. This Monte Carlo simulation confirms the lack of power of the ADF-GLS test for time series such as the global real rate when the span of the data is short.

The average estimated half-life across Monte Carlo simulations is about 3 periods (as the random series are generated to share the same features as the observed data). Thus, our estimated half-life of 3 periods is not incompatible with the fact that the test would need a relatively large sample (such as our full sample of 711 observations) to have large power.