Japan-IMF Scholarship Program for Asia 2022-2023
Basic Mathematics Aptitude Test
Test A
(Full score: 40)

Please note:

- You have 60 minutes to complete the test
- Calculators are not allowed

Name: _____

Country: _____

Reference Number: _____

Name:

Reference Number:

Please show all you work here and write your answers in the designated place

1. **(1 point)** Calculate $7 \div 2 \times (-4) + 12$.

Answer: _____

2. **(1 point)** Find the largest number out of $\frac{\sqrt{3}}{2}$, $\frac{\sqrt{6}}{3}$, and $\frac{2}{\sqrt{5}}$.

Answer: _____

3. **(2 points)** Calculate $\frac{\sqrt{12}}{\sqrt{3}-1} + \frac{\sqrt{12}}{\sqrt{3}+1}$.

Answer: _____

4. **(2 points)** Calculate $49^{\frac{1}{\log_8 7}}$.

Answer: _____

5. **(2 points)** Calculate $\frac{x^3-1}{x^2+x+1}$ for $x = 101$.

Answer: _____

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6. **(2 points)** Solve the following system of linear equations in x and y:

$$\begin{cases} 5x + 2y = 12 \\ 3x - 2y = 4 \end{cases}$$

Answer: _____

7. **(2 points)** Solve for x from the following equation

$$\frac{b}{x-a} + \frac{a}{x-b} = 2,$$

where a and b are parameters such that $a \neq 0$, $b \neq 0$, and $a \neq b$.

Answer: _____

8. **(2 points)** Find the first-order derivative of the function $f(x) = \dfrac{x^2+2}{x+1}$.

Answer: _____

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9. **(2 points)** For which x function $f(x) = \sqrt{2 + x - x^2}$ is well-defined?

Answer: _____

10. **(2 points)** Calculate the sum $\sum_{d=0}^{19} (3 + d)$.

Answer: _____

11. **(2 points)** Let $f(x) = a^x$, where $a > 0$ is a parameter. For which $a > 0$ function $f(x)$ is decreasing in x, and for which $a > 0$ this function is increasing?

Answer: _____

12. **(2 points)** Find the partial derivative $\frac{\partial^2 f(x,y)}{\partial x \partial y}$ for $f(x,y) = xy + \frac{x}{y}$.

Answer: _____

Please show all you work here and write your answers in the designated place

13. **(2 points)** Evaluate the integral $\int_1^8 x^{\frac{1}{3}} \mathrm{d}x$.

Answer:_____

14. **(2 points)** Find all local extrema of function $f(x) = x^3 - 6x^2 + 9x - 4$.

Answer:_____

15. **(2 points)** Assuming that n takes positive integer values, find $\lim_{n\to\infty} \frac{10000n}{n^2+1}$

Answer:_____

16. **(2 points)** Evaluate the limit $\lim_{x\to\infty} \frac{x}{x+\ln x}$.

Answer:_____

Please show all you work here and write your answers in the designated place

17. **(2 points)** Find the determinant of matrix

$$\begin{pmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{pmatrix}$$

Answer: _____

18. **(2 points)** For matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ find A^{-1}.

Answer: _____

19. **(2 points)** There are 5 organges and 4 apples in a box. Without looking inside the box, you take 2 random fruits from it. What is the probability that both fruits are oranges?

Answer: _____

Please show all you work here and write your answers in the designated place

20. **(2 points)** Consider all 4-digit numbers consisting only of digits 1, 2, and 3. What is the proportion of such numbers that start with digit 2?

Answer: _____

21. **(2 points)** Coordinates of points A and B in the three-dimensional space are given by $A = (8, 0, 1)$ and $B = (1, 1, -1)$. Find the distance between A and B.

Answer: _____