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Abstract

This paper assesses the optimal setting of fiscal spending and foreign exchange rate intervention policies in response to volatile foreign aid, in a small open economy model that incorporates typical features of low-income countries. Within a class of policy rules, it jointly considers the optimal aid spending and international reserve accumulation policies. The results show that it is optimal to adjust government spending gradually in response to unpredictable fluctuations in aid, while partially accumulating foreign exchange reserves to offset Dutch disease effects. Also, allocating relatively more of the government spending to productive public investment, and less to government consumption, is welfare improving.

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1 Introduction

Foreign aid is an important but volatile income source in low-income developing countries. Aid is highly procyclical and the volatility of aid receipts is about two to three times that of recipient countries' output, as found by Pallage and Robe (2001). The introduction of Poverty Reduction Strategy Papers (PRSPs) in 1999 and other donor initiatives did not reduce aid volatility. In fact, Bulíř and Hamann (2007) find that aid volatility increased substantially, on average, for the period between 2000 and 2003, particularly for non Highly Indebted Poor Countries.[1] Also, Bulíř and Hamann (2003) and Bulíř and Hamann (2008) document that aid is generally more volatile than fiscal revenues based on data from 1970s to early-2000s, especially in highly aid-dependent countries. Such a pattern continues to hold in recent data. For the period from 1990 to 2016, the average standard deviation of the aid-to-GNI ratio is 70 percent higher than that of the tax-to-GDP ratio, and only 7 of the 28 low-income countries (LICs) have more volatile tax revenues than aid.[2]

Aid volatility can hurt growth, reduce welfare, and add to macroeconomic instability. Lensink and Morrissey (2000) provide empirical evidence that uncertain aid reduces its effectiveness. Aid uncertainty can have different macroeconomic implications. Agénor and Aizenman (2010) show, for instance, that lack of predictability in aid disbursements can prevent LICs to invest productively to spur growth, leaving them in a poverty trap. Looking at welfare implications, Pallage and Robe (2003) argue that the welfare gain from removing consumption volatility in 33 low-income developing countries, often associated with aid volatility, is substantial—exceeding that from a permanent increase in one percentage point of growth. Along the same lines, using a model calibrated to Cote d'Ivoire, Arellano et al. (2009) find that donors can reduce aid by 8 percent and still provide the same level of welfare if aid is delivered steadily. Moreover, as governments in recipient low-income developing countries generally face challenges raising sufficient domestic revenues to cover expenditures, volatile aid could lead to fluctuating government spending and debt accumulation, affecting macroeconomic stability (Celasum and Walliser (2008)).

Given this evidence on unpredictable aid and its implications, does policy have a role to grapple with aid volatility? In this paper, we address this question by focusing on recipient countries' optimal policy setting, taking aid volatility as given. This contrasts with the current literature that has mostly focused on donors' behavior in coordinating aid disbursement and increasing the predictability of aid flows (Bulíř and Hamann (2003), Arellano et al. (2009)). In particular, we jointly consider the optimal degree of aid spending and reserve accumulation in response to fluctuating aid revenues. That is, we provide an answer to how fast a government should adjust spending in response to unpredictable aid flows as well as how much it should accumulate of foreign reserves, from the perspective of welfare analysis.

In the context of optimal policies that maximize welfare under aid volatility, we also revisit two issues typically associated with aid inflows: Dutch disease effects and the allocation between public consumption and investment. Foreign aid can give rise to Dutch disease—the negative effects of aid flows on the real exchange rate and traded goods production—which has received considerable attention by policy makers and in the literature (see, e.g., Torvik (2001), Adam and Bevan (2006), Agénor et al. (2008), and Rajan and Subramanian (2011)). During aid surge episodes in several

[1] PRSPs was one of the key initiatives introduced in late 1990s to address the problem of lack of donor coordination. See Appendix I in Bulíř and Hamann (2006) for a description of various aid initiatives introduced in 1990s and 2000s.

[2] Aid is measured by the net official development assistance consisting of concessional loans and grants from the World Development Indicators (World Bank (2018)). Tax revenue data are taken from the database of the World Economic Outlook (April, International Monetary Fund (2018)). Only LICs with at least 10 years of data are included in the calculation.

African countries with managed floats, concerns about the real exchange rate appreciation resulted in large accumulation of reserves (Berg et al. (2007)). This is not surprising. To mitigate aid-related Dutch disease effects, fiscal and/or reserve accumulation policies can be used to reduce real appreciation, as shown by Berg et al. (2010a).[3] However, such a policy response can also crowd out private consumption and investment, or generate higher inflation depending on whether the accumulation is sterilized (Berg et al. (2010a), Berg et al. (2015a)), affecting the effectiveness of aid and generating macroeconomic trade-offs. Another aspect to consider is the optimal government use of aid resources. Aid can finance the desperately needed public investment, as a "big push" to help LICs escape the poverty trap (Collier (2006)).[4] But it can also increase the provision of public goods to directly raise the living standards of a large poor population. The literature, however, is silent about the optimal allocation between government consumption and public investment. We address this issue from a welfare perspective.

The framework we use is a New Keynesian dynamic stochastic general equilibrium (DSGE) model of a small open economy, which incorporates several LIC-specific features and is based on Berg et al. (2010a), Berg et al. (2015b), and Shen et al. (2018). The model has a large share of financially constrained hand-to-mouth households, a closed capital account of the private sector, and low governance quality captured by public investment inefficiency. Dutch disease is modeled as learning-by-doing externalities such that real exchange rate appreciation can harm productivity growth in the traded goods sector (van Wijnbergen (1984)). Departing from the typical assumption that aid revenues are spent immediately and not accumulated in reserves, resulting in full absorption—an increase in the current account deficit, we allow for non-coordination between the government and the central bank in spending and absorption decisions, following Berg et al. (2010b) and Berg et al. (2015a). By assumption, aid is eventually absorbed and the government eventually spends all foreign aid. However, faced with aid fluctuations, the government can adjust expenditures immediately to reflect revenue changes, or it can do so gradually, by temporarily saving the aid, while at the same time the central bank can independently pursue a reserve accumulation policy. The government follows simple fiscal spending and reserve accumulation policy rules that capture various spending speeds and reserve accumulation fractions, which allows us to look for the optimal parameterization of these rules that maximizes households' welfare.

We follow the approach of Schmitt-Grohé and Uribe (2004), among others, to calculate a second-order accurate solution of the model and compute welfare measures to assess the optimality of fiscal and reserve accumulation policies. We decompose welfare measures into a "mean effect," which is generated by changes in the means of the variables affecting utility, and a "variance effect" or stabilization component, which is due to changes in the variances of these variables. The interaction between aid volatility and different parameterizations of fiscal and reserve accumulation rules induces mean and variance effects, which may offset or reinforce each other, contributing to the welfare ranking.

We find that facing volatile aid flows, it is optimal for the government to adjust its spending gradually, while the central bank follows a partial reserve accumulation policy to help contain exchange rate variability and associated Dutch disease effects. Adjusting public spending gradually

[3] See also Lama and Medina (2012) and Faltermeier et al. (2017), who study foreign exchange intervention to mitigate Dutch disease effects associated with terms of trade shocks.

[4] The positive link between public investment and growth has long been recognized (see, e.g., Chapter 12 of Agénor (2004) and International Monetary Fund (2014)), which suggests that aid should be spent in public investment. But, empirically and at more general level, whether foreign aid can promote economic growth remains an open question. For example, Rajan and Subramanian (2008) find little robust evidence for a positive relationship, but Arndt et al. (2010) find a significant causal effect of aid on growth over the long run.

increases the volatility of private consumption (and money holdings) but reduces the variability in public goods consumption—which affects utility—and labor supply. When combined, the overall stabilization effect raises households' welfare. This is reinforced by the mean effect of gradual spending policies: the increased variability of consumption can raise precautionary savings motives for savers and lead to higher capital accumulation, output, and consumption over the long run, which increases welfare. At the same time, accumulating reserves (partial absorption) in response to aid inflows limits the appreciation of the exchange rate and results in more muted responses in private consumption, investment, and output. Thus, partial absorption policies are associated with reduced volatility in macroeconomic variables and increased stability, which raises welfare. The reduced uncertainty, on the other hand, lowers precautionary savings and capital accumulation, generally reducing long-run consumption and welfare, through the mean effect. These opposing effects then create an overall *non-linear* welfare effect, such that only a limited accumulation of reserves is optimally desirable.

When considering the optimal allocation of government spending between productive public investment and public goods consumption, we find that relative to the historical average, shifting more of government spending towards public investment is beneficial. This reflects the balance between the two roles of government spending. Building of productive public capital (infrastructure) supports production capacity, and can also attract more private capital through complementary effects, thus allowing for higher output and consumption over the long run. The provision of public goods consumption—the alternative use of government spending—directly enhances households' utility. In the context of volatile aid, a higher share of public investment increases the volatility of private consumption but reduces the volatility in public goods consumption, inducing welfare gains or losses. The mean effects of a higher share, however, generate substantial welfare gains that guarantee an overall welfare improvement, supporting the shift of government spending towards public investment.

Our study is related to several papers that consider macroeconomic policy in relation to foreign aid. Adam et al. (2009), Buffie et al. (2008), and Berg et al. (2010b), among others, focus on the macroeconomic effects of various policy responses to aid inflows, but they do so without investigating the optimality of such policies and use models where sectoral output is exogenous or capital accumulation is absent.[5] Other work looks into optimal policy issues. Focusing solely on fiscal policy, Kimbrough (1986) finds that the optimal response to a surge in aid involves lowering tax rates and increasing government spending. When monetary policy is also considered, as in Gong et al. (2008), optimal policy suggests that the government should lower inflation and the income tax rate and increase public spending, which provides more incentives for private capital accumulation. Both analyses are conditional on an increase in aid and do not capture aid volatility more generally. They also do not consider the role of public investment.[6] Berg et al. (2010a) and Berg et al. (2015a) study the optimality of fiscal, monetary, and reserve accumulation policies in the same framework we use in this paper. However, their analysis assumes perfect foresight and therefore abstracts from the implications of aid volatility for the design of optimal policies. In their context, optimal policies tend to be "corner solutions"—e.g. full aid absorption or no accumulation of reserves—although this may depend on structural conditions of the economy. On aid volatility, Prati and Tressel

[5] There are also papers that investigate the macroeconomic effects of foreign aid in real models with capital. See, for instance, Adam and Bevan (2006), Agénor et al. (2008), Arellano et al. (2009), Cerra et al. (2009), and Chatterjee and Turnovsky (2007), among others. These papers, however, do not model the interaction of fiscal and monetary policies and, therefore, do not investigate their implications for the effects of volatile aid.

[6] Gong et al. (2008) allow public spending to enhance utility and production, but they do not explicitly distinguish between government consumption and public investment, which can accumulate into public capital.

(2006) use a two-period model to show that reserve accumulation or de-cumulation can undo the negative effects of fluctuating aid disbursements on the trade balance and exports and thus reduce the undesired consequences of aid volatility and the impact of Dutch disease. We take a more comprehensive approach and derive implementable optimal policies accounting for interactions among fiscal, monetary, and reserve accumulation rules and aid volatility.

The remainder of the paper is organized as follows. Section 2 outlines the model. Section 3 describes the solution method and calibration. In section 4, we discuss the welfare measures and present our results. A final section concludes.

2 Model Setup

The model includes two types of households (savers and hand-to-mouth consumers), two production sectors (producing tradable and non-tradable goods), the government and the monetary authority.

2.1 Households

A fraction f are savers (a) and the remaining $1-f$ are hand-to-mouth consumers (h). Only savers have access to domestic asset and capital markets, while the hand-to-mouth are liquidity constrained.

2.1.1 Savers

Savers derive utility from consumption of both private goods (c_t^a) and public goods (g_t^C), and real money balances (m_t^a), and disutility from supplying labor (l_t^a). They choose consumption (c_t^a), real money balances, labor, sector-specific investment ($i_t^{N,a}$ and $i_t^{T,a}$) and capital ($k_t^{N,a}$ and $k_t^{T,a}$), as well as one-period risk-free domestic government bonds ($b_t^{c,a}$) to maximize the expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left[\frac{(c_t^a)^{1-\sigma}}{1-\sigma} - \frac{(l_t^a)^{1+\psi}}{1+\psi} + \frac{(m_t^a)^{1-\xi}}{1-\xi} + \varkappa^G \frac{\left(g_t^C\right)^{1-\sigma}}{1-\sigma} \right]}_{\equiv U_t^a}, \tag{1}$$

subject to the budget constraint

$$\begin{aligned} & c_t^a + m_t^a + i_t^{N,a} + i_t^{T,a} + b_t^{c,a} + ac_t^{i,a} - \frac{R_{t-1} b_{t-1}^{c,a}}{\pi_t} \\ = & (1-\tau) \left(w_t l_t^a + r_t^N k_{t-1}^{N,a} + r_t^T k_{t-1}^{T,a} \right) + \frac{m_{t-1}^a}{\pi_t} + s_t rm^* + \Omega_t^a. \end{aligned} \tag{2}$$

E_t is the mathematical expectation conditional on information available at time t, β is the discount factor, σ, ψ, and ξ are the inverses of the elasticities of intertemporal substitution for consumption, labor, and money, and $\varkappa^G$ is the relative weight on public goods consumption in utility. Savers receive income from supplying labor and renting capital to firms, where w_t is the real wage rate, and r_t^N and r_t^T are the rental prices of capital in the non-traded and traded goods sectors. This income is taxed at a constant rate τ. They also receive dividends from firms Ω_t^a, as well as foreign remittances rm^*, expressed in units of the foreign good (denoted by $*$) and assumed to be constant. The real exchange rate, s_t, is in units of domestic consumption per unit of foreign good. Domestic government bonds, $b_t^{c,a}$ (the superscript c for consumers), pay a nominal rate R_t at $t+1$, and π_t is gross domestic inflation. We assume that investment is subject to adjustment costs, which are also sector specific and total up to $ac_t^{i,a} \equiv \frac{\kappa}{2}\left[\left(\frac{i_t^{N,a}}{k_{t-1}^{N,a}}-\delta\right)^2 k_{t-1}^{N,a}+\left(\frac{i_t^{T,a}}{k_{t-1}^{T,a}}-\delta\right)^2 k_{t-1}^{T,a}\right]$. The law of motion for capital is

$$k_t^{j,a}=(1-\delta)k_{t-1}^{j,a}+i_t^{j,a}, \quad j\in\{N,T\}, \tag{3}$$

where δ is the capital depreciation rate. Total investment by savers is then $i_t^a=i_t^{N,a}+i_t^{T,a}$. The household's optimal choices and all other equilibrium conditions are provided in Appendix A.

Consumption and investment are constant-elasticity-of-substitution (CES) aggregates of non-traded and traded goods, with the intratemporal elasticity of substitution χ and the degree of home bias φ,

$$c_t=\left[\varphi^{\frac{1}{\chi}}\left(c_t^N\right)^{\frac{\chi-1}{\chi}}+(1-\varphi)^{\frac{1}{\chi}}\left(c_t^T\right)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}. \tag{4}$$

The non-traded consumption bundle, c_t^N, is further assumed to be a CES aggregate of a continuum of non-traded goods varieties, $c_t^N(i)$, produced by monopolistically competitive firms indexed by $i\in[0,1]$

$$c_t^N=\left[\int_0^1 c_t^N(i)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where θ is the elasticity of substitution between varieties. The CES consumption basket implies that the price of c_t (CPI) is an index of the non-traded and traded goods prices, P_t^N and P_t^T, given by

$$P_t=\left[\varphi\left(P_t^N\right)^{1-\chi}+(1-\varphi)(P_t^T)^{1-\chi}\right]^{\frac{1}{1-\chi}}. \tag{6}$$

The relative prices of non-traded and traded goods to CPI are then defined as $p_t^N \equiv \frac{P_t^N}{P_t}$ and $s_t \equiv \frac{S_t P_t^*}{P_t}$ (the real exchange rate), where S_t is the nominal exchange rate and P_t^* is the price of foreign goods.

Households supply labor to both production sectors. Savers' total labor supply is

$$l_t^a=\left[(\varphi^l)^{-\frac{1}{\chi^l}}\left(l_t^{a,N}\right)^{\frac{1+\chi^l}{\chi^l}}+(1-\varphi^l)^{-\frac{1}{\chi^l}}\left(l_t^{a,T}\right)^{\frac{1+\chi^l}{\chi^l}}\right]^{\frac{\chi^l}{1+\chi^l}}, \tag{7}$$

where φ^l is the steady-state share of labor in the non-traded goods sector and $\chi^l>0$ is the elasticity of substitution between the two types of labor. The implied aggregate real wage index is

$$w_t=\left[\varphi^l\left(w_t^N\right)^{1+\chi^l}+(1-\varphi^l)\left(w_t^T\right)^{1+\chi^l}\right]^{\frac{1}{1+\chi^l}}, \tag{8}$$

where w_t^N and w_t^T are the wage rates in the two sectors.

2.1.2 Hand-to-Mouth Households

Hand-to-mouth consumers also derive utility from consumption of private goods $\left(c_t^h\right)$ and public goods (g_t^C), and disutility from supplied labor $\left(l^h\right)$, according to the periodic utility function

$$U_t^h = \left[\frac{\left(c_t^h\right)^{1-\sigma}}{1-\sigma} - \frac{\left(l^h\right)^{1+\psi}}{1+\psi} + \varkappa^G \frac{\left(g_t^C\right)^{1-\sigma}}{1-\sigma} \right]. \tag{9}$$

Unlike savers, hand-to-mouth consumers do not have access to capital markets and hence rely on after-tax labour income and foreign remittances as their only source of income. The level of private consumption is thus determined by the period-by-period budget constraint,

$$c_t^h = (1-\tau)\, w_t l^h + s_t r m^*. \tag{10}$$

2.2 Firms

The two production sectors have different market structures. Non-traded goods firms are assumed to be monopolistically competitive, since non-traded goods can only be produced domestically, and are subject to nominal price rigidities in the form of Rotemberg (1982) price adjustment costs. Traded goods firms are instead perfectly competitive facing flexible prices.

2.2.1 The Non-Traded Goods Sector

The monopolistically competitive firm $i \in [0,1]$ in the non-traded goods sector uses labor $l_t^N(i)$, private capital $k_{t-1}^N(i)$, and public capital k_{t-1}^G to produce goods with the technology

$$y_t^N(i) = z^N \left[k_{t-1}^N(i)\right]^{1-\alpha^N} \left[l_t^N(i)\right]^{\alpha^N} \left(k_{t-1}^G\right)^{\alpha^G}, \tag{11}$$

where z^N is a constant total factor productivity (TFP) term, specific to non-traded goods production, α^N corresponds to the labor share in non-traded output, and α^G is the elasticity of output with respect to public capital. The differentiated non-traded goods are aggregated into the non-traded good bundle, via the CES aggregator $y_t^N = \left[\int_0^1 y_t^N(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$. The associated demand function for each good i (coming from households and the government) is

$$y_t^N(i) = \left[\frac{p_t^N(i)}{p_t^N}\right]^{-\theta} y_t^N. \tag{12}$$

The firm chooses the price, labor, and capital to maximize its net present-value of profits

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda_t^a/\lambda_0^a\right) \underbrace{\left[(1-\iota)\left[p_t^N(i) y_t^N(i) - ac_t^p(i)\right] - w_t^N l_t^N(i) - r_t^N k_{t-1}^N(i) + \iota p_t^N y_t^N\right]}_{\equiv \Omega_t(i),\ \text{dividends}}, \tag{13}$$

subject to the production function (11) and the demand constraint (12). Profits are discounted by the stochastic discount factor $\beta^t\left(\lambda_t^a/\lambda_0^a\right)$, where λ_t^a is the savers' (firm owners') marginal utility of consumption. Price rigidity is introduced via the adjustment costs $ac_t^p(i) \equiv \frac{\zeta}{2}\left[\frac{\pi_t^N(i)}{\pi^N(i)} - 1\right]^2 p_t^N y_t^N$, where $\pi_t^N \equiv \frac{p_t^N}{p_{t-1}^N}\pi_t$ is non-traded goods inflation. We also introduce an implicit tax (cost) ι, which discourages firms from producing at a higher level. This is a shortcut to rationalize why, given the high marginal return to capital implied by capital scarcity, we do not observe a higher investment-to-output ratio in LICs. Unlike income taxes, the revenues collected via the tax ι do not enter the government budget but remain in the private sector. For simplicity, we assume they are rebated back to firms in lump-sum fashion. Total price adjustment costs, non-traded output, and dividends across all firms are $ac_t^p = \int_0^1 ac_t^p(i)di$, $y_t^N = \int_0^1 y_t^N(i)di$, and $\Omega_t = \int_0^1 \Omega_t(i)di$.

2.2.2 The Traded Goods Sector

Firms in the traded goods sector are perfectly competitive. The representative firm i uses labor $l_t^T(i)$, private capital $k_{t-1}^T(i)$, and public capital k_{t-1}^G to produce goods using the technology

$$y(i)_t^T = z_t^T \left[k(i)_{t-1}^T\right]^{1-\alpha^T} \left[l(i)_t^T\right]^{\alpha^T} \left(k_{t-1}^G\right)^{\alpha^G}. \tag{14}$$

The time-varying TFP of traded goods production, z_t^T, follows the process

$$\log\left(\frac{z_t^T}{z^T}\right) = \rho_{zT}\log\left(\frac{z_{t-1}^T}{z^T}\right) + \varkappa\log\left(\frac{y_{t-1}^T}{y^T}\right), \tag{15}$$

where $\rho_{zT} \in (0,1)$, $\varkappa > 0$, and $y_t^T = \int_0^1 y_t^T(i)di$ is total traded output. Variables without a time script indicate their deterministic steady-state levels. This specification implies learning-by-doing externalities capturing Dutch disease effects: When traded output falls, it affects TFP negatively with some persistence affecting all the firms of the sector, as in van Wijnbergen (1984).

Firms choose labor and capital to maximize period-t profits given by

$$(1-\iota)\, s_t y(i)_t^T - w_t^T l(i)_t^T - r_t^T k(i)_{t-1}^T + \iota y_t^T. \tag{16}$$

Total output produced in the economy in period t is defined as $y_t = p_t^N y_t^N + s_t y_t^T$.

2.3 The Public Sector

The public sector consists of the government and the central bank. Each period, the government receives tax revenues and foreign aid (a_t^*) and issues a constant amount of domestic debt (b), which is held by the central bank $\left(b_t^{cb}\right)$ and by those households with access to financial markets (b_t^c), hence $b = b_t^c + b_t^{cb}$.[7] Total expenditures include government consumption (g_t^C), public investment (g_t^I), and debt services. Note that interest is paid only on the portion of debt held by the private sector. To allow for the option of spending foreign aid inflows gradually, the government can deposit aid receipts with the central bank, and d_t represents the stock of government deposits at the central bank (expressed in units of domestic consumption). The government's flow budget constraint is

$$tax_t + \underbrace{b_t^c + b_t^{cb}}_{=b} + s_t a_t^* = p_t^G\left(g_t^C + g_t^I\right) + \left(d_t - \frac{d_{t-1}}{\pi_t}\right) + \frac{R_{t-1}b_{t-1}^c}{\pi_t} + \frac{b_{t-1}^{cb}}{\pi_t}, \tag{17}$$

where $tax_t = \tau\left(w_t l_t + r_t^N k_{t-1}^N + r_t^T k_{t-1}^T\right)$ and l_t, k_{t-1}^N, and k_{t-1}^T are the aggregate labor and capital stocks in the two sectors.

[7] Our focus is on aid; thus, we abstract from external public debt—an importance financing source in LICs.

The accumulation of government deposits is described by the following rule, as in Berg et al. (2010a):

$$d_t = (1-\rho_d)\, d + \rho_d d_{t-1} + (1-\gamma)\, s_t\, (a_t^* - a^*)\,. \tag{18}$$

In this setup, the government always spends the steady-state amount of aid. However, an increase in aid receipts above the log-run level may or may not be spent initially, depending on $\gamma \in [0,1]$. When $\gamma = 1$, the additional aid is spent immediately. If $\gamma < 1$, then the aid is initially accumulated (partially or fully) as deposits at the central bank, and it is only spent gradually over time, at a rate given by the persistence parameter $\rho_d \in (0,1)$. The spending of aid inflows is governed by both the γ and ρ_d parameters.[8]

Total government expenditure $g_t = g_t^C + g_t^I$ is also a CES aggregate of traded and non-traded goods, with elasticity of substitution χ and degree of home bias φ^G (possibly different from that of private households),

$$g_t = \left[\left(\varphi^G\right)^{\frac{1}{\chi}} \left(g_t^N\right)^{\frac{\chi-1}{\chi}} + (1-\varphi^G)^{\frac{1}{\chi}} \left(g_t^T\right)^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \tag{19}$$

where the implied relative price of the bundle is

$$p_t^G = \left[\varphi^G \left(p_t^N\right)^{(1-\chi)} + \left(1-\varphi^G\right) (s_t)^{1-\chi} \right]^{\frac{1}{1-\chi}}. \tag{20}$$

A share ϑ of changes in total government expenditures goes towards public investment, while the remaining share is spent on public goods consumption:

$$\left(g_t^I - g^I\right) = \vartheta\, (g_t - g)\,. \tag{21}$$

The accumulation of public capital is given by:

$$k_t^G = \left(1-\delta^G\right) k_{t-1}^G + \epsilon g_t^I \quad 0 < \epsilon < 1, \tag{22}$$

where δ^G is the depreciation rate of public capital and $\epsilon \in (0,1)$ captures the low efficiency characteristic of public investment in LICs, whereby one dollar of public investment expenditure delivers less than one dollar of public capital.

Aid follows an exogenous AR(1) process

$$\log\left(\frac{a_t^*}{a^*}\right) = \rho_a \log\left(\frac{a_{t-1}^*}{a^*}\right) + \varepsilon_t, \;\; \varepsilon_t \sim N\left(0, \sigma_a^2\right), \tag{23}$$

with persistence ρ_a and standard deviation of the shock, σ_a.[9]

The central bank conducts monetary and foreign reserves policies. Given less developed money and financial markets, many central banks in LICs target money in practice. Denoting foreign reserves with res_t^*, the central bank's balance sheet is

$$m_t - \frac{m_{t-1}}{\pi_t} + \left(d_t - \frac{d_{t-1}}{\pi_t}\right) = b_t^{cb} - \frac{b_{t-1}^{cb}}{\pi_t} + s_t \left(res_t^* - \frac{res_{t-1}^*}{\pi^*}\right). \tag{24}$$

[8] In the case of a decline in aid flows, the government either reduces its expenditures immediately (if $\gamma = 1$) or draws on existing deposits ($\gamma < 1$) to support existing spending levels.

[9] We assume that aid follows an exogenous, stochastic process. Although in reality aid is often procyclical, such that aid shocks can be correlated with other macroeconomic shocks in recipient countries, our assumption allows us to focus more cleanly on the implications of volatility in external government receipts. And the results can generalize to other sources of volatile receipts, such as natural resource revenues, that are subject to large fluctuations but due to volatile world market commodity prices.

Following Berg et al. (2010a), the reserves accumulation policy is given by the following rule

$$res_t^* = (1-\rho_{res})\,\overline{res}^* + \rho_{res} res_{t-1}^* + (1-\omega)\,(a_t^* - a^*)\,, \tag{25}$$

which describes the practice of adjusting foreign reserves in response to aid flows, aiming to offset the impact that aid spending has on the value of the currency and the associated Dutch disease effects. The degree of reserve accumulation in response to changes in aid is determined by the parameter $\omega \in [0,1]$. With $\omega = 1$, the aid is fully absorbed, as central banks do not adjust reserves in relation to aid flows, while $\omega < 1$ corresponds to partial absorption policies, whereby foreign exchange reserves increase temporarily during a rise in aid inflows. The additional reserves are eventually depleted at rate $\rho_{res} \in (0,1)$.

Finally, we assume that the central bank maintains a constant growth rate μ of nominal reserve money. Open market operations then imply

$$b_t^{cb} - \frac{b_{t-1}^{cb}}{\pi_t} = (\mu - 1)\,\frac{m_{t-1}}{\pi_t} + \left(d_t - \frac{d_{t-1}}{\pi_t}\right) - s_t\left(res_t^* - \frac{res_{t-1}^*}{\pi_t^*}\right). \tag{26}$$

2.4 Aggregation and Market Clearing

With two types of households, aggregate consumption and labor are computed as

$$x_t = f x_t^a + (1-f) x_t^h, \quad x \in \left\{c, c^N, c^T, l, l^N, l^T\right\}. \tag{27}$$

Since only savers have access to asset and capital markets, aggregate real money balances, investment, capital, debt, and dividends are determined as

$$x_t = f x_t^a, \quad x \in \left\{m, i^N, i^T, k^N, k^T, b^c, \Omega, ac^i\right\}, \tag{28}$$

while remittances are identical for all households, hence $rm_t^{*,a} = rm_t^{*,h} = rm_t^*$.

Finally, the market clearing condition of non-traded goods is

$$y_t^N = (p_t^N)^{-\chi}\left[\varphi\left(c_t + i_t + ac_t^i + ac_t^p\right) + \varphi^G (p_t^G)^{\chi} g_t\right], \tag{29}$$

and the balance of payment condition is

$$\underbrace{c_t + i_t + p_t^G g_t + ac_t^i + ac_t^p - y_t - s_t rm^*}_{\text{current account deficits}} = s_t\left(a_t^* - res_t^* + \frac{res_{t-1}^*}{\pi^*}\right).$$

3 Solution and Calibration

We use the algorithm of Schmitt-Grohé and Uribe (2004) to obtain a second-order accurate solution of the model and compute welfare measures. The use of a second-order solution is necessary because spurious welfare results may emerge from linear models that abstract from the effects of uncertainty on optimal decisions (Kim and Kim (2003)).

The model is at the quarterly frequency and calibrated based on available data for countries in Sub-Saharan Africa (SSA) over the period 2000 to 2015 (International Monetary Fund (2016) and The World Bank (2016)). Table 1 lists the key parameter values and aggregate ratios used in the calibration. In particular, we calibrate the values of consumption, private investment, government consumption, public investment, public debt, foreign exchange reserves, and foreign aid as a share of GDP, as well as the inflation level, to match the data average. To calibrate the aid process, we use the available data on foreign aid from 38 LICs and lower-middle income countries in SSA to estimate an AR(1) process for each country. ρ_a and σ_a are then calibrated to the mean of the two statistics. See Appendix B for details.

The discount factor $\beta = 0.98$ is consistent with an annual real interest rate of 8 percent. Based on the estimate by Ogaki et al. (1996) for developing countries, the intertemporal elasticity of substitution is set to 0.34, implying $\sigma = 2.94$. This suggests that consumption decisions are less on intertemporal smoothing considerations relative to households in developed countries, where the typical values are $\sigma = 1$ or 2. Without empirical evidence for the Frisch labor supply elasticity for SSA economies, we calibrate $\psi = 1/2$ for savers using the average estimate from developed economies based on macro level data (see, for example, Chetty et al. (2011) and Peterman (2016)). Together with hand-to-mouth households' inelastic labor supply, the average Frisch labor supply elasticity is 0.4.[10] The inverse of the intertemporal elasticity of real money balances $\xi = 5.27$ is endogenously determined given the calibrated quarterly nominal interest rate and the ratio of money balances to output. The weight attached to public goods consumption in utility, $\varkappa^G$, is 0.0051 such that the optimal ratio between private and public goods consumption is consistent with the data average.

We assume savers in our economy are a quarter of the population, $f = 0.25$. Based on data collected in 2011, Demirguc-Kunt and Klapper (2012) report that on average about 24 percent of adults in SSA have an account in a formal financial institution, although a wide variation exists, with 45 percent in the richest quintile of SSA countries and 12 percent in the poorest quintile. The intratemporal elasticity of substitution between labor of the two sectors is set to $\chi^l = 0.6$. Horvath (2000) estimates this elasticity to be 1 using the U.S. sectoral data. Artuc et al. (2015) estimate that on average labor mobility costs are 4.26 times of annual wages in SSA countries, and only 2.41 times in developed countries. The model assumes less labor mobility relative to developed countries. The elasticity of substitution between varieties of goods is set to $\theta = 6$, so a steady-state markup in the goods market is 20 percent, as calibrated in Galí and Monacelli (2005) for a small open economy, and the Rotemberg price adjustment parameter, $\zeta = 56.6$, matches the Calvo no-price change probability of 0.75, corresponding to an average price length of four quarters.

The degree of home bias in private consumption and investment is set to $\varphi = 0.6$, while for government purchases it is $\varphi^G = 0.7$. Since distribution costs can be high in rural Africa, we assume a slightly higher share than the typical value of 0.5 (Burstein et al. (2005)). We follow the convention to assume a higher degree of home bias in government purchases because a large part of government spending goes to pay for civil services. Together with the calibrated private consumption and investment shares to output in data (see Table 1), the model implies that almost 60 percent of labor works in the non-traded goods sector, and the value added by traded output in the steady state is 35 percent of GDP.

For the elasticity of substitution between traded and non-traded goods, we set $\chi = 0.44$, following Stockman and Tesar's (1995) estimate based on a sample of 30 countries including developing and developed countries. The labor income shares in non-traded and traded production are set to $\alpha^N = 0.45$ and $\alpha^T = 0.6$, following Buffie et al. (2012) for calibrating an average African economy.

[10] Goldberg (2016) estimates that the intertemporal elasticity of working probability in a daily labor market in rural Malawi is 0.15-0.17. The concept of her estimated elasticity—the elasticity of working with respect to a change in daily working wages—is different from the Frisch labor elasticity, though.

The depreciation rate of private capital is set to $\delta = 0.02$, corresponding to an annual depreciation rate of 8%, while that of public capital is set to 3.5% annually. These rates are slightly lower than usually assumed for developed economies, and reflect the nature of capital predominant in less developed economies, which typically has a longer life span (see for example, Arestoff and Hurlin (2006), Arslanalp et al. (2010), International Monetary Fund (2015)). The investment adjustment cost parameter is set to $\kappa = 1.4$, based on the only estimate we could locate for a developing country with the same specification (Mexico, Aguiar and Gopinath (2007)).

To calibrate the public investment efficiency ϵ, we refer to estimates in the literature: When the TFP growth rate is assumed to be zero, Pritchett (2000) estimates that the public investment efficiency is 0.49 for SSA economies, while Hurlin and Arestoff (2010) obtain a value of 0.4 for Colombia and Mexico. Our baseline calibration assumes $\epsilon = 0.4$.[11] The output elasticity with respect to public capital is $\alpha^G = 0.11$, which corresponds to an annual net rate of return to public capital of 19% in steady state, matching the evidence from lower-income countries, such as the rate of return of World Bank projects (International Bank for Reconstruction and Development and the World Bank (2010)). The implicit tax rate ι is 0.21, matching the investment to GDP ratio in the data. The income tax rate τ is computed from the government budget constraint, given the data on government expenditures and non-tax revenues as shares of GDP.

4 Analysis

We analyze the nature of optimal fiscal and reserve accumulation policies, when the economy experiences volatile foreign aid flows. Specifically, we compute the welfare maximizing parameterization of the policy rules in (18) and (25), in terms of the speed with which the government spends the aid (γ) and the degree of aid absorption (ω), and assess the welfare costs associated with departing from the optimal specification. We also investigate the optimal allocation of government expenditures between government consumption and public investment.

Welfare is measured as the *unconditional* expectation of discounted lifetime utility. For the savers in our economy, welfare is given by $W^a = E\sum_{t=0}^{\infty}\beta^t U_t^a$, and similarly for the hand-to-mouth consumers, $W^h = E\sum_{t=0}^{\infty}\beta^t U_t^h$. On the aggregate, we have $W = fW^a + (1-f)\,W^h$, which is the welfare measure used for searching over the policy parameter space.

We let the policy of immediate spending of aid and full absorption ($\gamma = 1$, $\omega = 1$) be the benchmark or *reference* policy regime, with the corresponding welfare measures for the two types of households denoted by $W^{a,\mathcal{R}}$ and $W^{h,\mathcal{R}}$. *Alternative* policy regimes are those of gradual spending of aid ($\gamma < 1$) and/or partial reserve accumulation ($\omega < 1$). They yield welfare $W^{a,\mathcal{A}}$ and $W^{h,\mathcal{A}}$. Then, following Schmitt-Grohé and Uribe (2007), the welfare benefit of an alternative policy Ξ^a $\left(\Xi^h\right)$ is expressed as the fraction of the consumption process under the reference policy regime that households must be given in order to be equally happy under the two policy specifications. For savers, this is determined by the following relationship:

$$W^{a,\mathcal{A}} \equiv E\sum_{t=0}^{\infty}\beta^t\left(c_t^{a,\mathcal{A}}, l_t^{a,\mathcal{A}}, m_t^{a,\mathcal{A}}, g_t^{C,\mathcal{A}}\right) = E\sum_{t=0}^{\infty}\beta^t U\left(\left(1+\Xi^a\right)c_t^{a,\mathcal{R}}, l_t^{a,\mathcal{R}}, m_t^{a,\mathcal{R}}, g_t^{C,\mathcal{A}}\right). \quad (30)$$

[11] The low public investment efficiency calibrated here is also consistent with an average low public investment and management index (PIMI) for low-income countries (Dabla-Norris et al. (2012)), which broadly assesses various components in public investment implementation, including appraisal, selection, budgeting, etc.

Given that utility is additively separable in its arguments, we can obtain an expression for Ξ^a as

$$\Xi^a = \left[\left(\frac{W^{a,\mathcal{A}} - W^{a,\mathcal{R}\_lmg^c}}{W^{a,\mathcal{R}\_c}} \right)^{\frac{1}{1-\sigma}} - 1 \right], \tag{31}$$

where

$$W^{a,\mathcal{R}\_c} = E \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^{a,\mathcal{R}} \right)^{1-\sigma}}{1-\sigma} \right] \tag{32}$$

and

$$W^{a,\mathcal{R}\_lmg^c} = E \sum_{t=0}^{\infty} \beta^t \left[-\frac{(l_t^a)^{1+\psi}}{1+\psi} + \frac{(m_t^a)^{1-\xi}}{1-\xi} + \varkappa^G \frac{\left(g_t^C \right)^{1-\sigma}}{1-\sigma} \right] \tag{33}$$

are the welfare terms under the reference regime due to private consumption alone and due to consumption, leisure, money holdings, and public goods consumption, respectively. For hand-to-mouth consumers, we have

$$W^{h,\mathcal{A}} \equiv E \sum_{t=0}^{\infty} \beta^t U \left(c_t^{h,\mathcal{A}}, l^h, g_t^{C,\mathcal{A}} \right) = E \sum_{t=0}^{\infty} \beta^t U \left(\left(1 + \Xi^h \right) c_t^{h,\mathcal{R}}, l^h, g_t^{C,\mathcal{A}} \right),$$

implying a welfare benefit measure:

$$\Xi^h = \left[\left(\frac{W^{h,\mathcal{A}} - W^{h,\mathcal{R}\_lg^c}}{W^{h,\mathcal{R}\_c}} \right)^{\frac{1}{1-\sigma}} - 1 \right], \tag{34}$$

where $W^{h,\mathcal{R}\_c} = E \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^{h,\mathcal{R}} \right)^{1-\sigma}}{1-\sigma} \right]$, while $W^{h,\mathcal{R}\_lg^c} = E \sum_{t=0}^{\infty} \beta^t \left[-\frac{\left(l^h \right)^{1+\psi}}{1+\psi} + \varkappa^G \frac{\left(g_t^C \right)^{1-\sigma}}{1-\sigma} \right]$. A positive Ξ^a $\left(\Xi^h \right)$ denotes that an alternative policy is welfare improving relative to the reference policy.

We further decompose welfare into a 'mean effect' (generated by changes in the means of the variables affecting utility) and a 'variance effect' or stabilization component (due to changes in the variances of these variables).[12] Firstly, by taking a second-order Taylor expansion of the utility function with respect to its arguments, we obtain the following expression for welfare, shown here for savers, where hatted variables denote log-deviations from the deterministic steady state (see Appendix C for more details):

$$\begin{aligned} W^a &= E \sum_{t=0}^{\infty} \beta^t U \left(c_t^a, l_t^a, m_t^a, g_t^C \right) \\ &= \frac{\overline{U}^a}{1-\beta} + E \sum_{t=0}^{\infty} \beta^t \left[(c^a)^{1-\sigma} \widehat{c}_t^a - (l^a)^{1+\psi} \widehat{l}_t^a + (m^a)^{1-\xi} \widehat{m}_t^a + \varkappa^G \left(g^C \right)^{1-\sigma} \widehat{g}_t^C \right] \\ &+ E \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left\{ \begin{array}{c} \left[(1-\sigma) (c^a)^{1-\sigma} \right] (\widehat{c}_t^a)^2 - \left[(1+\psi) (l^a)^{1+\psi} \right] \left(\widehat{l}_t^a \right)^2 + \left[(1-\xi) (m^a)^{1-\xi} \right] (\widehat{m}_t^a)^2 \\ + \left[\varkappa^G (1-\sigma) \left(g^C \right)^{1-\sigma} \right] \left(\widehat{g}_t^C \right)^2 \end{array} \right\}. \end{aligned}$$

[12] Similar decompositions are used by Kollmann (2002) and Kim and Kim (2018).

To capture the welfare consequences of changes in means and changes in volatility, we let $W^{a,M}$ and $W^{a,V}$ represent the two welfare components:

$$W^{a,M} = \frac{\overline{U}^a}{1-\beta} + E\sum_{t=0}^{\infty}\beta^t \left[(c^a)^{1-\sigma}\,\widehat{c}_t^a - (l^a)^{1+\psi}\,\widehat{l}_t^a + (m^a)^{1-\xi}\,\widehat{m}_t^a + \varkappa^G \left(g^C\right)^{1-\sigma}\widehat{g}_t^C \right];$$

$$W^{a,V} = \frac{\overline{U}^a}{1-\beta} + E\sum_{t=0}^{\infty}\beta^t \frac{1}{2} \left\{ \begin{array}{c} \left[(1-\sigma)(c^a)^{1-\sigma}\right](\widehat{c}_t^a)^2 - \left[(1+\psi)(l^a)^{1+\psi}\right]\left(\widehat{l}_t^a\right)^2 \\ +\left[(1-\xi)(m^a)^{1-\xi}\right](\widehat{m}_t^a)^2 + \left[\varkappa^G(1-\sigma)\left(g^C\right)^{1-\sigma}\right]\left(\widehat{g}_t^C\right)^2 \end{array} \right\}.$$

Similar welfare components are obtained for the hand-to-mouth households, whose utility depends entirely on consumption:

$$W^{h,M} = \frac{\overline{U}^h}{1-\beta} + E\sum_{t=0}^{\infty}\beta^t \left[\left(c^h\right)^{1-\sigma}\widehat{c}_t^h + \varkappa^G\left(g^C\right)^{1-\sigma}\widehat{g}_t^C \right];$$

$$W^{h,V} = \frac{\overline{U}^h}{1-\beta} + E\sum_{t=0}^{\infty}\beta^t \frac{1}{2} \left\{ \left[(1-\sigma)\left(c^h\right)^{1-\sigma}\right]\left(\widehat{c}_t^h\right)^2 + \left[\varkappa^G(1-\sigma)\left(g^C\right)^{1-\sigma}\right]\left(\widehat{g}_t^C\right)^2 \right\}.$$

Increased volatility is welfare reducing, while higher average consumption, including public goods consumption, raises welfare.[13] The welfare benefits of each component $\left(\Xi^M,\ \Xi^V\right)$ and for each type of household can be determined by applying the formulas in expressions (31) and (34).

4.1 Optimal Spending and Reserve Accumulation Policies

Starting from the reference policy (immediate aid spending with full absorption, $\gamma = 1$, $\omega = 1$), we explore the impact of alternative specifications of these policies. Table 2 gives the associated welfare benefits (costs, if negative) incurred when departing from the reference specification. On the aggregate, a more gradual spending of aid ($\gamma < 1$) and/or partial absorption through adjustment of foreign reserves ($\omega < 1$) results in welfare gains, with an optimal parameterization of $\gamma = 0$ and $\omega = 0.5$. Faced with volatile aid inflows, it is best to smooth the impact of aid changes on government expenditures by adopting a very gradual spending policy, while at the same time a certain degree of reserve accumulation helps maintain a more stable currency and overall supports the government's spending policy. The results reflect the stabilization role of the two policies, as captured by the volatility of aggregate variables, as well as the implied effects on long-run consumption levels that arise when people account for uncertainty in making decisions, as changes in volatility and overall uncertainty affect precautionary savings motives and the accumulation of capital. To have a better understanding of these results, we now look in more detail at how aid changes affect key macroeconomic variables under different policy specifications.

4.1.1 The Spending of Aid Inflows

Figures 1 and 2 show impulse responses to a 1 percent increase in aid, when the government either spends the aid inflows immediately ($\gamma = 1$) or more gradually, by initially saving it (partially or fully) as deposits ($\gamma = 0.5$ or $\gamma = 0$), while there is no accumulation of reserves by the central

[13] Hours worked and real money balances also affect the welfare of asset holders. However, they have a relatively small weight.

bank ($\omega = 1$). The solid lines correspond to the reference policy of immediate spending with no accumulation of reserves ($\gamma = 1$ and $\omega = 1$). The surge in aid leads to an increase in government expenditures which raises the demand for goods, with a higher propensity for non-tradable goods. Given the nominal inertia present in the non-traded goods sector, output in this sector expands and generates a rise in GDP, alongside an increase in labor and real wages, as a result of the higher demand for labor. At the same time, we observe a real appreciation of the currency due to the aid inflows and no accumulation of reserves, which negatively affects output in the tradable sector,[14] but allows for an increase in the trade deficit. Overall, private consumption rises initially but private investment falls in the short run mainly because of a substantial increase in the real interest rate.

A share of the higher government expenditures goes towards public investment, while the remaining share takes the form of increased current spending on public goods (g^C). The higher public capital contributes directly to the production of both traded and non-traded goods, but also indirectly, by raising the marginal product of private capital and encouraging private investment. This effect helps mitigate the initial crowding out of private investment, which is seen to rise beyond the initial periods.

Considering the impact of the shock for the different households in our economy, savers see a slight crowding-out of private consumption in the first quarter, which then rises in subsequent periods, while at the same time they increase hours worked (reducing leisure time) and suffer an initial decline in real money balances (due to the rise in inflation that the higher demand creates). For hand-to-mouth consumers, however, consumption is entirely determined by the after-tax wage income and the value of foreign remittances. While the domestic currency appreciation reduces the latter, this negative effect is outweighed by the increase in labor income due to higher real wages. Hence, hand-to-mouth consumers enjoy an increase in consumption when government spending rises. In addition, both types of households also benefit from the higher provision of public goods, which affects their utility directly.

A more gradual spending of aid ($\gamma < 1$) means that both government consumption and investment increase by less in the short run but by relatively more in the medium run, as the accumulated deposits are slowly used over time. The dashed impulse responses in Figures 1 and 2 illustrate the case of $\gamma = 0.5$ and the dash-dot lines the case of $\gamma = 0$.[15] Under a policy of full absorption of aid ($\omega = 1$), there is still a strong real appreciation of the currency and an increase in the trade deficit.

The lower increase in government spending implies more muted effects in the labor market and a slightly smaller increase in labor and output in the initial periods. However, the relative magnitude of these changes is such that there are gains for the private sector under a policy of gradual spending, as crowding-out effects are reduced or even eliminated. Total private investment falls by less or even rises in response to the aid shock, when aid spending is more spread over time (lower γ values). Together with relatively higher public investment over the medium run, this implies a larger accumulation of private and public capital which supports relatively *higher* GDP levels beyond the first periods. Asset holders are generally able to further increase their consumption levels, while also reducing hours worked and enjoying more leisure. Hand-to-mouth consumers see more muted changes in consumption, particularly in the short run, as a reflection of lower demand and reduced real wages.

Overall, a policy that delays aid spending, smoothing it over time, reduces crowding-out effects and leads to largely stronger responses in output, investment, and the consumption of savers, and

[14] Mild learning-by-doing externalities, as assumed in the calibration, worsen slightly these negative effects.

[15] Recall that even when $\gamma = 0$, there is gradual spending, since the spending of aid inflows is also governed by ρ_d. Our baseline sets $\rho_d = 0.9$, and the optimal policy exercise searches for the optimal γ.

is associated with increased volatility in these variables. Figure 7 shows the volatility of a set of variables, as measured by their standard deviation, for different values of the spending parameter γ—the red circles. A more gradual spending of aid (lower γ) raises the volatility of output and savers' private consumption and money holdings, while reducing the volatility of consumption of hand-to-mouth households, government consumption, and hours worked.[16]

In welfare terms, both the variance (stabilization) effect and the mean effect need to be considered. Regarding the variance effect, the increased volatility of private consumption (and money holdings), on the one hand, reduces welfare for asset holders. On the other hand, there are welfare gains from the reduced variability in public goods consumption and a more stable labor supply. Overall, from a stabilization point of view, a gradual adjustment in government spending raises welfare for savers, as indicated by the variance terms $\left(\Xi^{a,V}\right)$ in column 3 of Table 2. When $\gamma = 0.5$, the welfare gains due to reduced volatility are 0.650% (relative to a policy of immediate spending, $\gamma = 1$), while further smoothing the spending process ($\gamma = 0$) raises the welfare benefit to 0.874%. Similar results are obtained for the hand-to-mouth households, who also benefit from a policy of gradual spending of aid through increased stabilization effects, with the highest welfare gains of 0.061% under $\gamma = 0$.

Considering the long-run implications of gradual spending policies, welfare results indicate a positive mean effect. This results from the fact that the increased variability of consumption can raise precautionary savings motives for savers and lead to higher capital accumulation and output over the long run. Through increased tax revenues, the higher output can also support more public spending on consumption goods and investment, which further enhances demand and production capacity. This allows for higher long-run consumption levels. The welfare gains due to changes in means are increasing for more gradual spending policies, reaching up to 0.948% for savers and 0.171% for hand-to-mouth consumers when $\gamma = 0$.[17]

Overall, a policy that gradually adjusts government expenditures when faced with volatile aid revenues enhances households' welfare, through both stabilization and mean effects. Figure 9 provides a graphical illustration of these welfare benefits, which are independent of the central bank's reserve accumulation policy. The figure plots the three welfare measures (Ξ, Ξ^M, and Ξ^V) for the two types of households as γ varies from 0 to 1, and for different degrees of reserve accumulation (ω values). The largest welfare gain is associated with the most gradual adjustment of spending, when $\gamma = 0$, for all consumers.[18]

[16] Fluctuations in real money balances arise primarily from inflation dynamics, but have a relatively small effect on welfare.

[17] Our results highlight the long-run implications of increased volatility and precautionary saving motives, which raise capital investment, output, and consumption. We do not capture the short-run effects of increased uncertainty on investment, which are often discussed in the news and uncertainty literature. Increased unexpected uncertainty can make investment (and labor) decline in the short run (before rebounding to higher levels later), through a real-options effect that makes firms wait and see, pausing hiring and investment (as shown in Bloom (2009) and Bloom et al. (2018), in a model of the firm with fixed labor and investment costs and irreversibility). This uncertainty shock also brings forth volatility effects from expecting future higher fluctuations that dominate in the longer horizon, leading labor and investment to rise. The effects of raised uncertainty and increased precautionary savings may also depend on the nature of the assets into which these savings are channeled—a portfolio reallocation, away from capital and towards relatively safer assets (such as government bonds or foreign assets), may lead to lower capital investment in some cases (see, for example, Moldovan (2010), Fogli and Fabrizio (2015), and Cherif and Hasanov (2018)). As we focus on the role of volatile external resources for government financing and policy, we have abstracted from other sources of financing (such as through borrowing on the domestic or foreign markets).

[18] Our simulations in searching for the optimal spending parameter are conditional on a specific $\rho_d = 0.9$ in the government deposit rule, (6). While we do not consider the optimal value of ρ_d, a higher ρ_d (approaching 1)—implying an even more gradual spending path—should further raise welfare. We ran the model simulations for a

4.1.2 The Role of Reserve Accumulation

We now consider the role of the central bank's reserve accumulation policy in managing the effects of volatile foreign aid revenues. Figures 3 and 4 illustrate the impulse responses to a 1 percent increase in aid for different degrees of reserve accumulation—i.e., different ω values—and given an immediate spending of the aid $(\gamma = 1)$. A full absorption of aid occurs when foreign reserves do not change with changes in aid $(\omega = 1)$.

If the central bank departs from a policy of full absorption by increasing its holdings of foreign reserves to partially or fully reflect the value of aid inflows $(\omega < 1)$, as some central banks do, this then reduces the real appreciation of the domestic currency and limits the expansion of the trade deficit.[19] Given full spending, a smaller increase in trade deficits implies that private demand is reduced. We observe a smaller increase in private consumption (relative to the case of full absorption, $\omega = 1$), and private investment is crowded out by more and for longer, with subsequent negative effects on capital accumulation and output. The dash lines in Figure 3 show the case of $\omega = 0.5$ and the dash-dot lines that of $\omega = 0$. The more contained appreciation of the currency is beneficial for the traded goods sector which contracts by less in the short-run, but detrimental to the non-traded goods sector where output increases by less as a consequence of the relatively lower demand. Overall, there is a more muted response in output, labor, and real wages. The latter implies a much smaller increase in labor income and consumption for the hand-to-mouth consumers (despite a relatively higher value of foreign remittances), as seen in Figure 4.

At the same time, with a lower tax base due to the reserve accumulation policy, tax revenues rise slightly less in equilibrium implying that public investment and public goods consumption are also not as high. Consumption of asset holders is now crowded out by more in the initial periods, before rebounding to relatively lower levels, while the more muted increase in real wages restricts consumption for hand-to-mouth consumers. Thus, although this policy mitigates the Dutch disease effects, it also leads to relative declines in both private and public capital, affecting output and private consumption of all households, while the relatively lower levels of government consumption also affect households' utility.

As in the welfare analysis of public spending policies, we can consider the variance (stabilization) effect and the mean effect. In response to changes in aid, a partial absorption policy contributes to more muted responses in output, consumption (both private and public, and for both types of households), and labor, and is associated with reduced volatility in these variables. The starred lines in Figure 7 show the implied changes in volatility as foreign reserves are adjusted more in response to aid changes (i.e., lower ω values). Table 2 gives the associated welfare gains due to these positive stabilization effects. Conditional on full spending $(\gamma = 1)$, a partial absorption with $\omega = 0.5$ yields welfare gains of 0.420% for savers and 0.053% for hand-to-mouth consumers, increasing to 0.734% and 0.084% when $\omega = 0$. The same effects are also illustrated in Figure 10 which depicts the welfare gains/losses of absorption policies, as ω varies from 0 to 1, for different degrees of spending (γ values).

Considering the mean effect, the reduced uncertainty of partial absorption policies, however, implies less precautionary savings incentives and leads to lower accumulation of capital and lower levels of output and consumption over the long run, while leisure rises. These changes generally

different and higher $\rho_d = 0.95$. Our result that a most gradual spending of aid $(\gamma = 0)$ is optimal continues to hold, and the welfare benefits of even smoother spending through higher ρ_d are enhanced relative to those presented in Table 2. These results are available upon request.

[19] These excess reserves are ultimately slowly reduced, at a rate given by the persistence parameter ρ_{res} set at 0.9 in the calibration.

have a negative impact on welfare, as can be seen in Figure 10, although the relative magnitude of these effects varies slightly across consumers and with the degree of spending.

Hence, partial absorption policies have largely opposing effects on welfare, through welfare gains due to increased stabilization and welfare losses due to lower long-run consumption levels, leading to an overall slightly *non-linear* effect on welfare which differs across households and depends to some extent on the government's spending policy. The optimal degree of absorption is generally higher—i.e., higher ω—for hand-to-mouth consumers and under a policy of delayed spending of aid (low γ).

Overall, taking into account the combined effects of spending and reserve accumulation policies, a very gradual spending of aid flows together with some degree of accumulation of foreign exchange reserves (partial absorption) is optimal, with the optimal policy rule parameters $\gamma = 0$ and $\omega = 0.5$. The optimal degree of absorption reflects an average of a lower optimal level of 0.4 for asset holders and a higher level of 0.6 for hand-to-mouth consumers. This policy mix has similar effects for both types of households in the economy and increases the welfare of savers by 2.143%, relative to the policy of immediate spending and full absorption, and that of hand-to-mouth consumers by 0.264%.

4.2 Optimal Share of Public Investment

Another aspect that we consider is the relative role of public investment. Conditional on the optimal policy of gradual spending and partial absorption of aid ($\gamma = 0, \omega = 0.5$), we explore the relative role of public investment and seek to determine how much of the change in government expenditures, associated with changes in aid, should be allocated to public investment as opposed to public goods consumption. Under the benchmark calibration, the share of total expenditures going to public investment, ϑ, was set to match the average value found in the data, $\vartheta = 0.24$. We now use the same welfare criterion as above to find the optimal value of this share.

Figures 5 and 6 show the impulse responses to an increase in aid for different values of ϑ and Table 3 gives the associated welfare costs/benefits, measured relative to the benchmark. The results suggest that, faced with volatile aid flows, shifting relatively more of the aid towards public investment (increasing ϑ) is beneficial for both types of households, and particularly for hand-to-mouth consumers.

Public capital has a productive role, as direct input in production, but also indirectly by increasing the marginal product of private capital. A higher share of public investment (higher ϑ) then affects the accumulation of both public and private capital and subsequently goods production and household private consumption over the medium to long run. Given an increase in foreign aid, for example, we note larger increases in output and private consumption over longer time horizons, as a reflection of increased accumulation of capital. At the same time, a higher ϑ means that the provision of public goods $\left(g_t^C\right)$ is lower, negatively affecting households' utility. These effects are seen in the impulse responses in Figures 5 and 6. The solid lines depict the responses under the benchmark calibration $\vartheta = 0.24$, the dashed lines are for $\vartheta = 0.6$, while the dash-dot lines for $\vartheta = 0$.

Overall, a higher share of public investment is associated with increased volatility of private consumption but reduced volatility in public goods consumption, as illustrated in Figure 8. This leads to an overall improvement in welfare for asset holders but has a more non-linear and generally worsening effect on hand-to-mouth consumers, as shown by the variance terms $\Xi^{a,V}$ and $\Xi^{h,V}$ in Table 3 and the welfare cost plots in Figure 11. The mean welfare effects, however, suggest a general improvement in welfare, for higher ϑ—the higher variability in private consumption enhances precautionary savings motives, supporting a stronger accumulation of capital and higher long-run output and consumption.

With a relatively strong mean effect, it is generally welfare enhancing to shift relatively more (or all) of public spending towards infrastructure investment—with an optimal share $\vartheta = 0.8$ for asset holders, and $\vartheta = 1$ for hand-to-mouth consumers.

5 Conclusion

This paper addresses the aid volatility issue in low-income developing countries from the perspective of optimal policy rules. It jointly considers a government spending rule in terms of the fraction of aid receipts to be spent immediately and a reserve accumulation rule in terms of the fraction of aid receipts to be accumulated as foreign reserves, as in Berg et al. (2010b). The two policies in the model, as often in reality, need not be coordinated as aid can be used for increasing government spending and for increasing reserve accumulation. The welfare analysis is conducted using a second-order approximation approach of the model and the households' utility function, and considering both stabilization and mean effects on variables, as a result of volatility.

The results show that, in response to volatile aid inflows, the optimal combination is to adjust government spending gradually, while partially accumulating these flows in foreign exchange reserves. Gradual aid spending enhances stability in public goods consumption and labor supply, raising welfare. In addition, more gradual spending increases consumption variability for asset holders, which reinforces precautionary savings motives. This leads to higher investment, capital, output and, therefore, consumption over the long term, which raises welfare through a mean effect. Hence, in the context of volatile aid, gradual spending policies increase welfare through both stabilization and mean effects. Regarding reserve accumulation rules, in contrast, there are opposing effects. Partial absorption (accumulating reserves), on the one hand, has welfare gains from increased stabilization but, on the other hand, has welfare losses from the mean effect due to reduced precautionary saving motives. By considering both effects, the simulation analysis allows us to determine the optimal degree of reserve accumulation of aid flows.

On the optimal spending allocation between utility-generating government consumption and productivity-enhancing public investment, the paper finds that higher productive public investment is generally welfare improving. More public investment directly increases public capital and also crowds in private production factors, raising private consumption over the medium to long run. However, the explanation of why using aid for public investment is welfare improving differs from other studies, such as Berg et al. (2010a). In the context of volatile aid, a higher share of public investment is associated with increased volatility of private consumption but reduced volatility in public goods consumption, which may lead to welfare gains or losses. The mean effects of a higher share, however, point towards welfare improvements and can be strong enough to guarantee an overall gain in welfare, favoring the use of aid in public investment projects.

While the paper focuses on foreign aid, the results can generalize to other kinds of fluctuating external government revenues, such as natural resource revenues, that are also subject to large fluctuations due to volatile commodity prices. The optimal fiscal policy presented here favors gradual spending. This result is consistent with the recommendation of a sustainable investing approach in Berg et al. (2013), which supports more favorable macroeconomic outcomes in the case of large declines in commodity prices than immediate spending on the ground of limited absorptive capacity constraints in LICs.

Table 1: **Baseline Calibration**

parameters	values	
β	0.98	the discount factor
σ	2.94	inverse of intertemporal elasticity of substitution for consumption, savers
ψ	0.5	inverse of Frisch labor elasticity, savers
ξ	5.27	inverse of intertemporal elasticity of substitution for m_t, savers
$\varkappa^G$	0.0051	weight on public goods consumption in utility
φ	0.6	degree of home bias in private consumption and investment
φ^G	0.7	degree of home bias in government consumption and investment
χ	0.44	elasticity of substitution between traded and non-traded goods
θ	6	elasticity of substitution among non-traded goods
f	0.25	fraction of savers
κ	1.4	investment adjustment cost parameter
δ	0.02	depreciation rate of private capital
φ^l	0.59	steady-state labor share in non-traded goods sector
χ^l	0.6	elasticity of substitution between labor of two sectors
z^N	1	TFP in non-traded sector
ρ_{zT}	0.1	AR(1) in traded TFP
ν	0.1	learning-by-doing parameter
α^N	0.45	labor income share in non-traded output
α^T	0.60	labor income share in traded output
α^G	0.11	output elasticity with respect to public capital
δ^G	0.008	depreciation rate of public capital
ζ	56.6	price adjustment cost parameter
ι	0.21	implicit production cost parameter
ϵ	0.4	steady-state public investment efficiency
π	1.02	CPI inflation
ρ_d	0.9	AR(1) coefficient, government deposits rule
ρ_{res}	0.9	AR(1) coefficient, reserves accumulation rule
c/y	82.9%	private consumption to output ratio
i/y	14%	private investment to output ratio
g^C/y	14%	government consumption to output ratio
g^I/y	4.5%	public investment to output ratio
$\frac{b}{4y}$	33.2%	government debt to output ratio, annual rate
$\frac{d}{4y}$	5.7%	government deposits to output ratio
$\frac{s \cdot res^*}{4y}$	18%	foreign reserves to output ratio, annual rate
$\frac{s \cdot a^*}{y}$	5.3%	foreign aid to output ratio

	Asset holders			Hand-to-mouth consumers			Aggregate (% change)
	Ξ^a	$\Xi^{a,M}$	$\Xi^{a,V}$	Ξ^h	$\Xi^{h,M}$	$\Xi^{h,V}$	
	Gradual Spending of Aid						
$\gamma = 0.7 \mid_{\omega=1}$	0.828	0.387	0.440	0.083	0.056	0.027	0.169
$\gamma = 0.5 \mid_{\omega=1}$	1.251	0.597	0.650	0.132	0.092	0.041	0.267
$\gamma = 0.2 \mid_{\omega=1}$	1.683	0.838	0.836	0.196	0.141	0.055	0.391
$\gamma = \mathbf{0} \mid_{\omega=1}$	**1.831**	**0.948**	**0.874**	**0.232**	**0.171**	**0.061**	**0.459**
	Partial Absorption						
$\omega = 0.7 \mid_{\gamma=1}$	0.332	0.061	0.264	0.055	0.020	0.034	0.107
$\omega = 0.5 \mid_{\gamma=1}$	0.473	0.041	0.420	0.075	0.021	0.053	0.146
$\omega = 0.2 \mid_{\gamma=1}$	0.562	−0.081	0.621	0.079	0.005	0.074	0.156
$\omega = 0 \mid_{\gamma=1}$	0.539	−0.224	0.734	0.065	−0.018	0.084	0.131
	Gradual Spending and Partial Absorption						
$\gamma = \mathbf{0},\ \omega = \mathbf{0.5}$	**2.143**	**0.944**	**1.173**	**0.264**	**0.164**	**0.099**	**0.524**

Table 2: **Welfare changes of different fiscal spending and reserve accumulation policies.** The changes are relative to the reference policy of immediate spending of aid and full absorption (values are in percentage points). For each household type, the first column gives the overall welfare benefit/cost ($\Xi^a,\ \Xi^h$), the second column the corresponding effects due to changes in means ($\Xi^{a,M},\ \Xi^{h,M}$), and the third column those due to variance changes ($\Xi^{a,V},\ \Xi^{h,V}$). These are measured as shares of the consumption stream under the reference policies of immediate spending with full absorption ($\gamma = 1,\ \omega = 1$). The last column gives the simple percent change in aggregate welfare (where aggregate welfare is computed as the weighed average of welfare measures across the two types of households).

	Asset holders			Hand-to-mouth consumers			Aggregate (% change)
	Ξ^a	$\Xi^{a,M}$	$\Xi^{a,V}$	Ξ^h	$\Xi^{h,M}$	$\Xi^{h,V}$	
$\vartheta = 0$	−0.443	−0.165	−0.266	−0.308	−0.299	−0.008	−0.569
$\vartheta = 0.4$	0.226	0.081	0.138	0.249	0.248	0.002	0.455
$\vartheta = 0.6$	0.422	0.143	0.264	0.634	0.636	−0.002	1.146
$\vartheta = \mathbf{0.8}$	**0.505**	**0.151**	**0.335**	1.135	1.151	−0.014	2.032
$\vartheta = \mathbf{1}$	0.446	0.081	0.345	**1.816**	**1.861**	**−0.041**	**3.212**

Table 3: **Welfare changes of different shares of public investment in total government expenditures.** The changes are relative to the benchmark value of $\vartheta = 0.24$, under the optimal policy of gradual spending of aid and partial absorption (values are in percentage points). See Table 2 for notes explaining each column.

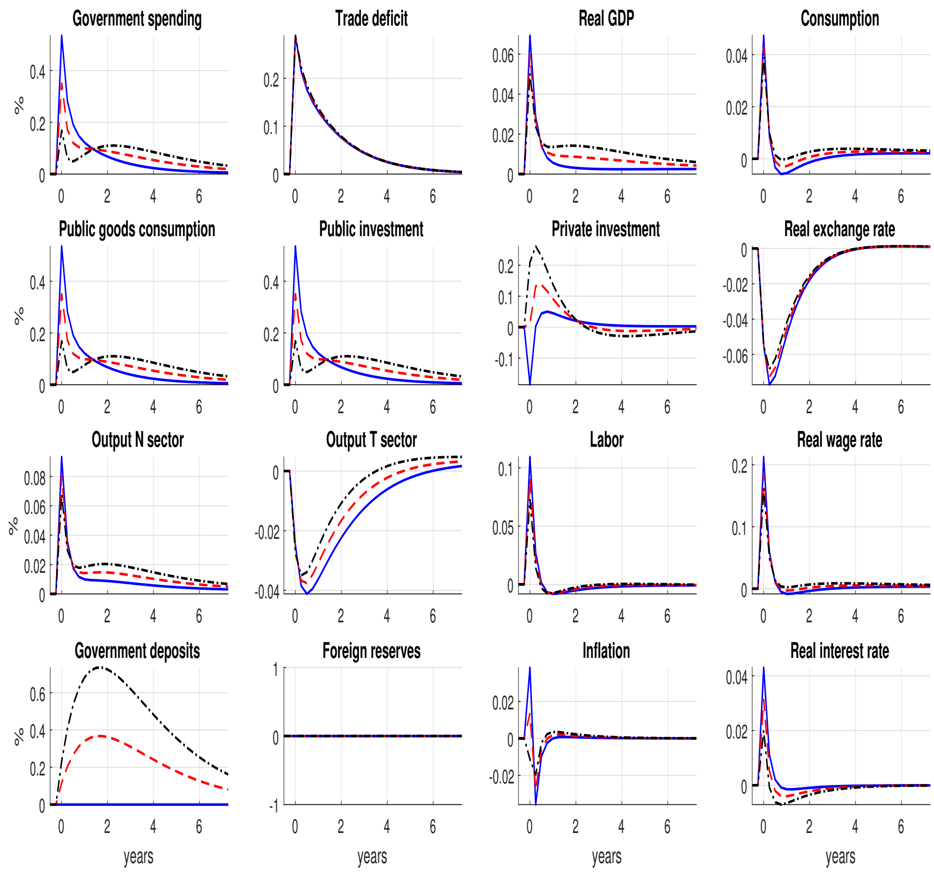

Figure 1: Impulse responses of aggregate variables for varying degrees of spending of aid, under full absorption ($\omega = 1$): immediate spending $\gamma = 1$ (solid lines) and more gradual spending $\gamma = 0.5$ (dash lines) and $\gamma = 0$ (dash-dot lines). Inflation and the interest rate are in annualized terms.

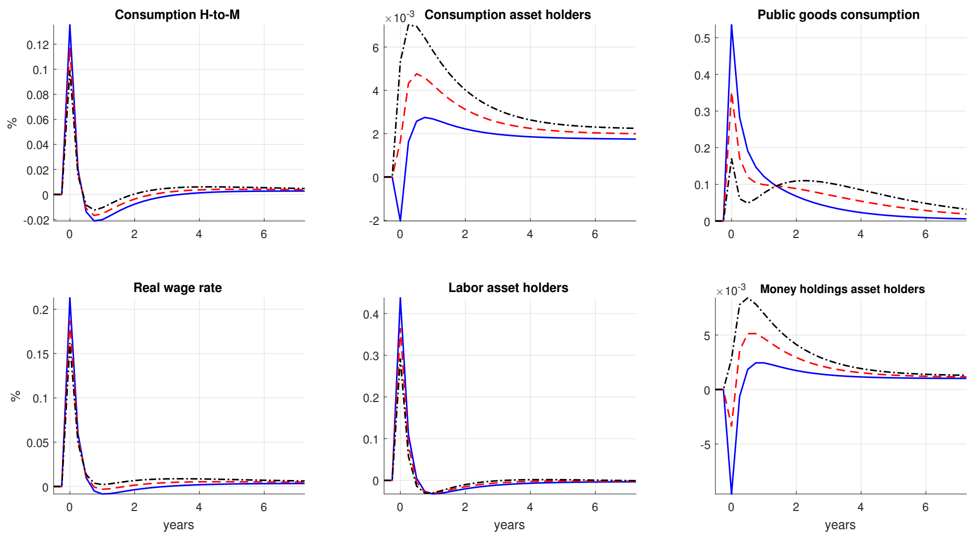

Figure 2: Impulse responses of household-specific variables for varying degrees of spending of aid, under full absorption ($\omega = 1$): immediate spending $\gamma = 1$ (solid lines) and more gradual spending $\gamma = 0.5$ (dash lines) and $\gamma = 0$ (dash-dot lines). 'H-to-M' refers to hand-to-mouth consumers.

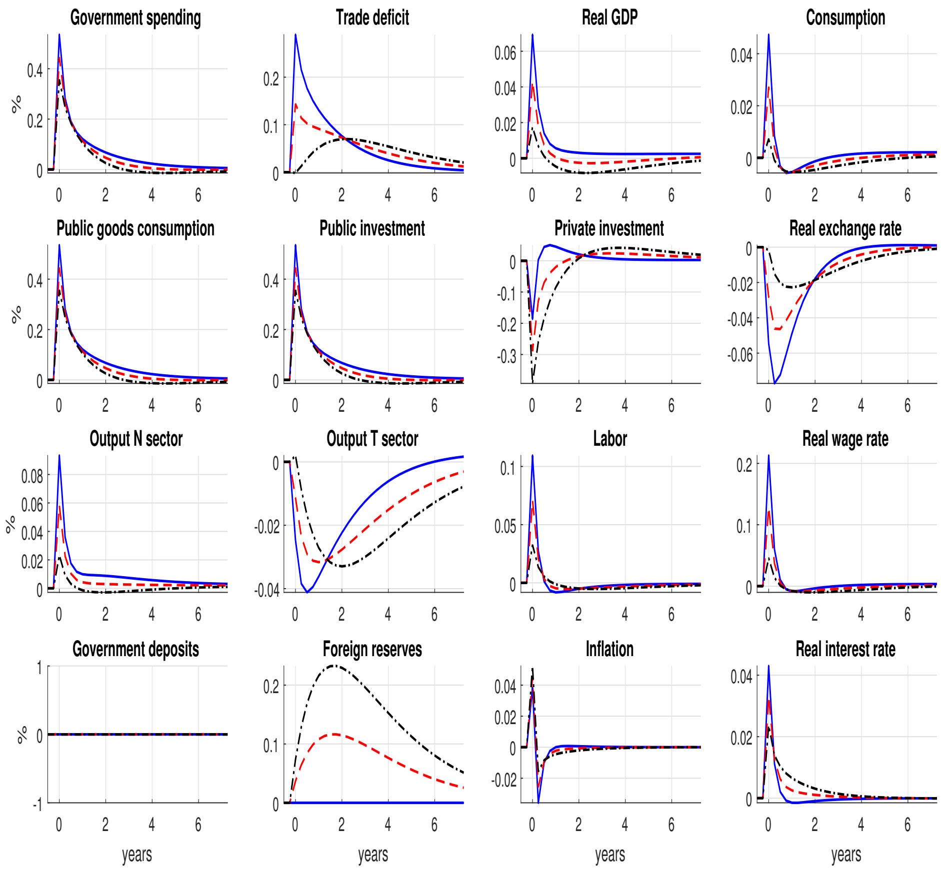

Figure 3: Impulse responses of aggregate variables for varying degrees of absorption (reserves accumulation) of aid, under immediate spending ($\gamma = 1$): full absorption $\omega = 1$ (solid lines) and partial absorption $\omega = 0.5$ (dash lines) and $\omega = 0$ (dash-dot lines). Inflation and the interest rate are in annualized terms.

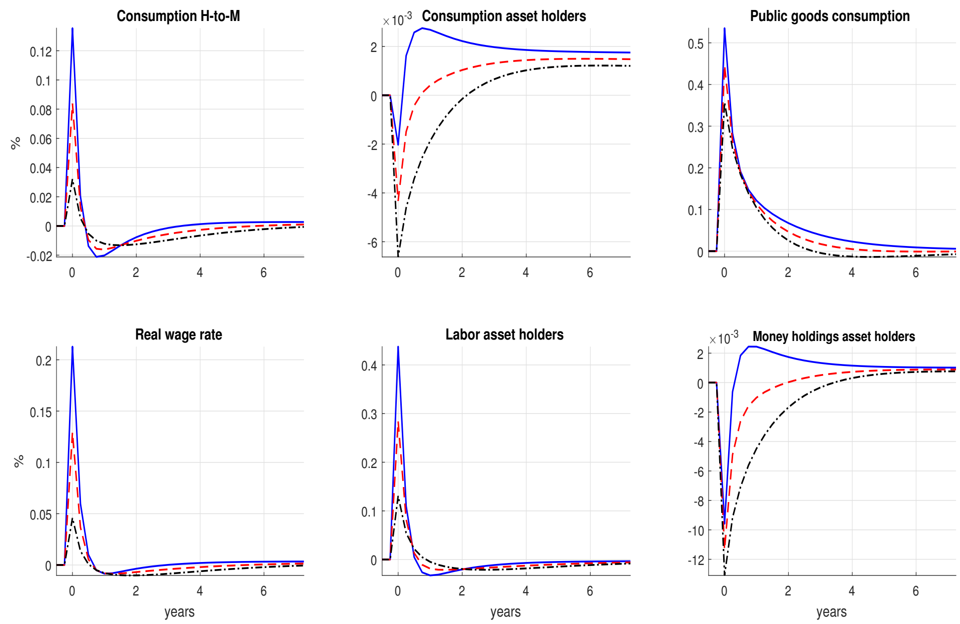

Figure 4: Impulse responses of household-specific variables for varying degrees of absorption (reserves accumulation) of aid, under immediate spending ($\gamma = 1$): full absorption $\omega = 1$ (solid lines) and partial absorption $\omega = 0.5$ (dash lines) and $\omega = 0$ (dash-dot lines). 'H-to-M' refers to hand-to-mouth consumers.

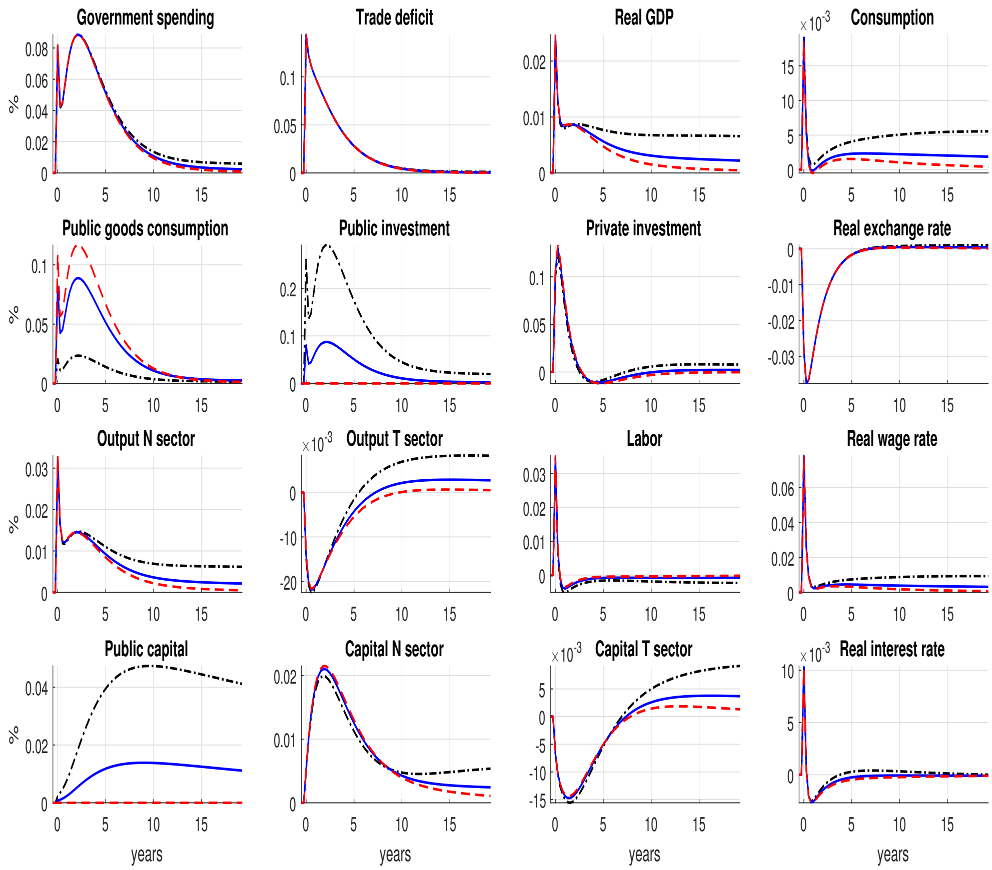

Figure 5: Impulse responses of aggregate variables for different shares of public investment in total government expenditures: $\vartheta = 0$ (dash lines), $\vartheta = 0.24$ (calibration value, solid lines), and $\vartheta = 0.8$ (dash-dot lines), under the optimal policy of gradual spending of aid, with partial absorption ($\gamma = 0$, $\omega = 0.5$). Inflation and the interest rate are in annualized terms.

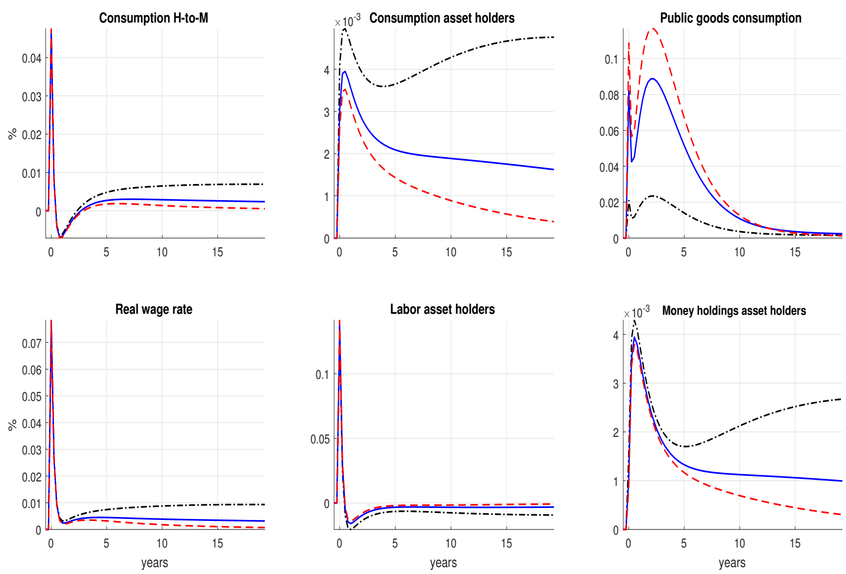

Figure 6: Impulse responses of household-specific variables for different shares of public investment in total government expenditures: $\vartheta = 0$ (dash lines), $\vartheta = 0.24$ (calibration value, solid lines), and $\vartheta = 0.8$ (dash-dot lines), under the optimal policy of gradual spending of aid, with partial absorption. 'H-to-M' refers to hand-to-mouth consumers.

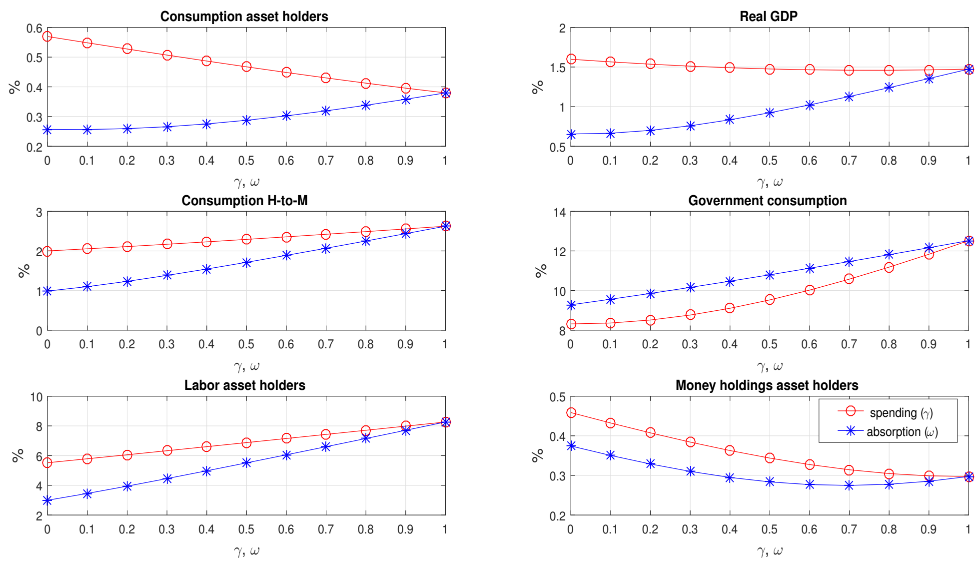

Figure 7: Volatility of selected variables for: (i) different degrees of spending of aid (varying γ), given full absorption—red circles; and (ii) varying degrees of absorption (varying ω), given immediate spending—blue stars. Volatility is measured as the standard deviation of fluctuations around the long-run average. 'H-to-M' refers to hand-to-mouth consumers.

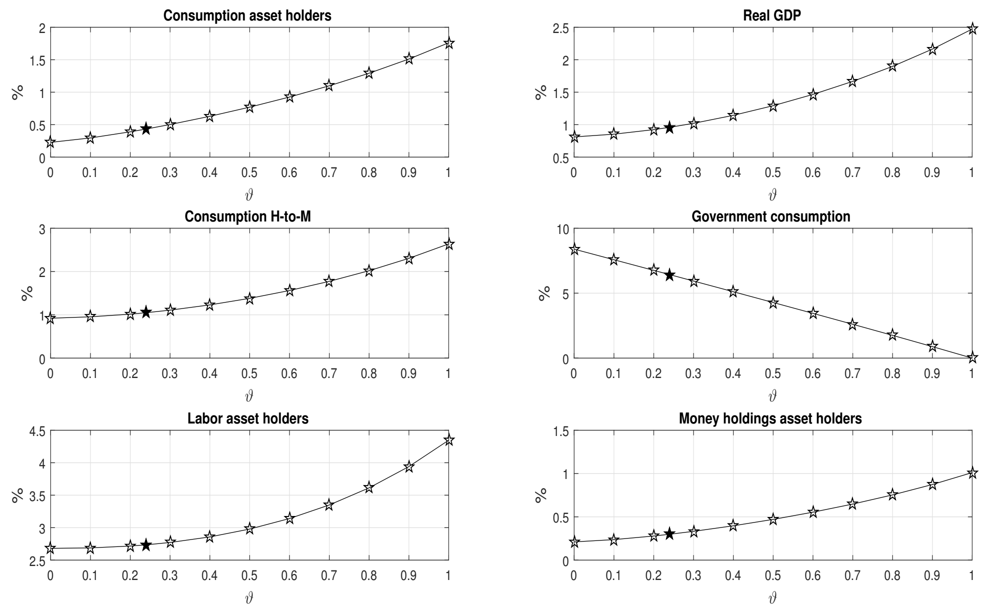

Figure 8: Volatility of selected variables for different shares of public investment in total government expenditures relative to the benchmark value of $\vartheta = 0.24$, under the optimal policy of gradual spending of aid, with partial absorption. Volatility is measured as the standard deviation of fluctuations around the long-run average. 'H-to-M' refers to hand-to-mouth consumers.

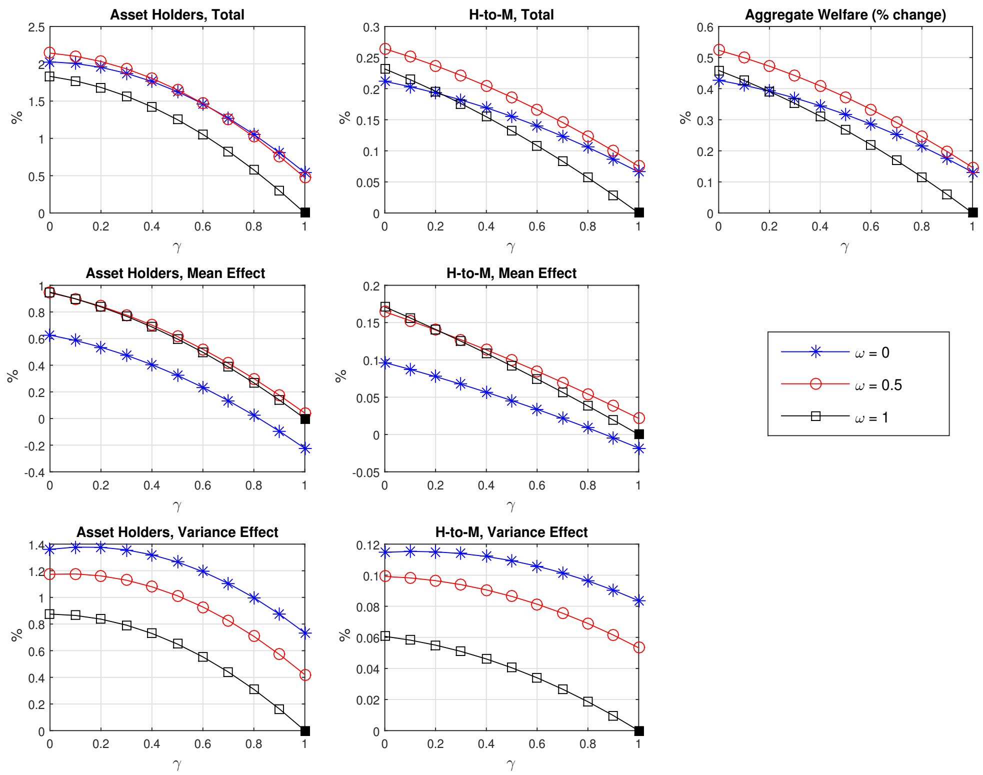

Figure 9: Welfare benefits/costs of gradual spending of aid flows for $0 \geq \gamma \geq 1$ under different degrees of absorption (reserves accumulation): full absorption ($\omega = 1$, squares) and partial absorption ($\omega = 0.5$, circles, and $\omega = 0$, stars). Welfare costs are measured relative to the policy of immediate spending and full absorption of aid ($\gamma = 1$, $\omega = 1$), denoted by the solid black square.

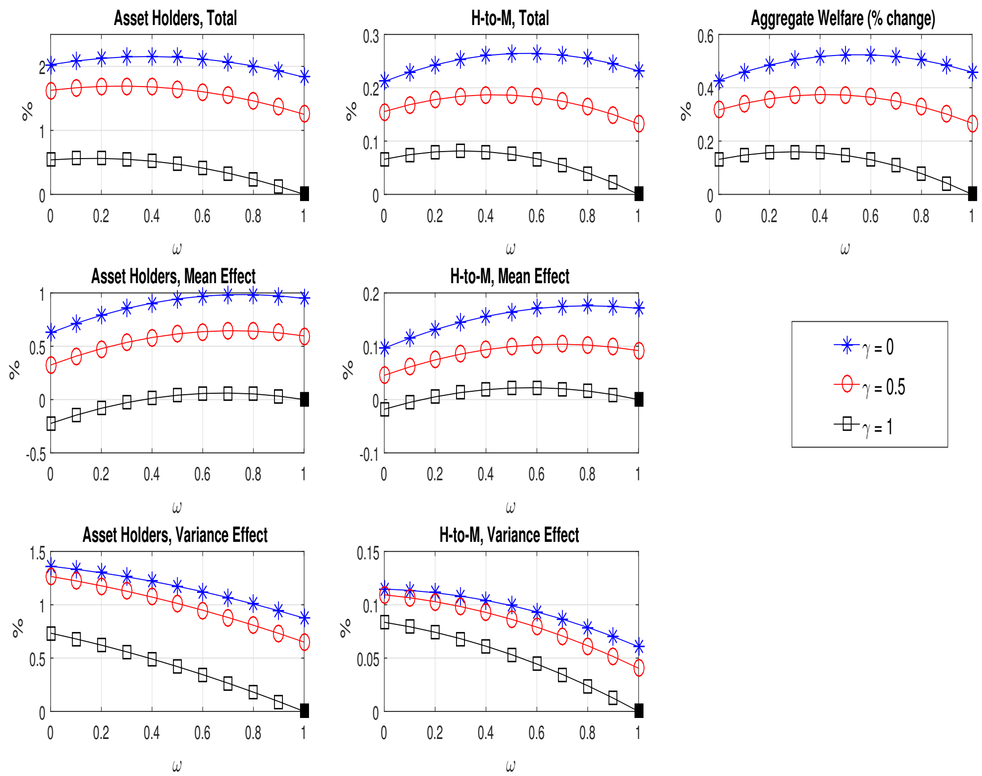

Figure 10: Welfare benefits/costs of reserve accumulation policies (aid absorption) - ω falls below 1, for different degrees of spending of aid: immediate spending ($\gamma = 1$, squares) and more gradual spending ($\gamma = 0.5$, circles, and $\gamma = 0$, stars). Welfare costs are measured relative to the policy of immediate spending and full absorption of aid ($\gamma = 1$, $\omega = 1$), denoted by the solid black square.

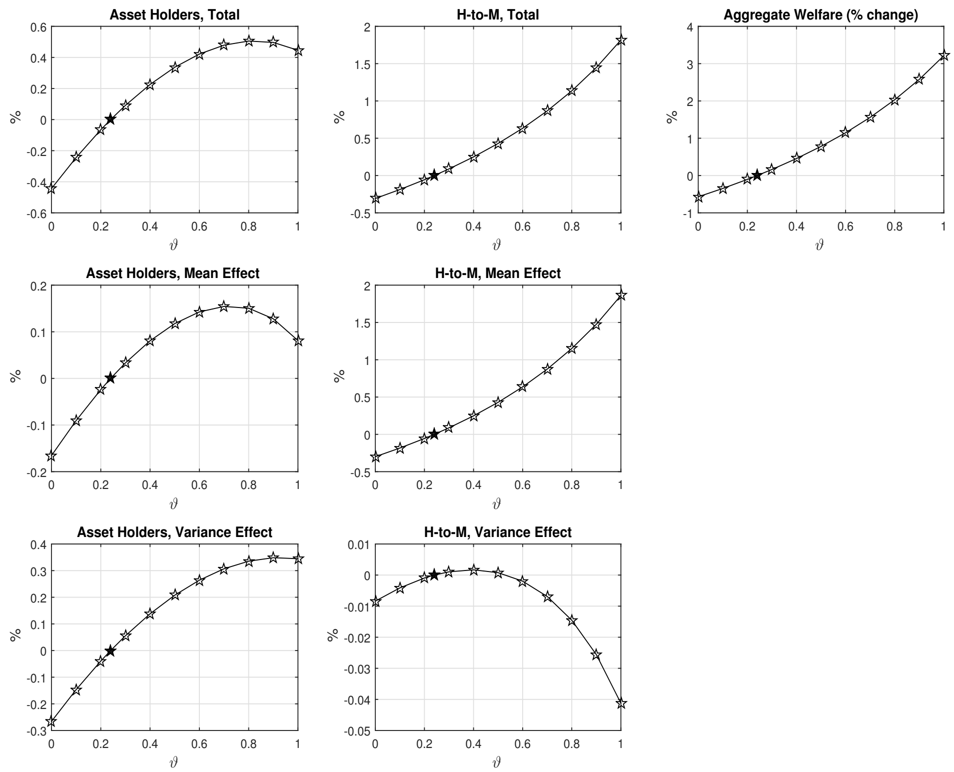

Figure 11: Welfare benefits/costs of different shares of public investment in total government expenditures, under the optimal fiscal and reserve accumulation policies. Welfare costs are measured relative to the benchmark calibration value of $\vartheta = 0.24$.

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A Equilibrium Conditions

This appendix provides the equilibrium conditions of the model. When solving the savers' utility maximization problem, let λ_t^a be the Lagrangian multiplier for the budget constraint and Q_t^N and Q_t^T the Lagrangian multipliers for the laws of motion of capital in the two production sectors. Then, Tobin's Q for k_t^N and k_t^T are $q_t^N \equiv \frac{Q_t^N}{\lambda_t^a}$ and $q_t^T \equiv \frac{Q_t^T}{\lambda_t^a}$.

A.1 Households

- First order condition (FOC) for private consumption asset holders, c_t^a

$$\lambda_t^a = (c_t^a)^{-\sigma} \tag{A.1}$$

- Labor supply asset holders, l_t^a

$$(l_t^a)^{\psi} = \lambda_t^a w_t (1-\tau) \tag{A.2}$$

- Euler equation

$$\lambda_t^a = \beta E_t \left(\lambda_{t+1}^a \frac{R_t}{\pi_{t+1}} \right) \tag{A.3}$$

- Money demand, m_t^a

$$(m_t^a)^{-\xi} = \lambda_t^a \left(\frac{R_t - 1}{R_t} \right) \tag{A.4}$$

- Hand-to-mouth consumers' budget constraint

$$c_t^h = (1-\tau) w_t l^h + s_t r m^* \tag{A.5}$$

- Households' FOC for capital non-traded sector, k_t^N

$$q_t^N = \beta E_t \frac{\lambda_{t+1}^a}{\lambda_t^a} \left[(1-\tau) r_{t+1}^N - \frac{\kappa}{2} \left(\frac{i_{t+1}^N}{k_t^N} - \delta \right)^2 + \kappa \left(\frac{i_{t+1}^N}{k_t^N} - \delta \right) \left(\frac{i_{t+1}^N}{k_t^N} \right) + q_{t+1}^N (1-\delta) \right] \tag{A.6}$$

- Households' FOC for capital traded sector, k_t^T

$$q_t^T = \beta E_t \frac{\lambda_{t+1}^a}{\lambda_t^a} \left[(1-\tau) r_{t+1}^T - \frac{\kappa}{2} \left(\frac{i_{t+1}^T}{k_t^T} - \delta \right)^2 + \kappa \left(\frac{i_{t+1}^T}{k_t^T} - \delta \right) \left(\frac{i_{t+1}^T}{k_t^T} \right) + q_{t+1}^T (1-\delta) \right] \tag{A.7}$$

- Households' FOC for investment non-traded sector, i_t^N

$$q_t^N = 1 + \kappa \left(\frac{i_t^N}{k_{t-1}^N} - \delta \right) \tag{A.8}$$

- Households' FOC for investment traded sector, i_t^T

$$q_t^T = 1 + \kappa \left(\frac{i_t^T}{k_{t-1}^T} - \delta \right) \tag{A.9}$$

- Capital accumulation non-traded sector, k_t^N

$$k_t^N = (1-\delta)k_{t-1}^N + i_t^N \tag{A.10}$$

- Capital accumulation traded sector, k_t^T

$$k_t^T = (1-\delta)k_{t-1}^T + i_t^T \tag{A.11}$$

- Labor supplied to the non-traded goods sector

$$l_t^N = \varphi^l \left(\frac{w_t^N}{w_t}\right)^{\chi^l} l_t \tag{A.12}$$

- Labor supplied to the traded goods sector

$$l_t^T = \left(1-\varphi^l\right) \left(\frac{w_t^T}{w_t}\right)^{\chi^l} l_t \tag{A.13}$$

A.2 Non-Traded Goods Firms

- Production of nontradables

$$y_t^N = z^N \left(l_t^N\right)^{\alpha^N} \left(k_{t-1}^N\right)^{1-\alpha^N} \left(k_{t-1}^G\right)^{\alpha^G} \tag{A.14}$$

- Pricing condition

$$\Pi_t^N = \beta E_t \left(\frac{\lambda_{t+1}^a}{\lambda_t^a} \Pi_{t+1}^N \frac{y_{t+1}^N}{y_t^N} \frac{p_{t+1}^N}{p_t^N}\right) + \frac{\theta}{\alpha^N \zeta \left(1-\iota\right)} \frac{w_t^N l_t^N}{p_t^N y_t^N} + \frac{1-\theta}{\zeta} \tag{A.15}$$

where $\Pi_t^N \equiv \frac{\pi_t^N}{\pi^N} \left(\frac{\pi_t^N}{\pi^N} - 1\right)$.

- Capital-labor ratio in non-traded goods production

$$(1-\alpha^N) w_t^N l_t^N = \alpha^N r_t^N k_{t-1}^N \tag{A.16}$$

A.3 Traded Goods Firms

- Production of tradables

$$y_t^T = z_t^T \left(l_t^T\right)^{\alpha^T} \left(k_{t-1}^T\right)^{1-\alpha^T} \left(k_{t-1}^G\right)^{\alpha^G} \tag{A.17}$$

- Labor demand tradable sector, l_t^T

$$w_t^T l_t^T = \left(1-\iota\right) s_t \alpha^T y_t^T \tag{A.18}$$

- Demand for capital tradable sector k_t^T

$$r_t^T k_{t-1}^T = \left(1-\iota\right) s_t (1-\alpha^T) y_t^T \tag{A.19}$$

- Time-varying TFP process

$$\log\left(\frac{z_t^T}{z^T}\right) = \rho_{zT} \log\left(\frac{z_{t-1}^T}{z^T}\right) + \varkappa \log\left(\frac{y_{t-1}^T}{y^T}\right) \tag{A.20}$$

A.4 The Government

- Government budget constraint

$$tax_t + \left(b_t^c + b_t^{cb}\right) + s_t a_t^* = p_t^G \left(g_t^C + g_t^I\right) + \frac{R_{t-1} b_{t-1}^c}{\pi_t} + \frac{b_{t-1}^{cb}}{\pi_t} \tag{A.21}$$

- Total tax receipts

$$tax_t = \tau \left(w_t l_t + r_t^N k_{t-1}^N + r_t^T k_{t-1}^T\right) \tag{A.22}$$

- Foreign aid process

$$\log\left(\frac{a_t^*}{a^*}\right) = \rho_a \log\left(\frac{a_{t-1}^*}{a^*}\right) + \varepsilon_t \tag{A.23}$$

- Total government debt

$$\overline{b} = b_t^c + b_t^{cb} \tag{A.24}$$

- Total government purchases

$$g_t = g_t^I + g_t^C \tag{A.25}$$

- Public investment rule

$$\left(g_t^I - g^I\right) = \vartheta \left(g_t - g\right) \tag{A.26}$$

- Law of motion for public capital

$$k_t^G = \left(1 - \delta^G\right) k_{t-1}^G + \epsilon_t g_t^I \tag{A.27}$$

- Public investment efficiency

$$\epsilon_t = \epsilon \frac{g^I}{g_t^I} + \tilde{\epsilon} \frac{g_t^I - g^I}{g_t^I} \tag{A.28}$$

- Government deposits process

$$d_t - d = \rho_d \left(d_{t-1} - d\right) + (1 - \gamma) s_t \left(a_t^* - a^*\right) \tag{A.29}$$

A.5 The Central Bank:

- Central bank's balance sheet:

$$m_t - \frac{m_{t-1}}{\pi_t} + \left(d_t - \frac{d_{t-1}}{\pi_t}\right) = b_t^{cb} - \frac{b_{t-1}^{cb}}{\pi_t} + s_t \left(res_t^* - \frac{res_{t-1}^*}{\pi^*}\right) \tag{A.30}$$

- Reserves accumulation process

$$res_t^* - res^* = \rho_{res} \left(res_{t-1}^* - res^*\right) + (1 - \omega) \left(a_t^* - a^*\right) - \omega_s \left(\pi_t^S - \pi^S\right) \tag{A.31}$$

- Monetary policy rule under full sterilization:

$$m_t = \mu \frac{m_{t-1}}{\pi_t} \tag{A.32}$$

A.6 Aggregate Constraints and Definitions:

- Aggregate consumption:

$$c_t = fc_t^a + (1-f)c_t^h \tag{A.33}$$

- Aggregate real money balances

$$m_t = fm_t^a \tag{A.34}$$

- Aggregate labor

$$l_t = fl_t^a + (1-f)l^h \tag{A.35}$$

- Real wage index

$$w_t l_t = w_t^N l_t^N + w_t^T l_t^T \tag{A.36}$$

- Total private investment

$$i_t = i_t^N + i_t^T \tag{A.37}$$

- Aggregate output

$$y_t = p_t^N y_t^N + s_t y_t^T \tag{A.38}$$

- Market clearing condition for non-traded goods

$$y_t^N \quad = \quad \left(p_t^N\right)^{-\chi} d_t^N \tag{A.39}$$

$$where \quad : \quad d_t^N = \varphi\left(c_t + i_t + ac_t^i + ac_t^p\right) + \varphi^G (p_t^G)^{\chi} g_t$$

and $ac_t^i = \frac{\kappa}{2}\left[\left(\frac{i_t^N}{k_{t-1}^N} - \delta\right)^2 k_{t-1}^N + \left(\frac{i_t^T}{k_{t-1}^T} - \delta\right)^2 k_{t-1}^T\right]$ and $ac_t^p = \frac{\zeta}{2}\left(\frac{\pi_t^N}{\pi^N} - 1\right)^2 p_t^N y_t^N$ are the investment and price adjustment costs, respectively.

- Current account deficit

$$ca_t^d = c_t + i_t + p_t^G g_t + ac_t^i + ac_t^p - y_t - s_t rm^* \tag{A.40}$$

- Balance of payment

$$ca_t^d = s_t\left(a^* - res_t^* + \frac{res_{t-1}^*}{\pi^*}\right) \tag{A.41}$$

- The relative price of consumption goods

$$1 = \left[\varphi\left(p_t^N\right)^{1-\chi} + (1-\varphi)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{A.42}$$

- The relative price of government purchases

$$p_t^G = \left[\varphi^G\left(p_t^N\right)^{(1-\chi)} + \left(1-\varphi^G\right)(s_t)^{1-\chi}\right]^{\frac{1}{1-\chi}} \tag{A.43}$$

- Inflation non-traded goods

$$\pi_t^N = \frac{p_t^N}{p_{t-1}^N}\pi_t \tag{A.44}$$

- Real interest rate

$$r_t = E_t\left(\frac{R_t}{\pi_{t+1}}\right) \tag{A.45}$$

B Calibrating the Aid Process

To calibrate the aid process, we use data of 38 LICs and lower-middle income countries in SSA. Aid is measured by the net official development assistance and official aid received in current US dollars from World Development Indicator (World Bank (2018)). Net official development assistance consists of disbursements of loans made on concessional terms (net of repayments of principal) and grants.

For each country, the data series is divided by the sample mean and taken logarithm to covert to percent deviation from the sample mean. Then, each converted series is fitted to an AR(1) process as specified in (23). The AR(1) coefficient ρ_a and standard deviation of the aid shocks σ_a are the averages of the estimates for all countries in the sample.

C Approximation of the Utility Function

Period-t utility is approximated by a second-order Taylor expansion around the deterministic steady state and, accounting for the fact that cross-products between private and public goods consumption, labor and real money balances are zero, we obtain the following expression (which we present for the more general specification of utility, which pertains to the savers in our economy. For the utility of hand-to-mouth consumers, only the terms in consumption remain, since these consumers do not hold money and supply labor inelastically):

$$\begin{aligned} U\left(c_t, l_t, m_t, g_t^C\right) \approx\ & \overline{U} + (U_c\ c)\frac{dc_t}{c} + (U_l l)\frac{dl_t}{l} + (U_m m)\frac{dm_t}{m} + \left(U_{g^C}\ g^C\right)\frac{dg_t^C}{g^C} \\ & + \frac{1}{2}\left[\begin{array}{c} \left(U_{cc}\ c^2\right)\left(\frac{dc_t}{c}\right)^2 + \left(U_{ll} l^2\right)\left(\frac{dl_t}{l}\right)^2 + \left(U_{mm} m^2\right)\left(\frac{dm_t}{m}\right)^2 \\ + \left(U_{g^C g^C}\ \left(g^C\right)^2\right)\left(\frac{dg_t^C}{g^C}\right)^2 \end{array}\right] \end{aligned}$$

The algebraic percent changes are then approximated by a second-order expansion in terms of logarithmic changes

$$\frac{dx_t}{x} \approx \widehat{x}_t + \frac{1}{2}\widehat{x}_t^2 \qquad where: \widehat{x}_t \equiv \ln x_t - \ln x$$

Making this substitution and keeping terms of order $O(2)$ and lower, the momentary utility has the following approximation:

$$\begin{aligned} U\left(c_t, l_t, m_t, g_t^C\right) \approx\ & \overline{U} + (U_c\ c)\,\widehat{c}_t + (U_l l)\,\widehat{l}_t + (U_m m)\,\widehat{m}_t + \left(U_{g^C}\ g^C\right)\widehat{g}_t^C \\ & + \frac{1}{2}\left\{\begin{array}{c} \left(U_c\ c + U_{cc}\ c^2\right)\left(\widehat{c}_t\right)^2 + \left(U_l l + U_{ll} l^2\right)\left(\widehat{l}_t\right)^2 + \left(U_m m + U_{mm} m^2\right)\left(\widehat{m}_t\right)^2 \\ + \left[U_{g^C}\ g^C + U_{g^C g^C}\ \left(g^C\right)^2\right]\left(\widehat{g}_t^C\right)^2 \end{array}\right\} \end{aligned}$$

Given the functional form adopted here, the approximation becomes:

$$\begin{aligned} U\left(c_t, l_t, m_t, g_t^C\right) \approx\ & \overline{U} + \left(c^{1-\sigma}\right)\widehat{c}_t - \left(l^{1+\psi}\right)\widehat{l}_t + \left(m^{1-\xi}\right)\widehat{m}_t + \varkappa^G\left(g^C\right)^{1-\sigma}\widehat{g}_t^C \\ & + \frac{1}{2}\left\{\begin{array}{c} \left[(1-\sigma)\,c^{1-\sigma}\right]\left(\widehat{c}_t\right)^2 - \left[(1+\psi)\,l^{1+\psi}\right]\widehat{l}_t^2 + \left[(1-\xi)\,m^{1-\xi}\right]\widehat{m}_t^2 \\ + \left[\varkappa^G(1-\sigma)\left(g^C\right)^{1-\sigma}\right]\left(\widehat{g}_t^C\right)^2 \end{array}\right\} \end{aligned}$$