



**WP/18/265**

# **IMF Working Paper**

---

Demographics, Pension Systems and the Saving-  
Investment Balance

by Hua Chai and Jun Il Kim

I N T E R N A T I O N A L M O N E T A R Y F U N D

**IMF Working Paper**

Fiscal Affairs Department

**Demographics, Pension Systems and the Saving-Investment Balance**

**Prepared by Hua Chai (FAD) and Jun Il Kim (IEO)**

Authorized for distribution by Era Dabla-Norris

November 2018

**IMF Working Papers describe research in progress by the author(s) and are published to elicit comments and to encourage debate.** The views expressed in IMF Working Papers are those of the author(s) and do not necessarily represent the views of the IMF, its Executive Board, or IMF management.

**Abstract**

This paper studies the effect of demographic change on national saving, global interest rates, and international capital flows, focusing on the role of the public pension system. We develop a small open economy overlapping generations model to illustrate the channels through which demographic variables and pension system generosity interact to affect both private and public saving behavior. We then extend this framework to a two-country setting and simulate scenarios of demographic change and pension reform. We find that the generosity of the pension system plays an important role in determining the movement of the global interest rate and patterns of international capital flows.

JEL Classification Numbers: E21, F21, F32, F41, H55

Keywords: demographics, pension, aging, saving, investment, current account

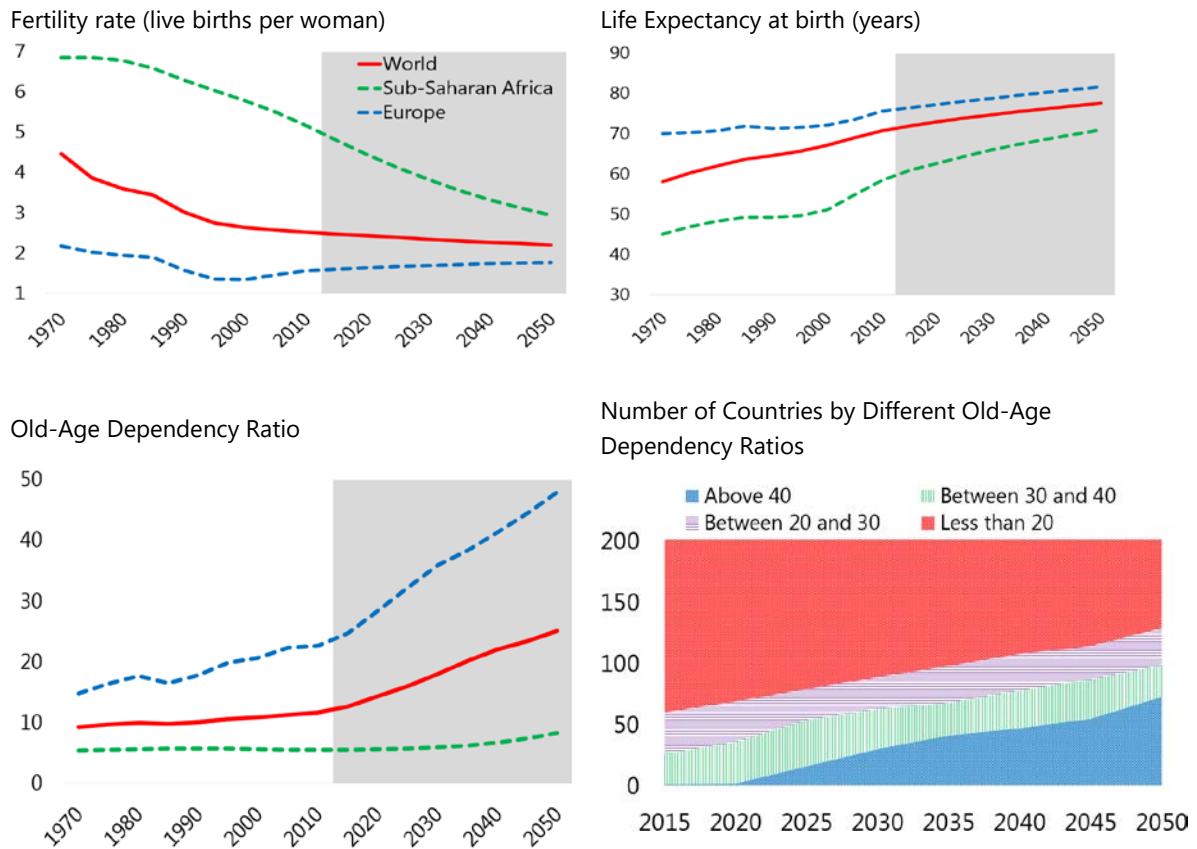
Authors' E-mail Address: [hchai@imf.org](mailto:hchai@imf.org) , [jkim2@imf.org](mailto:jkim2@imf.org)

<b>Content</b>	<b>Page</b>
Abstract .....	2
I. Introduction .....	4
II. Small Open Economy Model Setup.....	9
III. Steady-State Equilibrium .....	12
IV. A Two-Country Extension .....	15
V. Conclusion.....	18
References.....	20
 Figures	
1. Demographic Developments .....	4
2. Frequency Distribution of Coverage and Benefit Ratios Around the World, 2015 .....	5
3. Two-Country Model Simulation Scenario 1 .....	18
4. Two-Country Model Simulation Scenario 2 .....	19
5. Two-Country Model Simulation Scenario 3 .....	19

## I. INTRODUCTION

Many advanced and emerging market economies are facing rapid, unprecedented aging of their populations, driven by declining fertility and rising life expectancy, albeit at varying paces across countries (Figure 1). How will these asynchronous demographic developments affect national saving across countries? Clearly, the differential speed with which countries are experiencing population aging will shape the pattern of international capital flows, with implications for the marginal product of capital and thus the equilibrium international real interest rate.

Figure 1. Demographic Developments



Sources: UN World Population Prospects 2017; and IMF staff calculations.

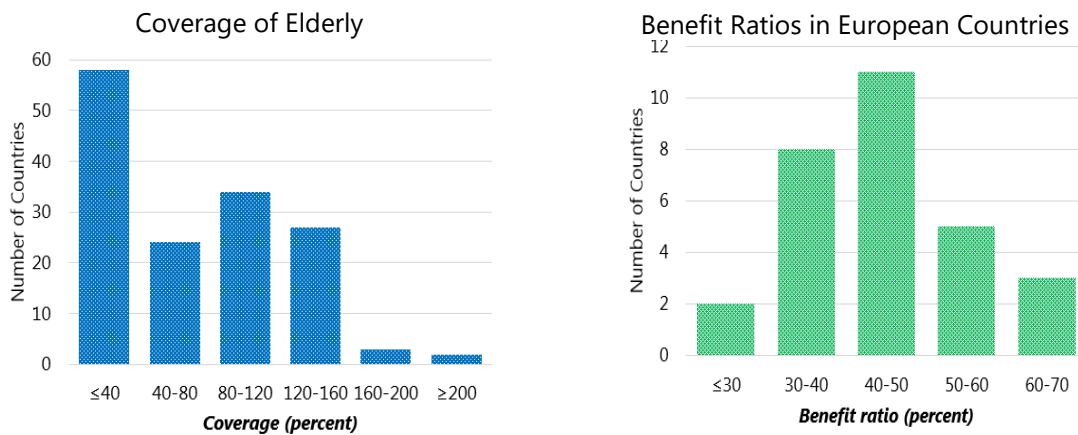
Notes: Population weights are used to sum across country groups.

National saving in an economy is equivalent to the sum of private and public savings. The impact of demographic changes on the private sector saving behavior, and the resulting patterns of international capital flows have been extensively studied in the theoretical literature (see for e.g. Brooks, 2003; Henriksen, 2002; Backus, Cooley and Henriksen, 2014). These studies are largely based on overlapping-generations (OLG) models of Samuelson (1958) and Auerbach and

Kotlikoff (1987).<sup>1</sup> Public saving, however, is typically not investigated in these studies, even though it also responds to shifts in demographic variables. For instance, rising public pension spending driven by population aging could depress public sector saving in the absence of commensurate adjustment in taxes and other spending. The magnitude of this effect depends to a large extent on the characteristics of the public pension system, such as coverage and generosity, which vary across countries (Figure 2; see also Amaglobeli and others, 2018).

Variation in the availability, structure, and coverage of pension systems around the world could also influence individuals' savings decisions. For instance, lack of development and coverage of pensions systems could cause households to save too little for retirement, particularly if financial markets do not offer appropriate saving instruments. Insufficient coverage, however, could also encourage excessive precautionary savings. Analyzing private and public savings in an integrated framework is thus essential to better understand the implications of demographics on national saving and international capital flows.

**Figure 2. Frequency Distribution of Coverage and Benefit Ratios Around the World, 2015**



Sources: World Bank; European Commission; authors' calculations.

Note. Coverage of the elderly is defined as the number of old-age pensioners divided by population 65 and above. The data are reported for 186 countries. Coverage can be above 100 percent in cases where people can retire before age 65. The benefit ratio is defined as the ratio between the average pension benefit and the economy-wide average wage. These data are reported for 31 countries in Europe.

<sup>1</sup> Alternative modeling approaches include the so-called perpetual youth model by Yarri (1965) and Blanchard (1985). Bloom, Canning, Mansfield and Moore (2007) use the perpetual youth model to study the effects of rising longevity on savings. Carvalho, Ferrero and Nechio (2016) use the model to investigate the effects of both rising longevity and decreasing fertility on the interest rate. Neither of these studies consider the impacts of demographic changes on public savings. The neoclassical growth model is also used as still another alternative, for example, as in Domeij and Floden (2006).

This paper is a theoretical attempt at studying the impact of demographic change on both private and public savings, and international capital flows, highlighting the role of public pension system design. We first present a tractable, small open economy three-period OLG model which features a public Pay-As-You-Go (PAYG) pension scheme, the most common form of public pension system in the world. In the model, economic agents live for three periods: as children, workers, and, if surviving into old age, retirees. The working age population are the only active economic decision makers. They optimally choose their labor supply, consumption, and saving. They also give birth to offspring at an exogenous fertility rate, and bear childrearing costs that are proportional to the number of offspring.<sup>2</sup> Economic agents survive into old age and become retirees with an exogenous probability of survival. The assets of those who do not survive into old age are redistributed to the workers as bequests, capturing within-family intergenerational transfers. The two exogenous demographic parameters, life expectancy (longevity) and the fertility rate, jointly determine the old-age dependency ratio. A higher survival probability translates into higher life expectancy at birth. Other than longevity risk, the model abstracts away from other uncertainties that could give rise to motives for precautionary saving. The small open economy takes the global interest rate as given.

The model features both private pensions and a public PAYG-defined benefits scheme that covers all retirees. Workers save to fully fund their private pension benefits. The public pension benefit that a retiree receives is proportional to the wage income while working. Given universal coverage, the public pension is considered more generous the higher the benefit ratio. For simplicity, we assume that the government levies an income tax on labor at an exogenous and constant rate to finance the public pension system, which can be financially unbalanced. The government can thus engage in borrowing or lending, and public saving reacts to changes in demographic trends and pension parameters. A pension deficit reduces public saving, whereas a surplus has the opposite effect.

We use analytical solutions from the simple model to compare steady states with different life expectancy and fertility rates. We show that demographic changes impact both public and private saving. A lower fertility rate increases private saving as households need to spend less on childcare. This effect is partially mitigated through a labor supply response: a smaller fraction of labor income devoted to childcare induces individuals to work more, thus increasing consumption. In line with theoretical models that capture lifecycle behavior, higher life expectancy at retirement raises private saving of the working age population to finance old-age consumption.<sup>3</sup> The impact of a longer lifespan on labor supply, however, depends on a number

---

<sup>2</sup> Having household saving and consumption decisions explicitly depend on the number of children in the household is a natural way to consider the impact of declining family size on saving rates.

<sup>3</sup> Our analysis suggests that the positive effect of higher life expectancy on private saving is attenuated in economies where within-family intergenerational transfers are an important source of income for the working age population.

of offsetting factors. Both lower fertility and higher life expectancy increase the old-age dependency ratio, and hence the share of dis-savers relative to savers in the economy. This demographic compositional effect serves to depress private saving. The overall impact on private saving thus depends on which effect dominates.<sup>4</sup> Higher old age dependency lowers public saving as public pension expenditure rises relative to tax revenue.

Public pension generosity has both a direct and an indirect effect on private saving. Firstly, public pension benefits and private saving are substitutes when it comes to providing old-age retirement income. Private saving directly depends on public pension generosity as higher pension benefits imply that individuals need to put less aside as private saving to finance consumption in the retirement.<sup>5</sup> This result is consistent with both theoretical and empirical findings that countries with PAYG systems tend to have lower private saving rates, the more generous the public pension benefits (Curtis and others, 2017, Rezk and others, 2009; Bloom and others, 2007). Secondly, there is an indirect effect, as the magnitude of the impact of aging on private saving depends on pension generosity, since demographic variables and pension generosity jointly determine the level of savings. In particular, higher generosity amplifies the positive effect on private saving of the working age population as life expectancy rises. Hence, economies with similar demographic trends could exhibit different private saving behavior owing to differences in pension system characteristics. At the same time, the negative impact of demographic changes on public saving is more pronounced in countries with relatively generous public pension systems.

Our model also sheds light on the impact of changes in the global interest rate on private savings. A lower return on assets discourages saving (the substitution effect). However, lower future income from savings can induce workers to reduce current consumption, thereby increasing saving (the income effect).<sup>6</sup> The overall impact depends on which effect dominates. In addition to these income and substitution effects, a lower interest rate also reduces the size of bequests relative to the size of the economy. Since bequests are another source of saving in the economy, a lower global interest rate serves to reduce private saving as a share of GDP. In this

---

<sup>4</sup>The private-saving-to-GDP ratio would also depend on the output impact of demographic changes. While our model suggests that the output effect exactly offsets the compositional effect, this result does not generalize in alternative model settings.

<sup>5</sup> This result follows from a direct income effect as well as an indirect labor supply effect. The latter stems from the fact that pension benefits in old age are proportional to labor income while working—higher pension generosity induces households to supply more labor. The worker compensates for the disutility of supplying labor by increasing consumption and reducing saving.

<sup>6</sup> The income effect is smaller the more generous is the pension system. Old age income consists of gross returns to private saving and public pension benefits. The income effect from changes in the interest rate only applies to the former. In other words, higher pension benefits translate into a smaller share of old age income subject to the income effect. The impact of the income effect will thus be smaller the more generous the public pension system.

framework, however, current account changes are solely determined by the saving side, as investment is independent of demographics given a constant global interest rate.

We then extend the small open economy model to a two-country setting to allow endogenous adjustment of the global interest rate to demographic shifts. We resort to numerical simulations to characterize the dynamic behavior of private and public savings, the interest rate, and the current account, as analytical solutions are not attainable. The numerical examples reveal some interesting results. Firstly, private and public saving can move in opposite directions in response to aging. The impact on national saving and the global interest rate thus depends on the relative strength of these two forces. Public pension generosity is crucial as it affects the responsiveness of both private and public savings. Secondly, countries with similar demographics could generate different saving behavior and capital flow patterns due to differences in pension systems generosity. Countries with a less generous public pension system tend to run current account surpluses due to a higher saving rate.

This paper is related to several strands of literature. The fact that PAYG public pension systems affect saving behavior has been recognized at least as early as Feldstein (1976). A more recent literature considers the effects of pension systems on saving as well as international capital flows. In a two-country OLG model, Eugeni (2015) demonstrates that the saving behavior of emerging economies and capital outflows to the United States can be attributed to weaknesses in their PAYG systems (e.g., poor coverage or low benefits). Staveley-O'Carrol and Staveley-O'Carrol (2017) develop a two-country OLG model that explores the relationship between the magnitude of pension guarantees and the resulting portfolio choices of workers. These papers, however, do not study demographics.

More closely related to this paper is the strand of literature that explores the effects of demographic change on saving and international capital flows in OLG models with a public pension system. Attanasio, Kitao and Violante (2006, 2007), Böersch-Supan, Ludwig and Winter (2006) and Attanasio, Bonfatti, Kitao and Weber (2016) develop quantitative multi-region models and examine consumption, saving, and international capital flows in regions differing in productivity and demographic trends. Barany, Coeurdacier and Guibaud (2016) investigate capital flows between economies differing in demographic variables as well as the ability to borrow inter-temporally and across generations through social security. They show that fast-growing emerging economies that are aging fast and have less developed credit markets and pension systems should be expected to export capital. Even though these studies explicitly model the public pension system and emphasize its impact on consumption and saving decision of private agents, the government budget is typically assumed to be balanced period-by-period. This assumption rules out responses of public saving to demographic developments and changes in pension generosity. In contrast, we allow pension spending to be partially financed by drawing down public financial resources, resulting in corresponding changes in public saving. We show that this change in public saving is not offset by the change in private saving, and hence affects the level of national saving.



The rest of the paper is organized as follows. Section II sets up the small open economy OLG model. Section III analyzes the steady-state equilibrium of the model. Section IV extends the model to a dynamic two-country setting and provide numerical examples illustrating how changing demographics and pension reforms affect private and public savings, the global interest rate and international capital flows. Section IV concludes.

## II. SMALL OPEN ECONOMY MODEL SETUP

In this section we set up a small open economy overlapping generations model to inspect the mechanisms by which pension system generosity and demographic variables affect private and public savings, while holding the interest rate fixed which greatly facilitates the analysis. We allow perfect capital mobility but shut down international movement of labor, abstracting away from the issue of labor migration. We keep assumptions simple for maximum tractability.

### ***Production and demographics***

Production is given by standard Cobb-Douglas function, where K denote capital and L labor.

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

Time is discrete. Agents live for at most three periods, namely as children, the young working-age (y) and old (o). The young give birth to offspring at a rate  $g_t$  in period t and survive into old age with constant probability  $\chi_t$ . That is,

$$N_{t+1}^y = g_t N_t^y,$$

$$N_{t+1}^o = \chi_t N_t^y.$$

Life expectancy at birth is therefore  $1 + \chi_t$ . It is straightforward to show that the old-age dependency ratio is  $\chi_t/g_t$ , increasing in longevity and decreasing in fertility. The population growth rate is equal to the fertility rate. Retirees consume all their private pension savings and public pension benefits in one period.

### ***Young agents' problem***

At time  $t$ , a representative young agent supplies labor  $l_t$  at the wage rate  $w_t$ , and pays labor income tax at rate  $\tau$  as well as childcare costs  $\eta$  as a share of labor income per child. The latter assumption captures the fact that childcare costs are generally positively correlated with the income level. The young also save  $a_{t+1}$  in competitive private pension funds at rate of gross return,  $R^*$ . The private pension funds can hold government bonds or invest in physical capital and rent to firms both domestically and abroad. Old people do not work and live off both private and public pension payments. The public pension payment is a fraction  $\phi$  of the old's labor income when young, i.e.,  $b_{t+1} = \phi w_t l_t$ . If a young person does not survive into old age, her

assets are distributed as unintentional bequests evenly among the working age population. The young's optimization problem is therefore:

$$\begin{aligned} & \max_{a_{t+1}, l_t^y} u(c_t^y, l_t) + \beta \chi_t u(c_{t+1}^o, 0) \\ \text{s. t. } & c_t^y = (1 - \tau - \eta g_t) w_t l_t + v_t - a_{t+1} \\ & c_{t+1}^o = R^* a_{t+1} + \phi w_t l_t \end{aligned}$$

The bequest,  $v_t$ , is given by:

$$v_t = \frac{(1 - \chi_{t-1}) R^* a_t}{g_{t-1}}. \quad (1)$$

The utility function is assumed to take the following form:

$$u(c, l) = \log\left(c - \frac{\theta}{1 + \varepsilon} l^{1 + \varepsilon}\right),$$

which is a special case of the Greenwood-Hercowitz-Huffman (GHH) utility function with the intertemporal elasticity of substitution being unity. The GHH formulation abstracts from wealth effects on labor supply. These preferences are widely used in the literature as it allows quantitative models to produce reasonably sized fluctuations in labor supply in the absence of either nominal price rigidities or labor market frictions. Solving the maximization problem reveals that labor supply is given by:

$$l_t = \left[ \left( \frac{\phi}{R^*} + 1 - \tau - \eta g_t \right) \frac{w_t}{\theta} \right]^{1/\varepsilon}.$$

Labor supply is increasing in the wage rate and pension generosity (the substitution effect). Note that there is no wealth effect given the way labor enters the utility function, and therefore the substitution effect dominates. The wage rate is constant in equilibrium. Higher life expectancy does not induce the young to supply more labor even though it increases the expected payoff of pension benefits by raising the effective discount factor. The reason is that the young would choose to suppress consumption to save more for retirement, raising the marginal utility of leisure and thus discouraging higher labor supply.<sup>7</sup> These two effects cancel out. A lower fertility rate reduces the effective tax rate on labor income, thereby encouraging more labor supply (see also Curtis and others, 2015). The young agent's saving is given by:

$$a_{t+1} = \left[ 1 - \tau - \eta g_t - \left( 1 + \frac{\varepsilon}{1 + \beta \chi} \right) \frac{1 - \tau - \eta g_t + \phi/R^*}{1 + \varepsilon} \right] w_t l_t + \frac{\beta \chi}{1 + \beta \chi} v_t. \quad (2)$$

<sup>7</sup> This result is specific to the GHH utility function, where  $U_{cl} > 0$ . With a separable utility function, for example,  $u(c, l) = c^{1-\sigma}/(1-\sigma) - \theta l^{1+\varepsilon}/(1+\varepsilon)$ , lower consumption this period does not affect the marginal utility of leisure to discourage labor supply. As a result, higher life expectancy leads to higher labor supply (see, e.g. He, Ning, and Zhu, 2018).

The savings of the young can be decomposed into two parts. The first part captures savings out of the present value of after-tax wage income and expected pension benefits, while the second part is the saved portion of received bequests. It is straightforward to show that given the size of wage income and bequests, higher life expectancy increases both components of savings as it effectively makes the young more patient. This effect is more pronounced with a more generous pension system. This is because a more generous pension system implies higher future income, and higher life expectancy suggests that a larger share of this future income is saved. Lower fertility frees up more income to be saved, although this effect is partially mitigated by the fact that it also increases labor supply, and hence induces more consumption (and less savings). A more generous pension increases resources in old age, thus reducing the incentive to save for the working age population. Of course, demographics and pension generosity may also affect wage income and bequests in equilibrium, which is examined below.

### ***The public PAYGO pension system***

There is a PAYGO public pension system with defined benefits. Total pension payout,  $B_t$ , is the product of benefits per retiree,  $b_t$ , and the number of retirees,  $N_t^o$ . The government issues one-period debt. Debt service in period  $t$  is  $R^*D_t$  (principal and interest). New borrowing is  $D_{t+1}$ . The government (flow) budget constraint is given by:

$$\tau w_t L_t + D_{t+1} - B_t - R^*D_t = 0.$$

The government's solvency constraint is:

$$\sum_{t=0}^{\infty} (R^*)^{-t} (\tau w_t L_t - B_t) \geq R^*D_0.$$

For simplicity, we assume  $D_0 = 0$ . The stock of public savings is the government's net asset position. Since all debt is one-period, it is simply given by:

$$S_t^G = -D_{t+1}.$$

Public saving is positive if the government is a net-lender and negative if it is a net-borrower. To focus on how characteristics of the pension system affect saving, we treat pension generosity and the tax rate as exogenous parameters, rather than the endogenous outcome of a government's optimization problem.

### III. STEADY-STATE EQUILIBRIUM

We assume away technological progress and set  $A_t = 1$ . In equilibrium, the capital/labor ratio is constant and pinned down by the global interest rate. Therefore, the wage rate is also constant. Individual labor supply is thus also constant and a function of demographic and pension parameters. This leads total labor supply, the capital stock and output to all grow at rate  $g_t$ . We now analyze a steady-state equilibrium.

#### Private saving

In equilibrium, the saving rate of a young individual is characterized by a first-order difference equation,

$$a_{t+1} = \left[ 1 - \tau - \eta g_t - \left( 1 + \frac{\varepsilon}{1 + \beta\chi} \right) \frac{1 - \tau - \eta g_t + \phi/R^*}{1 + \varepsilon} \right] w_t l_t + \frac{\beta\chi}{1 + \beta\chi} v_t,$$

where  $v_t$  is given by equation (1). At steady state, bequest is given by:

$$v = \frac{(1 - \chi)R^*a}{g}.$$

Combining the above two equations yields the steady state level of private saving per worker:

$$a = \frac{\left[ (1 - \tau - g\eta) - \frac{(1 - \tau - g\eta) + \phi/R^*}{1 + \varepsilon} \left( 1 + \frac{\varepsilon}{1 + \beta\chi} \right) \right]}{1 - \frac{(1 - \chi)R^*}{g} \left( \frac{\beta\chi}{1 + \beta\chi} \right)} w l.$$

Aggregate private saving is simply  $S_t^P = aN_t^Y$ , as the retirees consume all income and keep no savings. A higher old-age dependency ratio tends to lower aggregate saving due to the composition effect, as a higher share of the population become retirees. The private saving to output ratio is given by:

$$\frac{S^P}{Y} = \frac{aN^y}{Y} = \frac{\left[ (1 - \tau - g\eta) - \frac{(1 - \tau - g\eta) + \phi/R^*}{1 + \varepsilon} \left( 1 + \frac{\varepsilon}{1 + \beta\chi} \right) \right]}{1 - \frac{(1 - \chi)R^*}{g} \left( \frac{\beta\chi}{1 + \beta\chi} \right)} (1 - \alpha).$$

The above expression suggests that private saving decreases with the generosity of the public pension system. A lower fertility rate increases the private saving rate, as demonstrated in the section on the young agents' problem. In addition, lower fertility increases the size of bequests

relative to output. The impact of higher life expectancy on the private saving rate is ambiguous, as it decreases the size of bequests.

### Public saving

It is also straightforward to show that the tax revenue,  $T_t$ , and pension expenditure are constant as share of GDP.

$$\frac{T_t}{Y_t} = \tau(1 - \alpha)$$

$$\frac{B_t}{Y_t} = \frac{(1 - \alpha)\chi\phi}{g}$$

The budget constraint of the government can be re-written as a first-order difference equation in the debt-to-GDP ratio.

$$\frac{D_{t+1}}{Y_{t+1}} = R^* \frac{Y_t}{Y_{t+1}} \frac{D_t}{Y_t} + \frac{B_t - \tau w_t L_t}{Y_t} \frac{Y_t}{Y_{t+1}}.$$

We impose the restriction that  $g > R^*$  so that the debt-to-GDP ratio converges to a finite steady state.<sup>8</sup> It is straightforward to show that this ratio is:

$$\frac{D}{Y} = (1 - \alpha) \frac{\chi\phi - \tau g}{g - R^*}.$$

We also assume that  $\tau \geq \chi\phi/g$  to ensure that the government's solvency constraint is satisfied (as it runs a permanent primary surplus). These parameter restrictions imply that the public sector maintains positive saving at the steady state. The public saving to GDP ratio is given by:

$$\frac{S^G}{Y} = (1 - \alpha) \frac{\tau g - \chi\phi}{1 - R^*/g}.$$

Higher life expectancy and higher pension system generosity decrease public saving as more pension spending is required. The effect of lower fertility is indeterminate. It lowers public saving, on the one hand, since fewer workers pay taxes relative to the number of retirees. On the other hand, it also reduces the size of output.

---

<sup>8</sup> An alternative route would be to assume  $R^* > g$ , and then follow Bohn (2007) by assuming a tax policy rule where the tax rate positively responds to lagged debt-to-GDP ratio. This approach could potentially ensure the existence of a steady state even  $R^*$  is greater than  $g$ .

National saving is the sum of private and public saving. Higher pension generosity decreases both private and public savings, and hence lowers national saving. The effects of demographic change on national saving, on the other hand, is harder to ascertain theoretically.

### **Current account**

Capital flows freely across borders to ensure rate of return is equal to the global interest rate. The resulting capital-to-output ratio, given the Cobb-Douglas production function can be straightforwardly shown to be:

$$\frac{K}{Y} = \frac{\alpha}{R^* - 1 + \delta}$$

where  $\delta$  is the depreciation rate of capital. The capital to GDP ratio is independent of demographics, since any change in the labor force decreases capital and output at the same time without affecting the ratio of the two. The investment rate is given by:

$$\frac{I_t}{Y_t} = \frac{K_{t+1} - (1 - \delta)K_t}{Y_t} = \frac{\alpha(g - 1 + \delta)}{R^* - 1 + \delta}$$

The net foreign asset (NFA) position of the country at the end of the period is the difference between the value of its foreign assets and the value of domestic assets owned by foreigners. It is equivalent to the difference between the stock of national saving and the next period capital stock in the economy<sup>9</sup>:

$$NFA_t = S_t - K_{t+1}.$$

The current account is equal to the change in the net foreign asset position. Since the capital to output ratio is fixed and independent of demographics, the impact of demographics and pension generosity on the current account (as percent of GDP) come from the saving side. A reduction in the fertility rate raises the current account balance as it increases the stock of savings. The effect of longer life expectancy, however, is ambiguous.

The model in this section allows for permanent one-time "shocks" to longevity and fertility and explores the impact of these demographic changes on public and private saving. In reality, demographic changes are gradual shifts rather than sudden shocks. The simplified analysis here thus does not capture the transition dynamics of an aging economy, but nevertheless offers useful insights into long-run changes in saving rates and enables comparison of saving rates across economies with different demographic trends.

---

<sup>9</sup> Since there is only one financial asset in the model, namely a one-period bond, bearing interest at the global interest rate, we assume a country is either an issuer or a buyer on the international bond market, but not both.

#### IV. A TWO-COUNTRY EXTENSION

In this section we extend the model to a two-country setting to explore the effect of asynchronous demographic transition on the global interest rate and international capital flows. We also solve for the full dynamics instead of focusing on steady states only.

The household's problem and the production side of the model remain essentially the same as in the small open economy model, except that we abstract away from endogenous labor supply in this section and assume inelastic labor for ease of computation. The utility function therefore becomes simply  $u(c) = \log(c)$ . Private saving of a young worker in a given country is given by:

$$a_{t+1} = \left[ 1 - \tau - \eta g_t - \frac{1 - \tau - \eta g_t + \phi/R_{t+1}}{1 + \beta\chi_t} \right] w_t + \frac{\beta\chi}{1 + \beta\chi} v_t,$$

similar to equation (2), but now private saving decision is affected by the time-varying endogenous interest rate. The formulae for public saving remain the same.

In a two-country setting, the global interest is endogenously determined by the demand and supply of savings. In equilibrium, the combined total savings of the two countries should equal claims on total capital stock (in the next period), as claims between the government and the private sector offset each other. This capital market equilibrium condition can be stated as:

$$S_t^1 + S_t^2 = K_{t+1}^1 + K_{t+1}^2.$$

If the capital market clears, the world market for the good will clear by Walras' Law. The good market is in equilibrium when the combined output of the two economies is equal to the consumption of the current young and old, and investment in capital. The world capital stock is given at the beginning of each period and evolves reflecting investment in both countries. However, the allocation of capital across borders is endogenous, and needs to be solved in equilibrium.

Since the two-country model cannot be solved analytically, we provide three numerical examples. These examples should not be viewed as quantitative projections but rather as illustrations of the various general equilibrium forces at play. In all these examples, we assume symmetry of the two countries at the outset on steady state equilibrium, so that each has a net foreign asset position of zero and the interest rate is equal to that in a closed economy. One period in the model is interpreted as 30 years since an individual could live up to three periods. Parameter values used for these simulations are set as follows. Several are standard preference and technology parameters. The capital share parameter  $\alpha$ , is set to 0.33 as is standard in the literature. The discount factor  $\beta$ , is set to 0.74, based on an annual discount factor of 0.99. The depreciation rate  $\delta$  is set to 0.79, consistent with an annual depreciation rate of 0.05. The labor income tax rate is

set at 30 percent<sup>10</sup>. The per-child childcare share of labor income  $\eta$  is set to 0.1, which implies that 10 percent of a worker's lifetime labor income is spent on child-related expenses<sup>11</sup>. Initial values of demographic and policy variables are used to compute the starting point of the simulation. Initial population growth rate  $g$  is set to 1.35, consistent with the annual growth rate of the world population of 1.2 percent in 2016. Initial survival probability  $\chi$  is set to 0.5 to imply a life expectancy at birth of 75 years. Initial pension generosity  $\phi$  is set at 0.3<sup>12</sup>.

The first two scenarios showcase the effect of asynchronous aging on savings, interest rate and the current account. The last scenario demonstrates the effect of cross country difference in pension generosity.

### **Scenario 1. A permanent increase in life expectancy in country 1 in period 1.**

In this scenario, the survival probability rises from 0.5 to 0.7 in period 1 in country 1, which corresponds to a rise of life expectancy from 75 to 81 years. The new steady state features lower global aggregate savings and a higher interest rate.

Private saving (as share of GDP) jumps in period 1 as the working-age population save more for longer expected retirement in country 1. It then dips slightly in period 2 as the size of bequests is reduced. Nevertheless, private saving of country 1 stays on a much higher trajectory.

As private saving goes up in country 1, and public saving does not react in period 1, total saving increases, depressing the interest rate. Pension spending starts to pick up from period 2 onwards, lowering public saving in country 1, which outweighs the increase in private saving. As a result, total saving falls significantly in country 1 starting from period 2, pushing the interest rate back up. Both private and public saving increases in country 2 in response to interest rate dynamics. Country 1 runs an initial current account surplus but reverts to a deficit soon, mirroring the movement in its national saving.

---

<sup>10</sup> The average tax burden in OECD countries is 36 percent of labor income for a single worker and 27 percent for a one-earner couple with two children (source: OECD Taxing Wages 2018).

<sup>11</sup> We experimented with 0.05 and 0.2, and do not find any meaningful differences in the results of the simulations.

<sup>12</sup> The average benefit ratio in European countries is about 40-50 percent. We set a lower parameter value to reflect the fact that the benefit ratio in emerging and low-income economies are generally lower.



**Scenario 2. Country 1's fertility rate drops in period 1, followed by country 2 in the next period.<sup>13</sup>**

In this scenario, aging is driven by asynchronous reduction of fertility rates. In contrast to scenario 1, the interest rate at the new steady state is lower. This shows that the long-run effect of aging on the interest rate could depend on whether aging is driven by rising life expectancy or decreasing fertility.

During transition, the interest rate falls as the marginal return of capital declines due to slower growth of the labor force, initially in country 1, and subsequently in country 2. Lower returns discourage private saving in both countries, although this effect is partially offset by lower childcare spending. Lower fertility worsens the old-age dependency ratio and puts downward pressure on public saving. However, since output declines more sharply in this case, public saving as share of GDP in both countries are higher in the long run. In period 1, capital flows to country 2 from 1 to equalize the capital-labor ratio. Hence the public saving-to-GDP ratio in country 1 increases while the opposite holds for country 2. Country 1 runs a current account surplus in period 1 because of higher private saving and outflow of investment as the demographic transition starts there first and the pattern reverses as country 2 follows suit. Thereafter country 1 runs a dwindling current account deficit with country 2 mirroring this development. In both of the first two scenarios, the country that experiences aging first runs a temporary current account surplus.

**Scenario 3. Country 1's pension generosity decreases permanently in period 1.**

In this scenario, country 1 reforms the public pension system by cutting pension generosity from 0.2 to 0.1 in period 1. As shown in the small open economy model, a reduction in pension generosity induces higher private saving in country 1 and increases public saving due to lower pension expenditure. The interest rate declines in response to higher saving, lowering the trajectories of private and public saving in country 2.<sup>14</sup> Country 1 runs a current account surplus.

These numerical examples reveal some interesting results. First, private and public saving can move in opposite directions in response to aging. The movement of the global interest rate therefore would depend on the relative strength of these two forces. Public pension generosity may play an important role as it affects the responsiveness of both private and public savings. Secondly, countries with similar demographics could generate very different saving behavior and capital flow patterns owing to differences in pension system generosity.

---

<sup>13</sup> The drop of country 2's fertility rate is necessary to ensure equal long-run growth rates of the two economies.

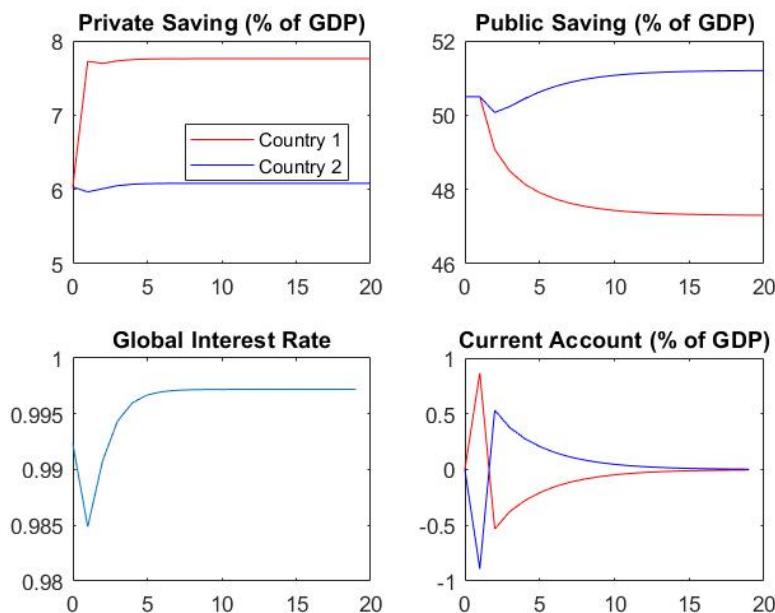
<sup>14</sup> The decline in public saving in response to lower global interest rate is due to the fact that the stock of public saving is positive (i.e. the government is a net lender rather than a net borrower) in the model. The opposite result would hold if the government is a net borrower.

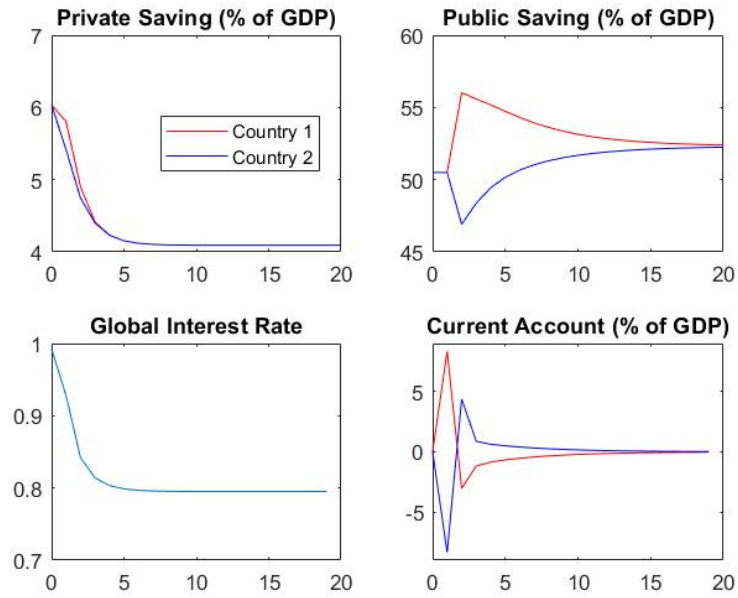
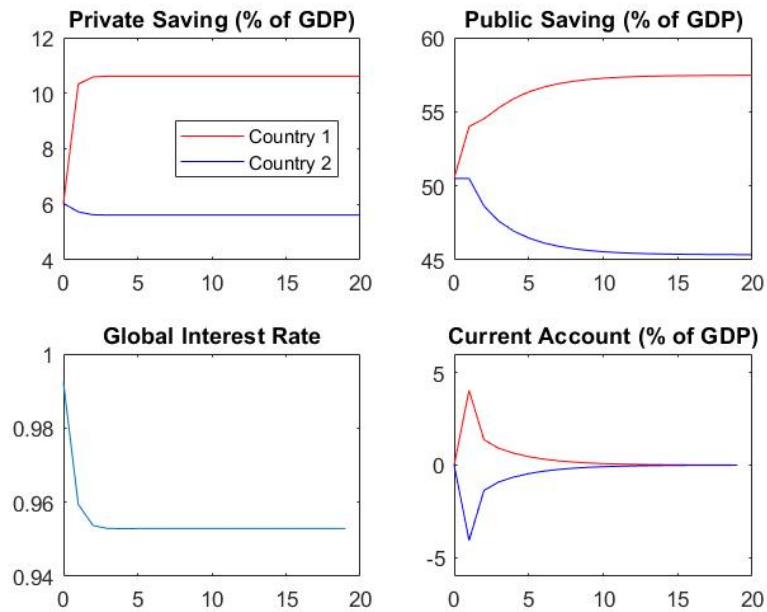
## V. CONCLUSION

In this paper, we develop a simple theoretical framework in which the demographic effects on private and public saving can be jointly studied. The generosity of public pension systems impacts both sources of saving in an economy. International capital flows are thus shaped by not only demographic variables but also cross-country differences in the public pension system. As for directions for future work, we intend to extend the current framework along a few dimensions. Firstly, the simple three-period structure can be extended to a multi-period one to unleash the full quantitative potential of the model and assess the quantitative importance of pension generosity in determining saving, interest rate and international capital flows. Second, our framework does not assume endogenous policy responses to demographic developments. This assumption is clearly unrealistic for countries with binding borrowing constraints. Either tax revenues would need to be raised or pension benefits need to be cut (see McGrattan and others, 2018). How countries react to the demographic challenge could be studied by incorporating endogenous fiscal policies choices into our framework.

The current framework is silent on how saving and international capital flows react to transitions from a predominantly PAYG public pension system to a fully-funded system. In general, a switch from an unfunded to a funded system would, absent other changes, require one generation of workers to pay for two retirements—their own and the generation before them—or would require several generations to bear the transition costs. Such a transition may well raise national saving by reducing the consumption of the transitional generations, but it will also make workers in those generations worse off. Future work could focus on examining this policy-relevant transition.

**Figure 3. Two-Country Model Simulation Scenario 1**



**Figure 4. Two-Country Model Simulation Scenario 2****Figure 5. Two-Country Model Simulation Scenario 3**

## REFERENCES

- Amaglobeli, D., H. Chai, E. Dabla-Norris, K. Dybczak, M. Soto, and A. Tieman, 2018, "The Future of Saving: The Role of Pension System Design in an Aging World", IMF Staff Discussion Note.
- Attanasio, O., A. Bonfatti, S. Kitao and G. Weber, 2016, "Global Demographic Trends: Consumption, Saving and International Capital Flows", *Handbook of the Economics of Population Aging*, Vol. 1, pp. 179-235.
- Attanasio, O., S. Kitao, and G. Violante, 2006, "Quantifying the Effects of the Demographic Transition in Developing Economies", *The B.E. Journal of Macroeconomics*, Vol. 6, Issue 1, pp. 1-44.
- Attanasio, O., S. Kitao, and G. Violante, 2007, "Global Demographic Trends and Social Security Reform", *Journal of Monetary Economics*, Vol. 54, pp. 144-198.
- Auerbach, A. and L. Kotlikoff, 1987, *Dynamic Fiscal Policy*, (Cambridge: Cambridge University Press).
- Backus, D., T. Cooley, and E. Henriken, 2014, "Demography and Low-frequency Capital Flows", *Journal of International Economics*, Vol. 92, pp. 94-102.
- Barany, Z., N. Coeurdacier, and S. Guibaud, 2016, "Fertility, Longevity, and Capital Flows", Working Paper.
- Blanchard, O., 1985, "Debt, Deficits and Finite Horizons", *Journal of Political Economy*, Vol. 93, Issue 2, pp. 223-247.
- Bloom, D., D. Canning, R. Mansfield and M. Moore, 2007, "Demographic Change, Social Security Systems and Savings", *Journal of Monetary Economics*, Vol. 54, Issue 1, pp. 92-114.
- Börsch-Supan, A., A. Ludwig and J. Winter, 2006, "Ageing, Pension Reform and Capital Flows: A Multi-Country Simulation Model", *Economica*, Vol. 73, pp. 625-658.
- Bohn, H., 2007, "Are Stationary and Cointegration Restrictions Really Necessary for the Intertemporal Budget Constraint?", *Journal of Monetary Economics*, Vol. 54, No. 7, pp. 1837-1847.
- Brooks, R., 2003, "Population Aging and Global Capital Flows in a Parallel Universe", *IMF Staff Paper*, Vol. 50, No. 2 (Washington: International Monetary Fund).
- Carvalho, C., A. Ferrero and F. Nechio, 2016, "Demographics and Real Interest Rates: Inspecting the Mechanism", *European Economic Review*, Vol. 88, pp 208-226.

- Curtis, C., S. Lugauer, and N. Mark, 2017, "Demographics and Aggregate Household Saving in Japan, China and India", *Journal of Macroeconomics*, Vol 52, pp. 175-191.
- Curtis, C., S. Lugauer, and N. Mark, 2015, "Demographic Patterns and Household Saving in China", *American Economic Journal: Macroeconomics*, Vol 7, No 2, pp. 58-94.
- Domeij, D. and M. Floden, 2006, "Population Aging and International Capital Flows", *International Economic Review*, Vol. 47, No. 3, pp. 1013-1032.
- Eugeni, S., 2015, "An OLG Model of Global Imbalances", *Journal of International Economics*, Vol. 95, pp. 83-97.
- Feldstein, M., 1976, "Social Security and Saving: The Extended Life Cycle Theory", *American Economic Review*, Vol. 66, No. 2, pp. 77-86.
- He, H., L. Ning, and D. Zhu, 2018, "The Impact of Rapid Aging and Pension Reform on Savings and the Labor Supply: The Case of China", forthcoming IMF Working Paper.
- Henriksen, E., 2002, "A Demographic Explanation of U.S. and Japanese Current Account Behavior", mimeo, Carnegie Mellon University.
- McGrattan, E., K. Miyachi, and A. Peralta-Alva, 2018, "On Financing Retirement, Health, and Long-term Care in Japan", Working Paper.
- Rezk, E., M. Irace, and V. Ricca, 2009, "Pension Funds' Contribution to the Enhancement of Aggregate Private Saving: A Panel Data Analysis for Emerging Economies", *SSRN Electronic Journal*, Available at SSRN: <https://ssrn.com/abstract=1992440> or <http://dx.doi.org/10.2139/ssrn.1992440>.
- Samuelson, P., 1958, "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money", *Journal of Political Economy*, Vol. 66, Issue 6, pp 467-482.
- Staveley-O'Carrol, J. and O. Staveley-O'Carrol, 2017, "Impact of Pension System Structure on International Financial Capital Allocation", *European Economic Review*, Vol. 95, pp. 1-22.
- Yarri, E., 1965, "Uncertain Lifetime, Life Insurance and the Theory of Consumer", *Review of Economic Studies*, Vol. 32, pp. 137-150.