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Optimal Macroprudential Policy and Asset Price Bubbles

by Nina Biljanovska, Lucyna Górnicka, and Alexandros Vardoulakis

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I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

Monetary and Capital Markets Department and Research Department

Optimal Macroprudential Policy and Asset Price Bubbles

Prepared by Nina Biljanovska, Lucyna Górnicka, and Alexandros Vardoulakis*

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Abstract

An asset bubble relaxes collateral constraints and increases borrowing by credit-constrained agents. At the same time, as the bubble deflates when constraints start binding, it amplifies downturns. We show analytically and quantitatively that the macroprudential policy should optimally respond to building asset price bubbles non-monotonically depending on the underlying level of indebtedness. If the level of debt is moderate, policy should accommodate the bubble to reduce the incidence of a binding collateral constraint. If debt is elevated, policy should lean against the bubble more aggressively to mitigate the pecuniary externalities from a deflating bubble when constraints bind.

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Contents

Page

1. Introduction.....	1
2. Literature.....	6
3. Model Economy.....	8
3.1. Small open economy with an asset price bubble	8
4. Optimal Macroprudential Policy.....	15
4.1. Time-consistent planner's problem	20
4.2. Optimal tax rate in the presense of a bubble	22
5. Quantitative Analysis.....	22
5.1. Calibration.....	22
5.2. Numerical results	24
6. Conclusions.....	30
References.....	30
Appendices.....	34

Figures

1. U.S. Credit Imbalances and Asset Price Overvaluations.....	2
2. Policy rules for borrowing and asset prices in the presence and absence of a bubble	25
3. Optimal tax on borrowing in the presence of an asset price bubble	19
4. Model simulations: Responses of key variables to a binding credit constraint	28
5. Model simulations: Social Planner's allocations.....	29

Table

1. Calibration.....	23
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1 Introduction

In the aftermath of the global financial crisis (GFC), policymakers and academics widely shared the view that policy should lean against financial imbalances. As a result, new macroprudential instruments to tackle credit imbalances have been introduced in many countries, including counter-cyclical capital buffers for banks and loan-to-value ratios for housing loans.¹

The literature has shown that surges in credit are often accompanied by hikes in asset prices, and that the interplay between the two may have large economic effects. For example, Mishkin (2011) and Jordà, Schularick and Taylor (2015) argue that credit-fueled asset price bubbles are more dangerous to financial stability and economic growth than bubbles not followed by debt build-ups. Fostel and Geanakoplos (2008) and Adrian and Shin (2009) show that the feedback loop between asset prices and credit can lead to procyclical leverage and to financial instability. Yet, despite the growing empirical evidence, there is no broadly accepted view on whether macroprudential policy should respond to asset price *beyond* its response to tackle credit imbalances (Barlevy, 2018). One reason for this may have been the modeling difficulties with incorporating asset price bubbles into theoretical models that are suitable for studying normative questions and conducting optimal policy analysis.²

We contribute to this discussion by proposing a theoretical framework that integrates a meaningful policy analysis into a model of asset price overvaluations. We use this framework to study the following questions. Should macroprudential policy respond to asset price overvaluations over and beyond its role in tackling excessive levels of credit?³ If yes, should macroprudential policy be more aggressive or more accommodative? Finally, to what extent does the optimal policy response to asset price overvaluations depend on the level of debt in the economy?

These considerations are not only of theoretical interest, but also of practical relevance as financial imbalances preceding crisis episodes are not always accom-

¹By credit imbalances we mean elevated levels of debt.

²An additional factor seems to be timely and precise identification of overvaluations. Concerns about the quality of existing tests for asset overvaluations have made many policymakers reluctant to automatically react to rapid asset price growth. This issue is, arguably, less concerning in the aftermath of the GFC, as policymakers and academics have placed more effort on detecting “valuation pressures” in asset prices. Some examples include the Office of Financial Research’s Financial System Vulnerabilities Monitor and the Shiller’s CAPE index (Cyclically Adjusted Price Earnings).

³We define asset price overvaluation as a positive deviation of the market price from its fundamental value. In the rest of the paper, we will use the terms asset price bubbles and price overvaluations interchangeably.

panied by overvaluations in asset prices. Figure 1 plots the credit-to-GDP gap (a measure of credit imbalances) and a composite index of house and equity price overvaluations for the United States. Credit imbalances were excessive both before the 1987 and the 2008 systemic crises, but only in the latter case they were accompanied by overvalued asset prices. Moreover, asset price overvaluations can emerge even when credit is subdued, such as before the dot-com bubble in the early 2000s and—more strikingly—since 2014.

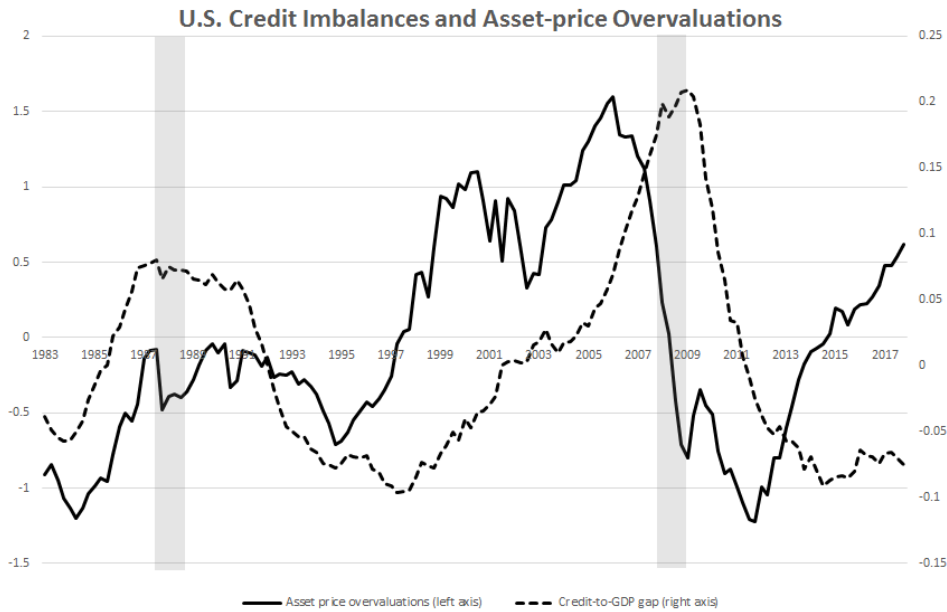


Figure (1) U.S. Credit Imbalances and Asset Price Overvaluations.

Note: The figure plots the credit-to-GDP gap and a measure of asset-price overvaluations for the U.S. from 1983 to 2018. The shaded areas correspond to the systemic crisis of 1987 and 2008 identified in Laeven and Valencia (2012). The credit-to-GDP gap is computed based on a smooth trend obtained through the Hodrick-Prescott filter with a smoothing parameter $\lambda = 400,000$ (after Basel Committee’s guidelines for setting the counter-cyclical capital buffer). Large positive (negative) values indicate excessive (subdued) credit. The asset price overvaluations index is a composite of overvaluations in equity and house prices computed as the percentiles of the price-to-earnings and price-to-rent ratios in the historical distributions for S&P 500 firms and the nationwide housing market, respectively. Note that the asset overvaluation measure assumes that price-to-earnings and price-to-rent ratios are stationary. However, these ratios may exhibit a trend within our sample. Thus, in our quantitative analysis, we calibrate our model using the identification of bubbles in Jorda, Schularick and Taylor, (2015), who take deviations of asset prices from the HP-filtered trend.

To answer the aforementioned questions, building on the work of Bianchi and Mendoza (2018) (henceforth Bianchi-Mendoza) and Miao and Wang (2018) (henceforth, Miao-Wang), we develop a dynamic stochastic general equilibrium model

with an occasionally binding collateral constraint and a rational stock price bubble. We then solve for the optimal time-consistent macroprudential policy of a planner who cannot commit to future policies, and derive analytically the policy instrument in the form of a tax on (new) borrowing that decentralizes the planner's allocations when an asset price bubble is present. Finally, we solve the model numerically by employing global solution methods to show how macroprudential policy should account for asset overvaluations over the credit cycle.

In the model, firms borrow for production purposes, but due to lack of commitment, borrowing is limited by collateral. Unlike Kiyotaki and Moore (1997) and Jermann and Quadrini (2012), who assume that borrowing is limited by the liquidation value of physical capital, we consider a setup where the total value of the firm can be pledged as collateral. The underlying idea is that lenders do not only confiscate the physical collateral if the firm does not honor its debt obligations, but also seize the ownership rights over the firm's operations. As a result, they can either dismantle the firm and liquidate its physical assets, or restructure its debt, hire a new manager and sell the restructured firm in the equity market. Hence, to the lenders the collateral value is equal to the market value of the firm, which is higher than the liquidation value as it may incorporate a bubble component. Households are willing to pay more than the fundamental value of the firm, because a higher market value allows the firm to borrow and invest more, which in turn makes the firm more valuable and justifies the originally elevated valuation. We show that this collateral constraint arises endogenously from an incentive compatibility constraint in an optimal contracting problem between borrowers and lenders, which is similar in spirit, but distinct from that in Miao-Wang. Finally, contrary to them, the constraint in our model binds only in some states.

Occasionally binding collateral constraints that incorporate asset prices justify macroprudential policy interventions, as Bianchi-Mendoza have emphasized.⁴ Atomistic agents neglect the effects of their actions on asset prices used as collateral, and consequently, on the tightness of the collateral constraint. This behavior generates a pecuniary externality from borrowing decisions. A social planner internalizes the effects of borrowing decisions on the incidence and tightness of a binding collateral constraint, and can choose a different level of borrowing to address the pecuniary externality.

When tackling the pecuniary externality arising from occasionally binding collateral constraints, the planner faces the following tradeoff: Higher borrowing pushes *current prices* up, alleviating the negative effects of a binding collateral

⁴See also, Lorenzoni (2008), Jeanne and Korinek (2010), and Korinek (2011).

constraint today, but also dampens *future prices*, thereby exacerbating the negative effects of binding collateral constraints in the future. The two opposing effects operate via the current and future Euler equations for capital investment, and the relative strength between the two determines the level of borrowing implemented by the planner.

The allocations chosen by the planner can be decentralized with a subsidy or a tax on borrowing. Importantly, if the collateral constraint does not bind today, then the only objective is to alleviate pecuniary externalities from binding constraints in the future, which calls for a positive tax on borrowing. In this case, the tax is interpreted as a purely macroprudential tax as it is imposed to address overborrowing during good times, i.e. when the collateral constraint does not bind, in order to alleviate the costs from deleveraging during bad times, i.e. when the collateral constraint becomes binding in the future.

Our paper shows that the non-linearity introduced by the occasionally binding constraints matters in the presence of an asset price bubble.⁵ The bubble raises the collateral value of the firm in the present period, and therefore, allows the firm to operate unconstrained for higher levels of debt compared to the bubbleless case. At the same time, the bubble amplifies pecuniary externalities once the collateral constraint starts binding since its value is linked to the rate of return on capital in the economy. Hence, when the constraint binds, the drop in consumption—due to the inability to borrow as much as desired—deflates the bubble, amplifying the pecuniary externality described above. We call this channel the *intensive margin* through which the bubble affects real outcomes and induces the planner to lean more aggressively against borrowing compared to the bubbleless economy.

The positive effect of the bubble on relaxing the collateral constraint and amplifying future pecuniary externalities should translate to a higher macroprudential tax. However, there is an additional channel through which the bubble affects real outcomes, which we call the *extensive margin*. The persistence of a bubble can relax collateral constraints in the future, such that the incidence of pecuniary externalities goes down at least for some states of the world. In other words, the bubble can make the collateral constraint non-binding for some shock realizations even if there are no other positive shocks to economic fundamentals.

The extensive margin operates in the opposite direction of the intensive margin. Thus, it is not clear whether the macroprudential tax should be higher or

⁵Guerrieri and Iacoviello (2017) emphasize the role of occasionally binding collateral constraints in generating asymmetric effects of house prices on economic activity, but abstract from asset price bubbles.

lower in the presence of a bubble. We show that the effect of asset price overvaluations on the optimal macroprudential policy is non-monotonic. This has important implications for the determination of countercyclical policies targeting credit imbalances employed by regulators globally. In particular, asset overvaluations should not only be used as an argument to lean more aggressively against the wind, but could also imply that regulators need not to worry as much about the build up of credit imbalances if the extensive margin dominates.

Our results also provide guidance on how the macroprudential tax depends on the underlying economic conditions. In particular, the extensive margin dominates the intensive margin when the level of debt is moderate: while the bubble relaxes the (marginally binding) collateral constraint, pecuniary externalities are not so severe as agents do not need to deleverage from high debt levels. Hence, macroprudential policy should be more accommodative, or in other words the macroprudential tax should be lower when the bubble is present for a given level of credit imbalances (proxied by the current level of debt). However, as credit imbalances grow, today's collateral constraint starts binding in the bubbleless economy, while it remains slack in the presence of a bubble. As a result, the borrowing tax starts decreasing in the former case, while it continues increasing in the latter, since the intensive margin becomes stronger as credit imbalances build up. Overall, asset overvaluations amplify externalities from high levels of credit imbalances, but at the same time they mitigate the adverse effects when imbalances are at a moderate level.⁶

We calibrate our model using OECD data and set the parameters governing the dynamics of bubbles to match the stylized facts in Jordà, Schularick and Taylor (2015). We find that the macroprudential tax in the presence of a bubble increases steadily as credit imbalances grow, reaching a peak of about 7% compared to 3% in the bubbleless economy. Moreover, the bubble allows current credit imbalances, as measured by the debt-to-GDP ratio, to increase by about 4 percentage points more compared to the bubbleless case, before the current collateral constraint starts binding.

In order to measure the real effects of the bubble, we simulate the economy and compare the outcomes when the collateral constraint starts binding in the presence and in the absence of a bubble. We find that when the initial level of debt is low and a bad shock forces the collateral constraint to bind, the reduction in consumption is smaller in the presence of a bubble that persists. However, if

⁶For low debt levels a policy intervention is not needed as collateral constraints do not bind in the present or in the future.

the initial level of debt is high, the reduction in consumption is higher with a bubble, which is in line with the aforementioned result that the intensive margin is stronger when the outstanding debt level is high.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 presents the baseline model. We analyze optimal policy in section 4. We show the numerical results in section 5. Finally, section 6 concludes.

2 Literature

The main focus of our paper is the design of optimal macroprudential policies in the presence of asset price bubbles and occasionally binding collateral constraints. We contribute to the literature by identifying an additional pecuniary externality that operates via stock price bubbles and show that policy should balance the counteracting impact of bubbles on relaxing financing frictions and on intensifying the pecuniary externality.

Our paper is related to several strands in the literature. First, it contributes to the literature on rational asset price bubbles. Papers on this topic differ in the friction that allows a bubble to exist in equilibrium, which matters also for how the bubble affects economic outcomes (Barlevy, 2018). In the early literature, including the seminal works by Samuelson (1958), Diamond (1965), and Tirole (1985), the bubble exists in equilibrium because of dynamic inefficiency. However, as argued in Abel, Mankiw, Summers and Zeckhauser (1989), real economies are dynamically efficient.⁷ Hence, most of the recent papers on rational bubbles have turned away from dynamic inefficiency and have considered other frictions to motivate bubbles' existence.

Some models attribute the bubbles' presence to financial frictions. In this class of models, an intrinsically worthless asset or a bubbly component of a productive asset can relax financial frictions by allowing agents to borrow more. Early examples include Kocherlakota (1992) and Santos and Woodford (1997). More recent work has emphasized the role of entrepreneurs and firms facing borrowing constraints, including Kocherlakota (2009), Farhi and Tirole (2012), Martin and Ventura (2012, 2016), Hirano and Yanagawa (2017) and Miao and Wang (2018). Other papers have showed that informational frictions (see Barlevy, 2015 for a survey) and agency problems (e.g. Allen and Gorton, 1993; Allen and Gale, 2000; Barlevy, 2014) can give rise to bubbles in equilibrium.

⁷Despite the predominant view that real economies are dynamically efficient, Geerolf (2013), more recently, has questioned the findings by Abel, Mankiw, Summers and Zeckhauser (1989).

Within the literature on rational asset price bubbles, the paper closest in spirit to ours is Miao and Wang (2018). They show that a *stock* price bubble can arise in equilibrium in a production economy with infinitely-lived agents. In their model, borrowing is restricted by a firm’s (market) value, and a bubble on the firm’s stock relaxes the borrowing constraint, increasing the borrowing capacity of the economy.⁸ The bubble exists in equilibrium as it provides a liquidity premium, which in turn yields a return on the bubble that is lower than the return on the stock, thereby satisfying the transversality condition. In our paper, we augment the model of Miao and Wang (2018) to allow for occasionally binding collateral constraints.

Our paper also contributes to the literature on optimal macroprudential policy. Macroprudential policy intervention is usually motivated by the presence of financial frictions, generating pecuniary externalities (Bianchi-Mendoza; Stein, 2012; Bianchi, 2011; Jeanne and Korinek, 2010), or by the presence of aggregate demand externalities (Eggertsson and Krugman, 2012; Korinek and Simsek, 2016; Farhi and Werning, 2016). Within this area of research, the paper that is most closely related to ours is Bianchi and Mendoza (2018). They also study the design of optimal macroprudential policy with commitment in a small open economy with occasionally binding collateral constraints, but they do not consider the effects of elevated asset prices on the optimal policy design. Another related paper is Biswas, Hanson and Phan (2019), who study the welfare effects of bubbles and subsequent policy intervention in an environment with downward wage rigidities. They find that policy should “lean against bubbles” because after a bubble’s collapse the aggregate economic activity dips below the pre-bubble trend. Although their policy result is reminiscent of ours, the key difference is that we find “leaning against bubbles” is only optimal when debt levels are high; otherwise, policy should be accommodative.

Finally, there are a few papers related to ours that consider the interaction of financial frictions and asset bubbles. Hirano and Yanagawa (2017) show that the effects of a bubble burst depend on the degree of pledgeability of the bubbly asset. They also show that bubbles can increase welfare, regardless of the effects of their bursts, as they relax financial frictions and allow for consumption smoothing

⁸While the focus in our paper and in Miao and Wang (2018) is on a stock price bubble, many papers have focused on studying the existence of pure bubbles, like money, in production economies. Pure bubbles can also provide liquidity by raising borrowers net worth (Caballero and Krishnamurthy, 2006; Farhi and Tirole, 2012; Kiyotaki and Moore, 2012; Martin and Ventura, 2012; Aoki, Nakajima and Nikolov, 2014; Ikeda and Phan, forthcoming). However, the borrowing constraints in models of pure bubbles are different than ours and that in Miao and Wang (2018) because they do not depend on the stock market value of the firm.

by credit-constrained agents. On the other hand, Chauvin, Laibson and Mollerstrom (2011) find that in the absence of financial frictions bubbles are always welfare-reducing as they magnify cyclical fluctuations of consumption. Martin and Ventura (2016) propose a new rationale for macroprudential regulation: Borrowing should be taxed (or subsidized) such that it replicates the optimal bubble in the economy, maximizing output and consumption. Aoki and Nikolov (2015) and Bengui and Phan (2018) study the effect of bubbles on risk-taking incentives and financial stability. Miao, Wang and Zhou (2015) find that loan-to-value limits and a property transaction tax can reduce the benefits of holding the bubbly asset, while Miao and Wang (2015) suggest that increasing bank capital requirements can prevent bubbles held by the financial sector. Compared to our study, the last two papers consider ex-post policy interventions as collateral constraints always bind; whereas we focus on ex-ante macroprudential regulation when constraints bind only occasionally. Moreover, policy analysis in those frameworks is based on a comparative statics exercise, whereas we study a fully-fledged Ramsey problem.

3 Model economy

We consider a small open economy with a rational bubble on a productive asset and an occasionally binding collateral constraint. The modeling framework is very similar to Bianchi-Mendoza, but in addition features a rational asset price bubble. We model the bubble as in Miao-Wang, who show that a rational stock price bubble can be supported in equilibrium in production economies with infinitely-lived agents. We proceed by first outlining the competitive economy (CE) equilibrium allocations. Subsequently in section 4, we analyze the time-consistent optimal policy following Bianchi-Mendoza and derive the optimal macroprudential tax on borrowing that decentralizes the planner's (SP) allocations.

3.1 Small open economy with an asset price bubble

The economy is populated by a continuum of mass one of two types of infinitely-lived representative agents: households and firms. Households consume, provide labor services, and are the owners of firms. They can also frictionlessly trade firm shares in the stock market. Firms own a production technology, which combines capital, labor and intermediate goods as inputs to production. They purchase and sell capital, borrow internationally in inter-temporal debt markets, and do not own or trade the shares of other firms in the stock market. In what follows we

define households' and firms' optimization problems and derive the competitive equilibrium.

Households. The representative household lives for infinite periods and maximizes the expected utility, which is a function of consumption, c_t , and labor, l_t ,

$$\max_{c_t, l_t, \eta_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t U(c_t - G(l_t)), \quad (1)$$

subject to the budget constraint,

$$c_t + \int_j (V_t^j - D_t^j) \eta_{t+1}^j dj = \int_j V_t^j \eta_t^j dj + w_t l_t, \quad (2)$$

where V_t^j denotes firm j 's cum-dividend equity value, and D_t^j is firm j 's dividend paid out in period t . The household starts the period with η_t^j shares of firm j , which it can trade in the stock market for shares of other firms. As a result, its end-period holdings of firm j 's equity are η_{t+1}^j . In addition to dividends, the household earns income from wages, denoted by w_t , for labor supplied to firms. The utility function $U(\cdot)$ is a standard concave, twice-continuously differentiable in both its arguments and satisfies the Inada conditions. As in Bianchi-Mendoza, preferences are defined over a composite commodity $c_t - G(l_t)$, where $G(l_t)$ is a convex function, strictly increasing and continuously differentiable.⁹

The first-order optimality conditions of the household are as follows

$$\lambda_t = U_{c,t}, \quad (3)$$

$$w_t = G_{l,t}, \quad (4)$$

$$V_t^j = \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} V_{t+1}^j \right) + D_t^j, \quad (5)$$

where λ_t is the Lagrange multiplier corresponding to the budget constraint (2), and $U_{c,t}$ and $G_{l,t}$ are the first-order derivatives of U and G with respect to c_t and l_t , respectively. Equation (3) denotes the marginal utility of the household with respect to consumption, and equation (4) denotes the household's optimal

⁹The formulation of this composite commodity is defined by Greenwood, Hercowitz and Huffman (1988) and removes the wealth effect on labor supply inducing a countercyclical increase in the labor supply during crises.

labor choice. The Euler equation (5) implies that a firm's equity is priced using the household's stochastic discount factor $\beta \frac{U_{c,t+1}}{U_{c,t}}$. The standard transversality condition is

$$\lim_{T \rightarrow \infty} \beta^T \frac{U_{c,T}}{U_{c,t}} V_T^j = 0, \quad (6)$$

where we have used the fact that $\eta_T^j = 1$ for all T and all j .

Firms. As all firms are the same in equilibrium, we explain the problem of a representative firm and drop the superscript j for notation simplicity. The firm's managers act in the best interest of shareholders (households) to maximize the market value of the firm,

$$V_t(k_t, L_t) = \max_{\substack{k_{t+1}, L_{t+1}, \\ v_t, l_t^d}} D_t + \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} V_{t+1}(k_{t+1}, L_{t+1}) \right), \quad (7)$$

where the two state variables are the stock of capital, k_t , and debt, L_t , from the last period.

The firm produces $y_t = z_t F(k_t, l_t^d, v_t)$ each period. The function $F(\cdot)$ is Cobb-Douglas and combines labor, l_t^d , with the stock of capital purchased in the previous period, k_t , and an intermediate good, v_t ; z_t is an aggregate productivity shock. Aggregate capital is in unit fixed supply: $K_t = 1$. The intermediate good is traded in competitive world markets at a fixed exogenous price, p_v . The profits of the firm are distributed as dividends, D_t , to households. The flow budget constraint of the firm is given by

$$D_t = y_t - p^v v_t - w_t l_t^d + \frac{L_{t+1}}{R} - L_t + q_t k_t - q_t k_{t+1}, \quad (8)$$

where L_t denotes the beginning-of-period holdings of one-period non-state contingent bonds, q_t is the price of capital, and R is the world-determined gross real interest rate taken as given in the small open economy. Hence, dividends are equal to what is left from the output after paying the factors of productions, v_t and l_t^d , and the net capital expenditure, $q_t(k_{t+1} - k_t)$, plus the net debt issuance, $L_{t+1}/R - L_t$.

We assume that a firm cannot raise equity and that its borrowing decision is limited by a collateral constraint, which is *endogenously* derived from an incentive compatibility constraint under limited commitment (see section A.1 of the Appendix). Similarly to Bianchi-Mendoza and Jermann and Quadrini (2012), we assume that the total liabilities of the firm at the beginning of the period comprise

of $\theta p_v v_t + L_{t+1}/R$. This assumption implies that, in addition to its inter-temporal borrowing, the firm also needs to finance ahead of production a portion $\theta \leq 1$ of the intermediate good purchases, $p_v v_t$. While L_{t+1} is an inter-temporal loan, $\theta p_v v_t$ is repaid within the same period and hence it does not bear any interest. Because both types of borrowing need to be collateralized, the collateral constraint takes the following form

$$\frac{L_{t+1}}{R_t} + \theta p^v v_t \leq m_t V_{t+1}(k_t, 0). \quad (9)$$

Constraint (9) limits the size of total debt to a fraction m_t of the firm's continuation value in the case when the firm defaults on its obligations. In other words, creditors can seize the entire firm, which is valued at the stock market (V_t), rather than seizing and liquidating only the firm's physical capital ($q_t k_t$), which is the case in Kiyotaki and Moore (1997), Jermann and Quadrini (2012) and Bianchi-Mendoza. This form of a collateral constraint ensures that a *stock price bubble*, i.e. a bubble on a productive asset, can be supported in equilibrium. The bubble has a positive value in equilibrium because it can relax the collateral constraint and allow the firm to borrow more, which in turn increases the firm's value supporting the initial (bubbly) valuation. While Miao-Wang assume that the constraint always binds, we allow it to bind only at times. We will show that the occasionally binding collateral constraint interacts with the bubble in a non-monotone way, altering the optimal policy response.¹⁰

Asset price bubble. To derive the value of the bubble, we solve the firm's dynamic programming problem (7) subject to (8) and (9), following the method of undetermined coefficients. Thus, we guess that the value function that the firm maximizes takes the following form

$$V(k_t, L_t) = a_t k_t + s_t L_t + b_t, \quad (10)$$

where a_t and s_t are coefficients associated with the fundamentals of the model. As

¹⁰A bubble on a productive asset would not be supported in equilibrium if the firm could only borrow against the liquidation value of physical capital. Instead, a typical collateral constraint akin to Kiyotaki and Moore (1997), where borrowing is limited up to the liquidation value of capital, can only ensure the existence of a pure bubble in equilibrium, i.e. a bubble on an intrinsically useless asset. See Miao-Wang for details. In our model, pure real estate bubbles cannot exist because the durable productive asset is part of the producing firm. However, we could easily introduce an additional intrinsically useless asset, akin to real estate in Miao, Wang and Zhou (2015), which can have a positive valuation as long as it serves as collateral. As a result, we would be able to study the differential liquidity properties of equity price bubbles and real estate bubbles.

in Miao-Wang, the third coefficient, b_t , is not related to the firm's fundamentals and is interpreted as a bubble component.

In section A.2 in the Appendix, we solve the dynamic programming problem of the firm and show that

$$a_t = [F_{k,t} + q_t(1 + m_t\mu_t)], \quad (11)$$

$$s_t = -1, \quad (12)$$

$$b_t = (1 + m_t\mu_t)\beta E_t\left(\frac{U_{c,t+1}}{U_{c,t}}b_{t+1}\right), \quad (13)$$

where $F_{k,t} \equiv F(k_t, l_t^d, v_t) - F_{l,t}l_t^d - F_{v,t}v_t$, $F_{l,t}$ and $F_{v,t}$ are the marginal products of capital, labor and the intermediate good, respectively; and μ_t is the Lagrange multiplier on the collateral constraint (9). After substituting the expressions for the coefficients attached to the firm's fundamental variables, (11) and (12), into the guess (10), the value function takes the following form

$$V_t(k_t, L_t) = [F_{k,t} + q_t(1 + m_t\mu_t)]k_t - L_t + b_t. \quad (14)$$

A positive b_t lifts the market value of the firm, and via the collateral constraint, it increases the firm's borrowing capacity. This can easily be seen by substituting the derived coefficients a_t , s_t and b_t into the collateral constraint (9), which then takes the following form

$$\frac{L_{t+1}}{R} + \theta p^v v_t \leq m_t \left[q_t k_t + \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right) \right]. \quad (15)$$

Constraint (15) depends on the capital stock at t , but also on the expected discounted value of the bubble component at $t+1$. Hence, the presence of a bubble can help relax the collateral constraint. The collateral value of the bubble will be denoted by

$$B_t \equiv \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right]. \quad (16)$$

Competitive equilibrium. The following proposition outlines the representative firm's optimality conditions; the proof is relegated to the Appendix A.3.

Proposition 1. *The representative firm chooses k_{t+1} , L_{t+1} , l_t^d , v_t to maximize its objective function (7), given the functional form (14), subject to the budget constraint (8) and the collateral constraint (15). In equilibrium, the optimality con-*

ditions (i)-(vi) below are satisfied:

(i) the Euler equation with respect to borrowing, L_{t+1} ,

$$1 = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} R + \mu_t, \quad (17)$$

ii) the Euler equation with respect to capital, k_{t+1} ,

$$q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} [z_{t+1} F_{k,t+1} + q_{t+1} (1 + m_{t+1} \mu_{t+1})] \right\}, \quad (18)$$

iii) the labor, l_t^d , optimality condition,

$$w_t = z_t F_{l,t}, \quad (19)$$

iv) the intermediate good, v_t , optimality condition,

$$p^v (1 + \theta \mu_t) = z_t F_{v,t}, \quad (20)$$

v) the bubble, B_t , accumulation process,

$$B_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \right], \quad (21)$$

vi) the complementarity slackness condition,

$$\mu_t \left[m_t (q_t k_t + B_t) - \frac{L_{t+1}}{R} - \theta p^v v_t \right] = 0. \quad (22)$$

The competitive equilibrium of the economy is defined as follows.

Definition. For given initial values of L_0 and exogenous processes $\{z_t, m_t\}_{t=1}^{\infty}$, a competitive equilibrium for the economy with a bubble on the productive asset and a collateral constraint is a sequence of allocations $\{c_t, l_t, l_t^d, v_t, y_t, B_t\}_{t=0}^{\infty}$, an asset profile $\{k_{t+1}, L_{t+1}\}_{t=0}^{\infty}$, and a price system $\{q_t, p^v, R\}_{t=0}^{\infty}$, such that

1. Given the price system $\{q_t, p^v, R\}_{t=1}^{\infty}$, the allocations and the asset profile solve the households' and firms' problems, i.e. conditions (3)-(5) and (17)-(22) are satisfied,

2. The markets for labor, capital, and equity clear, $l_t^d = l_t$, $k_t = K_t = 1$, $\eta_t^j = 1 \forall j$, and

3. The resource constraint holds, $c_t + L_t = L_{t+1}/R + z_t F(1, l_t, v_t) - p^v v_t$.

Two observations can be made from looking at the firm's optimality conditions.

First, the presence of the collateral constraint of the form (15) distorts both the optimal inter- and intra-temporal margins when binding. Condition (20), defining the choice of the intermediate good, embeds an additional cost, i.e. the cost of collateral financing equal to $\theta\mu_t p_v$. In addition, both Euler equations are distorted. The Euler equation for borrowing (17) implies that the marginal benefit from increasing borrowing today outweighs the expected future marginal cost by an amount equal to the shadow price of relaxing the collateral constraint. Similarly, the Euler equation with respect to capital (18), equating the marginal cost of an extra unit of capital with its marginal benefit, embeds an additional benefit that derives from relaxing the collateral constraint, valued at $m_{t+1}\mu_{t+1}q_{t+1}$. As pointed out in Bianchi-Mendoza, this equation is at the core of the mechanism through which the pecuniary externality operates: The choice of borrowing and consumption today influence the fundamental price of the asset, which in turn affects the tightness of the collateral constraint.

Second, both the fundamental asset price of capital q_t and the bubble component in the collateral constraint, B_t , depend on the the consumption choice of households, which is increasing in the firm's borrowing decision. In particular, higher current consumption (arising from higher borrowing) reduces the marginal utility of consumption and via equation (18) results in a higher price of capital. While this mechanism has been already highlighted by Bianchi-Mendoza, we show that it also operates via the bubble on the productive asset: a lower marginal utility of consumption, via equation (21) increases the size of the bubble. A larger bubble translates into higher stock market valuations that help relax the collateral constraint. As a result, the bubble increases the firm's borrowing capacity and dividend payouts to the households. This mechanism will be at the core of our policy analysis in section 4.

Bubble existence and multiplicity of equilibria. With regards to the bubble existence, two conditions in models with infinitely-lived agents need to hold so that a rational bubble can be supported in equilibrium. First, it has to be priced in equilibrium to ensure rationality; second, the transversality condition needs to be satisfied. Within our framework both of these conditions hold as long as the collateral constraint binds in at least one period. In particular, the bubble is priced by equation (21), and as long as $E_t\mu_{t+1} > 0$, the transversality condition (6) is satisfied as the growth rate of the bubble is lower than the return on the stock

priced by households. The reason is that the bubble carries a liquidity premium as it relaxes the collateral constraint.

With regards to the multiplicity of equilibria, note that both $b_t = 0$ and $b_t > 0$ are solutions in equilibrium. In the case when $b_t = 0$, the model reduces exactly to the one in Bianchi-Mendoza. The alternative solution is $b_t > 0$. As it is common in the rational bubbles' literature, we are agnostic about how the bubble is generated and focus on the case where the bubble appears at some point in time and persists thereafter. Note, however, that the value of the bubble is endogenously determined in equilibrium. Our formulation encompasses both a deterministic bubble, i.e. a bubble that is always present in every period once it emerges, as well as a stochastic bubble, i.e. a bubble that may burst with some exogenous probability in the next period and never re-emerge.¹¹ To see this, rewrite B_t in (21) as

$$B_t = \pi \cdot \beta E_t^\pi \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \middle| \text{Bubble persists at } t + 1 \right] \\ + (1 - \pi) \cdot \beta E_t^{1-\pi} \underbrace{\left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \middle| \text{Bubble bursts at } t + 1 \right]}_{=0}, \quad (23)$$

where π is the probability that a bubble that is present at t persists at $t + 1$ (or equivalently, $1 - \pi$ is the probability that the bubble bursts at $t + 1$). So, E_t in (21) is the unconditional expectation over all possible realizations including realizations where the bubble persists and realizations where the bubble bursts. As it is standard in the literature, we assume that once the bubble bursts, it cannot re-emerge again.¹² Naturally, setting $\pi = 1$ yields the deterministic bubble case.

4 Optimal macroprudential policy

To derive the optimal policy, we proceed by first formulating the planner's problem and then discussing the properties of the optimal taxation that implements the planner's solution.

¹¹See Blanchard and Watson (1982) and Weil (1987) for stochastic bubbles as well as Miao-Wang more recently.

¹²More recently, Guerron-Quintana, Hirano and Jinnai (2019) develop a model of recurrent bubbles in an environment with endogenous growth and infinitely-lived households.

4.1 Time-consistent planner's problem

The policy design follows the Ramsey approach, which consists of the social planner choosing policies, prices, and allocations in order to maximize the economy's social welfare function. In doing so, the planner has to respect all equilibrium conditions of the competitive economy in order to ensure that the allocations chosen can be implemented as allocations in the competitive economy.

Unlike in the standard Ramsey literature, where the planner optimally chooses distortionary policies intended to finance government expenditure, the planner in our model chooses a policy to alleviate the inefficiencies arising from pecuniary externalities. In particular, as in Bianchi-Mendoza, we assume that the only policy available to the planner is a tax on borrowing, τ .¹³ This instrument is Pigouvian in nature with the tax revenues being rebated lump-sum back to the private agents, T_t . The resource constraint of the decentralized economy then takes the following form

$$c_t + L_t(1 + \tau_{t-1}) + p^v v_t \leq \frac{L_{t+1}}{R} + z_t F(\cdot) + T_t, \quad (24)$$

where $T_t = \tau_{t-1} L_t$. (24) is obtained by adding (2) and (8), with the borrowing tax introduced, and using the equilibrium conditions.

The Euler equation with respect to borrowing (17) then becomes

$$U_{c,t} = \beta R(1 + \tau_t) E_t U_{c,t+1} + \mu_t U_{c,t}. \quad (25)$$

For simplicity of presentation, in the formulation of the Ramsey problem, we use a reduced set of competitive equilibrium conditions as constraints. In section A.5 in the Appendix we show that the planner's allocations obtained as a result of this "relaxed" planner's problem, excluding conditions (19), (20), and (25), are the same as those obtained by the "fully" constrained planner, i.e. one that incorporates all equilibrium conditions as constraints in the maximization problem.

In a bubbleless model, the set of competitive equilibrium equations faced by the planner comprises of (i) the resource constraint, (ii) the collateral constraint, (iii) the Euler equation with respect to capital. The inclusion of the last constraint reflects the fact that the planner has to respect competitive asset pricing in the economy. As pointed out by Bianchi-Mendoza, through this equation, the planner internalizes how private agents' choices affect equilibrium asset pricing. In the

¹³Alternative instruments that affect the inter-temporal margin can be used instead of the tax. The tax can also be imposed on the interest rate expenses.

presence of an asset price bubble, the set of constraints (i)-(iii) is expanded by an additional constraint, the bubble accumulation equation (21). Hence, the planner also internalizes how private agents' decisions affect the evolution of the bubble component, B , over time.

Finally, we assume that the planner does not have the technology to commit to future policies.¹⁴ Therefore, we solve for the optimal time-consistent macroprudential policy, taking into account the effects of the planner's current period choices on future planners'. As a result, the planner does not have an incentive to deviate from policy rules of previous social planners. The planner's maximization problem is given by

$$\max_{c_t, q_t, l_t, v_t, L_{t+1}, B_t} E_t \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

$$s.t. \quad c_t + L_t + p^v v_t \leq z_t F(1, l_t, v_t) + \frac{L_{t+1}}{R} \quad (\lambda_t^p) \quad (26)$$

$$\frac{L_{t+1}}{R} + \theta p^v v_t \leq m_t (q_t + B_t) \quad (\mu_t^p) \quad (27)$$

$$q_t U_{c,t} = \beta E_t \{U_{c,t+1} [(q_{t+1} + F_{k,t+1}) + m_{t+1} \mu_{t+1} q_{t+1}]\} \quad (\xi_t) \quad (28)$$

$$B_t = \beta E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \right], \quad (\psi_t) \quad (29)$$

where the Lagrange multipliers associated with each constraint are given in parentheses. Note that we distinguish between the Lagrange multipliers on the budget (resource) and collateral constraints in the competitive and planner's problem (λ_t and λ_t^p ; μ_t and μ_t^p , respectively). In sum, compared to the competitive economy, the planner's problem includes two additional constraints (28)-(29). Equation (28) allows the planner to respect competitive pricing of capital. Equation (29) allows the planner to internalize the existence and the accumulation process of the bubble.

The social planner's optimality conditions, after a few algebraic manipulations, take the following form:

¹⁴Bianchi-Mendoza show that the optimal policy under commitment is time inconsistent since asset prices are determined by a dynamic condition linking the present and future (expected) marginal utilities of consumption. Instead, they follow the time-consistent approach under which a planner cannot commit at t to the whole path of future policy choices.

$$c_t : \quad \lambda_t^p = U_{c,t} - \xi_t q_t U_{cc,t} - \psi_t E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \frac{U_{cc,t}}{U_{c,t}} \right], \quad (30)$$

$$q_t : \quad \xi_t U_{c,t} = m_t \mu_t^p, \quad (31)$$

$$B_t : \quad m_t \mu_t^p = \psi_t, \quad (32)$$

$$L_{t+1} : \quad \lambda_t^p = \beta R E_t [\lambda_{t+1} + \xi_t \Omega_{t+1} + \psi_t \mu_t^p \Delta_{t+1}], \quad (33)$$

$$l_t : \quad U_{c,t} G_{l,t} = -\lambda_t^p F_{l,t}, \quad (34)$$

$$v_t : \quad \mu_t^p = \frac{\lambda_t^p (F_{v,t} - p^v)}{\theta p^v}, \quad (35)$$

where Ω_{t+1} and Δ_{t+1} collect all partial derivatives with respect to L_{t+1} on the right-hand side of the capital-Euler and the bubble accumulation equations, capturing the impact of the planner's choice of L_{t+1} on the actions of future planners (reflecting the “time-consistency” nature of the policy rule).

We begin our analysis by comparing the planner's first-order condition with respect to consumption, both in the presence and in the absence of a bubble, to the corresponding condition of the competitive economy, $\lambda_t = U_{c,t}$.¹⁵ Without an asset price bubble, i.e. when $B_t = 0 \forall t$, this condition for the planner is given by

$$\lambda_t^p = U_{c,t} - \xi_t q_t U_{cc,t}, \quad (36)$$

which is the exact same condition as in Bianchi-Mendoza. In this case, the key difference between the CE and the Ramsey planner is that the latter's shadow value of wealth, in addition to the marginal utility of current consumption, also entails the amount by which an additional unit of consumption reduces today's marginal utility and relaxes the collateral constraint through its effect on prices ($-\xi_t q_t U_{cc,t}$).

When the bubble is present, the first-order condition (36) is augmented by an additional term, $-m_t \mu_t^p E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \frac{U_{cc,t}}{U_{c,t}} \right]$, which is positive since $U_{cc,t} < 0$, and can be obtained by substituting (32) in (30). This term captures the amount by which an increase in c_t reduces today's marginal utility of consumption, increasing the discounted value of the bubble (see equation 29), and relaxing the collateral constraint. Note that, irrespectively of the bubble's presence, if the collateral constraint does not bind at t , then $\mu_t^p = \xi_t = \psi_t = 0$, and the planner's

¹⁵The household's first-order condition with respect to consumption is the same regardless of whether there is a bubble or not.

first-order condition reduces to the one of the CE.

Next, consider the Euler condition with respect to borrowing. After substituting equation (30) in (33), the planner's Euler condition with respect to borrowing takes the following form

$$\begin{aligned}
& U_{c,t} - \xi_t q_t U_{cc,t} - \psi_t E_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \frac{U_{cc,t}}{U_{c,t}} \right] = \\
& \beta R E_t \left\{ U_{c,t+1} - \xi_{t+1} q_{t+1} U_{cc,t+1} - E_{t+1} \psi_{t+1} \left[\frac{U_{c,t+2}}{U_{c,t+1}} (1 + m_{t+2} \mu_{t+2}) B_{t+2} \frac{U_{cc,t+1}}{U_{c,t+1}} \right] \right\} \\
& + \beta R E_t [\xi_t \Omega_{t+1} + \psi_t \Delta_{t+1}] + \mu_t^p, \tag{37}
\end{aligned}$$

which we compare to the corresponding condition of the agents, (17). The best way to do the comparison is to highlight the pecuniary externalities operating through the *current* and the *future* price of capital and the bubble component of the firm's value in two cases: when the collateral constraint binds at t and when it does not.

1. The collateral constraint does not bind at t , but may bind at $t + 1$: We proceed by describing the effects via q_{t+1} and B_{t+1} . First, without an asset price bubble, the marginal cost of borrowing at t is higher for the social planner than for the private agents by an amount $\beta R E_t [\xi_{t+1} U_{cc,t+1} q_{t+1}]$. This term reflects the fact that the planner internalizes the effect of a larger debt at t on reducing the borrowing capacity at $t + 1$ through a lower price of capital, q_{t+1} , if the constraint binds at $t + 1$. This mechanism operates through equation (28): A binding collateral constraint at $t + 1$ results in lower consumption, higher marginal utility, and hence a lower future price of capital q_{t+1} as pointed out by Bianchi-Mendoza.

When an asset price bubble is present, the marginal cost of borrowing at t is higher by an additional amount $-\beta R m_{t+1} \mu_{t+1}^p E_{t+1} \left[\frac{U_{c,t+2}}{U_{c,t+1}} (1 + m_{t+2} \mu_{t+2}) B_{t+2} \frac{U_{cc,t+1}}{U_{c,t+1}} \right]$. This term is at the core of our optimal policy analysis and captures the impact of current borrowing on the future size of the bubble. The mechanism operates through equation (29): As more borrowing at t implies a larger reduction in consumption at $t + 1$ in the states when the collateral constraint binds, this will also deflate the bubble (which grows with consumption). Since elevated firm values help alleviate adverse welfare effects when the collateral constraint binds, the planner tries to avoid deflating the bubble exactly when borrowing capacity is

curtailed. As we will show in the next section, the planner can do so by leaning against debt build-up more aggressively ex-ante.

2. The collateral constraint binds at t and may bind at $t + 1$: We proceed by describing the effects via q_t , q_{t+1} and B_t , B_{t+1} . In the event the collateral constraint binds at t and may also bind at $t + 1$, then there are two opposing effects resulting from the borrowing decision that the planner has to take into account. In particular, the planner faces a tradeoff between choosing allocations such that it increases current prices, q_t and B_t , at the cost of potentially decreasing future prices, q_{t+1} and B_{t+1} . The mechanism operates as follows: More borrowing, accompanied by higher consumption at t , increases the price of physical capital q_t and relaxes the collateral constraint. At the same time, more borrowing and higher consumption at t may result in lower consumption and lower price of capital at $t + 1$ in the event the collateral constraint continues to bind in the future. This tradeoff is amplified in the presence of a bubble. The overall effect from borrowing on the price of capital and the bubble is therefore ambiguous when the collateral constraint binds at t and may bind at $t + 1$. Importantly, however, the effects in either direction are amplified because of the presence of the bubble. The planner chooses allocations such that it balances the benefits of increasing the current price of capital and inflating the bubble, and the cost of potentially decreasing future prices and deflating the bubble.

4.2 Optimal tax rate in the presence of a bubble

The optimal tax on debt can be derived by comparing the Euler equation for L_{t+1} of the Ramsey planner (37) with the corresponding equation of the agents incorporating the tax on borrowing (25). In section A.4 in the Appendix, we derive the general tax formula for the case when $\mu_t > 0$ and $E_t\mu_{t+1} > 0$. As hinted in the previous section, this tax will be set to balance the effects of borrowing on the *current* versus the *future* price of physical capital and the value of the bubble. In the current section, we focus on the macroprudential tax, which arises in the case when $\mu_t = 0$ and $E_t\mu_{t+1} > 0$. In these states the tax is given a macroprudential interpretation because it is set such that it affects credit during good times ($\mu_t = 0$) to avoid the negative effects from deleveraging during bad times ($E_t\mu_{t+1}$). The Lemma below defines the macroprudential tax in the presence of bubbles.

Lemma 2. *Given that credit is not constrained at t but may be constrained at $t + 1$, the macroprudential tax on borrowing is higher when bubbles are present at*

t. The general formula for the macroprudential tax is

$$\tau_t^{mp} = -\frac{E_t[\xi_{t+1}q_{t+1}U_{cc,t+1}] + E_{t+1}\psi_{t+1}\left[\frac{U_{c,t+2}}{U_{c,t+1}}(1 + m_{t+2}\mu_{t+2})B_{t+2}\frac{U_{cc,t+1}}{U_{c,t+1}}\right]}{E_t U_{c,t+1}}, \quad (38)$$

The derivation of τ_t^{mp} is relegated to section A.4 in the Appendix. The first component in the tax rate (38) matches the pecuniary externality operating via q_{t+1} . Intuitively, this component of the tax rate intends to hamper excessive borrowing in good times (i.e. when the collateral constraint does not bind) to lower the risk of future instability in bad times (when the collateral constraint starts to bind). In the absence of a bubble, the macroprudential tax on debt reduces just to this term, i.e. $\tau_t^{mp,nB} = -E_t[\xi_{t+1}q_{t+1}U_{cc,t+1}]/E_t U_{c,t+1}$, which is positive, as shown in Bianchi-Mendoza.

When a bubble is present at t , the macroprudential tax is amplified by the second term in the numerator in the tax equation (38). This term matches the pecuniary externality operating via B_{t+1} , which represents the novel component from the presence of the bubble. This term is positive, which means it amplifies the macroprudential tax rate that is tackling excessive borrowing alone ($\tau_t^{mp,nB}$).¹⁶ Intuitively, the goal of this component of the tax rate is to limit borrowing in good times to avoid the downward pressure a deflating bubble can have on the tightness of binding collateral constraints in the future. This result suggest that, for the same level of credit imbalances, the planner leans against new borrowing more aggressively if asset prices are elevated. Note that the tax accounts only for the collateral value of the bubble, B_{t+2} , and not the total bubble component in equity prices, b_{t+2} . To sum up, we will refer to this positive contribution of the bubble to the macroprudential tax as the *intensive margin*.

However, as discussed before, the mere presence of the bubble can make the collateral constraint also slack in some states. As a result, the *probability* of the collateral constraint binding in the future and the equilibrium allocations change when there is a bubble in the model. We will refer to this contribution of the bubble to the macroprudential tax as the *extensive margin*.

The intensive and the extensive margins can be isolated by computing the differential between the macroprudential taxes with (τ_t^{mp}) and without ($\tau_t^{mp,nB}$) a

¹⁶This can easily be shown by substituting for ψ_{t+1} from equation (32) and noting that $U_{cc,t+1} < 0$.

bubble, which is given by

$$\tau_t^{mp} - \tau_t^{mp,nB} = -\underbrace{\Delta \frac{E_t[\xi_{t+1} q_{t+1} U_{cc,t+1}]}{E_t U_{c,t+1}}}_{\text{Extensive margin}} - \underbrace{\frac{E_{t+1} \psi_{t+1}}{E_t U_{c,t+1}} \left[\frac{U_{c,t+2}}{U_{c,t+1}} (1 + m_{t+2} \mu_{t+2}) B_{t+2} \frac{U_{cc,t+1}}{U_{c,t+1}} \right]}_{\text{Intensive margin}}.$$

Since the sign of the differential tax rate is ambiguous, to verify which of the two effects—intensive vs. extensive margin—prevails and under what conditions, in the next section we solve a calibrated version of the model numerically.

5 Quantitative analysis

This section presents the quantitative implications of the model. We proceed by discussing the baseline calibration, and then turn to the numerical results.

5.1 Calibration

We calibrate most of the the parameters following Bianchi-Mendoza to allow for close comparisons of our results with their bubbleless economy. Thus, for calibration details of all non-bubble parameters we refer the reader to their section III.A. One exception is the global interest rate R , which in Bianchi-Mendoza follows an AR(1) process. In order to limit the number of states for the numerical solution, we keep R fixed at its long-term average level, calibrated to $\bar{R} = 1.01$.

The parameters associated with the asset price bubble are calibrated following the results in Jordà, Schularick and Taylor (2015), who study bubbles in equities and housing markets in 17 advanced countries over the past 140 years. They report summary statistics separately for house and for equity bubbles, in pre- and post-World War II periods. As Bianchi-Mendoza do not distinguish between housing versus other types of assets, we calibrate the bubble parameter π to the average duration of the two types of bubbles, weighted by their relative frequency. Given the sample period used for calibrating other model parameters, we focus on the post-World War II bubbles. The average duration of a bubble in this period is 2.3 years, which implies a survival probability of $\pi = 0.56$ in our model. For a period of price growth to be identified as a bubble, Jordà, Schularick and Taylor (2015) require that the log of real asset prices diverge by more than one standard deviation from a country-specific Hodrick-Prescott filtered trend ($\lambda = 100$, annual data). This implies an average deviation of real asset prices from their long-term trend of around 8 percent in the first year of the bubble. In the numerical

algorithm, we take the size of the bubble in the initial period as given and calibrate it so that it is equal to 8 percent of the price q_t on average across grid-points.

Table 1 summarizes all parameter values. The functional forms for preferences and technology are

$$U = \frac{(c - \chi \frac{l^{1+\omega}}{1+\omega})^{1-\sigma} - 1}{1-\sigma}, \quad \omega, \chi > 0, \sigma > 1,$$

$$F = e^z k^{\alpha_k} v^{\alpha_v} l^{\alpha_l}, \quad \alpha_k, \alpha_v, \alpha_l \leq 1.$$

Total factor productivity (TFP) follows an independent AR(1) process, given by

$$z_t = \bar{z} + \rho_z z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon)$$

The productivity shock is discretized using the Tauchen's quadrature method with three realizations z^l, \bar{z}, z^h , such that $z^l < \bar{z} < z^h$. The parameter in the collateral constraint, m_t , follows a two-state regime switching Markov process with two states $\{m^l, m^h\}$, where m^l denotes tight and m^h denotes normal credit conditions, respectively. This process is also assumed to be independent from the Markov process for z . Finally, $P_{x,y}$ in Table 1 denotes the transition probability from a state x to a state y .

Table (1) Calibration.

Parameters	Value
Risk aversion	$\sigma = 1$
Share of intermediate good in output	$\alpha_v = 0.45$
Share of labor in output	$\alpha_l = 0.352$
Share of assets in output	$\alpha_k = 0.008$
Labor disutility coefficient	$\chi = 0.352$
Fischer elasticity	$1/\omega = 2$
Working capital coefficient	$\theta = 0.16$
Tight credit regime	$m^l = 0.75$
Normal credit regime	$m^h = 0.90$
Global interest rate	$R = 1.01$
TFP process	$\rho_z = 0.78$
	$\sigma_\epsilon = 0.01$
Discount factor	$\beta = 0.95$
Transition probability, m^h to m^l	$P_{h,l} = 0.1$
Transition probability, m^l to m^l	$P_{l,l} = 0$
Bubble bursting probability	$\pi = 0.56$
Bubble size at period t (in percent of the price of capital)	8%

5.2 Numerical results

To solve the model, we use a global, non-linear solution algorithm. The CE solution is obtained by iterating over the first-order conditions, and the SP problem solution is obtained by applying a value function iteration algorithm.¹⁷ Both methods are augmented to incorporate the occasionally-binding collateral constraint, using a grid of 150 points for L_t over a range $[0, 0.11]$, with 12 states (3 productivity states, 2 credit condition states, and 2 bubble states: a bubbleless state and a bubbly state).

Policy functions and optimal macroprudential tax in the presence of asset price bubbles. We now move to the numerical analysis and discuss the optimal policy rules. Figure 2 shows new borrowing L_{t+1} and the equity price, which combines the fundamental price of capital (q_t) and the bubble component (B_t), as functions of the outstanding debt level, L_t , when financial conditions are favorable and productivity is low ($m_t = m^h$ and $z_t = z^l$). The top panels show the bubbleless state, and the bottom panels show the states with a bubble. CE (SP) decision rules are depicted in red (blue). The red (blue) vertical lines mark outstanding debt levels above which the collateral constraint starts to bind at t in the CE (SP).

There are notable differences between the SP and CE decision rules. The SP chooses new borrowing L_{t+1} that is always lower than in the CE, independently of the bubble's presence. It does so because it internalizes pecuniary externalities, and mitigates their negative impact on consumption, asset prices and welfare. The equity price is also lower under the SP than in the CE, which is consistent with lower borrowing and consumption. Moreover, the collateral constraint starts binding in the SP equilibrium for lower levels of debt than in the CE. As mentioned, this happens as lower SP borrowing results in lower consumption, putting a downward pressure on equity prices, thereby tightening the collateral constraint further.

¹⁷In order to obtain the solution for the competitive economy, we iterate the (competitive) Euler equation for borrowing, which does not incorporate the effect of borrowing decisions on prices. Value function iteration internalizes the effect of pecuniary externalities on welfare and, hence, yields the planner's solution. Given that we solve for time-consistent policies, we use a nested fixed point algorithm for the value function iteration. See Appendix A.6 for details on the numerical method.

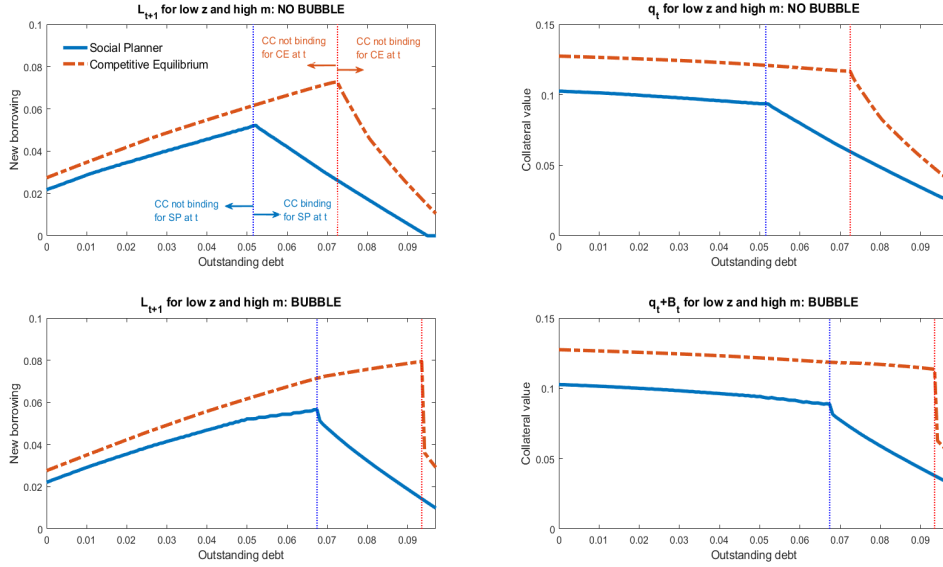


Figure (2) Policy rules for borrowing and asset prices in the presence and absence of a bubble.

Note: The figure plots the new borrowing L_{t+1} (panels on the left) and equity price (panels on the right) as a function of outstanding debt L_t , for $z_t = z^l$ and $m_t = m^h$. The top panels show the case without a bubble, and the bottom panels show the case with an asset price bubble. The blue lines correspond to the SP policy rules, while the red dashed lines represent the CE policy rules. The collateral constraint binds for outstanding debt levels to the right from the red (CE) and blue (SP) vertical lines.

With a bubble, the equity price is higher than under the no-bubble regime for both SP and CE. As a result, more debt can be supported as higher equity prices relax the collateral constraint. In particular, the collateral constraint starts binding at t for a threshold of debt that is higher for both CE and SP in the economy with a bubble compared to the one without (the vertical lines in Figure 2 move further to the right). The outstanding debt, L_t , can increase by about 40% more compared to the bubbleless case before the collateral constraint starts binding at t . This result is crucial for how the planner chooses to mitigate debt growth. As discussed earlier, reducing new borrowing alleviates pressures on future prices, but intensifies pressures on prices in the current period. The presence of a bubble amplifies the importance of this effect.

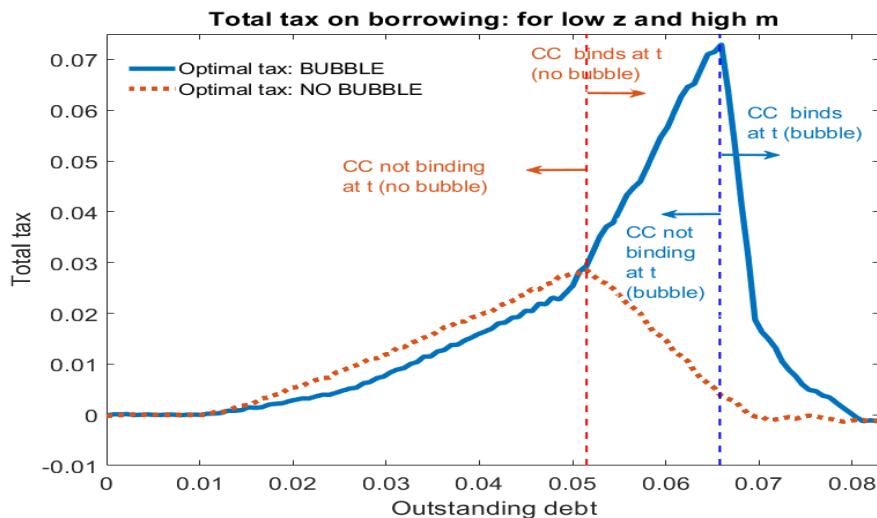


Figure (3) Optimal tax on borrowing in the presence of an asset price bubble.

Note: The figure plots the optimal tax in the absence (dotted red line) and in the presence (solid blue line) of a bubble as a function of debt outstanding, L_t .

Figure 3 plots the optimal tax on borrowing with (solid blue line) and without (red dotted line) a bubble as a function of outstanding debt, L_t . A couple of observations are worth noting.

First, for very low levels of debt the tax on borrowing is zero, independently of whether there is a bubble or not. The reason is that for such low levels of outstanding debt the collateral constraint does not bind at t or $t + 1$, irrespective of which state of the world realizes. In other words, there is not need for a policy intervention when credit imbalances are subdued.

Second, for low to medium levels of outstanding debt, the optimal tax is lower when the bubble is present: the extensive margin pushes the tax down and dominates the intensive margin. In other words, for low to moderate levels of outstanding debt, the bubble reduces the probability of the collateral constraint binding in the future (as it lifts up the level of debt above which the constraint binds, see Figure 2), while the future negative externalities from the bubble deflating in an event of a binding collateral constraint are still modest (given moderate new borrowing today). This justifies lifting the tax “later,” i.e. for higher levels of L_t when the bubble is present.

Third, at the level of outstanding debt, L_t , of around 0.05, or about 11% of output, the optimal tax starts to decline when there is no bubble present. At this level of L_t the collateral constraint becomes binding at t in the absence of the bubble, and the SP reduces the tax in order to weigh the benefit of relaxing the constraint today against the cost of it binding in the future. As discussed

in section 4 and shown analytically in section A.4 in the Appendix, when the collateral constraint binds at t and there is a positive probability it binds at $t + 1$, the planner will set the tax by weighing the effects of borrowing on asset prices at t and $t + 1$. The tax becomes smaller for higher levels of outstanding debt and can turn slightly negative in our calibration.

On the contrary, the optimal tax continues to increase in the presence of a bubble until the level of outstanding debt reaches levels of around 0.07, or about 15% of output. This is consistent with the bubble's positive effect on relaxing the collateral constraint at t . It is important to note that the tax on borrowing has a macroprudential interpretation as long as the collateral constraint does not bind at t ($\mu_t = 0$), but with a positive probability it may bind at $t + 1$, $E_t \mu_{t+1} > 0$. As Figure 3 shows, the *macroprudential tax* continues to increase to much higher levels in the bubbly economy (a maximum of about 7% compared to about 3% when there is no bubble) for high levels of debt outstanding. Intuitively, asset overvaluations amplify credit imbalances when the current level of debt is high, because the expected future costs of the bubble deflating are higher as well.

In sum, our results suggest that asset price overvaluations might not be enough by themselves to justify a tightening of macroprudential policy. This happens because for low or moderate levels of credit, the bubble's persistence into the next periods makes tight financial conditions less likely in the future, while the costs of the bubble deflating in the future are still low. As a result, under our calibration parameters, it is optimal not to lean against the bubble when debt is low. However, once debt increases sufficiently, macroprudential policy should remain active for longer and for higher realizations of debt outstanding. Hence, the planner should lean against debt build-up more aggressively compared to the bubbleless case.

Crisis scenario exercise. We also perform a simulation exercise to investigate the magnitude of the pecuniary externalities from the asset price bubble in the model.

The economy is simulated for 10 periods, where in the first period a bubble with a size of 8% deviation relative to the price of capital is present. The bubble's continuation probability is set at $\pi = 0.56$, and a 10-period path is simulated 100,000 times.¹⁸

Figure 4 shows the average values across simulations. We are interested in the

¹⁸We choose to simulate the economy for a shorter horizon, yet many times, because the bubble in our setting does not re-emerge after busting, generating an absorbing state. Thus, the distribution of outcomes would be biased towards the no-bubble outcomes if we simulated the economy for a longer horizon.

economy’s responses to a binding collateral constraint in two cases: (i) when there is no bubble (red solid lines), and (ii) when there is a bubble that persists throughout the period when the constraint binds (blue dashed lines). By comparing these two events we can verify whether the pecuniary externalities from a deflating asset price bubble are meaningful. The left-hand side of Figure 4 shows the responses of consumption, new borrowing and the equity price—all in terms of deviations from averages across all simulations—when the starting level of outstanding debt in the first period is “low.” The right-hand side graphs show responses of the same variables when the initial level of outstanding debt is “high.”

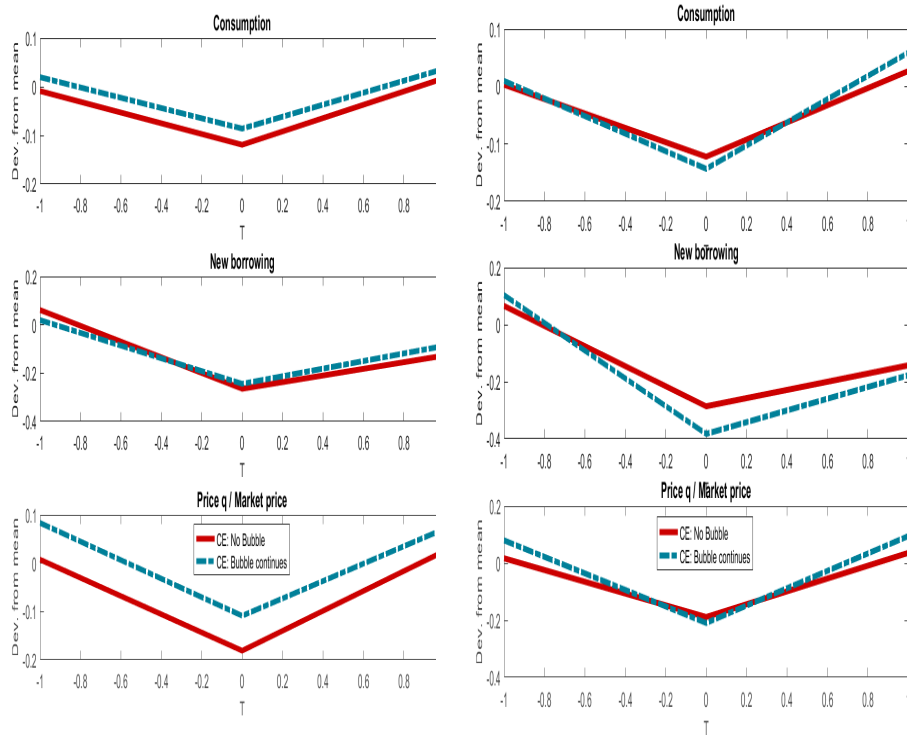


Figure (4) Model simulations: Responses of key variables to a binding credit constraint.

Note: The figure plots responses of consumption c_t , new borrowing L_{t+1} , and capital price q_t in the competitive economy (CE), in the event of the collateral constraint binding ($T = 0$ in the figure), in presence of a bubble that persists throughout the event (blue dashed line), and in absence of a bubble (red solid line). The left panels show responses q when the starting level of debt outstanding in the 10-period simulated path is low ($L_1 = 0.04$), the right panels—when the initial debt level is high ($L_1 = 0.08$). All responses are in terms of deviation from averages across all simulations.

Corroborating our previous results, Figure 4 shows that the bubble’s effects on the economy depend on the initial debt level. With a relatively subdued level of

debt, a bubble has an average positive impact on consumption and prices when the collateral constraint becomes binding. In contrast, when the level of debt is high, the bubble on average amplifies the negative effects of a binding constraint on consumption and prices.

Next, we compare the responses of the same variables in CE and SP across the states when the collateral constraint binds and the bubble persists in the competitive economy. The average responses of output, consumption, equity prices and new borrowing are depicted in Figure 5. As the figure clearly shows, the social planner prevents excessive borrowing in advance, and is able to considerably mitigate the impact of the binding constraint on debt deleveraging, consumption and output.

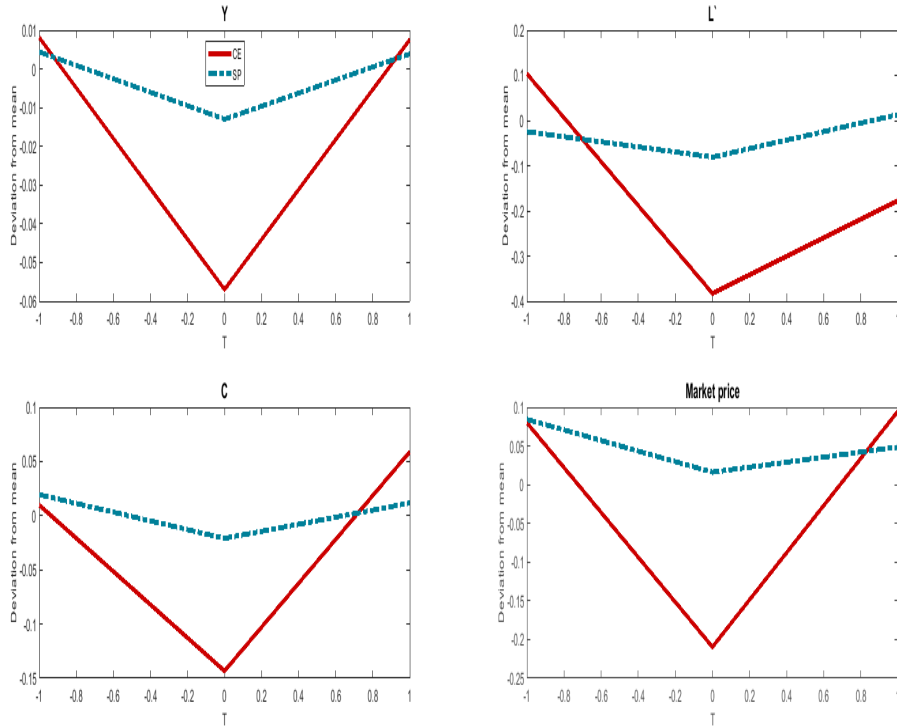


Figure (5) Model simulations: Social Planner's allocations.

Note: The figure plots responses of output y_t , new borrowing L_{t+1} , consumption c_t , and capital price q_t in competitive economy (CE, red solid line) and under a social planner (SP, blue dashed line), across states in which the collateral constraint binds ($T = 0$ in the figure), and when there is a bubble in competitive economy. The initial debt level was set to $L_1 = 0.08$. All responses are in terms of deviation from averages across all simulations.

6 Conclusions

We study optimal macroprudential policy when credit imbalances are accompanied by an asset price bubble. To this end, we augment the model of Bianchi-Mendoza, which features an occasionally binding collateral constraint with a rational equity price bubble akin to Miao-Wang. We show that the presence of a bubble generates an additional pecuniary externality, which requires further macroprudential intervention in order to avoid a bubble deflation in bad times when the bubble is most useful to relax collateral constraints. But, at the same time, the presence of the bubble alters equilibrium allocations, and by helping to keep collateral constraints slack for some shock realizations, it may result in a macroprudential tax on borrowing that is lower relative to the bubbleless case. Our quantitative results suggest that the optimal policy response depends in a non-monotone way on the outstanding level of debt in the economy. When credit imbalances are moderate, the optimal tax is lower in the presence of the bubble. However, when the credit imbalances are high, the optimal tax level is much higher than in the absence of the bubble, suggesting the presence of an amplifying effect from elevated credit imbalances and asset overvaluations.

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A Appendix

A.1 Derivation of Collateral Constraint

The collateral constrained (9) can be derived from a renegotiation of debt problem between a borrower and a lender. At the beginning of period t , after L_t has been repaid, the total liabilities of the borrower (firm) equal to $\theta p^v v_t + \frac{L_{t+1}}{R}$. Before production and investment in new capital take place, the borrower can decide to divert the borrowed funds. If the diversion does not take place at that specific point in time, there will be no opportunity for the borrower to divert within the same period. Using the threat to divert, the borrower can try to renegotiate her debt. Following Jermann and Quadrini (2012), we assume that the firm has full negotiation power. If the lender does not agree to renegotiate the debt, she can seize the firm and sell it in the equity market. The value of the seized firm is given by $m_t \beta E_t V_{t+1}(k_t, 0)$, where m_t denotes the probability of a successful seizure and reselling of the firm in the equity market.

It follows that for the firm managers the value from renegotiating its debt is given by

$$V^R = \theta p^v v_t + \frac{L_{t+1}}{R} - m_t \beta E_t \frac{U_{c,t+1}}{c_t} V_{t+1}(k_t, 0) + \beta E_t \frac{U_{c,t+1}}{c_t} V_{t+1}(k_{t+1}, L_{t+1}), \quad (\text{A.1})$$

where the first three terms denote the net renegotiated debt benefit, equal to the total liabilities net of the firm's expected value after funds have been diverted.

The value to the firm managers from avoiding renegotiation is given by

$$V^{NR} = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} V_{t+1}(k_{t+1}, L_{t+1}). \quad (\text{A.2})$$

The incentive compatibility constraint requires that the net renegotiation value is smaller or equal to the value from honoring the debt obligations, $V^{NR} \geq V^R$. This incentive compatibility constraint gives rise to the collateral constraint (9).

A.2 Conjecture and Verify: Firm's Value Function

Conjecture that the value function of the representative firm is given by $V_t(k_t, L_t) = a_t k_t + s_t L_t + b_t$, where a_t , s_t and b_t are coefficients that will be determined. Substituting this conjecture into the firm's optimization problem (7) and in constraints (8)-(9) yields the following optimization problem

$$a_t k_t + s_t L_t + b_t = \max_{k_{t+1}, L_{t+1}, l_t, v_t} D_t + \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} (a_{t+1} k_{t+1} + s_{t+1} L_{t+1} + b_{t+1}) \right) \quad (\text{A.3})$$

subject to

$$D_t = F(k_t, l_t^d, v_t) - p^v v_t - w_t l_t^d + \frac{L_{t+1}}{R} - L_t + q_t k_t - q_t k_{t+1}, \quad (\text{A.4})$$

and

$$\frac{L_{t+1}}{R} + \theta p^v v_t \leq m_t \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} (a_{t+1} k_t + b_{t+1}) \right). \quad (\text{A.5})$$

Taking first-order conditions with respect to $k_{t+1}, L_{t+1}, l_t^d, v_t$ respectively yields

$$q_t = \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} a_{t+1} \right), \quad (\text{A.6})$$

$$\frac{1}{R} = \frac{\mu_t}{R} - \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} s_{t+1} \right), \quad (\text{A.7})$$

$$w_t = F_{l,t}, \quad (\text{A.8})$$

$$p^v = F_{v,t} \frac{1}{1 + \theta \mu_t}. \quad (\text{A.9})$$

where μ_t is the Lagrange multiplier on the collateral constraint (A.5). Substituting these conditions back into (A.3) and simplifying yields

$$a_t k_t + s_t L_t + b_t = F(k_t, l_t^d, v_t) - F_{l,t} l_t^d - F_{v,t} v_t - L_t + q_t k_t \quad (\text{A.10})$$

$$+ \mu_t m_t \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} (a_{t+1} k_t + b_{t+1}) \right) + \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right) \quad (\text{A.11})$$

where we have used the complementary slackness condition

$$\mu_t \left[\theta p^v v_t + m_t \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} (a_{t+1} k_t + b_{t+1}) \right) \right] = 0 \quad (\text{A.12})$$

to substitute out the $\mu_t \frac{L_{t+1}}{R}$. Comparing LHS and RHS of equation (A.10) yields

$$a_t = F_{k,t} + q_t (1 + m_t \mu_t), \quad (\text{A.13})$$

$$s_t = -1, \quad (\text{A.14})$$

$$b_t = (1 + m_t \mu_t) \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right), \quad (\text{A.15})$$

where $F_{k,t} = F(k_t, l_t^d, v_t) - F_{l,t} l_t^d - F_{v,t} v_t$. Thus, the value function takes the form

$$V_t(k_t, L_t) = [F_{k,t} + q_t (1 + m_t \mu_t)] k_t - L_t + b_t. \quad (\text{A.16})$$

Moreover, using the above value function to substitute out a_{t+1} , yields the credit constraint

$$\frac{L_{t+1}}{R} + \theta p^v v_t \leq m_t (q_t k_t + B_t), \quad (\text{A.17})$$

where

$$B_t \equiv \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right). \quad (\text{A.18})$$

A.3 Proof of Proposition 1

Using that $a_{t+1} = F_{k,t+1} + q_{t+1}(1 + m_{t+1}\mu_{t+1})$ and $s_{t+1} = -1$, the first-order conditions (A.6)-(A.7) can be rewritten as

$$\mu_t = 1 - \beta R E_t \left(\frac{U_{c,t+1}}{U_{c,t}} \right), \quad (\text{A.19})$$

$$U_{c,t} q_t = \beta E_t U_{c,t+1} [z_{t+1} F_{k,t+1} + q_{t+1}(1 + m_{t+1}\mu_{t+1})], \quad (\text{A.20})$$

yielding, together with equations (A.8)-(A.9), the first-order conditions (17)-(20) in Proposition 1. To derive the bubble accumulation process (21), we use the definition of B_t in (A.18), where

$$b_t = (1 + m_t \mu_t) \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} b_{t+1} \right). \quad (\text{A.21})$$

Iterating this equation forward yields

$$b_{t+1} = (1 + m_{t+1} \mu_{t+1}) \beta E_{t+1} \left(\frac{U_{c,t+2}}{U_{c,t+1}} b_{t+2} \right) = (1 + m_{t+1} \mu_{t+1}) B_{t+1}. \quad (\text{A.22})$$

Substituting the expression for b_{t+1} from equation (A.22) into (A.18), yields (21).

A.4 Tax on borrowing

This section derives the tax on borrowing ($\mu_t > 0$, $E_t \mu_{t+1} > 0$) and the macroprudential tax ($\mu_t = 0$, $E_t \mu_{t+1} > 0$).

To derive the tax on borrowing, combine equations (25) and (37) and solve for τ_t , which yields

$$\begin{aligned} \tau_t = & - \frac{E_t [\xi_{t+1} q_{t+1} U_{cc,t+1} - \xi_t \Omega_{t+1}]}{E_t U_{c,t+1}} + \frac{\xi_t q_t U_{cc,t}}{\beta R E_t U_{c,t+1}} + \frac{\mu_t^p - U_{c,t} \mu_t}{\beta R E_t U_{c,t+1}} \\ & - \frac{E_{t+1} \psi_{t+1} \left[\frac{U_{c,t+2}}{U_{c,t+1}} (1 + m_{t+2} \mu_{t+2}) B_{t+2} \frac{U_{cc,t+1}}{U_{c,t+1}} \right]}{E_t U_{c,t+1}} \\ & + \frac{E_t \psi_t \left[\frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \frac{U_{cc,t}}{U_{c,t}} + \Delta_{t+1} \right]}{\beta R E_t U_{c,t+1}}. \end{aligned} \quad (\text{A.23})$$

The first line of the tax formula exactly equals the borrowing tax in Bianchi-

Mendoza.¹⁹ The second and third lines contain the additional terms that arise because of the bubble's presence. In general, the tax rate continues to have two components that match the q_t and q_{t+1} effects on the social marginal benefit of lower borrowing identified earlier, augmented by the additional terms B_t and B_{t+1} due to the bubble's presence.

To derive the optimal macroprudential tax, set $\mu_t^p = \mu_t = 0$, from which it also follows that $\xi_t = \psi_t = 0$. Then, the macroprudential tax is given by

$$\tau_t^{mp} = -\frac{E_t[\xi_{t+1}q_{t+1}U_{cc,t+1}]}{E_tU_{c,t+1}} - \frac{E_{t+1}\psi_{t+1}\left[\frac{U_{c,t+2}}{U_{c,t+1}}(1+m_{t+2}\mu_{t+2})B_{t+2}\frac{U_{cc,t+1}}{U_{c,t+1}}\right]}{E_tU_{c,t+1}}, \quad (\text{A.24})$$

which is exactly expression (38) given in section 4.2.

A.5 Equivalence Between the “Relaxed” and “Fully” Constrained Ramsey Problem

Section 4.1 considers a problem of the Ramsey planner which consists of maximizing the welfare of the private agents subject to a *subset* of the competitive equilibrium optimality conditions. In what follows, we show that the allocations obtained by the “relaxed” planner's optimization problem are equivalent to those obtained by a “fully” constrained planner, i.e. one that takes into account all competitive equilibrium conditions.

A “fully” constrained Ramsey planner solves the following maximization problem

$$\max_{\substack{c_t, q_t, B_t, l_t, v_t, \\ L_{t+1}, \tau_t, \mu_t}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to:

$$c_t + L_t + p^v v_t \leq z_t F(1, v_t, l_t) + \frac{L_{t+1}}{R} \quad (\lambda_t^p) \quad (\text{A.25})$$

$$\frac{L_{t+1}}{R} + \theta p^v v_t \leq m(q_t + B_t) \quad (\mu_t^p) \quad (\text{A.26})$$

$$q_t U_{c,t} = \beta E_t U_{c,t+1} [q_{t+1} + F_{k,t+1} + m_{t+1} \mu_{t+1} q_{t+1}] \quad (\xi_t) \quad (\text{A.27})$$

¹⁹The Lagrange multipliers on the collateral constraint μ_t and μ_t^p are given by (20) and (35), respectively. Note that in their case $\mu_t^p = U_{c,t} \mu_t$. The two multipliers could be equalized by introducing an additional instrument into the planner's problem. This would not modify any of the results relating to optimal macroprudential policy.

$$B_t = \beta E_t \frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \quad (\psi_t) \quad (\text{A.28})$$

$$z_t F_{l,t} = G_{l,t} \quad (\varphi_t) \quad (\text{A.29})$$

$$U_{c,t} = \beta R U_{c,t+1} (1 + \tau_t) + U_{c,t} \mu_t \quad (\gamma_t) \quad (\text{A.30})$$

$$z_t F_{v,t} = p^v (1 + \theta \mu_t) \quad (\eta_t) \quad (\text{A.31})$$

$$\mu_t \geq 0 \quad (\text{A.32})$$

$$\mu_t \left(\frac{L_{t+1}}{R} + \theta p^v v_t - m_t q_t - B_t \right) = 0 \quad (\text{A.33})$$

Proposition 3. *Constraints (A.29)-(A.31) of the “fully” constrained problem are not binding for the social planner. Therefore the solution to the problem yields equivalent results to those of the “relaxed” Ramsey problem.*

Proof. The first-order conditions with respect to τ_t is given by $\gamma_t \beta R E_t U_{c,t+1} = 0$, from which it follows that $\gamma_t = 0$ as $\beta R E_t U_{c,t+1} > 0$. The first-order condition with respect to μ_t is given by $\gamma_t = \eta_t p^v \theta$, from which it follows that $\eta_t = 0$. Finally, taking the first-order condition with respect to l_t and taking into account that $\gamma_t = \eta_t = 0$, yields

$$U_{c,t} G_{l,t} + \lambda_t^p - \varphi_t (G_{u,t} - F_{u,t}) - \xi_t q_t U_{cl,t} - \psi_t E_t \frac{U_{c,t+1}}{U_{c,t}} (1 + m_{t+1} \mu_{t+1}) B_{t+1} \frac{U_{cl,t}}{U_{c,t}} = 0. \quad (\text{A.34})$$

Substituting the first-order condition with respect to consumption (30) in (A.34), and using the properties of the utility function, yields $G_{l,t} = F_{l,t}$, hence $\varphi_t = 0$.

A.6 Numerical Algorithm

Competitive equilibrium. We solve for the CE using an Euler-equation iteration algorithm. In each iteration, we solve the system of equations presented below in a recursive form for each of 1800 gridpoints: 150 values of debt (L), and 12 states (3 states for productivity \times 2 states for pledgeable fraction of collateral \times 2 bubble states, i.e. bubble or no-bubble). Formally, we solve for the policy functions $\{\tilde{L}(L, \omega), c(L, \omega), q(L, \omega), l(L, \omega), v(L, \omega), \mu(L, \omega)\}$, and for $B(L, \omega)$ such that the equilibrium conditions below are satisfied

$$c(L, \omega) + L + p^v v(L, \omega) = zF(1, v(L, \omega), l(L, \omega)) + \frac{\tilde{L}(L, \omega)}{R}, \quad (\text{A.35})$$

$$\frac{\tilde{L}(L, \omega)}{R} + \theta p^v v(L, \omega) \leq m(q(L, \omega) + B(L, \omega)), \quad (\text{A.36})$$

$$B(L, \omega) = \beta E_{\omega'|\omega} \left(\frac{U_c(c(L', \omega') - G(l(L', \omega')))}{U_c(c(L, \omega) - G(l(L, \omega)))} b(L', \omega') \right), \quad (\text{A.37})$$

$$b(L, \omega) = (1 + m\mu(L, \omega)) \beta E_{\omega'|\omega} \left(\frac{U_c(c(L', \omega') - G(l(L', \omega')))}{U_c(c(L, \omega) - G(l(L, \omega)))} b(L', \omega') \right), \quad (\text{A.38})$$

$$\mu(L, \omega) = 1 - \beta R E_{\omega'|\omega} \frac{U_c(c(L', \omega') - G(l(L', \omega')))}{U_c(c(L, \omega) - G(l(L, \omega)))}, \quad (\text{A.39})$$

$$q(L, \omega) U_c(c(L, \omega) - G(l(L, \omega))) = \beta E_{\omega'|\omega} [U_c(c(L', \omega') - G(l(L', \omega')))] \times \\ (q(L', \omega') + z' F_k(1, v(L', \omega'), l(L', \omega')) + m' \mu(L', \omega') q(L', \omega')), \quad (\text{A.40})$$

$$z F_l(1, v(L, \omega), l(L, \omega)) = G_l(L, \omega), \quad (\text{A.41})$$

$$z F_v(1, v(L, \omega), l(L, \omega)) = p^v (1 + \theta \mu(L, \omega)), \quad (\text{A.42})$$

where $\tilde{L}(L, \omega)$ is the new borrowing, and x' denotes the next period realization of variable x . When there is no bubble, $b(L, \omega) = 0$.

The algorithm proceeds in the following steps:

1. For each gridpoint in L , conjecture future policy functions $L' = \tilde{L}(L, \omega)$, $c(L', \omega')$, $q(L', \omega')$, $l(L', \omega')$, $v(L', \omega')$, $\mu(L', \omega')$. For the first iteration use a guess. For further iterations define future policies as the solution to the current policy functions from the previous iteration (see step 3 below). As explained in section 5, we fix the size of the bubble today to 8% of the price of capital. In other words, we set $b(L, \omega)$ to a fixed, constant value across iterations, and use it to update $B(L, \omega)$ in each iteration using the fact that $B(L, \omega) = b(L, \omega)/(1 + m\mu(L, \omega))$.
2. Taking future policies from step 1 as given, for each gridpoint in L , solve (A.35)-(A.42) to obtain current policy functions $\tilde{L}(L, \omega)$, $c(L, \omega)$, $q(L, \omega)$, $l(L, \omega)$, $v(L, \omega)$, $\mu(L, \omega)$. We distinguish between cases that the collateral constraint binds and does not bind in the present:

- i. First, assume that the collateral constraint (A.36) binds and solve for the current policy functions. Then, check that $\mu(L, \omega) > 0$ using equation (A.39). If this is true, proceed to step 3; otherwise move to substep ii.
 - ii. If for a given gridpoint the collateral constraint in the present does not bind, solve the system of equations above for the current policy functions by setting $\mu(L, \omega) = 0$.
3. Use the optimal policy functions from substeps 2i or 2ii to update the (conjectured) future policy functions in step 1.
 4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i - 1$ and i it holds that $\sup_{L, \omega} \|x_i(L, \omega) - x_{i-1}(L, \omega)\| < \varepsilon$, where $x = \tilde{L}, c, q, l, v$. We set $\varepsilon = e^{-4}$, but we also confirm that the results do not change if we choose a stricter convergence criterion.

Social planner. We solve for the SP policy functions using a value function iteration, nested fixed point algorithm. In each iteration we solve for the value function using a fixed-grid optimization procedure as an inner loop. In the outer loop, we update future policies given the solution to the Bellman equation from the inner loop. As in Klein, Krusell and Ríos-Rull (2008) and Bianchi and Mendoza (2018), this procedure delivers time-consistent policies. The detailed steps are described below.

The value function representation of the SP's optimization problem is:

$$V(L, \omega) = \max_{\tilde{L}, c, l, v, q, B, \mu} (U(c(L, \omega) - G(l(L, \omega))) + \beta E_{\omega'|\omega}[V(L', \omega')]) \quad (\text{A.43})$$

subject to (A.44)-(A.50):

$$c(L, \omega) + L + p^v v(L, \omega) = zF(1, v(L, \omega), l(L, \omega)) + \frac{\tilde{L}(L, \omega)}{R} \quad (\text{A.44})$$

$$\frac{\tilde{L}(L, \omega)}{R} + \theta p^v v(L, \omega) \leq m(q(L, \omega) + B(L, \omega)) \quad (\text{A.45})$$

$$B(L, \omega) = \beta E_{\omega'|\omega} \left(\frac{U_c(c(L', \omega') - G(l(L', \omega')))}{U_c(c(L, \omega) - G(l(L, \omega)))} b(L', \omega') \right), \quad (\text{A.46})$$

$$b(L, \omega) = (1 + m\mu(L, \omega)) \beta E_{\omega'|\omega} \left(\frac{U_c(c(L', \omega') - G(l(L', \omega')))}{U_c(c(L, \omega) - G(l(L, \omega)))} b(L', \omega') \right), \quad (\text{A.47})$$

$$q(L, \omega) U_c(c(L, \omega) - G(l(L, \omega))) = \beta E_{\omega'|\omega} [U_c(c(L', \omega') - G(l(L', \omega')) \times (q(L', \omega') + z' F_k(1, v(L', \omega'), l(L', \omega')) + m' \mu(L', \omega') q(L', \omega'))], \quad (\text{A.48})$$

$$zF_l(1, v(L, \omega), l(L, \omega)) = G_l(L, \omega), \quad (\text{A.49})$$

$$zF_v(1, v(L, \omega), l(L, \omega)) = p^v(1 + \theta\mu(L, \omega)). \quad (\text{A.50})$$

The algorithm proceeds in the following steps:

1. In the outer loop, define future policies $V(L', \omega')$, $\tilde{L}(L', \omega')$, $c(L', \omega')$, $q(L', \omega')$, $l(L', \omega')$, $v(L', \omega')$, $\mu(L', \omega')$ as the solution to current policy functions from the previous iteration (see step 3 below) or the policy functions from the CE solution for the first iteration. Also, set $b(L, \omega)$ at a fixed value (calibrated to match the data).
2. In the inner loop, for each gridpoint of L , solve for policy functions $V(L, \omega)$, $\tilde{L}(L, \omega)$, $c(L, \omega)$, $q(L, \omega)$, $l(L, \omega)$, $v(L, \omega)$, $\mu(L, \omega)$, $B(L, \omega)$ that satisfy (A.43) - (A.50) given future policies from step 1. We distinguish between the case that the collateral constraint binds and the case that the collateral constraint does not bind today:
 - i. First, assume that the collateral constraint (A.45) does not bind, i.e. $\mu(L, \omega) = 0$. Then, the objective is to find the level of $\tilde{L}(L, \omega)$ that maximizes (A.43). To do that, we first solve for v and l using (A.49)-(A.50) and substitute out consumption using the budget constraint (A.44). Then, we compute $\tilde{L}(L, \omega)$ by calculating (A.45) for a subgrid of 5000 values of \tilde{L} and choosing the value with the highest $V(L, \omega)$: \tilde{L} matters for $V(L, \omega)$ not only because it determines current utility $U(c(L, \omega) - G(l(L, \omega)))$, but also because it is the future state variable, i.e. $L' = \tilde{L}(L, \omega)$. Thus, its choice determines the level of the continuation value $V(L', \omega')$. The policy function $V(L', \omega')$ assigning a value for different values L' is taken as given from the outer loop in step 1, but, in the inner loop, we choose the value of L' (\tilde{L}) that maximizes the sum of current utility and the continuation value. Finally, check if the optimal choice of $\tilde{L}(L, \omega)$ is lower than the borrowing limit computed from (A.45). If this is true, proceed to step 3; otherwise move to substep ii.
 - ii. Solve for the current policy functions given future polices from step 1, but set (A.45) to hold with equality. For each point on the subgrid of \tilde{L} values, calculate corresponding values of c , q , l , v , μ , B satisfying equations (A.44)-(A.50). Finally, choose the level of \tilde{L} for which $V(L, \omega)$ is the highest similar to substep i above.
3. Use the optimal policy functions from substeps 2i or 2ii to update the (conjectured) future policy functions in step 1.
4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i - 1$ and i it holds that $\sup_{L, \omega} \|x_i(L, \omega) - x_{i-1}(L, \omega)\| < \varepsilon$, where $x = \tilde{L}, c, q, l, v$, and $\varepsilon = e^{-4}$.