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Accounting for Macrofinancial Fluctuations and Turbulence

by Francis Vitek

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Accounting for Macrofinancial Fluctuations and Turbulence

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Abstract

This paper investigates the sources of macrofinancial fluctuations and turbulence within the framework of an approximate linear dynamic stochastic general equilibrium model of the world economy, augmented with structural shocks exhibiting potentially asymmetric generalized autoregressive conditional heteroskedasticity. Very strong evidence of asymmetric autoregressive conditional heteroskedasticity is found, providing a basis for jointly decomposing the levels and volatilities of key macrofinancial variables into time varying contributions from sets of shocks. Risk premia shocks are estimated to contribute disproportionately to cyclical output fluctuations and turbulence during swings in financial conditions, across the fifteen largest national economies in the world.

JEL Classification Numbers: C50; E30; E40; E52; E62; F40; G10; G21; G28

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I. INTRODUCTION

In recent decades, the world economy has experienced extended periods of cyclical expansion and tranquility, occasionally disrupted by bouts of cyclical contraction and turbulence. Indeed, the Global Financial Crisis abruptly ended the extended period of cyclical expansion and tranquility known as the Great Moderation, generating cyclical output contractions and financial market turbulence across major advanced and emerging market economies. The Euro Area Sovereign Debt Crisis generated further cyclical output contractions and financial market turbulence in some major advanced economies, while the Taper Tantrum precipitated cyclical contractions and turbulence in some major emerging market economies.

These occasional bouts of cyclical output contraction and financial market turbulence may have a common cause. In a recent paper, Adrian, Boyarchenko and Giannone (2017) find that a tightening of financial conditions is associated with a reduction in the conditional mean and an increase in the conditional variance of output growth in the United States. They argue that these adverse effects on the conditional distribution of output growth are generated by financial amplification mechanisms.

To investigate the sources of macrofinancial fluctuations and turbulence, this paper augments an approximate linear dynamic stochastic general equilibrium (DSGE) model of the world economy with structural shocks exhibiting potentially asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) effects. A refinement of the DSGE model documented in Vitek (2018), this model features a range of nominal and real rigidities, extensive macrofinancial linkages with both bank and capital market based financial intermediation, and diverse spillover transmission channels. Very strong evidence of asymmetric autoregressive conditional heteroskedasticity (ARCH) effects is found, providing a basis for jointly decomposing the levels and volatilities of output and financial conditions into time varying contributions from sets of shocks. Consistent with the finding of Adrian, Boyarchenko and Giannone (2017), risk premia shocks are estimated to contribute disproportionately to cyclical output fluctuations and turbulence during occasional abrupt swings in financial conditions, across the fifteen largest national economies in the world. This phenomenon struck all of the economies most affected by the Global Financial Crisis, the Euro Area Sovereign Debt Crisis, and the Taper Tantrum.

Accounting for the sources of macrofinancial fluctuations or turbulence within a DSGE framework is common. For example, Smets and Wouters (2007) decompose output growth fluctuations in the United States into contributions from various structural shocks using an approximate linear DSGE model. In another influential paper, Justiniano and Primiceri (2008) decompose output growth volatility in the United States into contributions from various structural shocks exhibiting symmetric stochastic volatility (SV) effects using an approximate linear DSGE model. Unlike these and related papers, this paper jointly analyzes the sources of macrofinancial fluctuations and turbulence in the world economy within a DSGE framework. To our knowledge, it is the first to add ARCH effects to a DSGE model. These are simpler to interpret than SV effects, as the conditional variances of the structural shocks are driven by the same innovations as their conditional means.

The organization of this paper is as follows. The next section develops the theoretical framework, while the following section describes the corresponding empirical framework. Estimation of this empirical framework is the subject of section four. Inference on the sources of macrofinancial fluctuations and turbulence is conducted in section five. Finally, section six offers conclusions and recommendations for further research.

II. The Theoretical Framework

Consider a finite set of structurally isomorphic national economies indexed by $i \in \{1,\ldots,N\}$ which constitutes the world economy. Each of these economies consists of households, developers, firms, banks, and a government. The government in turn consists of a monetary authority, a fiscal authority, and a macroprudential authority. Households, developers, firms and banks optimize intertemporally, interacting with governments in an uncertain environment to determine equilibrium prices and quantities under rational expectations in globally integrated output and financial markets. Economy $i*$ issues the quotation currency for transactions in the foreign exchange market.

A. The Household Sector

There exists a continuum of households indexed by $h \in [0,1]$. Households are differentiated according to whether they are credit constrained, and according to how they save if they are credit unconstrained, but are otherwise identical. Credit unconstrained households of type $Z = B$ and measure ϕ^B have access to domestic banks where they accumulate deposits, and to domestic property markets where they trade real estate, where $0 < \phi^B < 1$. In contrast, credit unconstrained households of type $Z = A$ and measure ϕ^A have access to domestic and foreign capital markets where they trade financial assets, where $0 < \phi^A < 1$. Finally, credit constrained households of type $Z = C$ and measure ϕ^C do not have access to banks or capital markets, where $0 < \phi^C < 1$ and $\phi^B + \phi^A + \phi^C = 1$. All households are originally endowed with one share of each domestic developer, firm and bank.

In a reinterpretation of the labor market in the model of nominal wage rigidity proposed by Erceg, Henderson and Levin (2000) to incorporate involuntary unemployment along the lines of Galí (2011), each household consists of a continuum of members represented by the unit square and indexed by $(f,g) \in [0,1]\times[0,1]$. There is full risk sharing among household members, who supply indivisible differentiated intermediate labor services indexed by $f \in [0,1]$, incurring disutility from work determined by $g \in [0,1]$ if they are employed and zero otherwise. Trade specific intermediate labor services supplied by bank intermediated, capital market intermediated, and credit constrained households are perfect substitutes.

Consumption and Saving

The representative infinitely lived household has preferences defined over consumption $C_{h,i,s}$, housing $H_{h,i,s}$, labor supply $\{L_{h,f,i,s}\}_{f=0}^{1}$, real property balances $A_{h,i,s+1}^{B,H}/P_{i,s}^{C}$, and real portfolio balances $A_{h,i,s+1}^{A,H}/P_{i,s}^{C}$ represented by intertemporal utility function

$$U_{h,i,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u\left(C_{h,i,s}, H_{h,i,s}, \{L_{h,f,i,s}\}_{f=0}^{1}, \frac{A_{h,i,s+1}^{B,H}}{P_{i,s}^{C}}, \frac{A_{h,i,s+1}^{A,H}}{P_{i,s}^{C}} \right), \tag{1}$$

where E_t denotes the expectations operator conditional on information available in period t, and $0 < \beta < 1$. The intratemporal utility function is additively separable and represents external habit formation preferences in consumption and labor supply,

$$\begin{aligned} u\left(C_{h,i,s}, H_{h,i,s}, \{L_{h,f,i,s}\}_{f=0}^{1}, \frac{A_{h,i,s+1}^{B,H}}{P_{i,s}^{C}}, \frac{A_{h,i,s+1}^{A,H}}{P_{i,s}^{C}} \right) = \nu_{i,s}^{C} \Bigg[\frac{1}{1-1/\sigma} \left(C_{h,i,s} - \alpha^{C} \frac{C_{i,s-1}^{Z}}{\phi^{Z}} \right)^{1-1/\sigma} + \frac{\upsilon_{i,s}^{H,H}}{1-1/\varsigma} (H_{h,i,s})^{1-1/\varsigma} \\ -\upsilon_{i,s}^{L,H} \int_{0}^{1} \int_{\alpha^{L} \frac{L_{f,i,s-1}^{Z}}{\phi^{Z}}}^{L_{h,f,i,s}} \left(g - \alpha^{L} \frac{L_{f,i,s-1}^{Z}}{\phi^{Z}} \right)^{1/\eta} dg df + \frac{\upsilon_{i,s}^{B,H}}{1-1/\mu} \left(\frac{A_{h,i,s+1}^{B,H}}{P_{i,s}^{C}} \right)^{1-1/\mu} + \frac{\upsilon_{i,s}^{A,H}}{1-1/\mu} \left(\frac{A_{h,i,s+1}^{A,H}}{P_{i,s}^{C}} \right)^{1-1/\mu} \Bigg], \end{aligned} \tag{2}$$

where $0 \le \alpha^{C} < 1$ and $0 \le \alpha^{L} < 1$. Endogenous preference shifters $\upsilon_{i,s}^{H,H}$, $\upsilon_{i,s}^{L,H}$, $\upsilon_{i,s}^{B,H}$ and $\upsilon_{i,s}^{A,H}$ depend on aggregate consumption or employment according to intratemporal subutility functions

$$\upsilon_{i,s}^{H,H} = \nu_{i}^{H,H} \left(\frac{C_{i,s}^{Z}}{\phi^{Z}} - \alpha^{C} \frac{C_{i,s-1}^{Z}}{\phi^{Z}} \right)^{-1/\sigma} (C_{i,s})^{1/\varsigma}, \tag{3}$$

$$\upsilon_{i,s}^{L,H} = \tilde{\mathcal{A}}_{i,s} \left(\frac{C_{i,s}^{Z}}{\phi^{Z}} - \alpha^{C} \frac{C_{i,s-1}^{Z}}{\phi^{Z}} \right)^{-1/\sigma} \left(\frac{L_{i,s} - \alpha^{L} L_{i,s-1}}{(L_{i,s}/\nu_{i,s}^{\mathcal{N}})^{\iota}} \right)^{-1/\eta}, \tag{4}$$

$$\upsilon_{i,s}^{B,H} = \nu_{i}^{B,H} \left(\frac{C_{i,s}^{Z}}{\phi^{Z}} - \alpha^{C} \frac{C_{i,s-1}^{Z}}{\phi^{Z}} \right)^{-1/\sigma} (C_{i,s})^{1/\mu}, \tag{5}$$

$$\upsilon_{i,s}^{A,H} = \nu_{i}^{A,H} \left(\frac{C_{i,s}^{Z}}{\phi^{Z}} - \alpha^{C} \frac{C_{i,s-1}^{Z}}{\phi^{Z}} \right)^{-1/\sigma} (C_{i,s})^{1/\mu}, \tag{6}$$

where $\iota > 0$. The intratemporal utility function is strictly increasing with respect to consumption if and only if serially correlated consumption demand shock $\nu_{i,s}^{C}$ satisfies $\nu_{i,s}^{C} > 0$. Given this parameter restriction, this intratemporal utility function is strictly increasing with respect to housing if and only if $\nu_{i}^{H,H} > 0$, is strictly decreasing with respect to labor supply if and only if serially correlated labor supply shock $\nu_{i,s}^{\mathcal{N}}$ satisfies $\nu_{i,s}^{\mathcal{N}} > 0$, is strictly increasing with respect to real property balances if and only if $\nu_{i}^{B,H} > 0$, and is strictly increasing with respect to real portfolio balances if and only if $\nu_{i}^{A,H} > 0$. Given these parameter restrictions, this intratemporal utility function is strictly concave if $\sigma > 0$, $\varsigma > 0$, $\eta > 0$ and $\mu > 0$. In steady state equilibrium, $\nu_{i}^{B,H}$ equates the marginal rate of substitution between real property balances and consumption to one, while $v_i^{A,H}$ equates the marginal rate of substitution between real portfolio balances and consumption to one.

The representative household enters period s in possession of previously accumulated property balances $A_{h,i,s}^{B,H}$ which yield return $i_{h,i,s}^{A^{B,H}}$, and portfolio balances $A_{h,i,s}^{A,H}$ which yield return $i_{h,i,s}^{A^{A,H}}$. Property balances are distributed across the values of bank deposits $B_{h,i,s}^{D,H}$ which bear interest at deposit rate $i_{i,s-1}^{D}$, and a real estate portfolio $S_{h,i,s}^{H,H}$ which yields return $i_{h,i,s}^{S^{H,H}}$. It follows that $(1+i_{h,i,s}^{A^{B,H}})A_{h,i,s}^{B,H}=(1+i_{i,s-1}^{D})B_{h,i,s}^{D,H}+(1+i_{h,i,s}^{S^{H,H}})S_{h,i,s}^{H,H}$. The value of this real estate portfolio is in turn distributed across the values of developer specific shares $\{V_{i,e,s}^{H}S_{h,i,e,s}^{H,H}\}_{e=0}^{1}$, where $V_{i,e,s}^{H}$ denotes the price per share. It follows that $(1+i_{h,i,s}^{S^{H,H}})S_{h,i,s}^{H,H}=\int_0^1(\Pi_{i,e,s}^{H}+V_{i,e,s}^{H})S_{h,i,e,s}^{H,H}de$, where $\Pi_{i,e,s}^{H}$ denotes the dividend payment per share. Portfolio balances are distributed across the values of internationally diversified short term bond $B_{h,i,s}^{S,H}$, long term bond $B_{h,i,s}^{L,H}$ and stock $S_{h,i,s}^{F,H}$ portfolios which yield returns $i_{h,i,s}^{B^{S,H}}$, $i_{h,i,s}^{B^{L,H}}$ and $i_{h,i,s}^{S^{F,H}}$, respectively. It follows that $(1+i_{h,i,s}^{A^{A,H}})A_{h,i,s}^{A,H}=(1+i_{h,i,s}^{B^{S,H}})B_{h,i,s}^{S,H}+(1+i_{h,i,s}^{B^{L,H}})B_{h,i,s}^{L,H}+(1+i_{h,i,s}^{S^{F,H}})S_{h,i,s}^{F,H}$. The values of these internationally diversified short term bond, long term bond and stock portfolios are in turn distributed across the domestic currency denominated values of economy specific short term bond $\{\mathcal{E}_{i,j,s}B_{h,i,j,s}^{S,H}\}_{j=1}^{N}$, long term bond $\{\mathcal{E}_{i,j,s}B_{h,i,j,s}^{L,H}\}_{j=1}^{N}$ and stock $\{\mathcal{E}_{i,j,s}S_{h,i,j,s}^{F,H}\}_{j=1}^{N}$ portfolios, where nominal bilateral exchange rate $\mathcal{E}_{i,j,s}$ measures the price of foreign currency in terms of domestic currency. It follows that $(1+i_{h,i,s}^{B^{S,H}})B_{h,i,s}^{S,H}=\sum_{j=1}^{N}\mathcal{E}_{i,j,s}(1+i_{j,s-1}^{S})B_{h,i,j,s}^{S,H}$ where $i_{j,s-1}^{S}$ denotes the economy specific yield to maturity on short term bonds, $(1+i_{h,i,s}^{B^{L,H}})B_{h,i,s}^{L,H}=\sum_{j=1}^{N}\mathcal{E}_{i,j,s}(1+i_{h,i,j,s}^{B^{L,H}})B_{h,i,j,s}^{L,H}$ where $i_{h,i,j,s}^{B^{L,H}}$ denotes the economy specific return on long term bonds, and $(1+i_{h,i,s}^{S^{F,H}})S_{h,i,s}^{F,H}=\sum_{j=1}^{N}\mathcal{E}_{i,j,s}(1+i_{h,i,j,s}^{S^{F,H}})S_{h,i,j,s}^{F,H}$ where $i_{h,i,j,s}^{S^{F,H}}$ denotes the economy specific return on stocks. The local currency denominated values of economy specific long term bond portfolios $\{B_{h,i,j,s}^{L,H}\}_{j=1}^{N}$ are in turn distributed across the values of economy and vintage specific long term bonds $\{\{V_{j,k,s}^{B}B_{h,i,j,k,s}^{L,H}\}_{k=1}^{s-1}\}_{j=1}^{N}$, where $V_{j,k,s}^{B}$ denotes the local currency denominated price per long term bond, with $V_{j,k,k}^{B}=1$. It follows that $(1+i_{h,i,j,s}^{B^{L,H}})B_{h,i,j,s}^{L,H}=\sum_{k=1}^{s-1}(\Pi_{j,k,s}^{B}+V_{j,k,s}^{B})B_{h,i,j,k,s}^{L,H}$, where $\Pi_{j,k,s}^{B}=(1+i_{j,k}^{L}-\omega^{B})(\omega^{B})^{s-k}V_{j,k,k}^{B}$ denotes the local currency denominated coupon payment per long term bond, and $i_{j,k}^{L}$ denotes the economy and vintage specific yield to maturity on long term bonds at issuance. In parallel, the local currency denominated values of economy specific stock portfolios $\{S_{h,i,j,s}^{F,H}\}_{j=1}^{N}$ are distributed across the values of economy, industry and firm specific shares $\{\{\{V_{j,k,l,s}^{S}S_{h,i,j,k,l,s}^{F,H}\}_{l=0}^{1}\}_{k=1}^{M}\}_{j=1}^{N}$, where $V_{j,k,l,s}^{S}$ denotes the local currency denominated price per share. It follows that $(1+i_{h,i,j,s}^{S^{F,H}})S_{h,i,j,s}^{F,H}=\sum_{k=1}^{M}\int_0^1(\Pi_{j,k,l,s}^{S}+V_{j,k,l,s}^{S})S_{h,i,j,k,l,s}^{F,H}dl$, where $\Pi_{j,k,l,s}^{S}$ denotes the local currency denominated dividend payment per share. During period s, the representative household receives profit income from banks $\Pi_{i,s}^{C}$, and supplies differentiated intermediate labor services $\{L_{h,f,i,s}\}_{f=0}^{1}$, earning labor income at trade specific nominal wages $\{W_{f,i,s}\}_{f=0}^{1}$. The government levies a tax on household labor income at rate $\tau_{i,s}^{L}$, and remits household type specific lump sum transfer payments $T_{h,i,s}^{Z}$. These sources of wealth are summed in household dynamic budget constraint:

$$A_{h,i,s+1}^{B,H}+A_{h,i,s+1}^{A,H}=(1+i_{h,i,s}^{A^{B,H}})A_{h,i,s}^{B,H}+(1+i_{h,i,s}^{A^{A,H}})A_{h,i,s}^{A,H}+\Pi_{i,s}^{C}+(1-\tau_{i,s}^{L})\int_0^1 W_{f,i,s}L_{h,f,i,s}df+T_{h,i,s}^{Z}-P_{i,s}^{C}C_{h,i,s}-i_{i,s}^{H}H_{h,i,s}. \tag{7}$$

According to this dynamic budget constraint, at the end of period s, the representative household holds property balances $A_{h,i,s+1}^{B,H}$ and portfolio balances $A_{h,i,s+1}^{A,H}$. Property balances are allocated across the values of bank deposits $B_{h,i,s+1}^{D,H}$ and the real estate portfolio $S_{h,i,s+1}^{H,H}$, that is

$A^{B,H}_{h,i,s+1} = B^{D,H}_{h,i,s+1} + S^{H,H}_{h,i,s+1}$. The value of this real estate portfolio is in turn allocated across the values of developer specific shares $\{V^H_{i,e,s} S^{H,H}_{h,i,e,s+1}\}^1_{e=0}$ subject to $S^{H,H}_{h,i,s+1} = \int_0^1 V^H_{i,e,s} S^{H,H}_{h,i,e,s+1} de$. Portfolio balances are allocated across the values of internationally diversified short term bond $B^{S,H}_{h,i,s+1}$, long term bond $B^{L,H}_{h,i,s+1}$ and stock portfolios $S^{F,H}_{h,i,s+1}$, that is $A^{A,H}_{h,i,s+1} = B^{S,H}_{h,i,s+1} + B^{L,H}_{h,i,s+1} + S^{F,H}_{h,i,s+1}$. The values of these internationally diversified short term bond, long term bond and stock portfolios are in turn allocated across the domestic currency denominated values of economy specific short term bond $\{\mathcal{E}_{i,j,s} B^{S,H}_{h,i,j,s+1}\}^N_{j=1}$, long term bond $\{\mathcal{E}_{i,j,s} B^{L,H}_{h,i,j,s+1}\}^N_{j=1}$ and stock $\{\mathcal{E}_{i,j,s} S^{F,H}_{h,i,j,s+1}\}^N_{j=1}$ portfolios subject to $B^{S,H}_{h,i,s+1} = \sum_{j=1}^N \mathcal{E}_{i,j,s} B^{S,H}_{h,i,j,s+1}$, $B^{L,H}_{h,i,s+1} = \sum_{j=1}^N \mathcal{E}_{i,j,s} B^{L,H}_{h,i,j,s+1}$ and $S^{F,H}_{h,i,s+1} = \sum_{j=1}^N \mathcal{E}_{i,j,s} S^{F,H}_{h,i,j,s+1}$, respectively. The local currency denominated values of economy specific long term bond portfolios $\{B^{L,H}_{h,i,j,s+1}\}^N_{j=1}$ are in turn allocated across the local currency denominated values of economy and vintage specific long term bonds $\{\{V^B_{j,k,s} B^{L,H}_{h,i,j,k,s+1}\}^s_{k=1}\}^N_{j=1}$ subject to $B^{L,H}_{h,i,j,s+1} = \sum_{k=1}^s V^B_{j,k,s} B^{L,H}_{h,i,j,k,s+1}$. In parallel, the local currency denominated values of economy specific stock portfolios $\{S^{F,H}_{h,i,j,s+1}\}^N_{j=1}$ are allocated across the local currency denominated values of economy, industry and firm specific shares $\{\{\{V^S_{j,k,l,s} S^{F,H}_{h,i,j,k,l,s+1}\}^1_{l=0}\}^M_{k=1}\}^N_{j=1}$ subject to $S^{F,H}_{h,i,j,s+1} = \sum_{k=1}^M \int_0^1 V^S_{j,k,l,s} S^{F,H}_{h,i,j,k,l,s+1} dl$. Finally, the representative household purchases final private consumption good $C_{h,i,s}$ at price $P^C_{i,s}$, and rents final housing service $H_{h,i,s}$ at price $\iota^H_{i,s}$.

Bank Intermediated Households

The representative bank intermediated household has a capitalist spirit motive for holding real property balances, independent of financing deferred consumption, motivated by Weber (1905). It also has a diversification motive over the allocation of real property balances across alternative assets which are imperfect substitutes, motivated by Tobin (1969). The set of assets under consideration consists of bank deposits and domestically traded real estate. Preferences over the real values of bank deposits $B^{D,H}_{h,i,s+1}/P^C_{i,s}$ and the real estate portfolio $S^{H,H}_{h,i,s+1}/P^C_{i,s}$ are represented by constant elasticity of substitution intratemporal subutility function

$$\frac{A^{B,H}_{h,i,s+1}}{P^C_{i,s}} = \left[(1-\phi^H)^{\frac{1}{\psi^H}} \left(\frac{B^{D,H}_{h,i,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^H-1}{\psi^H}} + (\phi^H)^{\frac{1}{\psi^H}} \left(\frac{1}{\nu^H_{i,s}} \frac{S^{H,H}_{h,i,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^H-1}{\psi^H}}\right]^{\frac{\psi^H}{\psi^H-1}}, \tag{8}$$

where serially correlated housing risk premium shock $\nu^H_{i,s}$ satisfies $\nu^H_{i,s} > 0$, while $0 \leq \phi^H < 1$ and $\psi^H > 0$. Preferences over the real values of developer specific shares $\{V^H_{i,e,s} S^{H,H}_{h,i,e,s+1}/P^C_{i,s}\}^1_{e=0}$ are in turn represented by constant elasticity of substitution intratemporal subutility function:

$$\frac{S^{H,H}_{h,i,s+1}}{P^C_{i,s}} = \left[\int_0^1 \left(\frac{V^H_{i,e,s} S^{H,H}_{h,i,e,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^H-1}{\psi^H}} de\right]^{\frac{\psi^H}{\psi^H-1}}. \tag{9}$$

In the limit as $\nu^{B,H}_i \to 0$ there is no capitalist spirit motive for holding real property balances, while in the limit as $\psi^H \to \infty$ there is no diversification motive over the allocation of real property balances across alternative assets which in this case are perfect substitutes.

In period t, the representative bank intermediated household chooses state contingent sequences for consumption $\{C_{h,i,s}\}_{s=t}^{\infty}$, housing $\{H_{h,i,s}\}_{s=t}^{\infty}$, labor force participation $\{\{N_{h,f,i,s}\}_{f=0}^{1}\}_{s=t}^{\infty}$, property balances $\{A_{h,i,s+1}^{B,H}\}_{s=t}^{\infty}$, bank deposit holdings $\{B_{h,i,s+1}^{D,H}\}_{s=t}^{\infty}$, and real estate holdings $\{\{S_{h,i,e,s+1}^{H,H}\}_{e=0}^{1}\}_{s=t}^{\infty}$ to maximize intertemporal utility function (1) subject to dynamic budget constraint (7), the applicable restrictions on financial asset holdings, and terminal nonnegativity constraints $B_{h,i,T+1}^{D,H} \geq 0$ and $S_{h,i,e,T+1}^{H,H} \geq 0$ for $T \to \infty$. In equilibrium, abstracting from the capitalist spirit motive for holding real property balances, the solutions to this utility maximization problem satisfy intertemporal optimality condition

$$\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C}(1+i_{h,i,t+1}^{A^{B,H}}) = 1, \tag{10}$$

which equates the expected present value of the gross real property return to one. In addition, these solutions satisfy intratemporal optimality condition

$$\frac{u_H(h,i,t)}{u_C(h,i,t)} = \frac{\iota_{i,t}^H}{P_{i,t}^C}, \tag{11}$$

which equates the marginal rate of substitution between housing and consumption to the real rental price of housing. Furthermore, these solutions satisfy intratemporal optimality condition

$$-\frac{u_{L_f}(h,f,i,t)}{u_C(h,i,t)} = (1-\tau_{i,t}^L)\frac{W_{f,i,t}}{P_{i,t}^C}, \tag{12}$$

which equates the marginal rate of substitution between leisure and consumption for the marginal trade specific labor force participant to the corresponding after tax real wage. Abstracting from risk premium shocks, the expected present value of the gross real property return satisfies intratemporal optimality condition

$$\begin{aligned}(1-\phi^H)&\left\{1+\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{u_C(h,i,t)}{u_{A^B}(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C}\left[(1+i_{h,i,t+1}^{A^{B,H}})-(1+i_{i,t}^D)\right]\right\}^{1-\psi^H} \\ &+\phi^H \int_0^1 \left\{1+\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{u_C(h,i,t)}{u_{A^B}(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C}\left[(1+i_{h,i,t+1}^{A^{B,H}})-\frac{\Pi_{i,e,t+1}^H+V_{i,e,t+1}^H}{V_{i,e,t}^H}\right]\right\}^{1-\psi^H} de = 1,\end{aligned} \tag{13}$$

which relates it to the expected present values of the gross real returns on bank deposits and real estate. Finally, abstracting from the portfolio diversification motive over the allocation of real property balances these solutions satisfy intratemporal optimality condition

$$\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C}\left[(1+i_{i,t}^D)-\frac{\Pi_{i,e,t+1}^H+V_{i,e,t+1}^H}{V_{i,e,t}^H}\right] = -\frac{u_{A^B}(h,i,t)}{u_C(h,i,t)}\left(1-\frac{1}{\nu_{i,t}^H}\right), \tag{14}$$

which equates the expected present values of the gross real risk adjusted returns on bank deposits and real estate. Provided that the intertemporal utility function is bounded and strictly concave, together with other optimality conditions, and transversality conditions derived from necessary complementary slackness conditions associated with the terminal nonnegativity constraints, these optimality conditions are sufficient for the unique utility maximizing state contingent sequence of bank intermediated household allocations.

Capital Market Intermediated Households

The representative capital market intermediated household has a capitalist spirit motive for holding real portfolio balances, independent of financing deferred consumption, motivated by Weber (1905). It also has a diversification motive over the allocation of real portfolio balances across alternative financial assets which are imperfect substitutes, motivated by Tobin (1969). The set of financial assets under consideration consists of internationally traded and local currency denominated short term bonds, long term bonds, and stocks. Short term bonds are discount bonds, while long term bonds are perpetual bonds with coupon payments that decay exponentially at rate ω^B where $0<\omega^B<1$, following Woodford (2001). Preferences over the real values of internationally diversified short term bond $B^{S,H}_{h,i,s+1}/P^C_{i,s}$, long term bond $B^{L,H}_{h,i,s+1}/P^C_{i,s}$ and stock $S^{F,H}_{h,i,s+1}/P^C_{i,s}$ portfolios are represented by constant elasticity of substitution intratemporal subutility function

$$\frac{A^{A,H}_{h,i,s+1}}{P^C_{i,s}}=\left[(\phi^A_M)^{\frac{1}{\psi^A}}\left(\frac{B^{S,H}_{h,i,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}+(\phi^A_B)^{\frac{1}{\psi^A}}\left(\frac{1}{\upsilon^B_{i,s}}\frac{B^{L,H}_{h,i,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}+(\phi^A_S)^{\frac{1}{\psi^A}}\left(\frac{1}{\upsilon^S_{i,s}}\frac{S^{F,H}_{h,i,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}\right]^{\frac{\psi^A}{\psi^A-1}}, \quad (15)$$

where internationally and serially correlated duration risk premium shock $\upsilon^B_{i,s}$ satisfies $\upsilon^B_{i,s}>0$, and internationally and serially correlated equity risk premium shock $\upsilon^S_{i,s}$ satisfies $\upsilon^S_{i,s}>0$, while $0\le\phi^A_M\le1$, $0\le\phi^A_B<1$, $0\le\phi^A_S<1$, $\phi^A_M+\phi^A_B+\phi^A_S=1$ and $\psi^A>0$. Preferences over the real values of economy specific short term bond $\{\mathcal{E}_{i,j,s}B^{S,H}_{h,i,j,s+1}/P^C_{i,s}\}^N_{j=1}$, long term bond $\{\mathcal{E}_{i,j,s}B^{L,H}_{h,i,j,s+1}/P^C_{i,s}\}^N_{j=1}$ and stock $\{\mathcal{E}_{i,j,s}S^{F,H}_{h,i,j,s+1}/P^C_{i,s}\}^N_{j=1}$ portfolios are in turn represented by constant elasticity of substitution intratemporal subutility functions

$$\frac{B^{S,H}_{h,i,s+1}}{P^C_{i,s}}=\left[\sum_{j=1}^N(\phi^B_{i,j})^{\frac{1}{\psi^A}}\left(\frac{1}{\nu^{\mathcal{E}}_{j,s}}\frac{\mathcal{E}_{i,j,s}B^{S,H}_{h,i,j,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}\right]^{\frac{\psi^A}{\psi^A-1}}, \quad (16)$$

$$\frac{B^{L,H}_{h,i,s+1}}{P^C_{i,s}}=\left[\sum_{j=1}^N(\phi^B_{i,j})^{\frac{1}{\psi^A}}\left(\frac{1}{\nu^{\mathcal{E}}_{j,s}}\frac{\mathcal{E}_{i,j,s}B^{L,H}_{h,i,j,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}\right]^{\frac{\psi^A}{\psi^A-1}}, \quad (17)$$

$$\frac{S^{F,H}_{h,i,s+1}}{P^C_{i,s}}=\left[\sum_{j=1}^N(\phi^S_{i,j})^{\frac{1}{\psi^A}}\left(\frac{1}{\nu^{\mathcal{E}}_{j,s}}\frac{\mathcal{E}_{i,j,s}S^{F,H}_{h,i,j,s+1}}{P^C_{i,s}}\right)^{\frac{\psi^A-1}{\psi^A}}\right]^{\frac{\psi^A}{\psi^A-1}}, \quad (18)$$

where serially correlated currency risk premium shocks $\nu_{j,s}^{\mathcal{E}}$ satisfy $\nu_{j,s}^{\mathcal{E}} > 0$, while $0 \le \phi_{i,j}^{B} \le 1$, $\sum_{j=1}^{N} \phi_{i,j}^{B} = 1$, $0 \le \phi_{i,j}^{S} \le 1$ and $\sum_{j=1}^{N} \phi_{i,j}^{S} = 1$. Finally, preferences over the real values of economy and vintage specific long term bonds $\{\{\mathcal{E}_{i,j,s} V_{j,k,s}^{B} B_{h,i,j,k,s+1}^{L,H} / P_{i,s}^{C}\}_{k=1}^{s}\}_{j=1}^{N}$ and economy, industry and firm specific shares $\{\{\{\mathcal{E}_{i,j,s} V_{j,k,l,s}^{S} S_{h,i,j,k,l,s+1}^{F,H} / P_{i,s}^{C}\}_{l=0}^{1}\}_{k=1}^{M}\}_{j=1}^{N}$ are represented by constant elasticity of substitution intratemporal subutility functions

$$\frac{\mathcal{E}_{i,j,s} B_{h,i,j,s+1}^{L,H}}{P_{i,s}^{C}} = \left[\sum_{k=1}^{s} (\phi_{i,j,k,s}^{B})^{\frac{1}{\psi^{A}}} \left(\frac{\mathcal{E}_{i,j,s} V_{j,k,s}^{B} B_{h,i,j,k,s+1}^{L,H}}{P_{i,s}^{C}} \right)^{\frac{\psi^{A}-1}{\psi^{A}}} \right]^{\frac{\psi^{A}}{\psi^{A}-1}}, \tag{19}$$

$$\frac{\mathcal{E}_{i,j,s} S_{h,i,j,s+1}^{F,H}}{P_{i,s}^{C}} = \left[\sum_{k=1}^{M} (\phi_{i,j,k}^{S})^{\frac{1}{\psi^{A}}} \int_{0}^{1} \left(\frac{\mathcal{E}_{i,j,s} V_{j,k,l,s}^{S} S_{h,i,j,k,l,s+1}^{F,H}}{P_{i,s}^{C}} \right)^{\frac{\psi^{A}-1}{\psi^{A}}} dl \right]^{\frac{\psi^{A}}{\psi^{A}-1}}, \tag{20}$$

where $0 \le \phi_{i,j,k,s}^{B} \le 1$, $\sum_{k=1}^{s} \phi_{i,j,k,s}^{B} = 1$, $0 \le \phi_{i,j,k}^{S} \le 1$ and $\sum_{k=1}^{M} \phi_{i,j,k}^{S} = 1$. In the limit as $\nu_{i}^{A,H} \to 0$ there is no capitalist spirit motive for holding real portfolio balances, while in the limit as $\psi^{A} \to \infty$ there is no diversification motive over the allocation of real portfolio balances across alternative financial assets which in this case are perfect substitutes.

In period t, the representative capital market intermediated household chooses state contingent sequences for consumption $\{C_{h,i,s}\}_{s=t}^{\infty}$, housing $\{H_{h,i,s}\}_{s=t}^{\infty}$, labor force participation $\{\{N_{h,f,i,s}\}_{f=0}^{1}\}_{s=t}^{\infty}$, portfolio balances $\{A_{h,i,s+1}^{A,H}\}_{s=t}^{\infty}$, short term bond holdings $\{\{B_{h,i,j,s+1}^{S,H}\}_{j=1}^{N}\}_{s=t}^{\infty}$, long term bond holdings $\{\{\{B_{h,i,j,k,s+1}^{L,H}\}_{k=1}^{t}\}_{j=1}^{N}\}_{s=t}^{\infty}$, and stock holdings $\{\{\{\{S_{h,i,j,k,l,s+1}^{H}\}_{l=0}^{1}\}_{k=1}^{M}\}_{j=1}^{N}\}_{s=t}^{\infty}$ to maximize intertemporal utility function (1) subject to dynamic budget constraint (7), the applicable restrictions on financial asset holdings, and terminal nonnegativity constraints $B_{h,i,j,T+1}^{S,H} \ge 0$, $B_{h,i,j,k,T+1}^{L,H} \ge 0$ and $S_{h,i,j,k,l,T+1}^{H} \ge 0$ for $T \to \infty$. In equilibrium, abstracting from the capitalist spirit motive for holding real portfolio balances, the solutions to this utility maximization problem satisfy intertemporal optimality condition

$$\mathrm{E}_{t} \frac{\beta u_{C}(h,i,t+1)}{u_{C}(h,i,t)} \frac{P_{i,t}^{C}}{P_{i,t+1}^{C}} (1 + i_{h,i,t+1}^{A^{A,H}}) = 1, \tag{21}$$

which equates the expected present value of the gross real portfolio return to one. In addition, these solutions satisfy intratemporal optimality condition

$$\frac{u_{H}(h,i,t)}{u_{C}(h,i,t)} = \frac{t_{i,t}^{H}}{P_{i,t}^{C}}, \tag{22}$$

which equates the marginal rate of substitution between housing and consumption to the real rental price of housing. Furthermore, these solutions satisfy intratemporal optimality condition

$$-\frac{u_{L_f}(h,f,i,t)}{u_{C}(h,i,t)} = (1-\tau_{i,t}^{L}) \frac{W_{f,i,t}}{P_{i,t}^{C}}, \tag{23}$$

which equates the marginal rate of substitution between leisure and consumption for the marginal trade specific labor force participant to the corresponding after tax real wage. Abstracting from risk premium shocks, the expected present value of the gross real portfolio return satisfies intratemporal optimality condition

$$\begin{aligned}
&\phi_M^A \sum_{j=1}^{N} \phi_{i,j}^B \left\{1 + \mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{u_C(h,i,t)}{u_{A^A}(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{h,i,t+1}^{A^{A,H}}) - (1+i_{j,t}^S) \frac{\mathcal{E}_{i,j,t+1}}{\mathcal{E}_{i,j,t}} \right] \right\}^{1-\psi^A} \\
&\quad + \phi_B^A \sum_{j=1}^{N} \phi_{i,j}^B \sum_{k=1}^{t} \phi_{i,j,k,t}^B \left\{1 + \mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{u_C(h,i,t)}{u_{A^A}(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{h,i,t+1}^{A^{A,H}}) - \frac{\Pi_{j,k,t+1}^B + V_{j,k,t+1}^B}{V_{j,k,t}^B} \frac{\mathcal{E}_{i,j,t+1}}{\mathcal{E}_{i,j,t}} \right] \right\}^{1-\psi^A} \\
&\qquad + \phi_S^A \sum_{j=1}^{N} \phi_{i,j}^S \sum_{k=1}^{M} \phi_{i,j,k}^S \int_0^1 \left\{1 + \mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{u_C(h,i,t)}{u_{A^A}(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{h,i,t+1}^{A^{A,H}}) - \frac{\Pi_{j,k,l,t+1}^S + V_{j,k,l,t+1}^S}{V_{j,k,l,t}^S} \frac{\mathcal{E}_{i,j,t+1}}{\mathcal{E}_{i,j,t}} \right] \right\}^{1-\psi^A} dl = 1,
\end{aligned} \tag{24}$$

which relates it to the expected present values of the gross real returns on domestic and foreign short term bonds, long term bonds, and stocks. In addition, abstracting from the portfolio diversification motive over the allocation of real portfolio balances these solutions satisfy intratemporal optimality condition

$$\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{i,t}^S) - (1+i_{j,t}^S) \frac{\mathcal{E}_{i,j,t+1}}{\mathcal{E}_{i,j,t}} \right] = -\frac{u_{A^A}(h,i,t)}{u_C(h,i,t)} \left(\frac{1}{\nu_{i,t}^{\mathcal{E}}} - \frac{1}{\nu_{j,t}^{\mathcal{E}}} \right), \tag{25}$$

which equates the expected present values of the gross real risk adjusted returns on domestic and foreign short term bonds. Furthermore, abstracting from the portfolio diversification motive over the allocation of real portfolio balances these solutions satisfy intratemporal optimality condition

$$\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{i,t}^S) - \frac{\Pi_{i,k,t+1}^B + V_{i,k,t+1}^B}{V_{i,k,t}^B} \right] = -\frac{u_{A^A}(h,i,t)}{u_C(h,i,t)} \frac{1}{\nu_{i,t}^{\mathcal{E}}} \left(1 - \frac{1}{\upsilon_{i,t}^B} \right), \tag{26}$$

which equates the expected present values of the gross real risk adjusted returns on domestic short and long term bonds. Finally, abstracting from the portfolio diversification motive over the allocation of real portfolio balances these solutions satisfy intratemporal optimality condition

$$\mathrm{E}_t \frac{\beta u_C(h,i,t+1)}{u_C(h,i,t)} \frac{P_{i,t}^C}{P_{i,t+1}^C} \left[(1+i_{i,t}^S) - \frac{\Pi_{i,k,l,t+1}^S + V_{i,k,l,t+1}^S}{V_{i,k,l,t}^S} \right] = -\frac{u_{A^A}(h,i,t)}{u_C(h,i,t)} \frac{1}{\nu_{i,t}^{\mathcal{E}}} \left(1 - \frac{1}{\upsilon_{i,t}^S} \right), \tag{27}$$

which equates the expected present values of the gross real risk adjusted returns on domestic short term bonds and stocks. Provided that the intertemporal utility function is bounded and strictly concave, together with other optimality conditions, and transversality conditions derived from necessary complementary slackness conditions associated with the terminal nonnegativity constraints, these optimality conditions are sufficient for the unique utility maximizing state contingent sequence of capital market intermediated household allocations.

Credit Constrained Households

In period t, the representative credit constrained household chooses state contingent sequences for consumption $\{C_{h,i,s}\}_{s=t}^{\infty}$, housing $\{H_{h,i,s}\}_{s=t}^{\infty}$, and labor force participation $\{\{N_{h,f,i,s}\}_{f=0}^{1}\}_{s=t}^{\infty}$ to maximize intertemporal utility function (1) subject to dynamic budget constraint (7), and the applicable restrictions on financial asset holdings. In equilibrium, the solutions to this utility maximization problem satisfy household static budget constraint

$$P_{i,t}^{C}C_{h,i,t} + \iota_{i,t}^{H}H_{i,t} = \Pi_{i,t} + (1-\tau_{i,t}^{L})\int_{0}^{1} W_{f,i,t}L_{h,f,i,t}df + T_{h,i,t}^{C}, \tag{28}$$

which equates the sum of consumption and housing expenditures to the sum of profit and disposable labor income net of transfer payments, where profit income $\Pi_{i,t}$ satisfies $\Pi_{i,t} = \Pi_{i,t}^{H} + \Pi_{i,t}^{S} + \Pi_{i,t}^{C}$. The evaluation of this result abstracts from international bank lending. Furthermore, these solutions satisfy intratemporal optimality condition

$$\frac{u_H(h,i,t)}{u_C(h,i,t)} = \frac{\iota_{i,t}^{H}}{P_{i,t}^{C}}, \tag{29}$$

which equates the marginal rate of substitution between housing and consumption to the real rental price of housing. Finally, these solutions satisfy intratemporal optimality condition

$$-\frac{u_{L_f}(h,f,i,t)}{u_C(h,i,t)} = (1-\tau_{i,t}^{L})\frac{W_{f,i,t}}{P_{i,t}^{C}}, \tag{30}$$

which equates the marginal rate of substitution between leisure and consumption for the marginal trade specific labor force participant to the corresponding after tax real wage. Provided that the intertemporal utility function is bounded and strictly concave, these optimality conditions are sufficient for the unique utility maximizing state contingent sequence of credit constrained household allocations.

Labor Supply

The unemployment rate $u_{i,t}^{L}$ measures the share of the labor force $N_{i,t}$ in unemployment $U_{i,t}$, that is $u_{i,t}^{L} = U_{i,t} / N_{i,t}$, where unemployment equals the labor force less employment $L_{i,t}$, that is $U_{i,t} = N_{i,t} - L_{i,t}$. The labor force satisfies $N_{i,t} = \int_{0}^{1} N_{f,i,t}df$.

There exist a large number of perfectly competitive firms which combine differentiated intermediate labor services $L_{f,i,t}$ supplied by trade unions of workers to produce final labor service $L_{i,t}$ according to constant elasticity of substitution production function

$$L_{i,t} = \left[\int_{0}^{1} (L_{f,i,t})^{\frac{\theta_{i,t}^{L}-1}{\theta_{i,t}^{L}}} df\right]^{\frac{\theta_{i,t}^{L}}{\theta_{i,t}^{L}-1}}, \tag{31}$$

where serially uncorrelated wage markup shock $\vartheta_{i,t}^{L}$ satisfies $\vartheta_{i,t}^{L} = \frac{\theta_{i,t}^{L}}{\theta_{i,t}^{L}-1}$ with $\theta_{i,t}^{L} > 1$ and $\theta_{i}^{L} = \theta^{L}$. The representative final labor service firm maximizes profits derived from production of the final labor service with respect to inputs of intermediate labor services, implying demand functions:

$$L_{f,i,t} = \left(\frac{W_{f,i,t}}{W_{i,t}} \right)^{-\theta_{i,t}^L} L_{i,t}. \tag{32}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final labor service firm generates zero profit, implying aggregate wage index:

$$W_{i,t} = \left[\int_0^1 (W_{f,i,t})^{1-\theta_{i,t}^L} df \right]^{\frac{1}{1-\theta_{i,t}^L}}. \tag{33}$$

As the wage elasticity of demand for intermediate labor services $\theta_{i,t}^L$ increases, they become closer substitutes, and individual trade unions have less market power.

In an extension of the model of nominal wage rigidity proposed by Erceg, Henderson and Levin (2000) along the lines of Smets and Wouters (2003), each period a randomly selected fraction $1-\omega^L$ of trade unions adjust their wage optimally, where $0 \leq \omega^L < 1$. The remaining fraction ω^L of trade unions adjust their wage to account for past consumption price inflation and trend productivity growth according to partial indexation rule

$$W_{f,i,t} = \left(\frac{P_{i,t-1}^C \tilde{A}_{i,t-1}}{P_{i,t-2}^C \tilde{A}_{i,t-2}} \right)^{\gamma^L} \left(\frac{\overline{P}_{i,t-1}^C \overline{A}_{i,t-1}}{\overline{P}_{i,t-2}^C \overline{A}_{i,t-2}} \right)^{1-\gamma^L} W_{f,i,t-1}, \tag{34}$$

where $0 \leq \gamma^L \leq 1$. Under this specification, although trade unions adjust their wage every period, they infrequently do so optimally, and the interval between optimal wage adjustments is a random variable.

If the representative trade union can adjust its wage optimally in period t, then it does so to maximize intertemporal utility function (1) subject to dynamic budget constraint (7), intermediate labor service demand function (32), and the assumed form of nominal wage rigidity. Since all trade unions that adjust their wage optimally in period t solve an identical utility maximization problem, in equilibrium they all choose a common wage $W_{i,t}^*$ given by necessary first order condition:

$$\frac{W_{i,t}^*}{W_{i,t}} = -\frac{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(h,i,s)}{u_C(h,i,t)} \theta_{i,s}^L \frac{u_{L_f}(h,f,i,s)}{u_C(h,i,s)} \left[\left(\frac{P_{i,t-1}^C \tilde{A}_{i,t-1}}{P_{i,s-1}^C \tilde{A}_{i,s-1}} \right)^{\gamma^L} \left(\frac{\overline{P}_{i,t-1}^C \overline{A}_{i,t-1}}{\overline{P}_{i,s-1}^C \overline{A}_{i,s-1}} \right)^{1-\gamma^L} \frac{W_{i,s}}{W_{i,t}} \right]^{\theta_{i,s}^L} \left(\frac{W_{i,t}^*}{W_{i,t}} \right)^{-\theta_{i,s}^L} L_{h,i,s}}{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega^L)^{s-t} \frac{\beta^{s-t} u_C(h,i,s)}{u_C(h,i,t)} (\theta_{i,s}^L - 1)(1-\tau_{i,s}^L) \frac{W_{i,s}}{P_{i,s}^C} \left[\left(\frac{P_{i,t-1}^C \tilde{A}_{i,t-1}}{P_{i,s-1}^C \tilde{A}_{i,s-1}} \right)^{\gamma^L} \left(\frac{\overline{P}_{i,t-1}^C \overline{A}_{i,t-1}}{\overline{P}_{i,s-1}^C \overline{A}_{i,s-1}} \right)^{1-\gamma^L} \frac{W_{i,s}}{W_{i,t}} \right]^{\theta_{i,s}^L - 1} \left(\frac{W_{i,t}^*}{W_{i,t}} \right)^{-\theta_{i,s}^L} L_{h,i,s}}. \tag{35}$$

This necessary first order condition equates the expected present value of the marginal utility of consumption gained from labor supply to the expected present value of the marginal utility cost incurred from leisure foregone. Aggregate wage index (33) equals an average of the wage set by the fraction $1-\omega^L$ of trade unions that adjust their wage optimally in period t, and the average of the wages set by the remaining fraction ω^L of trade unions that adjust their wage according to partial indexation rule (34):

$$W_{i,t} = \left\{ (1-\omega^L)(W_{i,t}^*)^{1-\theta_{i,t}^L} + \omega^L \left[\left(\frac{P_{i,t-1}^C \tilde{A}_{i,t-1}}{P_{i,t-2}^C \tilde{A}_{i,t-2}} \right)^{\gamma^L} \left(\frac{\overline{P}_{i,t-1}^C \overline{A}_{i,t-1}}{\overline{P}_{i,t-2}^C \overline{A}_{i,t-2}} \right)^{1-\gamma^L} W_{i,t-1} \right]^{1-\theta_{i,t}^L} \right\}^{\frac{1}{1-\theta_{i,t}^L}}. \quad (36)$$

Since those trade unions able to adjust their wage optimally in period t are selected randomly from among all trade unions, the average wage set by the remaining trade unions equals the value of the aggregate wage index that prevailed during period $t-1$, rescaled to account for past consumption price inflation and trend productivity growth.

B. The Construction Sector

The construction sector supplies housing services to domestic households. In doing so, developers obtain mortgage loans from domestic banks and accumulate the housing stock through residential investment.

Housing Demand

There exist a large number of perfectly competitive developers which combine differentiated intermediate housing services $H_{i,e,t}$ supplied by intermediate developers to produce final housing service $H_{i,t}$ according to constant elasticity of substitution production function

$$H_{i,t} = \left[\int_0^1 (H_{i,e,t})^{\frac{\theta_{i,t}^H - 1}{\theta_{i,t}^H}} de \right]^{\frac{\theta_{i,t}^H}{\theta_{i,t}^H - 1}}, \quad (37)$$

where endogenous rental price of housing markup shifter $\vartheta_{i,t}^H$ satisfies $\vartheta_{i,t}^H = \frac{\theta_{i,t}^H}{\theta_{i,t}^H - 1}$ with $\theta_{i,t}^H > 1$ and $\theta_i^H = \theta^H$. The representative final developer maximizes profits derived from production of the final housing service with respect to inputs of intermediate housing services, implying demand functions:

$$H_{i,e,t} = \left(\frac{\iota_{i,e,t}^H}{\iota_{i,t}^H} \right)^{-\theta_{i,t}^H} H_{i,t}. \quad (38)$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final developer generates zero profit, implying aggregate rental price of housing index:

$$\iota_{i,t}^H = \left[\int_0^1 (\iota_{i,e,t}^H)^{1-\theta_{i,t}^H} de \right]^{\frac{1}{1-\theta_{i,t}^H}}. \quad (39)$$

As the price elasticity of demand for intermediate housing services $\theta_{i,t}^H$ increases, they become closer substitutes, and individual intermediate developers have less market power.

Residential Investment

There exist continuums of monopolistically competitive intermediate developers indexed by $e \in [0,1]$. Intermediate developers supply differentiated intermediate housing services, but are otherwise identical. We rule out entry into and exit out of the monopolistically competitive intermediate construction sector.

The representative intermediate developer sells shares to domestic bank intermediated households at price $V^H_{i,e,t}$. Acting in the interests of its shareholders, it maximizes its pre-dividend stock market value, which abstracting from the capitalist spirit motive for holding real property balances equals the expected present value of current and future dividend payments

$$\Pi^H_{i,e,t} + V^H_{i,e,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda^B_{i,s}}{\lambda^B_{i,t}} \Pi^H_{i,e,s}, \tag{40}$$

where $\lambda^B_{i,s}$ denotes the Lagrange multiplier associated with the period s bank intermediated household dynamic budget constraint. The derivation of this result imposes a transversality condition that rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to profits $\Pi^H_{i,e,s}$, defined as the sum of earnings and net borrowing less residential investment expenditures:

$$\Pi^H_{i,e,s} = \iota^H_{i,e,s} H_{i,e,s} + (B^{C,D}_{i,e,s+1} - (1-\delta^M_{i,s})(1+i^M_{i,s-1}) B^{C,D}_{i,e,s}) - P^{I^H}_{i,s} I^H_{i,e,s}. \tag{41}$$

Earnings are defined as revenues from sales of differentiated intermediate housing service $H_{i,e,s}$ at rental price $\iota^H_{i,e,s}$.

Motivated by the collateralized borrowing variant of the financial accelerator mechanism due to Kiyotaki and Moore (1997), the financial policy of the representative intermediate developer is to maintain debt equal to a fraction of the value of the housing stock,

$$\frac{B^{C,D}_{i,e,s+1}}{P^{I^H}_{i,s} H_{i,e,s+1}} = \phi^D_{i,s}, \tag{42}$$

given by regulatory mortgage loan to value ratio limit $\phi^D_{i,s}$. Net borrowing is defined as the increase in mortgage loans $B^{C,D}_{i,e,s+1}$ from domestic banks net of writedowns at mortgage loan default rate $\delta^M_{i,s}$ and interest payments at mortgage loan rate $i^M_{i,s-1}$.

The representative intermediate developer enters period s in possession of previously accumulated housing stock $H_{i,e,s}$, which subsequently evolves according to accumulation function

$$H_{i,e,s+1} = (1-\delta^H) H_{i,e,s} + \mathcal{H}^H(I^H_{i,e,s}, I^H_{i,e,s-1}), \tag{43}$$

where $0 \le \delta^H \le 1$. Effective residential investment function $\mathcal{H}^H(I^H_{i,e,s}, I^H_{i,e,s-1})$ incorporates convex adjustment costs in the gross growth rate of the ratio of nominal residential investment to aggregate nominal output,

$$\mathcal{H}^{H}(I_{i,e,s}^{H},I_{i,e,s-1}^{H})=v_{i,s}^{I^{H}}\left[1-\frac{\chi^{H}}{2}\left(\frac{P_{i,s}^{I^{H}}I_{i,e,s}^{H}}{P_{i,s-1}^{I^{H}}I_{i,e,s-1}^{H}}\frac{P_{i,s-1}^{Y}Y_{i,s-1}}{P_{i,s}^{Y}Y_{i,s}}-1\right)^{2}\right]I_{i,e,s}^{H}, \tag{44}$$

where serially correlated residential investment demand shock $v_{i,s}^{I^H}$ satisfies $v_{i,s}^{I^H} > 0$, while $\chi^H > 0$. In steady state equilibrium, these adjustment costs equal zero, and effective residential investment equals actual residential investment.

In period t, the representative intermediate developer chooses state contingent sequences for residential investment $\{I_{i,e,s}^H\}_{s=t}^{\infty}$ and the housing stock $\{H_{i,e,s+1}\}_{s=t}^{\infty}$ to maximize pre-dividend stock market value (40) subject to housing accumulation function (43) and terminal nonnegativity constraint $H_{i,e,T+1} \geq 0$ for $T \to \infty$. In equilibrium, demand for the final residential investment good satisfies necessary first order condition

$$Q_{i,e,t}^{H}\mathcal{H}_{1}^{H}(I_{i,e,t}^{H},I_{i,e,t-1}^{H})+\mathrm{E}_{t}\frac{\beta\lambda_{i,t+1}^{B}}{\lambda_{i,t}^{B}}Q_{i,e,t+1}^{H}\mathcal{H}_{2}^{H}(I_{i,e,t+1}^{H},I_{i,e,t}^{H})=P_{i,t}^{I^{H}}, \tag{45}$$

which equates the expected present value of an additional unit of residential investment to its price, where $Q_{i,e,s}^H$ denotes the Lagrange multiplier associated with the period s housing accumulation function. In equilibrium, this shadow price of housing satisfies necessary first order condition

$$Q_{i,e,t}^{H}=\mathrm{E}_{t}\frac{\beta\lambda_{i,t+1}^{B}}{\lambda_{i,t}^{B}}\left\{P_{i,t+1}^{I^{H}}\left\{\frac{\iota_{i,e,t+1}^{H}}{P_{i,t+1}^{I^{H}}}-\phi_{i,t}^{D}\frac{P_{i,t}^{I^{H}}}{P_{i,t+1}^{I^{H}}}\left[(1-\delta_{i,t+1}^{M})(1+i_{i,t}^{M})-\frac{\lambda_{i,t}^{B}}{\beta\lambda_{i,t+1}^{B}}\right]\right\}+(1-\delta^{H})Q_{i,e,t+1}^{H}\right\}, \tag{46}$$

which equates it to the expected present value of the sum of the future marginal revenue product of housing, and the future shadow price of housing net of depreciation, less the product of the loan to value ratio with the spread of the effective cost of bank over capital market funding. Provided that the pre-dividend stock market value is bounded and strictly concave, together with other necessary first order conditions, and a transversality condition derived from the necessary complementary slackness condition associated with the terminal nonnegativity constraint, these necessary first order conditions are sufficient for the unique value maximizing state contingent sequence of intermediate developer allocations.

Housing Supply

In period t, the representative intermediate developer adjusts its rental price of housing to maximize pre-dividend stock market value (40) subject to housing accumulation function (43) and intermediate housing service demand function (38). We consider a symmetric equilibrium under which all developer specific endogenous state variables are restricted to equal their aggregate counterparts. It follows that all intermediate developers solve an identical value maximization problem, which implies that they all choose a common rental price of housing $\iota_{i,t}^{H,*}$ given by necessary first order condition:

$$\frac{\iota_{i,t}^{H,*}}{P_{i,t}^{I^H}} = \frac{\theta_{i,t}^{H}}{\theta_{i,t}^{H} - 1}\left[\phi_{i,t-1}^{D}(1-\delta_{i,t}^{M})(1+i_{i,t-1}^{M})\frac{P_{i,t-1}^{I^H}}{P_{i,t}^{I^H}} - (1-\delta^{H})\frac{Q_{i,e,t}^{H}}{P_{i,t}^{I^H}}\right]. \tag{47}$$

This necessary first order condition equates the marginal revenue gained from housing supply to the marginal cost incurred from construction. Aggregate rental price of housing index (39) satisfies $\iota_{i,t}^{H} = \iota_{i,t}^{H,*}$.

C. The Production Sector

The production sector supplies output goods for domestic and foreign absorption. In doing so, firms demand labor services from domestic households, obtain corporate loans from domestic and foreign banks, and accumulate the private physical capital stock through business investment.

The production sector consists of a finite set of industries indexed by $k \in \{1,\ldots,M\}$, of which the first M^* produce nonrenewable commodities. In particular, the energy commodity industry labeled $k=1$ and the nonenergy commodity industry labeled $k=2$ produce internationally homogeneous output goods for foreign absorption, while all other industries produce internationally heterogeneous output goods for domestic and foreign absorption.

Output Demand

There exist a large number of perfectly competitive firms which combine industry specific final output goods $\{Y_{i,k,t}\}_{k=1}^{M}$ to produce final output good $Y_{i,t}$ according to fixed proportions production function

$$Y_{i,t} = \min\left\{\frac{Y_{i,k,t}}{\phi_{i,k}^{Y}}\right\}_{k=1}^{M}, \tag{48}$$

where $0 \leq \phi_{i,k}^{Y} \leq 1$ and $\sum_{k=1}^{M}\phi_{i,k}^{Y} = 1$. The representative final output good firm maximizes profits derived from production of the final output good with respect to inputs of industry specific final output goods, implying demand functions:

$$Y_{i,k,t} = \phi_{i,k}^{Y} Y_{i,t}. \tag{49}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final output good firm generates zero profit, implying aggregate output price index

$$P_{i,t}^{Y} = \sum_{k=1}^{M}\phi_{i,k}^{Y} P_{i,k,t}^{Y}, \tag{50}$$

where $P_{i,k,t}^{Y} = P_{i,k,t}^{X}$ for $1 \leq k \leq M^*$. This aggregate output price index equals the minimum cost of producing one unit of the final output good, given the prices of industry specific final output goods.

There exist a large number of perfectly competitive firms which combine industry specific differentiated intermediate output goods $Y_{i,k,l,t}$ supplied by industry specific intermediate output good firms to produce industry specific final output good $Y_{i,k,t}$ according to constant elasticity of substitution production function

$$Y_{i,k,t} = \left[\int_0^1 (Y_{i,k,l,t})^{\frac{\theta^Y_{i,k,t}-1}{\theta^Y_{i,k,t}}} dl \right]^{\frac{\theta^Y_{i,k,t}}{\theta^Y_{i,k,t}-1}}, \tag{51}$$

where serially uncorrelated output price markup shock $\vartheta^Y_{i,k,t}$ satisfies $\vartheta^Y_{i,k,t} = \frac{\theta^Y_{i,k,t}}{\theta^Y_{i,k,t}-1}$ with $\theta^Y_{i,k,t} > 1$ and $\theta^Y_{i,k} = \theta^Y$, while $\theta^Y_{i,k,t} = \theta^Y_{k,t}$ for $1 \le k \le M^*$ and $\theta^Y_{i,k,t} = \theta^Y_{i,t}$ otherwise. The representative industry specific final output good firm maximizes profits derived from production of the industry specific final output good with respect to inputs of industry specific intermediate output goods, implying demand functions:

$$Y_{i,k,l,t} = \left(\frac{P^Y_{i,k,l,t}}{P^Y_{i,k,t}} \right)^{-\theta^Y_{i,k,t}} Y_{i,k,t}. \tag{52}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative industry specific final output good firm generates zero profit, implying industry specific aggregate output price index:

$$P^Y_{i,k,t} = \left[\int_0^1 (P^Y_{i,k,l,t})^{1-\theta^Y_{i,k,t}} dl \right]^{\frac{1}{1-\theta^Y_{i,k,t}}}. \tag{53}$$

As the price elasticity of demand for industry specific intermediate output goods $\theta^Y_{i,k,t}$ increases, they become closer substitutes, and individual industry specific intermediate output good firms have less market power.

Labor Demand and Business Investment

There exist continuums of monopolistically competitive industry specific intermediate output good firms indexed by $l \in [0,1]$. Intermediate output good firms supply industry specific differentiated intermediate output goods, but are otherwise identical. We rule out entry into and exit out of the monopolistically competitive industry specific intermediate output good sectors.

The representative industry specific intermediate output good firm sells shares to domestic and foreign capital market intermediated households at price $V^S_{i,k,l,t}$. Acting in the interests of its shareholders, it maximizes its pre-dividend stock market value, which abstracting from the capitalist spirit motive for holding real portfolio balances equals the expected present value of current and future dividend payments

$$\Pi^S_{i,k,l,t} + V^S_{i,k,l,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda^A_{i,s}}{\lambda^A_{i,t}} \Pi^S_{i,k,l,s}, \tag{54}$$

where $\lambda_{i,s}^{\mathcal{A}}$ denotes the Lagrange multiplier associated with the period s capital market intermediated household dynamic budget constraint. The derivation of this result imposes a transversality condition that rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to net profits $\Pi_{i,k,l,s}^{S}$, defined as the sum of after tax corporate earnings and net borrowing less business investment expenditures,

$$\Pi_{i,k,l,s}^{S} = (1-\tau_{i,s}^{K})(P_{i,k,l,s}^{Y}Y_{i,k,l,s} - W_{i,s}L_{i,k,l,s} - \Phi_{i,k,l,s}) + (B_{i,k,l,s+1}^{C,F} - (1-\delta_{i,s}^{C})(1+i_{i,s}^{C,E})B_{i,k,l,s}^{C,F}) - P_{i,s}^{I^K} I_{i,k,l,s}^{K}, \tag{55}$$

where $Y_{i,k,l,s} = \mathcal{F}(u_{i,k,l,s}^{K}K_{i,k,l,s}, \mathcal{A}_{i,s}L_{i,k,l,s})$. Corporate earnings are defined as revenues from sales of industry specific differentiated intermediate output good $Y_{i,k,l,s}$ at price $P_{i,k,l,s}^{Y}$ less expenditures on final labor service $L_{i,k,l,s}$, and other variable costs $\Phi_{i,k,l,s}$. The government levies a tax on corporate earnings at rate $\tau_{i,s}^{K}$.

Motivated by the collateralized borrowing variant of the financial accelerator mechanism due to Kiyotaki and Moore (1997), the financial policy of the representative industry specific intermediate output good firm is to maintain debt equal to a fraction of the value of the private physical capital stock,

$$\frac{B_{i,k,l,s+1}^{C,F}}{P_{i,s}^{I^K} K_{i,k,l,s+1}} = \phi_{i,s}^{F}, \tag{56}$$

given by regulatory corporate loan to value ratio limit $\phi_{i,s}^{F}$. Net borrowing is defined as the increase in corporate loans $B_{i,k,l,s+1}^{C,F}$ from domestic and foreign banks net of writedowns at corporate loan default rate $\delta_{i,s}^{C}$ and interest payments at effective corporate loan rate $i_{i,s}^{C,E}$. This corporate loan default rate applies uniformly to all corporate loans received from domestic and foreign banks.

The representative industry specific intermediate output good firm utilizes private physical capital $K_{i,k,l,s}$ at rate $u_{i,k,l,s}^{K}$ and rents final labor service $L_{i,k,l,s}$ to produce industry specific differentiated intermediate output good $Y_{i,k,l,s}$ according to production function:

$$\mathcal{F}(u_{i,k,l,s}^{K}K_{i,k,l,s}, \mathcal{A}_{i,s}L_{i,k,l,s}) = (u_{i,k,l,s}^{K}K_{i,k,l,s})^{\phi_i^K} (\mathcal{A}_{i,s}L_{i,k,l,s})^{1-\phi_i^K}. \tag{57}$$

This production function exhibits constant returns to scale, with $0 \le \phi_i^K \le 1$. Productivity $\mathcal{A}_{i,s}$ depends on the ratio of the public physical capital stock to the aggregate labor force,

$$\mathcal{A}_{i,s} = (\upsilon_{i,s}^{\mathcal{A}})^{\phi^{\mathcal{A}}} \left(\frac{K_{i,s}^{G}}{N_{i,s}}\right)^{1-\phi^{\mathcal{A}}}, \tag{58}$$

where internationally and serially correlated productivity shock $\upsilon_{i,s}^{\mathcal{A}}$ satisfies $\upsilon_{i,s}^{\mathcal{A}} > 0$, while $0 < \phi^{\mathcal{A}} \le 1$. Trend productivity $\tilde{\mathcal{A}}_{i,s}$ exhibits partial adjustment dynamics $\tilde{\mathcal{A}}_{i,s} = (\tilde{\mathcal{A}}_{i,s-1})^{\rho^{\mathcal{A}}} (\mathcal{A}_{i,s})^{1-\rho^{\mathcal{A}}}$, where $0 \le \rho^{\mathcal{A}} < 1$.

In utilizing private physical capital to produce output, the representative industry specific intermediate output good firm incurs a cost $\mathcal{G}(u_{i,k,l,s}^{K}, K_{i,k,l,s})$ denominated in terms of business investment,

$$\Phi_{i,k,l,s} = P^{I^K}_{i,s}\mathcal{G}(u^K_{i,k,l,s}, K_{i,k,l,s}) + F^F_{i,k,s}, \tag{59}$$

where industry specific fixed cost $F^F_{i,k,s}$ ensures that $\Phi_{i,k,s} = 0$. Following Christiano, Eichenbaum and Evans (2005), this capital utilization cost is increasing in the capital utilization rate at an increasing rate,

$$\mathcal{G}(u^K_{i,k,l,s}, K_{i,k,l,s}) = \mu^K_i\left[e^{\eta^K(u^K_{i,k,l,s}-1)} - 1\right]K_{i,k,l,s}, \tag{60}$$

where $\eta^K > 0$, while $\mu^K_i = \frac{\mu^K}{1-\tau_i}$ with $\mu^K > 0$. In steady state equilibrium, the capital utilization rate equals one, and the cost of utilizing private physical capital equals zero.

The representative industry specific intermediate output good firm enters period s in possession of previously accumulated private physical capital stock $K_{i,k,l,s}$, which subsequently evolves according to accumulation function

$$K_{i,k,l,s+1} = (1-\delta^K)K_{i,k,l,s} + \mathcal{H}(I^K_{i,k,l,s}, I^K_{i,k,l,s-1}), \tag{61}$$

where $0 \le \delta^K \le 1$. Building on Christiano, Eichenbaum and Evans (2005), effective business investment function $\mathcal{H}(I^K_{i,k,l,s}, I^K_{i,k,l,s-1})$ incorporates convex adjustment costs in the gross growth rate of the ratio of nominal business investment to aggregate nominal output,

$$\mathcal{H}(I^K_{i,k,l,s}, I^K_{i,k,l,s-1}) = v^{I^K}_{i,s}\left[1 - \frac{\chi^K}{2}\left(\frac{P^{I^K}_{i,s}I^K_{i,k,l,s}}{P^{I^K}_{i,s-1}I^K_{i,k,l,s-1}}\frac{P^Y_{i,s-1}Y_{i,s-1}}{P^Y_{i,s}Y_{i,s}} - 1\right)^2\right]I^K_{i,k,l,s}, \tag{62}$$

where serially correlated business investment demand shock $v^{I^K}_{i,s}$ satisfies $v^{I^K}_{i,s} > 0$, while $\chi^K > 0$. In steady state equilibrium, these adjustment costs equal zero, and effective business investment equals actual business investment.

In period t, the representative industry specific intermediate output good firm chooses state contingent sequences for employment $\{L_{i,k,l,s}\}^\infty_{s=t}$, the capital utilization rate $\{u^K_{i,k,l,s}\}^\infty_{s=t}$, business investment $\{I^K_{i,k,l,s}\}^\infty_{s=t}$, and the private physical capital stock $\{K_{i,k,l,s+1}\}^\infty_{s=t}$ to maximize pre-dividend stock market value (54) subject to production function (57), private physical capital accumulation function (61), and terminal nonnegativity constraint $K_{i,k,l,T+1} \ge 0$ for $T \to \infty$. In equilibrium, demand for the final labor service satisfies necessary first order condition

$$\mathcal{F}_{\mathcal{A}L}(u^K_{i,k,l,t}K_{i,k,l,t}, \mathcal{A}_{i,t}L_{i,k,l,t})\Psi_{i,k,l,t} = (1-\tau^K_{i,t})\frac{W_{i,t}}{P^Y_{i,k,t}\mathcal{A}_{i,t}}, \tag{63}$$

where $P^Y_{i,k,s}\Psi_{i,k,l,s}$ denotes the Lagrange multiplier associated with the period s production technology constraint. This necessary first order condition equates real marginal cost $\Psi_{i,k,l,t}$ to the ratio of the after tax industry specific real wage to the marginal product of labor. In equilibrium, the capital utilization rate satisfies necessary first order condition

$$\mathcal{F}_{u^K K}(u^K_{i,k,l,t}K_{i,k,l,t}, \mathcal{A}_{i,t}L_{i,k,l,t})\frac{P^Y_{i,k,t}\Psi_{i,k,l,t}}{P^{I^K}_{i,t}} = (1-\tau^K_{i,t})\frac{\mathcal{G}_{u^K}(u^K_{i,k,l,t}, K_{i,k,l,t})}{K_{i,k,l,t}}, \tag{64}$$

which equates the marginal revenue product of utilized private physical capital to its marginal cost. In equilibrium, demand for the final business investment good satisfies necessary first order condition

$$Q_{i,k,l,t}^{K}\mathcal{H}_1(I_{i,k,l,t}^{K},I_{i,k,l,t-1}^{K})+\mathrm{E}_t\frac{\beta\lambda_{i,t+1}^{A}}{\lambda_{i,t}^{A}}Q_{i,k,l,t+1}^{K}\mathcal{H}_2(I_{i,k,l,t+1}^{K},I_{i,k,l,t}^{K})=P_{i,t}^{I^K}, \tag{65}$$

which equates the expected present value of an additional unit of business investment to its price, where $Q_{i,k,l,s}^{K}$ denotes the Lagrange multiplier associated with the period s private physical capital accumulation function. In equilibrium, this shadow price of private physical capital satisfies necessary first order condition

$$\begin{aligned}Q_{i,k,l,t}^{K}=\mathrm{E}_t\frac{\beta\lambda_{i,t+1}^{A}}{\lambda_{i,t}^{A}}\Bigg\{P_{i,t+1}^{I^K}\Bigg\{&u_{i,k,l,t+1}^{K}\mathcal{F}_{u^K K}(u_{i,k,l,t+1}^{K}K_{i,k,l,t+1},\mathcal{A}_{i,t+1}L_{i,k,l,t+1})\frac{P_{i,k,t+1}^{Y}\Psi_{i,k,l,t+1}}{P_{i,t+1}^{I^K}}\\&-(1-\tau_{i,t+1}^{K})\mathcal{G}_K(u_{i,k,l,t+1}^{K},K_{i,k,l,t+1})-\phi_{i,t}^{F}\frac{P_{i,t}^{I^K}}{P_{i,t+1}^{I^K}}\left[(1-\delta_{i,t+1}^{C})(1+i_{i,t+1}^{C,E})-\frac{\lambda_{i,t}^{A}}{\beta\lambda_{i,t+1}^{A}}\right]\Bigg\}+(1-\delta^{K})Q_{i,k,l,t+1}^{K}\Bigg\},\end{aligned} \tag{66}$$

which equates it to the expected present value of the sum of the future marginal revenue product of private physical capital net of its marginal utilization cost, and the future shadow price of private physical capital net of depreciation, less the product of the loan to value ratio with the spread of the effective cost of bank over capital market funding. Provided that the pre-dividend stock market value is bounded and strictly concave, together with other necessary first order conditions, and a transversality condition derived from the necessary complementary slackness condition associated with the terminal nonnegativity constraint, these necessary first order conditions are sufficient for the unique value maximizing state contingent sequence of industry specific intermediate output good firm allocations.

Output Supply

In an extension of the model of nominal output price rigidity proposed by Calvo (1983) along the lines of Smets and Wouters (2003), each period a randomly selected fraction $1-\omega_k^Y$ of industry specific intermediate output good firms adjust their price optimally, where $0\leq\omega_k^Y<1$ with $\omega_k^Y=\omega^Y$ for $k>M^*$. The remaining fraction ω_k^Y of intermediate output good firms adjust their price to account for past industry specific output price inflation according to partial indexation rule

$$P_{i,k,l,t}^{Y}=\left(\frac{P_{i,k,t-1}^{Y}}{P_{i,k,t-2}^{Y}}\right)^{\gamma_k^Y}\left(\frac{\overline{P}_{i,k,t-1}^{Y}}{\overline{P}_{i,k,t-2}^{Y}}\right)^{1-\gamma_k^Y}P_{i,k,l,t-1}^{Y}, \tag{67}$$

where $0\leq\gamma_k^Y\leq 1$ with $\gamma_k^Y=0$ for $1\leq k\leq M^*$ and $\gamma_k^Y=\gamma^Y$ otherwise. Under this specification, optimal price adjustment opportunities arrive randomly, and the interval between optimal price adjustments is a random variable.

If the representative industry specific intermediate output good firm can adjust its price optimally in period t, then it does so to maximize pre-dividend stock market value (54) subject to production function (57), industry specific intermediate output good demand function (52), and the assumed form of nominal output price rigidity. We consider a symmetric equilibrium under which all industry and firm specific endogenous state variables are restricted to equal their industry specific aggregate counterparts. It follows that all intermediate output good firms that adjust their price optimally in period t solve an identical value maximization problem, which implies that they all choose a common price $P_{i,k,t}^{Y,*}$ given by necessary first order condition:

$$\frac{P_{i,k,t}^{Y,*}}{P_{i,k,t}^{Y}} = \frac{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega_k^Y)^{s-t} \frac{\beta^{s-t}\lambda_{i,s}^A}{\lambda_{i,t}^A} \theta_{i,k,s}^Y \Psi_{i,k,l,s} \left[\left(\frac{P_{i,k,t-1}^Y}{P_{i,k,s-1}^Y} \right)^{\gamma_k^Y} \left(\frac{\bar{P}_{i,k,t-1}^Y}{\bar{P}_{i,k,s-1}^Y} \right)^{1-\gamma_k^Y} \frac{P_{i,k,s}^Y}{P_{i,k,t}^Y} \right]^{\theta_{i,k,s}^Y} \left(\frac{P_{i,k,t}^{Y,*}}{P_{i,k,t}^Y} \right)^{-\theta_{i,k,s}^Y} P_{i,k,s}^Y Y_{i,k,s}}{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega_k^Y)^{s-t} \frac{\beta^{s-t}\lambda_{i,s}^A}{\lambda_{i,t}^A} (\theta_{i,k,s}^Y - 1)(1-\tau_{i,s}^K) \left[\left(\frac{P_{i,k,t-1}^Y}{P_{i,k,s-1}^Y} \right)^{\gamma_k^Y} \left(\frac{\bar{P}_{i,k,t-1}^Y}{\bar{P}_{i,k,s-1}^Y} \right)^{1-\gamma_k^Y} \frac{P_{i,k,s}^Y}{P_{i,k,t}^Y} \right]^{\theta_{i,k,s}^Y - 1} \left(\frac{P_{i,k,t}^{Y,*}}{P_{i,k,t}^Y} \right)^{-\theta_{i,k,s}^Y} P_{i,k,s}^Y Y_{i,k,s}}. \quad (68)$$

This necessary first order condition equates the expected present value of the after tax marginal revenue gained from output supply to the expected present value of the marginal cost incurred from production. Aggregate output price index (53) equals an average of the price set by the fraction $1-\omega_k^Y$ of intermediate output good firms that adjust their price optimally in period t, and the average of the prices set by the remaining fraction ω_k^Y of intermediate output good firms that adjust their price according to partial indexation rule (67):

$$P_{i,k,t}^Y = \left\{ (1-\omega_k^Y)(P_{i,k,t}^{Y,*})^{1-\theta_{i,k,t}^Y} + \omega_k^Y \left[\left(\frac{P_{i,k,t-1}^Y}{P_{i,k,t-2}^Y} \right)^{\gamma_k^Y} \left(\frac{\bar{P}_{i,k,t-1}^Y}{\bar{P}_{i,k,t-2}^Y} \right)^{1-\gamma_k^Y} P_{i,k,t-1}^Y \right]^{1-\theta_{i,k,t}^Y} \right\}^{\frac{1}{1-\theta_{i,k,t}^Y}}. \quad (69)$$

Since those intermediate output good firms able to adjust their price optimally in period t are selected randomly from among all intermediate output good firms, the average price set by the remaining intermediate output good firms equals the value of the industry specific aggregate output price index that prevailed during period $t-1$, rescaled to account for past industry specific output price inflation.

D. The Banking Sector

The banking sector supplies global financial intermediation services subject to financial frictions and regulatory constraints. In particular, banks issue risky mortgage loans to domestic developers at infrequently adjusted predetermined mortgage loan rates, as well as risky domestic currency denominated corporate loans to domestic and foreign firms at infrequently adjusted predetermined corporate loan rates, given regulatory loan to value ratio limits. They obtain funding from domestic bank intermediated households via deposits and from the domestic interbank market via loans, accumulating bank capital out of retained earnings given credit losses to satisfy a regulatory capital requirement.

Credit Demand

There exist a large number of perfectly competitive banks which combine economy specific local currency denominated final corporate loans $\{B_{i,j,t}^{C,F}\}_{j=1}^{N}$ to produce domestic currency denominated final corporate loan $B_{i,t}^{C,F}$ according to fixed proportions portfolio aggregator

$$B_{i,t}^{C,F} = \min\left\{\frac{\mathcal{E}_{i,j,t-1}B_{i,j,t}^{C,F}}{\phi_{i,j}^{F}}\right\}_{j=1}^{N}, \tag{70}$$

where $0 \leq \phi_{i,j}^{F} \leq 1$ and $\sum_{j=1}^{N}\phi_{i,j}^{F} = 1$. The representative global final bank maximizes profits derived from intermediation of the domestic currency denominated final corporate loan with respect to inputs of economy specific local currency denominated final corporate loans, implying demand functions:

$$B_{i,j,t}^{C,F} = \phi_{i,j}^{F}\frac{B_{i,t}^{C,F}}{\mathcal{E}_{i,j,t-1}}. \tag{71}$$

Since the portfolio aggregator exhibits constant returns to scale, in equilibrium the representative global final bank generates zero profit, implying aggregate effective gross corporate loan rate index:

$$1+i_{i,t}^{C,E} = \sum_{j=1}^{N}\phi_{i,j}^{F}(1+i_{j,t-1}^{C})\frac{\mathcal{E}_{i,j,t}}{\mathcal{E}_{i,j,t-1}}. \tag{72}$$

This aggregate effective gross corporate loan rate index equals the minimum cost of producing one unit of the domestic currency denominated final corporate loan, given the rates on economy specific local currency denominated final corporate loans.

There exist a large number of perfectly competitive banks which combine differentiated intermediate mortgage or corporate loans $B_{i,m,t+1}^{C^Z,B}$ supplied by intermediate banks to produce final mortgage or corporate loan $B_{i,t+1}^{C^Z,B}$ according to constant elasticity of substitution portfolio aggregator

$$B_{i,t+1}^{C^Z,B} = \left[\int_0^1 (B_{i,m,t+1}^{C^Z,B})^{\frac{\theta_{i,t+1}^{C^Z}-1}{\theta_{i,t+1}^{C^Z}}} dm\right]^{\frac{\theta_{i,t+1}^{C^Z}}{\theta_{i,t+1}^{C^Z}-1}}, \tag{73}$$

where $Z \in \{D,F\}$, while serially uncorrelated mortgage or corporate loan rate markup shock $\vartheta_{i,t+1}^{C^Z}$ satisfies $\vartheta_{i,t+1}^{C^Z} = \frac{\theta_{i,t+1}^{C^Z}}{\theta_{i,t+1}^{C^Z}-1}$ with $\theta_{i,t+1}^{C^Z} > 1$ and $\theta_i^{C^Z} = \theta^C$. The representative domestic final bank maximizes profits derived from intermediation of the final mortgage or corporate loan with respect to inputs of intermediate mortgage or corporate loans, implying demand functions

$$B_{i,m,t+1}^{C^Z,B} = \left(\frac{1+i_{i,m,t}^{f(Z)}}{1+i_{i,t}^{f(Z)}}\right)^{-\theta_{i,t+1}^{C^Z}} B_{i,t+1}^{C^Z,B}, \tag{74}$$

where $f(D)=M$ and $f(F)=C$. Since the portfolio aggregator exhibits constant returns to scale, in equilibrium the representative domestic final bank generates zero profit, implying aggregate gross mortgage or corporate loan rate index:

$$1+i_{i,t}^{f(Z)}=\left[\int_0^1 (1+i_{i,m,t}^{f(Z)})^{1-\theta_{i,t+1}^{C^Z}} dm\right]^{\frac{1}{1-\theta_{i,t+1}^{C^Z}}}. \tag{75}$$

As the rate elasticity of demand for intermediate mortgage or corporate loans $\theta_{i,t+1}^{C^Z}$ increases, they become closer substitutes, and individual intermediate banks have less market power.

Funding Demand and Bank Capital Accumulation

There exists a continuum of monopolistically competitive intermediate banks indexed by $m\in[0,1]$. Intermediate banks supply differentiated intermediate mortgage and corporate loans, but are otherwise identical. We rule out entry into and exit out of the monopolistically competitive intermediate banking sector.

The representative intermediate bank sells shares to domestic bank intermediated households at price $V_{i,m,t}^{C}$. Acting in the interests of its shareholders, it maximizes its pre-dividend stock market value, which equals the expected present value of current and future dividend payments:

$$\varPi_{i,m,t}^{C}+V_{i,m,t}^{C}=\mathrm{E}_t\sum_{s=t}^{\infty}\frac{\beta^{s-t}\lambda_{i,s}^{B}}{\lambda_{i,t}^{B}}\varPi_{i,m,s}^{C}. \tag{76}$$

The derivation of this result imposes a transversality condition that rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments $\varPi_{i,m,s}^{C}$, defined as profits derived from providing financial intermediation services less retained earnings $I_{i,m,s}^{B}$:

$$\begin{aligned}\varPi_{i,m,s}^{C}&=(B_{i,m,s+1}^{D,B}-(1+i_{i,s-1}^{D})B_{i,m,s}^{D,B})+(B_{i,m,s+1}^{B,B}-(1+i_{i,s-1}^{B})B_{i,m,s}^{B,B})\\&\quad-(B_{i,m,s+1}^{C^D,B}-(1-\delta_{i,s}^{M})(1+i_{i,m,s-1}^{M})B_{i,m,s}^{C^D,B})-(B_{i,m,s+1}^{C^F,B}-(1-\delta_{i,s}^{C,E})(1+i_{i,m,s-1}^{C})B_{i,m,s}^{C^F,B})-\varPhi_{i,m,s}^{B}-I_{i,m,s}^{B}.\end{aligned} \tag{77}$$

Profits are defined as the sum of the increase in deposits $B_{i,m,s+1}^{D,B}$ from domestic bank intermediated households net of interest payments at the deposit rate and the increase in net loans $B_{i,m,s+1}^{B,B}$ from the domestic interbank market net of interest payments at the interbank loans rate $i_{i,s-1}^{B}$, less the increase in differentiated intermediate mortgage loans $B_{i,m,s+1}^{C^D,B}$ to domestic developers net of writedowns at mortgage credit loss rate $\delta_{i,s}^{M}$ and interest receipts at mortgage loan rate $i_{i,m,s-1}^{M}$, less the increase in differentiated intermediate corporate loans $B_{i,m,s+1}^{C^F,B}$ to domestic and foreign firms net of writedowns at corporate credit loss rate $\delta_{i,s}^{C,E}$ and interest receipts at corporate loan rate $i_{i,m,s-1}^{C}$, less a cost of satisfying the regulatory capital requirement $\varPhi_{i,m,s}^{B}$.

The representative intermediate bank transforms deposit and money market funding into risky differentiated intermediate mortgage and corporate loans according to balance sheet identity:

$$B_{i,m,s+1}^{C^D,B} + B_{i,m,s+1}^{C^F,B} = B_{i,m,s+1}^{D,B} + B_{i,m,s+1}^{B,B} + K_{i,m,s+1}^{B}. \tag{78}$$

The bank credit stock $B_{i,s+1}^{C,B}$ measures aggregate bank assets, that is $B_{i,s+1}^{C,B} = B_{i,s+1}^{C^D,B} + B_{i,s+1}^{C^F,B}$, while the money stock $M_{i,s+1}^{S}$ measures aggregate bank funding, that is $M_{i,s+1}^{S} = B_{i,s+1}^{D,B} + B_{i,s+1}^{B,B}$ where $B_{i,s+1}^{B,B} = 0$. The bank capital ratio $\kappa_{i,s+1}$ equals the ratio of aggregate bank capital to assets, that is $\kappa_{i,s+1} = K_{i,s+1}^{B} / B_{i,s+1}^{C,B}$.

In transforming deposit and money market funding into risky mortgage and corporate loans, the representative intermediate bank incurs a cost of satisfying the regulatory capital requirement,

$$\Phi_{i,m,s}^{B} = \mathcal{G}^{B}(B_{i,m,s}^{C^D,B}, B_{i,m,s}^{C^F,B}, K_{i,m,s}^{B}) + F_{i,s}^{B}, \tag{79}$$

where fixed cost $F_{i,s}^{B}$ ensures that $\Phi_{i,s}^{B} = -I_{i,s}^{B}$. Motivated by Gerali, Neri, Sessa and Signoretti (2010), this regulation cost is decreasing in the ratio of bank capital to assets at a decreasing rate,

$$\mathcal{G}^{B}(B_{i,m,s}^{C^D,B}, B_{i,m,s}^{C^F,B}, K_{i,m,s}^{B}) = \mu^{C}\left[e^{(2+\eta^{C})\left(1-\frac{1}{\kappa_{i,s}^{R}}\frac{K_{i,m,s}^{B}}{B_{i,m,s}^{C^D,B}+B_{i,m,s}^{C^F,B}}\right)} - 1 \right] K_{i,m,s}^{B}, \tag{80}$$

given regulatory capital requirement $\kappa_{i,s}^{R}$, where $\eta^{C} > 0$ and $\mu^{C} > 0$. In steady state equilibrium, the bank capital ratio equals its required value, and the cost of regulation is constant.

The financial policy of the representative intermediate bank is to smooth retained earnings intertemporally, given credit losses. It enters period s in possession of previously accumulated bank capital stock $K_{i,m,s}^{B}$, which subsequently evolves according to accumulation function

$$K_{i,m,s+1}^{B} = (1-\delta_{i,s}^{B})K_{i,m,s}^{B} + \mathcal{H}^{B}(I_{i,m,s}^{B}, I_{i,m,s-1}^{B}), \tag{81}$$

where bank capital destruction rate $\delta_{i,s}^{B}$ satisfies $\delta_{i,s}^{B} = \chi^{C}(w_{i}^{C}\delta_{i,s}^{M} + (1-w_{i}^{C})\delta_{i,s}^{C,E})$ with $\chi^{C} > 0$, while mortgage loan weight w_{i}^{C} satisfies $0 < w_{i}^{C} < 1$. Effective retained earnings function $\mathcal{H}^{B}(I_{i,m,s}^{B}, I_{i,m,s-1}^{B})$ incorporates convex adjustment costs,

$$\mathcal{H}^{B}(I_{i,m,s}^{B}, I_{i,m,s-1}^{B}) = \left[1 - \frac{\chi^{B}}{2}\left(\frac{I_{i,m,s}^{B}}{I_{i,m,s-1}^{B}} - 1\right)^{2}\right] I_{i,m,s}^{B}, \tag{82}$$

where $\chi^{B} > 0$. In steady state equilibrium, these adjustment costs equal zero, and effective retained earnings equals actual retained earnings.

In period t, the representative intermediate bank chooses state contingent sequences for deposit funding $\{B_{i,m,s+1}^{D,B}\}_{s=t}^{\infty}$, net interbank market funding $\{B_{i,m,s+1}^{B,B}\}_{s=t}^{\infty}$, retained earnings $\{I_{i,m,s}^{B}\}_{s=t}^{\infty}$, and the bank capital stock $\{K_{i,m,s+1}^{B}\}_{s=t}^{\infty}$ to maximize pre-dividend stock market value (76) subject to balance sheet identity (78), bank capital accumulation function (81), and terminal nonnegativity constraints $B_{i,m,T+1}^{D,B} \geq 0$, $B_{i,m,T+1}^{B,B} \geq 0$ and $K_{i,m,T+1}^{B} \geq 0$ for $T \to \infty$. In equilibrium, the solutions to this value maximization problem satisfy necessary first order condition

$$1 + i_{i,t}^{D} = 1 + i_{i,t}^{B}, \tag{83}$$

which equates the deposit rate to the interbank loans rate. In equilibrium, retained earnings satisfies necessary first order condition

$$Q_{i,m,t}^{B}\mathcal{H}_{1}^{B}(I_{i,m,t}^{B},I_{i,m,t-1}^{B})+\mathrm{E}_{t}\frac{\beta\lambda_{i,t+1}^{B}}{\lambda_{i,t}^{B}}Q_{i,m,t+1}^{B}\mathcal{H}_{2}^{B}(I_{i,m,t+1}^{B},I_{i,m,t}^{B})=1, \tag{84}$$

which equates the expected present value of an additional unit of retained earnings to its marginal cost, where $Q_{i,m,s}^{B}$ denotes the Lagrange multiplier associated with the period s bank capital accumulation function. In equilibrium, this shadow price of bank capital satisfies necessary first order condition

$$Q_{i,m,t}^{B}=\mathrm{E}_{t}\frac{\beta\lambda_{i,t+1}^{B}}{\lambda_{i,t}^{B}}\left\{(1-\delta_{i,t+1}^{B})Q_{i,m,t+1}^{B}-\left\{\mathcal{G}_{3}^{B}(B_{i,m,t+1}^{C^{D},B},B_{i,m,t+1}^{C^{F},B},K_{i,m,t+1}^{B})+\left[\frac{\lambda_{i,t}^{B}}{\beta\lambda_{i,t+1}^{B}}-(1+i_{i,t}^{B})\right]\right\}\right\}, \tag{85}$$

which equates it to the expected present value of the future shadow price of bank capital net of destruction, less the sum of the marginal utilization cost of bank capital and the spread of the cost of deposit over interbank market funding. The evaluation of this result abstracts from risk premium shocks. Provided that the pre-dividend stock market value is bounded and strictly concave, together with other necessary first order conditions, and transversality conditions derived from the necessary complementary slackness conditions associated with the terminal nonnegativity constraints, these necessary first order conditions are sufficient for the unique value maximizing state contingent sequence of intermediate bank allocations.

Credit Supply

In an adaptation of the model of nominal output price rigidity proposed by Calvo (1983) to the banking sector along the lines of Hülsewig, Mayer and Wollmershäuser (2009), each period a randomly selected fraction $1-\omega^{C}$ of intermediate banks adjust their gross mortgage and corporate loan rates optimally, where $0\leq\omega^{C}<1$. The remaining fraction ω^{C} of intermediate banks do not adjust their loan rates,

$$1+i_{i,m,t}^{f(Z)}=1+i_{i,m,t-1}^{f(Z)}, \tag{86}$$

where $Z\in\{D,F\}$, while $f(D)=M$ and $f(F)=C$. Under this financial friction, intermediate banks infrequently adjust their loan rates, mimicking the effect of maturity transformation on the spreads between the loan and deposit rates.

If the representative intermediate bank can adjust its gross mortgage and corporate loan rates in period t, then it does so to maximize pre-dividend stock market value (76) subject to balance sheet identity (78), intermediate loan demand function (74), and the assumed financial friction. We consider a symmetric equilibrium under which all bank specific endogenous state variables are restricted to equal their aggregate counterparts. It follows that all intermediate banks that adjust their loan rates in period t solve an identical value maximization problem, which implies that they all choose common loan rates $i_{i,t}^{f(Z),*}$ given by necessary first order conditions

$$\frac{1+i_{i,t}^{f(Z),*}}{1+i_{i,t}^{f(Z)}}=\frac{\mathrm{E}_t\sum_{s=t}^{\infty}(\omega^C)^{s-t}\frac{\beta^{s-t}\lambda_{i,s}^B}{\lambda_{i,t}^B}\theta_{i,s}^{C^Z}\frac{(1+i_{i,s-1}^B)+\mathcal{G}_{h(Z)}^B(B_{i,m,s}^{C^D,B},B_{i,m,s}^{C^F,B},K_{i,m,s}^B)}{1+i_{i,s-1}^{f(Z)}}\left(\frac{1+i_{i,s-1}^{f(Z)}}{1+i_{i,t}^{f(Z)}}\right)^{\theta_{i,s}^{C^Z}}\left(\frac{1+i_{i,t}^{f(Z),*}}{1+i_{i,t}^{f(Z)}}\right)^{-\theta_{i,s}^{C^Z}}(1+i_{i,s-1}^{f(Z)})B_{i,s}^{C^Z,B}}{\mathrm{E}_t\sum_{s=t}^{\infty}(\omega^C)^{s-t}\frac{\beta^{s-t}\lambda_{i,s}^B}{\lambda_{i,t}^B}(\theta_{i,s}^{C^Z}-1)(1-\delta_{i,s}^{g(Z)})\left(\frac{1+i_{i,s-1}^{f(Z)}}{1+i_{i,t}^{f(Z)}}\right)^{\theta_{i,s}^{C^Z}-1}\left(\frac{1+i_{i,t}^{f(Z),*}}{1+i_{i,t}^{f(Z)}}\right)^{-\theta_{i,s}^{C^Z}}(1+i_{i,s-1}^{f(Z)})B_{i,s}^{C^Z,B}}, \tag{87}$$

where $g(D)=M$ and $g(F)=C,E$, while $h(D)=1$ and $h(F)=2$. These necessary first order conditions equate the expected present value of the marginal revenue gained from mortgage or corporate loan supply to the expected present value of the marginal cost incurred from intermediation. Aggregate gross mortgage or corporate loan rate index (75) equals an average of the gross mortgage or corporate loan rate set by the fraction $1-\omega^C$ of intermediate banks that adjust their loan rates in period t, and the average of the gross mortgage or corporate loan rates set by the remaining fraction ω^C of intermediate banks that do not adjust their loan rates:

$$1+i_{i,t}^{f(Z)}=\left[(1-\omega^C)(1+i_{i,t}^{f(Z),*})^{1-\theta_{i,t+1}^{C^Z}}+\omega^C(1+i_{i,t-1}^{f(Z)})^{1-\theta_{i,t+1}^{C^Z}}\right]^{\frac{1}{1-\theta_{i,t+1}^{C^Z}}}. \tag{88}$$

Since those intermediate banks able to adjust their loan rates in period t are selected randomly from among all intermediate banks, the average gross mortgage or corporate loan rate set by the remaining intermediate banks equals the value of the aggregate gross mortgage or corporate loan rate index that prevailed during period $t-1$.

E. The Trade Sector

The nominal effective exchange rate $\mathcal{E}_{i,t}$ measures the trade weighted average price of foreign currency in terms of domestic currency, while the real effective exchange rate $\mathcal{Q}_{i,t}$ measures the trade weighted average price of foreign output in terms of domestic output,

$$\mathcal{E}_{i,t}=\prod_{j=1}^{N}(\mathcal{E}_{i,j,t})^{w_{i,j}^T},\ \mathcal{Q}_{i,t}=\prod_{j=1}^{N}(\mathcal{Q}_{i,j,t})^{w_{i,j}^T}, \tag{89}$$

where the real bilateral exchange rate $\mathcal{Q}_{i,j,t}$ satisfies $\mathcal{Q}_{i,j,t}=\mathcal{E}_{i,j,t}P_{j,t}^Y/P_{i,t}^Y$, and bilateral trade weight $w_{i,j}^T$ satisfies $w_{i,i}^T=0$, $0\le w_{i,j}^T\le 1$ and $\sum_{j=1}^{N}w_{i,j}^T=1$. Furthermore, the terms of trade $\mathcal{T}_{i,t}$ equals the ratio of the internal terms of trade to the external terms of trade,

$$\mathcal{T}_{i,t}=\upsilon_t^T\frac{\mathcal{T}_{i,t}^X}{\mathcal{T}_{i,t}^M},\ \mathcal{T}_{i,t}^X=\frac{P_{i,t}^X}{P_{i,t}},\ \mathcal{T}_{i,t}^M=\frac{P_{i,t}^M}{P_{i,t}}, \tag{90}$$

where the internal terms of trade $\mathcal{T}_{i,t}^X$ measures the relative price of exports, and the external terms of trade $\mathcal{T}_{i,t}^M$ measures the relative price of imports, while $P_{i,t}$ denotes the price of the final noncommodity output good. Finally, under the law of one price for $1\le k\le M^*$,

$$P_{k,t}^Y=\sum_{i=1}^{N}w_i^Y\mathcal{E}_{i*,i,t}P_{i,k,t}^Y, \tag{91}$$

where $P_{k,t}^Y$ denotes the quotation currency denominated price of energy or nonenergy commodities, and world output weight w_i^Y satisfies $0<w_i^Y<1$ and $\sum_{i=1}^{N}w_i^Y=1$. Endogenous global terms of trade shifter $\upsilon_t^{\mathcal{T}}$ adjusts to ensure multilateral consistency in nominal trade flows, and in steady state equilibrium satisfies $\upsilon^{\mathcal{T}} = 1$.

The Export Sector

There exist a large number of perfectly competitive firms which combine industry specific final export goods $\{X_{i,k,t}\}_{k=1}^{M}$ to produce final export good $X_{i,t}$ according to fixed proportions production function

$$X_{i,t} = \min\left\{\frac{X_{i,k,t}}{\phi_{i,k}^{X}}\right\}_{k=1}^{M}, \tag{92}$$

where $X_{i,k,t} = Y_{i,k,t}$ for $1 \leq k \leq M^*$, while $0 \leq \phi_{i,k}^{X} \leq 1$ and $\sum_{k=1}^{M} \phi_{i,k}^{X} = 1$. The representative final export good firm maximizes profits derived from production of the final export good with respect to inputs of industry specific final export goods, implying demand functions:

$$X_{i,k,t} = \phi_{i,k}^{X} X_{i,t}. \tag{93}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final export good firm generates zero profit, implying aggregate export price index

$$P_{i,t}^{X} = \sum_{k=1}^{M} \phi_{i,k}^{X} P_{i,k,t}^{X}, \tag{94}$$

where $P_{i,k,t}^{X} = P_{i,k,t}^{Y}$ for $k > M^*$. This aggregate export price index equals the minimum cost of producing one unit of the final export good, given the prices of industry specific final export goods.

Export Demand

There exist a large number of perfectly competitive firms which combine industry specific differentiated intermediate export goods $X_{i,k,n,t}$ supplied by industry specific intermediate export good firms to produce industry specific final export good $X_{i,k,t}$ according to constant elasticity of substitution production function

$$X_{i,k,t} = \left[\int_0^1 (X_{i,k,n,t})^{\frac{\theta_{i,t}^{X}-1}{\theta_{i,t}^{X}}} dn\right]^{\frac{\theta_{i,t}^{X}}{\theta_{i,t}^{X}-1}}, \tag{95}$$

for $1 \leq k \leq M^*$, where serially uncorrelated export price markup shock $\vartheta_{i,t}^{X}$ satisfies $\vartheta_{i,t}^{X} = \frac{\theta_{i,t}^{X}}{\theta_{i,t}^{X}-1}$ with $\theta_i^{X} > 1$ and $\theta_i^{X} = \theta^{X}$. The representative industry specific final export good firm maximizes profits derived from production of the industry specific final export good with respect to inputs of industry specific intermediate export goods, implying demand functions:

$$X_{i,k,n,t} = \left(\frac{P_{i,k,n,t}^{X}}{P_{i,k,t}^{X}}\right)^{-\theta_{i,t}^{X}} X_{i,k,t}. \tag{96}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative industry specific final export good firm generates zero profit, implying industry specific aggregate export price index:

$$P^X_{i,k,t} = \left[\int_0^1 (P^X_{i,k,n,t})^{1-\theta^X_{i,t}} dn \right]^{\frac{1}{1-\theta^X_{i,t}}}. \tag{97}$$

As the price elasticity of demand for industry specific intermediate export goods $\theta^X_{i,t}$ increases, they become closer substitutes, and individual industry specific intermediate export good firms have less market power.

Export Supply

There exist continuums of monopolistically competitive industry specific intermediate export good firms indexed by $n \in [0,1]$. Intermediate export good firms supply industry specific differentiated intermediate export goods, but are otherwise identical. We rule out entry into and exit out of the monopolistically competitive industry specific intermediate export good sectors.

The representative industry specific intermediate export good firm sells shares to domestic capital market intermediated households at price $V^X_{i,k,n,t}$. Acting in the interests of its shareholders, it maximizes its pre-dividend stock market value, which equals the expected present value of current and future dividend payments:

$$\Pi^X_{i,k,n,t} + V^X_{i,k,n,t} = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda^A_{i,s}}{\lambda^A_{i,t}} \Pi^X_{i,k,n,s}. \tag{98}$$

The derivation of this result imposes a transversality condition that rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to profits $\Pi^X_{i,k,n,s}$, defined as earnings less industry specific fixed cost $F^X_{i,k,s}$:

$$\Pi^X_{i,k,n,s} = P^X_{i,k,n,s} X_{i,k,n,s} - \mathcal{E}_{i,i^*,s} P^Y_{k,s} X_{i,k,n,s} - F^X_{i,k,s}. \tag{99}$$

Earnings are defined as revenues from sales of industry specific differentiated intermediate export good $X_{i,k,n,s}$ at price $P^X_{i,k,n,s}$ less expenditures on energy or nonenergy commodity good $X_{i,k,n,s}$. The representative industry specific intermediate export good firm purchases the energy or nonenergy commodity good and differentiates it. Fixed cost $F^X_{i,k,s}$ ensures that $\Pi^X_{i,k,s} = 0$.

In an adaptation of the model of nominal import price rigidity proposed by Monacelli (2005) to the export sector, each period a randomly selected fraction $1-\omega^X$ of industry specific intermediate export good firms adjust their price optimally, where $0 \le \omega^X < 1$. The remaining fraction ω^X of intermediate export good firms adjust their price to account for past industry specific export price inflation, as well as the contemporaneous change in the domestic currency denominated price of energy or nonenergy commodities, according to partial indexation rule

$$P_{i,k,n,t}^{X} = \left[\left(\frac{P_{i,k,t-1}^{X}}{P_{i,k,t-2}^{X}} \right)^{1-\mu^X} \left(\frac{\mathcal{E}_{i,i^*,t} P_{k,t}^{Y}}{\mathcal{E}_{i,i^*,t-1} P_{k,t-1}^{Y}} \right)^{\mu^X} \right]^{\gamma^X} \left[\left(\frac{\bar{P}_{i,k,t-1}^{X}}{\bar{P}_{i,k,t-2}^{X}} \right)^{1-\mu^X} \left(\frac{\bar{\mathcal{E}}_{i,i^*,t} \bar{P}_{k,t}^{Y}}{\bar{\mathcal{E}}_{i,i^*,t-1} \bar{P}_{k,t-1}^{Y}} \right)^{\mu^X} \right]^{1-\gamma^X} P_{i,k,n,t-1}^{X}, \tag{100}$$

where $0 \le \gamma^X \le 1$ and $\mu^X \ge 0$. Under this specification, the probability that an intermediate export good firm has adjusted its price optimally is time dependent but state independent.

If the representative industry specific intermediate export good firm can adjust its price optimally in period t, then it does so to maximize pre-dividend stock market value (113) subject to industry specific intermediate export good demand function (111), and the assumed form of nominal export price rigidity. Since all intermediate export good firms that adjust their price optimally in period t solve an identical value maximization problem, in equilibrium they all choose a common price $P_{i,k,t}^{X,*}$ given by necessary first order condition:

$$\frac{P_{i,k,t}^{X,*}}{P_{i,k,t}^{X}} = \frac{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega^X)^{s-t} \frac{\beta^{s-t} \lambda_{i,s}^{A}}{\lambda_{i,t}^{A}} \theta_{i,s}^{X} \frac{\mathcal{E}_{i,i^*,s} P_{k,s}^{Y}}{P_{i,k,s}^{X}} \left\{ \left[\left(\frac{P_{i,k,t-1}^{X}}{P_{i,k,s-1}^{X}} \right)^{1-\mu^X} \left(\frac{\mathcal{E}_{i,i^*,t} P_{k,t}^{Y}}{\mathcal{E}_{i,i^*,s} P_{k,s}^{Y}} \right)^{\mu^X} \right]^{\gamma^X} \left[\left(\frac{\bar{P}_{i,k,t-1}^{X}}{\bar{P}_{i,k,s-1}^{X}} \right)^{1-\mu^X} \left(\frac{\bar{\mathcal{E}}_{i,i^*,t} \bar{P}_{k,t}^{Y}}{\bar{\mathcal{E}}_{i,i^*,s} \bar{P}_{k,s}^{Y}} \right)^{\mu^X} \right]^{1-\gamma^X} \frac{P_{i,k,s}^{X}}{P_{i,k,t}^{X}} \right\}^{\theta_{i,s}^{X}} \left(\frac{P_{i,k,t}^{X,*}}{P_{i,k,t}^{X}} \right)^{-\theta_{i,s}^{X}} P_{i,k,s}^{X} X_{i,k,s}}{\mathrm{E}_t \sum_{s=t}^{\infty} (\omega^X)^{s-t} \frac{\beta^{s-t} \lambda_{i,s}^{A}}{\lambda_{i,t}^{A}} (\theta_{i,s}^{X} - 1) \left\{ \left[\left(\frac{P_{i,k,t-1}^{X}}{P_{i,k,s-1}^{X}} \right)^{1-\mu^X} \left(\frac{\mathcal{E}_{i,i^*,t} P_{k,t}^{Y}}{\mathcal{E}_{i,i^*,s} P_{k,s}^{Y}} \right)^{\mu^X} \right]^{\gamma^X} \left[\left(\frac{\bar{P}_{i,k,t-1}^{X}}{\bar{P}_{i,k,s-1}^{X}} \right)^{1-\mu^X} \left(\frac{\bar{\mathcal{E}}_{i,i^*,t} \bar{P}_{k,t}^{Y}}{\bar{\mathcal{E}}_{i,i^*,s} \bar{P}_{k,s}^{Y}} \right)^{\mu^X} \right]^{1-\gamma^X} \frac{P_{i,k,s}^{X}}{P_{i,k,t}^{X}} \right\}^{\theta_{i,s}^{X}-1} \left(\frac{P_{i,k,t}^{X,*}}{P_{i,k,t}^{X}} \right)^{-\theta_{i,s}^{X}} P_{i,k,s}^{X} X_{i,k,s}}. \tag{101}$$

This necessary first order condition equates the expected present value of the marginal revenue gained from export supply to the expected present value of the marginal cost incurred from production. Aggregate export price index (112) equals an average of the price set by the fraction $1-\omega^X$ of intermediate export good firms that adjust their price optimally in period t, and the average of the prices set by the remaining fraction ω^X of intermediate export good firms that adjust their price according to partial indexation rule (115):

$$P_{i,k,t}^{X} = \left\{ (1-\omega^X)(P_{i,k,t}^{X,*})^{1-\theta_{i,t}^{X}} + \omega^X \left\{ \left[\left(\frac{P_{i,k,t-1}^{X}}{P_{i,k,t-2}^{X}} \right)^{1-\mu^X} \left(\frac{\mathcal{E}_{i,i^*,t} P_{k,t}^{Y}}{\mathcal{E}_{i,i^*,t-1} P_{k,t-1}^{Y}} \right)^{\mu^X} \right]^{\gamma^X} \left[\left(\frac{\bar{P}_{i,k,t-1}^{X}}{\bar{P}_{i,k,t-2}^{X}} \right)^{1-\mu^X} \left(\frac{\bar{\mathcal{E}}_{i,i^*,t} \bar{P}_{k,t}^{Y}}{\bar{\mathcal{E}}_{i,i^*,t-1} \bar{P}_{k,t-1}^{Y}} \right)^{\mu^X} \right]^{1-\gamma^X} P_{i,k,t-1}^{X} \right\}^{1-\theta_{i,t}^{X}} \right\}^{\frac{1}{1-\theta_{i,t}^{X}}}. \tag{102}$$

Since those intermediate export good firms able to adjust their price optimally in period t are selected randomly from among all intermediate export good firms, the average price set by the remaining intermediate export good firms equals the value of the industry specific aggregate export price index that prevailed during period $t-1$, rescaled to account for past industry specific export price inflation, as well as the contemporaneous change in the domestic currency denominated price of energy or nonenergy commodities.

The Import Sector

There exist a large number of perfectly competitive firms which combine the final noncommodity output good $Z_{i,t}^{h} \in \{C_{i,t}^{h}, I_{i,t}^{H,h}, I_{i,t}^{K,h}, G_{i,t}^{C,h}, G_{i,t}^{I,h}\}$ with the final import good $Z_{i,t}^{f} \in \{C_{i,t}^{f}, I_{i,t}^{H,f}, I_{i,t}^{K,f}, G_{i,t}^{C,f}, G_{i,t}^{I,f}\}$ to produce final private consumption, residential investment, business investment, public consumption or public investment good $Z_{i,t} \in \{C_{i,t}, I^H_{i,t}, I^K_{i,t}, G^C_{i,t}, G^I_{i,t}\}$ according to constant elasticity of substitution production function

$$Z_{i,t} = \left[(1-\phi_i^M)^{\frac{1}{\psi_i^M}} (Z^h_{i,t})^{\frac{\psi_i^M - 1}{\psi_i^M}} + (\phi_i^M)^{\frac{1}{\psi_i^M}} (\nu^M_{i,t} Z^f_{i,t})^{\frac{\psi_i^M - 1}{\psi_i^M}} \right]^{\frac{\psi_i^M}{\psi_i^M - 1}}, \tag{103}$$

where serially correlated import demand shock $\nu^M_{i,t}$ satisfies $\nu^M_{i,t} > 0$, while $0 \le \phi_i^M < 1$ and $\psi_i^M = \psi^M \left(1 - \frac{\theta^M}{\theta^M - 1} \frac{\overline{M}_{i,t}}{\overline{Y}_{i,t}} \right)$ with $\psi^M > 0$. The representative final absorption good firm maximizes profits derived from production of the final private consumption, residential investment, business investment, public consumption or public investment good, with respect to inputs of the final noncommodity output and import goods, implying demand functions:

$$Z^h_{i,t} = (1-\phi_i^M) \left(\frac{P_{i,t}}{P^Z_{i,t}} \right)^{-\psi_i^M} Z_{i,t}, \; Z^f_{i,t} = \phi_i^M \left(\frac{1}{\nu_i^M} \frac{P^M_{i,t}}{P^Z_{i,t}} \right)^{-\psi_i^M} \frac{Z_{i,t}}{\nu^M_{i,t}}. \tag{104}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final absorption good firm generates zero profit, implying aggregate private consumption, residential investment, business investment, public consumption or public investment price index:

$$P^Z_{i,t} = \left[(1-\phi_i^M)(P_{i,t})^{1-\psi_i^M} + \phi_i^M \left(\frac{P^M_{i,t}}{\nu_i^M} \right)^{1-\psi_i^M} \right]^{\frac{1}{1-\psi_i^M}}. \tag{105}$$

Combination of this aggregate private consumption, residential investment, business investment, public consumption or public investment price index with final noncommodity output and import good demand functions (104) yields:

$$Z^h_{i,t} = (1-\phi_i^M) \left[(1-\phi_i^M) + \phi_i^M \left(\frac{\mathcal{T}^M_{i,t}}{\nu_i^M} \right)^{1-\psi_i^M} \right]^{\frac{\psi_i^M}{1-\psi_i^M}} Z_{i,t}, \; Z^f_{i,t} = \phi_i^M \left[\phi_i^M + (1-\phi_i^M) \left(\frac{\mathcal{T}^M_{i,t}}{\nu_i^M} \right)^{\psi_i^M - 1} \right]^{\frac{\psi_i^M}{1-\psi_i^M}} \frac{Z_{i,t}}{\nu^M_{i,t}}. \tag{106}$$

These demand functions for the final noncommodity output and import goods are directly proportional to final private consumption, residential investment, business investment, public consumption or public investment good demand, with a proportionality coefficient that varies with the external terms of trade. The derivation of these results selectively abstracts from import demand shocks.

Import Demand

There exist a large number of perfectly competitive firms which combine economy specific final import goods $\{M_{i,j,t}\}_{j=1}^N$ to produce final import good $M_{i,t}$ according to fixed proportions production function

$$M_{i,t} = \min\left\{ \nu_{j,t}^{X} \frac{M_{i,j,t}}{\phi_{i,j}^{M}} \right\}_{j=1}^{N}, \tag{107}$$

where serially correlated export demand shock $\nu_{i,t}^{X}$ satisfies $\nu_{i,t}^{X} > 0$, while $\phi_{i,i}^{M} = 0$, $0 \leq \phi_{i,j}^{M} \leq 1$ and $\sum_{j=1}^{N} \phi_{i,j}^{M} = 1$. The representative final import good firm maximizes profits derived from production of the final import good with respect to inputs of economy specific final import goods, implying demand functions:

$$M_{i,j,t} = \phi_{i,j}^{M} \frac{M_{i,t}}{\nu_{j,t}^{X}}. \tag{108}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative final import good firm generates zero profit, implying aggregate import price index:

$$P_{i,t}^{M} = \sum_{j=1}^{N} \phi_{i,j}^{M} \frac{P_{i,j,t}^{M}}{\nu_{j}^{X}}. \tag{109}$$

This aggregate import price index equals the minimum cost of producing one unit of the final import good, given the prices of economy specific final import goods. The derivation of these results selectively abstracts from export demand shocks.

There exist a large number of perfectly competitive firms which combine economy specific differentiated intermediate import goods $M_{i,j,n,t}$ supplied by economy specific intermediate import good firms to produce economy specific final import good $M_{i,j,t}$ according to constant elasticity of substitution production function

$$M_{i,j,t} = \left[\int_{0}^{1} (M_{i,j,n,t})^{\frac{\theta_{i,t}^{M}-1}{\theta_{i,t}^{M}}} dn \right]^{\frac{\theta_{i,t}^{M}}{\theta_{i,t}^{M}-1}}, \tag{110}$$

where serially uncorrelated import price markup shock $\vartheta_{i,t}^{M}$ satisfies $\vartheta_{i,t}^{M} = \frac{\theta_{i,t}^{M}}{\theta_{i,t}^{M}-1}$ with $\theta_{i,t}^{M} > 1$ and $\theta_{i}^{M} = \theta^{M}$. The representative economy specific final import good firm maximizes profits derived from production of the economy specific final import good with respect to inputs of economy specific intermediate import goods, implying demand functions:

$$M_{i,j,n,t} = \left(\frac{P_{i,j,n,t}^{M}}{P_{i,j,t}^{M}} \right)^{-\theta_{i,t}^{M}} M_{i,j,t}. \tag{111}$$

Since the production function exhibits constant returns to scale, in equilibrium the representative economy specific final import good firm generates zero profit, implying economy specific aggregate import price index:

$$P_{i,j,t}^{M} = \left[\int_{0}^{1} (P_{i,j,n,t}^{M})^{1-\theta_{i,t}^{M}} dn \right]^{\frac{1}{1-\theta_{i,t}^{M}}}. \tag{112}$$

As the price elasticity of demand for economy specific intermediate import goods $\theta_{i,t}^{M}$ increases, they become closer substitutes, and individual economy specific intermediate import good firms have less market power.

Import Supply

There exist continuums of monopolistically competitive economy specific intermediate import good firms indexed by $n \in [0,1]$. Intermediate import good firms supply economy specific differentiated intermediate import goods, but are otherwise identical. We rule out entry into and exit out of the monopolistically competitive economy specific intermediate import good sectors.

The representative economy specific intermediate import good firm sells shares to domestic capital market intermediated households at price $V_{i,j,n,t}^{M}$. Acting in the interests of its shareholders, it maximizes its pre-dividend stock market value, which equals the expected present value of current and future dividend payments:

$$\Pi_{i,j,n,t}^{M} + V_{i,j,n,t}^{M} = \mathrm{E}_t \sum_{s=t}^{\infty} \frac{\beta^{s-t} \lambda_{i,s}^{A}}{\lambda_{i,t}^{A}} \Pi_{i,j,n,s}^{M}. \tag{113}$$

The derivation of this result imposes a transversality condition that rules out self-fulfilling speculative asset price bubbles.

Shares entitle households to dividend payments equal to profits $\Pi_{i,j,n,s}^{M}$, defined as earnings less economy specific fixed cost $F_{i,j,s}^{M}$:

$$\Pi_{i,j,n,s}^{M} = P_{i,j,n,s}^{M} M_{i,j,n,s} - \mathcal{E}_{i,j,s} P_{j,s}^{X} M_{i,j,n,s} - F_{i,j,s}^{M}. \tag{114}$$

Earnings are defined as revenues from sales of economy specific differentiated intermediate import good $M_{i,j,n,s}$ at price $P_{i,j,n,s}^{M}$ less expenditures on foreign final export good $M_{i,j,n,s}$. The representative economy specific intermediate import good firm purchases the foreign final export good and differentiates it. Fixed cost $F_{i,j,s}^{M}$ ensures that $\Pi_{i,j,s}^{M} = 0$.

In an extension of the model of nominal import price rigidity proposed by Monacelli (2005), each period a randomly selected fraction $1-\omega^{M}$ of economy specific intermediate import good firms adjust their price optimally, where $0 \le \omega^{M} < 1$. The remaining fraction ω^{M} of intermediate import good firms adjust their price to account for past economy specific import price inflation, as well as contemporaneous changes in the domestic currency denominated prices of energy and nonenergy commodities, according to partial indexation rule

$$P_{i,j,n,t}^{M} = \left[\left(\frac{P_{i,j,t-1}^{M}}{P_{i,j,t-2}^{M}} \right)^{1-\mu_i^M} \prod_{k=1}^{M^*} \left(\frac{\mathcal{E}_{i,i^*,t} P_{k,t}^{Y}}{\mathcal{E}_{i,i^*,t-1} P_{k,t-1}^{Y}} \right)^{\mu_{i,k}^{M}} \right]^{\gamma^M} \left[\left(\frac{\bar{P}_{i,j,t-1}^{M}}{\bar{P}_{i,j,t-2}^{M}} \right)^{1-\mu_i^M} \prod_{k=1}^{M^*} \left(\frac{\bar{\mathcal{E}}_{i,i^*,t} \bar{P}_{k,t}^{Y}}{\bar{\mathcal{E}}_{i,i^*,t-1} \bar{P}_{k,t-1}^{Y}} \right)^{\mu_{i,k}^{M}} \right]^{1-\gamma^M} P_{i,j,n,t-1}^{M}, \tag{115}$$

where $0 \le \gamma^{M} \le 1$, while $\mu_i^{M} = \sum_{k=1}^{M^*} \mu_{i,k}^{M}$ with $\mu_{i,k}^{M} = \mu^{M} \frac{\bar{M}_{i,k,t}}{\bar{M}_{i,t}}$ and $\mu^{M} \ge 0$. Under this specification, the probability that an intermediate import good firm has adjusted its price optimally is time dependent but state independent.

If the representative economy specific intermediate import good firm can adjust its price optimally in period t, then it does so to maximize pre-dividend stock market value (113) subject to economy specific intermediate import good demand function (111), and the assumed form of nominal import price rigidity. Since all intermediate import good firms that adjust their price optimally in period t solve an identical value maximization problem, in equilibrium they all choose a common price $P_{i,j,t}^{M,*}$ given by necessary first order condition:

$$\frac{P_{i,j,t}^{M,*}}{P_{i,j,t}^{M}}=\frac{\mathrm{E}_t\sum_{s=t}^{\infty}(\omega^M)^{s-t}\frac{\beta^{s-t}\lambda_{i,s}^A}{\lambda_{i,t}^A}\theta_{i,s}^M\frac{\mathcal{E}_{i,j,s}P_{j,s}^X}{P_{i,j,s}^M}\left\{\left[\left(\frac{P_{i,j,t-1}^M}{P_{i,j,s-1}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\mathcal{E}_{i,i^*,t}P_{k,t}^Y}{\mathcal{E}_{i,i^*,s}P_{k,s}^Y}\right)^{\mu_{i,k}^M}\right]^{\gamma^M}\left[\left(\frac{\bar{P}_{i,j,t-1}^M}{\bar{P}_{i,j,s-1}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\bar{\mathcal{E}}_{i,i^*,t}\bar{P}_{k,t}^Y}{\bar{\mathcal{E}}_{i,i^*,s}\bar{P}_{k,s}^Y}\right)^{\mu_{i,k}^M}\right]^{1-\gamma^M}\frac{P_{i,j,s}^M}{P_{i,j,t}^M}\right\}^{\theta_{i,s}^M}\left(\frac{P_{i,j,t}^{M,*}}{P_{i,j,t}^M}\right)^{-\theta_{i,s}^M}P_{i,j,s}^M M_{i,j,s}}{\mathrm{E}_t\sum_{s=t}^{\infty}(\omega^M)^{s-t}\frac{\beta^{s-t}\lambda_{i,s}^A}{\lambda_{i,t}^A}(\theta_{i,s}^M-1)\left\{\left[\left(\frac{P_{i,j,t-1}^M}{P_{i,j,s-1}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\mathcal{E}_{i,i^*,t}P_{k,t}^Y}{\mathcal{E}_{i,i^*,s}P_{k,s}^Y}\right)^{\mu_{i,k}^M}\right]^{\gamma^M}\left[\left(\frac{\bar{P}_{i,j,t-1}^M}{\bar{P}_{i,j,s-1}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\bar{\mathcal{E}}_{i,i^*,t}\bar{P}_{k,t}^Y}{\bar{\mathcal{E}}_{i,i^*,s}\bar{P}_{k,s}^Y}\right)^{\mu_{i,k}^M}\right]^{1-\gamma^M}\frac{P_{i,j,s}^M}{P_{i,j,t}^M}\right\}^{\theta_{i,s}^M-1}\left(\frac{P_{i,j,t}^{M,*}}{P_{i,j,t}^M}\right)^{-\theta_{i,s}^M}P_{i,j,s}^M M_{i,j,s}}. \quad (116)$$

This necessary first order condition equates the expected present value of the marginal revenue gained from import supply to the expected present value of the marginal cost incurred from production. Aggregate import price index (112) equals an average of the price set by the fraction $1-\omega^M$ of intermediate import good firms that adjust their price optimally in period t, and the average of the prices set by the remaining fraction ω^M of intermediate import good firms that adjust their price according to partial indexation rule (115):

$$P_{i,j,t}^M=\left\{(1-\omega^M)(P_{i,j,t}^{M,*})^{1-\theta_{i,t}^M}+\omega^M\left\{\left[\left(\frac{P_{i,j,t-1}^M}{P_{i,j,t-2}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\mathcal{E}_{i,i^*,t}P_{k,t}^Y}{\mathcal{E}_{i,i^*,t-1}P_{k,t-1}^Y}\right)^{\mu_{i,k}^M}\right]^{\gamma^M}\left[\left(\frac{\bar{P}_{i,j,t-1}^M}{\bar{P}_{i,j,t-2}^M}\right)^{1-\mu_i^M}\prod_{k=1}^{M^*}\left(\frac{\bar{\mathcal{E}}_{i,i^*,t}\bar{P}_{k,t}^Y}{\bar{\mathcal{E}}_{i,i^*,t-1}\bar{P}_{k,t-1}^Y}\right)^{\mu_{i,k}^M}\right]^{1-\gamma^M}P_{i,j,t-1}^M\right\}^{1-\theta_{i,t}^M}\right\}^{\frac{1}{1-\theta_{i,t}^M}}. \quad (117)$$

Since those intermediate import good firms able to adjust their price optimally in period t are selected randomly from among all intermediate import good firms, the average price set by the remaining intermediate import good firms equals the value of the economy specific aggregate import price index that prevailed during period $t-1$, rescaled to account for past economy specific import price inflation, as well as contemporaneous changes in the domestic currency denominated prices of energy and nonenergy commodities.

F. Monetary, Fiscal, and Macroprudential Policy

The government consists of a monetary authority, a fiscal authority, and a macroprudential authority. The monetary authority conducts monetary policy, while the fiscal authority conducts fiscal policy, and the macroprudential authority conducts macroprudential policy.

The Monetary Authority

The monetary authority implements monetary policy through control of the nominal policy interest rate. We differentiate between flexible inflation targeting, managed exchange rate, and fixed exchange rate regimes. Under a monetary union, the leader economy follows a modified flexible inflation targeting regime, while all other union members follow fixed exchange rate regimes.

Under a flexible inflation targeting or managed exchange rate regime, the nominal policy interest rate satisfies a monetary policy rule exhibiting partial adjustment dynamics of the form

$$\begin{aligned} i_{i,t}^{P} - \overline{i}_{i,t}^{P} &= \rho^{i}(i_{i,t-1}^{P} - \overline{i}_{i,t-1}^{P}) \\ &+(1-\rho^{i})\left[\xi^{\pi}\mathrm{E}_{t}(\pi_{i,t+1}^{C} - \overline{\pi}_{i,t+1}^{C}) + \xi^{Y}(\ln Y_{i,t} - \ln \tilde{Y}_{i,t}) + \xi_{j}^{\mathcal{E}}(\Delta \ln \mathcal{E}_{i,t} - \Delta \ln \overline{\mathcal{E}}_{i,t})\right] + \nu_{i,t}^{i^{P}}, \end{aligned} \tag{118}$$

where $0 \leq \rho^{i} < 1$, $\xi^{\pi} > 1$, $\xi^{Y} > 0$ and $\xi_{j}^{\mathcal{E}} \geq 0$. This rule prescribing the conduct of monetary policy is consistent with achieving some combination of inflation control, output stabilization, and exchange rate stabilization objectives. As specified, the deviation of the nominal policy interest rate from its steady state equilibrium value depends on a weighted average of its past deviation and its desired deviation. Under a flexible inflation targeting regime $j = 0$, and this desired deviation is increasing in the expected future deviation of consumption price inflation from its target value, as well as the contemporaneous output gap. We define the output gap as the deviation of output from its potential value, which we define as that output level consistent with full utilization of private physical capital and effective labor, given the private physical capital stock and effective labor force. For the leader economy of a monetary union, the target variables entering into its monetary policy rule are expressed as output weighted averages across union members. Under a managed exchange rate regime $j = 1$, and the desired deviation of the nominal policy interest rate from its steady state equilibrium value is also increasing in the contemporaneous deviation of the change in the nominal effective exchange rate from its steady state equilibrium value with $\xi_{j}^{\mathcal{E}} = \xi^{\mathcal{E}} > 0$. Deviations from this monetary policy rule are captured by mean zero and serially uncorrelated monetary policy shock $\nu_{i,t}^{i^{P}}$.

Under a fixed exchange rate regime, the nominal policy interest rate instead satisfies a monetary policy rule exhibiting feedback of the form

$$i_{i,t}^{P} - \overline{i}_{i,t}^{P} = (i_{k,t}^{P} - \overline{i}_{k,t}^{P}) + \xi^{\mathcal{E}_{k}}(\Delta \ln \mathcal{E}_{i,k,t} - \Delta \ln \overline{\mathcal{E}}_{i,k,t}), \tag{119}$$

where $\xi^{\mathcal{E}_{k}} > 1$. As specified, the deviation of the nominal policy interest rate from its steady state equilibrium value tracks the contemporaneous deviation of the nominal policy interest rate of the leader economy from its steady state equilibrium value one for one, and is increasing in the contemporaneous deviation of the change in the corresponding nominal bilateral exchange rate from its target value.

The Fiscal Authority

The fiscal authority implements fiscal policy through control of public consumption and investment, as well as the tax rates applicable to corporate earnings and household labor income. It also operates a budget neutral nondiscretionary lump sum transfer program that redistributes national financial wealth from capital market intermediated households to credit constrained households while equalizing steady state equilibrium consumption across households, as well as a discretionary lump sum transfer program that provides income support to credit constrained households. The fiscal authority can transfer its budgetary resources intertemporally through transactions in the domestic money and bond markets. Considered jointly, the rules prescribing the conduct of this distortionary fiscal policy are countercyclical, representing automatic fiscal stabilizers, and are consistent with achieving public and national financial wealth stabilization objectives.

Public consumption and investment satisfy countercyclical fiscal expenditure rules exhibiting partial adjustment dynamics of the form

$$\ln \frac{G_{i,t}^{Z}}{\bar{G}_{i,t}^{Z}} = \rho_G \ln \frac{G_{i,t-1}^{Z}}{\bar{G}_{i,t-1}^{Z}} + (1-\rho_G) \ln \frac{\tilde{Y}_{i,t}}{\bar{Y}_{i,t}} + \nu_{i,t}^{G^Z}, \tag{120}$$

where $Z \in \{C, I\}$, while $0 \le \rho_G < 1$. As specified, the deviation of public consumption or investment from its steady state equilibrium value depends on a weighted average of its past deviation and its desired deviation, which in turn tracks the contemporaneous deviation of potential output from its steady state equilibrium value one for one. Deviations from these fiscal expenditure rules are captured by mean zero and serially correlated public consumption or investment shock $\nu_{i,t}^{G^Z}$.

The tax rates applicable to corporate earnings and household labor income satisfy acyclical fiscal revenue rules of the form

$$\tau_{i,t}^{Z} - \tau_i = \rho_\tau (\tau_{i,t-1}^{Z} - \tau_i) + \nu_{i,t}^{\tau^Z}, \tag{121}$$

where $Z \in \{K, L\}$, while $0 < \tau_i < 1$ and $0 \le \rho_\tau < 1$. As specified, the deviations of these tax rates from their steady state equilibrium value depend on their past deviations. Deviations from these fiscal revenue rules are captured by mean zero and serially correlated corporate or labor income tax rate shock $\nu_{i,t}^{\tau^Z}$.

The ratio of nondiscretionary lump sum transfer payments to nominal output satisfies a nondiscretionary transfer payment rule that stabilizes national financial wealth of the form

$$\frac{T_{i,t}^{C,N}}{P_{i,t}^{Y} Y_{i,t}} - \frac{\bar{T}_{i,t}^{C,N}}{\bar{P}_{i,t}^{Y} \bar{Y}_{i,t}} = \zeta^{T^N} \left(\frac{A_{i,t}}{P_{i,t-1}^{Y} Y_{i,t-1}} - \frac{\bar{A}_{i,t}}{\bar{P}_{i,t-1}^{Y} \bar{Y}_{i,t-1}} \right), \tag{122}$$

where $\zeta^{T^N} > 0$. As specified, the deviation of the ratio of nondiscretionary lump sum transfer payments to nominal output from its steady state equilibrium value is increasing in the past deviation of the ratio of national financial wealth to nominal output from its target value. The ratio of discretionary lump sum transfer payments to nominal output satisfies a discretionary transfer payment rule that stabilizes public financial wealth of the form

$$\frac{T_{i,t}^{C,D}}{P_{i,t}^{Y} Y_{i,t}} - \frac{\bar{T}_{i,t}^{C,D}}{\bar{P}_{i,t}^{Y} \bar{Y}_{i,t}} = \zeta^{T^D} \left(\frac{A_{i,t}^{G}}{P_{i,t-1}^{Y} Y_{i,t-1}} - \frac{\bar{A}_{i,t}^{G}}{\bar{P}_{i,t-1}^{Y} \bar{Y}_{i,t-1}} \right) + \nu_{i,t}^{T}, \tag{123}$$

where $\zeta^{T^D} > 0$. As specified, the deviation of the ratio of discretionary lump sum transfer payments to nominal output from its steady state equilibrium value is increasing in the past deviation of the ratio of public financial wealth to nominal output from its target value. Deviations from this discretionary transfer payment rule are captured by mean zero and serially correlated transfer payment shock $\nu_{i,t}^{T}$.

The gross yield to maturity on short term bonds depends on the contemporaneous gross nominal policy interest rate according to money market relationship:

$$1+i_{i,t}^{S}=\upsilon_{i,t}^{i^{S}}(1+i_{i,t}^{P}). \tag{124}$$

Deviations from this money market relationship are captured by internationally and serially correlated credit risk premium shock $\upsilon_{i,t}^{i^{S}}$. In parallel, the gross nominal interbank loans rate depends on the contemporaneous gross yield to maturity on short term bonds according to interbank market relationship:

$$1+i_{i,t}^{B}=\upsilon_{i,t}^{i^{B}}(1+i_{i,t}^{S}). \tag{125}$$

Deviations from this interbank market relationship are captured by internationally and serially correlated liquidity risk premium shock $\upsilon_{i,t}^{i^{B}}$.

The fiscal authority enters period t in possession of previously accumulated financial wealth $A_{i,t}^{G}$ which yields return $i_{i,t}^{A^{G}}$. This financial wealth is distributed across the values of domestic short term bond $B_{i,t}^{S,G}$ and long term bond $B_{i,t}^{L,G}$ portfolios which yield returns $i_{i,t-1}^{S}$ and $i_{i,t}^{B^{L,G}}$, respectively. It follows that $(1+i_{i,t}^{A^{G}})A_{i,t}^{G}=(1+i_{i,t-1}^{S})B_{i,t}^{S,G}+(1+i_{i,t}^{B^{L,G}})B_{i,t}^{L,G}$, where $(1+i_{i,t}^{B^{L,G}})B_{i,t}^{L,G}=\sum_{k=1}^{t-1}(\Pi_{i,k,t}^{B}+V_{i,k,t}^{B})B_{i,k,t}^{L,G}$ with $\Pi_{i,k,t}^{B}=(1+i_{i,k}^{L}-\omega^{B})(\omega^{B})^{t-k}V_{i,k,k}^{B}$ and $V_{i,k,k}^{B}=1$. At the end of period t, the fiscal authority levies taxes on corporate earnings at rate $\tau_{i,t}^{K}$ and household labor income at rate $\tau_{i,t}^{L}$, generating tax revenues $T_{i,t}$. These sources of public wealth are summed in government dynamic budget constraint:

$$A_{i,t+1}^{G}=(1+i_{i,t}^{A^{G}})A_{i,t}^{G}+\sum_{k=1}^{M}\int_{0}^{1}\tau_{i,t}^{K}(P_{i,k,l,t}^{Y}Y_{i,k,l,t}-W_{i,t}L_{i,k,l,t})dl+\int_{0}^{1}\tau_{i,t}^{L}\int_{0}^{1}W_{f,i,t}L_{h,f,i,t}df\,dh-\int_{0}^{1}T_{h,i,t}^{Z}dh-P_{i,t}^{G^{C}}G_{i,t}^{C}-P_{i,t}^{G^{I}}G_{i,t}^{I}. \tag{126}$$

According to this dynamic budget constraint, at the end of period t, the fiscal authority holds financial wealth $A_{i,t+1}^{G}$, which it allocates across the values of domestic short term bond $B_{i,t+1}^{S,G}$ and long term bond $B_{i,t+1}^{L,G}$ portfolios, that is $A_{i,t+1}^{G}=B_{i,t+1}^{S,G}+B_{i,t+1}^{L,G}$ where $B_{i,t+1}^{L,G}=\sum_{k=1}^{t}V_{i,k,t}^{B}B_{i,k,t+1}^{L,G}$. It also remits household type specific lump sum transfer payments $\{T_{h,i,t}^{Z}\}_{h=0}^{1}$, which it allocates across nondiscretionary transfers $\{T_{h,i,t}^{Z,N}\}_{h=0}^{1}$ and discretionary transfers $\{T_{h,i,t}^{Z,D}\}_{h=0}^{1}$, that is $T_{h,i,t}^{Z}=T_{h,i,t}^{Z,N}+T_{h,i,t}^{Z,D}$ where $\int_{0}^{1}T_{h,i,t}^{Z,N}dh=0$ and $T_{h,i,t}^{B,D}=T_{h,i,t}^{A,D}=T_{h,i}^{C,D}=0$. Finally, the fiscal authority purchases final public consumption good $G_{i,t}^{C}$ at price $P_{i,t}^{G^{C}}$, and final public investment good $G_{i,t}^{I}$ at price $P_{i,t}^{G^{I}}$, accumulating the public physical capital stock $K_{i,t+1}^{G}$ according to $K_{i,t+1}^{G}=(1-\delta^{G})K_{i,t}^{G}+G_{i,t}^{I}$ where $0\leq\delta^{G}\leq 1$.

The Macroprudential Authority

The macroprudential authority implements macroprudential policy through control of a regulatory capital requirement and loan to value ratio limits. It imposes the regulatory capital requirement on lending by domestic banks, and the regulatory loan to value ratio limits on borrowing by domestic developers and firms.

The regulatory capital ratio requirement applicable to lending by domestic banks to domestic and foreign developers and firms satisfies a countercyclical capital buffer rule exhibiting partial adjustment dynamics of the form

$$\begin{aligned}\kappa_{i,t+1}^{R}-\kappa^{R} &= \rho_{\kappa}(\kappa_{i,t}^{R}-\kappa^{R})+(1-\rho_{\kappa})\Big[\zeta^{\kappa,B}(\Delta\ln B_{i,t+1}^{C,B}-\Delta\ln \overline{B}_{i,t+1}^{C,B})+\zeta^{\kappa,V^{H}}(\Delta\ln V_{i,t}^{H}-\Delta\ln \overline{V}_{i,t}^{H}) \\ &\quad +\zeta^{\kappa,V^{S}}(\Delta\ln V_{i,t}^{S}-\Delta\ln \overline{V}_{i,t}^{S})\Big]+\nu_{i,t}^{\kappa},\end{aligned} \tag{127}$$

where $0<\kappa^{R}<1$, $0\leq\rho_{\kappa}<1$, $\zeta^{\kappa,B}>0$, $\zeta^{\kappa,V^{H}}>0$ and $\zeta^{\kappa,V^{S}}>0$. As specified, the deviation of the regulatory capital ratio requirement from its steady state equilibrium value depends on a weighted average of its past deviation and its desired deviation. This desired deviation is increasing in the contemporaneous deviation of bank credit growth from its steady state equilibrium value, as well as the contemporaneous deviations of the changes in the prices of housing and equity from their steady state equilibrium values. Deviations from this countercyclical capital buffer rule are captured by mean zero and serially correlated capital requirement shock $\nu_{i,t}^{\kappa}$.

The regulatory loan to value ratio limits applicable to borrowing by domestic developers and firms from domestic and foreign banks satisfy loan to value limit rules exhibiting partial adjustment dynamics of the form

$$\phi_{i,t}^{Z}-\phi^{Z}=\rho_{\phi^{Z}}(\phi_{i,t-1}^{Z}-\phi^{Z})-(1-\rho_{\phi^{Z}})\Big[\zeta^{\phi^{Z},B}(\Delta\ln B_{i,t+1}^{C,Z}-\Delta\ln \overline{B}_{i,t+1}^{C,Z})+\zeta^{\phi^{Z},V}(\Delta\ln V_{i,t}^{f(Z)}-\Delta\ln \overline{V}_{i,t}^{f(Z)})\Big]+\nu_{i,t}^{\phi^{Z}}, \tag{128}$$

where $Z\in\{D,F\}$, while $f(D)=H$ and $f(F)=S$. As specified, the deviations of the regulatory loan to value ratio limits from their steady state equilibrium values depend on a weighted average of their past deviations and their desired deviations, where $0<\phi^{Z}<1$, $0\leq\rho_{\phi^{Z}}<1$, $\zeta^{\phi^{Z},B}>0$ and $\zeta^{\phi^{Z},V}>0$. These desired deviations are decreasing in the contemporaneous deviation of mortgage or nonfinancial corporate debt growth from its steady state equilibrium value, as well as the contemporaneous deviation of the change in the price of housing or equity from its steady state equilibrium value, respectively. Deviations from these loan to value limit rules are captured by mean zero and serially uncorrelated mortgage or corporate loan to value limit shock $\nu_{i,t}^{\phi^{Z}}$.

The loan default rates applicable to borrowing by domestic developers and firms from domestic and foreign banks satisfy default rate relationships exhibiting partial adjustment dynamics of the form

$$\delta_{i,t}^{Z}-\delta=\rho_{\delta}(\delta_{i,t-1}^{Z}-\delta)-(1-\rho_{\delta})\Big[\zeta^{\delta^{Z},Y}(\ln Y_{i,t}-\ln \tilde{Y}_{i,t})+\zeta^{\delta^{Z},V}(\Delta\ln V_{i,t}^{f(Z)}-\Delta\ln \overline{V}_{i,t}^{f(Z)})\Big]+\nu_{i,t}^{\delta^{Z}}, \tag{129}$$

where $Z\in\{M,C\}$, while $f(M)=H$ and $f(C)=S$. As specified, the deviations of the mortgage or corporate loan default rates from their steady state equilibrium value depend on a weighted average of their past deviations and their attractor deviations, where $0<\delta<1$, $0\leq\rho_{\delta}<1$, $\zeta^{\delta^{Z},Y}>0$ and $\zeta^{\delta^{Z},V}>0$. These attractor deviations are decreasing in the contemporaneous deviations of output from its potential value and the change in the price of housing or equity from its steady state equilibrium value, which affect the probability of default and loss given default, respectively. Deviations from these default rate relationships are captured by mean zero and serially uncorrelated mortgage or corporate loan default shock $\nu_{i,t}^{\delta^{Z}}$.

G. Market Clearing Conditions

A rational expectations equilibrium in this DSGE model of the world economy consists of state contingent sequences of allocations for the households, developers, firms and banks of all economies that solve their constrained optimization problems given prices and policies, together with state contingent sequences of allocations for the governments of all economies that satisfy their policy rules and constraints given prices, with supporting prices such that all markets clear.

Clearing of the final output good market requires that exports $X_{i,t}$ equal production of the domestic final output good less the total demand of domestic households, developers, firms and the government,

$$X_{i,t} = Y_{i,t} - C_{i,t}^{h} - I_{i,t}^{H,h} - I_{i,t}^{K,h} - G_{i,t}^{C,h} - G_{i,t}^{I,h}, \tag{130}$$

where $X_{i,t} = \sum_{j=1}^{N} X_{i,j,t}$ and $X_{i,j,t} = M_{j,i,t}$. Clearing of the final import good market requires that imports $M_{i,t}$ equal the total demand of domestic households, developers, firms and the government:

$$M_{i,t} = C_{i,t}^{f} + I_{i,t}^{H,f} + I_{i,t}^{K,f} + G_{i,t}^{C,f} + G_{i,t}^{I,f}. \tag{131}$$

In equilibrium, combination of these final output and import good market clearing conditions yields output expenditure decomposition,

$$P_{i,t}^{Y} Y_{i,t} = P_{i,t}^{C} C_{i,t} + P_{i,t}^{I} I_{i,t} + P_{i,t}^{G} G_{i,t} + P_{i,t}^{X} X_{i,t} - P_{i,t}^{M} M_{i,t}, \tag{132}$$

where the price of investment satisfies $P_{i,t}^{I} = P_{i,t}^{I^H} = P_{i,t}^{I^K}$ while investment satisfies $I_{i,t} = I_{i,t}^{H} + I_{i,t}^{K}$, and the price of public domestic demand satisfies $P_{i,t}^{G} = P_{i,t}^{G^C} = P_{i,t}^{G^I}$ while public domestic demand satisfies $G_{i,t} = G_{i,t}^{C} + G_{i,t}^{I}$. The price of domestic demand satisfies $P_{i,t}^{D} = P_{i,t}^{C} = P_{i,t}^{I} = P_{i,t}^{G}$ while domestic demand satisfies $D_{i,t} = C_{i,t} + I_{i,t} + G_{i,t}$.

Clearing of the final bank loan markets requires that mortgage loan supply equals the total demand of domestic developers, that is $B_{i,t+1}^{C^D,B} = B_{i,t+1}^{C,D}$, while corporate loan supply equals the total demand of domestic and foreign firms

$$B_{i,t+1}^{C^F,B} = \sum_{j=1}^{N} B_{j,i,t+1}^{C,F}, \tag{133}$$

where $B_{i,j,t+1}^{C^F,B} = B_{j,i,t+1}^{C,F}$. In equilibrium, clearing of the final corporate loan payments system implies that the corporate credit loss rate satisfies:

$$1 - \delta_{i,t}^{C,E} = \sum_{j=1}^{N} \frac{B_{j,i,t}^{C,F}}{B_{i,t}^{C^F,B}} (1 - \delta_{j,t}^{C}). \tag{134}$$

The derivation of this result equates the principal and interest receipts of the banking sector to the total domestic currency denominated principal and interest payments of domestic and foreign firms.

Let $A_{i,t+1}$ denote the net foreign asset position, which equals the sum of the financial wealth of households $A^H_{i,t+1}$, developers $A^D_{i,t+1}$, firms $A^F_{i,t+1}$, banks $A^B_{i,t+1}$ and the government $A^G_{i,t+1}$,

$$A_{i,t+1} = A^H_{i,t+1} + A^D_{i,t+1} + A^F_{i,t+1} + A^B_{i,t+1} + A^G_{i,t+1}, \tag{135}$$

where $A^H_{i,t+1} = A^{B,H}_{i,t+1} + A^{A,H}_{i,t+1} + V^C_{i,t}$, $A^D_{i,t+1} = -B^{C,D}_{i,t+1} - V^H_{i,t}$, $A^F_{i,t+1} = -B^{C,F}_{i,t+1} - V^S_{i,t}$ and $A^B_{i,t+1} = K^B_{i,t+1} - V^C_{i,t}$. Imposing equilibrium conditions on government dynamic budget constraint (126) reveals that the increase in public financial wealth equals public saving, or equivalently that the fiscal balance $FB_{i,t} = A^G_{i,t+1} - A^G_{i,t}$ equals the sum of net interest income and the primary fiscal balance $PB_{i,t} = T_{i,t} - T^{C,D}_{i,t} - P^G_{i,t}G_{i,t}$,

$$FB_{i,t} = \left[\frac{B^{S,G}_i}{A^G_i} i^S_{i,t-1} + \frac{B^{L,G}_i}{A^G_i} i^{L,E}_{i,t-1}\right] A^G_{i,t} + PB_{i,t}, \tag{136}$$

where $T_{i,t} = \tau^K_{i,t}(P^Y_{i,t}Y_{i,t} - W_{i,t}L_{i,t}) + \tau^L_{i,t}W_{i,t}L_{i,t}$, while nominal effective long term market interest rate $i^{L,E}_{i,t}$ satisfies $i^{L,E}_{i,t} = \omega^B i^{L,E}_{i,t-1} + (1-\omega^B)\left[\omega^B((1+i^L_{i,t}) + (1-\omega^B)) - 1\right]$. The derivation of this result abstracts from valuation gains on long term bond holdings, and imposes restrictions $B^{L,G}_{i,k,t} = B^{L,G}_{i,k-1,t}$, $B^{S,G}_{i,t} / A^G_{i,t} = B^{S,G}_i / A^G_i$ and $B^{L,G}_{i,t} / A^G_{i,t} = B^{L,G}_i / A^G_i$. Imposing equilibrium conditions on household dynamic budget constraint (7), and combining it with government dynamic budget constraint (136), developer dividend payment definition (41), firm dividend payment definition (55), bank dividend payment definition (77), bank balance sheet identity (78), output expenditure decomposition (132), and final corporate loan payments system clearing condition (134) reveals that the increase in net foreign assets equals national saving less investment expenditures, or equivalently that the current account balance $CA_{i,t} = \mathcal{E}_{i*,i,t}A_{i,t+1} - \mathcal{E}_{i*,i,t-1}A_{i,t}$ equals the sum of net international investment income and the trade balance $TB_{i,t} = \mathcal{E}_{i*,i,t}P^X_{i,t}X_{i,t} - \mathcal{E}_{i*,i,t}P^M_{i,t}M_{i,t}$:

$$CA_{i,t} = \left\{\sum_{j=1}^{N} w^A_j \left[(1+i^S_{j,t-1})\frac{\mathcal{E}_{i*,j,t}}{\mathcal{E}_{i*,j,t-1}} - 1\right]\right\}\mathcal{E}_{i*,i,t-1}A_{i,t} + TB_{i,t}. \tag{137}$$

The derivation of this result abstracts from international financial intermediation except via the money markets and imposes restriction $\mathcal{E}_{i,j,t-1}B^S_{i,j,t} / A_{i,t} = w^A_j$, where world capital market capitalization weight w^A_i satisfies $0 < w^A_i < 1$ and $\sum_{i=1}^{N} w^A_i = 1$. Multilateral consistency in nominal trade flows requires that $\sum_{j=1}^{N} TB_{j,t} = 0$.

III. The Empirical Framework

Estimation and inference are based on a linear state space representation of an approximate multivariate linear rational expectations representation of this DSGE model of the world economy, expressed as a function of its potentially heteroskedastic structural shocks. This multivariate linear rational expectations representation is derived by analytically linearizing the equilibrium conditions of the DSGE model around its stationary deterministic steady state equilibrium, and consolidating them by substituting out intermediate variables assuming small capital utilization costs and abstracting from the global terms of trade shifter. The response coefficients of these consolidated approximate linear equilibrium conditions are functions of behavioral parameters that have been restricted to coincide across economies—occasionally within groups sharing a structural characteristic—and economy specific structural characteristics implied by steady state equilibrium relationships. Except where stated otherwise, this steady state equilibrium features zero inflation, productivity and labor force growth, as well as public and national financial wealth.[2]

In what follows, $\hat{x}_{i,t}$ denotes the deviation of variable $x_{i,t}$ from its steady state equilibrium value x_i, while $\mathrm{E}_t\, x_{i,t+s}$ denotes the rational expectation of variable $x_{i,t+s}$ conditional on information available in period t. Bilateral weights $w_{i,j}^{Z}$ for evaluating the trade weighted average of variable $x_{i,t}$ across the trading partners of economy i are based on exports for $Z = X$, imports for $Z = M$, and their average for $Z = T$. In addition, bilateral weights $w_{i,j}^{Z}$ for evaluating the weighted average of variable $x_{i,t}$ across the lending destinations and borrowing sources of economy i are based on bank lending for $Z = C$ and nonfinancial corporate borrowing for $Z = F$. Furthermore, bilateral weights $w_{i,j}^{Z}$ for evaluating the portfolio weighted average of variable $x_{i,t}$ across the investment destinations of economy i are based on debt for $Z = B$ and equity for $Z = S$. Finally, world weights w_i^Z for evaluating the weighted average of variable $x_{i,t}$ across all economies are based on output for $Z = Y$ and capital market capitalization for $Z = A$. Auxiliary parameters λ^Z are theoretically predicted to equal one, and satisfy $\lambda = 0$ and $\lambda^Z > 0$.

A. Endogenous Variables

Core inflation depends on a linear combination of its past and expected future values driven by contemporaneous real unit labor cost according to Phillips curve

$$\hat{\pi}_{i,t} = \frac{\gamma^Y}{1+\gamma^Y\beta}\hat{\pi}_{i,t-1} + \frac{\beta}{1+\gamma^Y\beta}\mathrm{E}_t\hat{\pi}_{i,t+1} + \frac{(1-\omega^Y)(1-\omega^Y\beta)}{\omega^Y(1+\gamma^Y\beta)}\left[\ln\frac{\hat{W}_{i,t}\hat{L}_{i,t}}{\hat{P}_{i,t}\hat{Y}_{i,t}} + \ln\hat{\vartheta}_{i,t}^{Y}\right], \tag{138}$$

which determines the core price level $\ln\hat{P}_{i,t}$. Core inflation $\hat{\pi}_{i,t}$ satisfies $\hat{\pi}_{i,t} = \ln\hat{P}_{i,t} - \ln\hat{P}_{i,t-1}$. The output price level $\ln\hat{P}_{i,t}^{Y}$ depends on the contemporaneous core price level and internal terms of trade according to output price relationship:

$$\ln\hat{P}_{i,t}^{Y} = \ln\hat{P}_{i,t} + \frac{X_i}{Y_i}\ln\hat{\mathcal{T}}_{i,t}^{X}. \tag{139}$$

Output price inflation $\hat{\pi}_{i,t}^{Y}$ satisfies $\hat{\pi}_{i,t}^{Y} = \ln\hat{P}_{i,t}^{Y} - \ln\hat{P}_{i,t-1}^{Y}$. The consumption price level $\ln\hat{P}_{i,t}^{C}$ depends on the contemporaneous core price level and external terms of trade according to consumption price relationship:

$$\ln\hat{P}_{i,t}^{C} = \ln\hat{P}_{i,t} + \frac{M_i}{Y_i}\ln\hat{\mathcal{T}}_{i,t}^{M}. \tag{140}$$

Consumption price inflation $\hat{\pi}_{i,t}^{C}$ satisfies $\hat{\pi}_{i,t}^{C} = \ln\hat{P}_{i,t}^{C} - \ln\hat{P}_{i,t-1}^{C}$. The response coefficients of these relationships vary across economies with their trade openness.

[2] In steady state equilibrium $\nu_i^{\mathcal{A}} = \nu_i^{C} = \nu_i^{I^H} = \nu_i^{I^K} = \nu_i^{X} = \upsilon_i^{i^B} = \nu_i^{H} = \upsilon_i^{i^S} = \upsilon_i^{B} = \upsilon_i^{S} = \nu_i^{\mathcal{E}} = 1$, $\nu_i^{\delta^M} = \nu_i^{\delta^C} = \nu_i^{i^P} = \nu_i^{G^C} = \nu_i^{G^I} = \nu_i^{\tau^K} = \nu_i^{\tau^L} = \nu_i^{T} = \nu_i^{\kappa} = \nu_i^{\phi^D} = \nu_i^{\phi^F} = 0$, $\nu_i^{M} = \frac{\theta^{M}}{\theta^{M}-1}$, and $h_i^{\nu^{\mathcal{A}}} = h_i^{\nu^{\mathcal{N}}} = h_i^{\nu^{C}} = h_i^{\nu^{I}} = h_i^{\nu^{X}} = h_i^{\nu^{M}} = h_i^{\vartheta^{Y}} = h_i^{\vartheta^{L}} = h_i^{\vartheta^{X}} = h_i^{\vartheta^{M}} = h_k^{\vartheta^{Y}} = h_i^{\nu^{i,B}} = h_i^{\nu^{H}} = h_i^{\nu^{i,S}} = h_i^{\nu^{B}} = h_i^{\nu^{S}} = h_i^{\nu^{\mathcal{E}}} = h_i^{\vartheta^{C}} = h_i^{\nu^{\delta}} = h_i^{\nu^{i,P}} = h_i^{\nu^{G,C}} = h_i^{\nu^{G,I}} = h_i^{\nu^{\tau}} = h_i^{\nu^{T}} = h_i^{\nu^{\kappa}} = h_i^{\nu^{\phi}} = 0$.

Output $\ln \hat{Y}_{i,t}$ depends on contemporaneous domestic demand, exports and imports according to output demand relationship:

$$\ln \hat{Y}_{i,t} = \ln \hat{D}_{i,t} + \frac{X_i}{Y_i} \ln \frac{\hat{X}_{i,t}}{\hat{M}_{i,t}}. \tag{141}$$

Domestic demand $\ln \hat{D}_{i,t}$ depends on a weighted average of contemporaneous consumption, investment and public domestic demand according to domestic demand relationship:

$$\ln \hat{D}_{i,t} = \frac{C_i}{Y_i} \ln \hat{C}_{i,t} + \frac{I_i}{Y_i} \ln \hat{I}_{i,t} + \frac{G_i}{Y_i} \ln \hat{G}_{i,t}. \tag{142}$$

Investment $\ln \hat{I}_{i,t}$ depends on a weighted average of contemporaneous residential and business investment according to investment demand relationship:

$$\frac{I_i}{Y_i} \ln \hat{I}_{i,t} = \frac{I_i^H}{Y_i} \ln \hat{I}_{i,t}^H + \frac{I_i^K}{Y_i} \ln \hat{I}_{i,t}^K. \tag{143}$$

Public domestic demand $\ln \hat{G}_{i,t}$ depends on a weighted average of contemporaneous public consumption and investment according to public domestic demand relationship:

$$\frac{G_i}{Y_i} \ln \hat{G}_{i,t} = \frac{G_i^C}{Y_i} \ln \hat{G}_{i,t}^C + \frac{G_i^I}{Y_i} \ln \hat{G}_{i,t}^I. \tag{144}$$

The response coefficients of these relationships vary across economies with the composition of their domestic demand or their trade openness.

Consumption $\ln \hat{C}_{i,t}$ depends on a weighted average of its past and expected future values driven by a weighted average of the contemporaneous real property and portfolio returns according to consumption demand relationship:

$$\begin{aligned} \ln \hat{C}_{i,t} &= \frac{\alpha^C}{1+\alpha^C} \ln \hat{C}_{i,t-1} + \frac{1}{1+\alpha^C} \mathrm{E}_t \ln \hat{C}_{i,t+1} \\ &\quad -(1-\phi^C)\sigma \frac{1-\alpha^C}{1+\alpha^C} \mathrm{E}_t \left[\frac{\phi^B}{1-\phi^C} \hat{r}_{i,t+1}^{A^{B,H}} + \left(1 - \frac{\phi^B}{1-\phi^C}\right) \hat{r}_{i,t+1}^{A^{A,H}} - \ln \frac{\hat{v}_{i,t}^C}{\hat{v}_{i,t+1}^C} \right] + \phi^C \mathcal{P}_1(L) \ln \hat{C}_{i,t}^C. \end{aligned} \tag{145}$$

Reflecting the existence of credit constraints, consumption also depends on contemporaneous, past and expected future credit constrained consumption, where polynomial in the lag operator $\mathcal{P}_1(L) = 1 - \frac{\alpha^C}{1+\alpha^C} L - \frac{1}{1+\alpha^C} \mathrm{E}_t L^{-1}$. Credit constrained consumption $\ln \hat{C}_{i,t}^C$ depends on contemporaneous output and the terms of trade according to credit constrained consumption demand relationship

$$\begin{aligned} \ln \hat{C}_{i,t}^C &= \lambda_i^C \left(\frac{C_i}{Y_i}\right)^{-1} \left\{ (1-\tau_i) \left\{ \ln \hat{Y}_{i,t} + \frac{X_i}{Y_i} \ln \hat{\mathcal{T}}_{i,t} - \frac{1}{1-\tau_i} \left[\left(1 - \frac{W_i L_i}{P_i^Y Y_i}\right) \hat{\tau}_{i,t}^K + \frac{W_i L_i}{P_i^Y Y_i} \hat{\tau}_{i,t}^L \right] \right\} + \lambda_i^T \frac{1}{\phi^C} \frac{\hat{T}_{i,t}^C}{P_{i,t}^Y Y_{i,t}} \right. \\ &\quad \left. + \lambda \frac{M_i^S}{P_i^Y Y_i} \left[\ln \frac{\hat{M}_{i,t+1}^S}{\hat{P}_{i,t}^C} - \frac{1}{\beta} \left((\hat{i}_{i,t-1}^B - \hat{\pi}_{i,t}^C) + \ln \frac{\hat{M}_{i,t}^S}{\hat{P}_{i,t-1}^C} \right) + \frac{1-\beta}{\beta} \left(\ln \hat{Y}_{i,t} + \frac{X_i}{Y_i} \ln \hat{\mathcal{T}}_{i,t} \right) \right] - \lambda \frac{I_i}{Y_i} \ln \hat{I}_{i,t} \right\}, \end{aligned} \tag{146}$$

where economy specific auxiliary parameters $\lambda_i^C = \frac{1}{1-\tau_i}\frac{C_i}{Y_i}$ and $\lambda_i^T = 1-\tau_i$. Credit constrained consumption also depends on a weighted average of the contemporaneous corporate and labor income tax rates, as well as the contemporaneous transfer payment ratio. The response coefficients of this relationship vary across economies with the size of their government, their trade openness, and their labor income share.

Residential investment $\ln \hat{I}_{i,t}^H$ depends on a weighted average of its past and expected future values driven by the contemporaneous relative shadow price of housing according to residential investment demand relationship:

$$\ln \hat{I}_{i,t}^H = \frac{1}{1+\beta}\ln \hat{I}_{i,t-1}^H + \frac{\beta}{1+\beta}\mathrm{E}_t \ln \hat{I}_{i,t+1}^H + \frac{1}{\chi^H(1+\beta)}\ln\left(\hat{v}_{i,t}^{I^H}\frac{\hat{Q}_{i,t}^H}{\hat{P}_{i,t}^C}\right) + \mathcal{P}_2(L)\left(\ln \hat{Y}_{i,t} + \frac{X_i}{Y_i}\ln \hat{\mathcal{T}}_{i,t}\right). \quad (147)$$

Residential investment also depends on contemporaneous, past and expected future output and the terms of trade, where polynomial in the lag operator $\mathcal{P}_2(L) = 1 - \frac{1}{1+\beta}L - \frac{\beta}{1+\beta}\mathrm{E}_t L^{-1}$. Reflecting the existence of a financial accelerator mechanism, the relative shadow price of housing depends on its expected future value, as well as the contemporaneous real property return and mortgage loan rate, according to residential investment Euler equation

$$\begin{aligned}\ln\frac{\hat{Q}_{i,t}^H}{\hat{P}_{i,t}^C} = \mathrm{E}_t\Bigg\{&\beta(1-\delta^H)\ln\frac{\hat{Q}_{i,t+1}^H}{\hat{P}_{i,t+1}^C} - \left[(1-\phi^D)\left(\hat{r}_{i,t+1}^{A^{B,H}} + \frac{\hat{\phi}_{i,t}^D}{\phi^D}\right) + \phi^D\beta\frac{\theta^C}{\theta^C-1}\frac{1+\kappa^R(1-\beta(1-\chi^C\delta))}{\beta}\left(\hat{r}_{i,t}^M - \lambda\hat{\delta}_{i,t+1}^M + \lambda\frac{\hat{\phi}_{i,t}^D}{\phi^D}\right)\right] \\ &+\left[(1-\beta(1-\delta^H)) + \phi^D\beta\left(\frac{\theta^C}{\theta^C-1}\frac{1+\kappa^R(1-\beta(1-\chi^C\delta))}{\beta} - \frac{1}{\beta}\right)\right]\ln\frac{\hat{\iota}_{i,t+1}^H}{\hat{P}_{i,t+1}^C}\Bigg\} + \frac{\hat{\phi}_{i,t}^D}{\phi^D},\end{aligned} \quad (148)$$

which determines the shadow price of housing $\ln \hat{Q}_{i,t}^H$. The relative shadow price of housing also depends on the expected future real rental price of housing and the contemporaneous regulatory mortgage loan to value ratio limit. The real rental price of housing depends on the deviation of contemporaneous consumption from the past housing stock according to rental price of housing relationship

$$\ln\frac{\hat{\iota}_{i,t}^H}{\hat{P}_{i,t}^C} = \frac{1}{\varsigma}\ln\frac{\hat{C}_{i,t}}{\hat{H}_{i,t}}, \quad (149)$$

which determines the rental price of housing $\ln \hat{\iota}_{i,t}^H$. The housing stock $\ln \hat{H}_{i,t+1}$ is accumulated according to $\ln \hat{H}_{i,t+1} = (1-\delta^H)\ln \hat{H}_{i,t} + \delta^H \ln(\hat{v}_{i,t}^{I^H}\hat{I}_{i,t}^H)$.

Business investment $\ln \hat{I}_{i,t}^K$ depends on a weighted average of its past and expected future values driven by the contemporaneous relative shadow price of private physical capital according to business investment demand relationship:

$$\ln \hat{I}_{i,t}^K = \frac{1}{1+\beta}\ln \hat{I}_{i,t-1}^K + \frac{\beta}{1+\beta}\mathrm{E}_t \ln \hat{I}_{i,t+1}^K + \frac{1}{\chi^K(1+\beta)}\ln\left(\hat{v}_{i,t}^{I^K}\frac{\hat{Q}_{i,t}^K}{\hat{P}_{i,t}^C}\right) + \mathcal{P}_2(L)\left(\ln \hat{Y}_{i,t} + \frac{X_i}{Y_i}\ln \hat{\mathcal{T}}_{i,t}\right). \quad (150)$$

Business investment also depends on contemporaneous, past and expected future output and the terms of trade. Reflecting the existence of a financial accelerator mechanism, the relative shadow price of private physical capital depends on its expected future value, as well as the contemporaneous real portfolio return and effective corporate loan rate, according to business investment Euler equation

$$\ln\frac{\hat{Q}^K_{i,t}}{\hat{P}^C_{i,t}} = \mathrm{E}_t\left\{\beta(1-\delta^K)\ln\frac{\hat{Q}^K_{i,t+1}}{\hat{P}^C_{i,t+1}} - \left[(1-\phi^F)\left(\hat{r}^{A^{A,H}}_{i,t+1} + \frac{\hat{\phi}^F_{i,t}}{\phi^F}\right) + \phi^F\beta\frac{\theta^C}{\theta^C-1}\frac{1+\kappa^R(1-\beta(1-\chi^C\delta))}{\beta}\left(\hat{r}^{C,E}_{i,t+1} - \lambda\hat{\delta}^C_{i,t+1} + \lambda\frac{\hat{\phi}^F_{i,t}}{\phi^F}\right)\right] \right. \tag{151}$$
$$\left. + \left[(1-\beta(1-\delta^K)) + \phi^F\beta\left(\frac{\theta^C}{\theta^C-1}\frac{1+\kappa^R(1-\beta(1-\chi^C\delta))}{\beta} - \frac{1}{\beta}\right)\right]\left(\eta^K\ln\hat{u}^K_{i,t+1} - \frac{1}{1-\tau_i}\hat{\tau}^K_{i,t+1}\right)\right\} + \frac{\hat{\phi}^F_{i,t}}{\phi^F},$$

which determines the shadow price of private physical capital $\ln\hat{Q}^K_{i,t}$. The relative shadow price of private physical capital also depends on the expected future capital utilization and corporate tax rates, as well as the contemporaneous regulatory corporate loan to value ratio limit. The capital utilization rate $\ln\hat{u}^K_{i,t}$ depends on the contemporaneous real wage, as well as the deviation of the past private physical capital stock from contemporaneous employment, according to capital utilization relationship:

$$\ln\hat{u}^K_{i,t} = \frac{1}{1+\eta^K}\left(\ln\frac{\hat{W}_{i,t}}{\hat{P}^C_{i,t}} - \ln\frac{\hat{K}_{i,t}}{\hat{L}_{i,t}}\right). \tag{152}$$

The private physical capital stock $\ln\hat{K}_{i,t+1}$ is accumulated according to $\ln\hat{K}_{i,t+1} = (1-\delta^K)\ln\hat{K}_{i,t} + \delta^K\ln(\hat{v}^{I^K}_{i,t}\hat{I}^K_{i,t})$.

Exports $\ln\hat{X}_{i,t}$ depend on contemporaneous export weighted foreign imports according to export demand relationship:

$$\ln\hat{X}_{i,t} = \sum_{j=1}^{N} w^X_{i,j}\ln\frac{\hat{M}_{j,t}}{\hat{v}^X_{i,t}}. \tag{153}$$

Imports $\ln\hat{M}_{i,t}$ depend on contemporaneous domestic demand, as well as the external terms of trade, according to import demand relationship:

$$\ln\hat{M}_{i,t} = \ln\frac{\hat{D}_{i,t}}{\hat{v}^M_{i,t}} - \psi^M\ln\hat{\mathcal{T}}^M_{i,t}. \tag{154}$$

The response coefficients of the former relationship vary across economies with their trade pattern.

The nominal property return $\mathrm{E}_t\hat{i}^{A^{B,H}}_{i,t+1}$ depends on the contemporaneous nominal interbank loans rate according to property return function:

$$\mathrm{E}_t\hat{i}^{A^{B,H}}_{i,t+1} = \hat{i}^B_{i,t} + \phi^H\ln\hat{v}^H_{i,t}. \tag{155}$$

Reflecting the existence of a portfolio balance mechanism, the nominal property return also depends on the contemporaneous housing risk premium. The real property return $\mathrm{E}_t\hat{r}^{A^{B,H}}_{i,t+1}$ satisfies $\mathrm{E}_t\hat{r}^{A^{B,H}}_{i,t+1} = \mathrm{E}_t\hat{i}^{A^{B,H}}_{i,t+1} - \mathrm{E}_t\hat{\pi}^C_{i,t+1}$.

The nominal interbank loans rate $\hat{i}^B_{i,t}$ depends on the contemporaneous nominal short term bond yield adjusted by the liquidity risk premium according to interbank market relationship:

$$\hat{i}^B_{i,t} = \hat{i}^S_{i,t} + \ln\hat{\upsilon}^{i^B}_{i,t}. \tag{156}$$

The real interbank loans rate $\hat{r}_{i,t}^{B}$ satisfies $\hat{r}_{i,t}^{B} = \hat{i}_{i,t}^{B} - \mathrm{E}_t \hat{\pi}_{i,t+1}^{C}$. The liquidity risk premium $\ln \hat{\upsilon}_{i,t}^{i^B}$ satisfies dynamic factor process $\ln \hat{\upsilon}_{i,t}^{i^B} = \lambda_k^M \sum_{j=1}^{N} w_j^A \ln \hat{v}_{j,t}^{i^B} + (1-\lambda_k^M w_i^A) \ln \hat{v}_{i,t}^{i^B}$. The intensity of international interbank market contagion varies across economies, with $k=0$ for low interbank market contagion economies, $k=1$ for medium interbank market contagion economies, and $k=2$ for high interbank market contagion economies, where $\lambda_0^M < \lambda_1^M < \lambda_2^M$.

The price of housing $\ln \hat{V}_{i,t}^{H}$ depends on its expected future value driven by expected future developer profits, and the contemporaneous nominal interbank loans rate adjusted by the housing risk premium, according to housing market relationship:

$$\ln \hat{V}_{i,t}^{H} = \beta \mathrm{E}_t \ln \hat{V}_{i,t+1}^{H} + (1-\beta)\mathrm{E}_t \ln \hat{\Pi}_{i,t+1}^{H} - (\hat{i}_{i,t}^{B} + \ln \hat{v}_{i,t}^{H}). \tag{157}$$

Developer profits $\ln \hat{\Pi}_{i,t}^{H}$ depends on contemporaneous housing rental revenues according to developer profit function

$$\begin{aligned}\ln \hat{\Pi}_{i,t}^{H} = \lambda_i^H \left(\frac{\Pi_i^H}{P_i^Y Y_i} \right)^{-1} & \left\{ \frac{\iota_i^H}{P_i^Y} \frac{H_i}{Y_i} \ln(\hat{\iota}_{i,t}^{H} \hat{H}_{i,t}) \right. \\ & \left. + \lambda \frac{B_i^{C,D}}{P_i^Y Y_i} \left[\ln \hat{B}_{i,t+1}^{C,D} - (1-\delta)(1+i_i^M)(\hat{i}_{i,t-1}^{M} - \hat{\delta}_{i,t}^{M} + \ln \hat{B}_{i,t}^{C,D}) \right] - \lambda \frac{I_i^H}{Y_i} \ln(\hat{P}_{i,t}^{C} \hat{I}_{i,t}^{H}) \right\},\end{aligned} \tag{158}$$

where economy specific auxiliary parameter $\lambda_i^H = \frac{\Pi_i^H}{P_i^Y Y_i} \left(\frac{\iota_i^H}{P_i^Y} \frac{H_i}{Y_i} \right)^{-1}$.

The nominal portfolio return $\mathrm{E}_t \hat{i}_{i,t+1}^{A^{A,H}}$ depends on the contemporaneous nominal short term bond yield according to portfolio return function:

$$\mathrm{E}_t \hat{i}_{i,t+1}^{A^{A,H}} = \hat{i}_{i,t}^{S} + \phi_B^A \sum_{j=1}^{N} w_{i,j}^B \left(\ln \hat{\upsilon}_{j,t}^{B} + \lambda \frac{1-\phi_S^A}{\phi_B^A} \ln \frac{\hat{v}_{j,t}^{\mathcal{E}}}{\hat{v}_{i,t}^{\mathcal{E}}} \right) + \phi_S^A \sum_{j=1}^{N} w_{i,j}^S \left(\ln \hat{\upsilon}_{j,t}^{S} + \lambda \ln \frac{\hat{v}_{j,t}^{\mathcal{E}}}{\hat{v}_{i,t}^{\mathcal{E}}} \right). \tag{159}$$

Reflecting the existence of a portfolio balance mechanism, the nominal portfolio return also depends on contemporaneous domestic and foreign duration and equity risk premia. The response coefficients of this relationship vary across economies with their domestic and foreign bond and stock market exposures. The real portfolio return $\mathrm{E}_t \hat{r}_{i,t+1}^{A^{A,H}}$ satisfies $\mathrm{E}_t \hat{r}_{i,t+1}^{A^{A,H}} = \mathrm{E}_t \hat{i}_{i,t+1}^{A^{A,H}} - \mathrm{E}_t \hat{\pi}_{i,t+1}^{C}$.

The nominal short term bond yield $\hat{i}_{i,t}^{S}$ depends on the contemporaneous nominal policy interest rate adjusted by the credit risk premium according to money market relationship:

$$\hat{i}_{i,t}^{S} = \hat{i}_{i,t}^{P} + \ln \hat{\upsilon}_{i,t}^{i^S}. \tag{160}$$

The real short term bond yield $\hat{r}_{i,t}^{S}$ satisfies $\hat{r}_{i,t}^{S} = \hat{i}_{i,t}^{S} - \mathrm{E}_t \hat{\pi}_{i,t+1}^{C}$. The credit risk premium $\ln \hat{\upsilon}_{i,t}^{i^S}$ satisfies dynamic factor process $\ln \hat{\upsilon}_{i,t}^{i^S} = \lambda_k^B \sum_{j=1}^{N} w_j^A \ln \hat{v}_{j,t}^{i^S} + (1-\lambda_k^B w_i^A) \ln \hat{v}_{i,t}^{i^S}$. The intensity of international money market contagion varies across economies, with $k=0$ for low capital market contagion economies, $k=1$ for medium capital market contagion economies, and $k=2$ for high capital market contagion economies, where $\lambda_0^B < \lambda_1^B < \lambda_2^B$.

The nominal long term bond yield $\hat{i}_{i,t}^{L}$ depends on its expected future value, driven by the contemporaneous nominal short term bond yield adjusted by the duration risk premium, according to bond market relationship:

$$\hat{i}_{i,t}^{L} = \omega^{B}\beta \mathrm{E}_{t}\hat{i}_{i,t+1}^{L} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\left(\omega^{B} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\right)^{-1}(\hat{i}_{i,t}^{S} + \ln\hat{\upsilon}_{i,t}^{B}). \tag{161}$$

The real long term bond yield $\hat{r}_{i,t}^{L}$ depends on its expected future value, driven by the contemporaneous real short term bond yield adjusted by the duration risk premium, according to:

$$\hat{r}_{i,t}^{L} = \omega^{B}\beta \mathrm{E}_{t}\hat{r}_{i,t+1}^{L} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\left(\omega^{B} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\right)^{-1}(\hat{r}_{i,t}^{S} + \ln\hat{\upsilon}_{i,t}^{B}). \tag{162}$$

The term premium $\ln\hat{\mu}_{i,t}^{B}$ depends on its expected future value driven by the contemporaneous duration risk premium according to:

$$\ln\hat{\mu}_{i,t}^{B} = \omega^{B}\beta \mathrm{E}_{t}\ln\hat{\mu}_{i,t+1}^{B} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\left(\omega^{B} + \frac{1-\omega^{B}\beta}{\omega^{B}\beta}\right)^{-1}\ln\hat{\upsilon}_{i,t}^{B}. \tag{163}$$

The duration risk premium $\ln\hat{\upsilon}_{i,t}^{B}$ satisfies dynamic factor process $\ln\hat{\upsilon}_{i,t}^{B} = \lambda_{k}^{B}\sum_{j=1}^{N} w_{j}^{A}\ln\hat{v}_{j,t}^{B} + (1-\lambda_{k}^{B}w_{i}^{A})\ln\hat{v}_{i,t}^{B}$. The intensity of international bond market contagion varies across economies, with $k=0$ for low capital market contagion economies, $k=1$ for medium capital market contagion economies, and $k=2$ for high capital market contagion economies, where $\lambda_{0}^{B} < \lambda_{1}^{B} < \lambda_{2}^{B}$.

The price of equity $\ln\hat{V}_{i,t}^{S}$ depends on its expected future value driven by expected future nonfinancial corporate profits, and the contemporaneous nominal short term bond yield adjusted by the equity risk premium, according to stock market relationship:

$$\ln\hat{V}_{i,t}^{S} = \beta \mathrm{E}_{t}\ln\hat{V}_{i,t+1}^{S} + (1-\beta)\mathrm{E}_{t}\ln\hat{\Pi}_{i,t+1}^{S} - (\hat{i}_{i,t}^{S} + \ln\hat{\upsilon}_{i,t}^{S}). \tag{164}$$

Nonfinancial corporate profits $\ln\hat{\Pi}_{i,t}^{S}$ depends on contemporaneous nominal output and the corporate tax rate according to nonfinancial corporate profit function

$$\begin{aligned}\ln\hat{\Pi}_{i,t}^{S} = \lambda_{i}^{S}\left(\frac{\Pi_{i}^{S}}{P_{i}^{Y}Y_{i}}\right)^{-1}&\left\{(1-\tau_{i})\left[\ln(\hat{P}_{i,t}^{Y}\hat{Y}_{i,t}) - \lambda\frac{W_{i}L_{i}}{P_{i}^{Y}Y_{i}}\ln(\hat{W}_{i,t}\hat{L}_{i,t}) - \frac{1}{1-\tau_{i}}\left(1-\frac{W_{i}L_{i}}{P_{i}^{Y}Y_{i}}\right)\hat{\tau}_{i,t}^{K}\right]\right.\\ &\left.+\lambda\frac{B_{i}^{C,F}}{P_{i}^{Y}Y_{i}}\left[\ln\hat{B}_{i,t+1}^{C,F} - (1-\delta)(1+i_{i}^{C})(\hat{i}_{i,t}^{C,E} - \hat{\delta}_{i,t}^{C} + \ln\hat{B}_{i,t}^{C,F})\right] - \lambda\frac{I_{i}^{K}}{Y_{i}}\ln(\hat{P}_{i,t}^{C}\hat{I}_{i,t}^{K})\right\},\end{aligned} \tag{165}$$

where economy specific auxiliary parameter $\lambda_{i}^{S} = \frac{1}{1-\tau_{i}}\frac{\Pi_{i}^{S}}{P_{i}^{Y}Y_{i}}$. The response coefficients of this relationship vary across economies with the size of their government and their labor income share. The equity risk premium $\ln\hat{\upsilon}_{i,t}^{S}$ satisfies dynamic factor process $\ln\hat{\upsilon}_{i,t}^{S} = \lambda_{k}^{S}\sum_{j=1}^{N} w_{j}^{A}\ln\hat{v}_{j,t}^{S} + (1-\lambda_{k}^{S}w_{i}^{A})\ln\hat{v}_{i,t}^{S}$. The intensity of international stock market contagion varies across economies, with $k=0$ for low capital market contagion economies, $k=1$ for medium capital market contagion economies, and $k=2$ for high capital market contagion economies, where $\lambda_{0}^{S} < \lambda_{1}^{S} < \lambda_{2}^{S}$.

Under a flexible inflation targeting or managed exchange rate regime, the nominal policy interest rate $\hat{i}_{i,t}^{P}$ depends on a weighted average of its past and desired values according to monetary policy rule:

$$\hat{i}_{i,t}^{P} = \rho^{i}\hat{i}_{i,t-1}^{P} + (1-\rho^{i})(\xi^{\pi}\mathrm{E}_{t}\hat{\pi}_{i,t+1}^{C} + \xi^{Y}\ln\hat{\hat{Y}}_{i,t} + \xi_{j}^{\mathcal{E}}\Delta\ln\hat{\mathcal{E}}_{i,t}) + \hat{v}_{i,t}^{i^{P}}. \tag{166}$$

Under a flexible inflation targeting regime $j=0$, and the desired nominal policy interest rate responds to expected future consumption price inflation and the contemporaneous output gap. For the leader economy of a monetary union, the target variables entering into its monetary policy rule are expressed as output weighted averages across union members. Under a managed exchange rate regime $j=1$, and the desired nominal policy interest rate also responds to the contemporaneous change in the nominal effective exchange rate. Under a fixed exchange rate regime, the nominal policy interest rate instead tracks the contemporaneous nominal policy interest rate of the economy that issues the anchor currency one for one, while responding to the contemporaneous change in the corresponding nominal bilateral exchange rate, according to monetary policy rule:

$$\hat{i}_{i,t}^{P} = \hat{i}_{k,t}^{P} + \xi^{\mathcal{E}_{k}}\Delta\ln\hat{\mathcal{E}}_{i,k,t}. \tag{167}$$

It follows that under a fixed exchange rate regime, $\ln\hat{\mathcal{E}}_{i,k,t}=0$ in equilibrium, in the absence of asymmetric credit and currency risk premium shocks. The real policy interest rate $\hat{r}_{i,t}^{P}$ satisfies $\hat{r}_{i,t}^{P} = \hat{i}_{i,t}^{P} - \mathrm{E}_{t}\hat{\pi}_{i,t+1}^{C}$.

Bank credit depends on a weighted average of the contemporaneous money and bank capital stocks according to bank balance sheet identity

$$\ln\hat{B}_{i,t+1}^{C,B} = (1-\kappa^{R})\ln\hat{M}_{i,t+1}^{S} + \kappa^{R}\ln\hat{K}_{i,t+1}^{B}, \tag{168}$$

which determines the money stock $\ln\hat{M}_{i,t+1}^{S}$. Bank credit $\ln\hat{B}_{i,t+1}^{C,B}$ depends on a weighted average of contemporaneous mortgage debt, and the bank lending weighted average of contemporaneous domestic currency denominated domestic and foreign nonfinancial corporate debt, according to bank credit demand function:

$$\ln\hat{B}_{i,t+1}^{C,B} = w_{i}^{C}\ln\hat{B}_{i,t+1}^{C,D} + (1-w_{i}^{C})\sum_{j=1}^{N}w_{i,j}^{C}\ln\frac{\hat{B}_{j,t+1}^{C,F}}{\hat{\mathcal{E}}_{j,i,t}}. \tag{169}$$

Mortgage debt $\ln\hat{B}_{i,t+1}^{C,D}$ satisfies $\ln\hat{B}_{i,t+1}^{C,D} = \ln\hat{P}_{i,t}^{C} + \ln\hat{H}_{i,t+1} + \frac{\hat{\phi}_{i,t}^{D}}{\phi^{D}}$, while nonfinancial corporate debt $\ln\hat{B}_{i,t+1}^{C,F}$ satisfies $\ln\hat{B}_{i,t+1}^{C,F} = \ln\hat{P}_{i,t}^{C} + \ln\hat{K}_{i,t+1} + \frac{\hat{\phi}_{i,t}^{F}}{\phi^{F}}$. The bank capital ratio $\hat{\kappa}_{i,t+1}$ satisfies $\hat{\kappa}_{i,t+1} = \kappa^{R}(\ln\hat{K}_{i,t+1}^{B} - \ln\hat{B}_{i,t+1}^{C,B})$.

The nominal effective corporate loan rate $\hat{i}_{i,t}^{C,E}$ depends on the nonfinancial corporate borrowing weighted average of past domestic and foreign nominal corporate loan rates, adjusted for contemporaneous changes in nominal bilateral exchange rates, according to effective corporate loan rate function:

$$\hat{i}_{i,t}^{C,E} = \sum_{j=1}^{N}w_{i,j}^{F}\left(\hat{i}_{j,t-1}^{C} + \ln\frac{\hat{\mathcal{E}}_{i,j,t}}{\hat{\mathcal{E}}_{i,j,t-1}}\right). \tag{170}$$

The corporate credit loss rate $\hat{\delta}_{i,t}^{C,E}$ depends on the bank lending weighted average of contemporaneous domestic and foreign corporate loan default rates according to corporate credit loss rate function:

$$\hat{\delta}_{i,t}^{C,E} = \sum_{j=1}^{N} w_{i,j}^{C} \hat{\delta}_{j,t}^{C}. \tag{171}$$

The real effective corporate loan rate $\mathrm{E}_t \hat{r}_{i,t+1}^{C,E}$ satisfies $\mathrm{E}_t \hat{r}_{i,t+1}^{C,E} = \mathrm{E}_t \hat{i}_{i,t+1}^{C,E} - \mathrm{E}_t \hat{\pi}_{i,t+1}^{C}$.

The nominal mortgage and corporate loan rates $\hat{i}_{i,t}^{f(Z)}$ depend on a weighted average of their past and expected future values, driven by the deviation of the past nominal interbank loans rate from the contemporaneous nominal mortgage or corporate loan rate net of the contemporaneous mortgage or corporate credit loss rate, according to lending rate Phillips curves

$$\begin{aligned}\hat{i}_{i,t}^{f(Z)} = {} & \frac{1}{1+\beta}\hat{i}_{i,t-1}^{f(Z)} + \frac{\beta}{1+\beta}\mathrm{E}_t \hat{i}_{i,t+1}^{f(Z)} + \frac{(1-\omega^C)(1-\omega^C\beta)}{\omega^C(1+\beta)}\Bigg\{\left[\hat{i}_{i,t-1}^{B} - (\hat{i}_{i,t}^{f(Z)} - \hat{\delta}_{i,t}^{g(Z)})\right] \\ & - \frac{1-\beta(1-\chi^C\delta)}{1+\kappa^R(1-\beta(1-\chi^C\delta))}\left[\eta^C(\hat{\kappa}_{i,t} - \hat{\kappa}_{i,t}^{R}) - (\hat{\kappa}_{i,t}^{R} - \kappa^R \hat{i}_{i,t-1}^{B})\right] + \ln \hat{\vartheta}_{i,t}^{C^Z}\Bigg\},\end{aligned} \tag{172}$$

where $Z \in \{D, F\}$, while $f(D) = M$ and $f(F) = C$. The nominal mortgage and corporate loan rates also depend on the past deviation of the bank capital ratio from its required value, as well as the past deviation of the regulatory bank capital ratio requirement from its funding cost, where $g(D) = M$ and $g(F) = C,E$. The real mortgage and corporate loan rates $\hat{r}_{i,t}^{f(Z)}$ satisfy $\hat{r}_{i,t}^{f(Z)} = \hat{i}_{i,t}^{f(Z)} - \mathrm{E}_t \hat{\pi}_{i,t+1}^{C}$.

Bank retained earnings $\ln \hat{I}_{i,t}^{B}$ depends on a weighted average of its past and expected future values driven by the contemporaneous shadow price of bank capital according to retained earnings relationship:

$$\ln \hat{I}_{i,t}^{B} = \frac{1}{1+\beta}\ln \hat{I}_{i,t-1}^{B} + \frac{\beta}{1+\beta}\mathrm{E}_t \ln \hat{I}_{i,t+1}^{B} + \frac{1}{\chi^B(1+\beta)}\ln \hat{Q}_{i,t}^{B}. \tag{173}$$

The shadow price of bank capital $\ln \hat{Q}_{i,t}^{B}$ depends on its expected future value net of the expected future bank capital destruction rate, as well as the contemporaneous nominal interbank loans rate, according to retained earnings Euler equation:

$$\ln \hat{Q}_{i,t}^{B} = \mathrm{E}_t \left\{\beta(1-\chi^C\delta)(\ln \hat{Q}_{i,t+1}^{B} - \hat{\delta}_{i,t+1}^{B}) - \left[\hat{i}_{i,t}^{B} + (1-\beta(1-\chi^C\delta))\frac{\eta^C}{\kappa^R}(\hat{\kappa}_{i,t+1} - \hat{\kappa}_{i,t+1}^{R})\right]\right\}. \tag{174}$$

The shadow price of bank capital also depends on the contemporaneous deviation of the bank capital ratio from its required value. The bank capital stock $\ln \hat{K}_{i,t+1}^{B}$ is accumulated according to $\ln \hat{K}_{i,t+1}^{B} = (1-\chi^C\delta)(\ln \hat{K}_{i,t}^{B} - \hat{\delta}_{i,t}^{B}) + \chi^C\delta \ln \hat{I}_{i,t}^{B}$, where the bank capital destruction rate $\hat{\delta}_{i,t}^{B}$ satisfies $\hat{\delta}_{i,t}^{B} = \chi^C(w_i^C \hat{\delta}_{i,t}^{M} + (1-w_i^C)\hat{\delta}_{i,t}^{C,E})$.

The regulatory bank capital ratio requirement $\hat{\kappa}_{i,t+1}^{R}$ depends on a weighted average of its past and desired values according to countercyclical capital buffer rule:

$$\hat{\kappa}_{i,t+1}^{R}=\rho_{\kappa}\hat{\kappa}_{i,t}^{R}+(1-\rho_{\kappa})(\zeta^{\kappa,B}\Delta\ln\hat{B}_{i,t+1}^{C,B}+\zeta^{\kappa,V^{H}}\Delta\ln\hat{V}_{i,t}^{H}+\zeta^{\kappa,V^{S}}\Delta\ln\hat{V}_{i,t}^{S})+\hat{v}_{i,t}^{\kappa}. \tag{175}$$

The desired regulatory bank capital ratio requirement responds to contemporaneous bank credit growth, as well as to contemporaneous changes in the prices of housing and equity. The regulatory mortgage and corporate loan to value ratio limits $\hat{\phi}_{i,t}^{Z}$ depend on a weighted average of their past and desired values according to loan to value limit rules

$$\hat{\phi}_{i,t}^{Z}=\rho_{\phi^{Z}}\hat{\phi}_{i,t-1}^{Z}-(1-\rho_{\phi^{Z}})(\zeta^{\phi^{Z},B}\Delta\ln\hat{B}_{i,t+1}^{C,Z}+\zeta^{\phi^{Z},V}\Delta\ln\hat{V}_{i,t}^{f(Z)})+\hat{v}_{i,t}^{\phi^{Z}}, \tag{176}$$

where $Z\in\{D,F\}$, while $f(D)=H$ and $f(F)=S$. The desired regulatory mortgage or corporate loan to value ratio limit responds to contemporaneous mortgage or nonfinancial corporate debt growth, as well as to the contemporaneous change in the price of housing or equity, respectively. The mortgage and corporate loan default rates $\hat{\delta}_{i,t}^{Z}$ depend on a weighted average of their past and attractor values according to default rate relationships

$$\hat{\delta}_{i,t}^{Z}=\rho_{\delta}\hat{\delta}_{i,t-1}^{Z}-(1-\rho_{\delta})(\zeta^{\delta^{Z},Y}\ln\hat{\hat{Y}}_{i,t}+\zeta^{\delta^{Z},V}\Delta\ln\hat{V}_{i,t}^{f(Z)})+\hat{v}_{i,t}^{\delta^{Z}}, \tag{177}$$

where $Z\in\{M,C\}$, while $f(M)=H$ and $f(C)=S$. The attractor loan default rate depends on the contemporaneous output gap, as well as the contemporaneous change in the price of housing or equity, respectively.

The real effective wage depends on a weighted average of its past and expected future values driven by the contemporaneous and past unemployment rates according to wage Phillips curve

$$\begin{aligned}\ln\frac{\hat{W}_{i,t}}{\hat{P}_{i,t}^{C}\hat{\hat{\mathcal{A}}}_{i,t}}&=\frac{1}{1+\beta}\ln\frac{\hat{W}_{i,t-1}}{\hat{P}_{i,t-1}^{C}\hat{\hat{\mathcal{A}}}_{i,t-1}}+\frac{\beta}{1+\beta}\mathrm{E}_{t}\ln\frac{\hat{W}_{i,t+1}}{\hat{P}_{i,t+1}^{C}\hat{\hat{\mathcal{A}}}_{i,t+1}}\\&\quad-\frac{(1-\omega^{L})(1-\omega^{L}\beta)}{\omega^{L}(1+\beta)}\left[\frac{1}{\eta}\frac{1}{1-\alpha^{L}}(\hat{u}_{i,t}^{L}-\alpha^{L}\hat{u}_{i,t-1}^{L})-\ln\hat{\vartheta}_{i,t}^{L}\right]-\frac{1+\gamma^{L}\beta}{1+\beta}\mathcal{P}_{3}(L)\Delta\ln(\hat{P}_{i,t}^{C}\hat{\hat{\mathcal{A}}}_{i,t}),\end{aligned} \tag{178}$$

which determines the nominal wage $\ln\hat{W}_{i,t}$. The real effective wage also depends on contemporaneous, past and expected future consumption price inflation and trend productivity growth, where polynomial in the lag operator $\mathcal{P}_{3}(L)=1-\frac{\gamma^{L}}{1+\gamma^{L}\beta}L-\frac{\beta}{1+\gamma^{L}\beta}\mathrm{E}_{t}L^{-1}$. Wage inflation $\hat{\pi}_{i,t}^{W}$ satisfies $\hat{\pi}_{i,t}^{W}=\ln\hat{W}_{i,t}-\ln\hat{W}_{i,t-1}$, while the unemployment rate $\hat{u}_{i,t}^{L}$ satisfies $\hat{u}_{i,t}^{L}=\ln\hat{N}_{i,t}-\ln\hat{L}_{i,t}$.

The unemployment rate depends on its past value driven by contemporaneous employment and the real effective wage according to labor supply relationship

$$\hat{u}_{i,t}^{L}=\alpha^{L}\hat{u}_{i,t-1}^{L}-(1-\alpha^{L})\left[\iota\ln\frac{\hat{L}_{i,t}}{\hat{v}_{i,t}^{\mathcal{N}}}-\eta\left(\ln\frac{\hat{W}_{i,t}}{\hat{P}_{i,t}^{C}\hat{\hat{\mathcal{A}}}_{i,t}}-\lambda\frac{1}{1-\tau_{i}}\hat{\tau}_{i,t}^{L}\right)\right], \tag{179}$$

which determines the labor force $\ln\hat{N}_{i,t}$. The response coefficients of this relationship vary across economies with the size of their government.

Output depends on the contemporaneous utilized private physical capital stock and effective employment according to production function

$$\ln \hat{Y}_{i,t} = \left(1 - \frac{\theta^Y}{\theta^Y - 1}\frac{W_i L_i}{P_i^Y Y_i}\right)\ln(\hat{u}_{i,t}^K \hat{K}_{i,t}) + \frac{\theta^Y}{\theta^Y - 1}\frac{W_i L_i}{P_i^Y Y_i}\ln(\hat{\mathcal{A}}_{i,t}\hat{L}_{i,t}), \tag{180}$$

which determines employment $\ln \hat{L}_{i,t}$. The output gap $\ln \hat{\tilde{Y}}_{i,t}$ satisfies $\ln \hat{\tilde{Y}}_{i,t} = \ln \hat{Y}_{i,t} - \ln \hat{\bar{Y}}_{i,t}$, where potential output $\ln \hat{\bar{Y}}_{i,t}$ depends on the contemporaneous private physical capital stock and effective labor force according to

$$\ln \hat{\bar{Y}}_{i,t} = \left(1 - \frac{\theta^Y}{\theta^Y - 1}\frac{W_i L_i}{P_i^Y Y_i}\right)\ln \hat{K}_{i,t} + \frac{\theta^Y}{\theta^Y - 1}\frac{W_i L_i}{P_i^Y Y_i}\ln(\hat{\mathcal{A}}_{i,t}\hat{N}_{i,t}), \tag{181}$$

given that full utilization of private physical capital and effective labor is defined by $\ln \hat{u}_{i,t}^K = 0$ and $\hat{u}_{i,t}^L = 0$, respectively. Productivity $\ln \hat{\mathcal{A}}_{i,t}$ depends on the deviation of the past public physical capital stock from the contemporaneous labor force according to:

$$\ln \hat{\mathcal{A}}_{i,t} = \phi^{\mathcal{A}} \ln \hat{\upsilon}_{i,t}^{\mathcal{A}} + (1 - \phi^{\mathcal{A}}) \ln \frac{\hat{K}_{i,t}^G}{\hat{N}_{i,t}}. \tag{182}$$

Trend productivity $\ln \hat{\bar{\mathcal{A}}}_{i,t}$ depends on its past value driven by contemporaneous productivity according to $\ln \hat{\bar{\mathcal{A}}}_{i,t} = \rho^{\mathcal{A}} \ln \hat{\bar{\mathcal{A}}}_{i,t-1} + (1 - \rho^{\mathcal{A}}) \ln \hat{\mathcal{A}}_{i,t}$. The productivity shifter $\ln \hat{\upsilon}_{i,t}^{\mathcal{A}}$ satisfies dynamic factor process $\ln \hat{\upsilon}_{i,t}^{\mathcal{A}} = \lambda^{\mathcal{A}} \sum_{j=1}^{N} w_j^Y \ln \hat{v}_{j,t}^{\mathcal{A}} + (1 - \lambda^{\mathcal{A}} w_i^Y) \ln \hat{v}_{i,t}^{\mathcal{A}}$. The response coefficients of these relationships vary across economies with their labor income share.

The nominal bilateral exchange rate $\ln \hat{\mathcal{E}}_{i,i*,t}$ depends on its expected future value driven by the contemporaneous nominal short term bond yield differential adjusted by the currency risk premium differential according to foreign exchange market relationship:

$$\ln \hat{\mathcal{E}}_{i,i*,t} = \mathrm{E}_t \ln \hat{\mathcal{E}}_{i,i*,t+1} - \left[(\hat{i}_{i,t}^S - \ln \hat{v}_{i,t}^{\mathcal{E}}) - (\hat{i}_{i*,t}^S - \ln \hat{v}_{i*,t}^{\mathcal{E}})\right]. \tag{183}$$

Under a fixed exchange rate regime, the currency risk premium is endogenous and satisfies $(\hat{i}_{i,t}^S - \ln \hat{v}_{i,t}^{\mathcal{E}}) - (\hat{i}_{k,t}^S - \ln \hat{v}_{k,t}^{\mathcal{E}}) = \hat{i}_{i,t}^P - \hat{i}_{k,t}^P$, which ensures that $\ln \hat{\mathcal{E}}_{i,k,t} = 0$ in equilibrium, in the presence of asymmetric credit or currency risk premium shocks. The real bilateral exchange rate $\ln \hat{\mathcal{Q}}_{i,i*,t}$ satisfies $\ln \hat{\mathcal{Q}}_{i,i*,t} = \ln \hat{\mathcal{E}}_{i,i*,t} + \ln \hat{P}_{i*,t}^Y - \ln \hat{P}_{i,t}^Y$, while the terms of trade $\ln \hat{\mathcal{T}}_{i,t}$ satisfies $\ln \hat{\mathcal{T}}_{i,t} = \ln \hat{\upsilon}_t^T + \ln \hat{\mathcal{T}}_{i,t}^X - \ln \hat{\mathcal{T}}_{i,t}^M$, where the internal terms of trade $\ln \hat{\mathcal{T}}_{i,t}^X$ satisfies $\ln \hat{\mathcal{T}}_{i,t}^X = \ln \hat{P}_{i,t}^X - \ln \hat{P}_{i,t}$, and the external terms of trade $\ln \hat{\mathcal{T}}_{i,t}^M$ satisfies $\ln \hat{\mathcal{T}}_{i,t}^M = \ln \hat{P}_{i,t}^M - \ln \hat{P}_{i,t}$.[3]

Export price inflation depends on a linear combination of its past and expected future values, driven by the contemporaneous deviations of the core price level and domestic currency denominated prices of energy and nonenergy commodities from the price of exports, according to export price Phillips curve

[3] The nominal effective exchange rate $\ln \hat{\mathcal{E}}_{i,t}$ satisfies $\ln \hat{\mathcal{E}}_{i,t} = \ln \hat{\mathcal{E}}_{i,i*,t} - \sum_{j=1}^{N} w_{i,j}^T \ln \hat{\mathcal{E}}_{j,i*,t}$, while the real effective exchange rate $\ln \hat{\mathcal{Q}}_{i,t}$ satisfies $\ln \hat{\mathcal{Q}}_{i,t} = \ln \hat{\mathcal{Q}}_{i,i*,t} - \sum_{j=1}^{N} w_{i,j}^T \ln \hat{\mathcal{Q}}_{j,i*,t}$.

$$\hat{\pi}_{i,t}^{X} = \frac{\gamma^{X}(1-\mu^{X})}{1+\gamma^{X}\beta(1-\mu^{X})}\hat{\pi}_{i,t-1}^{X} + \frac{\beta}{1+\gamma^{X}\beta(1-\mu^{X})}\mathrm{E}_t\hat{\pi}_{i,t+1}^{X}$$
$$+\frac{(1-\omega^{X})(1-\omega^{X}\beta)}{\omega^{X}(1+\gamma^{X}\beta(1-\mu^{X}))}\left[\left(1-\sum_{k=1}^{M*}\frac{X_{i,k}}{X_i}\right)\ln\frac{\hat{P}_{i,t}}{\hat{P}_{i,t}^{X}} + \sum_{k=1}^{M*}\frac{X_{i,k}}{X_i}\left(\ln\frac{\hat{\mathcal{E}}_{i,i*,t}\hat{P}_{k,t}^{Y}}{\hat{P}_{i,t}^{X}} + \ln\hat{\vartheta}_{i,t}^{X}\right)\right] \tag{184}$$
$$+\left(1-\sum_{k=1}^{M*}\frac{X_{i,k}}{X_i}\right)\mathcal{P}_4(L)\hat{\pi}_{i,t} + \frac{\mu^{X}\gamma^{X}(1+\beta)}{1+\gamma^{X}\beta(1-\mu^{X})}\mathcal{P}_2(L)\sum_{k=1}^{M*}\frac{X_{i,k}}{X_i}\ln(\hat{\mathcal{E}}_{i,i*,t}\hat{P}_{k,t}^{Y}),$$

which determines the price of exports $\ln\hat{P}_{i,t}^{X}$. Export price inflation also depends on contemporaneous, past and expected future core inflation and the domestic currency denominated prices of energy and nonenergy commodities, where polynomial in the lag operator $\mathcal{P}_4(L) = 1 - \frac{\gamma^{X}(1-\mu^{X})}{1+\gamma^{X}\beta(1-\mu^{X})}L - \frac{\beta}{1+\gamma^{X}\beta(1-\mu^{X})}\mathrm{E}_t L^{-1}$. The response coefficients of this relationship vary across economies with their commodity export intensities. Export price inflation $\hat{\pi}_{i,t}^{X}$ satisfies $\hat{\pi}_{i,t}^{X} = \ln\hat{P}_{i,t}^{X} - \ln\hat{P}_{i,t-1}^{X}$.

Import price inflation $\hat{\pi}_{i,t}^{M}$ depends on a linear combination of its past and expected future values, driven by the contemporaneous deviation of the import weighted average domestic currency denominated price of foreign exports from the price of imports, according to import price Phillips curve

$$\hat{\pi}_{i,t}^{M} = \frac{\gamma^{M}(1-\mu_i^{M})}{1+\gamma^{M}\beta(1-\mu_i^{M})}\hat{\pi}_{i,t-1}^{M} + \frac{\beta}{1+\gamma^{M}\beta(1-\mu_i^{M})}\mathrm{E}_t\hat{\pi}_{i,t+1}^{M}$$
$$+\frac{(1-\omega^{M})(1-\omega^{M}\beta)}{\omega^{M}(1+\gamma^{M}\beta(1-\mu_i^{M}))}\left[\sum_{j=1}^{N}w_{i,j}^{M}\ln\frac{\hat{\mathcal{E}}_{i,j,t}\hat{P}_{j,t}^{X}}{\hat{P}_{i,t}^{M}} + \ln\hat{\vartheta}_{i,t}^{M}\right] + \frac{\mu^{M}\gamma^{M}(1+\beta)}{1+\gamma^{M}\beta(1-\mu_i^{M})}\mathcal{P}_2(L)\sum_{k=1}^{M*}\frac{M_{i,k}}{M_i}\ln(\hat{\mathcal{E}}_{i,i*,t}\hat{P}_{k,t}^{Y}), \tag{185}$$

which determines the price of imports $\ln\hat{P}_{i,t}^{M}$. Import price inflation also depends on the contemporaneous, past and expected future domestic currency denominated prices of energy and nonenergy commodities. The response coefficients of this relationship vary across economies with their trade pattern and commodity import intensities. Import price inflation $\hat{\pi}_{i,t}^{M}$ satisfies $\hat{\pi}_{i,t}^{M} = \ln\hat{P}_{i,t}^{M} - \ln\hat{P}_{i,t-1}^{M}$.

Public consumption and investment $\ln\hat{G}_{i,t}^{Z}$ depend on a weighted average of their past and desired values according to fiscal expenditure rules

$$\ln\hat{G}_{i,t}^{Z} = \rho_G\ln\hat{G}_{i,t-1}^{Z} + (1-\rho_G)\ln\hat{\bar{Y}}_{i,t} + \hat{\nu}_{i,t}^{G^Z}, \tag{186}$$

where $Z \in \{C, I\}$. Desired public consumption or investment tracks contemporaneous potential output one for one. The corporate and labor income tax rates $\hat{\tau}_{i,t}^{Z}$ depend on their past values according to fiscal revenue rules

$$\hat{\tau}_{i,t}^{Z} = \rho_\tau\hat{\tau}_{i,t-1}^{Z} + \hat{\nu}_{i,t}^{\tau^Z}, \tag{187}$$

where $Z \in \{K, L\}$. The nondiscretionary transfer payment ratio $\frac{\hat{T}_{i,t}^{C,N}}{P_{i,t}^{Y}Y_{i,t}}$ responds to the past net foreign asset ratio according to nondiscretionary transfer payment rule:

$$\frac{\hat{T}^{C,N}_{i,t}}{P^Y_{i,t}Y_{i,t}} = \zeta^{T^N}\frac{\hat{A}_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}}. \tag{188}$$

The discretionary transfer payment ratio $\frac{\hat{T}^{C,D}_{i,t}}{P^Y_{i,t}Y_{i,t}}$ responds to the past net government asset ratio according to discretionary transfer payment rule:

$$\frac{\hat{T}^{C,D}_{i,t}}{P^Y_{i,t}Y_{i,t}} = \zeta^{T^D}\frac{\hat{A}^G_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}} + \hat{\nu}^T_{i,t}. \tag{189}$$

The public physical capital stock $\ln\hat{K}^G_{i,t+1}$ is accumulated according to $\ln\hat{K}^G_{i,t+1} = (1-\delta^G)\ln\hat{K}^G_{i,t} + \delta^G\ln\hat{G}^I_{i,t}$.

The fiscal balance ratio $\frac{\widehat{FB}_{i,t}}{P^Y_{i,t}Y_{i,t}}$ depends on the contemporaneous primary fiscal balance ratio, as well as a weighted average of the past nominal short term bond yield and nominal effective long term market interest rate, and the past net government asset ratio adjusted by contemporaneous nominal output growth, according to government dynamic budget constraint:

$$\frac{\widehat{FB}_{i,t}}{P^Y_{i,t}Y_{i,t}} = \frac{1}{\beta}\frac{1}{1+g}\left[\frac{A^G_i}{P^Y_iY_i}\left(\frac{B^{S,G}_i}{A^G_i}\hat{i}^S_{i,t-1} + \frac{B^{L,G}_i}{A^G_i}\hat{i}^{L,E}_{i,t-1}\right) + (1-\beta)\left(\frac{\hat{A}^G_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}} - \frac{A^G_i}{P^Y_iY_i}\ln\frac{\hat{P}^Y_{i,t}\hat{Y}_{i,t}}{\hat{P}^Y_{i,t-1}\hat{Y}_{i,t-1}}\right)\right] + \frac{\widehat{PB}_{i,t}}{P^Y_{i,t}Y_{i,t}}. \tag{190}$$

In addition, the nominal effective long term market interest rate $\hat{i}^{L,E}_{i,t}$ depends on its past value driven by the contemporaneous nominal long term bond yield according to:

$$\hat{i}^{L,E}_{i,t} = \omega^B\hat{i}^{L,E}_{i,t-1} + \omega^B\beta(1-\omega^B)\left(\omega^B + \frac{1-\omega^B\beta}{\omega^B\beta}\right)\hat{i}^L_{i,t}. \tag{191}$$

Furthermore, the primary fiscal balance ratio $\frac{\widehat{PB}_{i,t}}{P^Y_{i,t}Y_{i,t}}$ depends on the contemporaneous deviation of real tax revenues from public domestic demand, as well as the contemporaneous terms of trade and discretionary transfer payment ratio, according to:

$$\frac{\widehat{PB}_{i,t}}{P^Y_{i,t}Y_{i,t}} = \frac{G_i}{Y_i}\left[\ln\frac{\hat{T}_{i,t}}{\hat{P}^Y_{i,t}} - \left(\ln\hat{G}_{i,t} - \frac{X_i}{Y_i}\ln\hat{\mathcal{T}}_{i,t}\right)\right] - \frac{\hat{T}^{C,D}_{i,t}}{P^Y_{i,t}Y_{i,t}}. \tag{192}$$

In addition, the deviation of tax revenues from nominal output depends on a weighted average of the contemporaneous corporate and labor income tax rates according to

$$\ln\frac{\hat{T}_{i,t}}{\hat{P}^Y_{i,t}\hat{Y}_{i,t}} = \frac{1}{\tau_i}\left[\left(1-\frac{W_iL_i}{P^Y_iY_i}\right)\hat{\tau}^K_{i,t} + \frac{W_iL_i}{P^Y_iY_i}\hat{\tau}^L_{i,t}\right], \tag{193}$$

which determines tax revenues $\ln\hat{T}_{i,t}$. Furthermore, the transfer payment ratio $\frac{\hat{T}^C_{i,t}}{P^Y_{i,t}Y_{i,t}}$ depends on the contemporaneous nondiscretionary and discretionary transfer payment ratios according to:

$$\frac{\hat{T}^C_{i,t}}{P^Y_{i,t}Y_{i,t}} = \frac{\hat{T}^{C,N}_{i,t}}{P^Y_{i,t}Y_{i,t}} + \frac{\hat{T}^{C,D}_{i,t}}{P^Y_{i,t}Y_{i,t}}. \tag{194}$$

Finally, the net government asset ratio $\frac{\hat{A}^G_{i,t+1}}{P^Y_{i,t}Y_{i,t}}$ depends on its past value adjusted by contemporaneous nominal output growth, as well as the contemporaneous fiscal balance ratio, according to:

$$\frac{\hat{A}^G_{i,t+1}}{P^Y_{i,t}Y_{i,t}} = \frac{1}{1+g}\left(\frac{\hat{A}^G_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}} - \frac{A^G_i}{P^Y_i Y_i}\ln\frac{\hat{P}^Y_{i,t}\hat{Y}_{i,t}}{\hat{P}^Y_{i,t-1}\hat{Y}_{i,t-1}}\right) + \frac{\widehat{FB}_{i,t}}{P^Y_{i,t}Y_{i,t}}. \tag{195}$$

The linearization of these relationships assumes nominal output growth at rate g in steady state equilibrium. Their response coefficients vary across economies with the size of their government, their trade openness, their labor income share, and the size and composition of their public financial wealth.

The current account balance ratio $\frac{\widehat{CA}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}}$ depends on the contemporaneous trade balance ratio and past net foreign asset ratio according to national dynamic budget constraint:

$$\frac{\widehat{CA}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}} = \frac{1-\beta}{\beta}\frac{1}{1+g}\frac{\hat{A}_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}} + \frac{\widehat{TB}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}}. \tag{196}$$

Furthermore, the trade balance ratio $\frac{\widehat{TB}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}}$ depends on the contemporaneous deviation of exports from imports, as well as the contemporaneous terms of trade, according to:

$$\frac{\widehat{TB}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}} = \frac{X_i}{Y_i}\left(\ln\frac{\hat{X}_{i,t}}{\hat{M}_{i,t}} + \ln\hat{\mathcal{T}}_{i,t}\right). \tag{197}$$

Finally, the net foreign asset ratio $\frac{\hat{A}_{i,t+1}}{P^Y_{i,t}Y_{i,t}}$ depends on its past value and the contemporaneous current account balance ratio according to:

$$\frac{\hat{A}_{i,t+1}}{P^Y_{i,t}Y_{i,t}} = \frac{1}{1+g}\frac{\hat{A}_{i,t}}{P^Y_{i,t-1}Y_{i,t-1}} + \frac{\widehat{CA}_{i,t}}{\mathcal{E}_{i*,i,t}P^Y_{i,t}Y_{i,t}}. \tag{198}$$

Multilateral consistency in nominal trade flows requires that the world output weighted average trade balance ratio equals zero,

$$\sum_{j=1}^N w^Y_j \frac{\widehat{TB}_{j,t}}{\mathcal{E}_{i*,j,t}P^Y_{j,t}Y_{j,t}} = 0, \tag{199}$$

which determines the global terms of trade shifter $\ln\hat{\upsilon}^{\mathcal{T}}_t$. It follows that the world output weighted average current account balance and net foreign asset ratios also equal zero. The linearization of these relationships assumes nominal output growth at rate g in steady state equilibrium. Their response coefficients vary across economies with their trade openness.

The prices of energy and nonenergy commodities $\ln\hat{P}^Y_{k,t}$ depend on a weighted average of their past and expected future values driven by contemporaneous world output weighted average real unit labor cost according to commodity price Phillips curves:

$$\begin{aligned}\ln\hat{P}^Y_{k,t} &= \frac{1}{1+\beta}\ln\hat{P}^Y_{k,t-1} + \frac{\beta}{1+\beta}\mathrm{E}_t\ln\hat{P}^Y_{k,t+1} \\ &\quad + \frac{(1-\omega^Y_k)(1-\omega^Y_k\beta)}{\omega^Y_k(1+\beta)}\sum_{i=1}^N w^Y_i\left[\ln\frac{\hat{W}_{i,t}\hat{L}_{i,t}}{\hat{P}_{i,t}\hat{Y}_{i,t}} + \lambda^Y\ln\frac{\hat{P}_{i,t}}{\hat{\mathcal{E}}_{i,i*,t}\hat{P}^Y_{k,t}} + \ln\hat{\vartheta}^Y_{k,t}\right] - \mathcal{P}_2(L)\sum_{i=1}^N w^Y_i\ln\hat{\mathcal{E}}_{i,i*,t}.\end{aligned} \tag{200}$$

The prices of energy and nonenergy commodities also depend on their deviation from the contemporaneous world output weighted average quotation currency denominated core price level. Finally, the prices of energy and nonenergy commodities depend on the contemporaneous, past and expected future world output weighted average nominal bilateral exchange rate. The response coefficients of these relationships vary across commodity markets $1 \leq k \leq M^*$, with $k=1$ for energy commodities and $k=2$ for nonenergy commodities.

B. Exogenous Variables

All structural shocks follow stationary first order autoregressive or white noise processes, generally with conditionally normally distributed heteroskedastic innovations. The conditional variances of these processes in turn follow a possibly asymmetric extension of the GARCH model due to Bollerslev (1986), building on the ARCH model introduced by Engle (1982). The asymmetric extension under consideration is the threshold GARCH model associated with Glosten, Jaganathan and Runkle (1993).

Conditional Means

The productivity $\ln \hat{\nu}_{i,t}^{\mathcal{A}}$ and labor supply $\ln \hat{\nu}_{i,t}^{\mathcal{N}}$ shocks follow stationary first order autoregressive processes with conditionally normally distributed heteroskedastic innovations:

$$\ln \hat{\nu}_{i,t}^{\mathcal{A}} = \rho_{\nu^{\mathcal{A}}} \ln \hat{\nu}_{i,t-1}^{\mathcal{A}} + \varepsilon_{i,t}^{\nu^{\mathcal{A}}},\ \varepsilon_{i,t}^{\nu^{\mathcal{A}}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{\mathcal{A}}}), \tag{201}$$

$$\ln \hat{\nu}_{i,t}^{\mathcal{N}} = \rho_{\nu^{\mathcal{N}}} \ln \hat{\nu}_{i,t-1}^{\mathcal{N}} + \varepsilon_{i,t}^{\nu^{\mathcal{N}}},\ \varepsilon_{i,t}^{\nu^{\mathcal{N}}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{\mathcal{N}}}). \tag{202}$$

In addition, the consumption demand $\ln \hat{\nu}_{i,t}^{C}$, residential investment demand $\ln \hat{\nu}_{i,t}^{I^H}$, business investment demand $\ln \hat{\nu}_{i,t}^{I^K}$, export demand $\ln \hat{\nu}_{i,t}^{X}$, and import demand $\ln \hat{\nu}_{i,t}^{M}$ shocks follow stationary first order autoregressive processes with conditionally normally distributed heteroskedastic innovations:

$$\ln \hat{\nu}_{i,t}^{C} = \rho_{\nu^{C}} \ln \hat{\nu}_{i,t-1}^{C} + \varepsilon_{i,t}^{\nu^{C}},\ \varepsilon_{i,t}^{\nu^{C}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{C}}), \tag{203}$$

$$\ln \hat{\nu}_{i,t}^{I^H} = \rho_{\nu^{I}} \ln \hat{\nu}_{i,t-1}^{I^H} + \varepsilon_{i,t}^{\nu^{I,H}},\ \varepsilon_{i,t}^{\nu^{I,H}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{I}}), \tag{204}$$

$$\ln \hat{\nu}_{i,t}^{I^K} = \rho_{\nu^{I}} \ln \hat{\nu}_{i,t-1}^{I^K} + \varepsilon_{i,t}^{\nu^{I,K}},\ \varepsilon_{i,t}^{\nu^{I,K}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{I}}), \tag{205}$$

$$\ln \hat{\nu}_{i,t}^{X} = \rho_{\nu^{X}} \ln \hat{\nu}_{i,t-1}^{X} + \varepsilon_{i,t}^{\nu^{X}},\ \varepsilon_{i,t}^{\nu^{X}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{X}}), \tag{206}$$

$$\ln \hat{\nu}_{i,t}^{M} = \rho_{\nu^{M}} \ln \hat{\nu}_{i,t-1}^{M} + \varepsilon_{i,t}^{\nu^{M}},\ \varepsilon_{i,t}^{\nu^{M}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{M}}). \tag{207}$$

The output price markup $\ln \hat{\vartheta}_{i,t}^{Y}$, wage markup $\ln \hat{\vartheta}_{i,t}^{L}$, export price markup $\ln \hat{\vartheta}_{i,t}^{X}$, import price markup $\ln \hat{\vartheta}_{i,t}^{M}$, and energy or nonenergy commodity price markup $\ln \hat{\vartheta}_{k,t}^{Y}$ shocks are conditionally normally distributed heteroskedastic innovations:

$$\ln \hat{\vartheta}_{i,t}^{Y} = \varepsilon_{i,t}^{\vartheta^{Y}},\ \varepsilon_{i,t}^{\vartheta^{Y}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\vartheta^{Y}}), \tag{208}$$

$$\ln\hat{\vartheta}_{i,t}^{L}=\varepsilon_{i,t}^{\vartheta^{L}},\ \varepsilon_{i,t}^{\vartheta^{L}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\vartheta^{L}}),\tag{209}$$

$$\ln\hat{\vartheta}_{i,t}^{X}=\varepsilon_{i,t}^{\vartheta^{X}},\ \varepsilon_{i,t}^{\vartheta^{X}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\vartheta^{X}}),\tag{210}$$

$$\ln\hat{\vartheta}_{i,t}^{M}=\varepsilon_{i,t}^{\vartheta^{M}},\ \varepsilon_{i,t}^{\vartheta^{M}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\vartheta^{M}}),\tag{211}$$

$$\ln\hat{\vartheta}_{k,t}^{Y}=\varepsilon_{k,t}^{\vartheta^{Y}},\ \varepsilon_{k,t}^{\vartheta^{Y}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{k,t}^{\vartheta^{Y}}).\tag{212}$$

Furthermore, the liquidity risk premium $\ln\hat{\nu}_{i,t}^{i^B}$, housing risk premium $\ln\hat{\nu}_{i,t}^{H}$, credit risk premium $\ln\hat{\nu}_{i,t}^{i^S}$, duration risk premium $\ln\hat{\nu}_{i,t}^{B}$, equity risk premium $\ln\hat{\nu}_{i,t}^{S}$, and currency risk premium $\ln\hat{\nu}_{i,t}^{\mathcal{E}}$ shocks follow stationary first order autoregressive processes with conditionally normally distributed homoskedastic or heteroskedastic innovations:

$$\ln\hat{\nu}_{i,t}^{i^B}=\rho_{\nu^{i,B}}\ln\hat{\nu}_{i,t-1}^{i^B}+\varepsilon_{i,t}^{\nu^{i,B}},\ \varepsilon_{i,t}^{\nu^{i,B}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i}^{\nu^{i,B}}),\tag{213}$$

$$\ln\hat{\nu}_{i,t}^{H}=\rho_{\nu^{H}}\ln\hat{\nu}_{i,t-1}^{H}+\varepsilon_{i,t}^{\nu^{H}},\ \varepsilon_{i,t}^{\nu^{H}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{H}}),\tag{214}$$

$$\ln\hat{\nu}_{i,t}^{i^S}=\rho_{\nu^{i,S}}\ln\hat{\nu}_{i,t-1}^{i^S}+\varepsilon_{i,t}^{\nu^{i,S}},\ \varepsilon_{i,t}^{\nu^{i,S}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{i,S}}),\tag{215}$$

$$\ln\hat{\nu}_{i,t}^{B}=\rho_{\nu^{B}}\ln\hat{\nu}_{i,t-1}^{B}+\varepsilon_{i,t}^{\nu^{B}},\ \varepsilon_{i,t}^{\nu^{B}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{B}}),\tag{216}$$

$$\ln\hat{\nu}_{i,t}^{S}=\rho_{\nu^{S}}\ln\hat{\nu}_{i,t-1}^{S}+\varepsilon_{i,t}^{\nu^{S}},\ \varepsilon_{i,t}^{\nu^{S}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{S}}),\tag{217}$$

$$\ln\hat{\nu}_{i,t}^{\mathcal{E}}=\rho_{\nu^{\mathcal{E}}}\ln\hat{\nu}_{i,t-1}^{\mathcal{E}}+\varepsilon_{i,t}^{\nu^{\mathcal{E}}},\ \varepsilon_{i,t}^{\nu^{\mathcal{E}}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{\mathcal{E}}}).\tag{218}$$

The mortgage loan rate markup $\ln\hat{\vartheta}_{i,t}^{C^D}$ and corporate loan rate markup $\ln\hat{\vartheta}_{i,t}^{C^F}$ shocks follow stationary first order autoregressive processes with conditionally normally distributed homoskedastic innovations, while the mortgage loan default $\hat{\nu}_{i,t}^{\delta^M}$ and corporate loan default $\hat{\nu}_{i,t}^{\delta^C}$ shocks are conditionally normally distributed homoskedastic innovations:

$$\ln\hat{\vartheta}_{i,t}^{C^D}=\rho_{\vartheta^{C}}\ln\hat{\vartheta}_{i,t}^{C^D}+\varepsilon_{i,t}^{\vartheta^{C,D}},\ \varepsilon_{i,t}^{\vartheta^{C,D}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i}^{\vartheta^{C}}),\tag{219}$$

$$\ln\hat{\vartheta}_{i,t}^{C^F}=\rho_{\vartheta^{C}}\ln\hat{\vartheta}_{i,t}^{C^F}+\varepsilon_{i,t}^{\vartheta^{C,F}},\ \varepsilon_{i,t}^{\vartheta^{C,F}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i}^{\vartheta^{C}}),\tag{220}$$

$$\hat{\nu}_{i,t}^{\delta^M}=\varepsilon_{i,t}^{\nu^{\delta,M}},\ \varepsilon_{i,t}^{\nu^{\delta,M}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i}^{\nu^{\delta}}),\tag{221}$$

$$\hat{\nu}_{i,t}^{\delta^C}=\varepsilon_{i,t}^{\nu^{\delta,C}},\ \varepsilon_{i,t}^{\nu^{\delta,C}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i}^{\nu^{\delta}}).\tag{222}$$

Finally, the monetary policy $\hat{\nu}_{i,t}^{i^P}$, mortgage loan to value limit $\hat{\nu}_{i,t}^{\phi^D}$ and corporate loan to value limit $\hat{\nu}_{i,t}^{\phi^F}$ shocks are conditionally normally distributed homoskedastic or heteroskedastic innovations, while the public consumption $\hat{\nu}_{i,t}^{G^C}$, public investment $\hat{\nu}_{i,t}^{G^I}$, corporate tax rate $\hat{\nu}_{i,t}^{\tau^K}$, labor income tax rate $\hat{\nu}_{i,t}^{\tau^L}$, transfer payment $\hat{\nu}_{i,t}^{T}$ and capital requirement $\hat{\nu}_{i,t}^{\kappa}$ shocks follow stationary first order autoregressive processes with conditionally normally distributed homoskedastic or heteroskedastic innovations:

$$\hat{\nu}_{i,t}^{i^P}=\varepsilon_{i,t}^{\nu^{i,P}},\ \varepsilon_{i,t}^{\nu^{i,P}}\,|\,\mathcal{I}_{t-1}\sim\mathcal{N}(0,h_{i,t}^{\nu^{i,P}}),\tag{223}$$

$$\hat{\nu}_{i,t}^{G^C} = \rho_{\nu^{G,C}} \hat{\nu}_{i,t-1}^{G^C} + \varepsilon_{i,t}^{\nu^{G,C}},\ \varepsilon_{i,t}^{\nu^{G,C}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{G,C}}), \tag{224}$$

$$\hat{\nu}_{i,t}^{G^I} = \rho_{\nu^{G,I}} \hat{\nu}_{i,t-1}^{G^I} + \varepsilon_{i,t}^{\nu^{G,I}},\ \varepsilon_{i,t}^{\nu^{G,I}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{G,I}}), \tag{225}$$

$$\hat{\nu}_{i,t}^{\tau^K} = \rho_{\nu^{\tau}} \hat{\nu}_{i,t-1}^{\tau^K} + \varepsilon_{i,t}^{\nu^{\tau,K}},\ \varepsilon_{i,t}^{\nu^{\tau,K}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{\tau}}), \tag{226}$$

$$\hat{\nu}_{i,t}^{\tau^L} = \rho_{\nu^{\tau}} \hat{\nu}_{i,t-1}^{\tau^L} + \varepsilon_{i,t}^{\nu^{\tau,L}},\ \varepsilon_{i,t}^{\nu^{\tau,L}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i,t}^{\nu^{\tau}}), \tag{227}$$

$$\hat{\nu}_{i,t}^{T} = \rho_{\nu^{T}} \hat{\nu}_{i,t-1}^{T} + \varepsilon_{i,t}^{\nu^{T}},\ \varepsilon_{i,t}^{\nu^{T}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i}^{\nu^{T}}), \tag{228}$$

$$\hat{\nu}_{i,t}^{\kappa} = \rho_{\nu^{\kappa}} \hat{\nu}_{i,t-1}^{\kappa} + \varepsilon_{i,t}^{\nu^{\kappa}},\ \varepsilon_{i,t}^{\nu^{\kappa}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i}^{\nu^{\kappa}}), \tag{229}$$

$$\hat{\nu}_{i,t}^{\phi^D} = \varepsilon_{i,t}^{\nu^{\phi,D}},\ \varepsilon_{i,t}^{\nu^{\phi,D}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i}^{\nu^{\phi}}), \tag{230}$$

$$\hat{\nu}_{i,t}^{\phi^F} = \varepsilon_{i,t}^{\nu^{\phi,F}},\ \varepsilon_{i,t}^{\nu^{\phi,F}} \mid \mathcal{I}_{t-1} \sim \mathcal{N}(0, h_{i}^{\nu^{\phi}}). \tag{231}$$

As an identifying restriction, all innovations are assumed to be independent, which combined with our distributional assumptions implies conditional multivariate normality.

Conditional Variances

The productivity $h_{i,t}^{\nu^{\mathcal{A}}}$, labor supply $h_{i,t}^{\nu^{\mathcal{N}}}$, consumption demand $h_{i,t}^{\nu^{C}}$, investment demand $h_{i,t}^{\nu^{I}}$, export demand $h_{i,t}^{\nu^{X}}$, and import demand $h_{i,t}^{\nu^{M}}$ conditional variances follow symmetric GARCH processes:

$$h_{i,t}^{\nu^{\mathcal{A}}} = \omega_{\nu^{\mathcal{A}}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{\mathcal{A}}})^2 + \beta_h h_{i,t-1}^{\nu^{\mathcal{A}}}, \tag{232}$$

$$h_{i,t}^{\nu^{\mathcal{N}}} = \omega_{\nu^{\mathcal{N}}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{\mathcal{N}}})^2 + \beta_h h_{i,t-1}^{\nu^{\mathcal{N}}}, \tag{233}$$

$$h_{i,t}^{\nu^{C}} = \omega_{\nu^{C}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{C}})^2 + \beta_h h_{i,t-1}^{\nu^{C}}, \tag{234}$$

$$h_{i,t}^{\nu^{I}} = \omega_{\nu^{I}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{I}})^2 + \beta_h h_{i,t-1}^{\nu^{I}}, \tag{235}$$

$$h_{i,t}^{\nu^{X}} = \omega_{\nu^{X}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{X}})^2 + \beta_h h_{i,t-1}^{\nu^{X}}, \tag{236}$$

$$h_{i,t}^{\nu^{M}} = \omega_{\nu^{M}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{M}})^2 + \beta_h h_{i,t-1}^{\nu^{M}}. \tag{237}$$

In addition, the output price markup $h_{i,t}^{\vartheta^Y}$, wage markup $h_{i,t}^{\vartheta^L}$, export price markup $h_{i,t}^{\vartheta^X}$, import price markup $h_{i,t}^{\vartheta^M}$, and energy or nonenergy commodity price markup $h_{k,t}^{\vartheta^Y}$ conditional variances follow symmetric GARCH processes:

$$h_{i,t}^{\vartheta^Y} = \omega_{\vartheta^Y} + \alpha_h (\varepsilon_{i,t-1}^{\vartheta^Y})^2 + \beta_h h_{i,t-1}^{\vartheta^Y}, \tag{238}$$

$$h_{i,t}^{\vartheta^L} = \omega_{\vartheta^L} + \alpha_h (\varepsilon_{i,t-1}^{\vartheta^L})^2 + \beta_h h_{i,t-1}^{\vartheta^L}, \tag{239}$$

$$h_{i,t}^{\vartheta^X} = \omega_{\vartheta^X} + \alpha_h (\varepsilon_{i,t-1}^{\vartheta^X})^2 + \beta_h h_{i,t-1}^{\vartheta^X}, \tag{240}$$

$$h_{i,t}^{\vartheta^M} = \omega_{\vartheta^M} + \alpha_h (\varepsilon_{i,t-1}^{\vartheta^M})^2 + \beta_h h_{i,t-1}^{\vartheta^M}, \tag{241}$$

$$h_{k,t}^{\vartheta^Y} = \omega_{\vartheta^Y,k} + \alpha_h (\varepsilon_{k,t-1}^{\vartheta^Y})^2 + \beta_h h_{k,t-1}^{\vartheta^Y}. \tag{242}$$

Furthermore, the housing risk premium $h_{i,t}^{\nu^H}$, credit risk premium $h_{i,t}^{\nu^{i,S}}$, duration risk premium $h_{i,t}^{\nu^B}$ and equity risk premium $h_{i,t}^{\nu^S}$ conditional variances follow asymmetric GARCH processes, whereas the currency risk premium $h_{i,t}^{\nu^\varepsilon}$ conditional variance follows a symmetric GARCH process:

$$h_{i,t}^{\nu^H} = \omega_{\nu^H} + \alpha_h (\varepsilon_{i,t-1}^{\nu^H})^2 + \gamma_h (\eta_{i,t-1}^{\nu^H})^2 + \beta_h h_{i,t-1}^{\nu^H}, \tag{243}$$

$$h_{i,t}^{\nu^{i,S}} = \omega_{\nu^{i,S}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{i,S}})^2 + \gamma_h (\eta_{i,t-1}^{\nu^{i,S}})^2 + \beta_h h_{i,t-1}^{\nu^{i,S}}, \tag{244}$$

$$h_{i,t}^{\nu^B} = \omega_{\nu^B} + \alpha_h (\varepsilon_{i,t-1}^{\nu^B})^2 + \gamma_h (\eta_{i,t-1}^{\nu^B})^2 + \beta_h h_{i,t-1}^{\nu^B}, \tag{245}$$

$$h_{i,t}^{\nu^S} = \omega_{\nu^S} + \alpha_h (\varepsilon_{i,t-1}^{\nu^S})^2 + \gamma_h (\eta_{i,t-1}^{\nu^S})^2 + \beta_h h_{i,t-1}^{\nu^S}, \tag{246}$$

$$h_{i,t}^{\nu^\varepsilon} = \omega_{\nu^\varepsilon} + \alpha_h (\varepsilon_{i,t-1}^{\nu^\varepsilon})^2 + \beta_h h_{i,t-1}^{\nu^\varepsilon}. \tag{247}$$

Finally, the monetary policy $\hat{v}_{i,t}^{i^P}$ conditional variance follows an asymmetric GARCH process, whereas the public consumption $\hat{v}_{i,t}^{G^C}$, public investment $\hat{v}_{i,t}^{G^I}$ and tax rate $\hat{v}_{i,t}^{\tau^K}$ conditional variances follow symmetric GARCH processes:

$$h_{i,t}^{\nu^{i,P}} = \omega_{\nu^{i,P}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{i,P}})^2 + \gamma_h (\eta_{i,t-1}^{\nu^{i,P}})^2 + \beta_h h_{i,t-1}^{\nu^{i,P}}, \tag{248}$$

$$h_{i,t}^{\nu^{G,C}} = \omega_{\nu^{G,C}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{G,C}})^2 + \beta_h h_{i,t-1}^{\nu^{G,C}}, \tag{249}$$

$$h_{i,t}^{\nu^{G,I}} = \omega_{\nu^{G,I}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{G,I}})^2 + \beta_h h_{i,t-1}^{\nu^{G,I}}, \tag{250}$$

$$h_{i,t}^{\nu^{\tau}} = \omega_{\nu^{\tau}} + \alpha_h (\varepsilon_{i,t-1}^{\nu^{\tau}})^2 + \beta_h h_{i,t-1}^{\nu^{\tau}}. \tag{251}$$

Given asymmetric innovation $\eta_{i,t}^Z = \max(0, \varepsilon_{i,t}^Z)$, if $\gamma_h > 0$ then a positive innovation raises its conditional variance more than a negative innovation of equal magnitude, and vice versa. The conditional covariance matrix is positive definite if $\omega_Z > 0$, $\alpha_h \geq 0$, $\alpha_h + \gamma_h \geq 0$ and $\beta_h \geq 0$ for all Z.

IV. Estimation

The traditional econometric interpretation of structural macroeconometric models regards them as representations of the joint probability distribution of the data. Adopting this traditional econometric interpretation, we jointly estimate subsets of the parameters and variables of our DSGE model of the world economy by full information maximum likelihood, conditional on calibrated values of its other parameters and observed values of its other variables.

In what follows, we employ a restricted version of the model that consolidates or eliminates those structural shocks that are weakly identified by our multivariate panel data set. In particular, the residential and business investment demand shocks are consolidated into an investment demand shock, while the corporate and labor income tax rate shocks are consolidated into a tax rate shock. These structural shocks are driven by heteroskedastic innovations with common conditional variances. Furthermore, the liquidity risk premium, mortgage and corporate loan rate markup, mortgage and corporate loan default, transfer payment, capital requirement, and mortgage and corporate loan to value limit shocks are eliminated. These structural shocks are driven by homoskedastic innovations. Given this stochastic structure, impulse response analysis could be based on the unrestricted version of the model.

A. Data Transformations

Estimation of our DSGE model of the world economy is conditional on the cyclical components of a total of 280 endogenous variables observed for fifteen economies over sample period 1999Q1 through 2017Q4. To ensure stochastic nonsingularity, the model is driven by a total of 281 structural shocks. The advanced and emerging market economies under consideration are Australia, Brazil, Canada, China, France, Germany, India, Italy, Japan, Korea, Mexico, Russia, Spain, the United Kingdom, and the United States. These are the fifteen largest national economies in the world. The observed macroeconomic and financial market variables under consideration are the core price level, the output price level, the consumption price level, the quantity of output, the quantity of private consumption, the quantity of exports, the quantity of imports, the nominal policy interest rate, the nominal short term bond yield, the nominal long term bond yield, the price of housing, the price of equity, the nominal bilateral exchange rate, the nominal wage, the unemployment rate, employment, the quantity of public consumption, the quantity of public investment, the fiscal balance ratio, and the prices of nonrenewable energy and nonenergy commodities. For the systemic advanced economies, the shadow nominal policy interest rate estimated by Krippner (2015) substitutes for the observed nominal policy interest rate during the effective lower bound period. For a detailed description of this multivariate panel data set, see Appendix A.

We estimate the cyclical components of all of the observed endogenous variables under consideration with the generalization of the filter described in Hodrick and Prescott (1997) due to Vitek (2014), which parameterizes the difference order associated with the penalty term determining the smoothness of the trend component. For those variables that exhibit long run trends, namely the core price level, the output price level, the consumption price level, the quantity of output, the quantity of private consumption, the quantity of exports, the quantity of imports, the price of housing, the price of equity, the nominal bilateral exchange rate, the nominal wage, employment, the quantity of public consumption, the quantity of public investment, and the prices of nonrenewable energy and nonenergy commodities, we set the difference order to two and the smoothing parameter to 16,000. In contrast, for those variables that do not exhibit long run trends, namely the nominal policy interest rate, the nominal short term bond yield, the nominal long term bond yield, the unemployment rate, and the fiscal balance ratio, we set the difference order to one and the smoothing parameter to 400.

B. Estimation Procedure

Let $\{\boldsymbol{x}_t\}_{t=1}^T$ denote a vector stochastic process consisting of N nonpredetermined endogenous variables, of which M are observed. This vector stochastic process satisfies second order stochastic linear difference equation

$$\boldsymbol{A}_0\boldsymbol{x}_t = \boldsymbol{A}_1\boldsymbol{x}_{t-1} + \boldsymbol{A}_2\mathrm{E}_t\boldsymbol{x}_{t+1} + \boldsymbol{A}_3\boldsymbol{v}_t, \tag{252}$$

where vector stochastic process $\{\boldsymbol{v}_t\}_{t=1}^T$ consists of K exogenous variables. This vector stochastic process satisfies stationary first order stochastic linear difference equation

$$\boldsymbol{v}_t = \boldsymbol{B}_1\boldsymbol{v}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{253}$$

where $\boldsymbol{H}_t^{-1/2}\boldsymbol{\varepsilon}_t \,|\, \mathcal{I}_{t-1} \sim \text{iid}\,\mathcal{N}(\boldsymbol{0}, \boldsymbol{I}_K)$ with $\mathcal{I}_t = \{\boldsymbol{y}_s\}_{s=1}^t$, which implies that $\boldsymbol{\varepsilon}_t \,|\, \mathcal{I}_{t-1} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{H}_t)$. If there exists a unique stationary solution to this multivariate linear rational expectations model, then it may be expressed as:

$$\boldsymbol{x}_t = \boldsymbol{C}_1\boldsymbol{x}_{t-1} + \boldsymbol{C}_2\boldsymbol{v}_t. \tag{254}$$

This unique stationary solution is calculated with the procedure due to Klein (2000).

The conditional covariance matrix $\boldsymbol{H}_t$ depends on its past value, driven by the outer product of past innovation vectors, as well as the outer product of past asymmetric innovation vectors, according to asymmetric multivariate GARCH process

$$\boldsymbol{H}_t = \boldsymbol{D}_0 + \boldsymbol{D}_1 \odot (\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}^{\mathsf{T}}) + \boldsymbol{D}_2 \odot (\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}^{\mathsf{T}}) + \boldsymbol{D}_3 \odot \boldsymbol{H}_{t-1}, \tag{255}$$

where $\boldsymbol{\eta}_t = \max(\boldsymbol{0}, \boldsymbol{\varepsilon}_t)$, while $\boldsymbol{D}_0$, $\boldsymbol{D}_1$, $\boldsymbol{D}_2$ and $\boldsymbol{D}_3$ are diagonal matrices of dimension K. This is the asymmetric extension due to Kroner and Ng (1998) of the constant conditional correlations model of Bollerslev (1990), under the special case in which all conditional correlations are zero. The conditional covariance matrix is positive definite if all diagonal elements of $\boldsymbol{D}_0$ are positive, while all elements of $\boldsymbol{D}_1$, $\boldsymbol{D}_1 + \boldsymbol{D}_2$ and $\boldsymbol{D}_3$ are nonnegative.

Let $\{\boldsymbol{y}_t\}_{t=1}^T$ denote a vector stochastic process consisting of M observed nonpredetermined endogenous variables, where $M \leq K$ to ensure stochastic nonsingularity. Also, let $\{\boldsymbol{z}_t\}_{t=1}^T$ denote a vector stochastic process consisting of N nonpredetermined endogenous variables and K exogenous variables. Given unique stationary solution (254), these vector stochastic processes have linear state space representation

$$\boldsymbol{y}_t = \boldsymbol{F}_1\boldsymbol{z}_t, \tag{256}$$

$$\boldsymbol{z}_t = \boldsymbol{G}_1\boldsymbol{z}_{t-1} + \boldsymbol{G}_2\boldsymbol{\varepsilon}_t, \tag{257}$$

where $\boldsymbol{z}_0 \sim \mathcal{N}(\boldsymbol{z}_{0|0}, \boldsymbol{P}_{0|0})$. The initial state vector is assumed to be independent from the state innovation vector, which combined with our distributional assumptions implies conditional multivariate normality.

Conditional on the parameters associated with this linear state space model, estimates of unobserved state vector $\boldsymbol{z}_t$ and its mean squared error matrix $\boldsymbol{P}_t$ may be calculated with the filter due to Kalman (1960) or the smoother associated with de Jong (1989), extended to account for the asymmetric multivariate GARCH process under consideration. In doing so, we approximate the expectation conditional on information available at time $t-1$ of this nested nonlinear function with the nested nonlinear function of expectations conditional on information available at time $t-1$ or $t-2$. Given initial conditions $\boldsymbol{z}_{0|0}$, $\boldsymbol{P}_{0|0}$, $\boldsymbol{\varepsilon}_{0|0}=\boldsymbol{0}$ and $\boldsymbol{H}_{0|-1}=\boldsymbol{H}_0$, estimates conditional on information available at time $t-1$ satisfy prediction equations

$$\boldsymbol{z}_{t|t-1}=\boldsymbol{G}_1\boldsymbol{z}_{t-1|t-1}, \tag{258}$$

$$\boldsymbol{H}_{t|t-1}=\boldsymbol{D}_0+\boldsymbol{D}_1\odot(\boldsymbol{\varepsilon}_{t-1|t-1}\boldsymbol{\varepsilon}_{t-1|t-1}^{\top})+\boldsymbol{D}_2\odot(\boldsymbol{\eta}_{t-1|t-1}\boldsymbol{\eta}_{t-1|t-1}^{\top})+\boldsymbol{D}_3\odot\boldsymbol{H}_{t-1|t-2}, \tag{259}$$

$$\boldsymbol{P}_{t|t-1}=\boldsymbol{G}_1\boldsymbol{P}_{t-1|t-1}\boldsymbol{G}_1^{\top}+\boldsymbol{G}_2\boldsymbol{H}_{t|t-1}\boldsymbol{G}_2^{\top}, \tag{260}$$

$$\boldsymbol{y}_{t|t-1}=\boldsymbol{F}_1\boldsymbol{z}_{t|t-1}, \tag{261}$$

$$\boldsymbol{Q}_{t|t-1}=\boldsymbol{F}_1\boldsymbol{P}_{t|t-1}\boldsymbol{F}_1^{\top}, \tag{262}$$

where $\boldsymbol{\eta}_{t|t}=\max(\boldsymbol{0},\boldsymbol{\varepsilon}_{t|t})$. Given these predictions and our distributional assumptions, estimates conditional on information available at time t satisfy updating equations

$$\boldsymbol{z}_{t|t}=\boldsymbol{z}_{t|t-1}+\boldsymbol{K}_t(\boldsymbol{y}_t-\boldsymbol{y}_{t|t-1}), \tag{263}$$

$$\boldsymbol{P}_{t|t}=\boldsymbol{P}_{t|t-1}-\boldsymbol{K}_t\boldsymbol{F}_1\boldsymbol{P}_{t|t-1}, \tag{264}$$

$$\boldsymbol{\varepsilon}_{t|t}=\boldsymbol{G}_2^{+}(\boldsymbol{F}_1^{+}\boldsymbol{y}_t-\boldsymbol{G}_1\boldsymbol{F}_1^{+}\boldsymbol{y}_{t-1}), \tag{265}$$

where $\boldsymbol{K}_t=\boldsymbol{P}_{t|t-1}\boldsymbol{F}_1^{\top}\boldsymbol{Q}_{t|t-1}^{-1}$. Given terminal conditions $\hat{\boldsymbol{z}}_{T+1|T}=\boldsymbol{0}$ and $\hat{\boldsymbol{P}}_{T+1|T}=\boldsymbol{0}$, estimates conditional on information available at time T satisfy computationally efficient smoothing equations

$$\hat{\boldsymbol{z}}_{t|T}=\boldsymbol{J}_t^{\top}\hat{\boldsymbol{z}}_{t+1|T}+\boldsymbol{F}_1^{\top}\boldsymbol{Q}_{t|t-1}^{-1}(\boldsymbol{y}_t-\boldsymbol{y}_{t|t-1}), \tag{266}$$

$$\boldsymbol{z}_{t|T}=\boldsymbol{z}_{t|t-1}+\boldsymbol{P}_{t|t-1}\hat{\boldsymbol{z}}_{t|T}, \tag{267}$$

$$\hat{\boldsymbol{P}}_{t|T}=\boldsymbol{J}_t^{\top}\hat{\boldsymbol{P}}_{t+1|T}\boldsymbol{J}_t-\boldsymbol{F}_1^{\top}\boldsymbol{Q}_{t|t-1}^{-1}\boldsymbol{F}_1, \tag{268}$$

$$\boldsymbol{P}_{t|T}=\boldsymbol{P}_{t|t-1}+\boldsymbol{P}_{t|t-1}\hat{\boldsymbol{P}}_{t|T}\boldsymbol{P}_{t|t-1}, \tag{269}$$

$$\boldsymbol{\varepsilon}_{t|T}=\boldsymbol{G}_2^{+}(\boldsymbol{z}_{t|T}-\boldsymbol{G}_1\boldsymbol{z}_{t-1|T}), \tag{270}$$

where $\boldsymbol{J}_t=\boldsymbol{G}_1(\boldsymbol{I}_{N+K}-\boldsymbol{P}_{t|t-1}\boldsymbol{F}_1^{\top}\boldsymbol{Q}_{t|t-1}^{-1}\boldsymbol{F}_1)$. Recursive forward evaluation of equations (258) through (265), followed by recursive backward evaluation of equations (266) through (269), yields conditional estimates of the unobserved state vector and its mean squared error matrix.

Let $\boldsymbol{\theta}\in\boldsymbol{\Theta}\subset\mathbb{R}^J$ denote a J dimensional vector containing the parameters associated with this linear state space model. Under our distributional assumptions, the full information maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ of this parameter vector maximizes conditional loglikelihood function

$$\mathcal{L}(\boldsymbol{\theta})=\sum_{t=1}^{T}\ell_t(\boldsymbol{\theta}), \tag{271}$$

where $\ell_t(\boldsymbol{\theta}) = -\frac{M}{2}\ln(2\pi) - \frac{1}{2}\ln|\boldsymbol{Q}_{t|t-1}| - \frac{1}{2}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})^{\mathsf{T}} \boldsymbol{Q}_{t|t-1}^{-1}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1})$. Under regularity conditions stated in Watson (1989), full information maximum likelihood estimator $\hat{\boldsymbol{\theta}}_T$ is consistent and asymptotically normal,

$$\sqrt{T}(\hat{\boldsymbol{\theta}}_T - \boldsymbol{\theta}_0) \overset{d}{\to} \mathcal{N}(\mathbf{0}, \mathcal{A}_0^{-1}\mathcal{B}_0\mathcal{A}_0^{-1}), \tag{272}$$

where $\boldsymbol{\theta}_0 \in \boldsymbol{\Theta}$ denotes the true parameter vector. Following Engle and Watson (1981), consistent estimators of $\mathcal{A}_0$ and $\mathcal{B}_0$ are given by

$$\hat{\mathcal{A}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T), \tag{273}$$

$$\hat{\mathcal{B}}_T = \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T)^{\mathsf{T}}, \tag{274}$$

where $\boldsymbol{a}_t(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1}^{\mathsf{T}}\boldsymbol{Q}_{t|t-1}^{-1}\nabla_{\boldsymbol{\theta}}\boldsymbol{y}_{t|t-1} + \frac{1}{2}\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}^{\mathsf{T}}(\boldsymbol{Q}_{t|t-1}^{-1} \otimes \boldsymbol{Q}_{t|t-1}^{-1})\nabla_{\boldsymbol{\theta}}\boldsymbol{Q}_{t|t-1}$ and $\boldsymbol{b}_t(\hat{\boldsymbol{\theta}}_T) = \nabla_{\boldsymbol{\theta}}\ell(\hat{\boldsymbol{\theta}}_T)$. If our distributional assumptions are satisfied, then the conditional information matrix equality holds, and $\mathcal{A}_0 = \mathcal{B}_0$.

C. Estimation Results

The set of parameters associated with our DSGE model of the world economy is partitioned into two subsets. Those parameters that determine conditional means are calibrated, whereas those that determine conditional variances are estimated.

Calibrated Parameters

The calibrated values of behavioral parameters lie within the range of estimates reported in the existing empirical literature where available, as reported in Table 2 of Appendix B. For example, the subjective discount factor parameter β is set to imply an annualized discount rate of 4 percent, while the habit persistence in consumption α^C and labor supply α^L parameters are both set to 0.80, and the intertemporal consumption σ and intratemporal labor supply η elasticity parameters are both set to 1.00. In addition, the adjustment cost parameters for residential χ^H and business χ^K investment are both set to 1.50, while the housing δ^H and physical capital δ^K stock depreciation parameters are both set to imply annualized depreciation rates of 10 percent. Furthermore, the partial indexation parameters for output price γ^Y, wage γ^L, export price γ^X and import price γ^M determination are all set to 0.80, while the nominal rigidity parameters for output price ω^Y, wage ω^L, export price ω^X and import price ω^M determination are all set to imply average reoptimization intervals of 6 quarters. Finally, the credit constrained household share parameter ϕ^C is set to 0.50, while the financial friction parameter for nominal mortgage and corporate loan rate determination ω^C is set to imply an average adjustment interval of 4 quarters, and the intratemporal import demand elasticity parameter ψ^M is set to 1.00.

The conduct of monetary policy is represented by a flexible inflation targeting regime in Australia, Canada, the Euro Area, Japan, Mexico, Russia, the United Kingdom and the United States, and by a managed exchange rate regime in Brazil, China, India and Korea, consistent with the de facto classification in IMF (2016). Within the Euro Area, the leader economy is Germany. The high interbank market contagion economies are the advanced economies, while the low interbank market contagion economy is China. The high capital market contagion economies are the emerging market economies with open capital accounts, while the low capital market contagion economy is China. The quotation currency for transactions in the foreign exchange market is issued by the United States.

All macroeconomic and financial great ratios are calibrated to match either their observed values, or the average of their observed values and their medians across economies, in 2016. Furthermore, all bilateral trade, bank lending, nonfinancial corporate borrowing, portfolio debt investment, and portfolio equity investment weights are calibrated to match their observed values in 2016, normalized to sum to one across economies. Finally, all world weights are calibrated to match their observed values in 2016, normalized to sum to one across economies.

Estimated Parameters

The estimated values of conditional variance parameters are reported in Table 3 of Appendix B. The conditional loglikelihood function is numerically maximized using a modified steepest ascent algorithm over effective sample period 1999Q3 through 2017Q4, with calculations performed to quadruple precision where necessary to ensure tolerable accuracy, and to double precision otherwise. The point estimates satisfy the sufficient conditions for positive definiteness of the conditional covariance matrix.

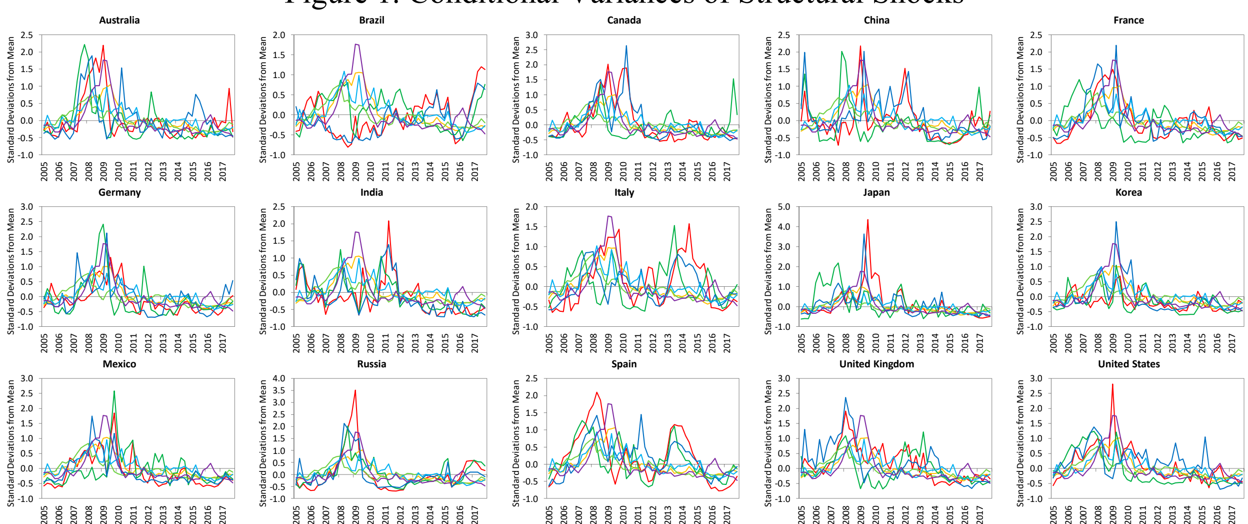
Allowing the structural shocks driving our DSGE model of the world economy to exhibit asymmetric ARCH effects yields an enormous improvement in its goodness of fit. Indeed, on the basis of a general to specific sequence of nested likelihood ratio tests, we find very strong evidence that these structural shocks exhibit ARCH effects. In particular, we reject the null hypothesis of no ARCH effects at all conventional levels of statistical significance, obtaining a point estimate for α_h of 0.15. This result corroborates the substantial symmetric SV effects in structural shocks documented in Justiniano and Primiceri (2008). Moreover, we find very strong evidence that these ARCH effects are asymmetric for some conditional variances, namely those of financial and monetary policy shocks. In particular, we reject the null hypothesis of symmetric ARCH effects at all conventional levels of statistical significance, obtaining a point estimate for γ_h of −0.11. It follows that positive financial and monetary policy shocks—which are associated with a tightening of financial conditions—are estimated to raise their conditional variances less than negative shocks of equal magnitude. Finally, we find little or no evidence of more persistent asymmetric GARCH effects, given these asymmetric ARCH effects.

Table 1. Model Specification Test Results

Restriction	LR-Statistic	P-Value
$\beta_h = 0$	6.77×10^{-3}	0.93441
$\gamma_h = 0 \mid \beta_h = 0$	$1.10 \times 10^{+2}$	0.00000
$\alpha_h = 0 \mid \gamma_h = \beta_h = 0$	$1.07 \times 10^{+3}$	0.00000

The estimated conditional variances of structural shocks exhibit substantial time variation. Indeed, volatility clustering is widespread across economies and types of structural shocks. Volatility generally peaked during the Global Financial Crisis, which was an episode of acute macroeconomic and financial market instability worldwide, accompanied by major disruptions to international trade and financial flows.

Figure 1. Conditional Variances of Structural Shocks



Note: Depicts the average standardized conditional variances of the domestic macroeconomic ■, foreign macroeconomic ■, domestic financial ■, foreign financial ■, domestic policy ■, foreign policy ■, and world terms of trade ■ shocks.

V. Inference

We account for macrofinancial fluctuations and turbulence with historical decompositions of output and financial conditions, which measure the time varying contributions of structural shocks to the levels and conditional variances of these variables. Our historical decomposition of levels is based on

$$\boldsymbol{z}_{t|T} = (\boldsymbol{G}_1)^t \boldsymbol{z}_{0|0} + \sum_{k=1}^{K} \sum_{s=0}^{t-1} (\boldsymbol{G}_1)^s \boldsymbol{g}_{2,k} \varepsilon_{k,t-s|T}, \tag{275}$$

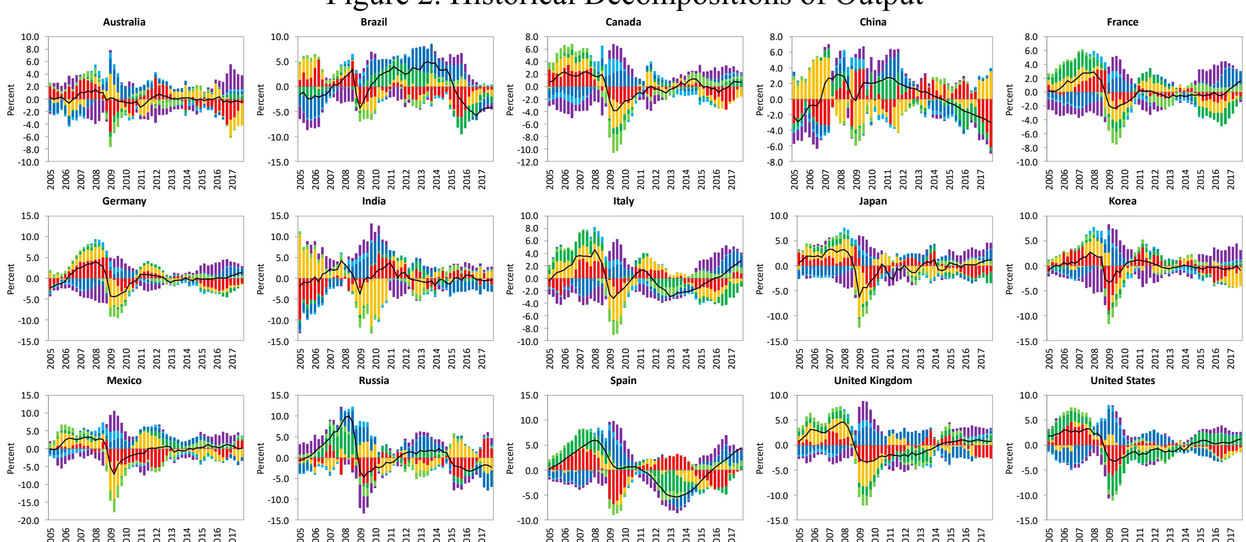
where $\boldsymbol{g}_{2,k}$ denotes column k of matrix $\boldsymbol{G}_2$. This result decomposes the smoothed state vector into the sum of deterministic and stochastic components, where the stochastic component equals the sum of contributions from contemporaneous and past smoothed innovations. In parallel, our historical decomposition of volatilities is based on:

$$\boldsymbol{P}_{t|t-1} \approx (\boldsymbol{G}_1)^t \boldsymbol{P}_{0|0} ((\boldsymbol{G}_1)^t)^\top + \sum_{k=1}^{K} \boldsymbol{g}_{2,k} \boldsymbol{g}_{2,k}^\top h_{k,t|t-1}. \tag{276}$$

This result decomposes the predicted covariance matrix of the state vector into the sum of deterministic and stochastic components, where the stochastic component equals the sum of contributions from contemporaneous conditional variances. Its derivation employs approximation $\varepsilon_{k,t-s} \in \mathcal{I}_{t-1}$ for all $k = 1,\ldots,K$ and $s = 1,\ldots,t-1$.

Our estimated historical decompositions of output attribute cyclical fluctuations primarily to economy specific combinations of domestic and foreign macroeconomic and financial shocks, generally mitigated by policy shocks. Indeed, during the buildup to the Global Financial Crisis, supportive macroeconomic and financial shocks caused a gradual synchronized global cyclical expansion. During the Global Financial Crisis, adverse macroeconomic and financial shocks concentrated in the United States caused an abrupt synchronized global cyclical contraction, mitigated by supportive policy shocks. In the aftermath of the Global Financial Crisis, supportive macroeconomic and financial shocks contributed to a synchronized global cyclical recovery. This recovery was derailed by adverse financial shocks in Italy and Spain during the Euro Area Sovereign Debt Crisis, and in Brazil and Russia in the wake of the Taper Tantrum with the collapse in commodity prices. The gradual global cyclical expansion underway in most advanced and emerging market economies has been driven by economy specific combinations of supportive macroeconomic, financial and policy shocks. On average over time, the relative contribution from domestic versus foreign shocks is decreasing across economies with their trade openness.

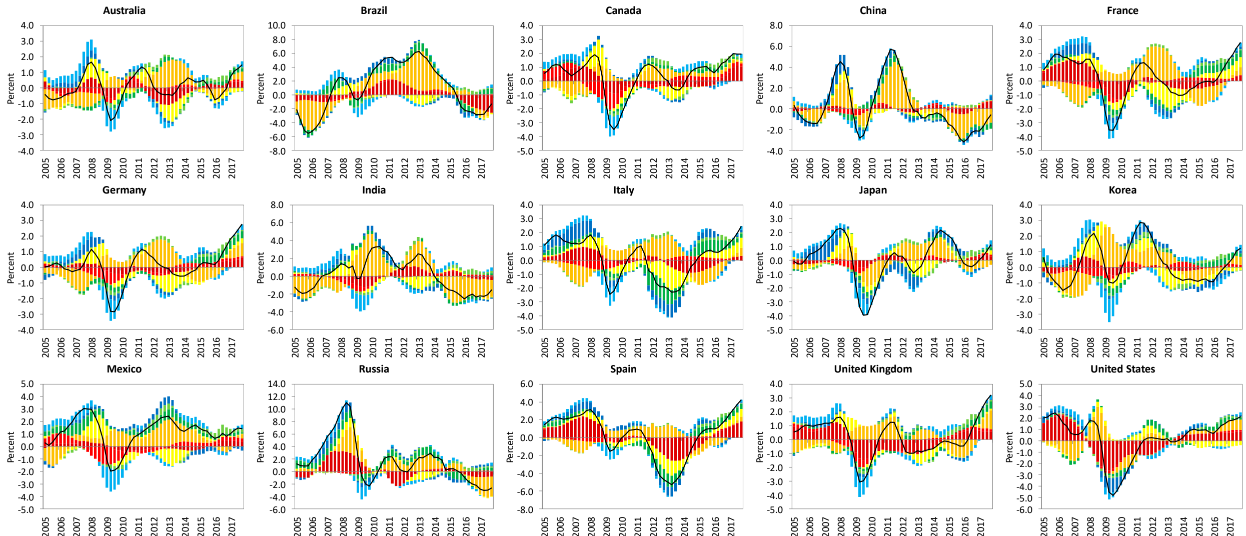
Figure 2. Historical Decompositions of Output



Note: Decomposes the cyclical component of output ■ into contributions from domestic macroeconomic ■, foreign macroeconomic ■, domestic financial ■, foreign financial ■, domestic policy ■, foreign policy ■, and world terms of trade ■ shocks.

The approximate multivariate linear rational expectations representation of our DSGE model does not imply a financial conditions index, conventionally defined as that linear combination of financial variables that drives output. Instead, the different components of private domestic demand are driven by different linear combinations of financial variables. We therefore infer a financial conditions index from our historical decomposition of output, as the contribution from financial and monetary policy shocks to it.

Figure 3. Historical Decompositions of Financial Conditions



Note: Decomposes the financial conditions index ■ into contributions from domestic housing risk premium ■, foreign housing risk premium ■, domestic monetary policy and credit risk premium ■, foreign monetary policy and credit risk premium ■, domestic duration risk premium ■, foreign duration risk premium ■, domestic equity risk premium ■, and foreign equity risk premium ■ shocks.

Across economies and over time, our estimated financial conditions indexes occasionally exhibit abrupt swings that tend to amplify cyclical output fluctuations. In particular, abrupt tightenings of financial conditions are associated with cyclical output contractions. Our estimated historical decompositions of financial conditions reveal the economy specific contributions of domestic and foreign financial and monetary policy shocks to these cyclical output fluctuations. They indicate that the compression, decompression and normalization of domestic housing risk premium shocks contributed substantially to the cyclical output expansions, contractions and recoveries experienced by Spain, the United Kingdom and the United States before, during and after the Global Financial Crisis, respectively. In parallel, these foreign housing risk premium shocks contributed substantially to the cyclical output expansions, contractions and recoveries experienced by Canada, Germany and Mexico before, during and after the Global Financial Crisis respectively, primarily transmitted via trade linkages. At the same time, the compression, decompression and normalization of domestic equity risk premium shocks in the United States generally contributed substantially to the cyclical output expansions, contractions and recoveries experienced worldwide before, during and after the Global Financial Crisis respectively, primarily transmitted via financial linkages to the rest of the world. These cyclical output dynamics induced by the loosening, tightening and normalization of financial conditions worldwide before, during and after the Global Financial Crisis respectively were usually mitigated by economy specific contributions from domestic monetary policy and credit risk premium shocks. During the Euro

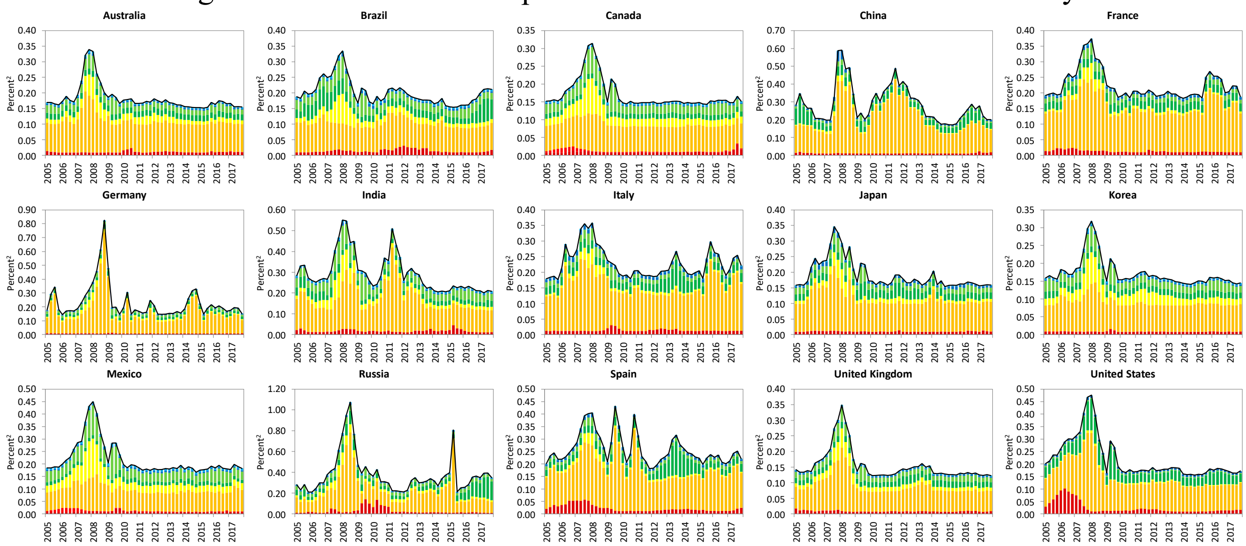
Area Sovereign Debt Crisis, domestic duration risk premium decompression in Italy and Spain contributed substantially to the further severe cyclical output contractions experienced there, while mitigating the mild cyclical output contractions experienced by France and Germany via safe haven capital inflows. Finally, in the wake of the Taper Tantrum, domestic duration and equity risk premium decompression contributed substantially to the further severe cyclical output contractions experienced by Brazil and Russia.

Figure 4. Historical Decompositions of Output Volatility

Note: Decomposes the conditional variance of the cyclical component of output ■ into contributions from the conditional variances of domestic macroeconomic ■, foreign macroeconomic ■, domestic financial ■, foreign financial ■, domestic policy ■, foreign policy ■, and world terms of trade ■ shocks.

Our estimated historical decompositions of output volatility explain its substantial variation across economies and over time. Across economies, output volatility is primarily generated by the conditional variances of macroeconomic shocks, where the relative contribution from the conditional variances of domestic versus foreign macroeconomic shocks is decreasing with trade openness. For major net commodity exporters like Australia, Canada and Russia, the conditional variances of world terms of trade shocks also contribute substantially to output volatility. Over time, the contribution from the conditional variances of domestic macroeconomic shocks to output volatility is essentially constant, whereas that from the conditional variances of foreign macroeconomic shocks exhibits considerable variation, peaking during episodes of trade disruption, which tend to coincide with bouts of financial market turbulence. Indeed, output volatility generally peaked worldwide during the Global Financial Crisis. Reflecting the strong identification of the structural shocks under consideration by our multivariate panel data set, the approximation errors realized in evaluating our historical decompositions of output volatility are negligible, amounting to only 0.03 percent of the conditional variance of output in absolute value, on average across economies and over time.

Figure 5. Historical Decompositions of Financial Conditions Volatility



Note: Decomposes the conditional variance of the financial conditions index ■ into contributions from the conditional variances of domestic housing risk premium ■, foreign housing risk premium ■, domestic monetary policy and credit risk premium ■, foreign monetary policy and credit risk premium ■, domestic duration risk premium ■, foreign duration risk premium ■, domestic equity risk premium ■, and foreign equity risk premium ■ shocks.

Although financial conditions volatility generally does not contribute substantially to output volatility across economies, this contribution does exhibit considerable variation over time. Our estimated historical decompositions of financial conditions volatility reveal the economy specific contributions from the conditional variances of domestic and foreign financial and monetary policy shocks to this time variation. They indicate that financial conditions volatility is primarily generated by the conditional variances of domestic monetary policy and credit risk premium shocks, and secondarily by the conditional variances of duration risk premium shocks, which contribute disproportionally during bouts of financial market turbulence. Indeed, financial conditions volatility spiked worldwide during the Global Financial Crisis. Across economies, the relative contribution from the conditional variances of domestic versus foreign duration risk premium shocks is increasing with financial depth and decreasing with financial openness, with the conditional variance of duration risk premium shocks in the United States making a dominant global contribution. In comparison, the conditional variances of housing and equity risk premia shocks contribute little to financial conditions volatility, and by implication output volatility, across economies and over time.

VI. Conclusion

This paper investigates the sources of macrofinancial fluctuations and turbulence within the framework of an approximate linear DSGE model of the world economy, augmented with structural shocks exhibiting potentially asymmetric GARCH effects. This DSGE model features a range of nominal and real rigidities, extensive macrofinancial linkages, and diverse spillover transmission channels. Very strong evidence of asymmetric ARCH effects is found, providing a basis for jointly decomposing the levels and volatilities of output and financial conditions into time varying contributions from sets of shocks. Risk premia shocks are estimated to contribute disproportionately to cyclical output fluctuations and turbulence during swings in financial conditions, across the fifteen largest national economies in the world.

Given that asymmetric ARCH effects can signal the omission of nonlinearities from the conditional mean function, it would be interesting to test whether they still exist after solving our DSGE model to second order, or generalizing it with time varying parameters. It would also be interesting to assess the relative empirical adequacy of asymmetric ARCH versus asymmetric SV effects as representations of volatility clustering in the structural shocks. These computationally demanding investigations remain objectives for future research.

Appendix A. Data Description

Estimation is based on quarterly data on a variety of macroeconomic and financial market variables observed for fifteen economies over sample period 1999Q1 through 2017Q4. The economies under consideration are Australia, Brazil, Canada, China, France, Germany, India, Italy, Japan, Korea, Mexico, Russia, Spain, the United Kingdom, and the United States. Where available, this data was obtained from the GDS and WEO databases compiled by the IMF, or from databases produced by Bloomberg and the Bank for International Settlements. Otherwise, it was extracted from the IFS database compiled by the IMF, or the ILO database produced by the International Labour Organization.

The macroeconomic variables under consideration are the core price level, the output price level, the consumption price level, the quantity of output, the quantity of private consumption, the quantity of exports, the quantity of imports, the price of housing, the nominal wage, the unemployment rate, employment, the quantity of public consumption, the quantity of public investment, the fiscal balance ratio, and the prices of nonrenewable energy and nonenergy commodities. The core price level is proxied by the seasonally adjusted core consumer price index, while the output price level is measured by the seasonally adjusted gross domestic product price deflator, and the consumption price level is proxied by the seasonally adjusted consumer price index. The quantity of output is measured by seasonally adjusted real gross domestic product, while the quantity of private consumption is measured by seasonally adjusted real private consumption expenditures. The quantity of exports is measured by seasonally adjusted real export revenues, while the quantity of imports is measured by seasonally adjusted real import expenditures. The price of housing is proxied by a broad residential property price index. The nominal wage is derived from the quadratically interpolated annual labor income share, while the unemployment rate is measured by the seasonally adjusted share of total unemployment in the total labor force, and employment is measured by seasonally adjusted total employment. The quantity of public consumption is measured by the quadratically interpolated annual real consumption expenditures of the general government, while the quantity of public investment is measured by the quadratically interpolated annual real investment expenditures of the general government, and the fiscal balance is measured by the quadratically interpolated annual overall fiscal balance of the general government. The prices of energy and nonenergy commodities are proxied by broad commodity price indexes denominated in United States dollars.

The financial market variables under consideration are the nominal policy interest rate, the nominal short term bond yield, the nominal long term bond yield, the price of equity, and the nominal bilateral exchange rate. The nominal policy interest rate is measured by the central bank policy rate, while the nominal short term bond yield is measured by the three month Treasury bill yield, and the nominal long term bond yield is measured by the ten year government bond yield. The price of equity is proxied by a broad stock price index denominated in domestic currency units, while the nominal bilateral exchange rate is measured by the domestic currency price of one United States dollar. All of these financial market variables are expressed as period average values.

Calibration is based on annual data obtained from databases compiled by the IMF where available, and from the Bank for International Settlements, the World Bank Group or the World Federation of Exchanges otherwise. Macroeconomic great ratios are derived from the WEO and WDI databases, while financial great ratios are also derived from the BIS, CPIS and WFE databases. Bilateral trade weights are derived for goods on a cost, insurance and freight basis from the DOTS database. Bilateral bank lending and nonfinancial corporate borrowing weights are derived on a consolidated ultimate risk basis from the BIS database. Bilateral portfolio debt and equity investment weights are derived from the CPIS and WFE databases.

Appendix B. Estimation Results

Table 2. Calibrated Parameter Values

α^C	0.80000	ϕ^A	0.95000	$\zeta^{\delta^M,V}$	0.02500
α^L	0.80000	ϕ^B	0.30000	$\zeta^{\delta^C,Y}$	$0.20000/S$
β	$1/(1+0.04000/S)$	ϕ^C	0.50000	$\zeta^{\delta^C,V}$	0.02500
g	$0.03999/S$	ϕ^D	0.70000	ζ^{T^N}	$0.00001/S$
χ^H	1.50000	ϕ^F	0.70000	ζ^{T^D}	$0.10000/S$
χ^K	1.50000	ϕ^H	0.75000	λ_0^M	$0.25000/w_{i*}^A$
χ^B	1.50000	ϕ_B^A	0.67500	λ_1^M	$0.50000/w_{i*}^A$
χ^C	$1/0.10000$	ϕ_S^A	0.12500	λ_2^M	$0.75000/w_{i*}^A$
δ	$0.01000/S$	ψ^M	1.00000	λ_0^B	$0.20000/w_{i*}^A$
δ^H	$0.10000/S$	$\rho^{\mathcal{A}}$	0.95000	λ_1^B	$0.40000/w_{i*}^A$
δ^K	$0.10000/S$	ρ^i	0.80000	λ_2^B	$0.60000/w_{i*}^A$
δ^G	$0.10000/S$	ρ_κ	0.80000	λ_0^S	$0.25000/w_{i*}^A$
η	1.00000	ρ_{ϕ^D}	0.80000	λ_1^S	$0.50000/w_{i*}^A$
η^K	1.00000	ρ_{ϕ^F}	0.40000	λ_2^S	$0.75000/w_{i*}^A$
η^C	1.25000	ρ_δ	0.60000	$\rho_{\nu,\mathcal{A}}$	0.40000
γ^Y	0.80000	ρ_G	0.80000	$\rho_{\nu,\mathcal{N}}$	0.40000
γ^L	0.80000	ρ_τ	0.80000	$\rho_{\nu,C}$	0.40000
γ^X	0.80000	σ	1.00000	$\rho_{\nu,I}$	0.60000
γ^M	0.80000	ς	0.15000	$\rho_{\nu,X}$	0.80000
ι	1.00000	θ^Y	$(1+0.15000)/0.15000$	$\rho_{\nu,M}$	0.80000
κ^R	0.10000	θ^C	$(1+0.01000/S)/(0.01000/$	$\rho_{\nu,i,B}$	0.80000
$\lambda^{\mathcal{A}}$	0.50000	ξ^π	2.00000	$\rho_{\nu,H}$	0.80000
λ^Y	0.01000	ξ^Y	$0.50000/S$	$\rho_{\nu,i,S}$	0.80000
μ^X	0.25000	$\xi^{\mathcal{E}}$	0.10000	$\rho_{\nu,B}$	0.80000
μ^M	0.15000	$\xi^{\mathcal{E}_k}$	2.00000	$\rho_{\nu,S}$	0.80000
ω^B	0.95000	$\zeta^{\kappa,B}$	$0.05000S$	$\rho_{\nu,\mathcal{E}}$	0.40000
ω^Y	$(6.00000-1)/6.00000$	ζ^{κ,V^H}	$0.02500S$	ρ_{ϱ^C}	0.60000
ω^L	$(6.00000-1)/6.00000$	ζ^{κ,V^S}	$0.01250S$	$\rho_{\nu,G,C}$	0.60000
ω^X	$(6.00000-1)/6.00000$	$\zeta^{\phi^D,B}$	$0.05000S$	$\rho_{\nu,G,I}$	0.60000
ω^M	$(6.00000-1)/6.00000$	$\zeta^{\phi^D,V}$	$0.02500S$	$\rho_{\nu,\tau}$	0.40000
ω^C	$(4.00000-1)/4.00000$	$\zeta^{\phi^F,B}$	$0.05000S$	$\rho_{\nu,T}$	0.80000
ω_1^Y	$(1.25000-1)/1.25000$	$\zeta^{\phi^F,V}$	$0.01250S$	$\rho_{\nu,\kappa}$	0.60000
ω_2^Y	$(1.50000-1)/1.50000$	$\zeta^{\delta^M,Y}$	$0.10000/S$		

Note: The calibration is a function of the seasonal frequency S, evaluated at $S=4$.

Table 3. Estimated Parameter Values

$\omega_{\nu,\mathcal{A}}$	$1.71\times10^{+0}$ ***	ω_{ϱ^X}	$1.95\times10^{+6}$ ***	$\omega_{\nu,G,C}$	1.95×10^{-1} ***
$\omega_{\nu,\mathcal{N}}$	$5.11\times10^{+0}$ ***	ω_{ϱ^M}	$4.69\times10^{+3}$ ***	$\omega_{\nu,G,I}$	$9.41\times10^{+0}$ ***
$\omega_{\nu,C}$	$1.22\times10^{+2}$ ***	$\omega_{\nu,H}$	1.94×10^{-1} ***	$\omega_{\nu,\tau}$	2.89×10^{-1} ***
$\omega_{\nu,I}$	$4.47\times10^{+1}$ ***	$\omega_{\nu,i,S}$	1.02×10^{-2} ***	$\omega_{\varrho^Y,k}$	$3.39\times10^{+1}$ ***
$\omega_{\nu,X}$	$7.27\times10^{+0}$ ***	$\omega_{\nu,B}$	4.54×10^{-1} ***	α_h	1.46×10^{-1} ***
$\omega_{\nu,M}$	$9.53\times10^{+0}$ ***	$\omega_{\nu,S}$	$2.25\times10^{+0}$ ***	γ_h	-1.10×10^{-1} ***
ω_{ϱ^Y}	$2.80\times10^{+2}$ ***	$\omega_{\nu,\mathcal{E}}$	$6.98\times10^{+0}$ ***		
ω_{ϱ^L}	$1.74\times10^{+3}$ ***	$\omega_{\nu,i,P}$	7.37×10^{-2} ***		

Note: Statistical significance at the 1, 5 and 10 percent levels is indicated by ***, ** and *, respectively.

References

Adrian, T., N. Boyarchenko and D. Giannone (2017), "Vulnerable Growth", *Federal Reserve Bank of New York Staff Report*, 794.

Black, F. (1976), "Studies of Stock Price Volatility Changes", *Proceedings from the American Statistical Association, Business and Economic Statistics Section*, pp. 177−181.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", *Journal of Econometrics*, Vol. 31, pp. 307−327.

Bollerslev, T. (1990), "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach", *Review of Economics and Statistics*, Vol. 72, pp. 498−505.

Calvo, G. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, Vol. 12, pp. 383−398.

Christiano, L., M. Eichenbaum and C. Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy*, Vol. 113, pp. 1−45.

de Jong, P. (1989), "Smoothing and Interpolation with the State-Space Model", *Journal of the American Statistical Association*, Vol. 84, pp. 1085−1088.

Engle, R. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation", *Econometrica*, Vol. 50, pp. 987−1008.

Engle, R. and M. Watson (1981), "A One-Factor Multivariate Time Series Model of Metropolitan Wage Rates", *Journal of the American Statistical Association*, Vol. 76, pp. 774−781.

Erceg, C., D. Henderson and A. Levin (2000), "Optimal Monetary Policy with Staggered Wage and Price Contracts", *Journal of Monetary Economics*, Vol. 46, pp. 281−313.

Galí, J. (2011), "The Return of the Wage Phillips Curve", *Journal of the European Economic Association*, Vol. 9, pp. 436−461.

Gerali, A., S. Neri, L. Sessa and F. Signoretti (2010), "Credit and Banking in a DSGE Model of the Euro Area", *Journal of Money, Credit and Banking*, Vol. 42, pp. 107−141.

Glosten, L., R. Jagannathan and D. Runkle (1993), "On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks", *Journal of Finance*, Vol. 48, pp. 1779−1801.

Hodrick, R. and E. Prescott (1997), "Post-War U.S. Business Cycles: A Descriptive Empirical Investigation", *Journal of Money, Credit and Banking*, Vol. 29, pp. 1−16.

Hülsewig, O., E. Mayer and T. Wollmershäuser (2009), "Bank Behavior, Incomplete Interest Rate Pass-Through, and the Cost Channel of Monetary Policy Transmission", *Economic Modelling*, Vol. 26, pp. 1310−1327.

International Monetary Fund (2016), *Annual Report on Exchange Arrangements and Exchange Restrictions*, Washington.

Justiniano, A. and G. Primiceri (2008), "The Time-Varying Volatility of Macroeconomic Fluctuations", *American Economic Review*, Vol. 98, pp. 604−641.

Kalman, R. (1960), "A New Approach to Linear Filtering and Prediction Problems", *Transactions ASME Journal of Basic Engineering*, Vol. 82, pp. 35−45.

Kiyotaki, N. and J. Moore (1997), "Credit Cycles", *Journal of Political Economy*, Vol. 105, pp. 211−248.

Klein, P. (2000), “Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model”, *Journal of Economic Dynamics and Control*, Vol. 24, pp. 1405−1423.

Krippner, L. (2015), *Term Structure Modeling at the Zero Lower Bound: A Practitioner's Guide*, Palgrave Macmillan.

Kroner, K. and V. Ng (1998), “Modelling Asymmetric Comovements of Asset Returns”, *Review of Financial Studies*, Vol. 11, pp. 817−844.

Monacelli, T. (2005), “Monetary Policy in a Low Pass-Through Environment”, *Journal of Money, Credit and Banking*, Vol. 37, pp. 1047−1066.

Smets, F. and R. Wouters (2003), “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”, *Journal of the European Economic Association*, Vol. 1, pp. 1123−1175.

Smets, F. and R. Wouters (2007), “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach”, *American Economic Review*, Vol. 97, pp. 586−606.

Tobin, J. (1969), “A General Equilibrium Approach to Monetary Theory”, *Journal of Money, Credit and Banking*, Vol. 1, pp. 15−29.

Vitek, F. (2014), “Policy and Spillover Analysis in the World Economy: A Panel Dynamic Stochastic General Equilibrium Approach”, *International Monetary Fund Working Paper*, 200.

Vitek, F. (2018), “The Global Macrofinancial Model”, *International Monetary Fund Working Paper*, 81.

Watson, M. (1989), “Recursive Solution Methods for Dynamic Linear Rational Expectations Models”, *Journal of Econometrics*, Vol. 41, pp. 65−89.

Weber, M. (1905), *The Protestant Ethic and the Spirit of Capitalism*, Charles Scribner's Sons.

Woodford, M. (2001), “Fiscal Requirements for Price Stability”, *Journal of Money, Credit and Banking*, Vol. 33, pp. 669−728.