Japan-IMF Scholarship Program for Asia 2021-2022
Basic Mathematics Aptitude Test
Test A

(Full score: 40)

Please Note:

- You have 60 minutes to complete
- Calculators are not allowed
- Please show all your work and write your answers in the designed space

Thank you

Country:______________________________

Reference Number:______________________

Name:________________________________

Referecene Number: Name:
Country:

Problem 1. Compute the following:

$$\frac{500+\frac{5}{2}}{5+\frac{10}{2}}-\frac{1}{4}$$

Answer:____________________

Problem 2. Assume that α is some constant which satisfies $1>\alpha>0$. Solve for x:

$$\frac{x^{\alpha-1}}{x^{\alpha}}=\frac{1}{1-x}$$

Answer:____________________

Problem 3. Let e denote Euler's constant. Solve for x:

$$\frac{e^{2x-5}}{e^{x}}=1$$

Answer:____________________

Problem 4. Solve for x:

$$2\ln(x+1)+\ln\left(\frac{1}{x+1}\right)=\ln(5)$$

Answer:____________________

Problem 5. Compute $f(11.5)$:

$$f(x)=\frac{4x^{2}(x+5)}{2x^{2}+10x}$$

Answer:____________________

Referecene Number: Name:
Country:

Problem 6. Assume that a and b are both positive constants and restrict $x > 0$. Under what conditions is $f(x)$ an increasing function?

$$f(x) = \frac{a+x}{b+x}$$

Answer:____________________

Problem 7. Assume that a, b, c are all constants and $a + b \neq 0$. Solve for x:

$$ax + bx = c$$

Answer:____________________

Problem 8. Suppose there are two goods (good 1 and good 2). Let x_i and p_i denote the quantity purchased and the price of good i respectively. The household's income is M. The household's budget constraint is

$$p_1 x_1 + p_2 x_2 = M.$$

Solve for x_2 as a function of income, prices, and x_1:

Answer:____________________

Problem 9. Assume that a and b are positive constants and restrict $x > 0$. Calculate the derivative of $f(x)$:

$$f(x) = \ln\left(ax^2 + bx\right)$$

Answer:____________________

Problem 10. Assume that $1 > \alpha > 0$, and that M and p are both positive parameters (constants). Calculate the derivative of $u(x)$:

$$u(x) = \ln\left[x^{\alpha}\right] + \ln\left[(M - px)^{1-\alpha}\right]$$

Answer:____________________

Referecene Number: Name:
Country:

Problem 11. Suppose that $A = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}$. Compute AB:

Answer:____________________

Problem 12. Suppose that a, b, c, d are all constants and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Write y_1 as a function of the constants and x_1 and x_2:

Answer:____________________

Problem 13. Solve for x_1, x_2, and x_3, where:

$$\begin{bmatrix} 1 & 5 & 10 \\ 2 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

Answer:____________________

Problem 14. Assume that $\kappa > 1$ and $b > 0$. Compute:

$$\int_b^\infty x \left(\frac{\kappa b^\kappa}{x^{\kappa+1}} \right) dx$$

Answer:____________________

Problem 15. Assume that

$$\int_{-\infty}^{\infty} f(z)\, dz = 1;$$
$$\int_{-\infty}^{a} f(z)\, dz = \frac{1}{3}.$$

Provide a numerical answer for

$$\int_a^\infty f(z)\, dz.$$

Answer:____________________

Referecene Number: Name:
Country:

Problem 16. Suppose that

$$S(\beta_0, \beta_1) = \sum_{i=1}^{N} (\beta_0 + \beta_1 x_i - y_i)^2 .$$

Compute the partial derivatives $\frac{\partial S(\beta_0,\beta_1)}{\partial \beta_0}$ and $\frac{\partial S(\beta_0,\beta_1)}{\partial \beta_1}$:

Answer:____

Problem 17. Refer to the previous question. Solve for the β_0 and β_1 which minimize $S(\beta_0, \beta_1)$ and denote them $\hat{\beta}_0$ and $\hat{\beta}_1$ respectively. Suppose that we have the following three pairs of data (x, y): $(0, 1)$, $(1, 1)$, $(2, 2)$ and hence $N = 3$. Given this data, provide a numerical answer for $\hat{\beta}_1$:

Answer:____

Problem 18. Suppose that you would like to build a fence for a farm. Because of regulations you can only build the fence in the form of a rectangle. You seek to maximize the area of farmland protected by the fence. The formula for the area (A) is

$$A = xy$$

where x is the length and y is the width of the fence. You must spend the entire budget to build the fence. You have \$100 to spend. Each meter of fencing costs \$2. Solve for the optimum x and y:

Answer:____

Problem 19. Suppose that we have 20 observations

$$X = \{10.4, 5.4, 11.8, 12.2, 9.9, 10.9, 10.5, 9.5, 12.2, 8.6, 8.9, 13.3, 11.2, 8.9, 13.5, 11.1, 7.0, 8.3, 6.7, 6.7\}$$

The mean of all of the observations is $\bar{x} = 9.85$. What is the sum of all of the observations?

Answer:____________________

Problem 20. Refer to the previous question. Let x_i denote each element in X (10.4, 5.4, ... 6.7). Calculate:

$$\sum_{i=1}^{20} (x_i - \bar{x})$$

Answer:____________________