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The Expansionary Lower Bound: Contractionary Monetary Easing and the Trilemma

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The Expansionary Lower Bound: Contractionary Monetary Easing and the Trilemma

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Abstract

We provide a theory of the limits to monetary policy independence in open economies arising from the interaction between capital flows and domestic collateral constraints. The key feature of our theory is the existence of an "Expansionary Lower Bound" (ELB), defined as an interest rate threshold below which monetary easing becomes contractionary. The ELB can be positive, thus acting as a more stringent constraint than the Zero Lower Bound. Furthermore, the ELB is affected by global monetary and financial conditions, leading to novel international spillovers and crucial departures from Mundell's trilemma. We present two models under which the ELB may arise, the first featuring carry-trade capital flows and the second highlighting the role of currency mismatches.

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1 Introduction

The large swings in capital flows during the global financial crisis and the concerns about international spillovers from the ongoing US monetary tightening have rekindled the debate on whether emerging markets (EMs) can retain monetary independence while having open capital accounts. According to Mundell's trilemma, monetary authorities in EMs can respond effectively to global financial and monetary shocks even if they are open to capital flows as long as they allow for exchange rate flexibility. Under this perspective, which is at the core of conventional open-economy models, movements in capital flows do not undermine the ability of monetary policy to ensure macro-economic stability.

However, growing skepticism against this benevolent view of capital flows has been voiced by both academics and policy makers (Blanchard et al., 2016; IMF, 2012; Obstfeld, 2015; Rajan, 2015; Rey, 2015, 2016; Arregui et al., 2018). These concerns stem in part from the observation that financial and monetary conditions in EMs are strongly affected by volatile international capital flows, raising doubts on whether monetary policy in EMs can effectively balance these pressures. Furthermore, monetary policy in EMs can itself generate swings in capital flows that may impair monetary transmission. For example, policy makers in EMs are often reluctant to lower interest rates during an economic downturn because they fear that, by spurring capital outflows, monetary easing may end up weakening, rather than boosting, aggregate demand.

An empirical analysis of the determinants of policy rates in EMs provides suggestive evidence about the tensions faced by monetary authorities, even in countries with flexible exchange rates. In Table 1, we regress policy rates for a sample of major EMs over Taylor-rule determinants as well as measures of global financial and monetary conditions.[1] The results reveal that, even after controlling for expected inflation and the output gap, monetary authorities in EMs tend to hike policy rates when the VIX or US policy rates increase. This is arguably driven by the desire to limit capital outflows and the depreciation of the exchange rate. These effects are highly statistically significant and economic sizable.[2] Furthermore, they are robust to using quarterly or monthly data, excluding one country at a time, and estimating the regressions in first differences.

In this paper we provide a theory that rationalizes how free capital mobility can hinder monetary policy independence in EMs, i.e. it can prevent monetary authorities from ensuring macro-economic

[1] The sample includes Brazil, China, India, Indonesia, Mexico, Russia, South Africa, Turkey and uses data from 2000 onward, both at quarterly and monthly frequency. The regressions are estimated with country-fixed effects. We use measures of expected inflation and GDP growth over the next year constructed with monthly data from Consensus Forecast. Forecasters are asked each month about their projections for the current year and the next one. We create indicators of inflation and growth over the next 12 months by taking a weighted average of the current and next year prediction based on the remaining months in the year. For example, in the month of September our averages use a weight of 3/12 on the current year and 9/12 on the following one. Regarding exchange rate, forecasters are asked about the changes over the next 12 months. The output gap is estimated using the HP filter. The US policy rate uses Wu and Xia (2016) shadow rate to account for changes in monetary policy during the zero lower bound period

[2] A one-standard-deviation increase in the VIX is associated with an increase of policy rates in EMs by about 50 basis points.

Table 1: Policy rate responses in EMs to global liquidity and monetary shocks

VARIABLES	(1) Quarterly	(2) Quarterly	(3) Quarterly	(4) Monthly
Expected inflation	1.13***	1.07***	1.02***	0.93***
	(0.07)	(0.07)	(0.06)	(0.04)
Output gap	0.14**	0.20***	-0.00	
	(0.07)	(0.07)	(0.05)	
VIX		0.06***	0.05***	0.05***
		(0.01)	(0.01)	(0.01)
U.S. policy rate			0.66***	0.64***
			(0.04)	(0.02)
Constant	2.48***	1.70***	1.46***	1.96***
	(0.44)	(0.48)	(0.38)	(0.23)
Observations	543	543	543	1,555
R-squared	0.311	0.330	0.570	0.502
Number of countries	8	8	8	8

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

stability even under a flexible exchange rate regime. This is because the interaction between capital flows and domestic collateral constraints can undermine monetary transmission. More specifically, our theory predicts the existence of an "Expansionary Lower Bound" (ELB) which is an interest rate threshold below which monetary easing becomes contractionary. The ELB constraints the ability of monetary policy to stimulate aggregate demand, placing an upper bound on the level of output achievable through monetary stimulus.

The ELB can occur at positive interest rates and is therefore a potentially tighter constraint for monetary policy than the Zero Lower Bound (ZLB). Furthermore, global monetary and financial conditions affect the ELB and thus the ability of central banks to support the economy through monetary accommodation. A tightening in global monetary and financial conditions leads to an increase in the ELB which in turn can force domestic monetary authorities to increase policy rates in line with the empirical evidence presented in 1.

We establish the conditions for the existence of the ELB in the context of two different models. This shows that the ELB can arise in various environments through the interaction of capital flows and domestic collateral constraints. In the first model the ELB arises because of the impact of monetary policy on carry-trade capital flows. The model features a small open economy populated by domestic borrowers and savers, in which collateral constraints take the form of leverage restrictions on the domestic banking sector. Banks collect deposits, invest in government bonds, and provide domestic loans. Government bonds are also held by foreign investors whose demand is increasing in the expected currency risk premium of domestic over foreign assets.

In the model, monetary easing triggers capital outflows since it reduces the excess return on domestic bonds. When the banks' leverage constraint is not binding, monetary easing has conventional expansionary effects as the banking sector can absorb the excess supply of bonds without jeopardizing its ability to provide loans. However, for a sufficiently strong monetary easing, capital outflows become large enough to push domestic banks against their leverage constraint. Once banks are constrained, further monetary easing can become contractionary. This is because to absorb the bonds liquidated by foreign investors, banks have to reduce private credit by increasing lending rates. If this credit crunch is sufficiently large, monetary easing becomes contractionary giving rise to the ELB.

In the second application the ELB arises because of the effects of currency mismatches on collateral constraints. This is a proverbial concern in EMs that in recent years have accumulated large amounts of US dollar debt attracted by low US rates (Acharya et al., 2015; McCauley, McGuire and Sushko, 2015). In the model, unhedged currency mismatches are held by domestic banks that borrow internationally in foreign currency and lend domestically in local currency. As in the first model, banks are subject to a leverage constraint that limits domestic lending to a certain multiple of bank capital.

When the leverage constraint is not binding, monetary accommodation is expansionary. Lower rates boost domestic demand and, by depreciating the exchange rate, they also strengthen foreign demand. However, a sufficiently large monetary easing can make the leverage constraint binding since the exchange rate depreciation reduces bank capital. From this point onward, if foreign-currency debt is sufficiently large, additional monetary easing becomes contractionary since banks can no longer freely intermediate foreign capital to provide domestic loans. This generates an increase in lending rates and a domestic credit crunch that contracts domestic demand and output.

A crucial aspect of our theory is that in both models the ELB is affected by global financial and monetary conditions. Under carry-trade capital flows, the ELB increases with a tightening of global financial conditions since foreign demand for domestic bonds weakens. In the presence of currency mismatches, the ELB rises instead with an increase in the foreign monetary policy rate due to the depreciation of the exchange rate. The increase in the ELB can in turn push EMs into a recession while central banks are forced to increase policy rates, in line with the evidence in Table 1. This is the case even in countries with flexible exchange rates, thus providing a crucial departure from Mundell's trilemma.

The existence of the ELB gives raise also to a novel inter-temporal trade-off for monetary policy. This is because, unlike the ZLB, the level of the ELB is affected by the monetary policy stance in previous periods through the effects on domestic lending, capital flows, and bank capital. In particular, a tighter ex-ante monetary policy tends to lower the ELB in subsequent periods. This calls for running the economy below potential by keeping a tighter monetary stance to lower the

ELB and allow for greater monetary space in the future. The negative correlation between ex-ante monetary policy and the ELB has the additional implication that monetary policy tends to become less effective in stimulating output even when the ELB does not bind. This is because the stimulative effects of monetary easing are partially offset by the expectation of a tighter future monetary stance due to the increase in the ELB.

The ELB provides also a rationale for alternative policy tools that can be used by domestic authorities to regain monetary space, especially unconventional monetary policies, capital controls, and macro-prudential measures. The effectiveness of these tools and the channels through which they operate depend on the determinants of the ELB. Balance-sheet operations by the central bank, including quantitative easing and foreign exchange intervention, are quite effective in overcoming the ELB due to carry-trade flows since they support credit supply by reducing the amount of government bonds held by banks. Capital controls are instead helpful in case of currency mismatches, since they can be used to decouple the exchange rate from the domestic monetary conditions. Interestingly, forward guidance is unable to ease the constraints imposed by the ELB, despite being quite effective in overcoming the ZLB. This is because the ELB is an endogenous interest threshold that increases with the expectation of looser monetary policy in the future.

The paper is structured as follows. After reviewing the relevant literature, we present the model with carry traders in section 2. We then analyze the model featuring currency mismatches in section 3. We summarize key findings and avenues for future research in the concluding section.

Literature review. The idea that domestic collateral constraints can alter the transmission of monetary policy is related to the literature spurred by the 1997 financial crisis in East Asia. Despite sound fiscal positions, East Asian countries suffered a severe crisis because the sharp depreciation of their exchange rates impaired the balance sheets of banks and firms with dollar liabilities. This led to the development of a third generation of currency crisis models to explain how the interplay between collateral constraints and currency mismatches can give rise to self-fulfilling currency runs (Krugman, 1999; Aghion, Bacchetta and Banerjee, 2000, 2001). Particularly related to our paper was the debate on the appropriate response of monetary policy, with some arguing in favor of monetary stimulus to support domestic demand, while others calling for monetary tightening to limit balance-sheet disruptions. These issues are analyzed in Céspedes, Chang and Velasco (2004), Christiano, Gust and Roldos (2004), and Gourinchas (2018). While these models can generate situations in which monetary easing is contractionary, the ability of the central bank to stabilize output is never constrained. Even when monetary easing is contractionary, monetary policy can still achieve any desired level of output by raising rather than lowering policy rates.

The global financial crisis led to renewed interest in how financial frictions can affect monetary transmission. Ottonello (2015), and Farhi and Werning (2016) show that currency mismatches and collateral constraints can considerably complicate the conduct of monetary policy. In these models, monetary easing remains expansionary, but by depreciating the exchange rate it tightens collateral constraints and forces a reduction in domestic consumption.[3] Therefore, monetary policy faces a trade-off between supporting output and stabilizing domestic consumption, even though it can still achieve any desired level of output. The interaction between monetary policy and collateral constraints is also analyzed in Fornaro (2015), but in a model where monetary easing relaxes domestic constraints.

We go beyond this literature by developing models in which the interplay between collateral constraints and capital flows does not only generate competing objectives for monetary authorities, but it even prevents monetary policy from achieving a unique target, namely output stabilization. This happens because in our models monetary policy itself determines whether collateral constraints are binding or not. This is essential to generate the ELB and thus place an upper bound on the level of output that monetary policy can achieve. Furthermore, while the preceding literature focused only on currency mismatches, we show that monetary policy can face limits in stimulating output also because of the impact of carry-trade capital flows on domestic collateral constraints.

The notion that monetary policy may become ineffective below a certain interest rate threshold is common to other two recent papers. Brunnermeier and Koby (2016) point out that monetary policy can become contractionary because it may impair bank profitability. This can in turn push banks against their leverage constraint at which point further monetary easing can lead to an increase in lending rates. Concerns about the impact on bank profitability are expressed also in Eggertsson, Juelsrud and Wold (2017), but in reference to the recent adoption of negative policy rates in several advanced economies. Since banks appear reluctant to lower deposit rates below zero, charging negative rates on bank reserve tends to reduce bank profits and lead to a contraction in credit supply. These papers use closed economy models which are therefore silent about the international aspects which are central to our analysis.

The paper is also closely related to a recent literature that analyzes the role of macro-prudential policies and capital controls in open economies, among which for example Jeanne and Korinek (2010), Bianchi (2011), Benigno et al. (2013) Benigno et al. (2016), and Korinek and Sandri (2016). These papers rationalize the use of these policy tools to correct externalities associated with collateral constraints in the context of real models. On the contrary, we work with a monetary model in which capital controls and macro-prudential policies are used to overcome the constraints imposed by the ELB. Closer to us, Aoki, Benigno and Kiyotaki (2016) analyze the tensions faced by monetary policy because of currency mismatches and the benefits from financial sector policies, but in a model where monetary easing remains expansionary.

We develop the analysis using models with collateral constraints and heterogeneity between

[3] Ottonello (2015) considers also an extension of his model in which collateral constraints limit the country's ability to import intermediate goods. In this case, monetary easing can in principle have contractionary effects on output by depreciating the exchange rate and tightening constraints. Nonetheless, in his calibration monetary accommodation remains expansionary.

constrained and unconstrained agents. The paper is thus related to a growing literature that analyzes monetary policy in models with incomplete financial markets and heterogeneous agents (Auclert, 2016; Gornemann, Kuester and Nakajima, 2016; Kaplan, Moll and Violante, 2016; McKay, Nakamura and Steinsson, 2016; Guerrieri and Lorenzoni, 2016; Werning, 2015). These models reveal important departures from the monetary transmission in representative agent models. For example, they tend to find a stronger responsiveness of consumption to income effects and uncover novel channels of transmission through redistribution effects. Nonetheless, in all these papers, monetary easing remains expansionary.

Finally, the paper is related to three streams of the empirical literature. One documents that EMs tend to resist large movements in exchange rates by displaying what Calvo and Reinhart (2002) referred to as "fear of floating". Consistent with this evidence, the ELB can induce monetary authorities in EMs to increase policy rates when global financial or monetary conditions tighten, thus leaning against sharp exchange rate movements. A second and more recent group of papers, among which (Bruno and Shin, 2015, 2017; Baskaya et al., 2017; Avdjiev and Hale, 2017), provide evidence about the large international spillovers from US monetary policy. These papers find that US monetary policy has pronounced effects on global financial intermediaries and in turn on international capital flows in line with the mechanisms underpinning our models. Third, our first model is related to the empirical literature that analyzes carry trade capital flows, including Lustig and Verdelhan (2007), Brunnermeier, Nagel and Pedersen (2008), Lustig, Roussanov and Verdelhan (2011), Menkhoff et al. (2012), and Corte, Riddiough and Sarno (2016).

2 The ELB under carry-trade capital flows

In this section, we present a model in which the ELB can emerge because of the effects of monetary policy on carry-trade capital flows. In the model, an interest rate cut reduces the expected excess return on domestic bonds and triggers a capital outflow. If large enough, the capital outflow tightens domestic collateral constraints and causes a domestic credit crunch which reduces aggregate demand and output. Monetary easing becomes therefore contractionary giving rise to an ELB.

2.1 Model setup

The model features a small open economy in which banks collect domestic deposits to provide loans and buy government bonds subject to a leverage constraint. Foreign investors supply funds to the small open economy by purchasing government bonds in proportion to their expected excess return over foreign assets. To ease notation, we present the model in a recursive infinite-horizon formulation. When solving it, we will assume that the model is in steady state from time 2 onward and focus on the equilibrium in the first two periods. We describe the the model in its most simple form by considering only the role of conventional monetary policy. In section 2.3, we incorporate fiscal and unconventional monetary policy tools to understand how they can help overcome the restrictions imposed by the ELB.

2.1.1 Household and corporate sector

The economy is populated by two types of households, borrowers and savers, whose variables are denoted with B and S superscripts, respectively. Borrowers and savers have identical preferences but heterogeneous income streams, such that at time 0 and 1 borrowers are borrowing and savers are saving. Households choose consumption to maximize the inter-temporal utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln C_t^i \tag{1}$$

where $i = \{B, S\}$ and β is the inter-temporal discount factor. The consumption index C_t^i is defined as $C_t^i = \left(C_{H,t}^i\right)^{1-\alpha} \left(C_{F,t}^i\right)^{\alpha}$, where the parameter $\alpha \in (0,1)$ reflects the degree of trade openness, and $C_{H,t}^i$ and $C_{F,t}^i$ are consumption aggregators of home and foreign goods.

Borrowers are subject to the following budget constraint

$$P_t C_t^B + L_{t-1} I_{t-1}^L = \Pi_t^B + L_t$$

where P_t is the aggregate price level, L_t are loans which carry the interest rate I_t^L, and Π_t^B is borrowers' total net income which includes both labor payments, profits from domestic firms, and lump-sum taxes.[4] Savers face a similar budget constraint

$$P_t C_t^S + D_t = \Pi_t^S + D_{t-1} I_{t-1}^D$$

where D_t are bank deposits that are remunerated at the interest rate I_t^D. Domestic households smooth consumption based on the Euler equations

$$\begin{aligned} 1 &= \beta_t^B I_t^L \mathbb{E}_t \left[P_t C_t^B / \left(P_{t+1} C_{t+1}^B \right) \right] \\ 1 &= \beta_t^S I_t^D \mathbb{E}_t \left[P_t C_t^S / \left(P_{t+1} C_{t+1}^S \right) \right] \end{aligned}$$

and allocate spending on Home goods according to $P_{H,t} C_{H,t}^i = (1-\alpha) P_t C_t^i$. Similarly, foreign households, denoted with an asterisk, smooth consumption according to $1 = \beta I_t^* \mathbb{E}_t \left[P_t^* C_t^* / \left(P_{t+1}^* C_{t+1}^* \right) \right]$ and spend on domestic goods an amount equal to $P_{H,t}^* C_{H,t}^* = \alpha P_t^* C_t^*$. We denote aggregate consumption and income by dropping the household-type superscript, so that $C_t = C_t^B + C_t^S$ and $\Pi_t = \Pi_t^B + \Pi_t^S$.

[4]We leave the details of the labor market concerning labor supply and wage setting unrestricted since they are not essential for the determination of the ELB. For example, the model can easily incorporate endogenous labor supply and sticky wages without altering its key results.

The production sector is composed of a continuum of monopolistically competitive firms which hire households to produce differentiated varieties of the domestic good.[5] Firms face downward sloping demand curves for their own variety and choose prices to maximize profits. Firms can set different domestic and foreign prices for their goods, so that the law of one price does not have to hold.[6] We allow monetary policy to have real affects in periods 0 and 1 by assuming that the prices of goods sold domestically and abroad are constant and equal to $\bar{P}_H$ and $\bar{P}^*_H$, respectively. Without loss of generality we normalize them to 1. We instead assume that prices are fully flexible from time 2 onward so that monetary policy has only nominal effects in the steady-state of the model.

2.1.2 Banking sector

Domestic banks use their networth N_t and collect domestic deposits to provide loans, buy domestic government bonds B_t, and hold central bank reserves R_t. The balance sheet of the representative bank is given by

$$N_t + D_t = L_t + B_t + R_t$$

Bank networth evolves according to

$$N_{t+1} = L_t I^L_t + B_t I^B_t + R_t I_t - D_t I^D_t \tag{2}$$

where I^B_t is the yield on government bonds and I_t is the policy rate, i.e. the remuneration rate on reserves.

We assume that banks are subject to a leverage constraint which prevents assets from exceeding a multiple of networth, according to

$$L_t + \lambda B_t \leq \phi N_t \tag{3}$$

where $\phi > 1$ and $\lambda \in (0,1)$, such that government bonds have a lower capital charge than domestic loans. This formulation can capture regulatory requirements that usually provide a preferential treatment to government bonds. Or it can be due to market forces that consider bonds as less risky than loans or more easily recoverable in case of bank failure. More formally, constraint (3) can be microfunded as the incentive compatibility constraint imposed to bankers by their creditors when

[5] Since firms produce differentiated varieties of the Home good, indexed by $j \in [0,1]$, the consumption aggregator for domestic goods is $C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$, where $\varepsilon > 1$ is the elasticity of substitution among varieties. A similar aggregator applies to $C^*_{H,t}$.

[6] This price-setting assumption is known as Local Currency Pricing (LCP) in contrast with Producer Currency Pricing (PCP). Under the latter, firms only choose the domestic-currency price of their goods and the law of one price holds. Notice that, since in our model the elasticity of substitution between domestic and foreign goods is one, the choice of LCP vs PCP only affects export quantities but not export revenues, which are the same under both assumptions. The model can be easily extended to incorporate PCP.

assets have different recovery values.[7] For the leverage constraint to be relevant, we assume that banks cannot issue new equity at time 0 and 1.

Banks act competitively and, since returns on their assets are riskless, they simply choose their balance sheets to maximize period-by-period networth subject to the leverage constraint. A no-arbitrage condition between household deposits and central bank reserves implies that the deposit rate is equal to the policy rate $I_t^D = I_t$. Lending rates and bond yields can instead increase above the policy rate because of the leverage constraint. The first order conditions with respect to loans and government bonds require that

$$\begin{aligned} I_t^L &\geq I_t \\ I_t^B &= \lambda I_t^L + (1-\lambda) I_t \end{aligned}$$

If the leverage constraint does not bind, lending and bond rates are equal to the policy rate I_t , so that any monetary policy change transmits one-for-one to all rates. If instead the constraint binds, the lending rate increases above the policy rate to ensure market clearing in the loan market. This gives rise to a lending spread that impairs the transmission of monetary policy. In fact, as we shall see below, a policy rate cut can even lead to an increase in lending rates so that monetary accommodation has contractionary effects on credit supply. When the leverage constraint binds, bond yields must also increase above the policy rate because of no-arbitrage between loans and bonds. The bond spread is proportional to the capital charge λ in the leverage constraint.

2.1.3 Foreign investors

The country can attract foreign capital by selling government bonds internationally. We assume that foreign capital is channeled through foreign financial intermediaries that finance the purchase of domestic bonds B_t^F by borrowing in foreign currency at the rate I_t^*, so that their balance sheet is given by $B_t^F + e_t B_t^* = 0$. These intermediaries earn an expected foreign-currency return equal to

$$V_t^* = B_t^* \mathbb{E}_t \left[\frac{e_t}{e_{t+1}} I_t^B - I_t^* \right]$$

In the spirit of Gabaix and Maggiori (2015), we assume that their intermediation capacity is limited by an agency friction due to their ability to divert funds. Rather than purchasing government bonds, foreign intermediaries can invest in foreign assets and divert a fraction $\gamma_t B_t^F$ of the proceeds, where the parameter $\gamma_t \geq 0$ controls the severity of the agency friction. Creditors can prevent foreign intermediaries from diverting money by constraining their balance sheets to satisfy the following

[7] Notice that, although we do not explicitly allow banks to borrow from foreign investors, we do not impose any restriction on the sign of B_t. Indeed, when $B_t < 0$ banks issue bonds that are perfectly substitutable with government bonds and effectively borrow from foreign investors. The implications of the model are unchanged since the collateral constraint still limits the substitutability between foreign funds and domestic deposits.

incentive compatibility condition

$$\mathbb{E}_t\left[\frac{e_t}{e_{t+1}}I_t^B - I_t^*\right] \geq \gamma_t B_t^F I_t^* \tag{4}$$

where the left and right-hand side expressions are the expected foreign-currency return for foreign intermediaries in case they invest in government bonds or divert money, respectively. Since the return from diverting funds is increasing in the size of the intermediaries' balance sheets, the incentive compatibility constraint is binding. Foreign demand for domestic government bonds is thus increasing in the expected excess return over foreign assets according to

$$B_t^F = \frac{1}{\gamma_t}\mathbb{E}_t\left[\frac{e_t}{e_{t+1}}\frac{I_t^B}{I_t^*} - 1\right] \tag{5}$$

The parameter γ_t determines the size of the intermediaries' balance sheets and is therefore an inverse measure of their risk-bearing capacity. The higher is γ_t, the higher is the required compensation per unit of risk. As $\gamma_t \uparrow \infty$, foreign demand shrinks to zero on matter the size of the excess return on domestic bonds. Vice versa, as $\gamma_t \downarrow 0$, the risk-bearing capacity is so high that any expected excess return is arbitraged away. In this case, Uncovered Interest Parity (UIP) holds as $I_t^B e_t/e_{t+1} \to I_t^*$. As we shall see when characterizing the model equilibrium, if $\gamma_t \in (0,1)$, the demand schedule in equation (5) generates carry-trade dynamics so that domestic monetary easing triggers capital outflows.[8]

The parameter γ_t is allowed to be stochastic to capture possible shocks to global liquidity conditions that can notoriously affect capital flows to EMs. For example, an increase in γ_t can reflect a rise in global risk aversion or in the perceived riskiness of EM government bonds. Modeling the exact source of shocks to γ_t goes beyond the scope of this paper since it does affect the implications for the ELB.

2.1.4 Public sector and market clearing

The public sector includes the central bank and the government. The central bank conducts monetary policy by setting the rate on reserves, I_t. To simplify the algebra, we abstract from balance-sheets operations by the central bank, considering the limit for $R_t \downarrow 0$. In Section 2.3, we relax this assumption and allow the central bank to use quantitative easing and foreign exchange intervention.

[8] This is not the case in specification used by Gabaix and Maggiori (2015), $B_t^F = \mathbb{E}_t\left[e_t - I_t^* e_{t+1}/I_t\right]/\gamma_t$, since monetary easing has no effects on capital flows. The key difference is that our model features foreign intermediaries that care about the foreign-currency return of their portfolio. Gabaix and Maggiori (2015) assume instead that intermediaries maximize the domestic return. This rather subtle difference has important implications for the elasticity of the exchange rate to changes in interest rates. In Gabaix and Maggiori (2015) a policy rate cut generates a proportional depreciation of the exchange rate that leaves the expected return on domestic bonds relative to foreign assets unchanged. As we shall see below, our formulation implies instead a lower responsiveness of the exchange rate so that monetary easing reduces the relative return on domestic bonds and triggers capital outflows.

Similarly, we start by assuming that the government simply rolls over the stock of public debt, B_t^G, that comes due each period

$$B_t^G = B_{t-1}^G I_{t-1}^B$$

and later extend the model to include taxes on domestic agents and capital flows. The model is closed by imposing market clearing conditions for domestic goods and government bonds

$$\begin{aligned} Y_{H,t} &= C_{H,t} + C_{H,t}^* && (6) \\ B_t^G &= B_t + B_t^F && (7) \end{aligned}$$

2.2 Model equilibrium

We assume that from time 2 onward the bank leverage constraint does not bind, prices are flexible, and the model is in steady state so that $I_t\beta = 1$. To ease notation and simplify the solution, we also set $\beta = 1$ which implies that in steady state agents spend all their income, $P_2C_2^i = \Pi_2^i$. We generalize the model results to the case in which $\beta < 1$ in Appendix A. The steady-state equilibrium can be easily characterized by considering that spending is also equal to the level of money supply, so that $P_2C_2^i = M_2^i$.[9] Using market clearing, which equates aggregate income in the Home economy with spending on Home goods, $\Pi_2 = (1-\alpha)M_2 + e_2\alpha M_2^*$, we can derive the steady-state level of the exchange rate, which is given by

$$e_2 = \frac{M_2}{M_2^*}$$

Without loss of generality, we normalize the steady-state money supply to 1 in both countries, such that $e_2 = 1$.

In the next section, we characterize the equilibrium in period 1, solving for the conditions under which the ELB may arise and showing how the ELB is affected by global conditions. We will then solve for the equilibrium at time 0, assuming that the bank leverage constraint does not bind and that global intermediaries can freely intermediate foreign funds under $\gamma_0 \downarrow 0$. This allows us to analyze how monetary authorities should set policy rates in tranquil times taking into account the possibility that the ELB may become binding in the future.

2.2.1 Model equilibrium at time 1

The level of domestic output at time 1 is determined by the consumption of home goods by domestic and foreign households. Using ω_2 to denote the share of steady-state output which is appropriated

[9] This can be rationalized in various ways, for example with a cash-in-advance constraint or money in the utility function.

by borrowers, $\omega_2 = \Pi_2^B / \Pi_2$, output can be expressed as

$$Y_{H,1} = (1-\alpha)\left(\frac{\omega_2}{I_1^L} + \frac{1-\omega_2}{I_1}\right) + \frac{\alpha}{I_1^*} \tag{8}$$

The first term on the right-hand side captures the consumption of domestic households, where the lending and deposit rates control the consumption of borrowers and savers, respectively. The second term on the right-hand side represents foreign demand which is not affected by the domestic policy rate because export prices are sticky in foreign currency.

Consider first the model implications if the bank leverage constraint does not bind, so that the lending rate is equal to the policy rate $I_1^L = I_1$. In this case, a policy rate cut not only increases savers' consumption by lowering deposit rates, but it also stimulates borrowers' consumption by reducing lending rates. Hence, monetary easing is expansionary, as it raises domestic demand and output.

The effect of a reduction in the policy rate on capital flows is less clear-cut. On the one hand, monetary easing boosts import consumption, thus leading to an increase in the demand for foreign funds holding the exchange rate constant. On the other hand, monetary accommodation reduces bond yields since $I_1^B = I_1$ and thus curbs the supply of foreign capital for a given level of the exchange rate. To restore equilibrium in the market for foreign funds, the exchange rate must necessarily depreciate. The effect on capital flows depends on the magnitude of the depreciation or, more specifically, on the elasticity of the exchange rate with respect to the policy rate. If the elasticity is larger than one, a reduction in the policy rate causes a proportionally larger depreciation of the exchange rate which increases the expected excess return on domestic bonds and attracts more inflows. If instead the elasticity is lower than one, a policy rate cut reduces the return of domestic bonds and therefore triggers capital outflows.

If the bank leverage constraint does not bind, the elasticity of the exchange rate with respect to the policy rate is given by

$$\varepsilon_I^e = 1 - \gamma_1 \frac{\mathbb{B}_1^F - B_1^F}{I_1 + \alpha\gamma_1} \frac{I_1^*}{e_1} \tag{9}$$

where $\mathbb{B}_1^F = B_0^F I_0$ are the government's foreign liabilities at the beginning of time 1. This expression shows that in our model, that assumes unitary elasticities of inter and intra-temporal substitution, the effect of monetary policy on capital flows depends on the sign of the current account which is equal to the net repayment of foreign debt, $\mathbb{B}_1^F - B_1^F$. If the country is running a current account deficit, the elasticity of the exchange rate is larger than one. In this case, monetary easing generates capital inflows, as it leads to a further deterioration of the current account. If the current account is instead in surplus, a reduction in the policy rate triggers capital outflows.

In turn, the current account crucially depends on global financial conditions captured by γ_1. Provided that the country enters period 1 with foreign debt, $\mathbb{B}_1^F > 0$, a higher γ_1 raises international borrowing costs and induces the country to deleverage by running a current account surplus. This lowers the elasticity of the exchange rate to the domestic policy rate, so that monetary easing generates capital outflows. The effects of γ_1 on the current account and thus on the elasticity are reversed if the country is a net debtor, $\mathbb{B}_1^F < 0$.[10]

The model can transparently illustrate the impact of monetary policy on capital flows since it allows for a closed-form solution of foreign bond holdings at the end of period 1. By equating demand and supply of foreign capital, we obtain:

$$B_1^F = \frac{\mathbb{B}_1^F}{1 + \gamma_1 \alpha / I_1} \tag{10}$$

Equation 10 shows that, in our setting, monetary easing increases capital outflows, i.e. it reduces B_1^F, as long as the country is a net debtor and γ_1 is strictly positive. As explained above, this is because in equilibrium a reduction in the domestic interest rate reduces the foreign-currency return of domestic bonds.

If banks are unconstrained, the capital outflows triggered by monetary easing do not impair monetary transmission. Domestic banks absorb the bonds sold by foreigners by increasing leverage without crowding out lending to the private sector. Banks finance the higher leverage by collecting more domestic deposits. This is possible since in equilibrium the deposit supply increases thanks to the expansionary effects of monetary policy on output and the increase in export revenues associated with the depreciation of the exchange rate. Therefore, when the leverage constraint does not bind, foreign financing can be freely substituted with domestic financing without impairing the transmission of monetary policy.

However, monetary easing can eventually push banks against their leverage constraint as they continue to increase their holdings of government bonds while foreigners pull out. The speed at which bank leverage increases in response to a reduction in the domestic policy rate depends not only on the size of capital outflows, but also on the effect of monetary easing on loan demand, which

[10] While in our setting the effect of monetary policy on capital flows depends on the sign of the current account, this result and the underlying intuition for the role played by (5) hold more generally. Consider the following generalization of the model. Let the equilibrium in the foreign funds market be described by the equation

$$B^F(Ie) = \mathbb{B}^F + \iota(I, e) - \xi(e)$$

where B^F is the supply function, which depends positively on the foreign-currency return of the domestic bond, proxied by Ie, $\mathbb{B}$ the beginning-of-period debt of the country, ι the value of imports, and ξ the value of exports, both measured in domestic currency. Then we can use the implicit function theorem to show that

$$\varepsilon_I^e = 1 - \frac{\left(\mathbb{B}^F - B^F\right)\varepsilon_e^{\xi} + \iota\left(\varepsilon_e^{\xi} - \varepsilon_I^{\iota} - \varepsilon_e^{\iota}\right)}{B^F \varepsilon_e^{B^F} + \xi \varepsilon_e^{\xi} - \iota \varepsilon_e^{\iota}}$$

where ε_z^x is the elasticity of x with respect to z (in absolute value, except for ε_e^{ι} whose sign can be positive or negative). Assume the elasticities of import and export are constant. Then the elasticity of the exchange rate with respect to the policy rate is decreasing in the current account. If $\mathbb{B}^F > 0$, a decrease in the desire of foreign investors to hold domestic bonds, that is an increase in γ in our model, reduces B^F and depreciates the exchange rate, reducing its elasticity.

is equal to

$$L_1 = \mathbb{L}_1 + \frac{\omega_2}{I_1^L} - \Pi_1^B \tag{11}$$

where $\mathbb{L}_1 = L_0 I_0^L$ is the outstanding stock of loans at the beginning of time 1. Monetary easing has ambiguous effects on loan demand. On the one hand, it stimulates borrowers' consumption, ω_2/I_1^L, by lowering lending rates. On the other hand, it raises borrowers' income, Π_1^B, by boosting output and export revenues. If the former effect is stronger, monetary easing raises loan demand which in turn accelerates the increase in bank leverage. If instead, borrowers' income increases faster than consumption, monetary easing reduces the equilibrium level of lending, slowing down the increase in leverage.[11]

To focus on the role of capital flows in affecting bank leverage and to allow for an analytical solution of the model, we assume that monetary policy has neutral effects on loan demand by setting $\Pi_1^B = \omega_2/I_1$. This ensures that borrowers have a constant discounted value of income over time so that they simply roll over their outstanding debt. Hence, their demand for credit does not respond to monetary policy. Under this assumption, monetary easing moves banks towards their leverage constraint by increasing their holdings of government bonds while lending to the private sector remains constant, $L_1 = \mathbb{L}_1$.

Using equations 7, 11, and 3, we can show that the leverage constraint is slack if and only foreigners purchase a sufficiently high level of domestic bonds

$$B_1^F \geq \underline{B}_1^F$$

where the variable $\underline{B}_1^F \equiv \mathbb{B}_1^G - (\phi N_1 - \mathbb{L}_1)/\lambda$ is the country's capital shortfall. This is the minimum amount of foreign capital which is needed to satisfy the domestic demand for credit by the private and public sectors, $\mathbb{L}_1 + \mathbb{B}_1^G$ at the prevailing policy rate. The bank leverage constraint limits the financial capacity of the country, that is its ability to collect deposits and transform them into credit. Thus, the country needs to attract foreign capital to absorb part of the public debt so that banks can supply enough credit to the private sector. Foreign capital is crucial because of the imperfect substitutability between domestic deposits and foreign funds. Deposits absorb financial capacity since they need to be intermediated by banks before they can be used to finance loans or government bonds. Foreign capital can instead directly fund government bonds without requiring domestic financial intermediation. We assume that the country's capital shortfall, $\underline{B}_1^F$, is bounded between

[11] If monetary easing leads to a decline in loan demand, it still determines an increase in bank leverage as long as borrowers' income at time 1 is not too large relative to income at time 2. This is consistent with the narrative that borrowers want to borrow to smooth consumption because their income is expected to growth over time. Formally, if we assume that borrowers receive a fraction ω_1 of aggregate income at time 1, the condition for monetary easing to push banks against their leverage constraint is $\omega_1 < \omega_2 - (\omega_2 - \lambda)\,\alpha\gamma_1 \mathbb{B}_0^F I_1^2 \left((I_1 + \alpha\gamma_1)^2 + \alpha\gamma_1 \mathbb{B}_0^F I_1^2\right)^{-1}$.

$(0,\mathbb{B}_1^F)$ which is required for the leverage constraint not to be always or never binding, irrespective of monetary policy.

By generating capital outflows and thus forcing a replacement of foreign funds with domestic funds, monetary easing moves banks towards their leverage constraint. The policy rate level at which the leverage constraint becomes binding is given by

$$I_1^{ELB} = \frac{\gamma_1 \alpha}{\mathbb{B}_1^F / \underline{B}_1^F - 1} \tag{12}$$

We refer to this interest rate threshold as the Expansionary Lower Bound. The higher the country's capital shortfall, the higher is the ELB since bonds have to pay a higher yield to attract sufficient capital inflows. The ELB is also increasing in the tightness of global financial conditions, captured by γ_1, as foreigners demand higher compensation to hold government bonds. In fact, if global financial conditions are tight enough, the ELB occurs at positive interest rates, $I_1^{ELB} > 1$, thus acting as a stronger constraint to monetary policy than the Zero Lower Bound. The ELB is instead declining in the foreign holdings of bonds at the beginning of time 1, $\mathbb{B}_1^F$. This is because a higher level of external debt depreciates the exchange rate which raises the foreign-currency return on domestic bonds and increases capital inflows.

If monetary easing continues below I_1^{ELB}, the economy experiences a credit crunch. Capital inflows are insufficient for banks to satisfy the domestic credit demand at the prevailing policy rate. Therefore, lending rates and bond yields have to increase above the policy rate, undermining the transmission of monetary policy. Their behavior can be characterized by considering the following equation which ensures that the level of foreign bond holdings, on the left-hand side, is consistent with the domestic leverage constraint, on the right-hand side:

$$\frac{1}{1+\gamma_1 \alpha / I_1^B} \left[\mathbb{B}_1^F + \alpha \frac{I_1^L - I_1}{I_1^B} \left(\frac{\lambda (1-\omega_2)}{I_1} - \frac{\omega_2 (1-\lambda)}{I_1^L} \right) \right] = \underline{B}_1^F + \frac{\omega_2}{\lambda} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right)$$

with $I_1^B = \lambda I_1^L + (1-\lambda) I_1$.

Using the implicit function theorem, the left derivative of the lending rate, evaluated at I_1^{ELB}, can be expresses as

$$\left. \frac{\partial I_1^L}{\partial I_1} \right|_{I_1 = \bar{I}_1}^{-} = 1 - \frac{1}{\lambda + \frac{\omega_2 \left[\mathbb{B}_1^F / \left(\alpha \lambda \underline{B}_1^F \right) - 1 \right] + \lambda}{\gamma_1 \underline{B}_1^F}} \tag{13}$$

This derivative is less than one, capturing the fact that, when banks are constrained, the lending rate rises above the policy rate. In fact, if global financial conditions are sufficiently tight, the derivative turns negative.[12] In this case, monetary easing leads to an increase in the lending rate, as illustrated in the left chart of Figure 1. Bond yields also increase above the policy rate because of the no-arbitrage condition between loans and bonds. However, monetary easing continues to reduce bond yields and trigger capital outflows even below the ELB.[13] In turn, capital outflows lead to the crowding out of domestic credit as lending rates increase. As previously discussed, this is because the leverage constraint creates a segmentation in financial markets that prevents the domestic economy from substituting foreign financing with domestic savings. The imperfect substitutability between domestic and foreign funds is the fundamental force that undermines monetary transmission and generates the ELB.

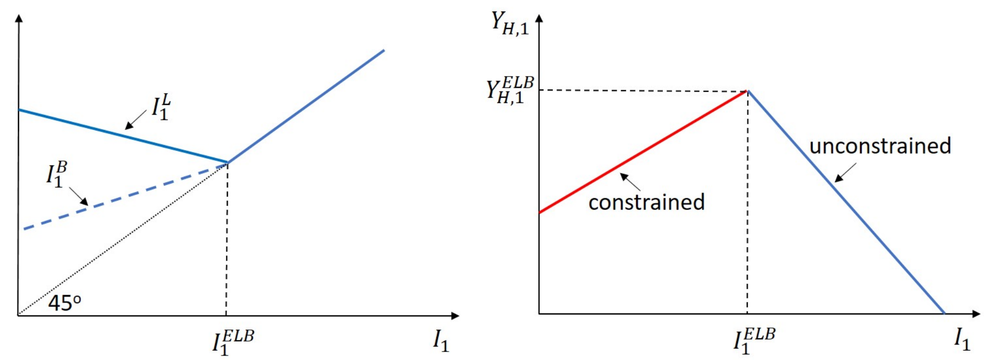

Figure 1: Monetary policy and the ELB.

By leading to an increase in lending rates once banks are constrained, monetary easing reduces borrowers' consumption. Nonetheless, monetary easing continues to stimulate savers' consumption since deposit rates decline in line with the policy rate. The ELB exists when the former effect prevails, so that monetary easing generates a contraction in aggregate demand and output. Formally, by differentiating equation (8), we can show that, once banks are constrained, monetary easing becomes contractionary if

$$-\omega_2 \left.\frac{\partial I_1^L}{\partial I_1}\right|^{-}_{I_1=I_1^{ELB}} > 1-\omega_2 \tag{14}$$

Intuitively, this condition requires that the increase in lending rates in response to monetary easing, which is controlled by global financial conditions γ_1, should be sufficiently strong relative to the share of aggregate demand arising from savers, $1-\omega_2$.

[12]This happens when $\gamma_1 > \frac{\omega_2\left[\mathbb{B}_1^F/\left(\alpha\lambda B_1^F\right)-1\right]+\lambda}{\underline{B}_1^F(1-\lambda)}$.

[13]Despite the emergence of a bond spread, the foreign-currency excess return on government bonds continues to decline because the exchange rate does not depreciate enough. This is because the contraction in import demand due to the increase in lending rates raises the current account and further reduces the elasticity of the exchange rate to policy rates.

If condition 14 is satisfied, the relationship between the domestic policy rate and output is non-monotonic, as shown in the right chart of Figure 1. The central bank is thus unable to raise output above the level associated with the ELB, which is given by

$$Y_{H,1}^{ELB} = \frac{1-\alpha}{I_1^{ELB}} + \frac{\alpha}{I_1^*} \tag{15}$$

as both a reduction and an increase in the policy rate around I_1^{ELB} leads to a contraction in aggregate demand. The ELB limits the ability of the central bank to stimulate aggregate demand and constrains the conduct of monetary policy. If the level of output targeted by the central bank is below $Y_{H,1}^{ELB}$, the ELB is not binding.[14] However, if the desired level of output is above $Y_{H,1}^{ELB}$, the optimal policy is to set the interest rate at I_1^{ELB} to stimulate output as much as possible. The central bank would never want to lower the policy rate below I_1^{ELB} since this would reduce output.

The existence of the ELB generates crucial departures from Mundell's trilemma since it can prevent monetary authorities from stabilizing output in response to global financial and monetary shocks. This is illustrated in Figure 2. The left chart considers the implications of changes in global financial conditions. In line with Mundell's trilemma, if banks are unconstrained, an increase in γ_1 does not affect output since the shock is entirely absorbed through a depreciation of the exchange rate. However, by triggering capital outflows an increase in γ_1 raises the ELB and lowers the maximum attainable level of output, as shown respectively in equations (12) and (15). If $Y_{H,1}^{ELB}$ falls below the desired level of output, the ELB becomes a binding constraint, forcing the central bank to increase rates and accept a decline in output. This result is consistent with the empirical evidence provided in Table 1, whereby emerging markets tend to hike rates when the VIX increases.

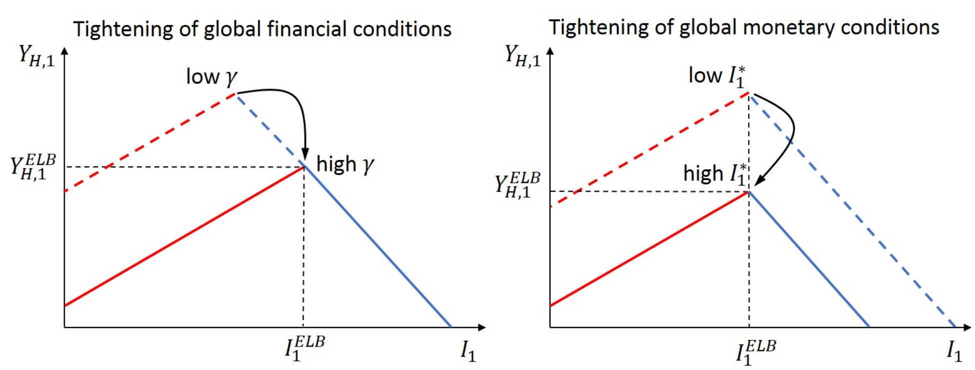

Figure 2: Global financial and monetary shocks in the presence of carry traders.

The ELB raises concerns also in reference to global monetary shocks, as illustrated in the right

[14] We do not characterize the optimal level of output since it would depend on the labor supply that we leave unrestricted. The model can be extended to incorporate different labor market structures without altering the results pertaining the ELB.

chart of Figure 2. Since export prices are sticky in foreign currency, an increase in the foreign policy rate I_1^*reduces foreign demand for home goods. This does not affect the level of the ELB which, as shown in equation (12), is not a function of I_1^*. However, it does reduce the maximum output level that can be achieved through monetary policy, thus potentially pushing the domestic economy into recession. In the next model where the ELB emerges because of currency mismatches, a foreign monetary tightening will also increase the ELB, thus accounting for the positive correlation between the policy rates in the US and emerging markets documented in Table 1.

2.2.2 Model equilibrium at time 0

In this section, we characterize the model equilibrium at time 0 to analyze the implications of the ELB for the ex-ante conduct of monetary policy. More specifically, we want to explore if and how the possibility that the ELB may arise in the future affects monetary policy decisions in earlier periods. We assume that at time 0 domestic and international financial conditions are favorable so that monetary policy operates in a conventional manner. Formally, we assume that the bank leverage constraint does not bind and we take the limit for $\gamma_0 \downarrow 0$. In this case, UIP holds and foreign investors are willing to supply any amount of capital that the country requires.

We assume that the only stochastic element at time 1 is the realization of the parameterγ_1 which follows a binary distribution. With probability ρ, γ_1 is sufficiently high to make the ELB a binding constraint for the central bank, thus forcing the monetary authority to set $I_1 = I_1^{ELB}$. With probability $1-\rho$ the realization of γ_1 is low, zero for simplicity, in which case the central bank is unconstrained and sets the time-1 policy rate at a certain optimal level I_1^{opt}. We also assume that borrowers have no income at time 0, $\Pi_0^B = 0$, so that they necessarily accumulate debt in this period.

How does monetary policy at time 0 affect monetary space at time 1, i.e. the level of the ELB? To answer this question, we need to understand how the policy rate I_0 affects the ratio $\mathbb{B}_1^F/\underline{B}_1^F$, which determines the ELB according to equation (12). By diving both terms by I_0, the ratio can be written as

$$\frac{\mathbb{B}_1^F}{\underline{B}_1^F} = \frac{B_0^F}{B_0^G - (\phi N_0 - L_0)/\lambda}$$

where we used $\mathbb{B}_1^G = B_0^G I_0$ and $N_1 = N_0 I_0$ since the bank leverage constraint does not bind at time 0. Therefore, time-0 monetary policy affects the ELB through the impact on capital inflows, B_0^F, and domestic lending, L_0, which are given by

$$B_0^F = \mathbb{B}_0^F + \frac{\alpha}{I_0}\left(\frac{1}{\mathbb{E}_0[I_1]} - \frac{1}{\mathbb{E}_0[1/e_1]}\right)$$

$$L_0 = \mathbb{L}_0 + \frac{\omega_2}{I_0 \mathbb{E}_0[I_1]}$$

To understand the impact of time-0 monetary policy on B_0^F and L_0, start by holding constant the expected policy rate and exchange rate at time 1. It is then easy to see that a tightening in monetary policy at time 0 reduces both capital inflows and domestic lending.[15] The reduction in capital inflows tends to raise the ELB since it implies a lower stock of external debt at time 1 and thus a more appreciated exchange rate which reduces foreign demand for government bonds. On the contrary, the reduction in domestic lending tends to lower the ELB since it allows banks to absorb more government bonds at time 1 given their leverage constraint. The overall impact on the ELB thus depends on the balance between these two effects.

Taking now into account also the effects that time-0 monetary policy has on the expected policy rate and exchange rate at time 1, the overall impact of I_0 on the ELB is proportional to the following expression

$$\frac{\partial I_1^{ELB}}{\partial I_0} \propto \left[\mathbb{B}_0^F + \frac{\rho \alpha \bar{\gamma}_1 \underline{B}_1^F I_1^{ELB}}{\mathbb{E}_0\left[\left(1+\bar{\gamma}_1 \underline{B}_1^F\right)/e_1\right]^2}\right] \frac{\partial L_0}{\partial I_0} - \lambda \underline{B}_1^F \frac{\partial B_0^F}{\partial I_0} < 0$$

As shown in Appendix A, the above expression implies that a monetary tightening at time 0 lowers the ELB at time 1.[16] The policy rate at time 0 is thus negatively correlated with the level of the ELB at time 1> This gives rise to a novel inter-temporal trade-off for monetary policy since when choosing policy rates at a given time, monetary authorities should be mindful about the implications for the ELB in the future. More specifically, the negative association between the ELB and ex-ante policy rates calls for keeping a somewhat tighter monetary stance when financial conditions are supportive – thus running the economy below potential – to lower the ELB and generate more monetary space in the future.

The negative association between I_0 and the ELB has also important implications for the effectiveness of monetary policy in periods when the ELB does not bind. In particular, monetary policy at time 0 becomes less powerful in stimulating output which is equal to

$$Y_{H,0} = \frac{1-\alpha}{I_0 \mathbb{E}_0\left[I_1\right]} + \frac{\alpha}{I_0^* I_1^*}$$

This is because a reduction in I_0, by raising the ELB, generates an expected tighter monetary stance in the future, $\mathbb{E}_0\left[I_1\right]$, which weakens the impact on output. Therefore, the ELB not only constraints monetary policy when it binds, but it also hinders monetary transmission when global financial conditions are supportive and banks are unconstrained.

[15] Monetary tightening reduces capital flows since the term in parenthesis is positive. To see this, note that $\mathbb{E}_0\left[I_1\right] = \rho I_1^{ELB} + (1-\rho) I_1^{opt}$,while $\mathbb{E}_0\left[1/e_1\right] = \rho I_1^{ELB} / \left(1 - \gamma_1 I_0 \frac{\phi N_0 - L_0 + \lambda B_0^G}{\lambda}\right) + (1-\rho) I_1^{opt}$. Thus $\mathbb{E}_0\left[1/e_1\right] > \mathbb{E}_0\left[I_1\right]$.

[16] Note that if we were to assume that $\gamma_0 > 0$, monetary tightening would lower the ELB even further. This is because an increase in I_0 would increase, rather than reduce, capital inflows because of carry-trade effects.

2.3 Policies to escape the ELB

In this section we expand the model to include a broad range of policy tools that may help overcome the ELB. We consider both fiscal policy and capital controls that imply the following budget constraint for the government

$$B_t^G = \mathbb{B}_t^G - T_t - \chi_t B_t^F$$

where T_t are lump-sum taxes on domestic households and χ_t is a tax on foreign capital inflows. Furthermore, we analyze the effects of changes in the balance sheet of the central bank which is given by

$$N_t^{CB} + R_t = B_t^{CB} + e_t X_t$$

where N_t^{CB} is networth, R_t are domestic reserves, B_t^{CB} are holdings of government bonds, and X_t are foreign reserves. Finally, we consider the impact of a recapitalization of the banking sector and of forward guidance, captured in the model through changes in the steady-state level of money supply M_2. We analyze primarily the effects of these tools at time 1 when the ELB binds, but also consider how some of them can be used preemptively at time 0. We discuss the results in intuitive terms referring the reader to Appendix A for formal derivations.

Regarding fiscal policy, since the ELB arises because public debt crowds out private lending, it may seem obvious that an increase in government taxes to reduce debt should alleviate the ELB. However, this is not necessarily the case since a tax-based fiscal consolidation has two effects. On the one hand, the reduction in public debt relaxes the bank leverage constraint in proportion to the capital requirement λ. On the other, a tax increase raises loan demand because of a Ricardian equivalence effect: despite higher taxes at time 1, borrowers want to maintain the same level of consumption by borrowing more. The aggregate demand for loans thus increases by the tax burden imposed on borrowers, T_t^B/T_t, for each unit of additional tax revenues. If $T_t^B/T_t > \lambda$, a tax-based fiscal consolidation ends up tightening collateral constraints, raising the ELB, and lowering output.

Fiscal consolidation can also be undertaken by taxing foreigners with a levy on capital inflows, χ_t . However, this reduces foreign holdings of government bonds, forcing banks to further curtail private lending to finance public debt. To lower the ELB, it is instead optimal to subsidize capital inflows, setting $\chi_1 < 0$. This entails an increase in public debt, but the effect is overall positive, lowering the ELB.

The model has also rich implications for the role of balance-sheet operations by the central bank. The need to relax bank leverage constraints provides a rationale for quantitative easing which involves the purchase of government bonds by the central bank B_1^{CB} against the increase in central bank reserves R_1. By doing so, the central bank acts as a financial intermediary for government bonds, thus releasing liquidity to the banking sector that can be used to extend credit to the private sector. Quantitative easing is thus an effective tool to lower the ELB and stimulate output. Note that this is the case even if part of the gains from quantitative easing are eroded by the actions of carry traders, since by lowering yields on government bonds, quantitative easing exacerbates capital outflows.

The central bank can also alleviate the ELB by engaging in unsterilized foreign exchange intervention. By purchasing foreign reserves X_1 against domestic reserves R_1, the central bank can depreciate the exchange rate, increase the expected return of government bonds for foreigners, and thus stimulate capital inflows. Finally, the central bank can also intervene through sterilized foreign exchange intervention, by selling foreign reserves and buying government bonds in line with Cavallino (2016). This operation can be seen as combining unsterilized intervention (selling FX reserves to reduce domestic reserves) with quantitative easing (increasing domestic reserves to buy bonds). In equilibrium, the latter effect prevails, so that sterilized intervention relaxes the ELB if the central bank reduces foreign reserves, despite the appreciation of the exchange rate.

Turning to forward guidance, this tool is quite effective in providing stimulus when the economy is at the ZLB, as for example discussed by (Krugman, Dominquez and Rogoff, 1998; Svensson, 2003; Eggertsson and Woodford, 2003). A pledge by the central bank to provide stronger monetary stimulus in the future can indeed increase current domestic spending. Does this logic apply also to the ELB? The answer is no because, unlike the ZLB, the ELB is an endogenous interest rate threshold that moves itself with forward guidance. An increase in M_2 does increase time-1 spending for a given policy rate I_1, but it also increases the ELB. This is because higher M_2 leads to a stronger depreciation of the exchange rate at time 2 than at time 1, thus reducing the foreign-currency return on domestic bonds and generating capital outflows. In the model, the overall effect of forward guidance is to increase the ELB, while leaving the level of output at the ELB unchanged.

A policy tool that is instead very effective in overcoming the ELB is the recapitalization of the banking sector, as also analyzed in Kollmann, Roeger and in't Veld (2012) and Sandri and Valencia (2013).[17] This is true even if the recapitalization is financed with lump-sum taxes on borrowers which can be captured in the model through an increase in loan repayments at the beginning of period 1, $\mathbb{L}_1$. This is because while lump-sum taxes increase one-to-one loan demand by borrowers, they increase lending supply by a greater factor thanks to bank leverage, i.e. $\phi > 1$. A bank recapitalization can thus lower the ELB and allow for greater monetary space.

For what concerns preemptive intervention at time 0, fiscal consolidation has similar effects than at time 1. In particular, fiscal consolidation can lower a future ELB only if the tax burden imposed on borrowers is smaller than the capital charge on government bonds, i.e. $T_0^B/T_0 < \lambda$. Taxes on capital inflows have instead ambiguous effects. On the one hand, they reduce public debt, thus lowering the ELB. On the other hand, they reduce capital inflows, thus raising the ELB. The overall

[17] We could also consider credit easing policies, whereby the government provides lending subsidies or operates as a financial intermediary as in Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012) and Negro et al. (2011). These measures would also help to relax lending constraints and stimulate aggregate demand.

effect depends on the parameters of the model and in particular on the probability that the ELB may bind in the future. Finally, if $\gamma_0 > 0$ as in Appendix A, foreign exchange intervention can also be helpful at time 0 since it can lower the ELB by depreciating the exchange rate and attracting more inflows.

3 The ELB and currency mismatches

In this section we present a second model to show that the ELB can also emerge because of the exposure of the domestic financial sector to currency mismatches.[18] By depreciating the exchange rate, monetary easing reduces bank networth and tightens leverage constraints, possibly leading to a domestic credit crunch and output contraction. As in the previous model, this gives rise to an ELB that places an upper bound on the level of output achievable through monetary accommodation. Others papers, in particular Céspedes, Chang and Velasco (2004), Christiano, Gust and Roldos (2004), and more recently Gourinchas (2018), already developed models in which collateral constraints associated with currency mismatches can generate contractionary effects from monetary easing. However, in those models monetary policy can still achieve any level of output since it does not affect whether constraints are binding or not. When constraints bind, monetary policy can indeed increase output without bounds by simply raising policy rates as much as needed. This is no longer possible in our framework, since monetary policy affects whether constraints bind or not. In particular, raising rates eventually makes leverage constraints no longer binding, at which point further interest rate hikes become contractionary.

3.1 Model setup

We consider again a small open economy in which households consume domestic and foreign goods. All households are now borrowers and raise domestic currency loans from local banks as described in the previous model. The corporate sector also mirrors the previous model, except that we now assume that foreign prices are sticky in local currency in period 0 and 1, $P^*_{H,t} = \bar{P}_H / e_t$ for $t = \{0, 1\}$. This leads to an additional expenditure-switching channel through which monetary policy stimulates demand for home goods by depreciating the exchange rate.

Unlike the previous model, we dispense from carry-trade capital flows by ruling out frictions in international financial markets so that UIP holds. Furthermore, we assume that banks finance

[18] We could also assume that unhedged exposures are actually borne by domestic non-financial firms. Emerging markets firms have indeed increased considerably the issuance of dollar bonds since the global financial crisis, as for example documented in Acharya et al. (2015) and McCauley, McGuire and Sushko (2015). We prefer our interpretation based on financial intermediaries for two reasons. First, even if currency mismatches are concentrated in the non-financial corporate sector, an exchange rate depreciation tends to ultimately generate losses in the financial sector, as firms default on their loans. Second, there is compelling empirical evidence (Caballero, Panizza and Powell, 2015; Bruno and Shin, 2015) that non-financial firms in emerging markets have behaved recently like financial intermediaries, by issuing dollar debt at low rates while holding large positions in domestically denominated financial assets.

themselves internationally by issuing foreign-currency debt. The balance sheet of the banking sector is thus given by

$$N_t + e_t D_t^* = L_t + R_t$$

where D_t^* is foreign-currency debt and the other variables are defined as in the previous model. Bank networth evolves according to

$$N_{t+1} = L_t I_t^L + R_t I_t - e_{t+1} D_t^* I_t^*$$

As before, banks are subject to a leverage constraint that limits lending to a certain multiple of networth:

$$L_t \leq \phi N_t \tag{16}$$

with $\phi \geq 1$. We abstract from the role of government debt by assuming that $\lambda = 0$.

Banks take interest rates as given and choose assets and liabilities to maximize networth. No arbitrage between central bank reserves and foreign currency debt implies the UIP condition, $\mathbb{E}_t\left[\left(e_t I_t - e_{t+1} I_t^*\right)\left(I_{t+1} + \phi \mu_{t+1}\right)\right] = 0$, where μ_{t+1} is the shadow cost of the leverage constraint. Furthermore, the first order condition with respect to domestic lending implies $I_{t+1}^L \geq I_t$. If the leverage constraint is not binding, the domestic lending rate is equal to the policy rate. If instead the constraint binds, the lending rate has to increase above the policy rate to equalize the demand for loans with the constrained supply level. The central bank conducts monetary policy by setting the interest rate on reserves. As in the previous model, we first abstract from the central bank's balance sheet by considering the limit of the model for $R_t \downarrow 0$.

3.2 Model equilibrium

The solution approach follows the one in the previous model. We assume that the economy is in steady-state from time 2 onward and that nominal spending is equal to money supply, in which case the exchange rate is pinned down by domestic and foreign money supply, $e_2 = M_2/M_2^*$. We first characterize the conditions for the existence of the ELB at time 1 and then analyze the implications for monetary policy at time 0.

3.2.1 Model equilibrium at time 1

The level of output at time 1 is equal to

$$Y_{H,1} = \frac{1-\alpha}{I_1^L} + e_1 \frac{\alpha}{I_1^*} \tag{17}$$

The first and second terms on the right-hand side capture nominal spending on home goods by domestic and foreign households, respectively. If banks are unconstrained, so that the lending rate is equal to the policy rate, $I_1^L = I_1$, monetary easing stimulates output through two channels. First, it boosts spending by domestic households by reducing lending rates. Second, it raises foreign demand through the depreciation of the exchange rate which is equal to $e_1 = I_1^*/I_1$.

Because of currency mismatches, the exchange rate depreciation caused by monetary easing leads to an erosion of bank networth which is given by

$$N_1 = \mathbb{L}_1 - e_1 \mathbb{D}_1^*$$

where $\mathbb{L}_1 \equiv L_0 I_0$ and $\mathbb{D}_1^* \equiv D_0 I_0^*$ are the loans and foreign-currency liabilities of the banking sector at the beginning of time 1. The networth loss leads to a tightening of the collateral constraint (16) that becomes binding for a sufficiently low domestic policy rate. Once the leverage constraint binds, banks lose the ability to freely intermediate foreign capital into domestic lending. Indeed, further monetary easing forces banks to reduce domestic lending as the economy experiences capital outflows. To preserve equilibrium in the credit market, the lending rate has to increase above the policy rate to satisfy

$$I_1^L = \frac{\alpha}{(\phi - 1)\mathbb{L}_1 - e_1\left(\phi \mathbb{D}_1^* - \frac{\alpha}{I_1^*}\right)} \tag{18}$$

The expression above shows that, when banks are constrained, monetary easing by depreciating the exchange rate may lead to an increase, rather than a decline, of the lending rate. On the one hand, the depreciation reduces credit supply through its impact on bank networth. On the other hand, it reduces credit demand by raising export revenues. If the former effect prevails, which occurs when foreign-currency debt is high enough to satisfy $\phi \mathbb{D}_1^* > \alpha / I_1^*$, the lending rate has to increase with monetary easing to preserve market clearing.

The increase in the lending rate, in turn, leads to a contraction in domestic spending. This negative effect on domestic demand has to be compared with the positive effect that monetary easing retains on foreign demand through the depreciation of the exchange rate. By plugging equation 18 into equation 17 we can show that the contractionary effect on domestic spending outweighs the expansionary effect on foreign demand if foreign-currency debt is sufficiently high to satisfy

$$\phi \mathbb{D}_1^* > \frac{\alpha}{(1-\alpha) I_1^*} \tag{19}$$

When this condition is satisfied, once the leverage constraint binds, monetary easing becomes contractionary giving rise to the following ELB

$$I_1^{ELB} = \frac{\phi \mathbb{D}_1^* I_1^*}{(\phi - 1)\mathbb{L}_1} \tag{20}$$

which prevents the central bank from increasing output above $Y_{H,1}^{ELB} = 1/I_1^{ELB}$.

The level of ELB depends on the extent of currency mismatches on banks' balance sheets, captured by the proportion of foreign-currency debt relative to domestic-currency loans. If mismatches are severe enough, the ELB can occur at positive interest rates, thus acting as a stronger constraint for monetary policy than the ZLB. Unlike the previous model, the level of ELB is now affected by global monetary conditions. An increase in the foreign policy rate depreciates the domestic currency, rises the ELB, and reduces the maximum level of output that monetary policy can achieve. This is illustrated in Figure 3. If collateral constraints are not binding, changes in foreign monetary policy do not affect domestic output since they are offset by exchange rate movements.[19] This is an implication of Mundell's trilemma whereby exchange rate flexibility insulates the country from foreign monetary conditions. Note that this is true even in the presence of currency mismatches, but only as long as constraints do not bind. However, by depreciating the domestic currency, an increase in foreign policy rates leads to an erosion in bank networth that tightens collateral constraints and raises the ELB. Therefore, for a large enough increase of the foreign policy rate, the ELB becomes binding and forces the domestic central bank to raise policy rates in line with the empirical evidence presented in Table 1.

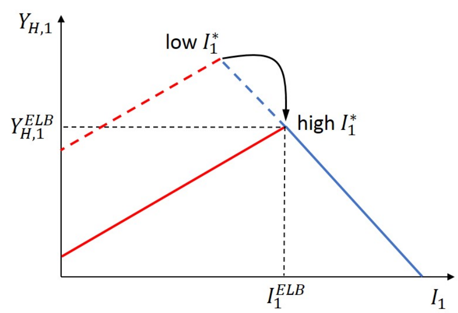

Figure 3: Foreign monetary shocks under currency mismatches.

3.2.2 Model equilibrium at time 0

We now analyze the model equilibrium at time 0 under the assumption that the bank leverage constraint does not bind. This allows us to analyze how the possibility that the ELB may bind in the future affects monetary policy when financial conditions are favorable. In doing so, we assume that the only stochastic element at time 1 is the foreign monetary policy rate I_1^*whose probability distribution can be left unspecified for the purpose of our analysis.[20] If the realization of I_1^* is sufficiently

[19] As shown in Appendix B, changes in foreign monetary policy can have effects on the domestic economy even when constraints are not binding if we allow for wealth effects by assuming $\beta < 1$. However, as long as constraints do not bind, the effects on domestic output can be offset with appropriate changes of the domestic policy rate.

[20] The narrative below only assumes that the ELB binds at time 1 with positive probability. This requires that in some instances the foreign monetary policy rates I_1^* is sufficiently high so that the ELB exists, thus satisfying condition (19),

low that the ELB does not bind, the domestic monetary authority can maintain output at a certain optimal level, $Y_{H,1}^{opt}$, by setting the policy rate at $I_1^{opt} = 1/Y_{H,1}^{opt}$. If instead $I_1^*$is high enough that the ELB increases above I_1^{opt}, the central bank finds it optimal to set the policy rate at the ELB, I_1^{ELB}. As shown below, to account for important transmission channels of monetary policy linked to currency mismatches, we allow the domestic inter-temporal discount factor at time 0, β_0, to possibly differ from one.

Consider first how monetary policy at time 0 affects the ELB at time 1. To do so, we need to understand the impact on the balance sheets of the banking sector at time 1. The equilibrium levels of foreign-currency debt and domestic-currency loans at the beginning of time 1 are given by

$$\mathbb{D}_1^* = \mathbb{D}_0^* I_0^* + \frac{\delta \alpha}{\mathbb{E}_0 [I_1^*]} \tag{21}$$

$$\mathbb{L}_1 = \mathbb{L}_0 I_0 + \frac{\delta \alpha}{\mathbb{E}_0 [I_1]} \tag{22}$$

where the parameter $\delta \equiv 1/\beta_0 - 1$ captures the impatience of domestic households relative to foreign agents. To understand the impact of time-0 monetary policy, start by holding constant the expected domestic interest rate at time 1, $\mathbb{E}_0 [I_1]$. In this case, a domestic monetary tightening at time 0 increases the loan repayments for banks at time 1, $\mathbb{L}_1$, since it leads to higher lending rates. At the same it does not generate an increase in foreign-currency debt, $\mathbb{D}_1^*$, which is insensitive to movements in the domestic policy rate.[21] Therefore, monetary tightening at time 0 leads to a reduction in the time-1 ELB as defined in equation (20). If $\delta > 0$, the impact on the ELB is magnified once we take into account the effects that monetary policy at time 0 has on $\mathbb{E}_0 [I_1]$. The first-round reduction in the ELB leads indeed to a decline in the expected level of the time 1 interest rate, $\mathbb{E}_0 [I_1]$ which in turn increases loan demand at time 0 as shown in equation (22). This raises loan repayments at the beginning of time 1 and further lowers the ELB.

As in the model with carry-trade capital flows, the negative correlation between the time-0 policy rate and the time-1 ELB generates an inter-temporal trade-off for monetary policy. Greater easing at time 0 reduces the space for monetary stimulus in the future by raising the ELB. This calls for keeping a tight domestic monetary stance when global monetary conditions are favorable to have more monetary space to absorb a future foreign monetary tightening. Furthermore, the negative correlation between ex-ante monetary policy and the ELB tends to weaken the transmission of monetary policy even when the ELB does not bind. To see this, note that time-0 output is given by

and is binding, i.e. the ELB is above the interest rate level I_1^{opt} which would bring output at a given optimal level.

[21] Changes in loan demand are accommodated through changes in the domestic-currency value of foreign-currency debt. For example, a monetary tightening leads to an appreciate of the domestic currency which reduces the domestic-currency value of foreign -currency debt in line with the contraction in domestic credit demand.

$$Y_{H,0} = \frac{1-\alpha}{\beta_0 I_0 \mathbb{E}_0 [I_1]} + e_0 \frac{\alpha}{I_0^* \mathbb{E}_0 [I_1^*]}$$

where the exchange rate is $e_0 = I_0^* \mathbb{E}_0 [I_1^*] / (I_0 \mathbb{E}_0 [I_1])$. The stimulative effect of a policy rate cut at time 0 is partially offset by an expected tightening of future monetary policy $\mathbb{E}_0 [I_1]$ due to the increase in the ELB. This weakens the impact on domestic demand since current consumption depends not only on the current interest rate, but also on the expected future monetary stance. Furthermore, the increase in $\mathbb{E}_0 [I_1]$ reduces foreign demand since it limits the depreciation of the time-0 exchange rate e_0.

The current model where the ELB is due to currency mismatches rather than carry-trade flows, provides also interesting insights about the ongoing debate on whether central banks in major advanced economies, notably the Fed, should internalize the effects of their monetary policy decisions on EMs. As shown in Figure (3), a tightening in the foreign policy rate increases the ELB and can push EMs into a recession. This seems to suggest that if foreign central banks care about global welfare, they should refrain from increasing rates sharply when the ELB binds in EMs. For example, if currency mismatches are associated with US dollars, the Fed should accept some overheating in the US to limit the adverse effects that a sharp monetary tightening would impose on EMs.

Note, however, that if the Fed is expected to follow this course of action, EMs have a perverse incentive to accumulate more foreign-currency debt. As shown in equation (21), foreign liabilities are indeed inversely proportional to the expected tightness of foreign monetary policy $\mathbb{E}_0 [I_1^*]$ if EMs are relatively impatient, so that $\delta > 0$. The model can therefore rationalize the growing concerns that EMs may be unable to insulate themselves from US monetary conditions, even if they have flexible exchange rates. However, it also shows that any commitment by the Fed to refrain from sharp policy rate increases to help EMs would be partially ineffective because it would led to an endogenous increase in foreign-currency borrowing.

There are two ways to limit this accumulation of additional foreign-currency debt. First, the expectation of a looser US monetary stance if the ELB binds in EMs could be offset with the promise of a tighter US monetary stance if the ELB does not bind. Doing so would prevent a reduction in the expected tightness of future US monetary policy $\mathbb{E}_0 [I_1^*]$ and thus avoid incentives for additional borrowing. Note that this policy implies a commitment by the Fed to keep a more stable US dollar, hiking policy rates by less when the US economy overheats and cutting them more moderately when the economy contracts. Second, policy makers in EMs can avoid additional foreign-currency debt by adopting macro-prudential regulations. The role of these tools and other policies is analyzed in the following section.

3.3 Policies to escape the ELB

In this section, we consider several policy tools that can be used to escape the ELB. As in the model with carry-trade capital flows, forward guidance is unable to deal with the ELB even it arises from currency mismatches. This is because the promise of a looser future monetary stance leads to an immediate depreciation of the exchange rate that tightens the bank leverage constraint. Formally, an increase in M_2 through forward guidance raises the ELB, while leaving the upper bound on the level of output attainable through monetary policy, $Y_{H,1}^{ELB}$, unchanged. Balance-sheet operations by the central bank are also ineffective in this model since the exchange rate is pinned down by the UIP condition and not by quantity conditions.

The relax the ELB in the presence of currency mismatches, policy markers can rely on the recapitalization of the banking sector which relaxes collateral constraints. Capital controls can also be effective since they can sever the link between the exchange rate and domestic monetary conditions. In particular, the government can stimulate capital inflows and support the domestic exchange rate by providing banks with a subsidy χ_1 on foreign currency debt. This places a wedge in the UIP condition, $e_1 = e_2(1-\chi_1)I_1^*/I_1$, that leads to an appreciate of the exchange rate, relaxes the ELB, and allows for greater monetary stimulus. The model provides also a rationale for macro-prudential capital controls that can be put in place in anticipation of the ELB becoming binding. As shown in Appendix B, by taxing capital inflows at time 0, policy makers can effectively reduce the amount of foreign currency debt carried into period 1. This lowers the time-1 ELB, I_1^{ELB}, and allows for a higher level of output.

4 Conclusion

In this paper, we provided a theory of the limits to monetary policy independence in open economies arising from the interaction between international capital flows and domestic collateral constraints. The key insight is that monetary policy can be constrained in its ability to stimulate output because of the existence of an Expansionary Lower Bound (ELB) which is an interest rate below which monetary easing becomes contractionary. The ELB places an upper bound on the level of output achievable through monetary policy and can thus prevent monetary authorities from responding effectively to global shocks. A tightening in global liquidity or monetary conditions may indeed raise the ELB and push emerging markets into a recession while central banks are forced to raise policy rates in line with the empirical evidence. Crucially, this is the case even in countries with flexible exchange rates, thus leading to crucial departures from Mundell's trilemma.

We showed that the conditions for existence of the ELB can be met under various circumstances, whenever monetary easing leads to a negative interaction between capital flows and collateral constraints. The ELB can for example arise because of carry-trade capital flows. In this case, monetary easing determines an outflow of capital which requires the domestic banking sector to absorb the bonds liquidated by foreign investors. This pushes banks against their leverage constraints and eventually forces them to reduce domestic credit. If the elasticity of capital flows to the domestic interest rate is sufficiently high, the credit crunch is so severe that monetary easing becomes contractionary, giving rise to the ELB.

Monetary policy can face an ELB also in the presence of currency mismatches. If the banking sector borrows abroad in foreign currency and lends domestically in local currency, monetary easing reduces bank networth and moves banks closer to their leverage constraint. Once the constraint binds, further monetary easing forces banks to reduce domestic credit and becomes contractionary if currency mismatches are sufficiently severe.

The models highlight a novel inter-temporal trade-off for monetary policy since the level of the ELB is affected by the past monetary stance. Tighter ex-ante monetary conditions tend to lower the ELB and thus create more monetary space to offset possible shocks. This observation has important normative implications since it calls for keeping a somewhat tighter monetary stance when global conditions are supportive to lower the ELB in the future.

Finally, the models have rich implications for the use of alternative policy tools that can be deployed to overcome the ELB and restore monetary transmission. In particular, the presence of the ELB calls for an active use of the central bank's balance sheet, for example through quantitative easing and foreign exchange intervention. Furthermore, the ELB provides a new rationale for capital controls and macro-prudential policies, as they can be successfully used to relax the tensions between domestic collateral constraints and capital flows. Fiscal policy can also help to overcome the ELB, while forward guidance is ineffective since the ELB increases with the expectation of looser future monetary conditions.

Looking ahead, the paper calls for more research along two fronts. First, it would be helpful to provide additional empirical evidence about the existence of the ELB. In principle, this would require showing that monetary easing is contractionary when domestic or international financial conditions are tight, for example as many argued during the Asian financial crisis. However, similar episodes are likely to be quite limited since, as described in the model, central banks do not have reasons to lower rates below the ELB when monetary easing becomes contractionary. It may thus be preferable to focus on how the ELB distorts the response of monetary policy to shocks in line with the suggestive evidence presented in the introduction. Second, future research can analyze the implications of the ELB using quantitative DSGE models. This would shed light on the exact circumstances under which the ELB may arise, on its level, and the extent to which monetary authorities should keep the economy below potential when the ELB is expected to bind in the near feature.

References

Acharya, Viral, Stephen Cecchetti, José De Gregorio, Sebnem Kalemli-Ozcan, Philip R. Lane, and Ugo Panizza. 2015. "Corporate Debt in Emerging Economies: A Threat to Financial Stability?" *Committee on International Economic Policy and Reform*.

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee. 2000. "A Simple Model of Monetary Policy and Currency Crises." *European Economic Review*, 44(4-6): 728 – 738.

Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee. 2001. "Currency Crises and Monetary Policy in an Economy with Credit Constraints." *European Economic Review*, 45(7): 1121–1150.

Aoki, Kosuke, Gianluca Benigno, and Nobuhiro Kiyotaki. 2016. "Monetary and financial policies in emerging markets." *Unpublished paper, London School of Economics*.

Arregui, Nicolas, Selim Elekdag, Gaston Gelos, Romain Lafarguette, and Dulani Seneviratne. 2018. "Can Countries Manage Their Financial Conditions Amid Globalization?" *IMF Working Paper No. 18/15*.

Auclert, Adrien. 2016. "Monetary Policy and the Redistribution Channel." *Manuscript*.

Avdjiev, Stefan, and Galina Hale. 2017. "U.S. monetary policy and fluctuations of international bank lending." *Manuscript*.

Baskaya, Yusuf Soner, Julian di Giovanni, Sebnem Kalemli-Ozcan, and Mehmet Fatih Ulu. 2017. "International spillovers and local credit cycles." *NBER Working Paper No. 23149*.

Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young. 2013. "Financial crises and macro-prudential policies." *Journal of International Economics*, 89(2): 453–470.

Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R Young. 2016. "Optimal capital controls and real exchange rate policies: A pecuniary externality perspective." *Journal of Monetary Economics*, 84: 147–165.

Bianchi, Javier. 2011. "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review*, 101(7): 3400–3426.

Blanchard, Olivier, Jonathan D. Ostry, Atish R. Ghosh, and Marcos Chamon. 2016. "Capital Flows: Expansionary or Contractionary?" *American Economic Review*, 106(6): 565–69.

Brunnermeier, Markus K., and Yann Koby. 2016. "The "Reversal Interest Rate": An Effective Lower Bound on Monetary Policy." *Manuscript*.

Brunnermeier, Markus K, Stefan Nagel, and Lasse H Pedersen. 2008. “Carry trades and currency crashes.” *NBER macroeconomics annual*, 23(1): 313–348.

Bruno, Valentina, and Hyun Song Shin. 2015. “Capital flows and the risk-taking channel of monetary policy.” *Journal of Monetary Economics*, 71: 119–132.

Bruno, Valentina, and Hyun Song Shin. 2017. “Global dollar credit and carry trades: a firm-level analysis.” *The Review of Financial Studies*, 30(3): 703–749.

Caballero, Julian, Ugo Panizza, and Andrew Powell. 2015. “The Second Wave of Global Liquidity: Why are Firms Acting like Financial Intermediaries?” *CEPR Working Paper 10926*.

Calvo, Guillermo A, and Carmen M Reinhart. 2002. “Fear of floating.” *The Quarterly Journal of Economics*, 117(2): 379–408.

Cavallino, Paolo. 2016. “Capital Flows and Foreign Exchange Intervention.” *Unpublished Manuscript*.

Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco. 2004. “Balance Sheets and Exchange Rate Policy.” *The American Economic Review*, 94(4): 1183–1193.

Christiano, Lawrence J., Christopher Gust, and Jorge Roldos. 2004. “Monetary Policy in a Financial Crisis.” *Journal of Economic Theory*, 119(1): 64–103.

Corte, Pasquale Della, Steven J Riddiough, and Lucio Sarno. 2016. “Currency premia and global imbalances.” *The Review of Financial Studies*, 29(8): 2161–2193.

Eggertsson, Gauti B., and Michael Woodford. 2003. “The Zero Bound on Interest Rates and Optimal Monetary Policy.” *Brookings Papers on Economic Activity*, 2003(1): 139–211.

Eggertsson, Gauti B., Ragnar E. Juelsrud, and Ella Getz Wold. 2017. “Are Negative Nominal Interest Rates Expansionary?” *NBER Working Paper No. 24039*.

Farhi, Emmanuel, and Ivan Werning. 2016. “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities.” *Econometrica*, 84(5): 1645–1704.

Fornaro, Luca. 2015. “Financial crises and exchange rate policy.” *Journal of International Economics*, 95(2): 202–215.

Gabaix, Xavier, and Matteo Maggiori. 2015. “International Liquidity and Exchange Rate Dynamics.” *Quarterly Journal of Economics*, 130(3): 1369–1420.

Gertler, Mark, and Peter Karadi. 2011. “A model of unconventional monetary policy.” *Journal of monetary Economics*, 58(1): 17–34.

Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto. 2012. "Financial crises, bank risk exposure and government financial policy." *Journal of Monetary Economics*, 59: 17–34.

Gornemann, Nils, Keith Kuester, and Makoto Nakajima. 2016. "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy." *International Finance Discussion Papers Number 1167*.

Gourinchas, Pierre-Olivier. 2018. "Monetary Policy Transmission in Emerging Markets: An Application to Chile." *Central Banking, Analysis, and Economic Policies Book Series*, 25: 279–324.

Guerrieri, Veronica, and Guido Lorenzoni. 2016. "Credit Crises, Precautionary Savings, and the Liquidity Trap." *Manuscript*.

IMF. 2012. "The Liberalization and Management of Capital Flows - An Institutional View." *IMF Policy Paper*.

Jeanne, Olivier, and Anton Korinek. 2010. "Excessive volatility in capital flows: A pigouvian taxation approach." *American Economic Review*, 100(2): 403–07.

Kaplan, Greg, Benjamin Moll, and Gianluca L. Violante. 2016. "Monetary Policy According to HANK." *NBER Working Paper No. 21897*.

Kollmann, Robert, Werner Roeger, and Jan in't Veld. 2012. "Fiscal Policy in a Financial Crisis: Standard Policy versus Bank Rescue Measures." *The American Economic Review*, 102(3): 77–81.

Korinek, Anton, and Damiano Sandri. 2016. "Capital controls or macroprudential regulation?" *Journal of International Economics*, 99: S27–S42.

Krugman, Paul. 1999. "Balance Sheets, the Transfer Problem, and Financial Crises." In *International Finance and Financial Crises*. , ed. Peter Isard, Assaf Razin and Andrew K. Rose, 31–55. Springer.

Krugman, Paul R., Kathryn M. Dominquez, and Kenneth Rogoff. 1998. "It's baaack: Japan's slump and the return of the liquidity trap." *Brookings Papers on Economic Activity*, 1998(2): 137–205.

Lustig, Hanno, and Adrien Verdelhan. 2007. "The cross section of foreign currency risk premia and consumption growth risk." *American Economic Review*, 97(1): 89–117.

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. 2011. "Common risk factors in currency markets." *The Review of Financial Studies*, 24(11): 3731–3777.

McCauley, Robert N., Patrick McGuire, and Vladyslav Sushko. 2015. "Global Dollar Credit: Links to US Monetary Policy and Leverage." *Economic Policy*, 30(82): 187–229.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson. 2016. "The Power of Forward Guidance Revisited." *American Economic Review*, 106(10): 3133–58.

Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf. 2012. "Carry trades and global foreign exchange volatility." *The Journal of Finance*, 67(2): 681–718.

Negro, Marco Del, Gauti Eggertsson, Nubuhiro Kiyotaki, and Andrea Ferrero. 2011. "The Great Escape? A Quantitative Evaluation of the Fed's Non-Standard Policies." *NY FED Staff Report no. 520*.

Obstfeld, Maurice. 2015. "Trilemmas and Trade-offs: Living with Financial Globalization." In *Global Liquidity, Spillovers to Emerging Markets and Policy Responses*. , ed. Claudio Raddatz, Diego Saravia and Jaume Ventura, Chapter 2, 13–78. Central Bank of Chile.

Ottonello, Pablo. 2015. "Optimal Exchange-Rate Policy Under Collateral Constraints and Wage Rigidities." *Manuscript*.

Rajan, Raghuram. 2015. "Competitive Monetary Easing: Is it Yesterday once More?" *Macroeconomics and Finance in Emerging Market Economies*, 8(1-2): 5–16.

Rey, Hélène. 2015. "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence." *NBER Working Paper 21162*.

Rey, Hélène. 2016. "International Channels or Transmission of Monetary Policy and the Mundellian Trilemma." *IMF Economic Review*, 64(1): 6–35.

Sandri, Damiano, and Fabian Valencia. 2013. "Financial Crises and Recapitalizations." *Journal of Money, Credit and Banking*, 45(s2): 59–86.

Svensson, Lars E.O. 2003. "Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others." *Journal of Economic Perspectives*, 17(4): 145–166.

Werning, Iván. 2015. "Incomplete Markets and Aggregate Demand." *NBER Working Paper 21448*.

Wu, Jing Cynthia, and Fan Dora Xia. 2016. "Measuring the macroeconomic impact of monetary policy at the zero lower bound." *Journal of Money, Credit and Banking*, 48(2-3): 253–291.

Appendix

A Generalization of the model with carry traders

The generalized equations of the carry-trade model are as follows. Borrowers and Savers budget constraints are

$$P_t C_t^B - \Pi_t^B = L_t - \mathbb{L}_t$$

$$P_t C_t^S - \Pi_t^S = -D_t + \mathbb{D}_t$$

where $\mathbb{L}_t = L_{t-1} I_{t-1}^L$, $\mathbb{D}_t = D_{t-1} I_{t-1}$, and net incomes are given by

$$\begin{aligned}
\Pi_t^B &= \omega_t^B P_{H,t} C_{H,t}^B + \omega_t^S P_{H,t} C_{H,t}^S + \omega_t^F e_t P_{H,t}^* C_{H,t}^* + \omega_t^G \left(P_{H,t} G_{H,t} - T_t\right) + \omega_t^N \left(\mathbb{N}_t - N_t\right) \\
\Pi_t^S &= \left(1-\omega_t^B\right) P_{H,t} C_{H,t}^B + \left(1-\omega_t^S\right) P_{H,t} C_{H,t}^S + \left(1-\omega_t^F\right) e_t P_{H,t}^* C_{H,t}^* \\
&\quad + \left(1-\omega_t^G\right) \left(P_{H,t} G_{H,t} - T_t\right) + \left(1-\omega_t^N\right) \left(\mathbb{N}_t - N_t\right)
\end{aligned}$$

where T_t are taxes. Their Euler equations are

$$\begin{aligned}
1 &= \beta I_t^L \mathbb{E}_t \left[P_t C_t^B / \left(P_{t+1} C_{t+1}^B\right)\right] \\
1 &= \beta I_t \mathbb{E}_t \left[P_t C_t^S / \left(P_{t+1} C_{t+1}^S\right)\right]
\end{aligned}$$

while their demand for Home goods is given by $P_{H,t}^i C_{H,t}^i = (1-\alpha) P_t^i C_t^i$. The balance sheet of the domestic banking sector is

$$N_t + D_t = L_t + B_t + R_t$$

therefore $\mathbb{N}_t = \mathbb{L}_t + \mathbb{B}_t + \mathbb{R}_t - \mathbb{D}_t$, where $\mathbb{B}_t = B_{t-1} I_{t-1}^B$, $\mathbb{R}_t = R_{t-1} I_{t-1}$, and $I_t^B = \lambda I_t^L + (1-\lambda) I_t$. Banks are subject to the leverage constraint $L_t + \lambda B_t \leq \phi N_t$. Foreign demand for domestic government bonds is

$$B_t^F = \frac{1}{\gamma_t} \mathbb{E}_t \left[\frac{e_t}{e_{t+1}} \frac{(1-\chi_t) I_t^B}{I_t^*} - 1\right]$$

where χ_t is a tax on capital inflows. The balance sheet of the domestic central bank is

$$N_t^{CB} + R_t = B_t^{CB} + e_t X_t$$

therefore $\mathbb{N}_t^{CB} = \mathbb{B}_t^{CB} + e_{t+1}\mathbb{X}_t - \mathbb{R}_t$, with $\mathbb{X}_t = X_{t-1} I_{t-1}^*$, where X_t are foreign reserves. Finally, the consolidated budget constraint of the public sector is

$$P_{H,t} G_{H,t} - T_t = B_t^G - \mathbb{B}_{t-1}^G + \chi_t B_t^F + \mathbb{N}_{t-1}^{CB} - N_t^{CB}$$

where $G_{H,t}$ is government spending, assumed to consist entirely of domestic goods. Market clearings require

$$\begin{aligned} Y_{H,t} &= C_{H,t} + C_{H,t}^* + G_{H,t} \\ B_t^G &= B_t + B_t^F + B_t^{CB} \end{aligned}$$

The aggregate budget constraint of the country is

$$\alpha P_t C_t = \alpha e_t P_t^* C_t^* + (1+\chi_t) B_t^F - \mathbb{B}_{t-1}^F + e_t (\mathbb{X}_{t-1} - X_t)$$

In a steady state with $\gamma_t = \chi_t = 0$, we have

$$e_t = \frac{\alpha \beta P_t C_t + (1-\beta) B_t^F}{\alpha \beta P_t^* C_t^* + (1-\beta) X_t}$$

with $B_t^F = \beta \mathbb{B}_2^F$ and $X_t = \beta \mathbb{X}_2$. Therefore, at time 2 we have

$$e_2 = \frac{\alpha M_2 + (1-\beta) \mathbb{B}_2^F}{\alpha M_2^* + (1-\beta) \mathbb{X}_2}$$

and

$$\begin{aligned} \Pi_2 &= M_2 + (1-\beta)(\mathbb{L}_2 - \mathbb{D}_2) \\ P_2 C_2^B &= \omega_2 M_2 - (1-\beta)\left[(1-\omega_2)\mathbb{L}_2 + \omega_2 \mathbb{D}_2\right] \\ P_2 C_2^S &= (1-\omega_2) M_2 + (1-\beta)\left[(1-\omega_2)\mathbb{L}_2 + \omega_2 \mathbb{D}_2\right] \end{aligned}$$

where we set $P_2 C_2 = M_2$, $P_2^* C_2^* = M_2^*$, and $\omega_2^B = \omega_2^S = \omega_2^F = \omega_2^G = \omega_2^N = \omega_2$.

At time 1, demand and supply for foreign funds are given by

$$(1+\chi_1) B_1^F = \mathbb{B}_1^F + \alpha \left(\frac{P_2 C_2^B}{\beta I_1^L} + \frac{P_2 C_2^S}{\beta I_1} \right) + e_1 \left(X_1 - \mathbb{X}_0 - \alpha \frac{P_2^* C_2^*}{\beta I_1^*} \right)$$

$$B_1^F = \frac{1}{\gamma_1} \mathbb{E}_1 \left[\frac{e_1}{e_2} \frac{(1-\chi_1) I_1^B}{I_1^*} - 1 \right]$$

while loan demand and deposit supply are

$$
\begin{aligned}
L_1 &= \mathbb{L}_1 + \frac{P_2 C_2^B}{\beta I_1^L} - (1-\alpha)\left(\omega_1^B \frac{P_2 C_2^B}{\beta I_1^L} + \omega_1^S \frac{P_2 C_2^S}{\beta I_1}\right) - \omega_1^F e_1 \frac{\alpha M_2^*}{\beta I_1^*} - \omega_1^G \left(P_{H,1} G_{H,1} - T_1\right) \\
D_1 &= \mathbb{D}_1 - \frac{P_2 C_2^S}{\beta I_1} + (1-\alpha)\left(\left(1-\omega_1^B\right) \frac{P_2 C_2^B}{\beta I_1^L} + \left(1-\omega_1^S\right) \frac{P_2 C_2^S}{\beta I_1}\right) \\
&\quad + \left(1-\omega_1^F\right) e_1 \frac{\alpha M_2^*}{\beta I_1^*} + \left(1-\omega_1^G\right)\left(P_{H,1} G_{H,1} - T_1\right)
\end{aligned}
$$

and the collateral constraint is

$$
(1+\chi_1)\,\lambda B_1^F \geq L_1 + \lambda\left(\mathbb{B}_1^G - \mathbb{N}_1^{CB} + P_{H,1} G_{H,1} - T_1 - R_1 + e_1 X_1\right) - \phi \mathbb{N}_1
$$

where we assumed the no-dividend policies $N_1 = \mathbb{N}_1$ and $N_1^{CB} = \mathbb{N}_1^{CB}$.

At time 0, demand and supply for foreign funds are given by

$$
(1+\chi_0)\, B_0^F = \mathbb{B}_0^F + \alpha\left(\frac{1}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1^L}{P_2 C_2^B}\right]} + \frac{1}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1}{P_2 C_2^S}\right]}\right) + e_0\left(X_0 - \mathbb{X}_0 - \frac{\alpha M_2^*}{\beta^2 I_0^* I_1^*}\right)
$$

$$
B_0^F = \frac{1}{\gamma_0}\mathbb{E}_0\left[\frac{e_0}{e_1}\frac{(1-\chi_0)\, I_0^B}{I_0^*} - 1\right]
$$

while loan demand and deposit supply are

$$
\begin{aligned}
L_0 &= \mathbb{L}_0 + \frac{1}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1^L}{P_2 C_2^B}\right]} - (1-\alpha)\left(\frac{\omega_0^B}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1^L}{P_2 C_2^B}\right]} + \frac{\omega_0^S}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1}{P_2 C_2^S}\right]}\right) \\
&\quad - \omega_0^F e_0 \frac{\alpha M_2^*}{\beta^2 I_0^* I_1^*} - \omega_0^G \left(P_{H,0} G_{H,0} - T_0\right) \\
D_0 &= \mathbb{D}_0 - \frac{1}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1}{P_2 C_2^S}\right]} + (1-\alpha)\left(\frac{1-\omega_0^B}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1^L}{P_2 C_2^B}\right]} + \frac{1-\omega_0^S}{\beta I_0 \mathbb{E}_0\left[\frac{\beta I_1}{P_2 C_2^S}\right]}\right) \\
&\quad + \left(1-\omega_0^F\right) e_0 \frac{\alpha M_2^*}{\beta^2 I_0^* I_1^*} + \left(1-\omega_0^G\right)\left(P_{H,0} G_{H,0} - T_0\right)
\end{aligned}
$$

and the collateral constraint is

$$
(1+\chi_0)\,\lambda B_0^F \geq L_0 + \lambda\left(\mathbb{B}_0^G - \mathbb{N}_0^{CB} + P_{H,0} G_{H,0} - T_0 - R_0 + e_0 X_0\right) - \phi \mathbb{N}_0
$$

where, again, we assumed the no-dividend policies $N_0 = \mathbb{N}_0$ and $N_0^{CB} = \mathbb{N}_0^{CB}$.

Alternative policy tools The baseline version of the model, presented in the main body of the paper, is recovered by setting $\beta = 1$, $\omega_1^B = \omega_1^F = 0$, and $\omega_1^S = \frac{1}{1-\alpha}\frac{\omega_2}{1-\omega_2}$. Then, we obtain $e_2 = M_2/M_2^*$and

$$
\begin{aligned}
P_2 C_2^B &= \omega_2 M_2 \\
P_2 C_2^S &= (1-\omega_2) M_2
\end{aligned}
$$

At time 1 we have

$$
\begin{aligned}
e_1 &= e_2 \frac{I_1^*}{I_1^B} \frac{1+\chi_1+\alpha\gamma_1\left[\frac{\omega_2 M_2}{I_1^L}+\frac{(1-\omega_2)M_2}{I_1}\right]+\gamma_1\mathbb{B}_1^F}{1-\chi_1^2-\gamma_1\frac{M_2}{M_2^*}\frac{I_1^*}{I_1^B}\left(X_1-\mathbb{X}_1-\alpha\frac{M_2^*}{\beta I_1^*}\right)} \\
B_1^F &= (1-\chi_1)\frac{\mathbb{B}_1^F+\alpha\left[\frac{\omega_2 M_2}{\beta I_1^L}+\frac{(1-\omega_2)M_2}{\beta I_1}\right]+\frac{1+\chi_1}{\gamma_1}}{1-\chi_1^2-\gamma_1\frac{M_2}{M_2^*}\frac{I_1^*}{I_1^B}\left(X_1-\mathbb{X}_0-\alpha\frac{M_2^*}{\beta I_1^*}\right)}-\frac{1}{\gamma_1} \\
L_1 &= \mathbb{L}_0+\omega_2 M_2\left(\frac{1}{I_1^L}-\frac{1}{I_1}\right)-\omega_1^G\left(P_{H,1}G_{H,1}-T_1\right)
\end{aligned}
$$

Thus, the ELB is given by

$$
\begin{aligned}
&\left(1-\chi_1^2\right)\mathbb{B}_0^F-(1+\chi_1)\frac{M_2}{I_1^{ELB}}\left(\alpha\chi_1+\mathbb{X}_0\frac{I_1^*}{M_2^*}\right)-\gamma_1 X_1\frac{M_2}{M_2^*}\frac{I_1^*}{I_1^{ELB}}\left(\mathbb{B}_0^F+\frac{\alpha M_2}{I_1^{ELB}}\right)+ \\
&+\left[1-\chi_1^2-\gamma_1\frac{M_2}{M_2^*}\frac{I_1^*}{I_1^{ELB}}\left(X_1-\mathbb{X}_0-\frac{\alpha M_2^*}{I_1^*}\right)\right]\left[\left(1-\frac{\omega_1^G}{\lambda}\right)\left(T_1-P_{H,1}G_{H,1}\right)+R_1-\underline{B}_1^F\right]=0
\end{aligned}
$$

We can use the implicit function theorem to prove the following comparative static results

$$
\begin{aligned}
\frac{\partial I_1^{ELB}}{\partial T_1} &= \left(\frac{1}{\underline{B}_1^F I_1^{ELB}}\right)^2\frac{\mathbb{B}_0^F}{\alpha\gamma_1}\frac{\omega_1^G-\lambda}{\lambda} \\
\frac{\partial I_1^{ELB}}{\partial \chi_1} &= \left(\frac{1}{I_1^{ELB}}\right)^3\frac{1}{\gamma_1\underline{B}_1^F} \\
\frac{\partial I_1^{ELB}}{\partial X_1^{Ster}} &= \left(\frac{1}{I_1^{ELB}}\right)^4\frac{1+\gamma_1\underline{B}_1^F}{\underline{B}_1^F} \\
\frac{\partial I_1^{ELB}}{\partial X_1^{Unster}} &= -\left(\frac{1}{I_1^{ELB}}\right)^3\frac{1+\gamma_1\underline{B}_1^F}{\alpha\gamma_1\underline{B}_1^F} \\
\frac{\partial I_1^{ELB}}{\partial R_1} &= -\left(\frac{1}{I_1^{ELB}}\right)^4\frac{\alpha\gamma_1\mathbb{B}_0^F}{\left(\mathbb{B}_0^F-\underline{B}_1^F\right)^2} \\
\frac{\partial I_1^{ELB}}{\partial M_2} &= \frac{M_2}{I_1^{ELB}}
\end{aligned}
$$

Now assume that all unconventional tools are set to zero at time 1 and $M_2 = M_2^* = 1$, such that $I_1^{ELB} = \alpha\bar{\gamma}_1 / \left(\frac{\lambda B_0^F}{\lambda B_0^G + L_0 - \phi N_0} - 1\right)$. At time 0 we have

$$
\begin{aligned}
L_0 &= \mathbb{L}_{-1} + \frac{\omega_2}{I_0 \mathbb{E}_0\left[I_1\right]} + \omega_0^G PB_0 \\
B_0^F &= \frac{\mathbb{B}_{-1}^F}{1+\chi_0} + \frac{\alpha}{1+\chi_0}\frac{1}{I_0\mathbb{E}_0\left[I_1\right]} + \frac{1}{I_0\mathbb{E}_0\left[\frac{1}{e_1}\right]}\frac{X_0-\alpha}{1-\chi_0^2} \\
B_0^G &= \mathbb{B}_{-1}^G - \chi_0 B_0^F - PB_0
\end{aligned}
$$

with $\mathbb{E}_0\left[\frac{1}{e_1}\right] = \rho \frac{I_1^{ELB}}{1+\gamma_1 I_0 \frac{L_0+\lambda B_0^G - \phi N_0}{\lambda}} + (1-\rho)\tilde{I}_1$ and PB_0 is the government's primary balance. We can use the implicit function theorem to show that, for any policy variable z we have

$$
\frac{\partial I_1^{ELB}}{\partial z} = \frac{\left[\rho\left(\frac{I_1^{ELB}}{1+\bar{\gamma}_1 \underline{B}^F}\right)^2 \frac{\mathbb{B}_0^F - \underline{B}^F}{\mathbb{E}_0\left[\frac{1}{e_1}\right]^2} + \mathbb{B}_0^F\right]\left(\frac{\partial L_0}{\partial z} + \lambda\frac{\partial B_0^G}{\partial z}\right) - \lambda \underline{B}^F \frac{\partial B_0^F}{\partial z}}{\lambda\frac{\left(\mathbb{B}_0^F - \underline{B}^F\right)^2}{\alpha\bar{\gamma}_1 I_0} + \lambda \underline{B}^F\left(\mathbb{B}_{-1}^F + \frac{\alpha}{I_0\mathbb{E}_0\left[\frac{1}{e_1}\right]^2}\frac{\rho}{1+\bar{\gamma}_1\underline{B}^F}\right) + \rho\frac{\mathbb{B}_0^F\omega_2 - \alpha\lambda\underline{B}^F}{I_0\mathbb{E}_0\left[I_1\right]^2}}
$$

A sufficient condition for the denominator to be positive is $\omega_2 > \lambda$. Therefore we obtain

$$
\begin{aligned}
\frac{\partial I_1^{ELB}}{\partial I_0} \propto &- \left[\left(\frac{I_1^{ELB}}{1+\bar{\gamma}_1\underline{B}^F}\right)^2 \frac{\rho\left(\mathbb{B}_0^F - \underline{B}^F\right)}{\mathbb{E}_0\left[\frac{1}{e_1}\right]^2} + \mathbb{B}_0^F\right]\frac{\omega_2}{I_0^2\mathbb{E}_0\left[I_1\right]} \\
&- \lambda\underline{B}^F \frac{\alpha}{I_0^2\mathbb{E}_0\left[\frac{1}{e_1}\right]^2}\frac{\rho I_1^{ELB}\bar{\gamma}_1\underline{B}^F}{1+\bar{\gamma}_1\underline{B}^F}\left(\frac{\mathbb{E}_0\left[\frac{1}{e_1}\right]}{\mathbb{E}_0\left[I_1\right]} + \frac{1}{1+\bar{\gamma}_1\underline{B}^F}\right) \\
\frac{\partial I_1^{ELB}}{\partial T_0} \propto &\left[\left(\frac{I_1^{ELB}}{1+\bar{\gamma}_1\underline{B}^F}\right)^2 \frac{\rho\left(\mathbb{B}_0^F - \underline{B}^F\right)}{\mathbb{E}_0\left[\frac{1}{e_1}\right]^2} + \mathbb{B}_0^F\right]\left(\omega_0^G - \lambda\right) \\
\frac{\partial I_1^{ELB}}{\partial \chi_0} \propto &-\lambda B_0^F\left[\left(\frac{I_1^{ELB}}{1+\bar{\gamma}_1\underline{B}^F}\right)^2 \frac{\rho\left(\mathbb{B}_0^F - \underline{B}^F\right)}{\mathbb{E}_0\left[\frac{1}{e_1}\right]^2} + \mathbb{B}_0^F\right] + \lambda\underline{B}^F\left[\mathbb{B}_{-1}^F + \frac{\alpha}{I_0\mathbb{E}_0\left[I_1\right]}\right] \\
\frac{\partial I_1^{ELB}}{\partial X_0} \propto &-\frac{\lambda\underline{B}^F}{I_0\mathbb{E}_0\left[\frac{1}{e_1}\right]}
\end{aligned}
$$

Generalized income profile Assume $\omega_1^B = \omega_1^S = \omega_1^F = \omega_1$ and set all the alternative policy tools to zero, with $M_2 = M_2^* = \beta = 1$. Then

$$
\begin{aligned}
e_1 &= \frac{I_1^*}{I_1^B} \frac{1 + \alpha\gamma_1 \left(\frac{\omega_2}{I_1^L} + \frac{1-\omega_2}{I_1}\right) + \gamma_1 \mathbb{B}_1^F}{1 + \frac{\alpha\gamma_1}{I_1^B}} \\
B_1^F &= \frac{\mathbb{B}_1^F + \alpha \left(\frac{\omega_2}{I_1^L} + \frac{1-\omega_2}{I_1} - \frac{1}{I_1^B}\right)}{1 + \frac{\alpha\gamma_1}{I_1^B}} \\
L_1 &= \mathbb{L}_1 + \frac{\omega_2}{I_1^L} - \omega_1 \left[(1-\alpha)\left(\frac{\omega_2}{I_1^L} + \frac{1-\omega_2}{I_1}\right) + \frac{\alpha e_1}{I_1^*}\right]
\end{aligned}
$$

while the banks' leverage constraint is $\lambda B_1^F - L_1 - \lambda \mathbb{B}_1^G + \phi \mathbb{N}_1 \geq 0$. In the unconstrained region we have

$$
\frac{\partial\left(\lambda B_1^F - L_1\right)}{\partial I_1} = \alpha\gamma_1 \mathbb{B}_1^F \frac{\lambda - \omega_1}{\left(I_1 + \alpha\gamma_1\right)^2} + \frac{\omega_2 - \omega_1}{I_1^2}
$$

which is positive if ω_1 is small relative to ω_2. When the constraint binds, the lending rate solves

$$
\begin{aligned}
&\left[\left(\omega_1 (1-\alpha) - 1 + \alpha\lambda\right) I_1^B - \alpha\gamma_1 (1-\omega_1)\right] \frac{\omega_2}{I_1^L} + \left[\left(\omega_1 (1-\alpha) + \alpha\lambda\right) I_1^B + \omega_1 \alpha\gamma_1\right] \frac{1-\omega_2}{I_1} + \\
&\qquad + \lambda I_1^B \left(\mathbb{B}_1^F - \underline{B}_1^F\right) - \alpha(\lambda - \omega_1) + \left(\omega_1 \mathbb{B}_1^F - \lambda \underline{B}_1^F\right) \alpha\gamma_1 = 0
\end{aligned}
$$

with $I_1^B = \lambda I_1^L + (1-\lambda) I_1$. Thus

$$
\begin{aligned}
\left.\frac{\partial I_1^L}{\partial I_1}\right|_{I_1 = I_1^{ELB}} &= -\frac{\left(\omega_1 (1-\alpha) + \alpha\lambda\right) \frac{\omega_2 - \lambda}{I_1} + (1-\lambda)(\omega_2 - \omega_1) \frac{\alpha\gamma_1}{I_1^2} + \lambda(1-\lambda) \underline{B}_1^F \frac{\alpha\gamma_1}{I_1}}{\left(1 - \alpha\lambda - \omega_1 (1-\alpha)\right) \frac{\omega_2 - \lambda}{I_1} + \lambda \frac{1-\omega_2}{I_1} + \alpha\gamma_1 (1-\omega_1) \frac{\omega_2}{I_1^2} + \lambda^2 \left(\mathbb{B}_1^F - \underline{B}_1^F\right)} + \\
&\quad + \frac{\frac{\omega_1}{I_1}\left[(1-\omega_2) \frac{\alpha\gamma_1}{I_1} + (1-\lambda)\left(1 + \alpha\gamma_1 \mathbb{B}_1^F\right)\right]}{\left(1 - \alpha\lambda - \omega_1 (1-\alpha)\right) \frac{\omega_2 - \lambda}{I_1} + \lambda \frac{1-\omega_2}{I_1} + \alpha\gamma_1 (1-\omega_1) \frac{\omega_2}{I_1^2} + \lambda^2 \left(\mathbb{B}_1^F - \underline{B}_1^F\right)}
\end{aligned}
$$

which is negative if ω_1 is small relative to ω_2.

Generalized discount factor β Assume $\omega_1^B = \omega_1^F = 0$, $\omega_1^S = \frac{1}{1-\alpha} \frac{P_2 C_2^B}{P_2 C_2^S}$, and set all the alternative policy tools to zero, with $M_2 = M_2^* = 1$. Then

$$
e_1 = \frac{I_1^*}{\alpha} \left(1 + \gamma_1 B_1^F\right) \left[\frac{\alpha}{I_1^B} + (1-\beta) B_1^F\right]
$$

and

$$B_1^F = \mathbb{B}_1^F + \alpha \frac{P_2 C_2^B}{\beta} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right) + \frac{\alpha}{\beta I_1} - \left(1 + \gamma_1 B_1^F\right) \left(\frac{\alpha}{\beta I_1^B} + \frac{1-\beta}{\beta} B_1^F \right)$$

while loan demand and deposit supply are

$$\begin{aligned} L_1 &= \mathbb{L}_1 + \frac{P_2 C_2^B}{\beta} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right) \\ D_1 &= \mathbb{D}_1 + (1-\alpha) \frac{P_2 C_2^B}{\beta} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right) - \frac{\alpha}{\beta I_1} + \left(1 + \gamma_1 B_1^F\right) \left[\frac{\alpha}{\beta I_1^B} + \frac{1-\beta}{\beta} B_1^F \right] \end{aligned}$$

with

$$\begin{aligned} P_2 C_2^B &= \omega_2 - (1-\beta) \left[(1-\omega_2) \mathbb{L}_2 + \omega_2 \mathbb{D}_2 \right] \\ P_2 C_2^S &= (1-\omega_2) + (1-\beta) \left[(1-\omega_2) \mathbb{L}_2 + \omega_2 \mathbb{D}_2 \right] \end{aligned}$$

$$P_2 C_2^B = \omega_2 - (1-\beta) \left[(1-\omega_2) \mathbb{L}_1 + \omega_2 \mathbb{D}_1 + \omega_2 \frac{\alpha \gamma_1}{\beta I_1} B_1^F + \omega_2 \left(1 + \gamma_1 B_1^F\right) \frac{1-\beta}{\beta} B_1^F \right] I_1$$

When the constraint does not bind

$$B_1^F - \mathbb{B}_1^F + \frac{\alpha \gamma_1}{\beta I_1} B_1^F + \frac{1-\beta}{\beta} B_1^F \left(1 + \gamma_1 B_1^F\right) = 0$$

and $L_1 = \mathbb{L}_1$. Therefore

$$\frac{\partial B_1^F}{\partial I_1} = \frac{\alpha \gamma_1 B_1^F / I_1^2}{\beta + \gamma_1 \frac{\alpha}{I_1} + \left(1 + 2\gamma_1 B_1^F\right)(1-\beta)}$$

which is positive. When the constraint binds, the lending rate solves

$$\lambda B_1^F - \frac{P_2 C_2^B}{\beta} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right) - \lambda \underline{B}_1^F = 0$$

where B_1^F solves

$$B_1^F - \mathbb{B}_1^F - \alpha \frac{P_2 C_2^B}{\beta} \left(\frac{1}{I_1^L} - \frac{1}{I_1} \right) - \frac{\alpha}{\beta I_1} + \left(1 + \gamma_1 B_1^F\right) \left(\frac{\alpha}{\beta I_1^B} + \frac{1-\beta}{\beta} B_1^F \right) = 0$$

and $I_1^B = \lambda I_1^L + (1-\lambda) I_1$. Thus

$$\left. \frac{\partial B_1^F}{\partial I_1} \right|_{I_1 = \bar{I}_1} = -\frac{1}{I_1} \frac{1}{I_1} \alpha \frac{-P_2 C_2^B + \lambda - (1-\lambda) \gamma_1 B_1^F}{1 + \gamma_1 \left(\frac{\alpha}{I_1} + 2(1-\beta) B_1^F \right)} - \frac{1}{I_1} \frac{1}{I_1} \alpha \frac{P_2 C_2^B - \lambda - \lambda \gamma_1 B_1^F}{1 + \gamma_1 \left(\frac{\alpha}{I_1} + 2(1-\beta) B_1^F \right)} \left. \frac{\partial I_1^L}{\partial I_1} \right|_{I_1 = \bar{I}_1}$$

and we obtain

$$\left.\frac{\partial I_1^L}{\partial I_1}\right|_{I_1=\bar{I}_1} = 1 - \frac{\gamma_1 B_1^F}{\frac{P_2 C_2^B}{\alpha\lambda\beta}\left[\beta\frac{\mathbb{B}_1^F}{B_1^F} - \alpha\lambda\beta + (1-\beta)\gamma_1 B_1^F\right] + \lambda\left(1+\gamma_1 B_1^F\right)}$$

which is negative is γ_1 is large enough.

B Generalization of the model with currency mismatches

The generalized equations of the carry trade model are as follows. The budget constraint of the representative agent is

$$P_t C_t - \Pi_t = L_t - \mathbb{L}_t$$

where $\mathbb{L}_t = L_{t-1} I_{t-1}^L$ and net income is

$$\Pi_t = P_{H,t} C_{H,t} + e_t P_{H,t}^* C_{H,t}^* - T_t + \mathbb{N}_t - N_t$$

Its Euler equation is

$$1 = \beta_t I_t \mathbb{E}_t \left[P_t C_t / (P_{t+1} C_{t+1})\right]$$

while its demand for Home goods is given by $P_{H,t} C_{H,t} = (1-\alpha) P_t C_t$. The balance sheet of the domestic banking sector is

$$N_t + e_t D_t^* = L_t + R_t$$

therefore $\mathbb{N}_t = \mathbb{L}_t + \mathbb{R}_t - e_t \mathbb{D}_t^*$, where the exchange rate satisfies the modified UIP condition

$$\mathbb{E}_t \left[\left(e_t I_t - e_{t+1} I_t^* (1-\chi_t)\right)\left(I_{t+1} + \phi\mu_{t+1}\right)\right] = 0$$

and μ_{t+1} is the shadow cost of the collateral constraints. Banks are subject to the leverage constraint $L_t \leq \phi N_t$. The balance sheet of the domestic central bank is

$$N_t^{CB} + R_t = e_t X_t$$

therefore $\mathbb{N}_t^{CB} = e_t \mathbb{X}_t - \mathbb{R}_t$, where X_t are foreign reserves. Finally, the consolidated budget constraint of the public sector is

$$T_t = N_t^{CB} - \mathbb{N}_t^{CB} + \chi_t e_t D_t^*$$

Market clearing requires $Y_{H,t} = C_{H,t} + C_{H,t}^*$. The aggregate budget constraint of the country is

$$\alpha P_t C_t = e_t \left[\alpha P_t^* C_t^* + (1-\chi_t) D_t^* - \mathbb{D}_t^* + \mathbb{X}_t - X_t\right]$$

In a steady state with $\chi_t = 0$, we have

$$e_t = \frac{\alpha \beta_1 P_t C_t}{\alpha \beta_1^* P_t^* C_t^* + (1-\beta)(X_t - D_t^*)}$$

with $D_t^* = \beta \mathbb{D}_2^*$ and $X_t = \beta \mathbb{X}_2$. Therefore, at time 2 we have

$$e_2 = \frac{\alpha M_2}{\alpha M_2^* + (1-\beta)(\mathbb{X}_2 - \mathbb{D}_2^*)}$$

and $\Pi_2 = M_2 + (1-\beta)\mathbb{L}_2$.

At time 1, loand demand is

$$L_1 = \frac{1}{1-\chi_1} \frac{\alpha M_2}{\beta_1 I_1^L} - \frac{e_1}{1-\chi_1}\left(\frac{\alpha M_2^*}{\beta_1^* I_1^*} - \chi_1 \mathbb{D}_1^*\right) + \mathbb{L}_1 + (N_1 - \mathbb{N}_1) + \frac{\chi_1}{1-\chi_1}(R_1 - \mathbb{R}_1)$$

while the collateral constraint is $L_1 \leq \phi(\mathbb{L}_1 + \mathbb{R}_1 - e_1 \mathbb{D}_1^*)$, with

$$e_1 = (1-\chi_1)\frac{I_1^*}{I_1} \frac{\alpha M_2}{\alpha M_2^* + (1-\beta)(\mathbb{X}_2 - \mathbb{D}_2^*)}$$

At time 0, loand demand is

$$L_0 = \frac{1}{1-\chi_0} \frac{1}{\beta_0 I_0} \frac{\alpha M_2}{\beta_1 \mathbb{E}_0\left[I_1^L\right]} - \frac{e_0}{1-\chi_0} \frac{1}{\beta_0^* I_0^*} \frac{\alpha M_2^*}{\beta_1^* \mathbb{E}_0\left[I_1^*\right]} + N_0 - \mathbb{N}_0$$

with

$$e_0 = (1-\chi_0)\frac{I_0^*}{I_0} \frac{\mathbb{E}_0\left[e_1(I_1 + \phi \mu_1)\right]}{\mathbb{E}_0\left[I_1 + \phi \mu_1\right]}$$

Alternative policy tools The baseline version of the model, presented in the main body of the paper, is recovered by setting $\beta_1 = \beta_2 = \beta_0^* = \beta_1^* = \beta_2^* = 1$. Then, we obtain $e_2 = M_2/M_2^*$. At time 1 we have

$$e_1 = (1-\chi_1)\frac{I_1^*}{I_1}\frac{M_2}{M_2^*}$$

Thus, the ELB is given by

$$I_1^{ELB} = \frac{M_2}{M_2^*} \frac{\frac{\alpha M_2^*}{1-\chi_1} - \alpha M_2^* + [\phi - \chi_1(\phi - 1)]\mathbb{D}_1^* \bar{I}_1^*}{(\phi - 1)(\mathbb{L}_1 + N_1 - \mathbb{N}_1) - \frac{\chi_1}{1-\chi_1} R_1 + \frac{\phi(1-\chi_1)+\chi_1}{1-\chi_1}\mathbb{R}_1}$$

and

$$
\begin{aligned}
\frac{\partial I_1^{ELB}}{\partial \chi_1} &= -\frac{(\phi-1)\,\mathbb{D}_1^* \bar{I}_1^* - \alpha}{(\phi-1)\,\mathbb{L}_1} \\
\frac{\partial I_1^{ELB}}{\partial M_2} &= \frac{I_1^{ELB}}{M_2} \\
\frac{\partial I_1^{ELB}}{\partial N_1} &= -\frac{I_1^{ELB}}{\mathbb{L}_1}
\end{aligned}
$$

Now assume that all unconventional tools are set to zero at time 1 and $M_2 = M_2^* = 1$, such that $I_1^{ELB} = \frac{\phi \mathbb{D}_1^* \bar{I}_1^*}{(\phi-1)\mathbb{L}_1}$. At time 0 we have

$$
\begin{aligned}
\mathbb{L}_1 &= \frac{\delta+\chi_0}{1-\chi_0}\frac{\alpha}{\mathbb{E}_0\,[I_1]} + N_0 I_0 \\
\mathbb{D}_1^* &= \frac{\delta+\chi_0}{(1-\chi_0)^2}\frac{\alpha}{\mathbb{E}_0\,[I_1^*]}
\end{aligned}
$$

with $\mathbb{E}_0\,[I_1] = \rho I_1^{ELB} + (1-\rho)\,\tilde{I}_1$. We can use the implicit function theorem to show that, for any policy variable z we have

$$
\frac{\partial I_1^{ELB}}{\partial z} \propto \frac{\phi}{\phi-1}\frac{\partial \mathbb{D}_1^*}{\partial z}\bar{I}_1^* - \frac{\partial \mathbb{L}_1}{\partial z} I_1^{ELB}
$$

Therefore we obtain

$$
\begin{aligned}
\frac{\partial I_1^{ELB}}{\partial I_0} &\propto -(\mathbb{L}_0 + N_0)\, I_1^{ELB} \\
\frac{\partial I_1^{ELB}}{\partial \chi_0} &\propto \frac{\phi \alpha \bar{I}_1^*}{\phi-1}\frac{1}{\mathbb{E}_0\,[I_1^*]}\left[\delta + \frac{(1+\delta)\,N_0 \mathbb{E}_0\,[I_1]\,I_0}{\alpha\delta + N_0 I_0 \mathbb{E}_0\,[I_1]}\right] \\
\frac{\partial I_1^{ELB}}{\partial \bar{I}_1^*} &\propto \frac{\phi \mathbb{D}_1^*}{(\phi-1)\,\mathbb{L}_1} - \frac{\phi}{\phi-1}\frac{\mathbb{D}_1^* \bar{I}_1^*}{\alpha\delta}\rho \mathbb{D}_1^* \\
\frac{\partial I_1^{ELB}}{\partial \mathbb{E}_0\,[I_1^*]} &\propto -\frac{\phi}{\phi-1}\frac{\alpha\delta}{\mathbb{E}_0\,[I_1^*]^2}\bar{I}_1^*
\end{aligned}
$$

Generalized betas Set all the alternative policy tools to zero. Then at time 1 we have

$$
\begin{aligned}
e_1 &= \frac{\alpha \beta_1^* I_1^* \left(\frac{1-\beta}{\beta_1 I_1^L} + \frac{1}{I_1}\right)}{\alpha\,(1-\beta+\beta_1^*) - (1-\beta)\,\beta_1^* \mathbb{D}_1^* I_1^*} \\
L_1 &= \mathbb{L}_1 - \alpha \frac{(1-\beta)\,\frac{\beta_1^*}{\beta_1}\frac{\mathbb{D}_1^* I_1^*}{I_1^L} + \alpha\left(\frac{1}{I_1} - \frac{1}{I_1^L}\frac{\beta_1^*}{\beta_1}\right)}{\alpha\,(1-\beta+\beta_1^*) - (1-\beta)\,\beta_1^* \mathbb{D}_1^* I_1^*} \\
D_1^* &= \frac{\frac{\alpha}{I_1^*}\left(I_1 - \frac{\beta_1 I_1^L}{\beta_1^*}\right) + \beta_1 I_1^L \mathbb{D}_1^*}{\beta_1 I_1^L + (1-\beta)\,I_1}
\end{aligned}
$$

When the constraint binds, the lending rate is

$$\frac{1}{I_1^L}=\frac{(\phi-1)\,\mathbb{L}_1\left[\alpha\,(1-\beta+\beta_1^*)-(1-\beta)\,\beta_1^*\mathbb{D}_1^*I_1^*\right]-\frac{\alpha}{I_1}\left(\phi\beta_1^*\mathbb{D}_1^*I_1^*-\alpha\right)}{\alpha\left[\alpha\frac{\beta_1^*}{\beta_1}+(\phi-1)\,\frac{1-\beta}{\beta_1}\beta_1^*\mathbb{D}_1^*I_1^*\right]}$$

which is decreasing in I_1 iff $\phi\mathbb{D}_1^*>\alpha/\left(\beta_1^*I_1^*\right)$. The ELB is given by

$$I_1^{ELB}=\frac{\alpha}{\beta_1}\frac{\alpha\,(\beta_1^*-\beta_1)+\left[(\phi-1)\,(1-\beta)+\phi\beta_1\right]\beta_1^*\mathbb{D}_1^*I_1^*}{(\phi-1)\,\mathbb{L}_1\left[\alpha\,(1-\beta+\beta_1^*)-(1-\beta)\,\beta_1^*\mathbb{D}_1^*I_1^*\right]}$$

The effect of monetary policy around the ELB is

$$\begin{aligned}\left.\frac{\partial Y_{H,1}}{\partial I_1}\right|_{I_1=I_1^{ELB}}^{-}=&-\left(\frac{1}{I_1^{ELB}}\right)^2\left[1-\alpha+\frac{\alpha^2\beta_1^*\frac{1-\beta}{\beta_1}}{\alpha\,(1-\beta+\beta_1^*)-(1-\beta)\,\beta_1^*\mathbb{D}_1^*I_1^*}\right]\left.\frac{\partial I_1^L}{\partial I_1}\right|_{I_1=I_1^{ELB}}^{-}\\&-\left(\frac{1}{I_1^{ELB}}\right)^2\frac{\alpha^2\beta_1^*}{\alpha\,(1-\beta+\beta_1^*)-(1-\beta)\,\beta_1^*\mathbb{D}_1^*I_1^*}\end{aligned}$$

Therefore output contracts in the constrained region iff

$$\left(\mathbb{D}_1^*I_1^*\right)^2-\frac{\alpha}{\phi}\left(\frac{1+\phi}{\beta_1^*}+\frac{\phi}{1-\beta}+\frac{\alpha}{1-\alpha}\frac{1-\beta}{\beta_1}\right)\mathbb{D}_1^*I_1^*+\frac{\alpha^2}{\phi}\left(\frac{1}{\beta_1^*}+\frac{1}{1-\beta}\right)\left(\frac{1}{\beta_1^*}+\frac{\alpha}{1-\alpha}\frac{1}{\beta_1}\right)<0$$

That is, if $\mathbb{D}_1^*I_1^*$ is high enough, but not too high.