Fiscal Rules and Market Discipline

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Abstract

Fiscal rules have proliferated as a way to limit public debt. Rules intend to impose fiscal discipline on governments that might be otherwise present-biased. However, lenders also discipline government borrowing through a market mechanism, with excessive debt penalized with higher interest rates. In this paper, we study the interaction between fiscal rules and market discipline in limiting government borrowing. We do so in a sovereign borrowing model with asymmetric information about governments' propensity to over-borrow and default. Governments may signal their fiscal rectitude by showing fiscal restraint and this can lead not only to over-borrowing as in traditional models, but also to under-borrowing, as governments attempt to signal their fiscal responsibility. In addition to its traditional role of restraining present-biased governments, fiscal rules also make signalling more difficult and may have the perverse effect of forcing prudent governments to save even more excessively than otherwise or alternatively hamper their ability to signal their rectitude entirely. Fiscal rules restrain impatient governments but penalize prudent governments. An optimal fiscal rule balances these trade-offs and will be binding at times, but will never be so tight as to naïvely push governments to "do the right thing".

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1 Introduction

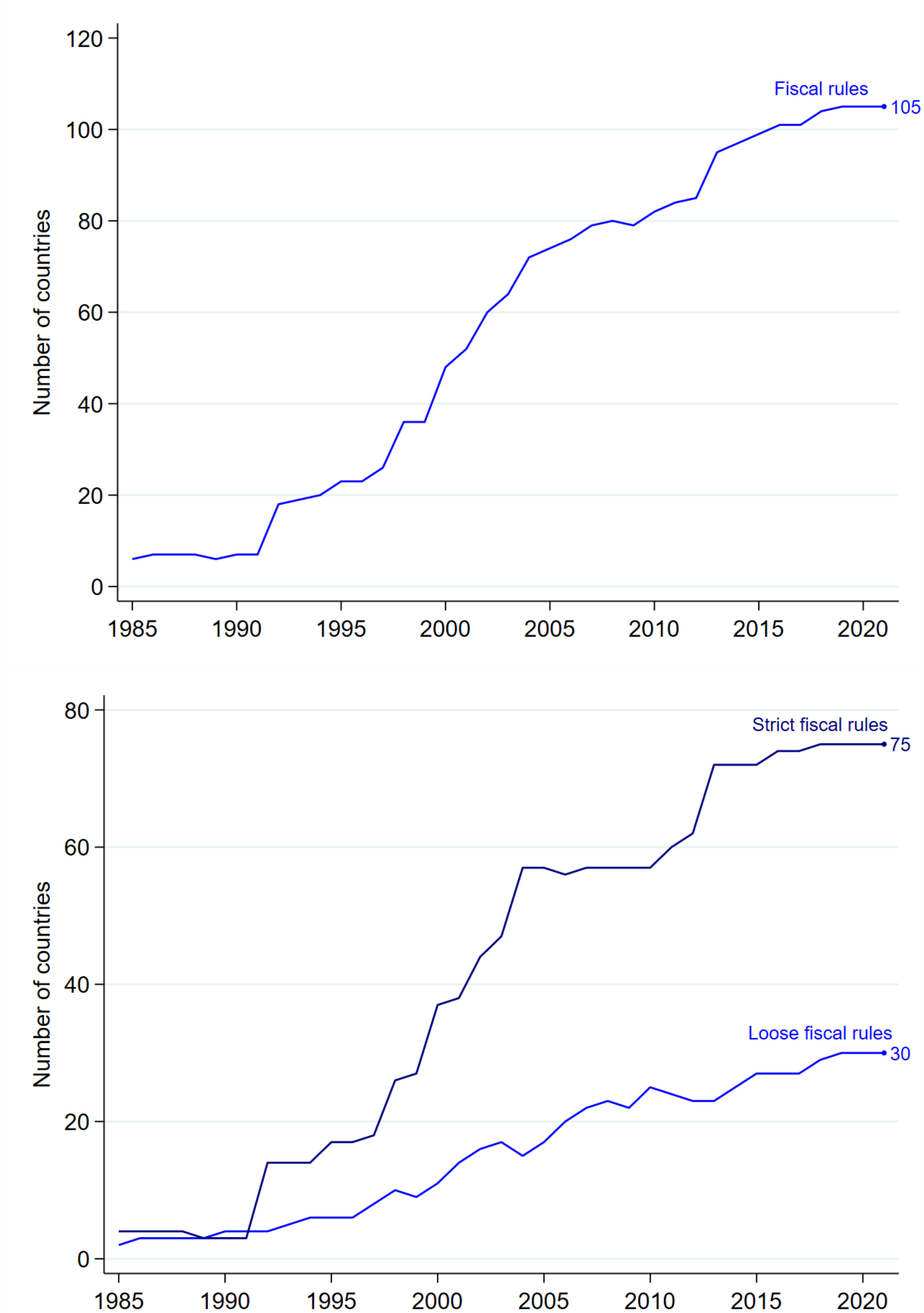
Over the past three decades, governments around the world have adopted fiscal rules to promote fiscal rectitude (Figure 1). These rules vary in their strictness, enforcement mechanisms, and whether they were self-motivated or required by international treaties. Rules stipulate limits on debt, deficits, and government spending. They aim to contain public debt growth and, in some cases, to help promote countercyclical fiscal policies. Perhaps the most prominent rules are those of the European Union and these are binding for its member states under the Maastricht treaty. The EU's rules are still evolving and continue to be a subject of debate among policymakers and economists (Zettelmeyer *et al.*, 2018; Bilbiie *et al.*, 2021; Ilzetzki, 2021; Arnold *et al.*, 2022; Thygesen *et al.*, 2022).

Debates and academic research have focused primarily on the tradeoff between credibility and flexibility in designing fiscal rules (Halac & Yared, 2014; Blanchard *et al.*, 2021; Campante *et al.*, 2021; Barnichon & Mesters, 2022). Stricter and less flexible rules are easier to monitor and may be more credible in ensuring long-run fiscal sustainability. However, they may lead to pro-cyclical fiscal policies with grave social consequences and can exacerbate business cycles. On the other hand, rules allowing more flexibility and "escape clauses" can be designed to permit more countercyclical policies, but they may lack credibility.

This paper investigates a different–and previously unexplored–tradeoff in the design of fiscal rules: the tradeoff between discipline imposed by markets and peers and those imposed by rules. The market mechanism imposes fiscal discipline by increasing interest rates when debt levels are elevated or the market questions the government's willingness to repay it debt. The desire to ensure cheaper funding and better market access gives governments an incentive to limit their borrowing. The common rationale for fiscal rules is that governments are more impatient (present-biased) than their citizens due to political economy forces (e.g. government turnover as in Alesina & Tabellini 1990 or legislative bargaining as in Azzimonti *et al.* 2016. Yared 2019 provides a survey of this literature.) These political forces not withstanding, governments may attempt to signal their fiscal rectitude and credibility by running lower deficits (or larger surpluses) to secure lower borrowing rates. This signalling motivation ensures a degree of fiscal discipline even absent fiscal rules. The premise of this paper is that fiscal rules interact with market discipline and signalling motivations in non-trivial ways. Ignoring this interaction may lead to sub-optimally designed fiscal rules.

We develop a model of asymmetric information, where governments differ in their degree of present bias. Policymakers know their own type, but markets cannot observe this directly, a reasonable assumption particularly when the policymaker is newly elected or facing new circumstances. The market must infer policymakers' type from their actions. Policymakers themselves

Figure 1: Fiscal Rules Have Proliferated



Note: The figure shows the number of countries with a fiscal rule. The top panel shows all types of fiscal rule. The bottom panel shows separately stricter and looser fiscal rules. Source of data and of rule classification: IMF Fiscal Affairs Division.

may then restrain their borrowing, even if they are present-biased, in order to signal to the market that they are fiscally responsible. We show that this may lead policymakers not only to over-borrow at times because of their present bias, but also under-borrow at other times to signal their fiscal responsibility to markets.

We then consider what happens when a debt limit is imposed on the government. Fiscal rules may affect the behavior of less-responsible governments by restricting their borrowing. In our setting this is always good for citizens' welfare because there is no uncertainty or asymmetric information about economic conditions. However, the debt limit has a negative externality for more responsible policymakers. These now now have to run even lower deficits to prove their rectitude and this may lead these governments to under-borrow (or over-save). Whether fiscal rules improve economic welfare becomes a quantitative question and depends on how strict the rules is. Nevertheless, the analysis in this paper demonstrates that it is never optimal to adopt a naïve fiscal rule that forces an irresponsible government to "do the right thing". The optimal fiscal rule will always give irresponsible governments some slack and this to benefit more responsible ones, whom the fiscal rule wouldn't appear to affect absent the analysis provide here.

This conference honors Ken Rogoff's illustrious research career and this paper is inspired by several of his contributions. First, this paper is most directly inspired by Ken's work on the optimal degree of commitment to an inflation target (Rogoff, 1985). Ken showed how an inflexible *monetary* policy rule can be sub-optimal. His rationale is the now-standard tradeoff between rules and discretion and presages the recent literature on fiscal rules. Second, this paper has a similar game theoretic framework as that of Ken's work on political budget cycles (Rogoff, 1990; Rogoff & Sibert, 1988). In Ken's papers, political budget cycles arise because election-year spending may signal a government's competence. In this paper, governments signal their fiscal responsibility by under-borrowing, a problem that is exacerbated in the presence of fiscal rules. Third, the paper relates to Ken's extensive body of work on sovereign debt and default (Bulow *et al.*, 1988, 1992; Bulow & Rogoff, 1988, 1989b,a, 1990, 1991; Reinhart & Rogoff, 2009). Finally, Ken's research on "Debt Intolerance" (Reinhart *et al.*, 2003) shows that governments struggle to improve their credit ratings even decades after a default, highlighting the difficulty competent governments face in signalling that they have turned a page.

This paper also relates to a more recent theoretical literature in political economy that investigates the effects of fiscal rules on economic outcomes and proposes optimal fiscal rules. This includes works by Halac & Yared (2014) and Barnichon & Mesters (2022). These articles investigate the tradeoff between commitment and flexibility. Asymmetric information is an important friction in Halac & Yared (2014), but the government has superior information about the state of the economy–and thus the need for policy discretion–in these studies. Piguillem & Riboni (2021) study how fiscal rules affect the politics of bargaining over budgets. Dovis & Kirpalani (2020) study a model of fiscal rules and asymmetric information in a federal setting, but there it is the credibility of the fiscal rule itself that is unknown. Local governments (e.g. EU member states) may breach the externally-imposed fiscal rule to test the central government's (e.g. the EU's) commitment to enforce the rule. There, a fiscal rule may lead counterintuitively to excess borrowing, while we show an opposing force whereby a fiscal rule could lead to excessive austerity. Amador & Phelan (2021) study a dynamic model of reputation that bears some resemblance to the framework we propose here, but they don't allow "responsible" governments to signal their type or consider the effects of fiscal rules.

2 The Model

An economy lasts for two periods, $t = 1, 2$. The economy consists of citizens and of a policymaker (PM), who chooses public goods in each period, g_1 and g_2, respectively. Citizens and the policymaker have preferences over public goods according to the utility function

$$u(g_1) + \beta^\theta u(g_2),$$

where u is increasing and concave. There are two types of policymaker, $\theta \in \{P, E\}$, which we will refer to as **P**rudent and **E**xtravagant. Nature selects the PM type prior to period 1 and selects a prudent type with probability $\Pr(\theta = P) = \pi$. The prudent and extravagant types differ in two ways. First, a prudent policymaker is more patient than an extravagant one: $\beta^P > \beta^E$. Second, they differ in their probability of of defaulting on the public debt in the second period. This probability is denoted by δ^θ and the extravagant type is more likely to default: $\delta^P < \delta^E$.[1]

The government receives an exogenous stream of revenues y_t in period t that can be transformed costlessly, one-to-one, into public goods. The government borrows in period 1 in a competitive lending market with risk-neutral international lenders, who face a gross funding cost of R. The government borrows by issuing bonds at a price q, so that it receives qb units of the consumption good in period 1 when it promises to repay b units of the good in period 2. We assume throughout that income growth in the second period (y_2/y_1) is sufficiently high and/or discount factors are sufficiently low so that all government types wish to borrow in equilibrium. We assume a minimal amount of public goods that must be provided in each period: $\underline{g} \geq 0$. This may represent non-discretionary spending that any government must provide, or the minimal amount

[1] These default probabilities are exogenous. The more impatient policymaker would default more frequently if default were endogenously determined in a longer-horizon model of strategic default.

of public goods that are politically feasible. With this in mind, the government faces the budget constraints

$$
\begin{aligned}
g_1 &\leq y_1 + qb \\
g_2 + b &\leq y_2 \\
b \geq -\max\{\frac{\underline{g} - y_1}{q}, 0\} &= \underline{b}(y_1, q).
\end{aligned}
$$

The first two constraints are the government's budget constraints in the two periods. The last constraint gives the minimal amount of borrowing the government requires to provide the minimum mandatory amount of public goods $\underline{g}$. Further, the government may face a fiscal rule in the form of a debt limit so that $b \leq \bar{b}$

A type θ PM chooses borrowing b^θ to maximize

$$U^\theta(b^\theta, q) = u(y_1 + qb^\theta) + \beta^\theta[(1 - \delta^\theta)u(y_2 - b^\theta) + \delta^\theta u(y_2)],$$

subject the minimal borrowing and the debt limit given above. As we will see, the bond price q is itself affected by the government's borrowing choice.

Let μ denote the market's beliefs about the PM's type, i.e. their subjective probability that they are lending to a prudent PM. The model has the following timing. In period 1:

1. The PM privately observes their type θ.
2. They request a borrowing amount b from the competitive market.
3. Given the borrowing request b and the market's beliefs μ about the PM's type θ, each lender L offers a bond price schedule $q^L(\mu)$.
4. The PM picks among the bond prices on offer, with $q(\cdot)$ denoting the lending rate offered by the chosen lender.

In period 2, the government defaults on the loan with probability δ^θ, and repays it with the remaining probability.

The equilibrium concept we employ is a Weak Perfect Bayesian Equilibrium, which is defined in our context as follows.

Definition 1 (Weak PBE) *A profile of policymaker strategies* $\{\sigma_\theta\}_{\theta \in \{P,E\}}$*, a bond price* q*, and a system of beliefs* μ *are a* weak perfect Bayesian equilibrium *if they satisfy*

1. *For every θ, every $b \in \sup(\sigma_\theta)$, and every feasible b'*

$$u(y_1 + q(\mu(b))b) + \beta^\theta[(1-\delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)] \geq$$
$$u(y_1 + q(\mu(b'))b') + \beta^\theta[(1-\delta^\theta)u(y_2 - b') + \delta^\theta u(y_2)]$$

2. *For every b, $q(\mu(b)) = \frac{1-\mu(b)\delta^P - (1-\mu(b))\delta^E}{R}$,*

3. *$\mu(b)$ is derived from $\{\sigma_\theta\}_{\theta \in \{P,E\}}$ using Bayes' rule whenever possible.*

The first of these conditions is a set of incentive-compatibility constraints that ensure that neither PM type has an incentive to deviate from their equilibrium strategy to another feasible strategy, given the bond price they will face at that debt level. The second condition follows from perfect competition in lending markets with risk-neutral lenders or a zero-profit condition. Lenders charge a risk premium over the risk-free rate of R to compensate for their expected losses due to default, but no more.

Throughout our analysis, we restrict attention to weak PBE's that satisfy the intuitive criterion, following Cho & Kreps (1987). In addition, we assume that the government never borrows so much that it cannot provide the minimal amount of public goods $\underline{g}$ in the second period.

We will first analyze a model with full information, in which the markets can observe whether a government is prudent or extravagant. We then analyze a model with asymmetric information. In each case, we will begin from a model with no fiscal rule and then impose a fiscal rule. To unify notation throughout, we let $b^\theta(\mu; \text{Inf}, \text{Rule})$ denote the optimal debt level for a policymaker with discount factor β^θ, when the lenders assign a probability μ that the policymaker is prudent, when information (*Inf*) is full or asymmetric: Inf $\in \{FI, AI\}$, and the fiscal rule (*Rule*) includes a debt limit or no rule: Rule $\in \{DL, NR\}$. When there is no room for confusion we omit the reference to the information structure and fiscal rule.

For simplicity we, we will assume that citizens have the same preferences as the prudent policymaker $\beta^C = \beta^P$. This contrasts with the existing literature on fiscal rules, in that we consider a policymaker that is a perfect agent of the citizens it represents. This helps highlight the distortions that fiscal rules introduce in this model as they will affect not only the "bad" extravagant PM, but also the "good" prudent one. The entire positive analysis goes through regardless of social preferences and we will state explicitly when normative results hang on this specific assumption.

2.1 Observable types - No borrowing limit

In a model with observable types, the financial market knows the default rate and therefore assigns the bonds of each type of PM a price reflecting their probability of default, so that a type θ PM can issue bonds at a price q^θ, given by

$$q^\theta = q(\mathbb{1}(\theta = P)) = \frac{1-\delta^\theta}{R}. \tag{1}$$

This is true regardless of the requested loan size, b, because of our simplifying assumption that default probabilities are contingent only on the government's type, not the amount borrowed. Assuming that the discount factors are low enough that both PM types want to borrow but high enough that they do not violate the minimal amount of public spending in the second period, the type θ PM issues b to maximize:

$$U^\theta(b, q^\theta) = u(y_1 + q^\theta b) + \beta^\theta[(1-\delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)].$$

Borrowing $b^\theta(\mathbb{1}(\theta = P); FI, NR)$ satisfies

$$u'(y_1 + q^\theta b^\theta(FI, NR)) = \beta^\theta R u'(y_2 - b^\theta(FI, NR)). \tag{2}$$

We omit the dependence of b on the market's beliefs μ but it should be clear that the market assigns a probability of one to the policymaker's correct type and this affects the policymaker only through the bond price q^θ. The effective interest rate applying to inter-temporal decisions is the same for both types of government and equal to the risk-free rate R because the risk premium exactly offsets default probabilities when types are observable. The bond price still has an endowment effect in period 1, giving the prudent government more resources for a given choice of outstanding debt. This gives the prudent government a lower incentive to borrow, complementing the the lower desire for borrowing due to its greater patience ($\beta^P > \beta^E$). This means that the extravagant government always borrows more than the prudent one when types are observable. We summarize this observation as follows:[2]

Observation 1 $b^E(0; FI, NR) > b^P(1; FI, NR)$

The proof to this and all following results are in Appendix A.

2.2 Observable types - Debt Limit

In this section we consider the effect of a debt ceiling. For this rule to have any implications, it must be binding for at least one type, so that $\bar{b} \leq b^E(0; FI, NR)$. With a binding constraint, the

[2] We drop the market's beliefs to make notation more concise, but it should be clear that the market assigns a probability one to the PM being its actual type. Formally $b^P(FI, NR) = b^P(1; FI, NR)$ and $b^E(FI, NR) = b^E(0; FI, NR)$.

extravagant type will borrow up to the limit: $b^E(0; FI, DL) = \bar{b}$, while the prudent type will be unaffected unless the debt limit is so tight that it is binding for them as well: $b^P(1; FI, DL) = \min\{b^P(FI, NR), \bar{b}\}$. This leads to:

Observation 2 *When types are observable, the best upper debt limit is given by $\bar{b} = b^P(0; FI, NR))$.*

This observation states that the optimal debt limit will force the extravagant type to borrow exactly as a prudent government would if it faced the lower bond price q^E. In other words, the rule aligns the extravagant government's actions with public preferences, although it takes into account the lower bond price that the extravagant government faces. This debt limit will not be binding for the prudent government, who borrows less at the higher bond price it faces ($b^P(1; FI, NR)) < b^P(0; FI, NR)))$.

This is the common rationale for a debt limit. Extravagant policymakers are present-biased relative to citizens' preferences and a debt limit can force them to choose a debt level more aligned with citizen welfare. The prudent PM is a perfect representative of citizens and there is no reason to alter her behavior. The optimal rule simply forces the extravagant type to behave prudently: to "do the right thing".

2.3 Unobservable types - No debt limit

We now turn to the case where types are privately observed by governments. Lenders must infer default probabilities from policymakers' actions, i.e. their requested borrowing. We restrict attention to equilibiria in pure strategies. There are two possible types of pure-strategy equilibria: pooling and separating. We start by considering the case of separating equilibria.

Separating Equilibrium

In a separating equilibrium, the PM's type is revealed to the market and each type is charged an bond price that accords with its objective default probability as in (1). The prudent PM can borrow at a more favorable bond price and she will never want to mimic the extravagant PM. Furthermore, the interest rate faced by the extravagant government is the worst that could arise given the lenders' updating rule. Hence in any separating equilibrium, we have $b^E(0; AI, NR) = b^E(0; FI, NR)$: The extravagant type borrows as it would in the full information equilibrium.

For equilibrium to be separating, the extravagant policymaker must be satisfied with this strategy and be unable to improve its fate by mimicking the prudent type.

$$U^E(b^E(0; AI, NR), q^E) \geq U^E(b^P(1; AI, NR), q^P) \tag{3}$$

The extravagant policymaker faces a tradeoff. It can borrow less than it desires but obtain the higher bond price afforded to prudent policymakers; or it can borrow its desired amount, but at the lower bond price that reflects the extravagant policymaker's higher default probability. This inequality states that the extravagant type must choose the latter and adopt the strategy "assigned" to its type, for an equilibrium to be separating.

It is possible that the prudent policymaker's full information borrowing $b^P(1; FI, NR)$ is already low enough to signal her type, i.e. 3 holds for $b^P(1; AI, NR) = b^P(1; FI, NR)$. Otherwise, she needs to borrow less to separate from the extravagant type and signal that she is prudent. In a separating equilibrium, this needs to be incentive compatible for the prudent PM, so it must be the case that

$$U^P(b^P(0; FI, NR), q^E) \leq U^P(b^P(1; AI, NR), q^P). \tag{4}$$

That is, the prudent government will prefer to request the loan amount that signals that they are prudent over the best outcome they could achieve if they were offered the lower bond price q^E.[3]

To formalize these notions and for future reference, it is useful to introduce additional notation. We denote by $\hat{b}^\theta(\mu; b^{\theta'}(\mu'; \text{Inf}, \text{Rule}))$ the debt level that will make a policymaker of type θ indifferent between being perceived as the prudent type with probability μ and borrowing $b^{\theta'}(\mu'; \text{Inf}, \text{Rule})$, and being perceived as being the prudent type with probability μ' and borrowing freely at the resultant bond price. As before, the debt level $b^{\theta'}(\mu'; \text{Inf}, \text{Rule})$ is the optimal borrowing of a policymaker of type θ', when the market assigns a probability of μ' that it is prudent, in an equilibrium of type *Inf* with a rule of type *Rule*.

Lemma 1 *In a model of asymmetric information with no fiscal rule, a separating equilibrium exists if and only if $\hat{b}^E(1, b^E(0; FI, NR)) \geq \underline{b}(y_1, q^P)$. Under the intuitive criterion, the unique separating equilibrium (if it exists) is characterized by*[4]

$$b^E(0; AI, NR) = b^E(0; FI, NR)$$

and

$$b^P(1; AI, NR) = \min\{\hat{b}^E(1, b^E(0; FI, NR)), b^P(1; FI, NR)\}$$

In a separating equilibrium, the extravagant type over-borrows, because of the standard present-bias problem. However, the prudent government may *under*-borrow to signal its prudence. Notice

[3] These are the off-the-equilibrium path beliefs that are most likely to ensure equilibrium existence so that any equilibrium policy must satisfy this condition.

[4] Formally this isn't a unique equilibrium but rather the unique pair of strategies in all separating equilibria. These strategies may be supported with a variety of off-the-equilibrium path beliefs, each formally leading to a different equilibrium.

that this occurs even though the prudent type is no more patient than are citizens. Instead, the prudent type is willing to borrow less than she (and her citizens) desire in order to signal her prudence and obtain more favorable lending conditions. This is a new friction relative to the existing literature and suggests that governments may not only over-borrow because they are present-biased, but also may under-borrow as they attempt to convince markets that they *aren't* present-biased.

Implication 1 *In a separating equilibrium, the exorbitant policymaker over-borrows relative to the social optimum, but the prudent policymaker (weakly) under-borrows.*

Signalling is a common explanation given by governments when implementing austerity programs, e.g. during the global financial crisis. Gaining financial investors' confidence was a central rationale for the UK government's austerity plans following the global financial crisis. The 2011 budget states that "There is a broad international consensus that advanced economies should put in place and begin implementing credible medium-term fiscal consolidation plans this year, in order to underpin market confidence." One can question whether such consensus existed in 2011, but this quote illustrates how financial market confidence has been used as a justification for deficit reduction. Following the fiscal event in the UK in 2022, a majority of UK economists surveyed in the survey of the Centre for Macroeconomics, felt that deficit reduction was important to restore the government's credibility (Ilzetzki & Jain, 2022).

Ben Bernanke, although not an advocate of austerity during the financial crisis, argued that "maintaining the confidence of the public and financial markets requires that policy makers begin planning now for the restoration of fiscal balance."[5] Former Treasury Secretary Robert Rubin, writing with later CBO and OMB chair Peter Orszag and Todd Sinai wrote in 2004 that "substantial deficits projected far into the future can cause a fundamental shift in market expectations and a related loss of confidence both at home and abroad." (Rubin *et al.*, 2004) Finance Minister Wolfgang Schäuble defended Euro-area deficit plans because they would promote "consumer and investor confidence." He states that "governments in and beyond the eurozone need not just to commit to fiscal consolidation... Countries faced with high levels of debt and deficits need to cut expenditures, increase revenues and remove the structural hindrances in their economies, however politically painful... The truth is that governments need the disciplining forces of markets.[6]

The case for confidence-building austerity is equally prevalent in emerging markets. Brazil was on the verge of financial crisis following the election of Lula in 2002. The (first) Lula administration lowered the deficits (of the consolidated public sector) from 4.2% of GDP in 2002 to 2.4% of GDP in 2004 and bringing down the debt-to-GDP ratio by 10 percentage points, largely aimed to calm

[5]Testimony on the Semiannual Monetary Policy Report to the Congress, Before the Committee on Financial Services. U.S. House of Representatives, Washington, DC, July 21, 2009.

[6]Financial Times, September 5, 2011

financial market turmoil.[7] In his address to the IMF in April 2003, Finance Minister Antonio Palocci Filho explained that "To shed concerns regarding debt sustainability, Brazil announced a half a percentage point of GDP increase in the primary surplus for 2003, bringing it to 4.25 percent. It reaffirmed its commitment to generate primary surpluses necessary to ensure a steady decline of the debt-to-GDP ratio over the medium term by maintaining a primary surplus target of 4.25 percent of GDP for 2004 and similar indicative target for 2005 and 2006." As a result, "Markets have responded positively to these initiatives. Spreads on Brazilian bonds have been cut from 2,400 to around 900 basis points, and there is still scope for further decline."[8] This impetus to cut deficits to signal fiscal responsibility has implications for the design of fiscal rules as we will shortly see.

Absent a lower limit $\underline{g}$ on public goods, Lemma 1 implies that the separating equilibrium characterized here always exists and it is the unique equilibrium. The prudent type is more patient than the extravagant type and will always be willing to borrow slightly less than the lowest level of debt that the extravagant type is willing to endure to obtain better borrowing terms. Consequently, it is always possible and desirable for the prudent type to signal its prudence. A separating equilibrium fails to materialize only when the prudent type is *unable* to reduce debt sufficiently to signal its type because of the minimal public good requirement $\underline{g}$. This explains the condition for equilibrium existence in the lemma. When the austerity required to signal the government's prudence is impossible because of the necessity of minimal public good provisions, pooling equilibria arise.

Pooling Equilibrium

In a pooling equilibrium, the prudent PM is unable to signal its type and the exorbitant PM successfully mimics her behavior. The market is unable to update its priors about the PM's type and buys bond at the price

$$q(\pi) = \frac{1 - \pi\delta^P - (1-\pi)\delta^E}{R}.$$

Pooling equilibria exist (if and) only when the the prudent PM is unable to signal her type, because she is constrained by the minimal public good requirement, as summarized in the following lemma.

Lemma 2 *In a model of asymmetric information with no fiscal rule, a pooling equilibrium exists if and only if* $\underline{b}(y_1, q^P) \geq \hat{b}^E(1, b^E(0; FI, NR))$.

A large set of pooling equilibria may arise in this model, even after eliminating equilibria using the intuitive criterion, as is common in models with asymmetric information. In a pooling equi-

[7] Source: IMF Article IV 2008.

[8] Statement by Antonio Palocci Filho, Minister of Finance, Brazil, International Monetary and Financial Committee Meeting, Washington, D.C., April 12, 2003.

librium, the market expects to see both the prudent the extravagant PMs to borrow a specific debt level $b^P(\pi; \text{AI}, \text{NR}) = b^E(\pi; \text{AI}, \text{NR})$. In the zero-probability (off-the-equilibrium-path) event that a government surprises the market with a different request for borrowing, the market concludes that this is an extravagant government. Fearing the penalizing interest rates they will face if they deviate, both PM types confirm the market's expectations and borrow $b^P(\pi; \text{AI}, \text{NR}) = b^E(\pi; \text{AI}, \text{NR})$. There may be large range of debt levels that can be supported in such an equilibrium.

We characterize the range of pooling equilibria explicitly in Appendix B. Three features of the pooling equilibria are noteworthy. First, the socially optimal debt level may be among the pooling equilibria for some parameter values. The social optimum is the debt level that would be chosen by a social planner, facing the interest rate $r(\pi)$, that has preferences like those of the prudent PM, but recognizes that prudent PMs will govern with probability π and exorbitant governments with probability $1 - \pi$. However, for other parameter values, the set of pooling equilibria will all involve over-borrowing relative to the social optimum; and for yet other parameter values, all pooling equilibria will involve under-borrowing. It is therefore impossible to make a general statements about whether governments will be borrowing excessively or insufficiently in a pooling equilibrium. And for any given parameter values, it is of course impossible to evaluate which of the many pooling equilbria will arise.

Second, the set of pooling equilibria may also include some very bad outcomes. For example, both PMs may incur as much debt as the exorbitant government would like to borrow (or even more). This may seem like an unrealistically perverse outcome, but note that the market cannot infer that a PM is prudent if she borrows less than this exorbitant amount. The market has no reason to believe that it isn't facing an exorbitant government if it observes a deviation from the equilibrium strategy, even when applying the intuitive criterion, because *both* PM types would benefit from such a deviation.

Third, even in the best outcome of a pooling equilibrium at the social "optimum", this is an optimum at a the "wrong" borrowing rate. The bond price $q(\pi)$ over-penalizes the prudent PM and under-penalizes the exorbitant one relative to their objective default probabilities. In fact, the prudent PM faces a *higher* effective interest rate than does the exorbitant PM: They face the same interest rate, but the exorbitant PM is more likely to default. The socially optimal debt level takes into account the two different interest rates that might arise and is sub-optimal ex-post regardless of who governs.

2.4 Unobservable types - Debt limit

We now turn to the implications of a fiscal rule in the form of a debt limit when governments have asymmetrical information about their propensity to borrow and default. We consider a debt limit that is sufficiently tight to bind for the extravagant type, otherwise it has no implications for the equilibrium.

Separating Equilibrium

As before, in a separating equilibrium the two PMs face different bond prices, given in (1). If the debt limit is lower than the amount of debt an extravagant PM would otherwise incur, the extravagant PM borrows to the limit: $b^E(0; AI, DL) = \bar{b} = b^E(0; FI, DL)$. This is precisely what a debt limit aims to achieve and counteracts the present-bias problem, as in the existing literature. However, changing the exorbitant PM's behavior also affects the prudent PM's strategy in a separating equilibrium and this may have perverse side-effects. This is the key insight of the paper and is summarized in the following lemma.

Lemma 3 *In a model of asymmetric information with a debt limit, a separating equilibrium exists if and only if $\underline{b}(y_1, q^P) \leq \hat{b}^E(1, b^E(0; FI, DL))$. Under the intuitive criterion the unique separating equilibrium (if it exists) is characterized by*

$$b^E(0; AI, DL) = \bar{b} = \hat{b}^E(1, b^E(0; FI, DL))$$

and

$$b^P(1; AI, DL) = \min\{\hat{b}^E(1, b^E(0; AI, DL)), b^P(1; FI, DL)\}$$

It is always the case that $\hat{b}^P(1, b^E(0; AI, DL)) \leq \hat{b}^P(1, b^E(0; AI, NR))$, so that the prudent type, who was already under-borrowing without the fiscal rule, will further decrease its debt when the debt limit is imposed. This is despite the fact that the debt constraint isn't binding when the prudent PM is in power. We summarize this in the following:

Implication 2 *In a separating equilibrium, a debt limit will (weakly) reduce the borrowing of both extravagant and prudent policymakers. This improves citizens' welfare when extravagant policymakers are in power, but hurts citizens' welfare when prudent governments are in power.*

The current literature and policy debate focuses on the implications of fiscal rules only when they are binding. We show here that a fiscal rule can distort policy even when it isn't binding. It can induce harsher fiscal austerity policies because the fiscal rule makes it more difficult for prudent governments to show their stripes. The fiscal rule brings the exorbitant policymakers' policies closer to those preferred by citizens. But the harm this causes the exorbitant PM makes it more attractive for him to mimic the prudent PM to obtain a better bond price. (It has to restrain its borrowing anyways, so it might as well borrow even less and get the lower interest rate while it's at it.)

This is a new informational friction relative to the existing literature. In existing models of fiscal policy with asymmetric information and fiscal rules, governments have private information about the *state of the economy*. Governments always want to over-borrow relative to citizens due to their present bias. The informational problem is that the government may be borrowing a large sum because this is required by economic circumstances, but it may be incurring large debts because it is present biased. The literature studies how flexible fiscal rules should be in light of this friction. Flexibility allows the government to respond to the state of the economy, on one hand, but it gives the government more leeway to over-borrow, on the other.

In contrast, in the model presented here, policymakers have private information about their propensity to borrow. There is no uncertainty nor private information about the state of the economy. The tradeoff a fiscal rule poses is different here. A fiscal rule limits present-biased policymakers' over-borrowing, but it may also reduce the borrowing of less present-biased policymakers, who are already under-borrowing. For simplicity, we assumed that the prudent policymaker is a perfect agent of citizens' policy preferences. Note, however, that under-borrowing could even arise if the prudent policymaker were present-biased relative to citizens. In this case, the prudent policymaker would indeed over-borrow if information were complete, but she could nevertheless under-borrow relative to citizens' preferences under asymmetric information, in the process of signalling her type.

Pooling

For completeness, we give conditions for the existence of a pooling equilibrium with a debt limit. As before, pooling equilibria arise (with the intuitive criterion) only when a separating equilibrium isn't feasible. When pooling equilibria exist, there are typically many such equilibria.

Lemma 4 *In a model of asymmetric information with a debt limit, a pooling equilibrium exists if and only if* $\underline{b}(y_1, q^P) \geq \hat{b}^E(1, b^E(0; FI, DL))$.

The main thing to note here is that the debt limit has decreased the parameter values in which a separating equilibrium exists and expanded the region where a pooling equilibrium exists. The intuition is that the debt limit requires the prudent PM to reduce its borrowing further to signal its prudence (the value of $\hat{b}^E(1, b^E(0; FI, DL))$ in Lemma 4 is lower than the value of $\hat{b}^E(1, b^E(0; FI, NR))$ in Lemma 2). This makes it more likely that the public good floor $\underline{g}$ is breached and signaling becomes infeasible. The fiscal rule can "jam the signal" and force the prudent PM into a pooling equilibrium.

3 Debt limits: A Cost-Benefit Analysis

The benefit of a debt limit in this model is clear and consistent with conventional wisdom and the existing literature. The debt limit restrains the extravangant policymaker's borrowing (in a separating equilibrium). The costs are twofold. First, the debt limit reduces the prudent PM's borrowing, which is already borrowing below the socially optimal debt level. Second, the debt limit makes a pooling equilibrium more likely. As previously noted, there is a large set of possible pooling equilibria, some which are Pareto-inferior to the the separating equilibrium. In all such equilibria, the prudent government is penalized with a lower bond price than merited by her default probability.

We stack up these costs against the benefits of the debt limit. We begin by analyzing the optimal debt limit in a separating equilibrium.

Optimal Debt Limit

The optimal debt limit from the perspective of citizens trades off the the benefits of reducing debt when the extravagant PM is in power with its costs when the prudent PM is in power.

Observation 3 *The socially optimal level of debt in a separating equilibrium is* $\bar{b} \in (b^P(0; FI, NR), b^E(0; FI, NR))$

This observation states that the optimal debt limit is strictly below the debt level that the extravagant policymaker would choose with full information or in a separating equilibrium without a debt limit. This means that the optimal debt limit does constrain the extravagant PM's borrowing.

However, the observation also states that the optimal debt limit is strictly lower than the debt level that the prudent policymaker would choose if it faced the extravagant government's bond price q^E. This would be the socially optimal thing to do if constraining the extravagant government had no impact on the prudent government's choice. Indeed, this was the optimal debt limit with full information, as noted in Observation 2. Despite this, a social planner should not choose a fiscal rule that imposes this debt level. A naïve debt limit that tries to get policymakers to do the "right thing" isn't good policy.

The proof of this result is in Appendix A, but it is worth explaining why these results hold, as they appear to be rather general. The intuition for these results rely on the envelope theorem. Imagine a debt limit that is just binding for the extravagant PM (it reduces his borrowing only marginally). The extravagant government is over-borrowing from the citizens' perspective, so that tightening the debt limit increases their utility when he is in power. Hence there is a positive benefit to imposing this debt limit. In a separating equilibrium, the extravagant type is choosing debt optimally from his own perspective. This means that minor deviations imposed by the debt limit have no effect, on the margin, on the extravagant government's utility. This, in turn, means that the debt limit makes the extravagant type no more tempted to mimic the prudent PM's policy and the prudent PM need not reduce her borrowing any further to signal her type. The implication is that this debt limit doesn't affect the prudent PM's behavior and imposes no negative externality on the prudent PM or on the public when the prudent PM is in power.

On the other hand, imagine a debt limit that forces extravagant policymakers to choose the naïve constrained social optimum, i.e. the prudent PM's (and public's) preferred debt under full information, but facing the extravagant government's bond price. This rule would impose the naïve constrained social optimum on the extravagant PM, but isn't binding for the prudent PM in a separating equilibrium. The prudent PM will over-save so as to signal its type and obtain the higher bond price. Now consider a marginal loosening of this debt limit. Minor deviations from the social optimum have negligible social costs so that the cost to the public of allowing the extravagant PM to borrow slightly more when he is in power is approximately zero. On the other hand, the extravagant PM is borrowing more than he would like due to the debt limit, so that loosening the debt limit increases his utility. This makes the extravagant type materially less inclined to mimic the prudent PM and allows the prudent PM to borrow more while still obtaining the higher bond price. Given that the prudent PM was under-borrowing to begin with, this change improves citizens' welfare when the prudent PM is in power. Thus a borrowing limit that attempts to impose the seemingly optimal policy on extravagant policymakers is too tight.

Signal Jamming

There is another potential cost to fiscal rules. We have seen that a fiscal rule decreases the parameter space where a separating equilibrium exits. So, for example, there is a smaller range of GDP growth y_2/y_1, for which the prudent PM can signal.

We have already pointed out that this could lead to a wide range of outcomes, some of which are extremely harmful. However, it is impossible to make a general comparison between social welfare in the separating equilibrium and in the wide range of pooling equilibria. There are some pooling equilibria that will dominate the (unique) separating equilibrium and others that are more harmful. To gain some intuition about the comparison between the two, consider the pooling equilibrium that emerges and adds to the set of potential pooling equilibria, due to a marginal tightening of the fiscal rule.

The marginal pooling equilibrium will be characterized with the borrowing rate $q(\pi)$ and both PMs borrowing at the rate $b^P = b^E = \hat{b}^E(\pi, b^E(0; AI, DL))$, i.e. the debt level that leaves the extravagant government indifferent between borrowing at the debt limit at the bond price q^P and borrowing $\hat{b}^E$ at the higher bond price $q(\pi)$. There are social costs and benefits when moving from a separating to a pooling equilibrium and these differ depending on the government who is in power. When the extravagant PM is governing, a pooling equilibrium lowers his borrowing. This may be in the the public benefit, because he is over-borrowing in a separating equilibrium. Further, the extravagant government obtains a lower interest rate. It is easy to show that the public is always better off in a pooling equilibrium than in a separating equilibrium if the extravagant PM is in power: If the extravagant government is willing to lower his borrowing to obtain improved lending terms, the public should prefer this *a fortiori*, as citizens are more patient than the extravagant government.

On the other hand, when the prudent PM is in power, the pooling equilibrium reduces public welfare. The prudent PM faces a higher interest rate and increases its borrowing in a pooling equilibirum, as compared to to a separating one. It can be shown that the public is always worse off in a pooling equilibrium relative to a separating equilibrium when the prudent PM is in power.

It is therefore ambiguous whether a separating equilibrium is preferable overall, but in all cases, the pooling equilibrium penalizes prudent PMs and rewards extravagant PMs. Perversely, the prudent PM faces a higher effective interest rate than the extravagant one does in a pooling equilibrium. Both PMs face the same bond price, but the extravagant type defaults more frequently. Fiscal rules increase the likelihood of pooling equilibria, which reward impatient and defaulting governments at the expense of more patient and those less prone to default. This could pose perverse incentives that go beyond the analysis of this paper, for example if the degree of fiscal prudence is a matter of public choice or once one takes the economic and social costs of default (Reinhart & Rogoff, 2009; Farah-Yacoub *et al.*, 2022).

4 Concluding Remarks

This paper studies the interaction between fiscal discipline achieved through market mechanism and discipline imposed by external rules. Market forces impose fiscal rectitude on governments. We have shown that the resulting rectitude could be so strict that it may cause a benevolent government to over-save (adopt austerity policies) to signal its fiscal prudence. Ignoring the discipline imposed by markets and policymakers' signalling incentives, it is correct that a fiscal rule should merely attempt to replicate socially desirable policies. However, in the presence of market discipline, imposing naïve fiscal rules can lead to unintended and undesirable consequences. These outcomes may occur even when the fiscal rule isn't binding and seemingly has no effect. Fiscal rules may make it more difficult for prudent governments to signal their rectitude and may incentivize them to impose harsher austerity policies. Alternatively, they may conclude that signaling their fiscal responsibility is too costly and accept the higher market interest rates that result.

A few caveats are due. First, the analysis assumes that the fiscal rule is external to the policymaking process. This is perhaps relevant for a European government in face of the external EU fiscal rules. This may also be relevant in cases where the fiscal rule requires a super-majority to reverse, for example if it is constitutionally enshrined. But in most cases, rules are set by the very policymakers that face them. For example, UK fiscal rules require a simple majority to change and this is the same majority required to pass any individual budget. It is an interesting question, worthy of future research, whether and how fiscal rules have any bite in these circumstances. One dimension that is relevant for the analysis we conducted here is the possibility that introducing, enforcing, violating, and overturning fiscal rules are themselves symbolic acts that have some signalling value.

Second, default probabilities are fixed for simplicity in the analysis presented here. Studying similar models with exogenous default is a more technically involved exercise and beyond the scope of the current paper. It is important to note that some results may be affected once endogenous sovereign default is considered. Endogenous default means that interest rates are a function not only of the government's "type" but also the amount of debt incurred. If this is the case, a fiscal rule has an additional advantage of reducing default rates. It is by no means straightforward to conclude how this additional margin affects governments' signalling incentives and we reserve this analysis for future research. However, the simple model outlined in this paper demonstrate a channel that should also play out in richer models.

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A Proofs

Proof of Observation 1. First notice that $q^E < q^P$, as $\delta^E > \delta^P$. Suppose for contradiction that $b^E(0; FI, NR) \leq b^P(1; FI, NR)$, then

$$\begin{aligned} &u'(y_1 + q^E b^E(0; FI, NR)) = \beta^E R u'(y_2 - b^E(0; FI, NR)) \\ <&\beta^P R u'(y_2 - b^P(1; FI, NR)) = u'(y_1 + q^P b^P(1; FI, NR)) < u'(y_1 + q^E b^E(0; FI, NR)), \end{aligned}$$

where the equalities follow from the first order conditions. The first inequality follows as $y_2 - b^E(0; FI, NR) \geq y_2 - b^P(1; FI, NR)$, $\beta^E < \beta^P$ and u is concave. The second inequality follows since $q^E < q^P$, $b^E(0; FI, NR) \leq b^P(1; FI, NR)$ and u is concave. Thus, we conclude that $b^E(0; FI, NR) > b^P(1; FI, NR)$ ■

Proof of Observation 2. If the social welfare planner could choose debt freely, it would maximize the following objective function when the PM is of type θ:

$$U^{SWP} = u(y_1 + q^\theta b) + \beta^P[(1 - \delta^\theta)u(y_2 - b) + \delta^\theta u(y_2)],$$

i.e. the social planner takes into account the borrowing rate and default rate of the PM that is in power, but chooses borrowing based on social preferences, reflected in the discount rate β^P.

The optimality condition gives

$$u'(y_1 + q^\theta b^{SWP}(\theta)) = \beta^P R u'(y_2 - b^{SWP}(\theta)), \tag{5}$$

where $b^{SWP}(\theta)$ gives the ideal borrowing when the PM is of type θ. This gives $b^{SWP}(P) = b^P(1, FI, NR)$, because the first order condition is identical to that of the prudent type when $q = q^P$. This also gives $b^{SWP}(E) = b^P(0, FI, NR)$, i.e. the debt that a prudent government would choose if the market perceived it to be extravagant and offered it the bond price q^E.

Optimal borrowing $b^{SWP}(\theta)$ is decreasing in q, so $b^P(1, FI, NR) < b^P(0, FI, NR)$ and the social planner can achieve the first best by imposing a debt limit of $\bar{b} = b^P(0, FI, NR)$, which will bind for the extravagant type, but won't affect the prudent government, who is already borrowing at the socially optimal rate given the bond price they face.

■

Proof of Lemma 1. This proof proceeds in steps. In step 1, we show that $b^P(1; AI, NR) \in A$, where $A = [\hat{b}^P(1, b^P(0; FI, NR)), \hat{b}^E(1, b^E(0; FI, NR))]$ is sufficient for inequality 3 and 4 to hold simultaneously. Step 2 establishes that this is also necessary. Finally, in step 3 we show that under the intuitive criterion any separating equilibrium results in the debt levels are as stated in Lemma 1.

In step 1 and 2, we will assume that the beliefs in the bonds market are given by $\mu(b) = 0$ for every $b \neq b^P(1; AI, NR)$.

Step 1: Consider $b \in A$. Note that $b \leq \hat{b}^E(1, b^E(0; FI, NR)) < b^E(0; FI, NR)$, and $U^E(\cdot, q^E)$ is increasing in the first argument for any $b' < b^E(0; FI, NR)$. Thus, it follows immediately that inequality 3 holds for $b^P(1; AI, NR) = b$. Similarly, $U^P(b^P(0; FI, NR), q^E) \leq U^P(b', q^P)$ for every $b' \geq \hat{b}^P(1, b^P(0; FI, NR))$.

Thus, all that remains to be shown is that $A \neq \emptyset$. Suppose for contradiction that $\hat{b}^P(1, b^P(0; FI, NR)) > \hat{b}^E(1, b^E(0; FI, NR))$. This implies that $U^E(\hat{b}^P(1, b^P(1; FI, NR)), q^P) > U^E(\hat{b}^E(1, b^E(0; FI, NR)), q^P)$. Furthermore, by definition $U^E(b^P(0; FI, NR), q^E) < U^E(b^E(0; FI, NR), q^P)$. Thus, $U^E(\hat{b}^P(1, b^P(0; FI, NR)), q^P) > U^E(b^P(0; FI, NR), q^E)$, which implies

$$
\begin{aligned}
&\beta^E(1-\delta^E)u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - \beta^P\delta^P u(y_2) \\
>&\beta^E(1-\delta^E)u(y_2 - b^P(0; FI, NR)) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - b^P(0; FI, NR)) - \beta^P\delta^P u(y_2) \\
\Rightarrow&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right] \\
>&\beta^P(1-\delta^P)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right] \\
>&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, NR))) - u(y_2 - b^P(0; FI, NR))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^P(1, b^P(0; FI, NR))$, and the third inequality follows since u is increasing, $\hat{b}^P(1, b^P(0; FI, NR)) < b^P(0; FI, NR)$, and $\beta^E(1-\delta^E) < \beta^P(1-\delta^P)$. Thus, we conclude that $b \in A$ is a sufficient condition for a separating equilibrium.

Step 2: To show that there exists no $b \notin A$ that satisfy both inequality 3 and 4, we note that there also exists $\hat{b}^P(1, b^P(0; FI, NR)) > b^P(0; FI, NR)$ and $\hat{b}^E(1, b^E(0; FI, NR)) > b^E(0; FI, NR)$. What remains to be shown is that $\hat{b}^P(1, b^P(0; FI, NR)) < \hat{b}^E(1, b^E(0; FI, NR))$, since

(1) $U^E(b^E(0; FI, NR), q^P) \leq U^E(b, q^P)$ if and only if $b \geq \hat{b}^E(1, b^E(0; FI, NR))$ for every $b > b^E(0; FI, NR)$, and

(2) $U^P(b^P(0; FI, NR), q^E) \geq U^P(b, q^P)$ if and only if $b \leq \hat{b}^P(1, b^P(0; FI, NR))$ for every $b > b^P(0; ;NR)$.

Suppose for contradiction that $\hat{b}^P(1, b^P(0; FI, NR)) \geq \hat{b}^E(1, b^E(0; FI, NR))$. This implies that $U^P(\hat{b}^E(1, b^E(0; FI, NR)), q^P) \geq U^P(b^E(0; FI, NR), q^E)$, and therefore:

$$
\begin{aligned}
&\beta^P(1-\delta^P)u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))+\beta^P\delta^P u(y_2)-\beta^E(1-\delta^E)u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-\beta^E\delta^E u(y_2)\\
\geq&\beta^P(1-\delta^P)u(y_2-b^E(0;FI,NR))+\beta^P\delta^P u(y_2)-\beta^E(1-\delta^E)u(y_2-b^E(0;FI,NR))-\beta^E\delta^E u(y_2)\\
\Rightarrow&\beta^P(1-\delta^P)\left[u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-u(y_2-b^E(0;FI,NR))\right]\\
\geq&\beta^E(1-\delta^E)\left[u(y_2-\hat{b}^E(1,b^E(0;FI,NR)))-u(y_2-b^E(0;FI,NR))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^E(1,b^E(0;FI,NR))$. Since $\hat{b}^E(1,b^E(0;FI,NR)) > b^E(0;FI,NR)$, and $\beta^E(1-\delta^E) < \beta^P)(1-\delta^P)$, this leads to a contradiction.

Step 3: Since $b^E(0;AI,NR)$ is the amount of bond issued by the extravagant government in every separating equilibrium, the intuitive criterion imply that $\mu(b)=1$ for every $b < \hat{b}^E(1,b^E(0;FI,NR)) < b^E(0;FI,NR)$. Thus, the only separating equilibrium (if it exists) that survives the intuitive criterion is the one stated in the Lemma. Furthermore, a separating only exists when $\underline{b}(y_1) \leq \hat{b}^E(1,b^E(0;FI,NR))$. ■

Proof of Lemma 2. Note that in order for b to be the equilibrium bond level issued in a pooling equilibrium the following two inequalities must hold:

$$
U^E(b^E(0;FI,NR),q^E) \leq U^E(b,q(\pi)), \tag{6}
$$
$$
U^P(b^P(0;FI,NR),q^E) \leq U^P(b,q(\pi)). \tag{7}
$$

We proceed in steps. In Step 1, we show that a necessary condition for inequalities 6 and 7 to hold simultaneously is for

$$
b \in A = [\hat{b}^E(\pi,b^E(0;FI,NR)),\hat{b}^P(\pi,b^P(0;FI,NR))]
$$

with $\hat{b}^E(\pi,b^E(0;FI,NR)) < b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) > b^P(0;FI,NR)$. Step 2 establishes that under the intuitive criterion no pooling equilibrium exists if $\underline{b}(y_1) < \hat{b}^E(1,b^E(0;FI,NR))$. In Step 3, we show that if $\underline{b}(y_1) \geq \hat{b}^E(1,b^E(0;FI,NR))$, then a pooling equilibrium exists.

Step 1: If $b \leq \hat{b}^E(\pi,b^E(0;FI,NR))$, then inequality 6 will be violated, and $b \geq \hat{b}^P(\pi,b^P(0;FI,NR))$ would violate inequality 7. Furthermore, if (1) $\hat{b}^E(\pi,b^E(0;FI,NR)) < b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) < b^P(0;FI,NR)$, then by the analogous arguments to Step 1 in the proof of Lemma 1, we have $\hat{b}^E(\pi,b^E(0;FI,NR)) > \hat{b}^P(\pi,b^P(0;FI,NR))$, and (2) $\hat{b}^E(\pi,b^E(0;FI,NR)) > b^E(0;FI,NR)$ and $\hat{b}^P(\pi,b^P(0;FI,NR)) > b^P(0;FI,NR)$, then by the analogous arguments to Step 2 in the proof of Lemma 1, we have $\hat{b}^E(\pi,b^E(0;FI,NR)) > \hat{b}^P(\pi,b^P(0;FI,NR))$.

Step 2: Since inequality 6 holds for any $b \in A$, then the intuitive criterion implies that $\mu(b)=1$

for every $b < \hat{b}^E(1, b^E(0; FI, NR))$. Thus, if $\underline{b}(y_1) < \hat{b}^E(1, b^E(0; FI, NR))$, then the Prudent government would deviate.

Step 3: Suppose that $\underline{b}(y_1) \geq \hat{b}^E(1, b^E(0; FI, NR))$. For $b^{Pool} = \max\{\underline{b}(y_1), \hat{b}^E(\pi, b^E(0; FI, NR))\}$ the intuitive criterion has no bite for feasible bond-levels. Thus,b^{Pool} as the equilibrium bond-level, and beliefs $\mu(b^{Pool}) = \pi$ and $\mu(b) = 0$ for any $b \neq b^{Pool}$ constitute a pooling equilibrium. This completes the proof. ■

Proof of Lemma 3. Whenever, $\bar{b} > b^P(0; FI, NR)$, the proof follows the same arguments as the proof of Lemma 1.

When $\bar{b} \leq b^P(0; FI, NR)$, we have $b^E(0; AI, DL) = b^P(0; FI, DL) = \bar{b}$. Suppose for contradiction that $\hat{b}^E(1, b^E(0; AI, DL)) < \hat{b}^P(1, b^P(0; FI, DL)) \leq \bar{b}$. This implies that $U^E(b^E(0; AI, DL), q^E) = U^E(\hat{b}^E(1, b^E(0; AI, DL)), q^P) < U^E(\hat{b}^P(1, b^P(0; FI, DL)), q^P)$. Thus we have:

$$
\begin{aligned}
&\beta^E(1-\delta^E)u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - \beta^P\delta^P u(y_2) \\
>&\beta^E(1-\delta^E)u(y_2 - b^P(0; FI, DL)) + \beta^E\delta^E u(y_2) - \beta^P(1-\delta^P)u(y_2 - b^P(0; FI, DL)) - \beta^P\delta^P u(y_2) \\
\Rightarrow&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right] \\
>&\beta^P(1-\delta^P)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right] \\
>&\beta^E(1-\delta^E)\left[u(y_2 - \hat{b}^P(1, b^P(0; FI, DL))) - u(y_2 - b^P(0; FI, DL))\right],
\end{aligned}
$$

where the first inequality follows from the definition of $\hat{b}^P(1, b^P(0; FI, DL))$, and the third inequality follows since u is increasing and $\hat{b}^P(1, b^P(0; FI, DL)) < b^P(0; FI, DL)$, and $\beta^E(1-\delta^E) < \beta^P(1-\delta^P)$. Thus, we have a contradiction.

The intuitive equilibrium again imply that the unique separating equilibrium is one in which $b^E(0; AI, DL) = \bar{b}$, and $b^P(1; AI, DL) = \hat{b}^E(1, b^E(0; AI, DL))$ ■

Proof of Lemma 4. This proof is analogous to the proof of Lemma 2. ■

Proof of Observation 3. First we observe that $b^P(0; FI, NR) < b^E(0; FI, NR)$. This follows directly from the first order conditions, and $\beta^E < \beta^P$.

Suppose that the socially optimal level of debt limit is $\bar{b} \geq b^E(0; FI, NR)$. Consider a decrease of the borrowing limit to $\bar{b}' = b^E(0; FI, NR) - \varepsilon$, where $\varepsilon > 0$ is small. Since $b^E(0; FI.NR)$ is the first best for the extravagant government, we have

$$U^E(b^E(0; FI, NR), q^E) \simeq U^E(\bar{b}', q^E) - \varepsilon U^E_2(b^E(0; FI, NR), q^E),$$

where the second term on the RHS is zero by the envelope theorem. Thus, the decrease to the borrowing limit has no effect on the utility of the extravagant government. By extension this has no effect on the equilibrium level of bonds issued by the prudent government compared to the case without a binding debt limit. The decrease in the bond-level issued by the extravagant government increases the social welfare, as $b^S < b^E(0; FI, NR)$.

Similarly, if $\bar{b} = b^S$, then the social cost of increasing the borrowing limit slightly will not impact the social welfare when the extravagant government is in power. However it will increase the utility of the extravagant government as $b^E(0; FI, NR) > b^S$. This in turn relaxes the constraint for the prudent government. As $b^P(1; FI, NR) < b^S$ this increases the social welfare. ∎

B Characterization of Pooling Equilibria

We now give an exact characterization of which pooling equilibria that survive the intuitive criterion given the minimum level of government spending, $\underline{g}$, and fiscal rule, $\bar{b}$. Note that for $\bar{b}$ sufficiently high this captures the case of in which there is no fiscal rule.

Note that $b^P(0; FI, NR) > b^P(\pi; FI, NR)$, since $q(\pi) > q^E$. If not, then

$$\beta^P R u'(y_2 - b^P(\pi; FI, NR)) = u'(y_1 + q(\pi) b^P(\pi; FI, NR)) \tag{8}$$

$$< u'(y_1 + q^E b^P(0; FI, NR)) = \beta^P R u'(y_2 - b^P(0; FI, NR)) \leq \beta^P R u'(y_2 - b^P(\pi; FI, NR)), \tag{9}$$

where the equalities follows from first order conditions, and the inequlities follows since u is concave, $b^P(0; FI, NR) \leq b^P(\pi; FI, NR)$ and $q(\pi) > q^E$.

As it will clearly not be optimal to impose a debt limit that restricts the debt such much such that it restricts the borrowing of the prudent type, when facing the worst possible bond price. Hence through out in this section we assume that $\bar{b} \geq b^P(0; FI, NR)$.

Let $\underline{g}$ and $\bar{b}$ be given. Denote by $b^P(0) = \max\{b^P(0; FI, NR), \underline{b}(y_1, q^E)\}$, and $b^E(0) = \min\{b^E(0; FI, NR), \bar{b}\}$.

Note, that when the minimum level for government spending restrictsthe choices of the Prudent government, when she is perceived as the Extravagant type ($b^P(0) > b^P(0; FI, NR)$), then highest level the Prudent government will be willing to choose when faced with the bond price of $q(\pi)$ increases ($\hat{b}^P(\pi, b^P(0)) > \hat{b}^P(\pi, b^P(0; FI, NR))$ for $\hat{b}^P(\pi, b^P(0)) > b^P(0)$, and $\hat{b}^P(\pi, b^P(0; FI, NR)) > b^P(0; FI, NR)$). Similarly, when the borrowing limit restrict the Extravagant government when perceived as the extravagant type, then the lowest level the extravagant government is willing to choose when faced with the bond price of $q(\pi)$ decreases ($b^E(0) < b^E(0; FI, NR)$ then $\hat{b}^E(\pi, b^E(0)) > \hat{b}^E(\pi, b^E(0; FI, NR))$ for $\hat{b}^E(\pi, b^E(0)) < b^E(0)$, and $\hat{b}^E(\pi, b^E(0; FI, NR)) < b^P(0; FI, NR)$).

By the same arguments as in the proof of Lemma 2 the range of possible bond levels that can be sustained in a pooling equilibrium with $\underline{g}$ and $\bar{b}$ is given by

$$A(\underline{g}, \bar{b}) = \left[\hat{b}^E(\pi, b^E(0)), \hat{b}^P(\pi, b^P(0))\right] \tag{10}$$

where $\hat{b}^E(\pi, b^E(0))$ is the lowest downward deviation that the Extravagant government will be willing to choose when faced with the bond price $q(\pi)$ ($\hat{b}^E(\pi, b^E(0)) < b^E(0)$) and $\hat{b}^P(\pi, b^P(0))$ is highest upward deviation that the Prudent government will be willing to choose when faced with the bond price of $q(\pi)$ instead of q^E ($\hat{b}^P(\pi, b^P(0)) > b^P(0)$). Furthermore, let

$$B(\underline{g}, \bar{b}) = [\underline{b}(y_1, q(\pi)), \bar{b}] \tag{11}$$

be the set of bond levels that are feasible when faced with the bond price of $q(\pi)$.

Thus, for $b^*(\pi)$ to be the bond level to be sustained as the bond level requested in a pooling equilibrium, then $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$.

So far we have been silent about the off-equilibrium beliefs. Suppose that $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$ is the bond-level requested by both types of governments in equilibrium. Let the beliefs be as follows:

$$\mu(b) = \begin{cases} \pi & \text{if } b = b^*(\pi) \\ 0 & \text{if } b > \hat{b}^P(1, b^*(\pi)) \\ 0 & \text{if } b \neq b^*(pi), b \geq \hat{b}^E(1, b^*(\pi)) \text{ and } b \leq \hat{b}^P(1, b^*(\pi)) \\ 1 & \text{if } b < \hat{b}^E(1, b^*(\pi)) \end{cases} \tag{12}$$

where the $\mu(b^*(\pi)) = \pi$ is given by Bayes rule, $\mu(b) = 0$ is $b > \hat{b}^P(1, b^*(\pi))$ and $\mu(b) = 1$ if $b < \hat{b}^E(1, b^*(\pi))$ follows from the restrictions imposed by the intuitive criterion. For the remaining of equilibrium values of b the intuitive criterion has no bite, so we assume the beliefs that will make deviations least attractive.

If $\hat{b}^E(1, b^*(\pi)) > \underline{b}(y_1, q^P)$, then the Prudent government will deviate to $b = \hat{b}^E(1, b^*(\pi)) - \varepsilon$ for $\varepsilon > 0$ small. Because of the restrictions imposed by the intuitive criterion, and sequential rationally, the bond price for b will be q^P, and thus will be a profitable deviation for the Prudent government. Such a deviation is not possible if $\hat{b}^E(1, b^*(\pi)) \leq \underline{b}(y_1, q^P)$. Since any bond level b for which the intuitive criterion would imply that $\mu(b) = 1$ are not feasible.

We conclude that the set of pooling equilibria are characterized by $b^*(\pi) \in A(\underline{g}, \bar{b}) \cap B(\underline{g}, \bar{b})$ for which $\hat{b}^E(1, b^*(\pi)) \leq \underline{b}(y_1, q^P)$.

To gain a better understanding of the set of pooling equilibria, we consider the effects of changes to $\underline{g}$ and $\bar{b}$.

First consider the comparative statics of a marginal increase in the minimum level of government spending $\underline{g}$:

$$\frac{\partial b^P(0)}{\partial \underline{g}} = \begin{cases} 0 & \text{if } b^P(0) \geq \underline{b}(y_1, q^E) \\ \frac{R}{1-\delta^E} > 0 & \text{if } b^P(0) < \underline{b}(y_1, q^E) \end{cases}, \tag{13}$$

and

$$\frac{\partial \underline{b}(y_1, q(\pi))}{\partial \underline{g}} = \frac{R}{1 - \pi\delta^P - (1-\pi)\delta^E} \in \left(0, \frac{R}{1-\delta^E}\right). \tag{14}$$

When $\frac{\partial b^P(0)}{\partial \underline{g}} > 0$, an increase in the minimum level of required government spending increases $A(\underline{g}, \bar{b})$ as the upper bound increases. This tends to increase the set of possible pooling equilibria on the margin. On the other hand, it decreases $B(\underline{g})$ so that the set of feasible pooling equilibrium may decrease.

Next, we consider how changes to the debt limit affects the set of pooling equilibria that can be sustained in equilibrium. The debt limit has a direct effect as follows:

$$\frac{\partial b^E(0)}{\partial \bar{b}} = \begin{cases} 0 & \text{if } b^E(0) < \bar{b} \\ -1 < 0 & \text{if } b^E(0) < \bar{b} \end{cases}. \tag{15}$$

This in turn decreases the lowest debt level the Extravagant government is willing to choose if she is perceived as the Prudent type for any bond levels when faced with the bond price $q(\pi)$. Thus, it increases the range of pooling equilibria possible.

In general, it is not possible to determine whether $b^P(\pi; FI, NR)$ is lower or higher than $\hat{b}^E(\pi, b^E(0))$. If $\hat{b}^E(\pi, b^E(0)) > b^P(\pi; FI, NR)$, then lowering the debt limit sufficiently will make it possible to sustain $b^P(\pi; FI, NR)$ in a pooling equilibrium. If $\hat{b}^E(\pi, b^E(0)) < b^P(\pi; FI, NR)$, then relaxing the the debt limit, will exclude suboptimal pooling equilibria. Note, however, whether or not the socially optimal level is feasible depends on the minimum level of government spending.