Debt, deficits and inflation in a low interest rate environment

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Zero real rates on government debt as a widow's cruse

- It seems that, when real rates on government debt are zero, debt finance is possible with no requirement for future primary surpluses to back the debt.
- Why, then, raise taxes when raising expenditures? Taxes distort, after all.
- This idea has recently emerged on the political left as "MMT", following a version of it articulated by VP Cheney: "Reagan taught us that deficits don't matter."

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The “intemporal government budget constraint”

Step 1: Invoke the actual GBC ($\tau = x - g$):

$$\dot{B} = rB - \tau \ .$$

Step 4: Solve forward

$$B_t = \int_0^\infty e^{-rs} \tau_{t+s} \, ds \ .$$

Step 2: General solution

$$B_t = \int_0^\infty e^{-rs}\tau_{t+s}\,ds + \kappa e^{rt}$$

Step 3: Invoke "transversality"

$$e^{-rt}B_t \xrightarrow[t\to\infty]{} 0$$

But step 3 is a mistake. Generally, if r is the real rate on government debt and r is less than the private discount rate β, it is possible for real debt to grow indefinitely at the rate r in equilibrium. The "intertemporal budget constraint" on the government derived this way is not only not a constraint, it need not even hold in equilibrium.

A simple equilibrium model with a liquidity premium on B and distorting tax

- Government debt in the budget constraint, as providing transaction services, so that it is return-dominated.
- Only input is labor, L.
- Just one kind of government liability: nominal, duration zero.

Private sector

$$\max_{C,L,B} \int_0^\infty e^{-\beta t}(\log C_t - L_t)\, dt$$

subject to

$$C \cdot (1 + \gamma v) + \frac{\dot{B}}{P} = (1 - \tau)L + \frac{rB}{P}$$

$$v = \frac{PC}{B}\,.$$

Private FOC's

$$
\begin{aligned}
\partial C: &\qquad \frac{1}{C} = \lambda \cdot (1 + 2\gamma v) \\
\partial L: &\qquad -1 = -\lambda \cdot (1 - \tau) \\
\partial B: &\qquad \frac{-\dot{\lambda} + \lambda\beta + \lambda\dot{P}/P}{P} = \frac{r\lambda}{P}\frac{\lambda\gamma v^2}{P} \\
TVC: &\qquad \frac{\lambda B}{P} e^{-\beta t} = \frac{B e^{-\beta t}}{(1-\tau)P} \xrightarrow[t \to \infty]{} 0
\end{aligned}
$$

Equation system

Solving private FOC's to eliminate λ:

$$C \cdot (1 + 2\gamma v) = 1 - \tau$$

$$\frac{\dot{C}}{C} + \frac{2\gamma \dot{v}}{1 + 2\gamma v} = \gamma v^2 - \beta - \frac{\dot{P}}{P} + r$$

Adding GBC and SRC:

$$SRC: \qquad C \cdot (1 + \gamma v) + G = L$$

$$GBC: \qquad \frac{\dot{B}}{P} + \tau L = G + \frac{rB}{P} \,.$$

Solving the GBC forward

To combine the GBC with the private TVC to get a valid equiibrium condition, we need to rearrange the GBC so that β appears in it (while at the same time writing it in terms of real debt $b = B/P$):

$$\dot{b} + b\frac{\dot{P}}{P} + \tau L = G - \frac{(\beta - r)B}{P} + \beta\frac{B}{P}\,.$$

We'll call the gap between the discount rate and the return on government debt, times real debt, seigniorage, σ. Then we can invoke the private TVC to solve the GBC forward as

$$b_t = \int_0^\infty e^{-\beta s}(G_{t+s} - \tau_{t+s}L_{t+s} - \sigma_{t+s})\,ds$$

Interpreting the solved-forward GBC

- Seigniorage and tax revenues together, discounted to the present at the rate β, determine the current real value of the debt.
- The seigniorage term is itself a kind of tax. It is income forgone by private agents in order to access liquidity services.
- The Friedman rule for zero-interest debt (money) is to set $r = 0$, which requires steady deflation at the rate $-\beta$.
- The informal argument is that nominal debt can be created by the government at no real cost, so ideally demand for it should be saturated.

The Friedman rule is costly

- In this model, the solved-forward GBC tells us that increases in debt, if they are not offset by inflation, must be financed either by higher future labor tax revenues or higher future seigniorage.

- In this model, because it is impossible to saturate demand for liquidity at a finite level of debt, the Friedman rule is not even feasible.

Comparing steady states with constant tax rate τ

- A sudden increase in γ, implying an increased demand for liquidity, will, with τ fixed, require sudden and large deflation, even though in this flexprice model it has no real effect.
- To avoid the sudden deflation, the government would have to run a very temporary, very large, flow deficit, financing a wealth transfer to the private sector, so that B/P can increase without a decrease in P.
- A permanent increase in g, with no accompanying increase in τ, requires a corresponding increase in seigniorage, which may require very high inflation and a large increase in the fraction of output absorbed by liquidity services.

Two interpretations of MMT?

- One would be that very large increases in deficits are perfectly justifiable when the economy is at the ZLB (spread between return on debt and real assets is high), though some modest tax increase is required later to avoid high inflation.

- Another would be that a g increase with no corresponding tax increase is feasible and good policy. This might be approximately true for small g increases, but not for major ones, e.g. a move from $g = .3$ to $g = .4$.

Example numerical solutions

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.97	1.31	0.97	-0.0194	-1.00	0.03	-0.0254	0.0007	0.76
0.01	0.94	2.70	0.94	-0.0188	-1.01	0.05	-0.0507	0.0035	0.37
0.1	0.87	5.27	0.88	-0.0173	-1.02	0.10	-0.0912	0.0162	0.19
1	0.72	9.27	0.78	-0.0139	-1.10	0.17	-0.1293	0.0722	0.11

Table 1: Optimal steady state with $G = 0$, $\beta = .02$

γ: transactions cost parameter; C: consumption; b/y: debt/output; L: labor; $\dot{P}/P$: inflation rate; U: utility; τ: labor tax rate; σ: seigniorage revenue; $\gamma v/(1 + \gamma v)$: proportion of consumption expenditure absorbed by transaction costs; P_0 : initial price level, assuming $M_0 = 1$; G non-productive government expenditure; β: discount rate.

Example numerical solutions

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.74	0.28	0.99	-0.0131	-1.29	0.26	-0.0037	0.0026	3.56
0.01	0.72	0.83	0.98	-0.0124	-1.30	0.27	-0.0103	0.0086	1.21
0.1	0.67	2.19	0.94	-0.0107	-1.34	0.29	-0.0234	0.0296	0.46
1	0.55	4.68	0.86	-0.0063	-1.46	0.32	-0.0295	0.1047	0.21

Table 2: Optimal steady state with $G = .25$, $\beta = .02$

See notes to Table 1

Example numerical solutions

γ	C	b/y	L	$\dot{P}/P$	U	τ	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.001	0.20	0.02	1.00	0.0623	-2.63	0.80	0.0013	0.0090	46.27
0.01	0.19	0.06	0.99	0.0655	-2.66	0.80	0.0042	0.0284	15.55
0.1	0.17	0.17	0.98	0.0755	-2.78	0.80	0.0129	0.0890	5.87
1	0.12	0.37	0.96	0.0937	-3.06	0.79	0.0342	0.2522	2.74

Table 3: Optimal steady state with $G = .8$, $\beta = .02$

See notes to Table 1

Example numerical solutions

G	τ	C	b/y	L	$\dot{P}/P$	U	σ	$\frac{\gamma v}{1+\gamma v}$	P_0
0.20	0.22	0.77	1.00	0.97	-0.0141	-1.238	-0.0142	0.0076	1.00
0.25	0.27	0.72	0.83	0.98	-0.0124	-1.304	-0.0103	0.0086	1.21
0.20	0.27	0.73	2.51	0.93	-0.0192	-1.247	-0.0481	0.0029	0.40
0.25	0.22	0.71	0.15	1.00	0.2081	-1.336	0.0310	0.0456	6.71

Table 4: Optimal and suboptimal financing of G

See notes to Table 1 for variable definitions. Lines 1 and 2 show solutions with optimal tax rates for the given G values. Lines 3 and 4 are solutions for given G and τ, with no optimization. Comparing lines 1 and 4 shows the change in going from $G = .20$ with optimal τ to $G = .25$ with unchanged τ. Comparing lines 2 and 3 shows the reverse case.

Conclusion: Lessons from this exercise

- The reasoning behind the Friedman rule relies on an environment with low distortion from other taxes.

- If all you have available are labor taxes, the inflation tax may be useful, but in this paper's model it cannot generate much revenue without imposing very high real costs. This seems inherent in the fact that the liquidity services of government debt must be a small fraction of output at modest levels of inflation.

- If real yields on debt are low because of the liquidity services of government debt, this represents a fiscal resource, but also a distorting tax, not a widow's cruse.