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## Relationship Between Short-Term Interest Rates and Excess Reserves: A Logistic Approach

by Romain Veyrune, Guido della Valle, and Shaoyu Guo

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I N T E R N A T I O N A L M O N E T A R Y F U N D

**IMF Working Paper**

Monetary and Capital Markets Department

**Relationship Between Short-Term Interest Rates and Excess Reserves: A Logistic Approach**

**Prepared by Romain Veyrune, Guido della Valle, and Shaoyu Guo<sup>1</sup>**

Authorized for distribution by Simon Gray

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**ABSTRACT**

This paper models the relationship between short-term rates and excess reserves in an interest rate corridor as a logistic function estimated for the Eurosystem. The estimate helps to identify conditions in which short-term rates become unanchored, that is, they move away from the policy rates and become more volatile within the interest rate corridor defined by the interest rates of the central bank's standing facilities. These conditions are attributed to coordination failures among counterparties at open market operations under fixed-rate and full-allotment procedures in the context of segmented markets. A model of the functioning of segmented markets describes how "un-anchoring" takes place when counterparties pursue bidding strategies optimal from an individual perspective but sub-optimal from an aggregate perspective.

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## I. INTRODUCTION

Based on the IMF Annual Report on Exchange Rate Arrangements and Exchange Restrictions, 45 central banks out of 172 were using short-term interest rates as their operational monetary policy target in 2016. The monetary policy implementation of these central banks aimed at steering short-term rates close to a certain level (or policy rate) and at minimizing their volatility around that level. These central banks seek to influence the interest rate term structure via their control of short-term rates, and via some degree of transparency on their monetary policy reaction function.

This paper attempts to model the relationship between excess reserves and short-term interest rates in a symmetric interest rate corridor. The assumption is that the relationship could be modeled as a bivariate logistic function, which is defined in such a way that short-term rates converge to the top of the interest rate corridor when excess reserves are increasingly negative; converge to the bottom of the corridor when excess reserves increase; and take the value 0.5 when there are no excess reserves.

This paper, then, estimates the bivariate logistic relationship between excess reserves and short-term interest rates in a symmetric interest rate corridor. These estimates are conducted based on the data published by the Eurosystem. They allow the position of short-term rates in the interest rate corridor to be determined and predicted depending on the level of excess reserves. The estimated function also allows to determine for which level of excess reserves short-term rates are anchored to one of the three characteristic points of the logistic function (higher edge, mid-corridor, or lower edge) or are “un-anchored.” Short-term rates become “un-anchored” when they diverge from the main policy rates and fluctuate in the corridor between one of the corridor edges and the mid-point or the other edge.

This “un-anchoring” is undesirable from a monetary policy transmission mechanism perspective. The policy stance, conveyed via a policy rate in normal circumstances, becomes less precisely defined as short-term rates fluctuate in the corridor. Furthermore, short-term rate volatility and the uncertainty regarding the level of short-term rates increase the liquidity premium, which is transmitted to the cost of funding in the economy via the interest rate term structure.

The logistic function contributes to estimate the short-term interest rate reaction function as excess reserves vary. It may, therefore, help a central bank to predict the excess reserves ranges in which short-term interest rates become “un-anchored.” A central bank wishing to avoid the negative consequences of this un-anchoring process may, thus, use the logistic function predictions to accelerate the transition from one state to another in which anchoring takes place to another characteristic point of the logistic function.

The paper also tests the impact of market segmentation on the relationship between excess reserves and short-term interest rates in the money market. In perfectly efficient markets, short-term rates should drop to the deposit facility rate for small amounts of excess reserves. However, market segmentation reduces the market efficiency in reallocating excess reserves, and, thus, changes the speed of the convergence to the edges of the corridor. In addition, the introduction of the fixed-rate full allotment method—that is, the regular central bank’s

refinancing operation allotment method in which the central bank fulfills all banks' bids at the refinancing operations at a fixed rate as long as banks can provide sufficient eligible collateral—to accommodate the demand for excess reserves in a segmented market could lead to episodes of “coordination failure” among banks bidding at the central bank's main open market operation. This coordination failure could contribute to a greater volatility of short-term rates and excess reserves.

Coordination failure occurs when a group of counterparties could achieve a more efficient equilibrium in which they all could benefit, but fail to do so because they are unable to coordinate their decision-making because of market segmentation. In this context, coordination failure refers to individual bids at the central bank's refinancing operation that, although individually optimal, lead to a sub-optimal outcome from a system perspective.

Like many other central banks, the European Central Bank (ECB) introduced several measures to increase reserves in the aftermath of the global financial crisis, leading to a large and increasing amount of excess reserves, standing at the beginning of 2018 above 1 trillion euro. When these measures will be discontinued, excess reserves are expected to decline. We argue that their decline will, at a certain point, represent a challenge for monetary policy implementation to effectively steer short-term rates to the central bank's desired level and to minimize the volatility of short-term rates. This was already briefly experienced at the beginning of 2014 when the 3-year longer-term refinancing operations were being repaid and before the ECB launched a new package of easing measures.

Our choice of the Eurosystem for this study is motivated by the data available on the ECB's website for long time series (since 1999). The Eurosystem experience has the advantage of having equal periods of low and stable excess reserves (1999–2008) and periods of large and volatile excess reserves (2009–2018). For an added analytical dimension, the Eurosystem also experienced market segmentation in the wake of the global crisis and changed its refinancing operation allotment method. These two topics are also explored in this paper.

Although the paper focuses on the ECB, similar considerations are applicable to different central banks with comparable implementation frameworks. They, too, might anchor short-term rates to either their main open market operation rate or the deposit facility rate, but may experience a higher level of interest rate volatility for intermediate levels of excess reserves. The shape and the parameters of the logistic function and the excess reserve level, where volatility is experienced, would need to be locally estimated, as they vary from country to country. In addition, the logistic function could also help central banks with operational targets such as reserve money or the exchange rate to estimate the impact of their operational target on short-term rates as these central banks cannot be indifferent to the level and volatility of short-term interest rates.

The rest of the paper is organized as follows: Section II explains the concept of steering short-term rates in a corridor and presents a brief review of the literature. Section III presents the demand for excess reserves model, based on a logistic function, and a model for the pricing of interbank transactions in a segmented market. Section IV estimates the logistic relationship between short-term rates and excess reserves under different periods, to

determine the impact of market segmentation on short-term rates convergence toward the deposit facility rate.

## **II. STEERING SHORT-TERM RATES IN AN INTEREST RATE CORRIDOR**

### **A. General Considerations**

To steer short-term rates, central banks influence the stock of banks' reserves in their accounts via the allotment of their open market operations (net reserves supply), define the interest rates at which reserves are supplied or withdrawn via open market operations, and determine the interest rate at which excess reserves are remunerated. Banks' demand for refinancing arises from autonomous factors, which are (1) the items in the central banks' balance sheets that can have an impact on banks' reserves at the central bank, but are not under the direct control of the central bank; (2) the reserve requirement; and (3) a possible demand in the market for excess reserves for various reasons, including precautionary purposes.

In an extreme scenario of null reserve requirements and neutral autonomous factors<sup>2</sup>, in which interbank transactions are settled in central bank money, banks would still demand a minimum level of reserves to minimize the risk of settlement failures. Such structural reserve demand is normally steady over time and can be considered a function of the volume of interbank payments, wholesale funding, liquidity of the money market, predictability of liquidity needs, banks' ease of access to wholesale funding, and the rate of remuneration of the reserve balances.

Depending on each central bank balance sheet structure, autonomous factors, net of the reserve requirement, and the demand for excess reserves could create either a demand for refinancing at the central bank or demand for short-term investment at the central bank (absorbing operations) due to involuntary excess reserves at the aggregated level. For the rest of this paper, we assume that the autonomous factors, the reserve requirement, and the demand for excess reserves create a demand for refinancing in the market.

If the central bank provides the exact amount of refinancing necessary to fulfill banks' demand, and is expected to continue to do so, short-term rates will remain close to the rate at which the central bank provides its refinancing. This assumes that the central bank could predict with sufficient accuracy the banks' demand for refinancing over a (usually short) period. In addition, the market should seamlessly match individual banks' excess reserves with other banks' residual refinancing needs in between market operation, as the open market operations are calibrated based on the market aggregated refinancing needs. In other words, when the demand for excess reserves is low and stable, it is predictable for the central bank.

Central banks have additional tools, namely reserve requirement averaging and standing facilities, to relax the assumption of perfect liquidity forecasting and perfect market functioning. Central banks usually permit reserve requirement to be averaged over a long enough maintenance period (the period during which the reserve requirement should be

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<sup>2</sup> Autonomous factors create neither a need for refinancing nor excess reserves.

fulfilled on average) to absorb these forecast errors, that is, a shortfall on one day compared to the average can be compensated during the rest of the period without market impact.

Central banks provide standing facilities that can be accessed, in principle, at any point in time by the commercial banks for potentially unlimited amounts. The standing facilities determine a floor and a ceiling for the interbank market (that is, an interest rate corridor). The corridor avoids excessively large jumps in short-term rates in the event of unexpected autonomous factor developments. The floor or lower edge of the corridor is a deposit facility on which banks can deposit unlimited amounts of reserves with a remuneration equal to the deposit facility rate set by the central bank (the lowest interest rate in the corridor). The ceiling or higher edge of the corridor is the lending facility from which banks can borrow at a predetermined rate (the highest rate in the corridor) amounts only limited by the eligible collateral they can pledge. These facilities are usually overnight, as the objective is to contain short-term interest rates in a predetermined range, and are activated upon counterparties' demand.

The width of the corridor is also an important factor to steer short-term rates. A narrow corridor would contain short-term rates volatility but would reduce banks' incentive to trade in the interbank market. If the width of the corridor is too narrow compared with credit risk premia and market transaction costs, a bank with excess liquidity would have no incentive to transact in the market as opposed to deposit the funds at the central bank's deposit facility. The central bank, in this scenario, would intermediate the market taking up extra credit risk and losing the signaling and controlling functions that the interbank market can provide. On the contrary, if the corridor were too wide, there would be greater scope for the interbank market, but the short-term interest rate volatility could be excessive.

Central banks operating an interest rate corridor usually aim at steering short-term rates in the middle of the corridor. Central banks, primarily in advanced economies, used to implement interest rate steering under "neutral liquidity allotment" (that is, to keep excess reserves low and stable), as it was the method that maximized opportunities for market transactions because it kept liquidity conditions balanced on an aggregated basis. In different terms, the neutral liquidity allotment could be presented as the allotment of the central bank's operation that balances the demand and the supply in the interbank market or the allotment that keeps short-term rates stable in the middle of the interest rate corridor. In well-functioning markets, the demand for excess reserves is limited; therefore, it was enough for the central bank to keep excess reserves low and stable to steer short-term rates toward the middle of the corridor.

With the onset of the global financial crisis, perceived increases in credit and liquidity risks among counterparties led to market segmentation, and, thus, reduced the market's ability to reallocate excess reserves among market participants. In other words, the demand for excess reserves became large, volatile, and unpredictable. The liquidity distribution across banks became an issue since the central bank could no longer rely on the market to redistribute liquidity seamlessly between banks with excess reserves and other banks with refinancing needs.

## B. Literature Review

There have been several studies of short-term rate dynamics in the Eurosystem related to the literature on monetary policy implementation, in the tradition of Poole (1968). Like us, Moschitz (2004), Beirne (2012), and Valimaki (2001 and 2008) studied the demand and supply of reserves as the main determinants of short-term rates. Valimaki (2001 and 2008) and Bindseil (2017) used a logistic representation of an interest rate corridor, but did not estimate it empirically.

Bech and Monnet (2015) exhibited the relationship between excess reserves and EONIA rates based on Eurosystem data but did not provide any estimate of the relationship under different states. They also presented a “directed-search” model of the functioning of the interbank money market that may contribute explaining the observed EONIA rate pattern as excess reserves vary. However, their model did not account for deviations of the interbank rate from the main policy rate of the central bank and for impairment in the transmission mechanism due to counterparty risk. In the model presented in this paper, such counterparty risk compounds the coordination failure among counterparties in their central bank’s bidding and hinders the redistribution of reserves from counterparties with excess reserves to those with a reserve shortage.

Vari (2016), on the contrary, shows that counterparty risk engenders market fragmentation and disrupts the monetary policy transmission mechanism. He also shows how excess liquidity arises endogenously once fragmentation is introduced into the standard theoretical models of monetary policy implementation.

Expectations of changes in the policy rate are also an important source of volatility in short-term rates. They have been studied by Valimaki (2001) and Moschitz (2004). However, the Eurosystem isolated banks’ demand for refinancing from expectation of policy rate changes by synchronizing decisions on policy rate with operations in March 2004. The Eurosystem reduced the maturity of its operation from two weeks to one week and synchronized the changes in its policy rate with the start of new maintenance periods. Thus, expectations of changes in the policy rate should not have an impact on banks’ bidding at current operation and on short-term rates until the end of the current maintenance period.

Moschitz (2004), Nautz and Offermanns (2007), and Wurtz (2003) focused their attention on the spread between short-term rates and the ECB’s main refinancing operation (MRO) minimum bid rate. The MRO has been the main monetary policy instrument used by the ECB to steer short-term interest rates. It has a 1-week maturity and is conducted on a weekly basis by the ECB.<sup>3</sup> Before the global financial crisis, this spread was considered to be important for monetary policy signaling (that is, short-term rate un-anchoring concerns). However, the debate around the risk of un-anchoring short-term rates was less intense after the global financial crisis while the risk of un-anchoring did not disappear and has even materialized several times during the periods of moderate excess reserves.

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<sup>3</sup> General documentation on the Eurosystem operations can be found on the ECB website: <https://www.ecb.europa.eu/ecb/legal/1002/1014/html/index-tabs.en.html>.

The relative advantages of fixed rate versus variable rate tenders have been extensively studied in the literature (Bindseil 2002). However, the literature usually examines the relative advantages of the two allotment methods, without full allotment, as the debate revolved around the overbidding at the MRO during 1999–2000, when the ECB implemented a fixed-rate fixed allotment procedure. In 2010, Catalão-Lopes included full allotment in her model. She considered whether full allotment resolves the issue arising from a possible ECB calibration error under fixed-rate allotment. In this paper, we consider whether the sum of counterparties' bids under full allotment provides the calibration necessary to keep short-term rates stable and anchored at the policy rate.

Bindseil and Jablecki (2011) studied the optimal size of the interest rate corridor based on the trade-off between short-term rate volatility and interbank market functioning. They establish that interbank transactions depend on transaction costs in the market, plus the expected cost of two-sided recourse to the standing facilities, that is, there would be no transactions in the market if the corridor were narrower than transaction costs in the market. Their main conclusion is that wider corridors between standing facilities are associated with greater interbank trading volumes and greater volatility of overnight rates.

Heider, Hoerova, and Holthausen (2009) studied the impact of counterparty risk on reserve hoarding behaviors by cash-rich banks. The paper peripherally addressed the impact of market segmentation on the relationship between short-term-rates and excess reserves. However, it mainly focused on longer unsecured maturities, which include a credit risk premium that could be calculated as the difference between them and overnight interest rate swaps with the same tenor.

Durre, Maddaloni, and Mongelli (2014) date the beginning of the global market impairment in August 2007. At that time, concerns regarding rising counterparty risk fueled an increase in unsecured market rates for term-transactions. However, overnight rates remained anchored to the policy rate until the introduction of fixed-rate full allotment in October 2008 led to a notable increase in the level of excess reserves. From August 2007 to October 2008, fine-tuning operations were more frequent, and the ECB accommodated the front-loading of the reserve requirement by banks in the allotment of its main refinancing operations (Cassola and Huetl 2010), but the operational framework remained overall unchanged as the ECB was still targeting neutral liquidity conditions on average during the maintenance period.

Bindseil and Lamoot (2011) raised the issue of the impact of Basel III liquidity requirements on banks' liquidity risk management and the demand at central bank refinancing operations. They predicted that the new requirements could increase the reliance of weaker banks on central bank operations, based on illiquid collateral. The liquidity requirements could also lead to a more frequent recourse to the central bank's standing facilities not related to aggregate liquidity conditions but aimed at complying with liquidity ratios. In other words, they suspected that the new liquidity regulation framework would increase the demand for excess reserves in the market.

### III. LOGISTIC MODELING OF THE INTEREST RATE CORRIDOR

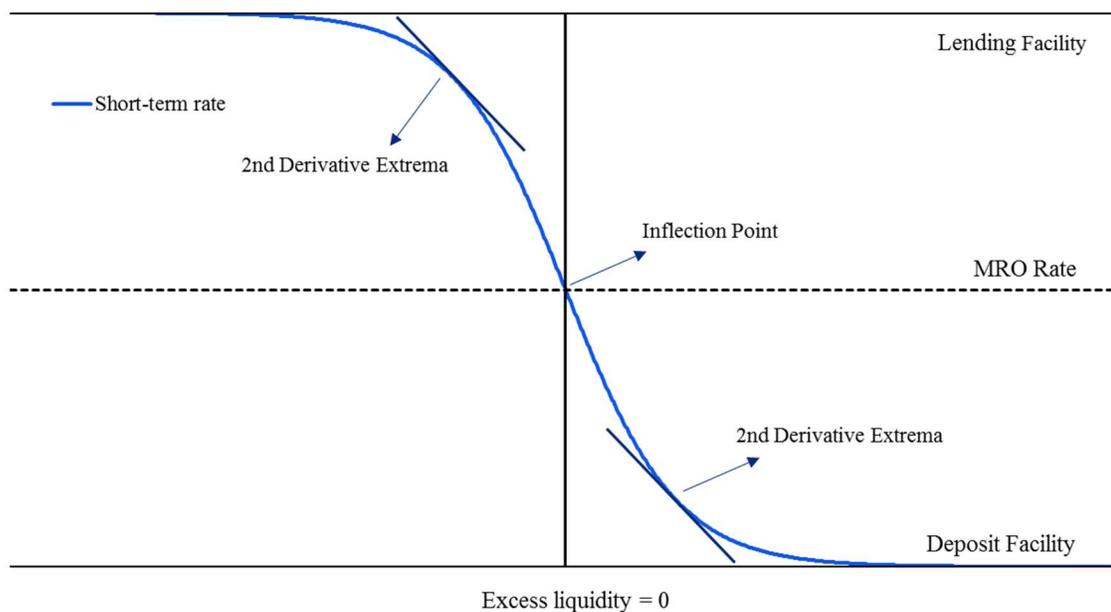
#### A. Stylized Representation of the Demand for Excess Reserves

The logistic function is a continuous bivariate function that is commonly used to represent non-linear growth. The logistic function is the only one that allows positive and negative values for the explanatory variables, asymptotic convergence for large positive and negative values of the explanatory variable, and a mid-rate outcome when the explanatory variable reaches zero. Other functions, such as exponential, logarithm, and power functions, do not present the properties necessary to model and estimate an interest rate corridor.

In the extreme case of perfectly functioning money markets, there should be no demand for excess reserves, that is, any reserve supplied at the central bank's operations in excess of the demand arising from the autonomous factors and the reserve requirement should lead to a drop in the price of excess reserve to the deposit facility rate, and vice-versa: any shortage would lead to an immediate jump to the lending facility rate. Then, the demand function for excess reserves would be a step function.

However, we argue that constraints regarding credit limits in a segmented market and coordination failure (as discussed in subsection C., below) have led to a non-null demand for excess reserves, and, thus, give a logistic shape to the excess reserves demand function, as depicted in Figure 1. Counterparties with different credit limits and refinancing needs must choose between (1) obtaining refinancing at the central bank; (2) obtaining refinancing in the market; (3) depositing excess reserves with the Eurosystem; and (4) lending excess reserves in the interbank market.

Figure 1. Stylized Logistic Representation of Short-Term Rates in an Interest Rate Corridor



Source: Authors' calculations.

The logistic function incorporates four main characteristics:

- Short-term interbank rates are confined between the rates of the two overnight standing facilities, that is, two asymptotes, which are defined by the ECB lending facility rate and the ECB deposit facility rate. This is supported by all available evidence.
- Depending on the level of excess liquidity, the short-term rates can be anchored at the deposit facility rate, the lending facility rate, or the midpoint of the corridor, which is the minimum bid rate of the ECB MRO. High excess reserves should anchor short-term rates to the deposit facility. A neutral allotment should keep short-term rates near the middle of the corridor. Finally, reserve shortage, that is, a scenario in which the central bank regularly supplies via regular refinancing operations fewer reserves than needed, should anchor short-term rates to the lending facility rate. The available evidence supports the first and second scenarios, whereas the third scenario has never been experienced in the Eurosystem (but it has occurred elsewhere).
- The inflection point represents the level of excess reserves above which short-term rates decrease and below which short-term rates increase. In other words, it represents excess reserves for a neutral liquidity allotment. In Figure 1, this level is zero, but it will be empirically estimated, as the neutral liquidity allotment can deviate from zero due to the demand for excess reserves.
- The short-term rate sensitivity to excess reserves diminishes as excess reserves reach high levels. The maximum of logistic function's second derivative indicates the reserve level after which the marginal effect of excess reserves injection or the reduction of the short-term rate diminishes as reserves accumulate. Higher values of this benchmark suggest a slower convergence of short-term rates to the deposit facility rate.

The logistic function is consistent with the models of Bech and Monnet (2015) and Vari (2016). The models presented in these papers, in fact, derive theoretical relationships between excess reserves and interest rate that are close to the logistic function.

The logistic function assumes a symmetric response to excess reserves or reserves' shortages. Reserves' shortages result from insufficient refinancing at the central bank open market operations. Counterparties, thus, face the choice among (1) obtaining refinancing at the open market operation (partially); (2) obtaining refinancing in the market; and (3) obtaining refinancing at a marginal lending facility. In the absence of stigma attributed to the lending facility, there is no reason, a priori, to have a different type of short-term rate response to reserve shortage compared to excess reserves. However, due to data limitations, it is not possible to test short-term rate response to a reserves' shortage scenario. In fact, based on the Eurosystem experience on which the logistic function has been estimated, there has never been a period in which the central bank systematically offered fewer reserves than needed, so that short-term rates drifted toward the lending facility rate.

## B. Logistic Function Parameters

We use a four-parameter logistic function to examine the relationship between short-term rates and excess reserves in the Eurosystem. In this section, we first introduce the basic three-parameter logistic function. Then, we discuss the four-parameter generic form proposed by Oliver (1969). Last, we present our modification of the Oliver's form and the economic implications behind our model.

In economic contexts, the three-parameter logistic function is frequently used:

$$(1) \quad y = \frac{\kappa}{1 + \beta e^{-ax}}$$

It arises as the solution to the differential equation:

$$(2) \quad \frac{dy}{dx} = \frac{\alpha y}{\kappa} (\kappa - y)$$

This logistic function is confined between two asymptotes:  $y = 0$  and  $y = \kappa$ , with an inflection point (or mid-corridor) at  $y = \frac{\kappa}{2}$ . It is, by itself, a generalization of the two-parameter logistic function, with  $\kappa = 1$ , which lies between  $y = 0$  and  $y = 1$ , and has an inflection point at  $y = 0.5$ .

Oliver (1969) among others proposed a four-parameter logistic function that vertically shifts the three-parameter function with a constant term:

$$(3) \quad y = \gamma + \frac{\kappa}{1 + \beta e^{-ax}}$$

It results from the differential equation:

$$(4) \quad \frac{dy}{dx} = \frac{\alpha}{\kappa} (\gamma + \kappa - y)(y - \gamma)$$

This function lies between asymptotes  $y = \gamma$  and  $y = \gamma + \kappa$ , with an inflection point at  $y = \gamma + \frac{\kappa}{2}$ ,  $\kappa \neq 0$ .

Our modification of Oliver (1969) is as follows:

$$(5) \quad y = \gamma + \frac{\kappa - \gamma}{1 + \beta e^{-ax+c}}$$

with the corresponding differential equation:

$$(6) \quad \frac{dy}{dx} = \frac{\alpha}{\kappa - \gamma} (\kappa - y)(y - \gamma)$$

Our first change from Oliver (1969) is to replace the numerator  $\kappa$  with  $\kappa - \gamma$ . This modification allows us to estimate the lower asymptote  $\gamma$  directly while keeping the upper bound fixed at  $\kappa$ . Oliver's logistic function has upper and lower boundaries at  $\kappa + \gamma$  and  $\gamma$ , a parallel upward shift of  $\kappa$  and 0 by  $\gamma$  (the constant Oliver added). For our purpose, we would like to keep the upper asymptote (that is, lending facility) binding at  $\kappa$  but allow the lower (that is, deposit facility) to be estimated empirically. Empirical evidence has suggested that the market rates are always above the deposit facility rate in the euro area, even within the context of a large amount of excess reserves.

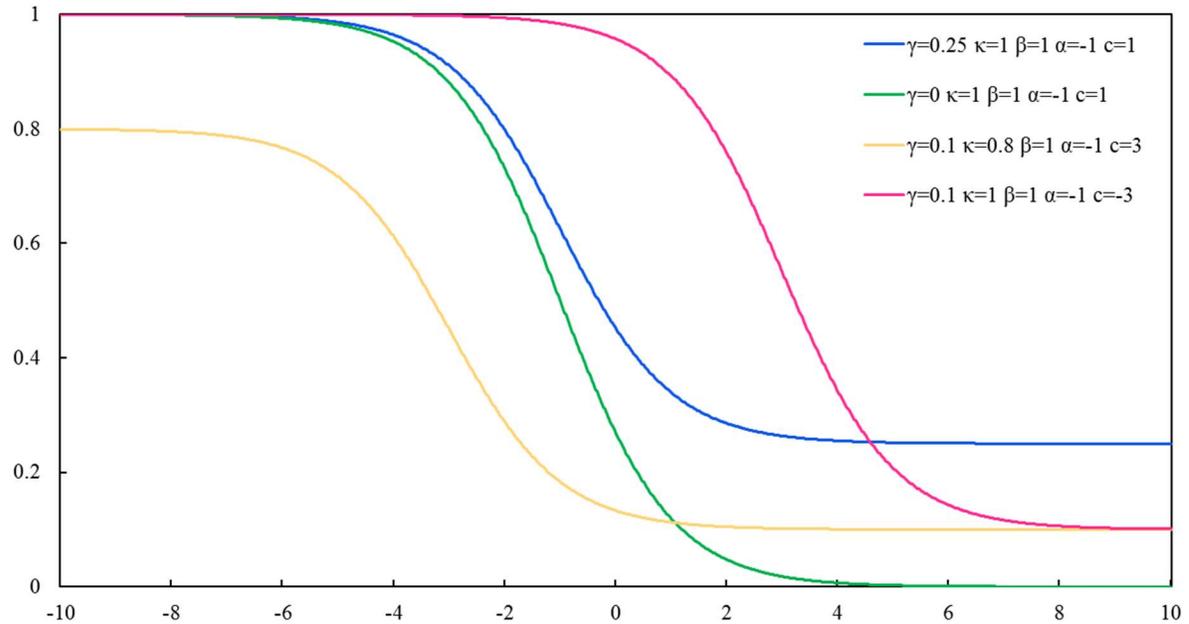
The four-parameter logistic function now lies between asymptotes  $y = \gamma$  and  $y = \kappa$ .  $\beta$  is set to 1 without losing generality for our purpose. The inflection appears at  $y = \frac{r+k}{2}$ ,  $k \neq r$ .

Compared to other analyses reflected in the literature, our model highlights two features:

- First, it allows the lower-bound of the corridor to be estimated (that is,  $\gamma$ ). We take into consideration the possible mismatch between the lower-bound and the deposit facility rate, due to the minimum remuneration that banks require for participating in the market instead of keeping their excess reserves at the deposit facility. (Empirical calibration of the floor provides evidence in that regard.)
- Second, the function reveals horizontal movements along the corridor, which is useful in computing empirically the short-term rate when excess reserves are null. When  $x = 0$ ,  $y = \gamma + \frac{\kappa - \gamma}{1 + \beta e^c}$  would become the predicted mid-corridor intercept.

The main advantage of the four-parameter generic form is to provide flexibility regarding the shape of the logistic function. Figure 2 provides examples of different parameterizations of the logistic function, based on equation 5. In the next section, we will estimate the parameters of equation 5, including the inflection points (that is, short-term rate for zero excess reserves) and the lower asymptote (that is, short-term rates for large excess reserves).

Figure 2. Examples of Logistic Function Parameterization



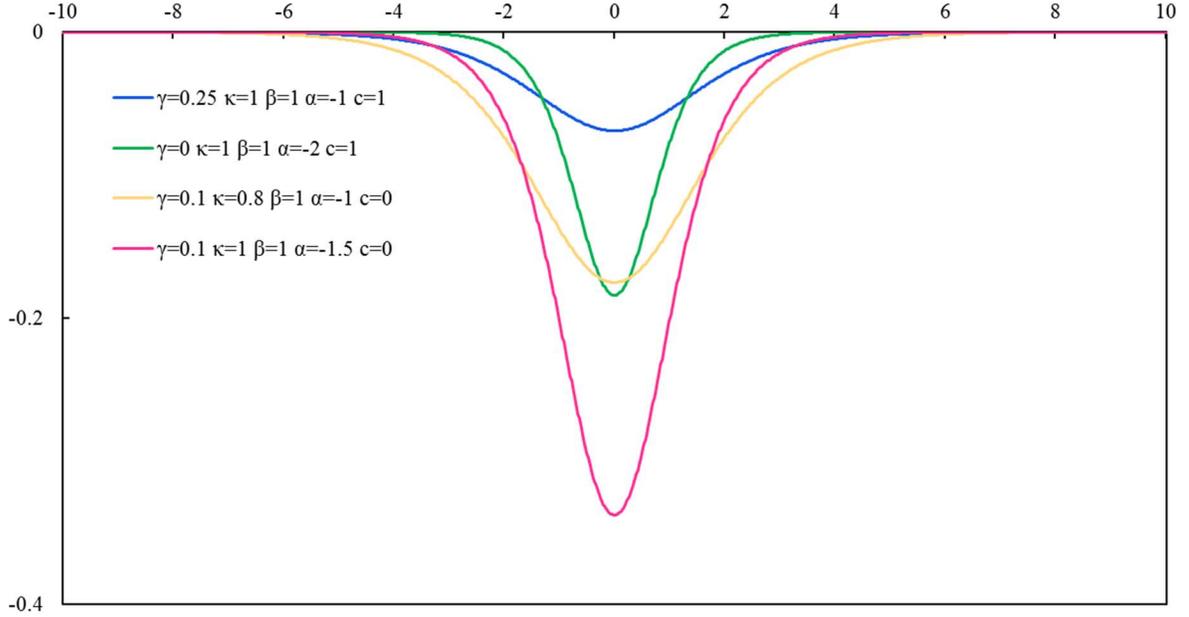
Source: Authors' calculations.

Differentiating equation (5) with respect to  $x$  formalizes a response function of the short-term rate to given levels of excess reserves (assuming  $\beta = 1$ ):

$$(7) \quad \frac{dy}{dx} = \frac{\alpha(\kappa - \gamma)e^{\alpha x + c}}{(e^{\alpha x} + e^c)^2}$$

The first derivative of the logistic function provides an indication of the convergence speed in the corridor and demand for excess reserves. Figure 3 presents the first derivative of the logistic function parameterized on Figure 2. The first derivative indicates the elasticity of short-term rates for a given level of excess reserves. The maximum of the first derivative indicates the inflection point, that is, the highest elasticity of short-term rates to excess reserves. In a well-functioning market, the elasticity distribution should show a thin and deep distribution around zero (the pink line in Figure 3), reflecting a rapid convergence toward the edges of the corridor. In a segmented market, the distributions are expected to be wider and potentially deviating from zero, reflecting a slower convergence toward the edges of the corridor and a significant demand for excess reserves (the blue and yellow lines in Figure 3).

Figure 3. Examples of Logistic Function—First Derivative Parameterization



Source: Authors' calculations.

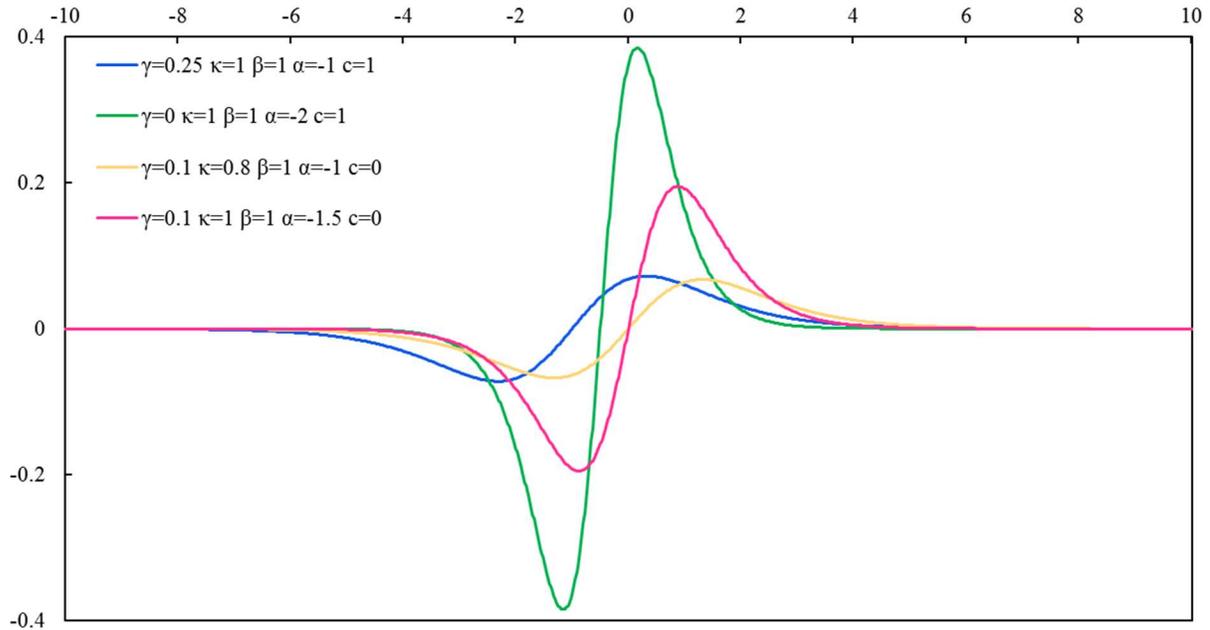
The elasticity function (that is, the first derivative of the logistic function), on either side of the inflection point, can be further divided into two segments by the extrema of its derivative, that is, the second derivative of the logistic function (Figure 4). On the right-hand side, for example, before reaching the extrema, elasticity increases exponentially (accelerates). After the threshold, however, the elasticity increased only at decelerating rates, suggesting that the elasticity has reached its maximum acceleration. For our purpose, we locate the maxima of the second derivative and adopt its corresponding levels of excess reserves as a benchmark to determine the excess reserves level beyond which short-term rates are anchored to the deposit facility rate. We argue that, when the benchmark is closer to zero, the money market is characterized by less segmentation. Formally, we find the global maximum and minimum of the second derivative of the logistic function (that is, the third derivative equals zero). Hence, we further differentiate equation (7) with respect to  $x$  to obtain the second derivative of  $y$  as a function of  $x$ :

$$(8) \quad \frac{d^2y}{dx^2} = \frac{\alpha^2(\gamma - \kappa)e^{\alpha x + c}(e^{\alpha x} - e^c)}{(e^{\alpha x} + e^c)^3}$$

with maximum and minimum appearing at:

$$(9) \quad x_{max/min} = \frac{\log[(2 \pm \sqrt{3})e^c]}{\alpha}$$

Figure 4. Examples of Logistic Function—Second Derivative Parameterization



Source: Authors' calculations.

### C. Model for the Pricing of Excess Reserves in the Interbank Market

We examine the convergence process of short-term rates toward the deposit facility rate. Under fixed-rate full allotment, counterparties having access to the facilities of the central bank should be able to keep short-term market rates close to the deposit facility rate to minimize their overall short-term borrowing costs. This entails bid coordination at the MRO based on information on the expected reserve supply disclosed in the autonomous factor forecast. However, restrictive credit limits in a segmented market complicate the interpretation of autonomous factor forecasting: specifically, the ability to appraise the reserve supply in the market. This is an uncertainty that exacerbates the risk of coordination failures.

#### Market segmentation and short-term rates

In a well-functioning money market, banks with excess reserves would be willing to lend in the interbank market at any rate above the deposit rate, plus a spread representative of the transaction costs, including capital charges, to minimize their opportunity costs and minimize the amount of excess reserves remunerated at the deposit facility rate. Hence, low levels of excess reserves in the system should be able to drive down short-term rates to the deposit facility rate.

In contrast, in a segmented interbank market, some banks are less willing to lend excess reserves because of counterparty risk concerns, and are more inclined to keep excess reserves at the Central Bank if they cannot find suitable investment opportunities compliant with their credit risk policy. Therefore, in addition to the stock of excess reserves, credit limits should

be factored in to determine the supply of excess reserves available to the market. Credit limits could not be precisely predicted, because they depend on banks' estimation of counterparty risk in the market. Therefore, short-term interest rates could deviate significantly from the deposit facility rate while there are non-trivial excess reserves in the system on an aggregated basis. The short-term rate formation in a segmented market is explained below.

We assume a banking system composed by three groups of banks:

*L1*: are banks with reserves well in excess of minimum reserve requirements.

*L2*: are banks with good market access and creditworthiness, with funding needs they can fulfill either via central bank refinancing operations or the money market.

*S*: are banks with impaired market access, limited borrowing capabilities in the money market, and substantial funding needs.

Furthermore, we assume that there is a central bank providing refinancing on a fixed-rate full allotment basis to solvent banks against adequate collateral at a rate  $R_c$  significantly above the rate  $R_d$  at which it remunerates excess reserve balances. We also assume an overnight money market so that banks should first decide whether to borrow from the central bank and later whether to borrow overnight in the market until the next operation.

We finally assume that the excess reserves held by *L1* ( $ER$ ) are larger than the funding needs of *L2* ( $BL2$ ) and the funding needs of *S* ( $BS$ ):  $ER > BL2 + BS$ .

In such a situation, *L1* would have obvious financial incentives to redeploy the excess liquidity at a rate higher than  $R_d$ , while *L2* and *S* would have a financial incentive to borrow at a rate lower than  $R_c$ .

In the absence of market segmentation, the competition among *L1* banks, and their attempt to avoid the opportunity cost of idle balances remunerated at  $R_d$ , would drive the short-term money market rates close to  $R_d$  (one could say at a rate equal  $R_d$  plus transaction costs). A surplus of excess reserves over the sum of  $BL2 + BS$  would in principle be sufficient to engender this pattern, as an efficient market would smoothly redistribute reserves from banks with a reserve surplus to banks with a reserve shortage. In such a scenario, central bank borrowing would be zero.

Let's now introduce credit limits, that is, a constraint representative of market segmentation.

We model the constraint as an overall credit line ( $CLL$ ) that *L1* have toward *S*, whereas *L1* to *L2* is unconstrained and takes place at a rate close to  $R_d$ . *L1* is ready to lend to *S* only  $CLL < BS$ . Credit risk is factored into the price of interbank transactions only to a limited extent. It is mostly factored into credit limits, whether trading with a given counterparty is allowed or not. This assumption is consistent with the behavior observed and reported by most banks post crises. The credit limit could also derive from collateral constraints in case banks of the group *S* have collateral that is not be accepted to secure transactions by banks in the group *L1*, but is accepted to secure refinancing at the central bank.

Finally, the demand for excess reserves could also arise from precautionary motives on the part of banks in groups  $L2$  and  $S$ . This would increase their respective refinancing needs  $BL2$  and  $BS$ . The precautionary demand would compound the restrictive credit limits and serve to squeeze the net supply of reserve.

On a weekly basis,  $S$  needs to decide how much to borrow from the central bank ( $Bcb$ ) and the residual they may borrow from  $L1$  ( $BL1$ ) under the constraint that  $Bcb + BL1 = BS$  and  $BL1 \leq CLL$ .

In this scenario, we assume that  $BL1 < CLL$ . The short-term money market rate at which  $L1$  lend to  $S$  will converge toward  $Rd$  and the rate at which  $L1$  lend to  $L2$  ( $RI$ ). In other words,  $S$ 's residual borrowing needs on the market are lower than what  $L1$  are ready to invest ( $CLL$ ), and drive the rates down, as  $L1$  scramble to invest excess liquidity.

On the contrary, when  $BL1 > CLL$ , the short-term money market rate will spike close to  $Rc$  or even above it, as  $S$  will be scrambling in the market to fulfill their residual borrowing needs. In this scenario,  $S$  will borrow at a high rate ( $Rh$ ), and the weighted average money market rate will be the average between  $Rh$  and  $Rd$ , at which  $L1$  lend to  $L2$ .

### **Coordination failure and short-term rates**

Under fixed-rate full allotment, total refinancing becomes the sum of the choices made by each bank treasurer regarding the management of their accounts at the central bank. Banks' treasurers typically aim at keeping a certain level of reserves on their account at the central bank to maintain a minimum precautionary amount for settlement purposes and to comply with the required reserve average by the end of the maintenance period.

Therefore, before each operation, the treasurer needs to estimate the amount needed to satisfy its reserve target, and then to decide whether it is preferable to obtain refinancing (1) at the central bank refinancing operation at a predetermined  $Rc$  rate for an unlimited amount (assuming collateral is not a constraint); or (2) in the market for a rate that is not known ex-ante ( $R$ ).  $R$  will depend on the reserves available to be lent in the market, which are influenced by both developments in autonomous factors and the available credit lines from cash-rich banks in the  $L1$  group. The problem for banks in the groups  $L2$  and  $S$  is to combine the two sources of funding to minimize their funding costs, knowing that market funding could be cheaper than central bank funding, though not in all circumstances, and that there might be dire reputational and financial consequences if they are unable to fulfil their funding needs.

However, while most banks forecast their refinancing needs over the maturity of the MRO, they usually do not know about the refinancing needs of other banks in the market and their intended bid amounts. Thus, the sum of individually rational bids at the main refinancing operation may not deliver the appropriate allotment on an aggregated basis to keep stable short-term rates at the deposit facility rate.

The publication of autonomous factor forecast by the central bank provides each bidder with information on expected market conditions, thereby encouraging more informed bidding at the operation on the basis of a better awareness of the expected market liquidity conditions.

However, market segmentation complicates the interpretation of the forecast, as counterparties should factor in restrictive credit limits to obtain the actual change in the reserve supply as related to any predicted changes in autonomous factors.

Under ideal conditions, with perfect coordination between the banks in the  $S$  and  $L2$  groups, they will borrow from the central bank an amount  $Bcb^*$  at  $Rc$ , so that the residual to be borrowed from  $L1$  ( $BLI^* = Bs - Bcb^*$ ) is equal to the amount counterparties in  $L1$  are ready to lend ( $CLL$ ) for a given expected change in available reserves. In this circumstance,  $S$  could borrow the rest from  $L1$  at a low rate  $Rd$ . This would be the perfect strategy to minimize their borrowing costs based on the information provided by the publication of the central bank's autonomous factor forecast.

Coordination failure could happen if counterparties misjudge the expected reserve supply during the MRO week. This may be due to insufficient monitoring of the autonomous factor forecasts or inaccurate estimation about market access. For instance, when banks in the  $S$  and  $L2$  groups have experienced low rates in the market  $Rd$  for a while, they may tend to become complacent regarding their market access and borrow from the central bank in aggregate  $Bcb < Bcb^*$  so that  $BLI > BLI^*$  and the short-term money market will jump to  $Rh > Rc$ .

On the contrary, once they have experienced  $Rh$  over a period, counterparties may tend to overreact based on an adaptive behavior and borrow in aggregate  $Bcb > Bcb^*$  at the next refinancing operation, so that  $BLI < BLI^*$  and the short-term money market will be  $Rd$ . Therefore, they end up borrowing less from the market at  $Rd$  than they could have and more at the central bank for  $Rc$ , thereby paying premia equal to  $(Bcb - Bcb^*) * (Rc - Rd)$ .

Although this insufficient monitoring of the autonomous factor forecasts may appear irrational, it should be recalled that monitoring entails a cost in terms of dedicated resources. There might be counterparties that consider the regular monitoring costs and the costs for consequent elaboration of more sophisticated bidding strategies greater than its benefits. In addition, the main reason why banks may misjudge the reserve supply in the market is because they do not know and can hardly know reserve demand and supply patterns of the other banks in their own group even using the central bank's autonomous factor forecasts, which reflect expected liquidity for the market at the aggregated level and irrespectively of the subgroups of market participants.

An additional problem for banks in the  $S$  group is that they cannot easily observe  $Rh$ , but only the weighted average between  $Rh$  and  $Rd$ , as calculated and communicated by the central bank. In this scenario, an increase in the volume lent to  $L2$  has a downward effect on the weighted average, which  $S$  could erroneously interpret as signaling easier market conditions and a greater willingness of  $L1$  to lend.

Averaging of the reserve requirement could absorb some of the coordination failures. Counterparties do not necessarily need to bid up in the market if their reserve shortfall vis-à-vis the reserve requirement could be compensated at the next operation. However, this supposes that the reserve requirement provides enough averaging margin compared to the incompressible reserves, which are defined as the minimum reserves that counterparties will keep in any situation for settlement purposes. Averaging also works less well at the end of

the maintenance period. At 1 percent of the basis, which includes overnight deposits, deposits with agreed maturity or periods of notice up to two years, debt securities issued with maturity up to two years, and money market papers, the reserve requirement in the Eurosystem provides limited room for averaging.

#### IV. EMPIRICAL RESULTS

##### A. Estimate of the Logistic Function from 1999 to 2018

The Eurosystem has publicly available data on liquidity conditions since January 1999. As of end-February 2018, these data are available for 249 consecutive maintenance periods. During this period, the Eurosystem experienced three regimes in terms of excess reserves: (1) a “neutral allotment” period (January 1999–October 2008), during which the Eurosystem provided calibrated refinancing at a fixed or variable rate with a view to keeping excess reserves low and stable; (2) moderate excess reserve periods (October 2008–December 2011, and March 2014–March 2015), during which the Eurosystem introduced fixed-rate full allotment for its MRO in response to euro area money market segmentation after the beginning of the global financial crisis; and (3) high excess reserve regimes (January 2012–February 2014 and since April 2015), during which the Eurosystem injected large amounts of excess reserves based on long-term refinancing operations and asset purchases programs (Figure 5).

We use maintenance period averages for excess reserves. As the reserve requirement is applied on average and remunerated at the MRO rate, “real” excess reserves could be assessed only at the end of maintenance periods, as excess reserves on a given day during the period could be compensated afterward. As such, only excess reserves at the end of the maintenance period represent a financial cost for the banks (that is, the difference between the MRO rate and the deposit facility rate).

Similarly, we use the volume-weighted maintenance period average for the Euro Overnight Index Average (EONIA) rate<sup>4</sup> to smooth out intra-maintenance period rate volatility. EONIA presented some volatility in the short term that mainly corresponds to phenomena that do not fundamentally affect the ECB’s ability to steer short-term rates on average over the maintenance period. The EONIA averaging over the maintenance period minimizes recurring spikes, such as those at the end of the month, which reflect drops in market turnover due to market participants’ end-of-period balance sheet presentation considerations (also called “window dressing” (Moschitz 2004)). These spikes are highly predictable for market participants.

Averaging the EONIA over the maintenance period also removes intra-maintenance period interest rate patterns in short-term rates, which reflect the strategies that banks adopted to fulfill the reserves requirement. Such strategies include, for instance, reserves front-loading—that is, a counterparties preference for over-fulfilling reserve requirements at the

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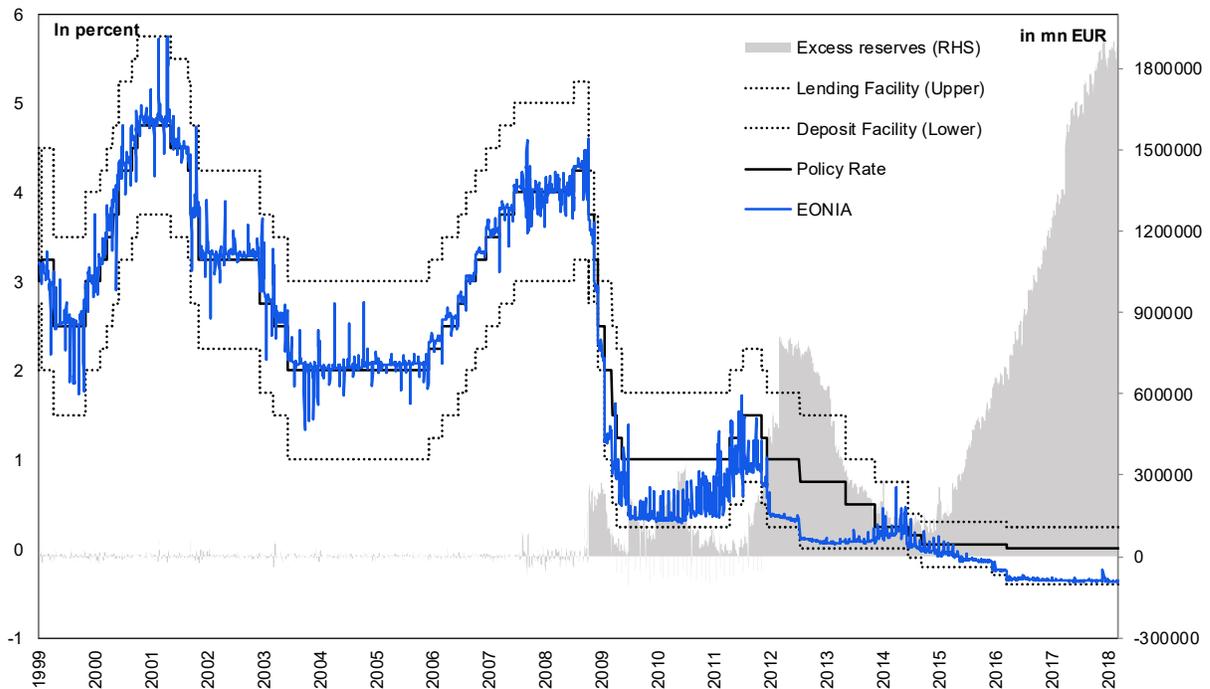
<sup>4</sup> EONIA is the volume-weighted average rate of overnight, unsecured interbank lending reported daily by a panel of the main market participants in the overnight market in the euro area. It is used as an indication of short-term rates in the euro area and is the implicit operational target of the ECB (Bindseil 2017).

beginning of the maintenance period to minimize the risk of under-fulfillment during the maintenance period—experienced in 2007 due to heightened concerns about market access in the months leading up to the global financial crisis (Cassola and Huetl 2010).

The interest rate corridor, the width of which has changed several times since 1999, has been normalized between zero and 1. For this paper, we keep the corridor symmetric, as it was in the Eurosystem until November 2013. The introduction of an asymmetric corridor, in fact, may have a larger impact on the short-term interbank rate under neutral liquidity conditions when there is an almost symmetric probability of recourse to the marginal lending facility and to the deposit facility at the end of the maintenance period. In these conditions, the short-term interest rate in an asymmetric corridor may diverge from the main policy rate as it should correspond to the weighted average between the marginal lending facility rate and the deposit facility rate, with the weights being the perceived probabilities of the banking system of being short or long liquidity at the end of the maintenance period (Bindseil and Jablecki 2011). However, with very large excess liquidity and fixed-rate and full allotment, the probability of being short of reserves at the end of the maintenance period and of a recourse to the marginal lending facility drops towards zero except for a handful of counterparties so that its level has a negligible influence on the short-term money market rates. In other words, the asymmetric corridor introduced as of November 2013 does not influence the results of the analysis under the liquidity conditions in place since the asymmetric corridor was introduced.

Figure 5. Policy Rates, EONIA, and Excess Reserves

(January 1999 to February 2018)



Sources: ECB data warehouse, authors' calculations.

Table 1 presents the results of our estimates of the logistic function. The sample consists of 249 maintenance periods, from January 1999 to February 2018. It includes the lower asymptote ( $\gamma$ ), the constant ( $c$ ), and the coefficient linking excess reserves with short-term rates ( $\alpha$ ). The full sample estimate shows three significant parameters, and a good fit, as indicated by the regression R-squared.

- The lower asymptote ( $\gamma$ ) indicates the rate toward which EONIA converges for the highest levels of excess reserves registered over the period. To lend reserves in the market instead of simply depositing them on the deposit facility, market participants always require some pick-up in remuneration as compared to the deposit facility rate. This spread, thus, reflects counterparty risk, transaction costs, such as brokerage fees and settlement costs, or market microstructure, such as special business relationships among banks and with central clearing institutions, all of which may lead to idiosyncratic pricing arrangements. Our estimate of the lower asymptote quantifies these factors at 8.6 percent of the normalized corridor, assuming that the difference between the deposit and the lending facility rate is 100 percent.
- The constant ( $c$ ) provides an estimate of the intercept of the logistic function, that is, an estimate of EONIA if excess reserves are null. The intercept is estimated at 0.52, slightly above the middle of the corridor. The more fundamental explanation of the intercept deviation from the middle of the corridor (i.e. the policy rate) is that it represents the counterparty risk in the market because MROs are collateralized transactions while EONIA is an average of unsecured transactions. It could also reflect the cost to transact in the market. Finally, the monthly average could slightly deviate from the minimum bid rate because of daily spikes due to calendar effects and other one-off effects.
- The coefficient ( $\alpha$ ) indicates the EONIA for a given level of excess reserves. A higher coefficient reflects a faster decline in EONIA as excess reserves increase, that is, a faster convergence process. Based on our estimate, one billion in additional excess reserves lead to a decreased of EONIA by 1.05 percent in the normalized corridor on average. As the regression is nonlinear, the actual decrease of EONIA as excess reserves increased also depends on the level of excess reserves.

The logistic function allows to estimate the demand for excess reserves at the MRO minimum bid rate. Excess reserves at the MRO minimum bid rate are the excess reserves predicted by the logistic function if the interest rate is 0.5, i.e., in the middle of the interest rate corridor. During the neutral allotment period, some counterparties had a demand for excess reserves, which was factored in the MRO allotment. As a result, EONIA remained close to the MRO minimum bid rate while there were some amounts of excess reserves prevailing at the end of the maintenance period. The demand of excess reserves at the MRO minimum bid rate is estimated at EUR 7.05 billion (Table 1).

We tested the robustness of the estimations with a full sample, using data in daily and MRO-cycle frequencies. Different frequencies do not appear to lead to strongly different estimates as the sheer number of observations in the estimation alleviate intra-maintenance period volatility with higher-frequency data. The most notable difference is the asymptotic

convergence that reaches a lower level for the estimate with daily frequency (7.4 percent of corridor as opposed to 8.6 percent with maintenance period frequency). This is due to the end of period EONIA spikes that tend to increase the maintenance period and MRO EONIA averages above the typical daily EONIA rate. The R-square under daily frequency is slightly lower than the estimates based on maintenance period and MRO maturity frequency, due to more noise in the data at high frequency. By the same token, MRO frequency appears marginally better than maintenance period frequency.

Figure 6 presents the predicted EONIA for a given level of excess reserves at the end of the maintenance period, as well as actual EONIA weighted average rates during the maintenance period. It shows that the logistic relationship assists in predicting the level of short-term rates in the interest corridor when the central bank changes the level of excess reserves through its operations, or as they fluctuate under the effect of the autonomous factors.

Table 1. Estimates of the Logistic Function—Different Data Frequency

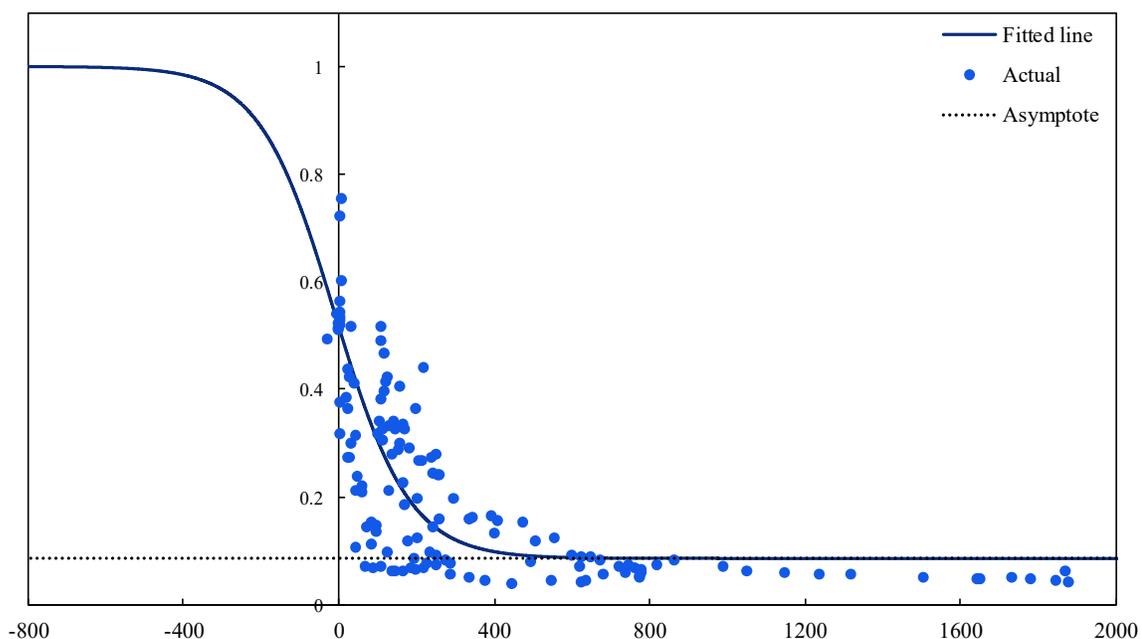
	Maintenance -period	Daily	Weekly
Lower asymptote $\gamma$	0.0860*** (6.234)	0.0738*** (22.68)	0.0938*** (15.09)
Coefficient $\alpha$	1.04e-05*** (11.95)	1.07e-05*** (44.68)	1.45e-05*** (21.06)
Constant $c$	0.115*** (2.523)	0.109*** (9.60)	0.106*** (4.82)
R-squared	0.951	0.932	0.953
Observations	249	4935	965
Predicted mid-corridor intercept	0.52	0.51	0.52
Excess reserves at inflection point (bn EUR)	-11.0	-10.0	-7.5
2nd-derivative maxima (bn EUR)	115.5	112.9	98.1
Excess reserves at minimum bid rate (bn EUR)	7.05	4.7	7.0

t-statistics in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Source: Authors' calculations.

Figure 6. Estimation of the Logistic Function—January 1999 to February 2018



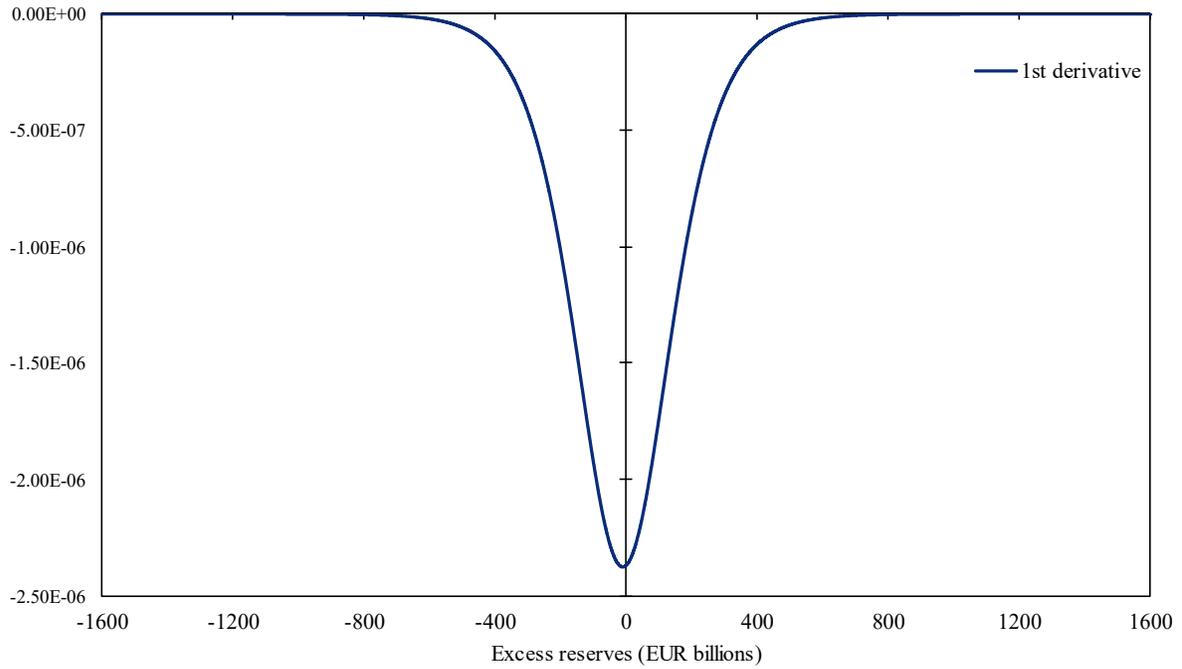
Note: excess reserves at the end of the maintenance period are on the x-axis and EONIA maintenance period average is on the y-axis. EONIA is normalized in a 0 to 1 corridor, representing the spread between the deposit facility rate (0) and the lending facility rate (1).

Source: Authors' calculations.

The inflection point of the estimated logistic function is indicated by its first derivative. Short-term rates are estimated to start declining if excess reserves increased from EUR -11 billion (Figure 7 and Table 1). Conversely, they would start increasing for a reserve shortage of more than EUR -11 billion. As result of the small demand for excess reserves from January 1999 to October 2008, EONIA remained slightly above the MRO minimum bid rate for small amount of excess reserves, which positions the inflection point in negative excess reserves territory. In term of rate, the inflection point is at 0.54 percent of the corridor, slightly above the logistic function intercept presented in Table 1 (that is, 0.52).

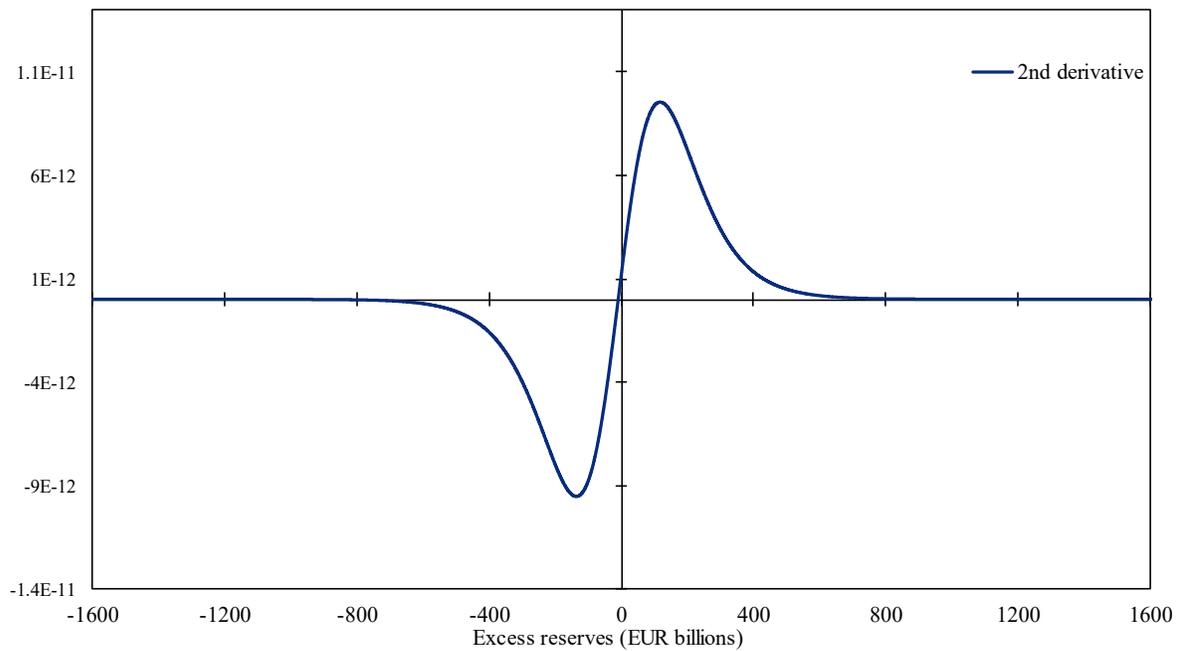
The second derivative maximum indicates that EONIA was the most responsive to excess reserves around the excess reserves level of EUR 111 billion. From zero to EUR 111 billion in excess reserves, EONIA response to additional excess reserves increases with the level of excess reserves. Beyond EUR 111 billion, the marginal effect of excess reserves on short-term rate start declining. Ultimately, EONIA becomes unresponsive to excess reserves for high levels of excess reserves as it has converged to the estimated asymptote (8.6 percent).

Figure 7. First Derivative of the Logistic Function— January 1999 to February 2018



Source: Authors' calculations.

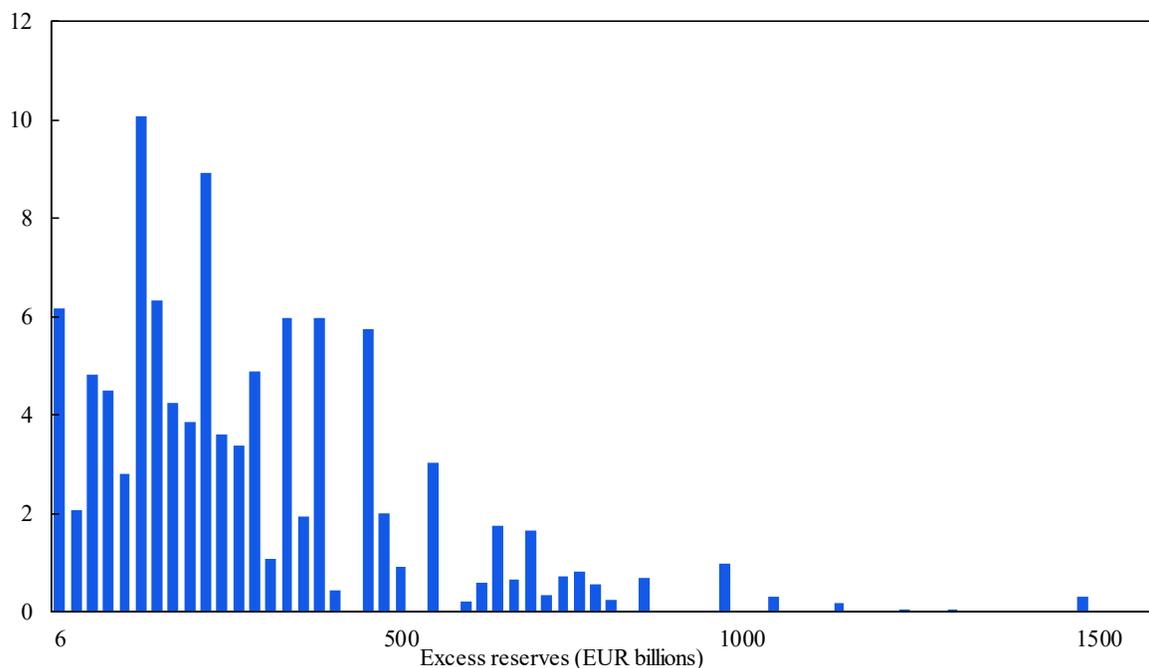
Figure 8. Second Derivative of the Logistic Function— January 1999 to February 2018



Source: Authors' calculations.

The predictive capacity of the logistic function is lower for intermediate levels of excess reserves. Figure 9 shows the absolute deviation between the predicted and actual values for short-term rates. Short-term rates are well-anchored to the MRO minimum bid rate when excess reserves are small (about EUR 7 billion). On the other hand, short-term rates are well-anchored to the deposit facility rate when excess reserves are above EUR 400 billion. In between, deviations are larger. Between EUR 7 billion and EUR 400 billion, short-term rates are, thus, anchored neither to the MRO minimum bid rate nor to the deposit facility rate (that is, they are “un-anchored”), as also highlighted in the January 2014 ECB monthly bulletin on “Recent Developments in Excess Liquidity and Money Market Rates.”

Figure 9. Absolute Deviation Between Predicted and Actual Short-Term Rates  
(in percent of the interest rate corridor)



Source: Authors' calculations.

This result indicates that factors other than excess reserves contribute to determine short-term rates for moderate levels of excess reserves. Short-term rates can be significantly different for the same level of excess reserves in the range of moderate excess reserves. The capacity of excess reserves to determine the level of short-term rates is, thus, reduced because other variables, which are not included in the estimate, such as market segmentation, influence the pricing of short-term rates (the level of tradable reserves in the market) in addition to the absolute level of excess reserves. It also makes more difficult for market participants to anticipate market rates based on expected development in excess reserves, potentially adding a liquidity premium to the money market term structure.

## B. Impact of Market Segmentation on the Relationship Between Short-Term Rates and Excess Reserves

This section focuses on the impact of market segmentation on the shape of the logistic function. Credit limits influence the amount of excess reserves that can be traded in the market, leading to higher short-term rates for a given level of excess reserves, thereby altering the shape of the logistic function. However, credit limits and, thus, tradable excess reserves, cannot be directly quantified. The approach adopted in this section is to compare the shape of the logistic function estimates in different subperiods, for which there is anecdotal evidence of change in risk perception in the euro area money market.

Market segmentation in the Eurosystem is usually considered to have increased with the sovereign crisis in Europe in 2011-12. Therefore, we split the sample between October 2008 to January 2012 and February 2012 to February 2018. The latter period was chosen considering the allotment of the ECB very long-term operations,<sup>5</sup> which represent an additional policy response to the fixed-rate and full allotment, introduced in October 2018, to address increasing market stress. Fixed-rate full allotment applies during both periods, with the main difference being a change in the risk sentiment in the market. While excess reserves were higher during the second period, the two periods experienced several maintenance periods with similar levels of excess reserves, allowing to compare EONIA for similar level of excess reserves under different market circumstances based on the estimates of logistic functions.

The change in market sentiment appears to have a rather dramatic impact on the parameters of the logistic function. Table 2 presents the results of the subperiod estimates as well as the full period estimate.

- The asymptotic convergence is estimated at its lowest point (6.7 percent) during the period January 2012 to February 2018 because this period experienced significantly more excess reserves than any other period in the Eurosystem history. As such, it could be interpreted as an empirical estimate of the incompressible spread between EONIA and the deposit facility rate. The introduction of the negative deposit rate in June 2014 has not changed the asymptotic convergence of short-term rates although it is responsible for higher rates for a given level of excess reserves during few maintenance periods after the introduction of the negative rate as it took some time for some counterparties to accept lending at negative rate in the market.
- The coefficient  $\alpha$  is lower during the period January 2012 to February 2018 than during the period October 2008 to December 2011, meaning that EONIA was higher on average for similar level of excess reserves during the second period than during the first. This provides some evidences of the impact of market segmentation on market

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<sup>5</sup> The very long-term operation is a 3-year, fixed-rate, and full allotment operation decided by the ECB Governing Council on 8 December 2011 and allotted on 21 December 2011 for a total amount of EUR 489 billion. A second 3-year operation allotted on 29 February 2012 provided additional EUR 529 billion.

rates for similar level of excess reserves (for which the two logistic functions overlap).

- The mid-corridor intercepts are estimated at 0.6, above the middle of the interest rate corridor, for both subperiods. The mid-corridor intercepts estimated for both subperiods are higher than the one estimated for the full period, reflecting the increase in risk perception in the market, especially during the second period.

Table 2. Estimates of the Logistic Function—October 2008 to December 2011 Versus January 2012 to February 2018

	Full sample	Oct. 2008–Dec. 2011	Jan. 2012–May 2017
Lower asymptote $\gamma$	0.0860*** (6.234)	0.1332*** (7.330)	0.0674*** (4.740)
Coefficient $\alpha$	1.04e-05*** (11.95)	4.99e-05** (2.450)	9.10e-06*** (7.600)
Constant $c$	0.115*** (2.523)	-0.141 (-0.240)	-0.344*** (-2.090)
R-squared	0.951	0.855	0.942
Observations	249	41	86
Predicted mid-corridor intercept	0.52	0.60	0.61
Excess reserves at inflection point (EUR bn)	-11.0	3.0	37.5
2nd-derivative maxima (EUR bn)	115.5	29.2	182.4
Excess reserves at minimum bid rate (EUR bn)	7.05	9.0	53.6

t-statistics in parentheses

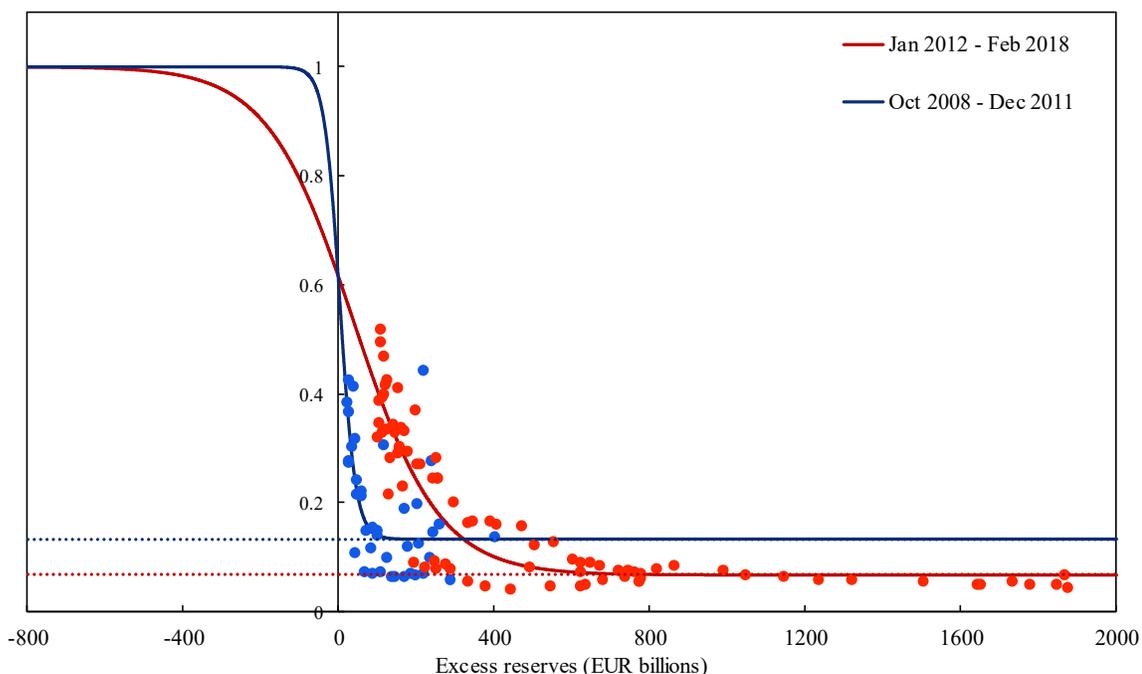
\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Source: Authors' calculations.

Figure 10 shows a rapid convergence of short-term rates to the deposit facility from October 2008 to December 2011, when the market functioned relatively well, while the convergence is slower from January 2012 to February 2018, as market segmentation deteriorated. In the context of restrictive credit lines, short-term rates have been higher when there was higher risk-aversion for the same level of excess reserves. In fact, from October 2008 to December 2011, the logistic function was close to having the step-form that would reflect well-

functioning markets—although some volatility could be noted in the second part of 2011 as funding pressures were building up in the market.

Figure 10. Estimation of the Logistic Function—October 2008 to December 2011 Versus January 2012 to February 2018.



Note: excess reserves at the end of the maintenance period are on the x-axis and EONIA maintenance period average is on the y-axis. EONIA is normalized in a 0 to 1 corridor, representing the spread between the deposit facility rate (0) and the lending facility rate (1).

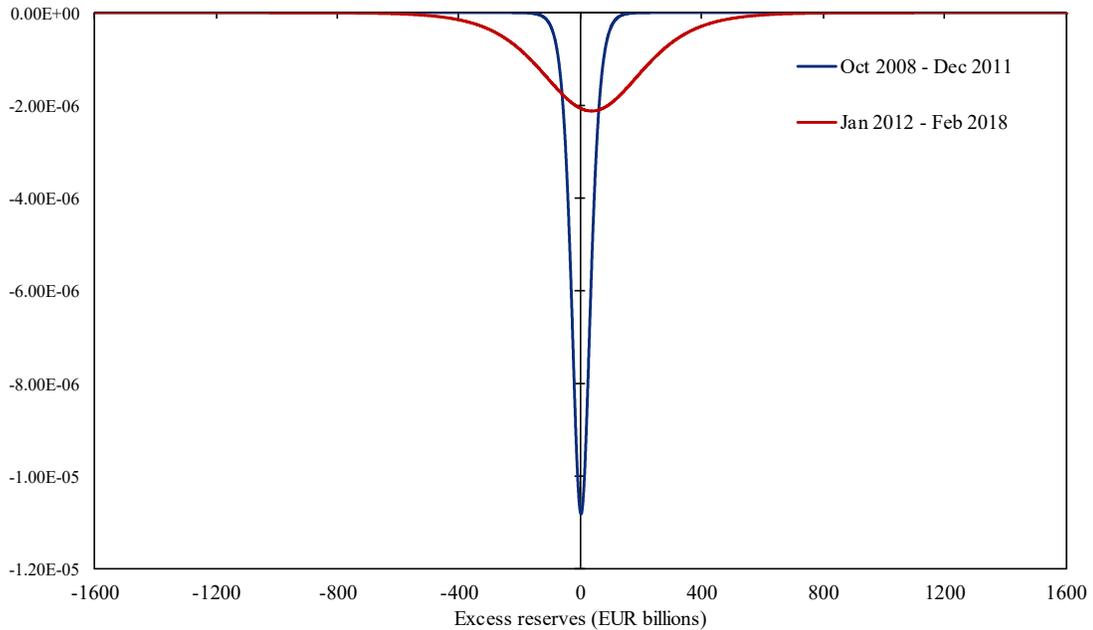
Source: Authors' calculations.

The distribution of the first derivatives indicates that short-term rates would decrease for a level close to zero excess reserves during the period October 2008 to December 2011, reflecting a still limited demand for excess reserves (the excess reserves at the MRO minimum bid rate standing at EUR 7.6 billion). The limited demand for excess reserves supports the assumption that the market was still reallocating reserves efficiently during this period. On the other hand, during the period January 2012 to February 2018, short-term rates would decrease only after EUR 37.5 billion. In addition, the estimated logistic function predicts more than EUR 55 billion in excess reserves at the minimum bid rate of the MRO during this period, reflecting a larger demand for excess reserves (Figure 11).

From October 2008 to December 2011, the second derivative spikes to its maximum for limited amounts of excess reserves, reflecting a rapid convergence to the deposit facility rate, when market circumstances are favorable (Figure 12). Table 3 indicates that the maximum of the marginal impact of excess reserves on short-term rates was reached at EUR 29.2 billion of excess reserves during the first period. During the period January 2012 to February 2018, the level of excess reserve at which the marginal impact of excess reserves reaches its

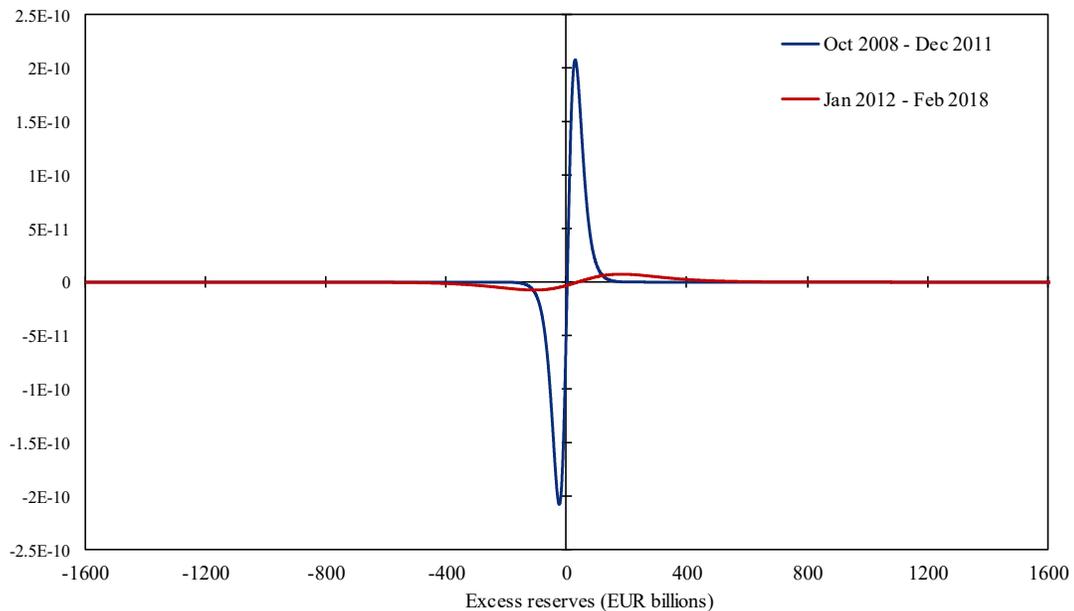
maximum increases to more than EUR 182.4 billion, reflecting the slower convergence process on the backdrop of a more segmented market.

Figure 11. First Derivative of the Logistic Function—October 2008 to December 2011  
Versus January 2012 to February 2018



Source: Authors' calculations.

Figure 12. Second Derivative of the Logistic Function—October 2008 to December 2011  
Versus January 2012 to February 2018



Source: Authors' calculations.

## Coordination failures

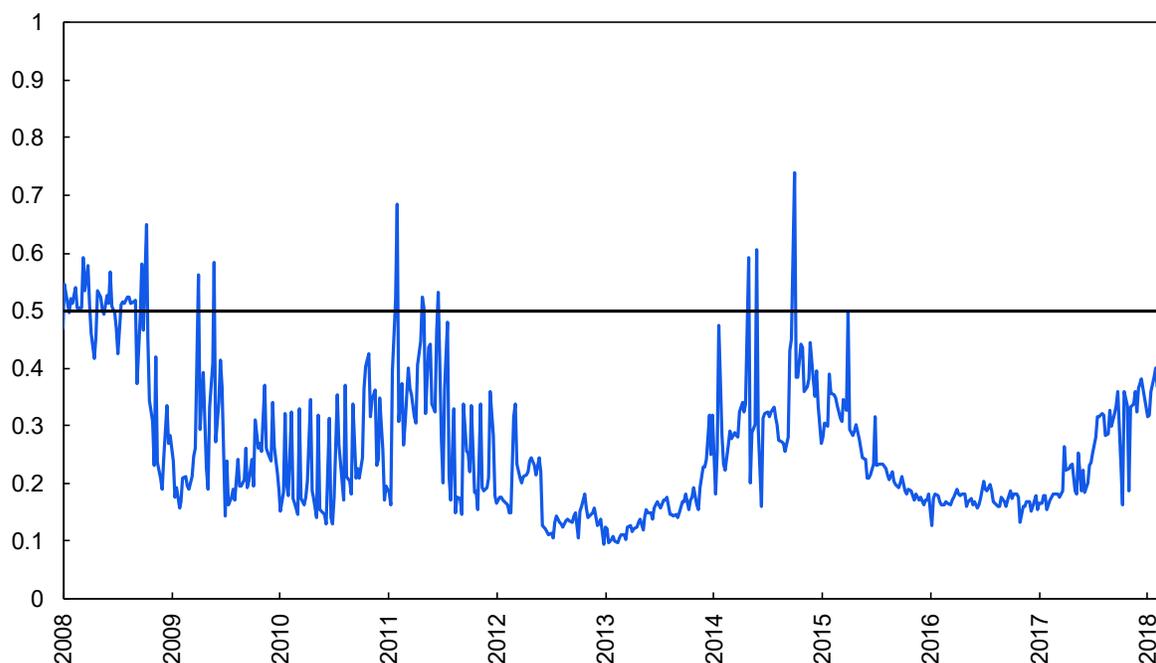
In theory, fixed-rate full allotment should keep short-term rates stable. As discussed in section C, counterparties' optimal strategy (that is, coordination) to minimize their borrowing costs based on the Eurosystem autonomous factor forecasts should stabilize the short-term rates close to the deposit facility rate. However, our findings support the relevance of scenarios in which coordination failures under fixed-rate full allotment engender deviations from the deposit facility rate.

The mere fact that short-term rates deviated notably from the deposit facility rate (and with some volatility) under fixed-rate full allotment is already an empirical evidence of coordination failures. The deviations from the deposit facility tend to be larger when bidding at the MRO under fixed-rate full allotment was the main driver of excess reserves (periods without other reserve-supplying operations, such as long-term refinancing and outright assets purchases). Figure 6 and 13 show notable differences in short-term rates for similar level of excess reserves between EUR 7 billion and 400 billion of excess reserves.

Market segmentation contributes to coordination failures. According to the definition, segmentation represents some counterparties unwillingness to cooperate with others. It also makes it more difficult for counterparties to interpret the impact of expected developments in excess reserves on short-term rates because only an unobservable part of those reserves is tradable, thereby complicating the bidding at the MRO. Counterparties were overall successful, with few exceptions, at keeping short-term rates close to the lower end of the interest rate corridor from October 2008 to December 2011. On the other hand, the difference between short-term rates and the deposit rate was larger during the period January 2012 to February 2018 after market risk perception increased in the market, underlining the impact of market segmentation on coordination failures.

Figure 13 provides further evidences of coordination failures based on the average cost of short-term borrowing. It shows the average rate of the MRO refinancing and EONIA weighted by the MRO allotment and EONIA volume. The weighted average is normalized such as 0.5 is the MRO minimum bid rate and zero is the deposit facility rate. The clearest (and more dramatic) indication of coordination failures are the nine episodes when the weighted average rate of MRO refinancing and EONIA borrowing jumped above the MRO rate. Indeed, counterparties could have paid less if they had borrowed the total amount at the MRO (which is already a coordination failure). Furthermore, even during the period of relatively lower market segmentation (i.e. before 2011), the counterparties' bidding never fully stabilized the average cost of their short-term borrowing close to the deposit facility rate.

Figure 13. Average Costs of Short-term Borrowing  
(in percent)



Note: X-axis is the normalized interest rate corridor in which is plotted the weighted average rate of MRO refinancing and EONIA borrowing.

Sources: ECB data warehouse, authors' calculations.

### **Possible other factors influencing the shape of the logistic function**

The introduction of regulations, such as the Liquidity Coverage Ratio (LCR) may have also contributed to the modifying pattern, since the LCR may have increased the banks' demand for excess reserves, counted as high-quality liquid assets (HQLA) under Basel III for banks having reached the inflow cap. Under the LCR, banks need to keep HQLA equal to at least 100 percent of the net cash outflows over the next 30 calendar days. The net cash outflows are equal to gross outflows minus gross inflows. However, the latter are capped to a maximum of 75 percent of the total expected cash outflows. Hence, a bank that has reached the 75 percent maximum inflow threshold would see its LCR ratio declining if it extended an interbank loan, even if interbank loans with a maturity of 30 days or less were eligible inflows in the denominator.

By the same token, new capital requirements under Basel III and the Capital Requirement Directive have increased the cost of interbank lending, if the capital opportunity cost is factored in. This may result, *ceteris paribus*, in higher spreads between interbank rates and the deposit facility rate as a consequence of lower opportunity costs.

## V. CONCLUSION

We argue that the conditions in which short-term rates get un-anchored can be attributed to entangled operational and market functioning issues. We traced their origin to coordination failures among counterparties at the central bank's open market operations under fixed-rate full allotment (that is, an operational issue). However, the introduction of fixed-rate full allotment itself could be traced to the impossibility of accurately forecasting the demand for excess reserves in a segmented market (that is, a market functioning issue). In turn, such market segmentation exacerbates the risk of coordination failures, as it makes the supply of excess reserves in the market more difficult to predict for counterparties.

The findings of this paper may have several policy implications going forward. As long as the current excess reserve conditions prevail, the ECB will communicate its policy stance and will steer short-term rates via the deposit facility rate. When the ECB, in the future, unwinds its unprecedented monetary policy stimulus and allows excess liquidity to decline, it should pay attention to the interest rate volatility for intermediate levels of excess reserves. Under these conditions, it should assess the instruments at hand to minimize this undesirable volatility, via, for instance, accelerating the transition to a new targeted steady state.

In this context, market functioning will likely shape policy implementation. If markets experience de-segmentation, short-term rates could remain stuck at a small premium over the deposit rate, even though excess reserves decline to low levels. In this case, the un-anchoring of the rates could be avoided at moderate levels of excess reserves. On the other hand, if the market remains segmented or if market segmentation increases further, excess reserves would have to be kept at a relatively high level to keep short-term rates anchored to the deposit facility, or else the ECB should consider alternative instruments for avoiding a prolonged period of interest rate volatility.

Certain actions could contribute to keeping stable short-term rates under fixed-rate full allotment. First, the publication of autonomous factor forecasts will become increasingly important as excess reserves decline, to facilitate counterparties' coordination and guide their bidding in a de-segmenting market. Second, higher reserve requirements than the current historic low would accelerate the reduction of excess reserves and facilitate a return to the neutral allotment. Higher reserve requirements, in fact, would provide more averaging room to absorb coordination failures under fixed-rate full allotment (especially since the maintenance period has been extended from four to six weeks). Finally, continuing the negative deposit facility rate, especially as the MRO rate is increased (that is, a negative deposit facility rate associated with an increase in the interest rate corridor) would create financial incentives to accelerate market de-segmentation, contributing *in fine* to the stability of short-term rates.

The logistic function presented in this paper is a useful tool for any central bank conducting monetary policy, regardless of their frameworks. The logistic function helps primarily central banks that steer short-term rates in an interest rate corridor as anchoring short-term rates is their operational target. However, it also supports central bank operating under different frameworks with different operational targets, such as reserve money or the exchange rate, as the level and volatility of short-term rates has an influence of their ability to achieve their

operational target. An estimate of the responsiveness of short-term rate to excess reserves is, thus, also important for them. The logistic function estimate also contributes to the authorities' monitoring of market functioning, in particular the changes in risk perception in the market, by observing the changes in the shape of the function.

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